Chapter 3
Stochastic Optimal Planning of Distribution System Considering Integrated Photovoltaic-Based DG and D-STATCOM

3.1 Distributed Network

The electric distributed network mainly delivers the electric power from the high-voltage transmission system to the consumers. In electric distribution networks, the R/X ratio is significantly high compared to transmission systems; hence, power loss is high (about 10–13% of the generated power). Moreover, poor quality of power including the voltage profile and voltage stability issues may arise. Therefore, the power quality of the service is based on the continuity of power and maintaining the supply voltage within certain limits with a specified frequency. Radial distribution system (RDS) includes various loads such as commercial, industrial, residential, etc. [136].

The inclusion of shunt capacitors and distributed flexible AC transmission system (D-FACTS) devices can significantly enhance the performance of electric distribution networks by providing the required reactive power compensation. D-FACTS include different members such as distributed static compensator (D-STATCOM), photovoltaic-distributed generation (PV-DG). Optimal allocation of these controllers in the electric distribution networks is a strenuous task for researchers for power loss minimizing, voltage profile improvement, voltage stability enhancement, reducing the system overall costs, and maximizing the system load ability and reliability.

3.2 Backward/Forward Sweep (BFS) Algorithm

The BFS algorithm is one of the most common methods used for load flow distribution systems because it is simple, fast, and robust in convergence and has a low memory requirement for processing with high efficiencies and solution accuracy. The BFS algorithm involves mainly three basic steps based on Kirchhoff's current law (*KCL*) and Kirchhoff's voltage law (*KVL*). The three steps are named the

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nodal current calculation, the backward sweep, and the forward sweep, and they are repeated until the convergence is achieved. The BFS utilizes a simple and flexible radial distribution system numbering scheme to number each branch in the feeder, lateral, and sublateral [136]. The BFS algorithm can be applied to find the load flow results using the following steps:

Step 1: Initialization which include the following:

- The distribution system line and load data.
- The base power and base voltage.
- Calculate the base impedance.
- Calculate the per-unit values of line and load data.
- Take the voltage for all bus's float voltage (1 p.u.).
- Set convergence tolerance $\epsilon = 0.0001$ and $\Delta V_{\max} = 0$.

Step 2: Radial Distribution System Numbering Scheme

The numbering scheme aims to give a number to each section in the distribution system, where a section is a part of a feeder, lateral, or sublateral that connects two buses in the distribution system. The total number of sections $N_{\mathrm{Sec}}^{\mathrm{Total}}$ of a distribution system can be calculated as:

$$N_{\mathrm{Sec}}^{\mathrm{Total}} = N_{\mathrm{bus}}^{\mathrm{Total}} - 1 \tag{3.1}$$

where $N_{\mathrm{bus}}^{\mathrm{Total}}$ is the total number of buses. Each section will carry a number which is one less than its receiving end bus number; for example, the number of sections that connects the sending end p and the receiving end q in Fig. 3.1 can be calculated as:

$$N_{\mathrm{Sec}/P-q} = N_{\mathrm{bus}/q} - 1 \tag{3.2}$$

where $N_{\mathrm{Sec}/P-q}$ is the section number between buses p and q and $N_{\mathrm{bus}/q}$ is the number of bus q. Now, the radial distribution system numbering scheme should be applied on the distribution system to give a number to each section in the system.

Step 3: Nodal Current Calculation

At iteration k, the nodal current injection at node i due to loads and any other shunt elements can be calculated as:

$$I_i^k = \left(\frac{S_I}{V_i^{(k-1)}}\right) - (Y_i)\left(V_i^{(k-1)}\right) \tag{3.3}$$

where I_i^k is the current injection at node I, S_I is the specified power injection at node I, $V_i^{(k-1)}$ is the voltage at node I at iteration $k-1$, and Y_i is the sum of all shunt elements at node i.

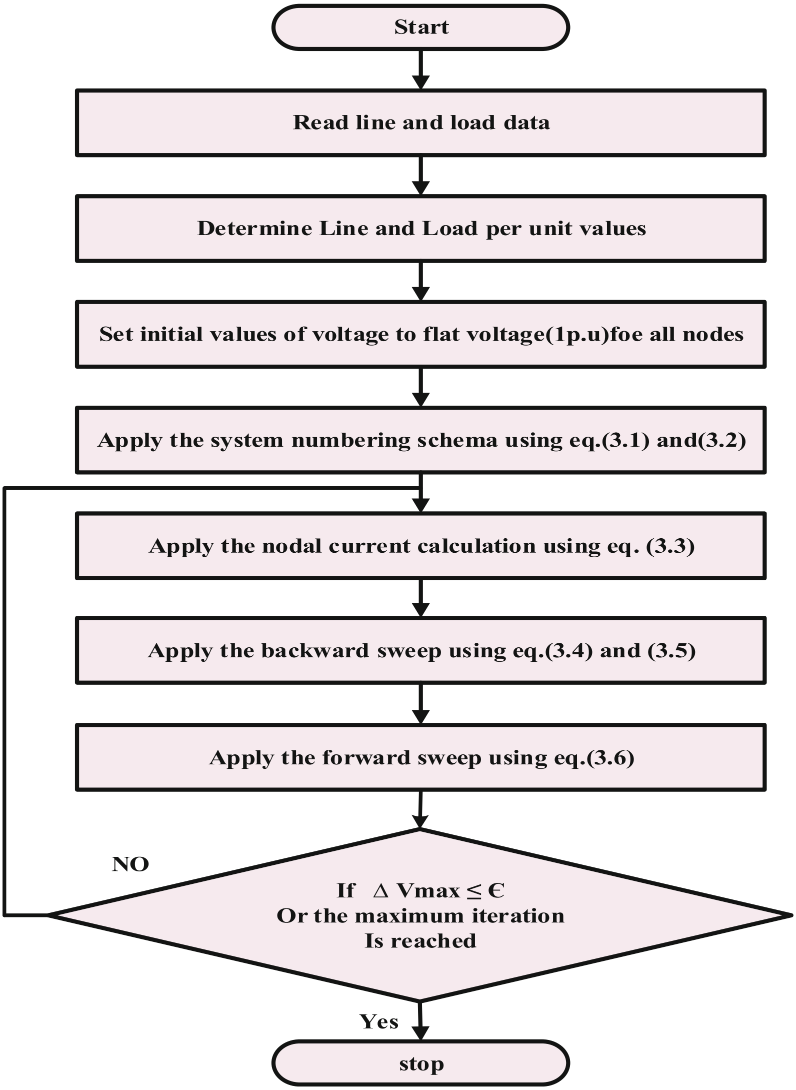

Fig. 3.1 Flow chart of BFS load flow

Step 4: Backward Sweep

At iteration *k*, start from the branches at the end nodes and move toward the branches connected to the substation. Hence, all branch currents can be calculated by applying the Kirchhoff Current Law (*KCL* (and then the powers through these branches can be determined as:

$$I_L^{(K)} = -I_j^{(K)} - \sum_{m=1}^{M} \left(\frac{S_m}{V_j^{(K)}} \right) \tag{3.4}$$

$$S_L^{(K)} = \left(V_j^{(K)} + Z_L \times I_L^{(K)} \right) \left(I_L^{(K)} \right) \tag{3.5}$$

where $I_L^{(K)}$ is the current flow in branch L at iteration k, $I_j^{(K)}$ is the current injected due to shunt elements at bus j, M is the number of branches connected to bus j, Sm is the complex power at the sending end of branch m, $V_j^{(K)}$is the voltage at bus j, $S_L^{(K)}$is the power flow in branch L, and Z_L is the impedance of branch L.

Step 5: Forward Sweep
At iteration k, the nodal voltages are updated in a forward sweep starting from the branches in the first section toward those in the last by applying the Kirchhoff Volt Law) KVL). For a branch L connected sending end p and receiving end q, the voltage at receiving end at iteration k can be calculated as:

$$V_q^{(K)} = \left(V_p^{(K)} - Z_L * I_L^{(K)} \right) \tag{3.6}$$

where $V_p^{(K)}$ and $V_q^{(K)}$ are the voltages at sending and receiving ends, respectively.

