Chapter 4
Development of a Type-2 Fuzzy Bi-level Programming Model Coupling MCDA Analysis for Water Resources Optimization Under Uncertainty

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Abstract In view of the contradiction of water use among water use departments, the optimal allocation of water resources can be effectively solved and the efficient utilization of water resources can be realized. The goal of this study is to combine the interval type-2 fuzzy algorithm with bi-level programming (BP) method, propose a type-2 fuzzy bi-level model (T2FBL), and apply it to the water resources planning system in the southern Fujian Province. The main innovations and contributions of this study is that a new type-two fuzzy bilayer model (T2FBL) is proposed by combining type-two fuzzy theory with BP method and applied to water resources planning. The new model balances the needs of decision-makers (DMs) at different levels in southern Fujian Province. The upper level decision aims to maximize the overall social benefit of the region by minimizing the regional water shortage, while the lower level decision focuses on achieving the economic benefit goal by improving the net benefit of regional water supply. The MCDA method is adopted to further analyze the results of water resources allocation. In the future direction, water supply to the service industry can be strengthened. Ideally, 73.79% water supply to the tertiary industry can achieve a scenario with the maximum target benefit. T2FBL model can obtain the results of water resources allocation among different agricultural, industrial and tertiary industries, and reveal the relationship between various competitive users. These findings can provide an in-depth understanding of the interaction between agricultural, industrial and tertiary industry water allocation and provide technical support for water resource planning issues.

Keywords Type-2 fuzzy sets · Bi-level programming · MCDA · Water resources

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4.1   Introduction

With the rapid development of social economy, the contradiction between the increase of human's demand for water and the decrease of water supply is becoming more and more prominent. In view of the contradiction of water use among water use departments, the optimal allocation of water resources can be effectively solved and the efficient utilization of water resources can be realized (Zhou et al. 2017). In the 1970s, D. H. Murks proposed linear decision programming for water resources systems, and Joeres (Joeres et al. 1971) applied linear programming theory to Baltimore's multi-source water supply research. After a long period of continuous research, the optimal allocation method of water resources gradually tends to mature.

Generally, decision making problems are often compounded by uncertainties related to benefits/costs, water availabilities, environmental capacities and objectives (Huang et al. 1993; Li et al. 2008; Yeomans 2008). Many methods have been proposed to handle uncertainties existing in water resources management systems (Zhang et al. 2017; Zhang and Guo 2016). To reflect the tradeoff between two decision-making levels with different concerns, the bi-level programing (BP) approach with a two-level structure was developed for supporting regional water policies formulation (Camacho-Vallejo et al. 2015). A leader-follower decision-making strategy was incorporated into the optimization process of BP to generate satisfied decision-making plan for both the upper-level and the lower-level decision makers. When dealing with a system problem, different decision-makers (DMs) generally have different perspectives and starting points, which leads to conflicts in the design of ideal solutions for certain variables. Although the BP algorithm can effectively balance the needs of decision-makers at different levels, it has limitations in representing uncertain parameters in the planning system. Therefore, different types of fuzzy numbers, interval numbers and BP linear programming can be combined to address the uncertainty in the system, and a suitable algorithm can be used to convert the fuzzy number and interval number into clear values and obtain an effective optimization plan (Ma et al. 2016).

Zadeh (Chen 2013) proposed type-2 fuzzy sets (T2FSs) whose membership grades themselves are type-1 fuzzy sets (T1FSs). As an extension of T1FSs, T2FSs are characterized by primary and secondary membership functions, which can provide greater freedom and flexibility for DMs to better express their uncertain judgments (Chen and Lee 2010). Although T2FS owns a wide range of advantages, it is usually accompanied by high computational difficulty, which results in its low practicality. Currently, trapezoidal interval type-2 fuzzy set (TrIT2FS) (Srinivasan and Geetharamani 2016), as a special form of T2FS, has been extensively applied to fuzzy decision making problems. Hence, this paper employs TrIT2FSs to represent the pairwise comparisons (PCs) of objects and the evaluations of alternatives on criteria.

