Structural-Parametric Synthesis of the Planar Four-Bar and Six-Bar Function Generators with Revolute Joints

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Abstract. This paper presents a structural-parametric synthesis of the four-bar and Stephenson II, Stephenson III six-bar function generating linkages. Four-bar linkage is formed by connecting two coordinate systems with given angles of rotation using a negative closing kinematic chain (CKC) of the **RR** type. Six-bar linkages are formed by connecting two coordinate systems using two CKCs: a passive CKC of the **RRR** type and a negative CKC of the **RR** type. The negative CKC of the **RR** type imposes one geometrical constraint to the relative motion of the links, and its geometric parameters are defined by least-square approximation. Passive CKC of the **RRR** type does not impose a geometrical constraint, and the geometric parameters of its links are varied.

Keywords: Function generator · Structural-parametric synthesis · Least-square approximation

1 Introduction

The first studies on the design of function generating linkages are due to A. Svoboda [1, 2], who designed a Watt II function generator for generation of the logarithmic function. Kinematic synthesis (dimensional or parametric synthesis) of mechanisms, including function generating linkages, is carried out on the basis of the kinematic geometry of finite positions of a rigid body, approximation methods (polynomials) and computers [3]. The kinematic geometry of finite positions of a rigid body, which in the case of plane motion is known as the Burmester theory [4] is used for the synthesis of function generators in the works of Hunt [5], Bottema and Roth [6], Angeles and Bai [7, 8], Pira and Cunaku [9], McCarthy and Soh [10] and others. Synthesis of mechanisms by kinematic geometry is clarity and simple. However, these methods are applicable for a limited number of positions. For the kinematic synthesis of mechanisms with unlimited positions of the output links, the approximation methods are used, the foundations of which were laid

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by Chebyshev [11]. Approximation (algebraic, optimization) methods for the kinematic synthesis of four-bar and six-bar function generators Watt II, Stephenson II, Stephenson III were used in the works of Freudenstein [12], Hartenberg and Denavit [13], McLarnan [14], Kiper [15], Hwang and Chen [16], Bulatovic et al. [17], Plecnik and McCarthy [18–20] and others. In [18–20] the polynomial homotopic software Bertini [21] was used.

At the intersection of kinematic geometry and approximation synthesis, a new direction in the kinematic synthesis of mechanisms: approximation kinematic geometry, has been created; Sarkissyan, Gupta, Roth [22–24]. Based on approximation kinematic geometry by Baigunchekov, Laribi, Carbone, et al. [25, 26] the parallel mechanism and manipulator are synthesized. In this work, a structural-parametric synthesis of four-bar and six-bar function generators is carried out, where the structures and geometric parameters of the links of the synthesized mechanisms are determined in arbitrary given discrete values of the input and output links angles.

2 Structural Synthesis of the Planar Four-Bar and Six-Bar Function Generators with Revolute Joints

According to the developed principle of mechanism formation, they are formed by connecting the output object to the base using active, passive and negative CKCs [25].

The output object of a function generating linkage is a link that performs a given rotary or translational motion relative to the base at a given motion of the input link. Let the input link and the output object perform rotational movements. We choose as the input and output links, the coordinate systems Ax_1y_1 and Bx_2y_2, rotating relative to the base with rotation angles φ_i and ψ_i (Fig. 1 a).

If we connect the planes of two moving coordinate systems Ax_1y_1 and Bx_2y_2 by a negative CKC *CD* of the binary link **RR** type, then we get a structural diagram of four-bar function generator *ACBD*. The connection of the planes of two moving coordinate systems Ax_1y_1 and Bx_2y_2 by the binary link *CD* of the type **RR** is possible when the plane of the moving coordinate system Bx_2y_2 has a circular point (a point moving along a circle) *D* in relative motion to the coordinate system Ax_1y_1, or vice versa, i.e. when there is a circular point *C* in the plane of the coordinate system Ax_1y_1 in relative motion with respect to the coordinate system Bx_2y_2.

