Chapter 5
Mathematics Education in Different Times and Cultures

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Abstract In this work we will present some references on the trends of mathematics education throughout its process of adaptation to the new times that must look at their reflection in the mirror of the cultures they represent, always considering their vertiginous changes. A multicultural and intercultural approach in the classroom thanks to the study of some cases that occurred in developing countries will make us understand certain ways of objectifying mathematical knowledge based on a dialogue between social and school practices. Counting on the idea that all peoples have developed a mathematical thought in relation to their vital context, geography, history, cosmogony and their mythical stories, both theoretical and practical knowledge can be used to solve their daily problems.

Thus, certain skills which are studied, they also include symbolic systems, spatial designs, techniques, construction practices, calculation methods, models and measurements of time and space, specific forms of reasoning, and other problems and activities, all of them developed in different towns, such as case studies in developing countries: the use of social and school practices within an indigenous school, Afro-Colombian studies and basic standards in competencies with intercultural dialogue and some mathematical competencies in the project Chair of Afro-Colombian Studies (CEA).

Keywords Multicultural · Intercultural · Practical knowledge · Mathematized society · Dynamic process

Introduction

Competencies to evaluate mathematical applications and their potential usefulness are now a fundamental precondition for planning within political administrations and democratic citizen participation. The new challenge is to determine what kind of

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knowledge about social knowledge is needed in a mathematized society and how its components can be acquired. Different institutions in many countries have been working, for decades, to give a new approach to the teaching of mathematics by using several forms of learning and working on the interaction between the history of mathematics within mathematics education from different points of view and in multiple, eminently practical ways, (relationships between mathematics and other disciplines, technology in the mathematics classroom, diversity and interculturality. . .) (MEC, 2006).

Max Scheler (1874–1928), a German philosopher of great importance in ethics and philosophical anthropology, as well as the development of phenomenology, used it to study emotional phenomena and their respective internationalities (values) with which he developed a very solid, original personalist foundation of ethics: the realization of values was concretized in human models that invited to their follow-up. Scheler distinguished three types of knowledge: inductive, essential - also named 'phenomenological structure'- and metaphysical (Scheler, 1934).

When this classification is applied to mathematics we can say that the knowledge of the Egyptians was practical: they were interested in the solution to concrete problems which were generated by social organizations such as architectural constructions, taxes, commercial transactions, establishment of limits in crop fields next to each flood of the river Nile, etc.. . . It was a useful knowledge to modify and improve the world for it to be 'organized' and inhabited. Later, Greek era started. Greeks were a typical example of cultured people in all aspects of knowledge, especially in mathematics, who studied in order to understand nature and universe, apart from capturing its harmony, reasoning activity, etc. Some of the most representative people regarding this era of mathematics were par excellence Apollonius of Perga (2nd century B.C.), author of books on conics, Erastothenes of Cyrene (2nd century B.C.) who ingeniously knew how to measure the radius of the Earth from the distance existing between Alexandria and Siena and their latitudes, and finally Claudius Ptolemy (2nd century B.C.) author of the *Almagest*, a work that systematized all the knowledge about astronomy by that time.

Along with other authors, usually compilers of earlier works, people like Pappus (3rd century), Diophante (3rd century) or Proclus (5th century) need to be mentioned here. This way we will go back to the Middle Age, when most part of the Western world began to worry about the knowledge of salvation whose essential interest was to participate in the existence and constitution of things (Santaló, 1994).

Since the Romans, contents of the trivium – grammar, history and dialectics, also named 'the art of word' – were the knowledge that was the object of formal teaching; and contents of the quadrivium – arithmetic, geometry, astronomy and music, also named 'the art of thought'- were institutionalized by Plato in his work *The Republic*. This knowledge did not have its own purpose, for which a limited use was not needed in order to increase the knowledge already acquired or to promote sciences by themselves, but it was enough to make an inventory of the inheritance of the past in encyclopedias, more or less extensive (Bréhier, 1926). This led to the fact that no original mathematics was produced. Whereas sciences related to 'the divine' were highly developed, other sciences -and especially mathematics- were reduced to the

reproduction of the classical works of the Greeks which was often misrepresented and poorly understood. These works were *Etymologies of San Isidoro* (570–636, bishop of Seville, Spain) and *On the Nature of Things* by Bede the Venerable (672–735). They were very famous as they pretended to contain all the knowledge of that time. But they did not contain any important mathematical novelties despite of being very extended texts for many centuries. Precisely, this respect and cult of exaggeration for notable predecessors were the reasons for which mathematics education was reduced to a continuous repetition of the classical works. Mathematics went into decline due to the lack of creative vitality, returning back even to the level of previous centuries. And it is for centuries when the difference between mathematics for researchers -or trained mathematicians- and mathematics for users and the proportion of both trends in elementary education has caused a great controversy and discrepancies amongst mathematicians, as well as amongst psychologists and educators.

In the eighteenth century the efficacy of Euclid's Elements in the learning of geometry began to be questioned when Alexis Claude Clairaut (1713–1765) said in a geometry text for high school: '*... although geometry itself is abstract, it must nevertheless be admitted that the difficulties experienced by those beginning to apply it stem most often from the way it is taught. ... It always begins with a large number of preliminary definitions, demands, axioms and principles, which seem to promise nothing but antipathy to the reader...'* (Claireaut, 1741).

What Kind of Knowledge Is Necessary in a Mathematized Society?

Taking a leap in time, mathematics is currently considered the most powerful social media for planning, optimizing, directing, representing and communicating man-made social issues. Thanks to the development of modern information and communication technologies based on mathematics, this social impact has reached its highest splendour: mathematics is now universally used in all fields of society and there is hardly a political decision-making process in which mathematics is not used as the rational argument and objective basis for political judgments and power relations to be replaced. However, it becomes increasingly tedious and sometimes even impossible for the ordinary citizen to follow these developments in mathematics as well as in mathematical applications, and to evaluate their social use in an appropriate way, since specialization and segmentation of mathematical applications are often extremely difficult to understand. The main perception of its purpose and the basic knowledge of its general importance are frequently confronted with a complete lack of knowledge of concrete examples of its impact. But competencies to assess mathematical applications, and so their potential utility or their problematic effects, are now a necessary precondition for political executive and democratic citizen participation.

Some Historical Considerations of Mathematics in Relation to the New Social Visions of the World

In 2004, in Huelva (Spain), when Cristine Keitel presented her work about historical considerations of mathematics in relation to new social visions of the world *The teaching of mathematics in the knowledge society or how much mathematics does a prime minister need?* she highlighted:

(a) Mathematics as a distinctive tool for solving problems in social practices and as means of social power.
(b) Mathematics as a theoretical system and universal cosmic vision with a new perspective.
(c) Mathematics as a human effort and driving force for scientific and social development in general.
(d) Mathematics as rationality and common sense where it played a decisive role: in the thirteenth century universities created the interest for the Renaissance movement in ancient culture, and the printing press made them be available for a larger audience and the media development revolution.
(e) The knowledge of non-European cultures.
(f) The growing social needs for teaching mathematics led to mathematics education as a public task.