Therefore, the goal of this study is to combine the interval type-2 fuzzy algorithm with BP method, propose a T2FBL model, and apply it to the water resources planning system in the southern Fujian Province, and analyze and suggest the water resources situation in the southern Fujian Province under the application analysis of MCDA

(Multiple Criteria Decision Analysis). The main innovations and contributions of this study are as follows: (1) a new T2FBL model is proposed by combining type-two fuzzy theory with BP method and applied to water resources planning. (2) The new model balances the needs of decision-makers at different levels in southern Fujian Province. The upper level decision-makers aim to maximize the overall social benefit of the region by minimizing the regional water shortage, while the lower level decision-makers focus on achieving the economic benefit goal by improving the net benefit of regional water supply. (3) By collecting the regional water resources data of the Minnan region of Fujian Province from 2016 to 2020, the water resources utilization situation of the three locations was analyzed. (4) T2FBL model can obtain the results of water resources allocation among different agricultural, industrial and tertiary industries, and reveal the relationship between various competitive users. (5) The MCDA method was used to further analyze the water resources allocation results.

4.2 Methodology

4.2.1 Bi-level Programming Model

Bi-level programming is a mathematical optimization model with a two-level hierarchical structure. The upper and lower level problems have their own objective functions and constraints. The objective function and constraints of the upper-level problem are not only related to the upper-level decision variables, but also depend on the optimal solution of the lower-level problem, which in turn is affected by the upper-level decision variables. It can simultaneously consider the interests of both the global and the individual, ensure that the global is first and the local is second, and the final decision result is often a mutually coordinated scheme between all layers, which can well solve complex practical problems (Lv et al. 2009). The general form of bi-level programming model is as follows:

$$
\begin{cases}
\min f_u(x, y), \\
\min f_l(x, y), \\
\end{cases}
$$
$$
s.t.\ A(x, y) \le B, i = 1, 2, \ldots, m \tag{4.1}
$$

x is the upper decision variable. y is the lower level decision variable; $f_u(x, y)$ is the upper objective function of the model. $f_l(x, y)$ is the lower objective function of the model. $A(x, y)$ is the constraint condition.

IT2FN is introduced into the bi-level programming model, and the uncertainty parameters in the model are represented by interval type-2 fuzzy numbers. The type-2 fuzzy bilevel model can be expressed as follows:

$$\begin{cases} \min f_u(x, y), \\ \min f_l(x, y) \end{cases}$$
$$s.t.\ \tilde{A}(x, y) \le \tilde{B}, i = 1, 2, \ldots, m \tag{4.2}$$

where $\tilde{A}$ and $\tilde{B}$ are the IT2FNs and $\tilde{\le}$ represents the fuzzy sorting relationship between the left and right sides of the equation.

4.2.2 Type 2 Fuzzy Sets

Let X be the universe of discourse. A type-2 fuzzy sets (T2FSs) $\tilde{A}$ can be represented by the type-2 membership function $\mu_{\tilde{A}}(x, u)$ such that:

$$\tilde{A} = \{(x, u), \mu_{\tilde{A}}(x, u) | x \in X, u \in [0, 1]\} \tag{4.3}$$

The IT2FSs are usually taken in some simplified formations in applications (Aviso et al. 2010). As shown in Fig. 4.1. Let $\tilde{A} = [A^L, A^U]$ be IT2FS on X. Let $\tilde{A}^L = (a_2^L, a_3^L, \alpha_L, \beta_L)$ and $\tilde{A}^U = (a_2^U, a_3^U, \alpha_U, \beta_U)$ be the lower and upper trapezoidal fuzzy number, respectively, with respect to $\tilde{A}$ defined on the universe of discourse X, where $a_2^L \le a_3^L \cdot a_2^U \le a_3^U \cdot \alpha_L, \alpha_U \ge 0$, and $\beta_L, \beta_U \ge 0$, $[a_2^L, a_3^L]$ is the core of $\tilde{A}^L$, and $\alpha_L, \beta_L \ge 0$ are the left-hand and right-hand spreads and $[a_2^U, a_3^U]$ is the core of $\tilde{A}^U$, and $\alpha_U, \beta_U \ge 0$ are the lefthand and right-hand spreads. The membership functions of x in $\tilde{A}^L$ and $\tilde{A}^U$ are expressed as follows:

$$\underline{u}_{\tilde{A}}(x) = \begin{cases} \frac{(x - a_2^L + \alpha_L)}{\alpha_L}, & a_2^L - \alpha_L \le x \le a_2^L, \\ 1, & a_2^L \le x \le a_3^L, \\ -\frac{(x - a_3^L - \beta_L)}{\beta_L}, & a_3^L \le x \le a_3^L + \beta_L, \\ 0, & otherwise, \end{cases} \tag{4.4}$$

$$\overline{u}_{\tilde{A}}(x) = \begin{cases} \frac{(x - a_2^U + \alpha_U)}{\alpha_U}, & a_2^U - \alpha_U \le x \le a_2^U, \\ 1, & a_2^U \le x \le a_3^U, \\ -\frac{(x - a_3^U - \beta_U)}{\beta_U}, & a_3^U \le x \le a_3^U + \beta_U, \\ 0, & otherwise, \end{cases} \tag{4.5}$$

Fig. 4.1 The upper membership function $\tilde{A}^U$ and the lower membership function $\tilde{A}^L$ of IT2FS $\tilde{\tilde{A}}$
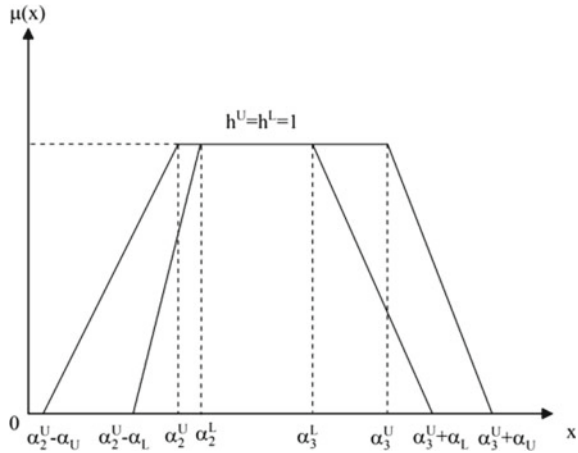

4.2.3 Multiple Criteria Decision Analysis (MCDA)

A Multi Criteria Decision Analysis (MCDA), is a decision-making analysis that evaluates multiple (conflicting) criteria as part of the decision-making process. MCDA resembles a cost–benefit analysis, but with the notable advantage of not being solely limited to monetary units for its comparisons. When making comprehensive or important decisions, multiple criteria and levels of scale need to be accounted for. Analytic hierarchy process (AHP) is a commonly used MCDA method. The AHP is a method that provides a choice between different alternatives (strategies, investments, etc.) based on a series of criteria or variables, which are usually in conflict. The method consists of the following steps.

The starting points are the different possible alternatives that a decision maker faces, shown in Fig. 4.2. The criteria used to make the selection are then specified. Once the alternatives are known and the criteria have been defined, the relative importance of each criterion is determined and weighed. Taking into account the criteria and their weights, the different alternatives are assessed obtaining n matrices, as many as the criteria. From each one of them, the eigenvector is calculated which will show the weight of the alternative as a function of each criterion. From the two previous processes two additional matrices are obtained. A column matrix n × 1 with the weights for the criteria (for n criteria) and another matrix m × n formed by the weights of the different alternatives taking into account all the criteria and their importance. The product of both matrices will produce a matrix that will prioritize the different alternatives taking into account all the criteria and their importance.

The result is a matrix w showing the relative importance of each alternative. This figure is the one that allows the ranking of all alternatives from least to most desirable, and quantifies the interest of each alternative with respect to the others as a function of all criteria and their importance, therefore allowing us to choose the best.