If none of the planes of the moving coordinate systems Ax_1y_1 and Bx_2y_2 do not have circular points in relative motion, then the planes of the two coordinate systems are connected by a passive CKC *CDE* of the **RRR** type dyad. As a result, we obtain a structural diagram of the five-bar mechanism *ACDEB* with two degrees of freedom (Fig. 1b).

To form six-bar function generators from this five-bar linkage we connect its non-adjacent links by a binary link *FG* of the type **RR**, having one negative degree of freedom, defined by Chebyshev formula [27].

$$F = 3n - 2p_5, \tag{1}$$

where $n = 1$ is the number of moving link, $p_5 = 2$ is the number of kinematic pairs of the fifth class, then $F = -1$.

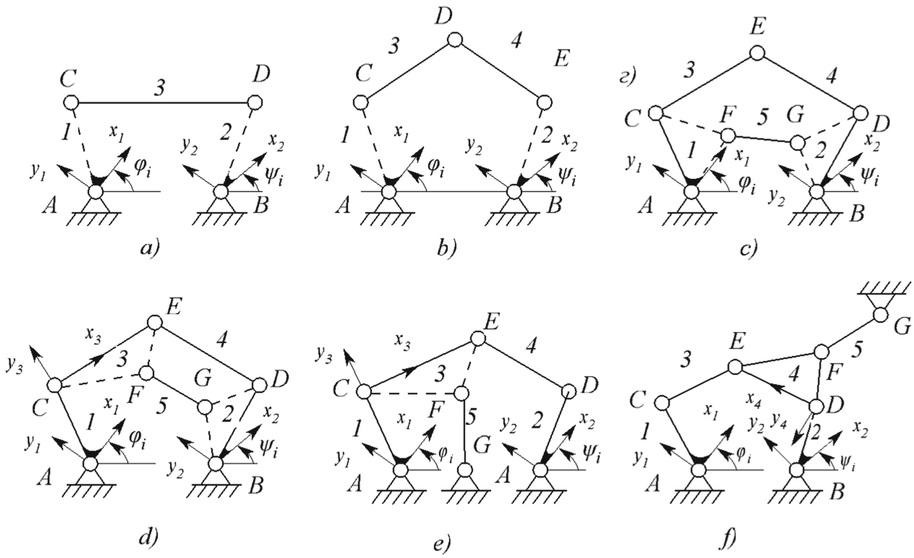

Fig. 1. Structural synthesis of planar function generators

Consequently, the negative CKC *FG*, imposing one geometrical constraint on the five-bar linkages with two DOF, forms six-bar function generators with one DOF. Figures 1c–f show the structural diagrams of the formed six-bar function generators. If links 1 and 2 of the five-bar linkage are connected by a binary link *FG*, we get a Stephenson I mechanism. If links 3 and 2 of the five-bar linkage are connected by a binary link *FG*, we get a Stephenson II mechanism. When a link 3 of the five-bar linkage is connected to the base by a binary link *FG*, we get a Stephenson III mechanism. When link 4 of the five-bar linkage is connected to the base by a binary link *FG*, we get a different type of six-bar function generator.

3 Parametric Synthesis of a Four-Bar Function Generator

Let given the function

$$\psi_i = f(\varphi_i), \tag{2}$$

where $i = 1,2,\ldots,N$; N is the number of finite positions of the movable planes Ax_1y_1 and Bx_2y_2. It is necessary to determine the synthesis parameters (geometric parameters) of the four-bar function generator that implements function (2). The synthesis parameters are: $x_C^{(1)}, y_C^{(1)}, x_D^{(2)}, y_D^{(2)}$ and l_{CD}, where $x_C^{(1)}, y_C^{(1)}$ and $x_D^{(2)}, y_D^{(2)}$ are the coordinates of the joints C and D in coordinate systems Ax_1y_1 and Bx_2y_2, respectively, l_{CD} is the length of the CD link.

