Chapter 16
Pre-service Teachers' Self-Efficacy for Teaching Mathematical Modelling

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Abstract We focus on the professionalisation of pre-service teachers through reflective practice when they train for mathematical modelling. To do so, we consider their self-efficacy beliefs as an important aspect of professional competence for teaching mathematical modelling. A pre-post design was used to examine the extent to which self-efficacy of mathematics pre-service teachers for mathematical modelling can be increased through a variety of different teaching–learning laboratories. Clearer effects could be seen when the pre-service teachers themselves created modelling tasks for use with grade nine students.

Keywords Mathematical modelling · Professional competence · Self-efficacy · Pre-service teacher · Teacher training · Teaching–learning laboratories

16.1 Introduction

Self-efficacy expectations represent an empirically founded characteristic of professional competence (Kunter, 2013). The term self-efficacy expectancy is understood as an evaluation of one's own effectiveness in certain situations. Tschannen-Moran and Woolfolk Hoy (2001, p. 783) characterise this as follows: "A teacher's efficacy belief is a judgement of his or her capabilities to bring about desired outcomes of

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student engagement and learning, even among those students who may be difficult or unmotivated".

Self-efficacy expectations can be concretised in terms of teachers' beliefs about their own efficacy in teaching mathematical modelling processes. Activities occurring in such processes are determined in terms of content by the facets of modelling-specific subject didactic knowledge. One of the main activities of the teacher during cooperative modelling processes is the diagnosis of the solution process. Since the diagnostic component has connections to both the intervention- and task-related knowledge facets, self-efficacy expectations are operationalised via the assessment of one's own ability to diagnose learners' performance potential in the modelling process (Wess et al., 2021a).

Learners' modelling process is characterised by different activities and cognitive processes in the different phases of the modelling process. Therefore, different diagnostic processes by the teachers are necessary in the different modelling phases the learners are currently in. This justifies the assumption that the teacher's self-efficacy also differs depending on the modelling phase. With regard to the learners' activities and the associated diagnostics, phases that are unspecific to the modelling process and in which the activities can be comprehended on the basis of written materials (mathematical work) can be distinguished from phases that are specific to the modelling process and in which cognitive processes predominate (simplifying/structuring; mathematising; interpreting; validating). The self-efficacy expectations for mathematical modelling are therefore conceptualised for the diagnosis of performance potentials for the learners' activities in the modelling process.

16.2 Theoretical Background

16.2.1 Modelling Competence

In recent years, numerous ideas about mathematical modelling and its associated translation processes have emerged in the mathematics education discussion about teaching close to reality.

The entire modelling process is often idealised as a modelling cycle. The literature therefore contains various modelling cycles. Blum and Leiß (2007) created such a modelling cycle from a cognitive perspective (see Fig. 16.1). For this purpose, a modelling cycle previously created by Blum (Blum & Kirsch, 1989) and further developed by different researchers was extended by the situation model. The situation model describes the mental representation of the situation by the individual. The creation of a mathematical model was addressed in detail, and the process of the individual creating the model was set out in greater detail.

This modelling cycle (Fig. 16.1) describes the various sub-processes of modelling more accurately and in greater detail than many other modelling cycles. Therefore,