When Wilhelm von Humboldt established the state educational system in Prussia at the beginning of the nineteenth century, he created the idea of the *Bildung*–word generally translated as 'training' in English and Spanish, which do not totally capture its meaning. *Bildung* encompasses learning as universal as possible with a great emphasis on humanities such as philosophy, history, literature, art, music. . . but also on mathematics and science (Keitel, 2004).

Combination of Education and Knowledge Needed to Prosper in Today's Society

There are many definitions of *Bildung*. The European Bildung Network describes it like 'the combination of education and knowledge needed to prosper in our society, and the moral and emotional maturity for team working while having personal autonomy at the same time' (Cofunded by Erasmus, 2021).

The reason for which it is called the 'Bildung Rose' (Andersen, 2019, 2020) and not 'the Society Rose' is because we need to understand the following domains of our society for us to prosper (Fig. 5.1).

Mathematics has become a subject in higher education because of its formal educational qualities like educating the mind to be independent from a direct utilitarian perspective, and also because it promotes general attitudes of support for science-driven scientific and technological development.

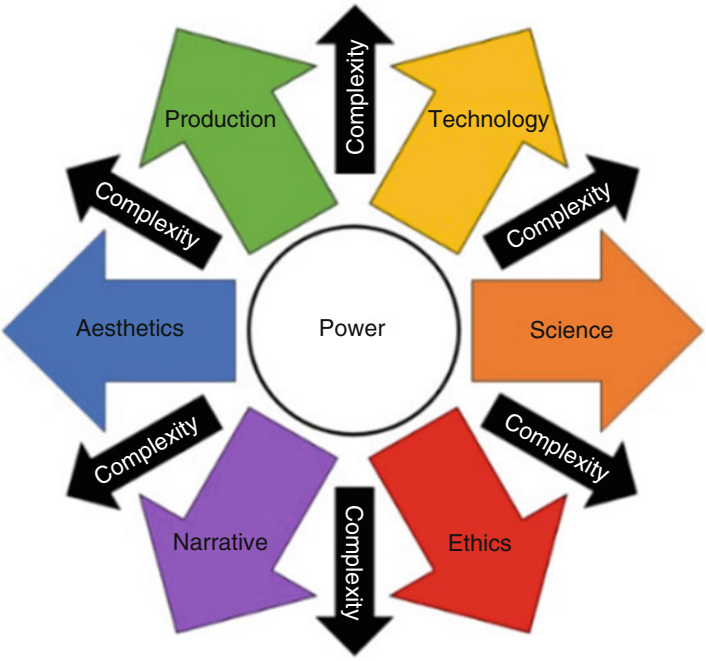

Fig. 5.1 Bildung Rose. (https://nordicbildung.org/papers/the-bildung-rose/)

In the 19th and 20th centuries, mathematics became the main driving force of almost all scientific and technological developments: mathematical and scientific models and their transformation into technology had their impact not only on the natural and social sciences and on economics, but also on all activities of social, professional and daily life. This impact was rapidly increased by the development of new information and communication technologies based on mathematics, which radically changed the social organization of work and our perceptions about knowledge or techniques to a point not fully explored yet.

It can become part of the learning experience in different ways when the concept of ethnomathematics (mathematical ideas from different cultural groups) are included in the curricular texts. From here, incorporated into the mathematics texts in the Western class, these are transformed with the following variations of form and achievement of objectives (Dickenson-Jones, 2008). The research carried out by this author provides a conceptual model that illustrates different modes of transformation that can occur when indigenous cultural practices are incorporated into the texts of the mathematics curriculum. To illustrate aspects of the model, the cited work: (a) includes an example of the Australian Aboriginal practice of throwing a return boomerang. With this, she gets the main actors (students and teachers), from constructive criticism working in the classroom, to reflect on the ways in which ethnomathematical ideas are modified when they are used in class; (b) the cultural praxis is also altered, allowing the teacher to elaborate and build their own curricular

texts that contain the different models of transformation. In short, the aim of the model developed in this research is not to criticize the potential educational value of ethnomathematical ideas in curriculum. Rather, the model provides a tool that may allow teachers of mathematics to identify the different kinds of representations of indigenous cultural practices in ethnomathematical curriculum and to choose those that are the most suitable for their own classroom environment.

On the one hand, mathematics being taken as a human activity in a social environment is determined by social structures, hence it is not disinterested or politically neutral. As opposed to it, the continuous application of mathematical models, being considered as universal problem-solving procedures, provides not only descriptions and predictions of social actions, but also prescriptions. Social and political decisions are translated into facts, restrictions or prescriptions that individuals or collective human behaviour must follow. Then how can we deal with these demands? What will keep the necessary knowledge, as this must be provided and the appropriate form of education must be offered, if mathematics plays such an important role in society and if the individual is supposed to act, knowingly, in accordance with the individual and social interests?

Since the end of the twentieth century, new perspectives on the social role of knowledge of mathematics and general education have been developed and have gained acceptance and political support: the 'Knowledge Society' and the 'Teaching of mathematics for all'.

In the last 60 years there have been many profound changes in the teaching of mathematics. Due to the efforts that the international community of experts in mathematics education is still making with the aim of finding suitable models, it is evident that we are witnessing a situation of experimentation and continuous change (Romero, 2016, 2019). Back in the 1960s and 1970s, the renovating movement towards modern mathematics already brought with it an important transformation of teaching, both in its nature and in its new content (De Guzmán, 2001), where some details are provided in Sect. 3.2. from the section developed in the section of this article indicated ut-infra, *Brief reflection on trends to face challenges in mathematics education.*

One of the most widespread general trends today, related to mathematics education in different times and cultures at least, is about the emphasis on the transmission of the thought processes of mathematics rather than on the mere transfer of the content (Keitel, 2006). Mathematics is, above all, a know-how science as the method clearly predominates over the content.

It is necessary for T/L model to be adapted to new times in different cultures for new approaches to be restructured. A brief reflection on the current problems of mathematics education will lead us to face challenges (Karp, 2016) by overcoming particular ad-hoc situations, which had already appeared thousands of years ago, but when they were approached from a generalist or universal point of view, different forms of approaches appeared in solving problems associated with the endemic characteristics of each area.

It is important to add that when different approaches are being re-examined to E/A, we need to weigh the role of multicultural orientation on mathematics itself at the time of teaching and learning by our students.

Regarding the question about why a multicultural orientation is needed we find an interesting answer in the works *Teaching Mathematics Concepts Using a Multicultural Approach* (Uy, 2000) and *Multicultural Math Classroom* (Zaslavsky, 1996). Also, a multicultural approach to science is generally important -and particularly to the teaching of mathematics- because it produces the necessary humanization of science. In my most sincere opinion, the scientificity of humanities, situated far from ordinary citizens, should be more humane and familiar, as well as much closer and friendly. This would mean humanizing the mathematical lessons and their content. People around the world have developed mathematical practices according to their needs and interests, especially for practical, aesthetic, and recreational ones.