Step 6: Check the Voltage Mismatches
After the previous steps are computed, the voltage mismatches for all nodes are calculated; for example, the voltage mismatch at bus i at iteration k can be calculated as:

$$\Delta V_i^{(K)} = \left\| V_i^{(K)} \right\| - \left\| V_i^{(K-1)} \right\| \tag{3.7}$$

After calculating the voltage mismatches, check the convergence of the voltage as:

$$\Delta V_i^{(K)} > \Delta V_{\max} \text{ then make} \Delta V_{\max} = \Delta V_i^{(K)} \tag{3.8}$$

If $\Delta V_{\max} \geq \in$, go to step 8, otherwise increment the iteration number, and go to step 3.

Step 7: Check Stopping Criterion
The program will be terminated when the maximum iteration is reached or the convergence from the voltage mismatches is verified.

3.3 Forward/Backward Power Flow

The main feature of the radial distribution network is that it has a high R/X ratio. Thus, the conventional power flow method cannot solve the power flow for these networks because these methods suffer from low convergence characteristics. As a result, the forward/backward (FB) load flow method is more applicable than the Newton–Raphael method for solving power flow in distribution grids. The power of forward and backward depends upon the power of Kirchhoff's current and voltage laws. Fig. 3.2 depicts a simple radial network. By applying Kirchhoff's law to the depicted Fig. 3.2, the following equations can be obtained, which describe the forward–backward method.

$$\bar{I_L} = \left(\frac{\bar{S_{L,n}}}{\bar{V_n}}\right)^* \tag{3.9}$$

where $\overline{S_{L,n}}$, $\bar{I_L}$, and $\bar{V_n}$ denote the apparent power current, and voltage, t bus n, respectively. The backward step of the FB method depends upon calculating the branch current as follows:

$$\bar{I_n} = \bar{I_k} + \sum_{p \in M} \bar{I_p} \tag{3.10}$$

The forward step is based on calculating and updating the buses voltage as follows:

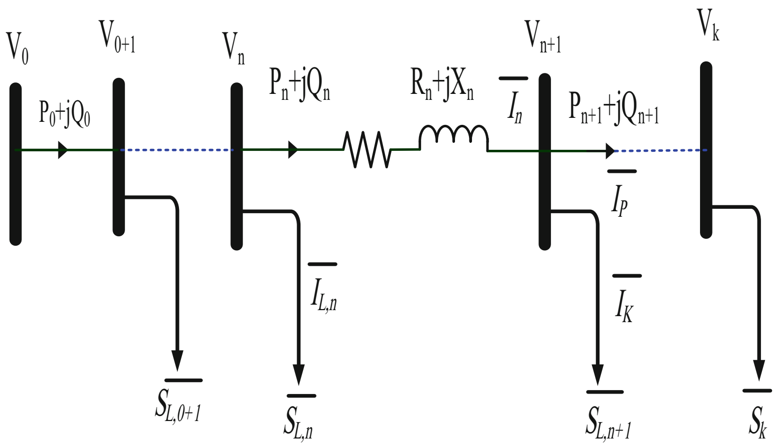

Fig. 3.2 A radial distribution network

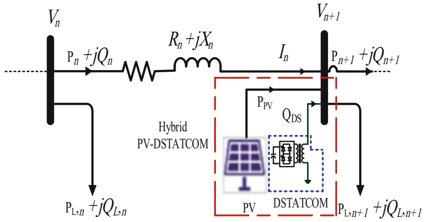

Fig. 3.3 The system with the PV and D-STATCOM

$$\overline{V_{n+1}} = \overline{V_n} - (R_n + jX_n)\overline{I_n} \tag{3.11}$$

In the case of the inclusion of the hybrid PV-DG and D-STATCOM as depicted in Fig. 3.3, the power flow is expressed as follows:

$$P_{n+1} = P_n - P_{L,n+1} - R_{n,n+1}\left(\frac{P_n^2 + jQ_n^2}{|V_n|^2}\right) + P_{\mathrm{PV}} \tag{3.12}$$

$$Q_{n+1} = Q_n - Q_{L,n+1} - X_{n,n+1}\left(\frac{P_n^2 + jQ_n^2}{|V_n|^2}\right) + Q_{\mathrm{DS}} \tag{3.13}$$

where X and R denote the reactance and resistance of the transmission line, respectively. Q_{Ds} is the reactive power injected by D-STATCOM, while P_{PV} is the active power inserted by PV unit. The active power loss of line n, n + 1 is calculated as follows:

$$P_{\mathrm{Loss}} = R_{n,n+1}\left(\frac{P_n^2 + jQ_n^2}{|V_n|^2}\right) \tag{3.14}$$

3.4 Problem Formulation

The aim of incorporating the PV-DG and D-STATCOM is reducing the annual cost, voltage deviations and improve system stability. Thus, the considered objective function is a multi-objective function which described as follows:

3.4.1 The Objective Functions

This book presents a single objective function and multi-objective function that can be discussed as follows:

3.4.1.1 Single Objective Function

The studied single objective function is the total active power loss of the system and can be formulated as follows:

$$F_{\text{obj}} = \sum\nolimits_{l=1}^{nl} \left(P_{\text{loss}}(l)\right) \tag{3.15}$$

where *nl* is the total number of branches in RDS. The active power losses of branch *n* and *n* + 1 is given as follows:

$$P_{\text{loss}} = R_{n,n+1}\left(\frac{P_n^2 + jQ_n^2}{|V_n|^2}\right) \tag{3.16}$$

The total power loss equals to summation of power losses in all branches as follows:

$$P_{\text{Total\_loss}} = \sum_{i=1}^{\text{NT}} P_{\text{loss},i} \tag{3.17}$$

where NT is number of branches. The total active and reactive power losses of a radial distribution system can be written as:

$$\text{Total } P_{\text{Loss}} = \sum_{q=2}^{N_b} \sum_{k=1}^{N_b-1} \left(\frac{P_{\text{eff}/q}^2 + Q_{\text{eff}/q}^2}{V_q^2}\right) * R_k \tag{3.18}$$

$$\text{Total } Q_{\text{Loss}} = \sum_{q=2}^{N_b} \sum_{k=1}^{N_b-1} \left(\frac{P_{\text{eff}/q}^2 + Q_{\text{eff}/q}^2}{V_q^2}\right) * X_k \tag{3.19}$$

where $P_{\text{eff}/q}^2$ and $Q_{\text{eff}/q}^2$ are the total effective active and reactive power loads beyond the node *q,* respectively. *Nb* is the total number of system buses, while *Nb* − 1 refers to the total number of system branches.

3.4.1.1.1 Voltage Stability Index

In this book, voltage stability index (VSI) is used to indicate the security level of distribution system. VSI determines the sensitivity of each bus system to voltage collapse by Eq. (3.20) [137]. If the bus has high value of VSI, the bus is more stable and the possibilities of voltage collapse at this bus is weak. The optimal allocation of DERs and capacitors in distribution network increase the value of VSI for each bus in RDS. Summation of voltage stability index is the total value of VSI for all buses in RDS [138].

$$\mathrm{VSI}_{(n+1)} = |V_n|^4 - 4(P_{mn+1}X_n - Q_{n+1}R_n)^2 - 4(P_{n+1}X_n + Q_{n+1}R_n)|V_n|^2 \quad (3.20)$$

$$\mathrm{VSI}_{(n+1)} = \left(\frac{I}{\mathrm{SI}n}\right), n = 2, 3 \ldots\ldots .n \quad (3.21)$$

where $\mathrm{VSI}_{(n+1)}$ represents the VSI for bus (r) and V_n represents the voltage at bus (s) while X_n and R_n represent the reactance and the resistance between bus (s) and bus (r). But P_{mn+1} and Q_{n+1} represent the active and reactive power injection from bus (r) to RDS. The maximum value of SIn gives minimum value of objective function (VSI). So, minimum values of objective function indicate improvement of voltage stability index. This can be accomplished by maximizing the total voltage stability index deviations according to:

$$\mathrm{TVSI} = 91.25 \times \sum_{i=1}^{\mathrm{Ns}} \sum_{h=1}^{24} \sum\nolimits_{n=1}^{\mathrm{NB}} \mathrm{VSI}_n \quad (3.22)$$

where

$$\mathrm{VSI}_n = |V_n|^4 - 4(P_{mn+1}X_n - Q_{n+1}R_n)^2 - 4(P_{n+1}X_n + Q_{n+1}R_n)|V_n|^2 \quad (3.23)$$

where VSI is the voltage stability index of the nth bus.