Consider the movement of the coordinate system Bx_2y_2 relative to the coordinate system Ax_1y_1. In this case, point D moves along a circle centered at point C and with radius l_{CD}. Let's derive an equation

$$(x_{D_i}^{(1)} - x_C^{(1)})^2 + (y_{D_i}^{(1)} - y_C^{(1)})^2 - l_{CD}^2 = 0, \tag{3}$$

where

$$\begin{bmatrix} x_{D_i}^{(1)} \\ y_{D_i}^{(1)} \end{bmatrix} = \begin{bmatrix} \cos\varphi_i & \sin\varphi_i \\ -\sin\varphi_i & \cos\varphi_i \end{bmatrix} \cdot \begin{bmatrix} X_{D_i} - X_A \\ Y_{D_i} - Y_A \end{bmatrix}, \tag{4}$$

$$\begin{bmatrix} X_{D_i} \\ Y_{D_i} \end{bmatrix} = \begin{bmatrix} X_B \\ Y_B \end{bmatrix} + \begin{bmatrix} \cos\psi_i - \sin\psi_i \\ \sin\psi_i \cos\psi_i \end{bmatrix} \cdot \begin{bmatrix} x_D^{(2)} \\ y_D^{(2)} \end{bmatrix}. \tag{5}$$

Equation (3) is an equation of a geometrical constraint imposed by a binary link CD of the type **RR** on the movements of two moving coordinate systems Ax_1y_1 and Bx_2y_2. The left side of Eq. (3) will be denoted by Δq_i and called the weighted difference function

$$\Delta q_i = (x_{D_i}^{(1)} - x_C^{(1)})^2 + (y_{D_i}^{(1)} - y_C^{(1)})^2 - l_{CD}^2. \tag{6}$$

Parametric synthesis of a four-bar function generator is to determine five geometric parameters $x_D^{(2)}, y_D^{(2)}, l_{CD}^2$ from the minimum of function (6).

Substituting expressions (4) and (5) into Eq. (3), we obtain

$$\begin{aligned} \Delta q_i = 2\{&[-(X_A - X_A)\cos\varphi_i - (Y_B - Y_A)\sin\varphi_i]\cdot x_C^{(1)} + [(X_B - X_A)\sin\varphi_i - (Y_B - Y_A)\cos\varphi_i]\cdot y_C^{(1)} \\ &+ [\frac{1}{2}(x_C^{(1)^2} + y_C^{(1)^2} + x_D^{(2)^2} + y_D^{(2)^2} - l_{CD}^2)] + [(X_B - X_A)\cos\psi_i + (Y_B - Y_A)\sin\psi_i]\cdot x_D^{(2)} \\ &+ [-(X_B - X_A)\sin\psi_i + (Y_D - Y_A)\cos\psi_i]\cdot y_D^{(2)} + [-\cos(\psi_i - \varphi_i)\cdot(x_C^{(1)}\cdot x_D^{(2)} + y_C^{(1)}\cdot y_D^{(2)})] \\ &+ [\sin(\psi_i - \varphi_i)\cdot(x_C^{(1)}\cdot y_D^{(2)} - y_C^{(1)}\cdot x_D^{(2)})] + \frac{1}{2}[(X_B - X_A)^2 + (Y_B - Y_A)^2]\}. \end{aligned} \tag{7}$$

If we introduce the notations

$$\begin{aligned} &p_1 = x_C^{(1)}, p_2 = y_C^{(1)}, p_3 = \frac{1}{2}(x_C^{(1)^2} + y_C^{(1)^2} + x_D^{(2)^2} + y_D^{(2)^2} - l_{CD}^2), p_4 = x_D^{(2)}, p_5 = y_D^{(2)}, \\ &f_{1i} = -(X_B - X_A)\cos\varphi_i - (Y_B - Y_A)\sin\varphi_i, f_{2i} = (X_B - X_A)\sin\varphi_i - (Y_B - Y_A)\cos\varphi_i, \\ &f_3 = 1, f_{4i} = (X_B - X_A)\cos\psi_i - (Y_B - Y_A)\sin\psi_i, f_{5i} = -(X_B - X_A)\sin\psi_i + (Y_B - Y_A)^2\cos\psi_i, \\ &f_{6i} = -\cos(\psi_i - \varphi_i), f_{7i} = \sin(\psi_i - \varphi_i), f_{0i} = -\frac{1}{2}[(X_B - X_A)^2 + (Y_B - Y_A)^2], \end{aligned}$$