Fig. 4.2 Graphical representation of a hierarchy
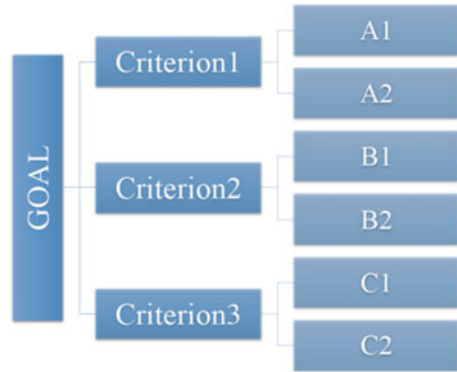
4.3 Application

4.3.1 Statement of Problem

The Southern Fujian Triangle is located on the southeast coast of Fujian Province, between the Jiulong River, the largest river system in the province, and the Jinjiang River, the third largest river system. Xiamen, Quanzhou and Zhangzhou belong to the southern Fujian region, which faces Taiwan across the sea and is the central part of the economic zone on the west coast of the Straits. It connects the Jiulong River and Jinjiang River basins together, so that Xiamen, Quanzhou and Zhangzhou occupy a unique position in the town system of southern Fujian. As shown in Fig. 4.3. Jiulong River is the second largest river in Fujian Province, second only to Minjiang River. The total area of Xia, Quan and Zhang is 25,180 km^2, accounting for 20.98% of the province. In 2020, the GDP of southern Fujian reached 2108.829 billion Yuan, accounting for 48.03% of Fujian Province, and has become the "locomotive" of Fujian Province's economic development.

However, Xiamen, Quanzhou and Zhangzhou are areas with water scarcity. With the further development of economy, how to protect water resources and rationally allocate water resources is the key to the development of this region. The optimal allocation of water resources in the water receiving area will have a positive impact on the social economy, ecological environment and sustainable utilization of water resources in the region.

4.3.2 Interval Type-2 Fuzzy Bi-level Programming Model

There are three sources of water supply in Fujian Province, which are: surface water supply, underground water supply and other water supply, and surface water supply is divided into water supply from storage projects, water supply from diversion projects

Fig. 4.3 Study area

and water supply from lifting projects. The water use in Fujian Province is mainly divided into three categories according to different uses, namely: agricultural water, industrial water and domestic water.

According to the requirements of sustainable development, the optimal allocation model of water resources should be a multi-objective model including social, economic and ecological environment. According to the characteristics of water resources system in southern Fujian, social benefits and economic benefits are selected as objective functions, and corresponding constraints are established. Three cities in southern Fujian are regarded as K sub-regions with N public water sources, I(k) independent water sources and J(k) water use departments, and the corresponding objective functions are established.

(1) Social benefit objectives. The minimum regional total water shortage is adopted to indirectly reflect social benefits:

$$\min f_1(x) = \sum_{k=1}^{K} \sum_{j=1}^{J} \left(D_j^k - \sum_{i=1}^{I} x_{ij}^k \right) \tag{4.6}$$

where D_j^k is the water demand of j users in k sub-areas; x_{ij}^k is the water supply from water source i to user j in subzone k.

(2) Economic benefit objectives. It is expressed as the largest net benefit generated by regional water supply:

$$\max f_2(x) = \sum_{k=1}^{K} \sum_{j=1}^{J} \sum_{i=1}^{I} \left(b_{ij}^k - c_{ij}^k \right) x_{ij} \tag{4.7}$$

where x_{ij} is the water supply from source i to user j; b_{ij}^k and c_{ij}^k are the unit water supply benefit coefficient and water supply consumption coefficient of water source i to user j in subarea k, respectively.

For the above objective functions, the relevant constraints are established.

(1) Constraint of water supply.

$$\sum_{j=1}^{J} x_{ij}^k \leq W_i^k \tag{4.8}$$

where W_i^k is the water supply from source I to subarea K.

(2) Water demand capacity constraints of users.

$$D_{\min}^{k'} \leq \sum_{j=1}^{J} \sum_{i=1}^{I} x_{ij}^k \leq D_{\max}^k \tag{4.9}$$

where $D_{\min}^{k'}$ and $D_{\max}^k$ are the minimum and maximum water demand of k subareas, respectively.

(3) Water transmission capacity constraint of water transmission system.

$$x_{ij}^k \leq Q_{i\,\max}^k \tag{4.10}$$

where $Q_{i\,\max}^k$ is the maximum water transfer capacity of the source to the K subareas.