3.4.1.1.2 Voltage Deviation

Bus system voltage is an indication to power equality and security indices of the power system. Therefore, any change in bus voltage affects the performance of power system that is calculated by the voltage deviation (VD). The voltage deviation of the system is defined as:

$$\mathrm{VD} = \sum\nolimits_{h=1}^{n} |V_n - 1| \quad (3.24)$$

where n is number of system nodes and $V_n = 1$ p. u

$$\mathrm{TVD} = 91.25 \times \sum_{i=1}^{Ns} \sum_{h=1}^{24} \sum\nolimits_{n=1}^{NB} |V_n - 1| \quad (3.25)$$

where TVD denotes the total voltage deviations; N_{B} denotes the number of buses in the grid.

3.4.1.1.3 The Total Annual Cost

In this book, the considered objective function is minimizing the total annual cost considering the seasonal and hourly variations of the load and the solar irradiance are changed. The year includes four seasons (winter, spring, summer, and fall). The load demand and irradiance are changed seasonally and hourly; this can represent by daily carve for each season (24 × Ns) where *Ns* equals to 4 which denotes to number of seasons per year and it is repeated 91.25 times to represent 1 year (91.25 × 24 × *N*s = *8760 h*). The objective function includes the annual energy loss cost (C_{Loss}), the cost of the purchasing electric power from grid (C_{Grid}), PV units cost (C_{PV}), and D-STATCOM cost (C_{DST}), and it can be represented as follows:

$$C_{\mathrm{PV\&DST}} = \min\ \left(C_{\mathrm{Loss}} + C_{\mathrm{Grid}} + C_{\mathrm{PV}} + C_{\mathrm{DST}}\right) \tag{3.26}$$

It should be pointed out here that the load demand is varied daily and seasonally per year. Thus, the cost of energy losses can be expressed as follows:

$$C_{\mathrm{Loss}} = K_{\mathrm{loss}} \times 91.25 \times \sum_{i=1}^{N\mathrm{s}} \sum_{h=1}^{24} \sum_{j=1}^{\mathrm{NT}} P_{\mathrm{Loss}(i,h,j)} \tag{3.27}$$

where K_{loss} represents the cost of energy loss. *N*s is the number of seasons per year, which is 4. NT is the number of the branches of the grid. 91.25 denotes the number of days per season. The second term of (3.26), which represents the purchased energy from the grid, can be described as follows:

$$C_{\mathrm{Grid}} = K_{\mathrm{Grid}} \times 91.25 \times \sum_{i=1}^{N\mathrm{s}} \sum_{h=1}^{24} P_{\mathrm{Grid}(i,h)} \tag{3.28}$$

where K_{Grid} is the cost of the purchasing power in *\$/kW* P_{Grid} is the captured power from the grid. The cost of the PV unit (C_{PV}) is divided into two costs including the fixed cost ($C_{\mathrm{PV}}^{\mathrm{Fixed}}$) and the variable cost ($C_{\mathrm{PV}}^{\mathrm{Variable}}$). These costs can be calculated as follows:

$$C_{\mathrm{PV}}^{\mathrm{Fixed}} = \mathrm{CRF} \times C_{\mathrm{PV}} \times P_{\mathrm{pr}} \tag{3.29}$$

where C_{PV} is the purchased cost of the PV unit in \$/kW; P_{pr} is the rated power of the PV unit. CRF denotes the capital recovery factor, which can be founded as follows:

$$\mathrm{CRF} = \frac{\alpha \times (1+\alpha)^{\mathrm{NP}}}{(1+\alpha)^{S} - 1} \tag{3.30}$$

where α denotes the rate of interest on capital investing of the connected PV. NP is the lifetime of the PV-DG in years. The variable cost that related to the PV is associated with output kWh as follows:

$$C_{\mathrm{PV}}^{\mathrm{Variable}} = C_{\mathrm{O\&M}} \times \sum_{i=1}^{N\mathrm{s}} \sum_{h=1}^{24} P_{\mathrm{PV}(i,h)} \tag{3.31}$$

$C_{\mathrm{O\ \&\ M}}$ is the operation and maintenance costs which is associated with the output kWh of the PV unit P_{PV} represents the PV output power, which can be given as follows:

$$P_{\mathrm{PV}} = \begin{cases} P_{\mathrm{sr}}\left(\dfrac{{g_{\mathrm{s}}}^2}{G_{\mathrm{STD}} \times X_{\mathrm{c}}}\right) & \text{for } 0 < g_{\mathrm{s}} \le X_{\mathrm{c}} \\ P_{\mathrm{sr}}\left(\dfrac{g_{\mathrm{s}}}{G_{\mathrm{STD}}}\right) & \text{for } X_{\mathrm{c}} \le g_{\mathrm{s}} \le G_{\mathrm{STD}} \\ P_{\mathrm{sr}} & g_{\mathrm{STD}} \le g_{\mathrm{s}} \end{cases} \tag{3.32}$$

where g_{s} denotes the solar irradiance in W/ m^2. G_{std} denotes the solar irradiance in a familiar natural environment of about 1000 W/m^2. X_{c} is a certain irradiance point. The cost of D-STATCOM is calculated as follows:

$$C_{\mathrm{DST}} = K_{\mathrm{DS}} \times Q_{\mathrm{DS}} \times \frac{(1+B)^{\mathrm{ND}} \times B}{(1+B)^{\mathrm{ND}} - 1} \tag{3.33}$$

where Q_{DS} is the rated of the D-STATCOM in kVar, K_{DS} denotes the capital cost of D-STATCOM in \$/KVar, ND is the lifetime of the D-STATCOM in years, and B is of the D-STATCOM.

3.4.1.2 Two-Objective Function

This function can be formulated as follows:

$$F_1 = \frac{P_{\mathrm{T,loss}}}{(P_{\mathrm{T,loss}})_{\mathrm{base}}} \tag{3.34}$$

$$F_2 = \frac{\mathrm{VD}}{(\mathrm{VD})_{\mathrm{base}}} \tag{3.35}$$

$$F_3 = \frac{1}{\sum_{i=1}^{nb} |\mathrm{VSI}(i)|_{\mathrm{base}}} \tag{3.36}$$

The generalized objective function can be formulated as follows:

$$F = H_1F_1 + H_2F_2 + H_3F_3 \tag{3.37}$$

where H_1, H_2, and H_3 are weighting factors. Value of any weighting factor is selected based on the relative important on the related objective function with others objective functions. The sum of the absolute values of the weighting factors in Eq. (3.38) assigned to all impacts should add up to one as follows:

$$|H_1| + |H_2| + |H_3| = 1 \tag{3.38}$$

3.4.1.2.1 Multi-Objective Function

This function can be formulated as follows:

$$F_1 = \frac{P_{\text{T,loss}}}{(P_{\text{T,loss}})_{\text{base}}} \tag{3.39}$$

$$F_2 = \frac{\text{VD}}{(\text{VD})_{\text{base}}} \tag{3.40}$$

$$F_3 = \frac{1}{\sum_{i=1}^{nb} |\text{VSI}(i)|_{\text{base}}} \tag{3.41}$$

$$F_4 = \frac{C_{\text{PV\&DST}}}{\text{Cost}_{\text{base}}} \tag{3.42}$$

The generalized objective function can be formulated as follows:

$$F = H_1F_1 + H_2F_2 + H_3F_3 + H_4F_4 \tag{3.43}$$

where H_1, H_2, and H_3 are weighting factors. Value of any weighting factor is selected based on the relative important on the related objective function with others objective functions. The sum of the absolute values of the weighting factors in Eq. (3.44) assigned to all impacts should add up to one as follows:

$$|H_1| + |H_2| + |H_3| + |H_4| = 1 \tag{3.44}$$

3.4.1.2.2 The Total Annual Cost

$$\min\ F(X,U)=\min(\mathrm{Cost_{loss}}+\mathrm{Cost_{Grid}}+\mathrm{Cost_{PV}}+\mathrm{Cost_{DS}}) \tag{3.45}$$

where

$$\mathrm{Cost_{loss}}=C_{\mathrm{E}}\times 91.25\times\sum_{i=1}^{Ns}\sum_{h=1}^{24}P_{\mathrm{Total\_loss}(i,h)} \tag{3.46}$$

$$\mathrm{Cost_{Grid}}=C_{\mathrm{Grid}}\times 91.25\times\sum_{i=1}^{Ns}\sum_{h=1}^{24}P_{\mathrm{Grid}(i,h)} \tag{3.47}$$

$$\mathrm{Cost_{PV}}=\mathrm{Cost_{PV}^{Fixed}}+\mathrm{Cost_{PV}^{Variable}} \tag{3.48}$$

$$\mathrm{Cost_{PV}^{Fixed}}=\mathrm{CRF}\times C_{\mathrm{PV}}\times P_{\mathrm{sr}} \tag{3.49}$$

$$\mathrm{Cost_{PV}^{Variable}}=C_{\mathrm{O\&M}}\times\sum_{i=1}^{Ns}\sum_{h=1}^{24}P_{\mathrm{PV}(i,h)} \tag{3.50}$$

$$\mathrm{CRF}=\frac{\gamma\times(1+\gamma)^{\mathrm{S}}}{(1+\gamma)^{\mathrm{S}}-1} \tag{3.51}$$

$$\mathrm{Cost_{DS}}=C_{\mathrm{DS}}\times Q_{\mathrm{DS}}\times\frac{(1+B)^{\mathrm{ND}}\times B}{(1+B)^{\mathrm{ND}}-1} \tag{3.52}$$

where

C_{E} is the cost of energy loss in \$/kWh.
C_{Grid} is the purchasing cost per electric power in \$/kWh.
C_{DS} is the capital cost of D-STATCOM in \$/kVAR
$C_{\mathrm{O\ \&\ M}}$ is the operation & maintenance cost of the PV in \$/kWh.
CRF is the capital recovery factor.
γ is the rate of interest on capital investment of the installed PV.
S is the lifetime of PV in years.
B is the asset rate of return.
ND is the lifetime of D-STATCOM in years.
$\mathrm{Cost_{PV}^{Fixed}}$ is the fixed cost of the PV unit.
$\mathrm{Cost_{PV}^{Variable}}$ is the variable cost of the PV unit.
P_{Grid} is the imported power from the grid per hour.
P_{PV} is the generated power by the PV per hour.
P_{sr} is the rated power of the PV unit.
Q_{DS} is the rated reactive power of the D-STATCOM in kVAR.

3.5 System Constraints

This book determines the optimal allocation of DERs and capacitors in RDS under equality and inequality constraints that can be discussed as follows:

3.5.1 Equality Constraints

$$P_{\text{Slack}} + \sum_{i=1}^{\text{NPV}} P_{\text{PV},i} = \sum_{i=1}^{\text{NT}} P_{\text{loss},i} + \sum_{i=1}^{\text{NB}} P_{\text{L},i} \tag{3.53}$$

$$Q_{\text{Slack}} + \sum_{i=1}^{\text{NDS}} Q_{\text{DS},i} = \sum_{i=1}^{\text{NT}} Q_{\text{loss},i} + \sum_{i=1}^{\text{NB}} Q_{\text{L},i} \tag{3.54}$$

where P_{Slack} and Q_{Slack} represent the substation active and reactive powers, respectively. P_{L} and Q_{L} represent the active load demand and reactive load demand, respectively. NPV and NDS denote the number of PV and D-STATCOM units, respectively.

3.5.2 Inequality Constraints

3.5.2.1 Bus Voltage Constraints

The bus system voltage must be operated between the maximum operating voltage (V_{max}) and the minimum operating voltage ($V_{\min}$):

$$V_{\min} \leq V_i \leq V_{\max} \tag{3.55}$$

$$I_n \leq I_{\max,n} \quad n = 1, 2, 3 \ldots, \text{NT} \tag{3.56}$$

$$\sum\nolimits_{i=1}^{N_{\text{PV}}} P_{\text{PV}} \leq \rho \times \sum_{i=1}^{\text{NB}} P_{\text{L},i} \tag{3.57}$$

$$\sum_{i=1}^{\text{NDS}} Q_{\text{DST},i} \leq \sum_{i=1}^{\text{NB}} Q_{\text{L},i} \tag{3.58}$$

3.5.2.2 Line Capacity Limits

In this book, the maximum branch currents of the studied RDS are taken into account as follows [139]:

$$I_n \leq I_{\max,n} \quad n = 1, 2, 3 \ldots, \mathrm{NT} \tag{3.59}$$

where $I_{\max,n}$ is the maximum allowable current through branch (n).

3.5.2.3 Penetration Level

$$\sum_{i=1}^{N_{\mathrm{PV}}} P_{\mathrm{PV}} \leq \rho \times \sum_{i=1}^{\mathrm{NB}} P_{\mathrm{L},i} \tag{3.60}$$

$$\sum_{i=1}^{\mathrm{NDS}} Q_{\mathrm{DST},i} \leq \sum_{i=1}^{\mathrm{NB}} Q_{\mathrm{L},i} \tag{3.61}$$

where ρ denotes the penetration level of the PV unit. min and max are subscripts that indicate the lower and upper allowed levels.

3.5.2.4 Load Demand Modeling

As the load demand is uncertain, normal probability distribution function (pdf) is employed to model it at each node. The normal pdf of the uncertain load demand can be computed as follows [88]:

$$f_n(l) = \frac{1}{\sigma_l \sqrt{2\pi}} \times \exp\left[-\left(\frac{l - \mu_l}{2\sigma_l^2}\right)\right] \tag{3.62}$$

where $f_n(l)$ represents the normal pdf of the load demand; μ_l and σ_l represent the mean and standard deviation of the load demand for each time. The occurrence probability of a segment during a specific hour can be computed as follows:

$$\mathrm{prob}_i^l = \int_{l_i}^{l_{i+1}} f_n(l) \mathrm{d}l \tag{3.63}$$

where l_i and l_{i+1} are the starting and ending points of the interval i and prob_i^l represents the probability occurrence of interval i.

3.6 Uncertainty Modeling

The uncertainties in the PV and load demand models are described in this book. In this work, the probabilistic generations of each PV unit and the load demand have been modeled based on the hourly historical data of the location under research. Three years of hourly historical data of load demand and solar irradiance have been considered in this chapter. Based on this, every single year has been split into four seasons. To describe the stochastic behavior of the PV and load demand during each season, a day within that season (24 − h) is considered. Thus, each year has 96 time periods (4 seasons × 24 − h). For each season, the probability distribution function (pdf) of each perod can be obtained by utilizing the data related to the same hours of the day. Accordingly, each period has 270 samples of solar irradiance and load demand (3 years × 3 months per season ×30 days per month) to generate the corresponding hourly pdfs. The probabilistic model of the PV system and load demand can be characterized as follows:

3.6.1 Modeling of the Solar Irradiance

The solar irradiance data of each hour have been used to generate a beta pdf for that hour and can be described as follows [87, 140]:

$$f_b(g_s) = \begin{cases} \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha).\Gamma(\beta)} g_s^{(\alpha-1)}.(1-g_s)^{(\beta-1)}, & 0 \leq g_s \leq 1; \alpha, \beta \geq 0 \\ 0, & \text{Otherwise} \end{cases} \tag{3.64}$$

where $f_b(g_s)$ is the beta pdf of the solar irradiance; Γ is the gamma function; α and β represent the beta parameters for each period. These parameters can be computed by utilizing the historical data as follows [88, 141]:

$$\beta = (1-\mu) \times \left(\frac{\mu \times (1+\mu)}{\sigma^2} - 1 \right) \tag{3.65}$$

$$\alpha = \frac{\mu \times \beta}{1-\mu} \tag{3.66}$$

where μ and σ are the mean and standard deviation of the solar irradiance for each period. The continuous beta pdfs are divided into many segments where each segment generates a mean value and a probability of occurrence. The occurrence probability of a segment during a specific hour can be determined by:

$$\text{prob}_i^{g_\text{s}} = \int_{g_{\text{s},i}}^{g_{\text{s},i+1}} f_\text{b}(g_\text{s})\text{d}g_{\text{s,i}} \tag{3.67}$$

where $g_{\text{s},\ i}$ and $g_{\text{s},\ i+1}$ represent, respectively, the start and end points of the interval i. $\text{prob}_i^{g_\text{s}}$ represents the probability occurrence of interval i. Based on the generated beta pdf of the solar irradiance of a period, the output power PV for the states of this period can be calculated using (3.32).