then Eq. (7) takes the form

$$\begin{aligned} \Delta q_i = 2[&p_1\cdot f_{1i} + p_2\cdot f_{2i} + p_3\cdot f_3 + p_4\cdot f_{4i} + p_5\cdot f_{5i} \\ &+(p_1\cdot p_4 + p_2\cdot p_5)\cdot f_{6i} + (p_1\cdot p_5 - p_2\cdot p_4)\cdot f_{7i} - f_{0i}]. \end{aligned} \tag{8}$$

Equation (8) is linear in the following two groups of synthesis parameters

$$\mathbf{p}^{(1)} = [p_1,\ p_2,\ p_3]^T, \mathbf{p}^{(2)} = [p_4,\ p_5,\ p_3]^T \tag{9}$$

and is represented in two linear forms

$$\begin{aligned} \Delta q_i^{(1)} = 2[&p_1(f_{1i} + p_4\cdot f_{6i} + p_5\cdot f_{7i}) + p_2(f_{2i} + p_5\cdot f_{6i} - p_4\cdot f_{7i}) \\ &+p_3\cdot f_3] + [p_4\cdot f_{4i} + p_5\cdot f_{5i} - f_{0i}] \end{aligned} \tag{10}$$

and

$$\Delta q_i^{(2)} = 2[p_4(f_{4i} + p_1 \cdot f_{6i} - p_2 \cdot f_{7i}) + p_5(f_{5i} + p_2 \cdot f_{6i} + p_1 \cdot f_{7i}) \\ + p_3 \cdot f_3 + [p_1 \cdot f_{1i} + p_2 \cdot f_{2i} - f_{0i}]. \tag{11}$$

Let us introduce the notations

$$\mathbf{g}_{1i}^{(1)} = [g_{1i}^{(1)}, g_{2i}^{(1)}, g_{3i}^{(1)}]^T, \mathbf{g}_{1i}^{(2)} = [g_{1i}^{(2)}, g_{2i}^{(2)}, g_{3i}^{(2)}]^T,$$

where

$$g_{1i}^{(1)} = f_{1i} + p_4 \cdot f_{6i} + p_5 \cdot f_{7i}, g_{2i}^{(1)} = f_{2i} + p_5 \cdot f_{6i} - p_4 \cdot f_{7i}, g_{3i}^{(1)} = p_3 \cdot f_3,$$
$$g_{1i}^{(2)} = f_{4i} + p_1 \cdot f_{6i} - p_2 \cdot f_{7i}, g_{2i}^{(2)} = f_{5i} + p_2 \cdot f_{6i} + p_1 \cdot f_{7i}, g_{3i}^{(2)} = p_3 \cdot f_3,$$
$$g_{0i}^{(1)} = p_4 \cdot f_{4i} + p_5 \cdot f_{5i} - f_{0i}, g_{0i}^{(2)} = p_1 \cdot f_{1i} + p_2 \cdot f_{2i} - f_{0i}.$$

Then Eqs. (10) and (11) take the form

$$\Delta q_i^{(k)} = 2(\mathbf{g}_i^{(k)^T} \cdot \mathbf{p}^{(k)} - g_{0i}^{(k)}), k = 1, 2. \tag{12}$$