Table 4.1 Input the water demand of each sub-district under different water use sectors

	Water demand from 2016 to 2020			
	Agriculture	Industry	Tertiary industry	Sum of max values
Xiamen	(1.016, 1.680)	(0.936, 1.788)	(1.608, 3.048)	6.516
Quanzhou	(6.336, 12.684)	(6.656, 18.720)	(3.392, 5.676)	37.080
Zhangzhou	(8.968, 16.104)	(2.720, 8.724)	(1.792, 2.976)	27.804

Table 4.2 Input the economic benefit values of each subdistrict under different water use sectors

	Economic benefits from 2016 to 2020			
	Agriculture	Industry	Tertiary industry	Sum
Xiamen	19.0	1584.5	1053.0	2656.5
Quanzhou	24.5	473.5	644.5	1142.5
Zhangzhou	35.5	403.0	578.5	1017

(4) Non-negative constraints on variables.

$$x_{ij}^k \geq 0 \tag{4.11}$$

As shown in Tables 4.1 and 4.2, the statistical data were entered into the model.

4.4 Results and Discussion

4.4.1 Results Analysis

Under different satisfaction levels, there are some changes in the regional water shortage and the net benefits of regional water supply. As shown in Fig. 4.4. When the satisfaction level is 0.9, the water shortage is the smallest and the total amount of available water resources is maximized, which maximizes the net benefits of water supply in southern Fujian, which is 1,951.172 billion yuan. When the satisfaction is 0.2, the total water shortage increases to 10.28146 billion m^3 and the net benefit of water supply decreases to 17,550.56 billion yuan, indicating that the value of satisfaction is inversely proportional to the risk of water shortage. This means that the higher the satisfaction level, the closer the total amount of available water is to the minimum. The lower the satisfaction level, the closer the total available water resources are to the maximum value of the statistics.

Figure 4.5 shows the overall water supply structure in southern Fujian under different planning periods and satisfaction levels. When the satisfaction level is the highest, the proportion of agricultural water use in the three cities is the lowest and the proportion of industrial water use is the highest. With the decrease of satisfaction,
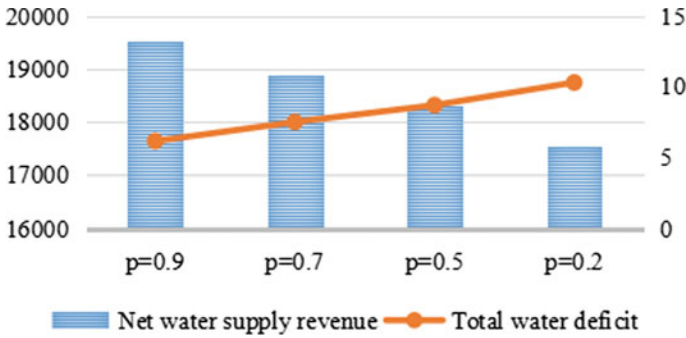
Fig. 4.4 Regional changes in water scarcity and net benefits of water supply
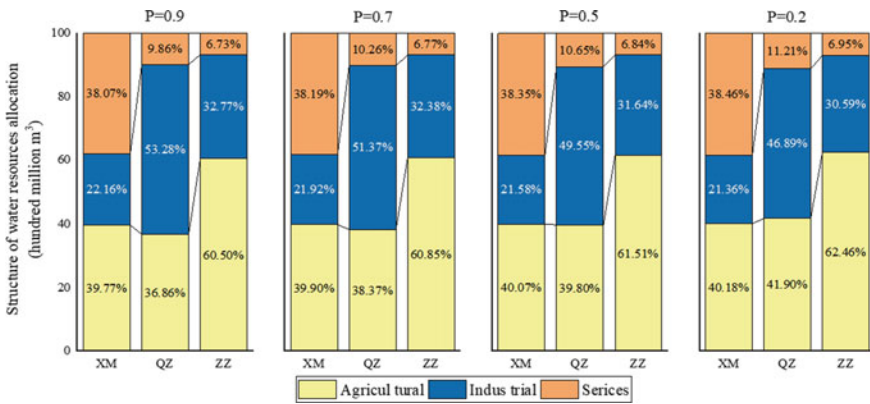

Fig. 4.5 Water distribution structure in each city in three industries at different α-cut

the proportion of agricultural water use gradually increased, while the proportion of industrial water use decreased. This means that when the water scarcity is minimum, the benefit is highest and the optimal allocation is reached, the benefit from industrial water is greater than that from agricultural water. The government departments in southern Fujian should control the proportion of industrial water consumption, focusing on adjusting the industrial structure and developing high-tech industries.