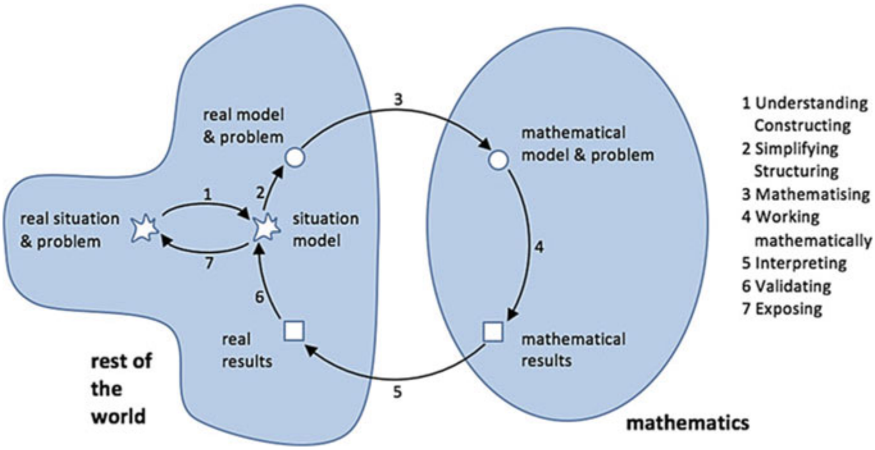

Fig. 16.1 Modelling cycle according to Blum and Leiß (2007, p. 221)

we use this cycle for our further consideration. The ability to perform such a sub-process can be seen as a special competence of modelling (Kaiser, 2007; Maaß, 2006). Students should be able to translate between reality and mathematics in both directions and work within the mathematical model. Niss et al. (2007) defined modelling competence as follows:

> Mathematical modelling competency means the ability to identify relevant questions, variables, relations or assumptions in a given real world situation, to translate these into mathematics and to interpret and validate the solution of the resulting mathematical problem in relation to the given situation, as well as the ability to analyse or compare given models by investigating the assumptions being made, checking properties and scope of a given model, etc. (Niss et al., 2007, p. 12)

Promoting the ability to process real-world problems with mathematical tools is therefore a central goal of modelling in school.

The definition describes the so-called global modelling competence by which specific sub-processes can be identified by means of an atomistic perspective. Thus, Blum (2015) understood modelling competence as the ability to construct, use or adapt mathematical models by carrying out process steps adequately and appropriate to the problem, as well as analysing or comparing given models. Modelling competence is therefore not a one-dimensional construct but one that can be interpreted as a combination of different sub-competencies. These sub-competencies could be characterised as presented in Table 16.1. By means of detailed descriptions, the definition of sub-competencies becomes obvious. Thus, an extensive list of modelling competencies can be obtained. Working mathematically has been included in the list of sub-competencies for the sake of completeness. However, it should be remembered that mathematical work is not as typical for modelling processes as, for example,

Table 16.1 Sub-competencies involved in modelling

Sub-competency	Description
Constructing	Students construct their own mental model from a given problem and thus formulate an understanding of the problem
Simplifying	Students identify relevant and irrelevant information from a real problem
Mathematising	Students translate specific, simplified real situations into mathematical models (e.g. terms, equations, figures, diagrams, functions)
Working mathematically	Students work with mathematical methods in the mathematical model and get mathematical solutions
Interpreting	Students relate results obtained from manipulation within the model to the real situation and thus obtain real results
Validating	Students judge the real results obtained in terms of plausibility
Exposing	Students relate the results obtained in the situational model to the real situation, and thus obtain an answer to the problem

Source Greefrath et al. (2013) and Greefrath and Vorhölter (2016)

mathematising or validating. By using different modelling cycles, other competencies emphasising other aspects of modelling could occur (Greefrath & Vorhölter, 2016).

In addition, metacognitive competences are necessary for the appropriate performance of modelling processes (Stillman, 2011). Lack of metacognition, such as controlling the solution process (Kaiser, 2007) or reflecting its appropriateness (Blomhøj & Jensen, 2003), can lead to problems in the modelling process.

The question of how modelling processes can be designed is closely related to perspectives on mathematical modelling as well as the goals pursued with the integration of mathematical modelling into mathematics education by using modelling tasks.

For teacher training in modelling, modelling tasks play an important role. Looking back at the modelling-specific task categories, it can be seen, according to Maaß (2010), that the nature of the relationship with reality—more precisely the context of the situation, its authenticity, and its relevance for students—seems to be very important for an adequate analysis of reality-related tasks. At the interface of the special and general task criteria, the dimension of the cognitive elements of the modelling cycle—in particular, the partial steps of modelling—is highlighted as a characteristic examination feature.