To determine the vectors $\mathbf{p}^{(k)}$ of synthesis parameters, we minimize function (12) by the least square optimization, i.e. derive the sums

$$S^{(k)} = \sum_{i=1}^{N} (\Delta q_i^{(k)})^2 \tag{13}$$

and consider the necessary conditions for the minimum of function (13) over two groups of synthesis parameters $\mathbf{p}^{(k)}$:

$$\frac{\partial S^{(1)}}{\partial p_1} = 0, \frac{\partial S^{(1)}}{\partial p_2} = 0, \frac{\partial S^{(1)}}{\partial p_3} = 0 \tag{14}$$

and

$$\frac{\partial S^{(2)}}{\partial p_4} = 0, \frac{\partial S^{(2)}}{\partial p_5} = 0, \frac{\partial S^{(2)}}{\partial p_3} = 0. \tag{15}$$

Conditions (14) and (15) lead to the following two systems of linear equations for two groups of synthesis parameters

$$\mathbf{D}^{(1)} \cdot \mathbf{p}^{(1)} = \mathbf{d}^{(1)} \tag{16}$$

and

$$\mathbf{D}^{(2)} \cdot \mathbf{p}^{(2)} = \mathbf{d}^{(2)}, \tag{17}$$

where

$$\mathbf{D}^{(1)} = \sum_{i=1}^{N} \begin{bmatrix} g_{1i}^{(1)^2} & g_{1i}^{(1)} \cdot g_{2i}^{(1)} & g_{1i}^{(1)} \\ g_{1i}^{(1)} \cdot g_{2i}^{(1)} & g_{2i}^{(1)^2} & g_{2i}^{(1)} \\ g_{1i}^{(1)} & g_{2i}^{(1)} & 1 \end{bmatrix}, \quad \mathbf{d}^{(1)} = \sum_{i=1}^{N} \begin{bmatrix} g_{1i}^{(1)} \cdot g_{0i}^{(1)} \\ g_{2i}^{(1)} \cdot g_{0i}^{(1)} \\ g_{0i}^{(1)} \end{bmatrix}, \tag{18}$$

$$\mathbf{D}^{(2)} = \sum_{i=1}^{N} \begin{bmatrix} g_{1i}^{(2)^2} & g_{1i}^{(2)} \cdot g_{2i}^{(2)} & g_{1i}^{(2)} \\ g_{1i}^{(2)} \cdot g_{2i}^{(2)} & g_{2i}^{(2)^2} & g_{2i}^{(2)} \\ g_{1i}^{(2)} & g_{2i}^{(2)} & 1 \end{bmatrix}, \quad \mathbf{d}^{(2)} = \sum_{i=1}^{N} \begin{bmatrix} g_{1i}^{(2)} \cdot g_{0i}^{(2)} \\ g_{2i}^{(2)} \cdot g_{0i}^{(2)} \\ g_{0i}^{(2)} \end{bmatrix}. \quad (19)$$

It is easy to show that the Hessian of matrices $\mathbf{D}^{(1)}$ and $\mathbf{D}^{(2)}$ is positive definite together with the principal minors [23], and then the solutions of the systems of linear Eqs. (16) and (17) correspond to the minimum of function (13). Therefore, for given values of the vector parameters $\mathbf{p}^{(2)} = [p_4, p_5, p_3]^T$, the vector parameters $\mathbf{p}^{(1)} = [p_1, p_2, p_3]^T$ are determined by solving the system of linear Eq. (16). Based on the obtained values of the vector parameters $\mathbf{p}^{(2)}$, the vector parameters $\mathbf{p}^{(1)}$ are determined from the system of linear Eq. (16). In this case, the sequence of function $S^{(k)}$ values will be decreasing and have a limit as a sequence bounded from below. This allows using the linear iteration method based on kinematic inversion to solve the quadratic optimization problem.