The book *Teaching with a Multicultural Perspective: A Practical Guide, 2nd Edition* (Davidman & Davidman, 1994) provides a clear model with objectives leading to the creation of effective instructions in culturally diverse settings. For its part, ethnomathematics reflects on the role of this multicultural perspective in different environments. You can, for a better understanding, read the Chap. 8 of this same book (Appelbaum & Stathopoulou, 2022), History of Ethnomathematics: Recent developments,

Based on a set of field-tested planning questions and linked to some curricular case studies, as well as educator profiles and activities, it makes a series of clear and compelling connections between multicultural theory and practice. Some authors think that mathematics does not seem to be a promising subject for multicultural education because mathematical truths are universally valid. However, all cultures produce mathematical ideas, just as they produce different languages and social systems too. Besides, it must be recognized the history of mathematics as well as the social and political values that have shaped the development of mathematics curricula (Nelson, 1993). For this, an analysis of the different approaches is essential to enrich a multicultural approach for both students and teachers, since they understand each other and value themselves much better by promoting critical thinking because the multicultural teaching of mathematics provokes in students a stimulus of depth and expansion of knowledge: thinking, learning and applying concepts is done from multiple perspectives.

Brief Reflection on Trends to Face Challenges in Mathematics Education

In relation to the knowledge of our society, it seems reasonable that the educational system participates in its generation and promotion, as well as its democratization forces the principles of merit and effort to be a reference in these environments. And as a result, the State is the backbone of the national system by defining the contents

of teaching thanks to the core subjects. As indicated by the OECD, '*. . .qualifying the system through external evaluations at the end of the cycles and reinforcing its presence in the autonomous sphere through high inspection has not been exercised and has generated educational lack of control. . . The reality, however, has actually been distorting the teaching and its results. . .*' (Luengo, 2018).

Education as a Dynamic Process

Education has a truly, fascinating aspect: it is an active, dynamic process, in constant construction, and undoubtedly highly contextualized. This characteristic makes changes and contributions to be introduced in order to ensure its permanent renewal.

From this point of view, research reappears as an inherent activity in the educational process; despite the scientific research is more or less rigorous, it must always be oriented to find answers for real needs and problems which are faced by everybody in a certain field of science.

Then an important reflection comes up in the field of research: which lines or fields of knowledge should we orient our research efforts to? Areas related to technology and experimental sciences have traditionally enjoyed the privilege of greater attention in this aspect, unlike what has happened in the field of humanistic sciences and education.

Nowadays, we find a reality which is sufficiently verified in studies and research, as an evidence of the enormous educational deficiencies of our population: graduates of basic, intermediate and higher level struggling to write a letter, to make themselves understood, to express an idea, to make estimates, to propose reasoning or problem solving, analysis or synthesis of particular situations. . . We definitely see here intellectual limitations that lead us to ask ourselves: where is the origin of these problems? Technology? Or maybe our human, social and educational reality?

It is about something beyond the investigation of deep, rugged problems, or the study of simple situations and daily practices, which are common aspects to our educational work and our sociocultural reality. This is an important source of research, even more than a science with a high value of social intersubjectivity such as mathematics and mathematics education in general.

Researching in the field of mathematics education has been favoured by the rise of 'research in education'; it has established itself as a field of study which has progressively evolved in order to be positioned from a philosophical perspective, much more than from a scientific one reaching 'the necessary experience' to have an identity of its own. The growing concern for mathematicians and educators about what kind of mathematics is taught in school, how this subject is learned and what and how it should be taught, has represented the main stimulus for the configuration and delimitation of the problems about this field of study and the adequate methods for its knowledge and intervention (Castro, 2007).

There is a world-wide community of educators, researchers, departments, institutions, etc., which is concerned about study and research in this field. They have

also contributed powerfully to the constitution of 'the new discipline scientist that deals with problems related to Mathematics Education' (de Guzmán, 1996; Godino, 2004). Efforts to strengthen academic cooperation and the linkage amongst different working groups have found some bases in the actions of groups and institutions such as the International Commission on Mathematical Instruction, the National Science Foundation (NSF), the UNESCO, the International Congress of Mathematics Education (ICME), the Inter-American Committee on Mathematics Education (CIAEM), the Commission Internationale pour l' Étude et l'Amélioration de l'Enseignement des Mathématiques (CIEAEM), or the Commission for the Study and Improvement of Mathematics Teaching or the Latin American Meeting of Educational Mathematics (RELME).

Current Trends in Mathematical Education

Miguel de Guzmán, a Spanish professor and president of the International Commission on Mathematical Instruction (ICMI), made at the end of the 90 s an interesting reflection on some aspects of the panorama of mathematics education in an excellent article entitled *Current trends in mathematical education*:

- Why is teaching mathematics a difficult task?
- Why is it necessary to study the change in the Didactics of Mathematics within each period?
- General trends in mathematics education
- Changes in methodological principles
- Some current trends in the contents

Amongst all the aspects that have been extracted from his approach and recommendations, it must be highlighted the initial and permanent training of mathematics teachers, research in mathematics education, mathematics education of society, popularization of mathematics and attention to early talent in mathematics.

Below I will present a summary scheme of this contribution by providing some ideas that complement the work referred to.

Mathematics is a way of approach to reality. It provides important elements for the development of the capacity for rational argumentation, reflective abstraction and the increase of the skills which are needed to solve problems not only in the school environment, but also widely applied and transferred to other fields of knowledge. These aspects constitute valid arguments for the mathematical education and, consequently, for the promotion and stimulation of research initiatives in this field, both of studies about pure research (epistemology and structure of science) and of those which are closest to teaching practice (planning, teaching strategies, development and use of resources and evaluation) that could be classified as applied research. Research in the field of mathematics education represents an alternative that could contribute not only to the development and stimulation of investigative skills of those people who assume it, but would also broaden the horizons of the

didactic-pedagogical analysis criteria, which favour the prospective, strategic and tactical vision of this science that are needed for all professionals, and especially for those in the educational field (Castro, 2007).

In order to delve into this question, I would like to add some comments about the thought of pedagogical renewal. This concept, which promotes a deepening of active and emancipatory practices in education if it is taken as a tendency, has experienced some historical moments of different intensity and in many ways all over the world. Its trace throughout history has penetrated into the world of education and pedagogy up to our days. It needs to be mentioned that the case of Spain -with a background of important history in Europe and in the rest of the world- is not a preferential study aim in this work, as in different regions the required renovation has not been carried out in the same way. From a critical perspective, it must be reviewed what the concept of contemporary pedagogical renewal has meant, and what it has also meant for Spanish education since many times its footprint has been diluted within the recent history of the country.

The fact of valuing the action of the Pedagogical Renewal Movements of that time can provide us with crucial clues and reflections on the situation of education at the procedural moment and today too, since many elements of its problem keep persisting and have not been addressed with sufficient details and depth although exponential progress has been made. Far from posing here a nostalgic anecdote of what the pedagogical renewal of the last decades meant -and still means- we must weigh up the results obtained as indicated ut-supra (Lorenzo, 2016).

, shows us that some of the changes which had been introduced in the 70 s turned out to be not successful like, for instance, when elementary mathematics lost content as geometry and it was replaced by algebra. Besides, drawbacks that arose with the introduction of modern mathematics far outweighed the questionable advantages that had been thought to achieve, for example, the rigor in the foundation and the understanding of mathematical structures.