3.6.2 Load Demand Modeling

As the load demand is uncertain, normal pdf is employed to model it at each node. The normal pdf of the uncertain load demand can be computed as follows [142, 143]:

$$f_n(l) = \frac{1}{\sigma_l\sqrt{2\pi}} \times \exp\left[-\left(\frac{l-\mu_l}{2\sigma_l^2}\right)\right] \tag{3.68}$$

where $f_n(l)$ represents the normal pdf of the load demand; μ_l and σ_l represent the mean and standard deviation of the load demand for each time period. The occurrence probability of a segment during a specific hour can be computed as follows:

$$\text{prob}_i^l = \int_{l_i}^{l_{i+1}} f_n(l)\text{d}l \tag{3.69}$$

where l_i and l_{i+1} are the starting and ending points of the interval i. prob_i^l represents the probability occurrence of interval i.

3.7 Sensitivity Analysis

Sensitivity analysis is applied for all system buses using a forward–backward power flow algorithm and then the buses are arranged in descending order according to their sensitivity values. The buses with high sensitivity values can be defined as a candidate bus for DER and capacitor installation in RDS. The loss sensitivity analysis is used to reduce the search agent of the presented optimization technique and the total simulation time. In this book, loss sensitivity factor (LSF) is utilized to assign the candidate buses for allocation of shunt compensators which prone to more loss reduction with reactive power injection [15]:

$$P_{\text{loss}} = R_{k,k+1}\left(\frac{P_k^2 + Q_k^2}{|V_k|^2}\right) \tag{3.70}$$

Thus, LSF is computed by taking the derivative of the power loss Eq. (3.64) with respect to reactive power injection as shown:

$$\text{LSF} = \frac{\partial P_{\text{loss}}}{\partial Q_k} = R_{k,k+1}\left(\frac{2Q_k}{|V_k|^2}\right) \tag{3.71}$$

LSF is calculated for all nodes and arrange this value in descending order then normalize the bus voltages as follows:

$$V_{\text{norm}} = \frac{|V_k|}{0.95} \tag{3.72}$$

All buses that have V_{norm} value less than 1.01 and high values of *LSF* are considered the most candidate buses for inclusion of compensators.

Example 3.1
Obtain the total active power losses, voltage stability index, voltage deviations, and the objective function for the IEEE-30 bus test system by the backward/forward method. The base voltage and power are 11 kV and 100 MVA, respectively. The line data are in Ω, and the bus data are in kW and kVAR. Bus 1 is taken as the slack bus with its voltage adjusted to 1.05 p.u. (see the MATLAB M-file code in Appendix A)

Solution The calculation of the total active power losses, voltage stability index, and voltage deviations depends on the following values:

Plossb = 805.733
VSIb = 24.9867
VDb = 1.0669
$w1 = 0.5$
$w2 = 0.25$
$w3 = 0.25$

The system with PV and DSTATCOM can be obtained by applying the ALO algorithm (see the MATLAB M-file code in Appendix A). Therefore, the voltage magnitudes in per unit after running the M-file code are as follows:

$$o = (w1 \times \mathrm{TotalActiveLoss\_KW/Plossb}) + (w2 \times \mathrm{VDD/VDb})$$
$$+ (w2 \times 1/\mathrm{Sum\_VSI})$$

$$F_1 = \frac{P_{\mathrm{T,loss}}}{(P_{\mathrm{T,loss}})_{base}} = F_2 = \frac{\mathrm{VD}}{(\mathrm{VD})_{\mathrm{base}}} = F_3 = \frac{1}{\sum_{i=1}^{\mathrm{nb}} |\mathrm{VSI}(i)|_{\mathrm{base}}}$$

The objective function is formulated as follows:

$$F = H_1F_1 + H_2F_2 + H_3F_3 = 0.0980$$

where H_1, H_2, and H_3 are weighting factors. Summation of the weight factors assigned to all impacts must add up to one as:

$$|H_1| + |H_2| + |H_3| = 1$$

The best optimal value of the objective function found by ALO is: 0.098006

F_1 represents total active power losses reduction, and it can be found as follows:

$$F_1 = \frac{P_{\mathrm{T,loss}}}{(P_{\mathrm{T,loss}})_{\mathrm{base}}} = \mathrm{TotalActiveLoss\_KW} = 115.9892$$

F_2 represents the improving of the voltage profile, which is satisfied by reducing the summation of voltage deviations in RDS and it can be given as follows:

$$F_2 = \frac{\mathrm{VD}}{(\mathrm{VD})_{\mathrm{base}}} = \mathrm{VDD} = 0.0740$$

where F_3 represents the voltage stability enhancement which can be achieved by improvement of voltage stability index (*VSI*) as follows:

$$F_3 = \frac{1}{\sum_{i=1}^{\mathrm{nb}} |VSI(i)\,|_{\mathrm{base}}} =$$

bus_stability_index =

2.0000	0.9937
3.0000	1.0027
4.0000	1.0059
5.0000	0.9986
6.0000	0.9936
7.0000	0.9902

8.0000	0.9894
9.0000	0.9882
10.0000	0.9887
11.0000	0.9882
12.0000	0.9876
13.0000	0.9876
14.0000	0.9879
15.0000	0.9865
16.0000	0.9878
17.0000	0.9889
18.0000	1.0041
19.0000	1.0064
20.0000	1.0133
21.0000	1.0100
22.0000	1.0039
23.0000	0.9993
24.0000	0.9951
25.0000	0.9848
26.0000	0.9839
27.0000	0.9834
28.0000	0.9833
29.0000	0.9823
30.0000	0.9822

Sum_VSI = 28.7976

Example 3.2

Find the first loss sensitivity index (LSI) for the 10-bus distribution system for case load flow general shown in Fig. 3.4. The base voltage and power are 23 kV and 100 MVA, respectively. The line data are in Ω and the bus data are in kW and kVAR.

Solution The calculation of LSI depends on the values of voltage magnitudes at load buses, total effective active and reactive powers at each load bus, and the resistance of system lines. The values of the voltage magnitudes case load flow general. Therefore, the voltage magnitudes in per unit after running the M-file code are as follows:

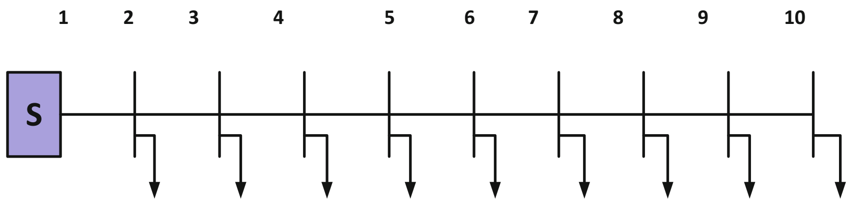

Fig. 3.4 Single line diagram of the 10-bus radial distribution system

$$V_{\text{mag}} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \\ V_9 \\ V_{10} \end{bmatrix} = \begin{bmatrix} 1.0000 \\ 0.9929 \\ 0.9874 \\ 0.9634 \\ 0.9480 \\ 0.9172 \\ 0.9072 \\ 0.8890 \\ 0.8587 \\ 0.8375 \end{bmatrix} \text{p.u}$$