4 Parametric Synthesis of the Six-Bar Function Generators

Parametric synthesis of six-bar function generators (Fig. 1c–f) consists of the parametric synthesis of the passive CKC *CED* and the negative CKC *FG*. Synthesis parameters of the passive CKC *CED* of all six-bar function generators (Fig. 1c–f) are $x_C^{(1)}, y_C^{(1)}, x_D^{(2)}, y_D^{(2)}, l_{CE}, l_{ED}$, where $x_C^{(1)}, y_C^{(1)}$ and $x_D^{(2)}, y_D^{(2)}$ are the coordinates of the joints *C* and *D* in the coordinate systems Ax_1y_1 and Bx_2y_2 of the links 1 and 2, respectively, l_{CE} and l_{ED} are the lengths of the *CE* and *ED* links. Since the passive CKC *CED* of the type **RRR** has zero degree of freedom and it does not impose a geometrical constraint on the motion of the coordinate systems Ax_1y_1 and Bx_2y_2, the geometric parameters of their links are varied, and the synthesis parameters of the negative CKC *FG* are approximated. For the parametric synthesis of Stephenson II (Fig. 1d), Stephenson III (Fig. 1e) function generator shown in Fig. 1f, we determine the positions of the links *CE* and *ED* of the passive CKC *CED* by the equations

$$\varphi_{3i} = \varphi_{(CD)_i} + cos^{-1} \frac{l_{CE}^2 + l_{(CD)_i}^2 - l_{ED}^2}{2l_{CE} \cdot l_{(CD)_i}}, \quad (20)$$

$$\varphi_{4i} = \text{tg}^{-1} \frac{Y_{E_i} - Y_{D_i}}{X_{E_i} - X_{D_i}}, \quad (21)$$

where

$$l_{(CD)_i} = [(X_{D_i} - X_{C_i})^2 + (Y_{D_i} - Y_{C_i})^2]^{\frac{1}{2}}, \quad (22)$$

$$\varphi_{(CD)i} = \text{tg}^{-1} \frac{Y_{D_i} - Y_{C_i}}{X_{D_i} - X_{C_i}}, \quad (23)$$

$$\begin{bmatrix} X_{C_i} \\ Y_{C_i} \end{bmatrix} = \begin{bmatrix} X_A \\ Y_A \end{bmatrix} + \begin{bmatrix} \cos\varphi_i & -\sin\varphi_i \\ \sin\varphi_i & \cos\varphi_i \end{bmatrix} \cdot \begin{bmatrix} x_C^{(1)} \\ y_C^{(1)} \end{bmatrix}, \quad (24)$$

$$\begin{bmatrix} X_{D_i} \\ Y_{D_i} \end{bmatrix} = \begin{bmatrix} X_B \\ Y_B \end{bmatrix} + \begin{bmatrix} \cos\psi_i & -\sin\psi_i \\ \sin\psi_i & \cos\psi_i \end{bmatrix} \cdot \begin{bmatrix} x_D^{(2)} \\ y_D^{(2)} \end{bmatrix}. \tag{25}$$

The synthesis parameters for the negative CKC *FG* of the Stephenson I mechanism (Fig. 1c) are $x_F^{(1)}, y_F^{(1)}, x_G^{(2)}, y_G^{(2)}$, which are determined similarly to the parametric synthesis of the four-bar function generator (Fig. 1a). Therefore, the functionality of the Stephenson I mechanism is the same as the functionality of the four-bar function generator.

The synthesis parameters for the negative CKC *FG* of the Stephenson II function generator (Fig. 1d) are $x_F^{(3)}, y_F^{(3)}, x_G^{(2)}, y_G^{(2)}, l_{FG}$, where $x_F^{(3)}, y_F^{(3)}$ and $x_G^{(2)}, y_G^{(2)}$ are the coordinates of the joints *F* and *G* in coordinate systems Cx_3y_3 and Bx_3y_3 of the links 3 and 2, respectively. For parametric synthesis of the link *FG*, we consider the movement of the coordinate system Bx_3y_3 relative to the coordinate system Cx_3y_3 and derive the equation of the geometrical constraint

$$(x_{G_i}^{(3)} - x_F^{(3)})^2 + (y_{G_i}^{(3)} - y_F^{(3)})^2 - l_{FG}^2 = 0, \tag{26}$$