The integration of historical and epistemological questions in the teaching and learning of mathematics constitutes a possible natural way of discovering, a better understanding of the specific parts of mathematics, and also a greater awareness of what part of mathematics is part of knowledge. This is something that practically everyone has an idea of (Santaló, 1994).

In mathematics education, mathematics is the result of contributions that have been taken from many different cultures, as well as its constant dialogue with other scientific disciplines, philosophy, arts and technology and being constantly updated with changes throughout history. These all facts are a constant force of encouragement and scientific, technical support, and so artistic and social development (Clark et al., 2016). As mathematics is fundamental to modern society, these facts will improve mathematics education since the historical and epistemological questions of mathematics are equally important because when its harmony is mixed with other areas of knowledge, they make a greater connection between humanities and sciences.

New Challenges in the Teaching of Mathematics

New challenges related to the teaching of mathematics that the international community had to face were passionately carried out (De Guzmán, 2001) on the benefits and deficits of the trends that were making their own way over time. As significant changes are taking place more and more in mathematics education, in the most developed countries there has been a growing interest of a large number of mathematicians who have focused in the problems currently posed at different levels, from the primary education to university.

The work of Gravemeijer (1994) that can be contemplated in the Education and Social Environment project (OSM), which finally appeared in the publication of the textbook series *Rekenen en Wiskunde*, sets the tone for the elaboration of a plan of mathematics studies that embodies the main characteristics of realistic mathematics education with the aim of changing educational practice in schools.

Besides, NCTM (2000) asserted that educational technology (computers) could help to teach content as it could offer students a linkage from concrete to abstract thinking by enabling them to observe and create multiple representations of mathematical ideas.

Also, HAL needs to be mentioned, a multidisciplinary open access that can be used for the deposit and dissemination of scientific material, research documents, whether published or not, which may come from teaching and research institutions of public or private research centers. Since 2000 it indicates the topics of TSG 25 (ICME 13) (Clark et al., 2016) in which the state of the art on the relationships between history and mathematics pedagogy is described in order to enlighten and provide insight into the following general questions:

- Which history is suitable, pertinent, and relevant to Mathematics Education (ME)?
- Which role can History of Mathematics (HM) play in ME?
- What extent has HM been integrated to in ME (curricula, textbooks, educational aids/resource material, teacher education)?
- How can this role be evaluated and assessed and what extent does it contribute to the teaching and learning of mathematics?

Highly-prestigious mathematicians have advocated as a priority the advancement in mathematics education from different topics mentioned above:

- *Mathematics Education and Language Diversity* (Barwell et al., 2016). Here the authors introduce challenges and possibilities for the development of indigenous mathematics education in multilingual contexts, as well as recent movements towards the recognition of linguistic and cultural diversity and opening spaces for new debates on education that must be recovered, and a cultural identity to be reclaimed.
- *Why including the History of Mathematics in Mathematics Education* (Clark et al., 2016; Palenzuela, 2017; Tzanakis et al., 2002; Arcavi, 1991; Ernest, 1998; Furinghetti & Somaglia, 1997; de Guzmán, 1993).

- The cultural, humanistic, interdisciplinary character and the possibility of its curricular organization are shown with the hope for the concern and need for the use of the history of mathematics as a didactic resource in the classroom to be sparked (Arcavi, 1991; Fauvel, 1991).
- The proper use of the history of mathematics helps to teach the subject. But in these times of mathematics for all, the history of mathematics is the most important thing, like an integral part of the subject TO HAVE A WIDER perspective OF IT and to present a more complete picture of what mathematics is FOR the public community (Siu & Siu, 1979; Siu, 1997).
- Studying the history of mathematics rather than just using it as a tool, and this means trying to understand it as a historian does. Then we can realize that mathematics is something that humans do and therefore it helps us to understand human identity itself. This way the history of mathematics in mathematics education has potentially made us fuller human beings, which is known in the heart of the educational tradition as 'liberal arts' (arts of a free human being). By considering the nature of liberal arts we can better understand the significance of the history of mathematics in mathematics education (Fried, 2018).
- The twenty-first century faces enormous challenges that increasingly and universally come from interrelated societies. Thus, it is possible to observe significant economic changes in the globalization of capitals and markets, both industrial and financial; or extremely dynamic advances in science and technology, in mechanical, virtual and spatial spheres, etc. All of this has affected all living beings that are part of this reality. Our cultures have absorbed all these new globalized aspects, by making them their own and interchangeable: communications are almost instantaneous in real time; the mobility of information is surprising; war conflicts between countries being broadcast on television; literacy is linked to the computer; virtual reality is almost the new company of children, much closer than manual, collective creation games. We are undoubtedly immersed in a scientific-technical revolution that means a new way of producing and thinking about what we are surrounded by. The theoretical-practical needs and problems have demanded epistemological changes and ruptures, and even rationality. According to what it was suggested by Thomas Kuhn, an American physicist, philosopher of science and historian who is known for his contribution to the change of orientation of philosophy and scientific sociology in the 1960s, we can say that we are facing paradigm changes as a result of scientific revolutions. At the end of the twentieth century these partial, disciplinary scientific movements began not only to be interrelated, but also to be measured as a single movement. In this sense, some authors began to identify and deepen the reflections on synchronic similarities, despite the different plots and knowledge problems. And all of this has turned out to be the configuration and denomination of a new generalizing scientific paradigm, which is capable of encompassing all sciences in general -and mathematics in particular- as it is the emergence of this new complex or complex paradigm. The scientific paradigm of complexity (Romero, 2020) comes up to overcome the historical insufficiency, like the classical paradigm and its

Fig. 5.2 Complexity Theory. (*Quaderni di Ricerca in Didattica (Mathematics), Numero Speciale N°. 7, 127–138. Università di Palermo)*

corresponding assessment of the notion of simplicity and domination of man towards nature (Bacon, 1620a, b; Menna & Salvatico, 2000) (Fig. 5.2).

Therefore, it goes beyond the identification of complexity as 'something which is complicated' and, on the contrary, the transcendence is about affirming that the complex is an attribute of reality and it is irreducible to discrete entities too. There are proposals that value the dialectical units of the simple and the complex, the validation of chance, uncertainty, chaos, indeterminacy and emergence, non-linearity, etc. (Taeli, 2010). However, this paradigm of complexity not only conforms what reality is like to the ontological vision, but it also demands an epistemological coherence, complex thinking or non-classical rationality to be increasingly accepted. For decades, numerous journals, some open access or not, have addressed the main topics of mathematics education whose aims were to share, disseminate and discuss current trends, research results, experiences and perspectives in a wide range of mathematics education, mathematics teaching, development in mathematics instruction, the innovations in mathematics learning and current trends in mathematics

education research, etc. Obviously, due to reasons of length I will only cite some of these most popular journals:

- Educational Studies in Mathematics

  (An International Journal)[1]

- Digital Experiences in Mathematics Education[2]
- Journal for Research in Mathematics Education[3]
- Canadian Journal of Science, Mathematics and Technology Education[4]
- International Journal of Mathematical Education in Science and Technology[5]
- ZDM – Mathematics Education[6]
- International Electronic Journal of Mathematics Education[7]
- Teaching Mathematics and Its Applications[8]
- The Journal of Mathematical Behaviour[9]
- Research in Mathematics Education[10]
- Journal of Mathematics Teacher Education[11]

Multicultural and Intercultural Approaches in the Classroom

Not many research projects, curriculum development efforts, or cross-cultural collaborations in mathematics education take seriously the notion that potentially confusing and complex multiplicities of cultures and identities manifest in what might at first glance be taken as a single "culture." and monolithic. in contemporary, postcolonial, Creole "intercultural" contexts,... (Appelbaum, 2008; Valero & Stentoft, 2010; Swanson & Appelbaum, 2012; Chronaki, 2005).