The total effective active power at each load bus in per unit can be determined as:

$$P_{\text{eff}} = \begin{bmatrix} P_{\text{eff}/2} \\ P_{\text{eff}/3} \\ P_{\text{eff}/4} \\ P_{\text{eff}/5} \\ P_{\text{eff}/6} \\ P_{\text{eff}/7} \\ P_{\text{eff}/8} \\ P_{\text{eff}/9} \\ P_{\text{eff}/10} \end{bmatrix} = \begin{bmatrix} P_2 + P_3 + P_4 + P_5 + P_6 + P_7 + P_8 + P_9 + P_{10} \\ P_3 + P_4 + P_5 + P_6 + P_7 + P_8 + P_9 + P_{10} \\ P_4 + P_5 + P_6 + P_7 + P_8 + P_9 + P_{10} \\ P_5 + P_6 + P_7 + P_8 + P_9 + P_{10} \\ P_6 + P_7 + P_8 + P_9 + P_{10} \\ P_7 + P_8 + P_9 + P_{10} \\ P_8 + P_9 + P_{10} \\ P_9 + P_{10} \\ P_{10} \end{bmatrix} /(1000 \times \text{MV}Ab)$$

$$= \begin{bmatrix} 0.12368 \\ 0.10528 \\ 0.09548 \\ 0.07758 \\ 0.06160 \\ 0.04550 \\ 0.03770 \\ 0\text{n}26\text{m}0 \end{bmatrix}$$

Similarly, the total effective reactive power at each load bus in per unit can be obtained as:

$$Q_{\text{eff}} = \begin{bmatrix} Q_{\text{eff}/2} \\ Q_{\text{eff}/3} \\ Q_{\text{eff}/4} \\ Q_{\text{eff}/5} \\ Q_{\text{eff}/6} \\ Q_{\text{eff}/7} \\ Q_{\text{eff}/8} \\ Q_{\text{eff}/9} \\ Q_{\text{eff}/10} \end{bmatrix} = \begin{bmatrix} Q_2+Q_3+Q_4+Q_5+Q_6+Q_7+Q_8+Q_9+Q_{10} \\ Q_3+Q_4+Q_5+Q_6+Q_7+Q_8+Q_9+Q_{10} \\ Q_4+Q_5+Q_6+Q_7+Q_8+Q_9+Q_{10} \\ Q_5+Q_6+Q_7+Q_8+Q_9+Q_{10} \\ Q_6+Q_7+Q_8+Q_9+Q_{10} \\ Q_7+Q_8+Q_9+Q_{10} \\ Q_8+Q_9+Q_{10} \\ Q_9+Q_{10} \\ Q_{10} \end{bmatrix} /(1000 \times \text{MV}Ab)$$

$$= \begin{bmatrix} 0.04186 \\ 0.03726 \\ 0.03386 \\ 0.02940 \\ 0.01100 \\ 0.00500 \\ 0.00390 \\ 0.00330 \\ 0.00200 \end{bmatrix} \text{p.u}$$

The resistances of system lines in per unit can be calculated as:

$$\mathrm{LSI}_1 = \frac{\partial P_{\mathrm{Loss}k}}{\partial V_q}$$

$$= -2 \times R_k \times \left(\frac{P^2_{(\mathrm{eff}/q)} + Q^2_{(\mathrm{eff}/q)}}{V_q^3} \right)$$

$$R = \begin{bmatrix} R_{1-2} \\ R_{2-3} \\ R_{3-4} \\ R_{4-5} \\ R_{5-6} \\ R_{6-7} \\ R_{7-8} \\ R_{8-9} \\ R_{9-10} \end{bmatrix} \times \frac{\mathrm{MVA}b}{\mathrm{kV}b^2} = \begin{bmatrix} 0.123 \\ 0.014 \\ 0.746 \\ 0.698 \\ 1.983 \\ 0.905 \\ 2.055 \\ 4.795 \\ 5.343 \end{bmatrix} \times \frac{100}{23^2} \text{p.u}$$

$$= \begin{bmatrix} 0.0233 \\ 0.0026 \\ 0.1411 \\ 0.1320 \\ 0.3749 \\ 0.1711 \\ 0.3885 \\ 0.9065 \\ 10101 \end{bmatrix} \text{p.u}$$

Now, the LSI at bus 2 can be calculated as:

$$
\begin{aligned}
LSI^{\mathrm{Bus2}} &= \frac{\partial P_{\mathrm{Loss}/1-2}}{\partial V_2} \\
&= -2 \times R_{1-2} \times \left(\frac{P^2_{(\mathrm{eff}/2)} + Q^2_{(\mathrm{eff}/2)}}{V_2^3}\right) \\
&= -2 \times 0.0233 \times \left(\frac{0.12368^2 + 0.04186^2}{0.9929^3}\right) \\
&= -0.00081165\ \text{p.u.}
\end{aligned}
$$

Similarly, the LSI1 for the rest of loads buses are:

$$
\begin{array}{llll}
\mathrm{LSI}_1^{\mathrm{Bus3}} = -6.8581 \times 10^{-5} & \text{p.u.} , & LSI_1^{\mathrm{Bus}\ 4} = -0.0032384 & \text{p.u.} \\
\mathrm{LSI}_1^{\mathrm{Bus}\ 5} = -0.00213310 & \text{p.u.} , & LSI_1^{\mathrm{Bus}\ 6} = -0.0038051 & \text{p.u} \\
\mathrm{LSI}_1^{\mathrm{Bus}\ 7} = -0.00096061 & \text{p.u.} , & LSI_1^{\mathrm{Bus}\ 8} = -0.0015889 & \text{p.u} \\
\mathrm{LSI}_1^{\mathrm{Bus}\ 9} = -0.00199670 & \text{p.u.} , & LSI_1^{\mathrm{Bus}\ 10} = -0.00093873 & \text{p.u}
\end{array}
$$

Example 3.3

Obtain the total active power losses, voltage stability index, voltage deviations for the 10-bus distribution system with PV and DSTATCOM shown in Fig. 3.4. The base voltage and power are 23 kV and 100 MVA, respectively. The line data are in Ω and the bus data are in kW and kVAR.

Solution The calculation of the total active power losses, voltage stability index, and voltage deviations depends on the following values:

Plossb = 783.669
VSIb = 6.6131
VDb = 0.6988
$w1 = 0.5$
$w2 = 0.25$
$w3 = 0.25$

The system with PV and DSTATCOM can be obtained by applying the ALO algorithm. The system with PV and DSTATCOM can be obtained by applying the ALO algorithm as following:

F_1 represents total active power losses reduction and it can be found as follows: $F_1 = \frac{P_{\mathrm{T,loss}}}{(P_{\mathrm{T,loss}})_{\mathrm{base}}} =$

===

bus no. mag_V angle_V Pg_KW Qg_KVAR

===

BusOut =

1.0e+04 *

0.0001	0.0001	0	0	0
0.0002	0.0001	−0.0000	0	0
0.0003	0.0001	0.0000	0	0
0.0004	0.0001	0.0000	0	0
0.0005	0.0001	0.0001	0	0
0.0006	0.0001	0.0001	1.2368	0
0.0007	0.0001	0.0000	0	0.4186
0.0008	0.0001	−0.0000	0	0
0.0009	0.0001	−0.0001	0	0
0.0010	0.0001	−0.0001	0	0

==

Line no. from to mag_I(p.u.) Ploss(KW) Qloss(KVAR)

==

LineFlow =

1.0000	1.0000	2.0000	0.0498	0.0578	0.1936
2.0000	2.0000	3.0000	0.1456	0.0561	2.4259
3.0000	3.0000	4.0000	0.2478	8.6596	13.9821
4.0000	4.0000	5.0000	0.4313	24.5628	21.3975
5.0000	5.0000	6.0000	0.6420	154.5320	134.6223
6.0000	6.0000	7.0000	0.5697	55.5445	48.3844
7.0000	7.0000	8.0000	0.3799	56.0771	31.7603
8.0000	8.0000	9.0000	0.2677	64.9770	36.8022
9.0000	9.0000	10.0000	0.1686	28.7298	16.2720