where

$$\begin{bmatrix} x_{G_i}^{(3)} \\ y_{G_i}^{(3)} \end{bmatrix} = \begin{bmatrix} \cos\varphi_{3i} & \sin\varphi_{3i} \\ -\sin\varphi_{3i} & \cos\varphi_{3i} \end{bmatrix} \cdot \begin{bmatrix} X_{G_i} - X_{C_i} \\ Y_{G_i} - Y_{C_i} \end{bmatrix}, \tag{27}$$

$$\begin{bmatrix} X_{G_i} \\ Y_{G_i} \end{bmatrix} = \begin{bmatrix} X_B \\ Y_B \end{bmatrix} + \begin{bmatrix} \cos\psi_i & -\sin\psi_i \\ \sin\psi_i & \cos\psi_i \end{bmatrix} \cdot \begin{bmatrix} x_G^{(2)} \\ y_G^{(2)} \end{bmatrix}. \tag{28}$$

Further, the synthesis parameters of the link *FG* are determined similarly to the determination of the synthesis parameters of the link *CD* of a four-bar function generator.

The synthesis parameters for the binary link *FG* of the Stephenson III function generator (Fig. 1e) and the mechanism shown in Fig. 1f are $x_F^{(3)}, y_F^{(3)}$ - for the Stephenson III mechanism and $x_F^{(4)}, y_F^{(4)}$- for the mechanism shown in Fig. 1f, and X_G, Y_G, l_{FG} are for both mechanisms, where $x_F^{(3)}, y_F^{(3)}$ and $x_F^{(4)}, y_F^{(4)}$ are the coordinates of the joint *F* in the coordinate systems Cx_3y_3 and Dx_4y_4, respectively, X_G and Y_G are the coordinates of the joint *G* in the absolute coordinate system *OXY*. For the parametric synthesis of the link *FG* of the Stephenson III function generator and the mechanism shown in Fig. 1f, we derive the following geometrical constraint equation

$$(X_{F_i} - X_G)^2 + (Y_{F_i} - Y_G)^2 - l_{FG}^2 = 0, \tag{29}$$

where for the Stephenson III function generator

$$\begin{bmatrix} X_{F_i} \\ Y_{F_i} \end{bmatrix} = \begin{bmatrix} X_{C_i} \\ Y_{C_i} \end{bmatrix} + \begin{bmatrix} \cos\varphi_{3i} & -\sin\varphi_{3i} \\ \sin\varphi_{3i} & \cos\varphi_{3i} \end{bmatrix} \cdot \begin{bmatrix} x_F^{(3)} \\ y_F^{(3)} \end{bmatrix}, \tag{30}$$

for the mechanism shown in Fig. 1f:

$$\begin{bmatrix} X_{F_i} \\ Y_{F_i} \end{bmatrix} = \begin{bmatrix} X_{D_i} \\ Y_{D_i} \end{bmatrix} + \begin{bmatrix} \cos\varphi_{4i} & -\sin\varphi_{4i} \\ \sin\varphi_{4i} & \cos\varphi_{4i} \end{bmatrix} \cdot \begin{bmatrix} x_F^{(4)} \\ y_F^{(4)} \end{bmatrix}. \tag{31}$$

Further, the synthesis parameters of the binary link *FG* are determined similarly to the parametric synthesis of the four-bar function generator.

5 Conclusion

Structural synthesis of four-bar and six-bar function generators with revolute joints has been carried out. A four-bar function generator is formed by connecting two rotating coordinate systems with given rotation angles using a binary link of the type **RR**, which is a negative CKC that imposes one geometrical constraint. Six-bar function generators are formed by connecting these two rotationally moving coordinate systems using a passive CKC of the type **RRR** and by connecting non-adjacent links of the resulting five-bar linkage by binary link of the type **RR**. As a result, Stephenson I, Stephenson II, Stephenson III function generators were formed. Passive CKC of the type **RRR** does not impose a geometrical constraint on the movement of two moving coordinate systems and its geometric parameters are varied. Geometric parameters of the negative CKC of the type **RR** are determined by least-square approximation. In this case, the least-square approximation problem is reduced to a simple kinematic inversion problem based on linear iteration.

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