[1] https://www.springer.com/journal/10649

[2] https://www.springer.com/journal/40751

[3] https://pubs.nctm.org/view/journals/jrme/jrme-overview.xml

[4] https://www.springer.com/journal/42330

[5] https://www.tandfonline.com/action/journalInformation?show=aimsScope&journalCode=tmes20

[6] https://www.springer.com/journal/11858

[7] https://www.iejme.com/

[8] https://academic.oup.com/teamat

[9] https://www.journals.elsevier.com/the-journal-of-mathematical-behavior

[10] https://www.tandfonline.com/action/journalInformation?show=aimsScope&journalCode=rrme20

[11] https://www.springer.com/journal/10857

In (Swanson & Appelbaum, 2012), the analysis of the debates around the links between mathematics and democracy among the population can be pointed out; in general, they have not been explicitly tested well. A critical relationship around democracy in mathematics education can compromise the direction of its goals. Consequently, in the work cited, it is notable to point out the issues that arise, and that I personally want to review, quote: "*. . . what if the "option" of not participating in experiences of mathematics education, or in its (re) direction, was also a critical relationship with mathematics education? What if this rejection and disobedience to the evocative power of mathematics were a democratic action?. . .*"

In Chap. 8 of this book (Appelbaum & Stathopoulou, 2022) it is analyzed how the few projects and study referents of the complexity of the notion of multiculturalism and interculturalism have not been clearly the object of investigation.

In this section we will do a brief analysis of different outlooks in multicultural and intercultural approaches in the classroom. We would like to make sure that making use of multicultural and intercultural approaches (Holm & Zilliacus, 2009) enriches both students and teachers since they are better understood and valued by promoting critical thinking, as students think about this topic more deeply and broadly. Multiculturalism is increasingly present in the classroom and this creates the need for teachers to have new challenges. Traditionally, students who participate in a multicultural class have to make an effort at the time of learning to live with unknown people, culture and language.

As a consequence, this implies that teachers must acquire the task and the commitment to direct students through that 'strange', unknown route which allows them to achieve the goals initially proposed thanks to specialized learning techniques. Classrooms with dynamic activities, both inside and outside the campus, are more likely to boost student achievement and to improve the negative aspects that were observed in multicultural classrooms in the past (Merfat, 2015) (Fig. 5.3).

When it comes to talking about multiculturalism and interculturality (Stan, 2020; Latour, 2010; Shkelzen, 2017), a brief reflection must be made since both concepts, in my opinion, present some differences between each other to be understood and effectively approached by the promotion of what democratic values may be.

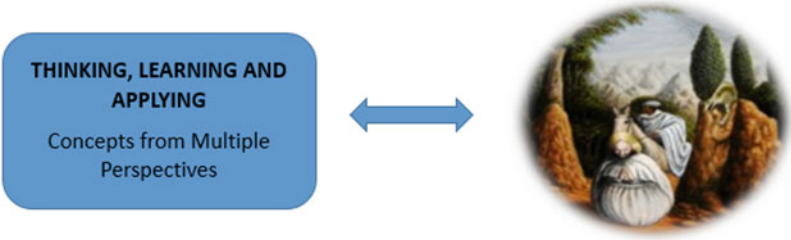

Fig. 5.3 S. Freud. (*St. George and the dragon*. Jósean Figueroa)

These differences mainly lie in the fact that interculturality is a much broader term that even encompasses multiculturalism.

Therefore, it would be better to focus on the concept of 'intercultural education'. A fully inclusive school goes one step beyond multicultural education. This is necessary to incorporate criticism and reflection so that students may be able to acquire values, such as respect for diversity, equality and dignity, apart from having a more empathetic vision to live with society as well. Some guidelines to follow would be:

- Educational proposals of a social nature
- Highlighting the similarities between cultures, and not only the differences
- Rejecting the hierarchy of cultures.
- Fostering relationships between individuals, groups and institutions of various cultures.
- Establishing common languages and shared rules that allow exchanges and facilities for interpersonal relationships, as well as the power of decision and participation.

And here mathematics education must play an important role. In order to achieve an education where all students are integrated, it is necessary that both in classrooms and in educational centers (Cole, 1996), formal and non-formal or intercultural, all groups from different cultures may be able to coexist based on equal treatment and respect for difference.

Diversity is a defining element of the dynamics of history and cultural richness sustained by the identities that are integrated into its cultural heritage linked to the social weft. The story of this diversity goes through the fight against racism, discrimination and stereotypes, along with equal access and enjoyment of Human Rights and Fundamental Freedoms. December 10, 2021 marked the 73rd anniversary of the adoption of the Universal Declaration of Human Rights (Resolution, 217-III, 1948) by the General Assembly. According to UNESCO, in a society where cultural diversity is part of the common heritage of mankind, communication and interaction between cultures is an essential need like biological diversity between living organisms, basic educational work to coexist freely, fair, egalitarian and dignified in everyone's society that must be an indispensable objective of every Curriculum Project. Interculturality means the relationship between two or more cultures that establish ties in a horizontal, non-hierarchical way, creating synergies and sharing different points of view (Reynolds-Case, 2013). In a school where there is an increasing confluence of cultures due to higher rates of immigration, the educational response to this interculturality must be adequate and adapted. And this is possible when an Intercultural Education (IE) is applied in a school environment that benefits the coexistence between people of different cultures.

As Heptalogue of IE Principles (Fig. 5.4)

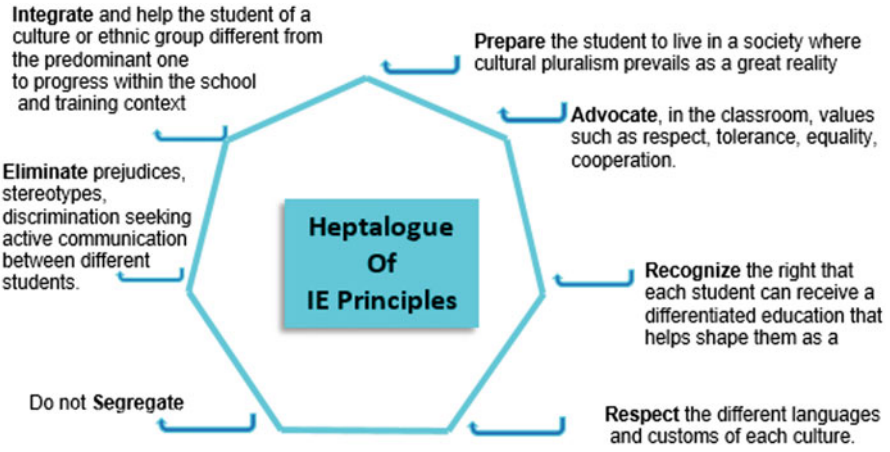

Fig. 5.4 Heptalogue of IE Principles. (Heptalogue of IE Principles)

How to Apply an Intercultural Education in a Mathematics Classroom

The presence of a high number of students from different cultures in the classroom makes the work of the teacher more important when it comes to facilitate communication and relationships between students in order to provide a positive intercultural mathematics education. This is considered as a mission and a challenge in which all members of the educational community must collaborate like center staff, environment, family, etc.