=====================================

========================

Minimum voltage (p.u.): 0.97963 @ bus 10
Maximum voltage (p.u.) excluding the slack bus: 1.03928 @ bus 7
Total active load demand (KW): 12368.000
Total reactive load demand (KVAR): 4186.000
Total active loss (KW): 393.197
Total reactive loss (KVAR): 305.840

F_2 represents the improving of the voltage profile, which is satisfied by reducing the summation of voltage deviations in RDS and it can be given as follows:

$$F_2 = \frac{\text{VD}}{(\text{VD})_{\text{base}}} =$$

0.0393 − 0.0306i
−0.1447 + 0.0155i
−0.2428 + 0.0493i
−0.4213 + 0.0926i
−0.5811 + 0.2731i
0.4502 + 0.3491i
0.3774–0.0435i
0.2651–0.0376i
0.1670–0.0235i

$$\text{VDD} = 0.1435$$

where F_3 represents the voltage stability enhancement which can be achieved by improvement of voltage stability index (*VSI*) as follows:

$$F_3 = \frac{1}{\sum_{i=1}^{\text{nb}} |\text{VSI}(i)\ |_{\text{base}}} =$$

bus_stability_index =

2.0000	0.9987
3.0000	0.9995
4.0000	1.0177
5.0000	1.0449
6.0000	1.1757
7.0000	1.1666
8.0000	1.0973
9.0000	0.9890
10.0000	0.9197

VSI_min =

0.9197

Sum_VSI =

9.4092

Example 3.4
Find the best optimal value of the objective function by ALO for the 12-bus distribution shown in Figs. 3.5 and 3.6 system with PV and DSTATCOM and obtain optimal sizing and location. The base voltage and power are 11 kV and 100 MVA, respectively. The line data are in Ω and the bus data are in kW and kVAR.

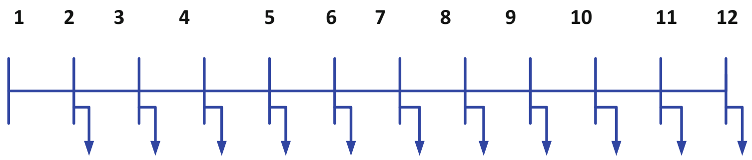

Fig. 3.5 Single line diagram of the 12-bus radial distribution system

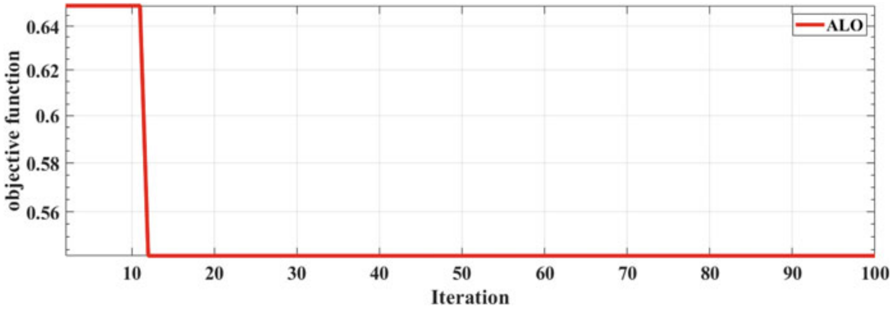

Fig. 3.6 The convergence characteristic of the ALO objective function

Solution The calculation of the objective function and obtainment of optimal sizing and location depend on the following values:

Plossb = 20.7138
VSIb = 9.4954
VDb = 0.4020
$w1 = 0.5$
$w2 = 0.25$
$w3 = 0.25$

Objective Function
The objective function is formulated as follows:

$$F = H_1F_1 + H_2F_2 + H_3F_3$$

where H_1, H_2, and H_3 are weighting factors. Summation of the weight factors assigned to all impacts must add up to one as:

$$|H_1| + |H_2| + |H_3| = 1$$

F_1 is the first objective of the multi objective function which represents total active power losses reduction, and it can be found as follows:

$$F_1 = \frac{P_{\mathrm{T,loss}}}{(P_{\mathrm{T,loss}})_{\mathrm{base}}} = 20.7138$$

F_2 represents the improving of the voltage profile, which is satisfied by reducing the summation of voltage deviations in RDS and it can be given as follows:

$$F_2 = \frac{\mathrm{VD}}{(\mathrm{VD})_{\mathrm{base}}} = 0.4020$$

where F_3 represents the voltage stability enhancement which can be achieved by improvement of voltage stability index (*VSI*) as follows:

$$F_3 = \frac{1}{\sum_{i=1}^{\mathrm{nb}} |\mathrm{VSI}(i)\ |_{\mathrm{base}}} = 9.4954$$

The best optimal value of the objective function found by ALO is: 0.54249

=======================================

Vbus =

1.0000 + 0.0000i
0.9999–0.0000i
1.0005–0.0004i
0.9922 + 0.0027i
0.9815 + 0.0064i
0.9783 + 0.0076i
0.9755 + 0.0086i
0.9671 + 0.0126i
0.9592 + 0.0163i
0.9563 + 0.0175i
0.9555 + 0.0179i
0.9552 + 0.0180i

IB =

0.0001 − 0.0001i
−0.0005 + 0.0005i
0.0035–0.0032i
0.0029–0.0027i
0.0026–0.0023i
0.0024–0.0022i
0.0019–0.0016i

```
 0.0014–0.0012i
 0.0010–0.0008i
 0.0006–0.0005i
 0.0002–0.0002i

   VDD =

0.2790

   VDD2 =

0.0101

   V_mag =

1.000000000000000
0.999851179490182
1.000522169547643
0.992174014649502
0.981552809884612
0.978307002323343
0.975554937949232
0.967221279945066
0.959289963540436
0.956509763408107
0.955622909823004
0.95541672750196

   MinVoltage =

0.9554

   MaxVoltage =

1

   bus_stability_index =

 2.0000   0.9994
 3.0000   1.0021
 4.0000   0.9689
 5.0000   0.9279
 6.0000   0.9160
 7.0000   0.9057
 8.0000   0.8748
 9.0000   0.8465
```

```
10.0000    0.8370
11.0000    0.8339
12.0000    0.8332

    VSI_min =

0.8332

    Sum_VSI =

9.9454

    Q_Loss_sensetivity_index =

 3.0000   -0.0011
 1.0000    0
 2.0000    0.0001
12.0000    0.0003
11.0000    0.0012
 7.0000    0.0037
10.0000    0.0038
 6.0000    0.0043
 4.0000    0.0112
 9.0000    0.0114
 8.0000    0.0123
 5.0000    0.0142

    Voltage_Loss_sensetivity_index =

 5.000000000000000   -0.000083889212794
 4.000000000000000   -0.000078289523576
 8.000000000000000   -0.000045921717473
 9.000000000000000   -0.000031972916046
 6.000000000000000   -0.000022897234630
 7.000000000000000   -0.000018119035210
10.000000000000000   -0.000007500821015
11.000000000000000   -0.000001448307598
 3.000000000000000   -0.000000993839255
12.000000000000000   -0.000000105584233
2.000000000000000   -0.000000041850221
1.000000000000000    0
```

```
=========================================
bus no.   mag_V   angle_V   Pg_KW   Qg_KVAR
=========================================
BusOut =

 1.0000   1.0000    0    0    0
 2.0000   0.9999   -0.0003    0    0
 3.0000   1.0005   -0.0202   435.0000   405.0000
 4.0000   0.9922   0.1545    0    0
 5.0000   0.9816   0.3757    0    0
 6.0000   0.9783   0.4432    0    0
 7.0000   0.9756   0.5023    0    0
 8.0000   0.9672   0.7491    0    0
 9.0000   0.9593   0.9743    0    0
10.0000   0.9565   1.0482    0    0
11.0000   0.9556   1.0713    0    0
12.0000   0.9554   1.0782    0    0

=========================================
Line no.   from to mag_I(p.u.) Ploss(KW) Qloss(KVAR)
=========================================
LineFlow =

 1.0000    1.0000    2.0000   0.0002   0.0021    0.0009
 2.0000    2.0000 3.0000   0.0007   0.0497    0.0207
 3.0000    3.0000    4.0000   0.0047   3.8838    1.6184
 4.0000    4.0000    5.0000 0.0040   4.1171    1.7163
 5.0000    5.0000    6.0000   0.0035   1.1200    0.4663
 6.0000    6.0000    7.0000   0.0033   0.8838    0.3678
 7.0000    7.0000    8.0000   0.0025   2.2208    0.6128
 8.0000    8.0000    9.0000   0.0018   1.5336    0.4341
 9.0000    9.0000   10.0000   0.0012   0.3587    0.1015
10.0000   10.0000   11.0000   0.0007   0.0692    0.0196
11.0000   11.0000   12.0000   0.0002   0.0050    0.0014

=========================================
Minimum voltage (p.u.): 0.95542 @ bus 12
Maximum voltage (p.u.) excluding the slack bus: 1.00000 @ bus 1
Total active load demand (KW): 435.000
Total reactive load demand (KVAR): 405.000
Total Active Loss (KW): 14.244
```