According to the observation data, when the satisfaction is 0.9, the optimal water allocation of Quanzhou is the closest to the statistical data, so it can be concluded that the water allocation structure of Quanzhou is relatively excellent. The proportions were 9.86, 53.28 and 36.86% respectively. It is the highest proportion of industrial water among the four satisfaction levels. Moreover, due to the development of industry, the economic benefits of industrial water allocation are gradually increasing, which means that the water resources needed to increase the same industrial output value as in the past are gradually decreasing.

This phenomenon also reflects the gradual development of Fujian's industrial industry in the direction of environmental protection and high technology. The tertiary industry and domestic water use generally showed a downward trend, indicating that the development of Fujian's service industry also showed an upward trend year by year. Another factor related to this phenomenon is that government departments pay attention to saving water and have made a series of policies to save water. With the increase of available water resources, the overall water resources planning structure has no obvious fluctuation, only the industrial water consumption shows an upward trend. The reason is that the economic benefit of industrial water distribution is more prominent than that of tertiary industry and agriculture. The model would prioritize allocating more water to industry to maximize economic benefits.

4.4.2 Discussion Based on MCDA

Contributions to solving complex natural resource issues can come from pooling expert opinion, using the Delphi method (Miller 1984). This technique brings together diverse expert opinion on specific, unresolved issues, with the goal of achieving agreement–as close to consensus as possible while respecting minority views. The Delphi method is adopted to value the weights.

The proportion of agricultural output value to gross domestic product is an important index to measure the development stage of agricultural industry. Generally speaking, the summary of expert data shows that the proportion of agriculture in economically developed countries and regions is less than 10%, the proportion of industrial manufacturing is 20–30%, and the proportion of service industry is more than 60%. According to the Delphi method, the weight distribution of agricultural, industrial and service standards under the objective of economic benefits is carried out, and the judgment matrix is generated, and the consistency test is carried out to obtain CR < 1, the ratio of economic benefits of the three is 9.45, 16.76 and 73.79%. This provides a development idea for the water supply structure in southern Fujian. While ensuring the water supply and economic benefits, more water supply will be used to develop the service industry, industry and construction industry, so that the city can achieve the goal of faster economic development while ensuring the quality of life of residents.

4.5 Conclusion

In this paper, a bi-level planning method based on T2FBL is proposed to optimize the water resources planning system. T2FBL method comprehensively considers the interests of decision makers at different levels, and uses two kinds of fuzzy ranking algorithms to solve the uncertainty in water resources planning. This method can not only reflect the degree of risk corresponding to uncertain parameters in the system,

but also balance the interest demand between two levels of decision makers. The new model balances the needs of decision-makers at different levels in southern Fujian Province. The upper level decision-makers aim to maximize the overall social benefit of the region by minimizing the regional water shortage, while the lower level decision-makers focus on achieving the economic benefit goal by improving the net benefit of regional water supply. The weight analysis of the data results was carried out through MCDA, and finally the following results were obtained:

(i)   Southern Fujian gives priority to adjusting the industrial water supply structure, focuses on adjusting the industrial structure, develops high-tech industries, and greatly alleviates the water pressure.
(ii)   This paper adopts T2FBL to calculate the uncertain parameters in the system. As the α-cut decreases and the total amount of water available to the system increases, the concomitant water shortage risk also increases. In this study, water resources allocation schemes under different satisfaction levels in three cities were calculated.
(iii) This paper adopts MCDA to conduct weighted analysis on the data results, and concludes that the future direction of developing into a developed city can strengthen the water supply to the service industry, and 73.79% water supply to the tertiary industry can achieve a scenario with the maximum target benefit.

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