The permanent training in mathematics by the teacher must keep being the transmission of pedagogical competences about how to program the classroom in a way that might be focused on interculturality, how to work with positive education, moderate conflicts, as well as proposing the resolution to problems associated with coexistence of cultures by presenting mathematical situations that are adapted to the procedural moment in which it is found. Thanks to a positive activity the teacher must be willing to successfully communicate her intercultural message, and promulgating confidence and security at the same time.

Multicultural Mathematics: Teaching Mathematics from a Global Perspective

For so long many researchers have been working on the need for teaching mathematics from a multicultural perspective. This approach can be applied to any school curriculum. Recognizing and appreciating the cultural heritage of minority students helps to build their confidence and pride. A multicultural approach to mathematics requires the acknowledgement of history of mathematics and recognizing the social and political values that shape the mathematics curriculum (Roth & Radford, 2011).

How to Work with Intercultural Education

The introduction of intercultural content in the didactic units of the different curricular areas, especially in mathematics, should serve to carry out cooperative activities in work groups by adapting them to the classroom and by developing materials that can favour their application (Salazar, 2009).

In the activities whose aims are the interaction and feedback between students of different ethnicities and cultures, an attempt will be made for an emotional development and a relational learning by contributing to the aforementioned values that promote reflection, dialogue, harmony, listening to the other person, empathy, as well as to adapt their skills to different environments, etc. (Velasco & Jablonska, 2010). A work that, if applied from the earliest educational stage, improves not only the quality of teaching but also enriches the very multicultural society where we find –and will find-ourselves. (CIEAEM, 70-Mostaganem, 2018).

Definitely, being the mestizo society, the multicultural reality is a condition of the way of life of the human species: we live in multicultural societies. As Gairín (2005) said, '. . .*all societies have been multicultural*. . .' (sic). On the other hand, Ainscow (2001) made a very serious approach in his book *Development of inclusive schools. Ideas, proposals and experiences to improve school institutions*. This book offers many ideas, reflections, proposals and experiences to make schools and classrooms more inclusive, capable of reaching out to those students who encounter difficulties in their participation and learning. Also, in the interesting work *Developing inclusive education systems: What are the levers for change?* Ainscow (2004) argues that inclusion is the main challenge facing education systems around the world. This reflection provides a framework for determining factors that can help to facilitate systems in a more inclusive direction. As a consequence, strategies for developing inclusive practices must involve interruptions at the time of thinking to foster an exploration of overlooked possibilities.

Figure 5.5 helps us to get focused on the factors that influence inclusive developments within an education system. The debate on interculturality has strongly penetrated into the educational field in southern European countries. In Spain, the real emergence took place in 1992 and the initiatives of the administrations and

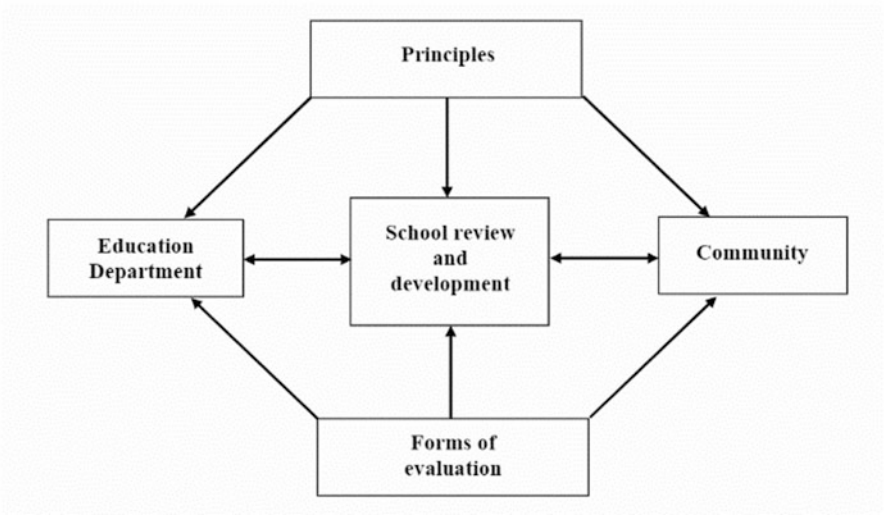

Fig. 5.5 Levers for change. Inclusive development. (Ainscow, 2004)

universities, publications and scientific meetings have been multiplying since then. I agree with the idea of interculturality of Xavier Besalú, 2002: '*The best intercultural education is the full social recognition of cultural minorities. We cannot respond to political and social problems with educational solutions alone. Pedagogizing the most complex issues and challenges is still a cheap and easy resource, serving only to divert attention from the true causes of these problems. However, education can favor a rational understanding of conflicts and avoid irrational and unfair explanations and solutions*. . .' (sic)

It is about a new space, still non-existent, governed by new rules, born from negotiation and joint creativity (Carbonell, 1998). And, consequently, what pedagogical policies, teaching organizations and methodologies can concur to the achievement of fair, adequate solutions. In education there are three major theoretical-practical perspectives on how to approach interculturality in school: compensatory, multicultural and intercultural education.

Cases in Developing Countries

There are many works and investigations that have been carried out in this respect of mathematics, diversity and cultural education. Today, in Latin America there are more than 600 indigenous tribes and minority groups, historically marginalized by educational systems and social policy, who obtain less successful results of learning achievement in mathematics. In order to promote equity in education and the relevance of learning and teaching processes, it is necessary to incorporate the

richness and cultural diversity of peoples into them. The works of the Peruvian Association for Research in Mathematical Education (Bonilla et al., 2018) highlight the importance of developing IBE-Intercultural Bilingual Education (López & Küper, 1999) and designing processes that use local mathematical knowledge, by making known the situation of Intercultural Bilingual Education in Chile (Peña & Hueitra, 2016) and Peru (Cáceres et al., 2016), as well as the bilingualism of deaf people in Brazil (Andreis-Witkoski, 2020), by developing aspects of its mathematical dimension and proposing alternatives which contribute to solve problems that may arise. Also, in Ecuador (MEE, 2013; Conejo, 2008; Bonilla et al., 2018) the 1IBE System Model (MOSEIB) allows the promotion and development of the ancestral language and culture thanks to active, student-focused proposals by considering their learning rhythms, which is a contextualized learning in the social, mental, cultural and linguistic fields.