Total Reactive Loss (KVAR): 5.360
Total energy losses cost ($): 7486.611

===

$o =$

0.5425

Location_PV1 =

3

Location_PV2 =

3

Location_DSTATCOM1 =

3

Location_DSTATCOM2 =

3

Size_PV1 =

435

Size_PV2 =

435

Size_DSTATCOM1 =

405

Size_DSTATCOM2

405

Example 3.5
Find the second loss sensitivity index (LSI2) as in for the 10-bus distribution system. The base voltage and power are 23 kV and 100 MVA, respectively. The line data are in Ω and the bus data are in kW and kVAR (Fig. 3.7).

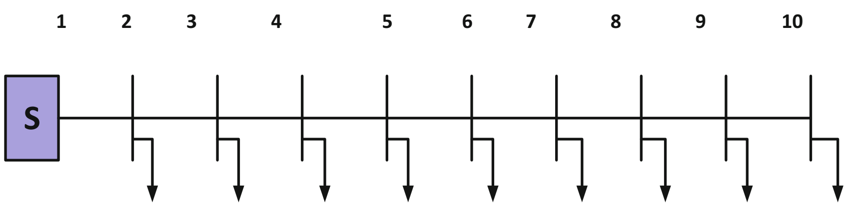

Fig. 3.7 Single line diagram of the 10-bus radial distribution system

Solution The calculation of LSI2 depends on the values of voltage magnitudes at load buses, total effective reactive power at each load bus, and the resistance of system.

$$V_{\text{mag}} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \\ V_9 \\ V_{10} \end{bmatrix} = \begin{bmatrix} 1.0000 \\ 0.9929 \\ 0.9874 \\ 0.9634 \\ 0.9480 \\ 0.9172 \\ 0.9072 \\ 0.8890 \\ 0.8587 \\ 0.8375 \end{bmatrix} \text{p.u.}$$

The total effective active power at each load bus in per unit can be determined as:

$$P_{\text{eff}} = \left[P_{\frac{\text{eff}}{2}} \, P_{\frac{\text{eff}}{3}} P_{\frac{\text{eff}}{4}} P_{\frac{\text{eff}}{5}} P_{\frac{\text{eff}}{6}} P_{\frac{\text{eff}}{7}} P_{\frac{\text{eff}}{8}} P_{\frac{\text{eff}}{9}} P_{\frac{\text{eff}}{10}} \right]$$

$$= \begin{bmatrix} P_2 + P_3 + P_4 + P_5 + P_6 + P_7 + P_8 + P_9 + P_{10} \\ P_3 + P_4 + P_5 + P_6 + P_7 + P_8 + P_9 + P_{10} \\ P_4 + P_5 + P_6 + P_7 + P_8 + P_9 + P_{10} \\ P_5 + P_6 + P_7 + P_8 + P_9 + P_{10} \\ P_6 + P_7 + P_8 + P_9 + P_{10} \\ P_7 + P_8 + P_9 + P_{10} \\ P_8 + P_9 + P_{10} \\ P_9 + P_{10} \\ P_{10} \end{bmatrix} =$$

$$= \left[\begin{matrix} 1840 + 980 + 1790 + 1598 + 1610 + 780 + 1150 + 980 + 1640 \\ 980 + 1790 + 1598 + 1610 + 780 + 1150 + 980 + 1640 \\ 1790 + 1598 + 1610 + 780 + 1150 + 980 + 1640 \\ 1598 + 1610 + 780 + 1150 + 980 + 1640 \\ 1610 + 780 + 1150 + 980 + 1640 \\ 780 + 1150 + 980 + 1640 \\ 1150 + 980 + 1640 \\ 980 + 1640 \\ 1640 \end{matrix} \right. \qquad /10^5$$

$$= \begin{bmatrix} 0.12368 \\ 0.10528 \\ 0.09548 \\ 0.07758 \\ 0.06160 \\ 0.04550 \\ 0.03770 \\ 0.02620 \\ 0.01640 \end{bmatrix} \text{p.u}$$

Similarly, the total effective reactive power at each load bus in per unit can be obtained as:

$$Q_{\text{eff}} = \begin{bmatrix} Q_{\text{eff}/2} \\ Q_{\text{eff}/3} \\ Q_{\text{eff}/4} \\ Q_{\text{eff}/5} \\ Q_{\text{eff}/6} \\ Q_{\text{eff}/7} \\ Q_{\text{eff}/8} \\ Q_{\text{eff}/9} \\ Q_{\text{eff}/10} \end{bmatrix}$$

$$= \begin{bmatrix} Q_2 + Q_3 + Q_4 + Q_5 + Q_6 + Q_7 + Q_8 + Q_9 + Q_{10} \\ Q_3 + Q_4 + Q_5 + Q_6 + Q_7 + Q_8 + Q_9 + Q_{10} \\ Q_4 + Q_5 + Q_6 + Q_7 + Q_8 + Q_9 + Q_{10} \\ Q_5 + Q_6 + Q_7 + Q_8 + Q_9 + Q_{10} \\ Q_6 + Q_7 + Q_8 + Q_9 + Q_{10} \\ Q_7 + Q_8 + Q_9 + Q_{10} \\ Q_8 + Q_9 + Q_{10} \\ Q_9 + Q_{10} \\ Q_{10} \end{bmatrix} / (1000 \times \text{MVA}b)$$

$$= \begin{bmatrix} 0.04186 \\ 0.03726 \\ 0.03386 \\ 0.02940 \\ 0.01100 \\ 0.00500 \\ 0.00390 \\ 0.00330 \\ 0.00200 \end{bmatrix} \text{p.u}$$

The resistances of system lines in per unit can be calculated as:

$$R = \begin{bmatrix} R_{1-2} \\ R_{2-3} \\ R_{3-4} \\ R_{4-5} \\ R_{5-6} \\ R_{6-7} \\ R_{7-8} \\ R_{8-9} \\ R_{9-10} \end{bmatrix} \times \frac{\mathrm{MV}Ab}{\mathrm{kV}b^2} = \begin{bmatrix} 0.123 \\ 0.014 \\ 0.746 \\ 0.698 \\ 1.983 \\ 0.905 \\ 2.055 \\ 4.795 \\ 5.343 \end{bmatrix} \times \frac{100}{23^2} \mathrm{p.u}$$

$$= \begin{bmatrix} 0.0233 \\ 0.0026 \\ 0.1411 \\ 0.1320 \\ 0.3749 \\ 0.1711 \\ 0.3885 \\ 0.9065 \\ 1.0101 \end{bmatrix} \mathrm{p.u}$$

The calculation of LSI2 depends on the values of voltage magnitudes at load buses, total effective reactive power at each load bus and the resistance of system lines. Therefore, the LSI2 at bus 2 can be calculated as:

$$\begin{aligned} \mathrm{LSI}_1^{\mathrm{Bus}}2 &= \frac{\partial P_{\mathrm{Loss}/1-2}}{\partial Q_{\mathrm{eff}/2}} \\ &= \frac{2 \times Q_{\mathrm{eff}/2 \times R_{1-2}}}{V_2^2} \\ &= \frac{2 \times 0.04186 \times 0.0233}{0.9929^2} \\ &= 0.001979 \text{ p.u.} \end{aligned}$$