Not wanting to enter into controversy about the consideration of a developing country, and due to obvious reasons of space and the singularity of the research which has been carried out, we will present with certain details, as a significant example in the context of the issue addressed, an investigation that makes us focus our attention on social practices in an indigenous community.

Case (a): The Use of Social and School Practices Within an Indigenous School

The excellent ethnomathematical work (Jaramillo, 2009, 2013, 2014) by the University of Antioquia (Colombia) exemplifies other understandings about the forms of objectification of mathematical knowledge from a dialogue between social practices and school practices within the indigenous school

This is an investigation that analyzes the relationship that can appear between the social practices of planting by the Tule and Embera-Chamí indigenous peoples and the production of mathematical knowledge related to measurement in an indigenous school context. An interesting conclusion is the presentation of latent intercultural tensions that have not been solved yet, neither for researchers in this field nor for indigenous peoples (Fig. 5.6).

> In this research project, the question that invited us was related to what interrelationships are woven, through mathematical knowledge, between the teaching processes and the learning processes, within the mathematics classroom. . . . A practice that makes it possible to explain the relationships between human action and social, cultural, political and historical situations, where such actions take place, and that enable the emergence of such knowledge. For its part, we understood the classroom as an encounter, where various subjects converge in a given space, time and sociocultural context - the three historical and political - where an interlocutory event has to take place, from the perspective of Geraldi (2000), around specific knowledge . . . (sic)

Jaramillo highlights in this analysis some ideas about the sociocultural perspective in mathematics education: knowledge is produced from the subject in their

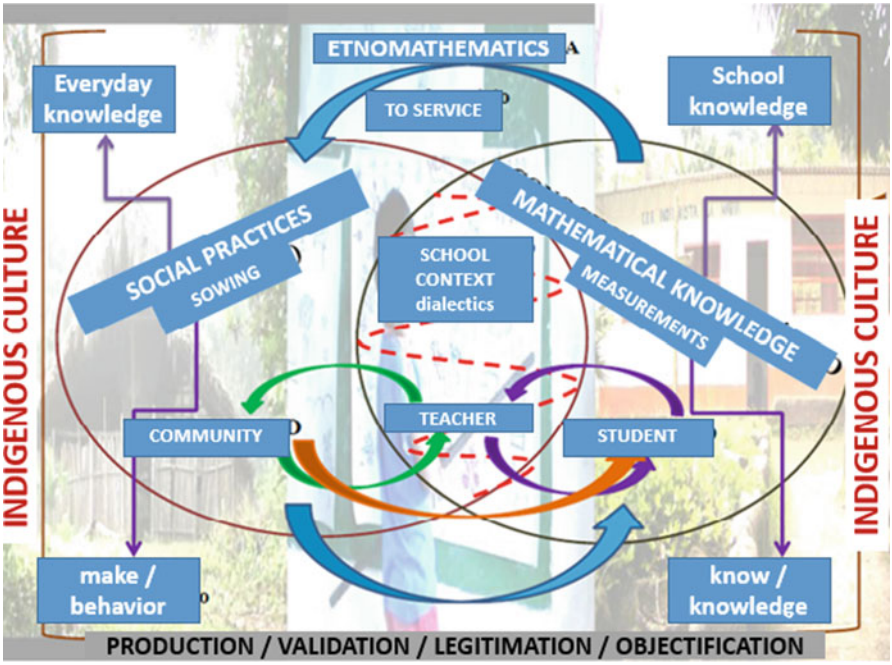

Fig. 5.6 Scheme Berrío Londoño, 2009. (Content translation: Romero, 2021)

interrelations with the world as a social activity, whose production and legitimation is the result of the explanation of different social practices in which the subjects are involved, based on the senses and shared meanings, thus respecting the different knowledge constituted by the various sociocultural groups.

From this sociocultural perspective, and according to some authors like Radford (2008), mathematics is seen as a product of human activity that is formed during the development of solutions to problems which are created in the interactions that produce the human way of living socially, always in a given time and context.

When it comes to teaching and learning mathematics, there are some important and basic relationships between culture, the curriculum and mathematics education to be taken into consideration. The discussion of these relationships can be possible thanks to some questions (Jaramillo, 2011):

- What sociocultural factors that can make possible mathematical knowledge must be considered, by influencing the educational processes of the teaching/learning of mathematics binomial when they are introduced in the classroom?
- What would happen if we looked at mathematics from social practices instead of looking at social practices from mathematics?
- How can mathematical activity be understood in the production and objectification of mathematical knowledge?

- What are the relationships between knowledge, behaviour and culture in the objectification of mathematical knowledge?

I personally agree with the conclusion that Radford reached to when he stated: *'... These questions are not new in education, perhaps they are for mathematical educators, they try to rescue subjectivity when practicing pedagogy, in their praxis, in mathematics. In this sociocultural perspective of mathematics education, the recovery of the subject and subjectivity in the educational act is sought. Use is made of the dyad used by* Freire *(2000), complaint-advertisement. A reality is denounced to announce a possibility, a utopia or a dream...'*. In this sense, Radford (2008) drew attention to the need to think about mathematics on a basis which would assume knowing as the result of human activity from a historical, social and cultural point of view.

Case (b): Afro-Colombian Studies and Basic Standards in Competencies

Intercultural Dialogue

This look at ethnomathematics and mathematics education in Colombia (Blanco-Álvarez et al., 2014) presents an interesting document about the path travelled by students, teachers and researchers (Oliveras et al., 2013) in the field of ethnomathematics and the relationships with mathematics education. It is about reflection, study and travel through different cultures: (a) making specific, curricular proposals that legitimize intercultural dialogue regarding the different ways of being and doing in a complex, diverse country like Colombia; (b) creating and proposing public policies in accordance with local needs and interests.

Besides, it is also interesting the study of the Chair of Afro-Colombian Studies (CEA) in the context of education for citizenship and coexistence, which has been carried out under the direction of Ángela Patricia Valencia Salas. This study has been stated despite the sociocultural character of mathematics since the seventeenth century:

They have been introduced as a universal, decontextualized and abstract knowledge, based on formal logic. Their cryteria have been applied to the thought systems of other non-Western cultures, and this way it has been argued that their explanations of the world are inconsistent, absurd, false or pre-logical (Páramo, 1996).

As we indicated in the introduction, in this work, all peoples have developed a mathematical thought in relation to their vital context, both from a theoretical and practical point of view, in short: a knowledge with its own logic.

Thus, ethnomathematics shows us a variety of models and explanations of the world, not only ours, but also of multiple others, and this is why teaching provides the student with tools for understanding and intercultural dialogue. Therefore, it is possible to carry out a research within Afro-descendant communities, street children,

indigenous communities, mathematicians, carpenters, bricklayers, peasants, dressmakers, etc., or any other cultural group whose practices will turn out to be another cultural group who will study according to the interests of the communities who the research may be carried out with. (Blanco-Álvarez, 2006, 2011, 2012; Blanco-Álvarez et al., 2014).

The aim of this relationship between mathematical knowledge and reality is constantly being researched, and also to highlight the knowledge of peoples that make possible the search and proposals of new pedagogical perspectives in order to make from the teaching of mathematics an inclusive, intercultural and participatory process (Jaramillo, 2011) which may incorporate the contributions of the African, Afro-diasporic and, especially, Afro-Colombian peoples. African and Afro-diasporic ethnomathematics is the set of mathematical ideas that African-origin different cultures have developed, based on the experience that men and women have had with their environment and relationships; and these ideas are also materialized in mathematical activities such as counting, measuring, estimating, classifying, or predicting, amongst others.

Eugenio Nkogo (2001), in his book *Systematic Synthesis of African Philosophy*, described the wisdom of the Dogon people, a community from Mali that contained precise and detailed data on the solar system.

The synthesis of African philosophy provides surprising data on the origin of Western culture. The common thread lies in the fact that the Egyptian and Greek cultural nourishing source was in African culture: the first essential achievements of the human condition, oral and written language, the emergence of systematized thought, as well as the ethical and artistic dimension of the human race took place in African territories. In great depth, Eugenio Nkogo showed that humanistic civilization came from Africa to the West through Egypt and Rome (Nkogo, 2001, p. 196–197).

The work of the CEA rescues the sociocultural character (Valero, 2004) and mathematics by taking into consideration: what is the use of rescuing the sociocultural character of mathematics for? Why is it useful to the teaching of mathematics the re-establishment of its sociocultural character? This can be used to make visible the mathematical knowledge which is present in all cultures as a way to provoke critical reflections in students that may allow them to recognize their own mathematical knowledge, their ability to think mathematically about reality and to solve everyday situations and problems.

It is part of an educational policy whose main purpose is to highlight, to transmit and to recover the contributions of Afro-descendant peoples to knowledge, values, or relationships between communities and nature which had been invisible, undervalued or denied:

The history of mathematics has not set apart from these processes of little visibility. For example, Pythagoras and Thales of Miletus, the great Greek mathematicians who were of Phoenician descent, learned and worked with wise African mathematicians; it is unknown that the oldest mathematical object of mankind (dated 35.000 years B.C.) was found in Eswatini (ancient Swaziland, Southern Africa): a

chuck bone fragment marked with 29 notches that served to keep time (Ikuska.com, 2013), which is also a calendar still used by some Khoisan groups in Namibia.

Some Mathematical Competencies in the Project (CEA)

Competencies that include symbolic systems, spatial designs, techniques, construction practices, calculation methods, models and measurements of time and space, specific forms of reasoning, and other problems and material activities, were developed in different peoples of the African continent. Also, it shows how this type of thinking is linked to the sociocultural practices of African peoples and responds to their needs, for example: (a) order, classification and set theory: a mathematical critique of race and gender; (b) spatial thinking: geometric figures, patterns, symmetry and asymmetry. Throughout history human beings learned to use geometry in work contexts and needs. In this sense, it seems that geometric exploration was par excellence a mathematical activity in the history of central and southern Africa. A reflection on this can be appreciated in the artistic and geometric work of mat and basket weaving, ceramics, beadwork, painting, wall ornamentation, hair braiding, tattoos, wood sculpture and architecture, by specifically exploring rotational symmetry patterns. Amongst these geometric explorations we must mention: *sipatsi*, *sona* and *litema*.

1. Tonga women from the province of Inhambane (South Mozambique) use a diagonal weaving technique to make handbags called *sipatsi*. These crafts are made with various mental calculations before crisscrossing starts. If there is no exact or positive solution, an approximate solution will be sought. (Gerdes, 2004, 2007, 2014) (Figs. 5.7, and 5.8).
2. The study of the *sona* drawings (*lusona*, singular) leads us to the historical dimension of ethnomathematics. This tradition of sand designs was developed amongst the Chokwe of North East Angola (Gerdes, 2007; Chavey, 2009) (Figs. 5.9, and 5.10).
3. This is another example of creation of geometric patterns that are used by some peoples who live in South Africa and Lesotho today. The *litema* are creations

Fig. 5.7 *Sipatsi* tissue

Fig. 5.8 Base of *sipatsi* basket

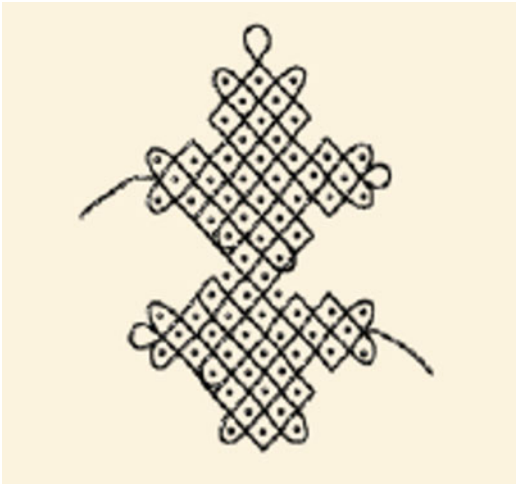

Fig. 5.9 *Sona* Non-monolinear

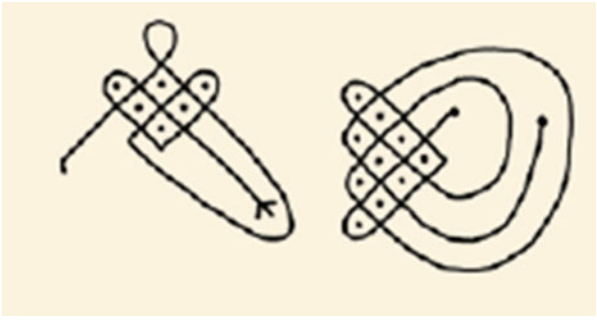

Fig. 5.10 *Sona* basic design Triangular

Fig. 5.11 *Litema*

mainly made by women to decorate their homes, and the motifs come up from the observation of nature and its surroundings (Fig. 5.11).

Conclusion

Unanimity is noticeable in the opinion of the world-wide mathematical community about the creation of a way to approach reality thanks to mathematics, by providing important elements for the development of the capacity related to rational argumentation, reflective abstraction and the increase of the skills which are needed for problem solving, not only in the school environment but also of wide application and transfer to other fields of knowledge. It represents a valid argument for mathematics education and for the promotion and revitalization of research initiatives in this field too, both in studies related to epistemology and those ones which are close to teaching practice. The advance in mathematics education has been consolidated as a field of study, which has been progressively evolving in order to make from itself a scientific perspective rather than a philosophical one, thus reaching 'the necessary experience' to have its own identity.

Finally, as evidenced by reading other chapters of this book, it is important to value the need for a thorough analysis of the diversity of cultures so that they are included in curricular designs as well as taken into account by the educational community, which means that a multicultural approach can potentially enrich both students and teachers. We must focus our attention on a school where every day there is more confluence of cultures and the educational response to this interculturality must be adequate and adapted. And this is possible when an education that promotes coexistence between people of different cultures is applied, presented as a heptalogue. Also, and by way of example, some experiences carried out in disadvantaged environments in developing countries have been presented, such as the use of social and school practices within an indigenous school.

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