Modelling tasks include an authentic context (Maaß, 2010; Siller & Greefrath, 2020). Realistic contexts, which should be relevant to learners' present or future life, enable learners to use their everyday knowledge to find a solution. Furthermore, modelling tasks can stimulate various activities when they are being solved. The more sub-competencies (Kaiser, 2007) are addressed, and the more clearly this is done, the greater the opportunity for students to find their own solutions. Hence we can summarise various criteria for modelling tasks (Greefrath et al., 2017; Wess &

Greefrath, 2019). The first of these is openness. The problem allows for different solutions and approaches at different levels. The openness of a task, in the sense of multiple approaches and solutions (Schukajlow et al., 2015), is an essential feature of modelling tasks. The second is authenticity. This is the question of whether the context is really related to an actual situation and if the task is authentic with regard to the application of mathematics in a concrete situation. The third criterion, relevance, is about the question of whether the context is relevant to the students themselves. The task is then seen by the students as interesting, closely related to their everyday life or relevant to it. Fourthly, it is desirable that as many sub-competencies of mathematical modelling as possible are taken into account. The problem then promotes cognitive elements in the form of sub-competencies of mathematical modelling.

16.2.2 Professional Competence

Professional competence is a much discussed topic (Cochran-Smith & Fries, 2001; Darling-Hammond & Bransford, 2005) and has been measured globally in various large-scale studies (Blömeke et al., 2014; Kunter et al., 2013). The dimensions for the subject of mathematics range from knowledge of mathematical content to pedagogical knowledge and affective aspects of teachers with the aim of bringing them together.

The professional competence of a teacher is to be understood within this concept of competence, which is based on different professional requirements, since motivational, volitional and social aspects play a role, in addition to cognitive performance dispositions (Weinert, 2001).

Professional competence is a concept used to describe the skills teachers need to meet their professional requirements. Several aspects are emphasised, including a commitment to service to others, as in a "calling", and an understanding of a scientific or theoretical nature. It also emphasises the exercise of judgement under conditions of unavoidable uncertainty. Thus, the need to learn from experience also arises when theory and practice interact (Shulman, 1998). Building on Shulman (1986, 1987), a distinction in the aspect of a teacher's professional knowledge is made between content knowledge, pedagogical content knowledge, curricular knowledge, and pedagogical-psychological knowledge.

Teachers' perspectives, however, are not assigned to professional knowledge in the currently discussed conceptualisations, but to certain constructs, beliefs, attitudes, or values (Baumert & Kunter, 2013). Pre-service teachers acquire a basic scientific knowledge in their own subject. They serve society in their respective field of education through their activity and have a significant influence on the individuals they educate. They see themselves as lifelong learners and work professionally with colleagues to ensure the quality of school education. According to these characteristics, teaching can be clearly described as a profession, and professional competence can be seen, in terms of the concept of competence, as a combination of specific declarative and procedural knowledge, professional values, beliefs and goals, as well as motivational orientations and professional self-regulation skills (Baumert & Kunter, 2013).

Specific competences in this way have been described differently in various conceptualisations. In principle, these models have the goal of covering the central areas of teachers' competence.

In the context of the professionalisation of mathematics pre-service teachers, the question of the existence and structure of specific professional competence is also raised to verify skill gains in specific areas. Due to the numerous requirements in the care of cooperative modelling processes and "the strong implantation of real-world problem solving [...] into the curricula" (Schwarz et al., 2008, p. 788), it makes sense to differentiate professional competence in the field of mathematical modelling (Borromeo Ferri & Blum, 2010). In this way, the conceptualisation of a structural model for teaching mathematical modelling will be presented. A structural model describing and relating professional competence for teaching mathematical modelling has been developed and empirically confirmed. With regard to professional knowledge, an interpretation of the facets of the pedagogical content knowledge can be made taking into account Borromeo Ferri and Blum's (2010) competence dimensions. Thus, a description of pre-service teachers' modelling-specific professional competence can be achieved with the help of a structural model and associated empirical validation (Wess et al., 2021b). Regarding the necessary professional competences for the teaching of mathematical modelling (cf. Fig. 16.2), in addition to beliefs/values/goals and motivational orientations, pedagogical content knowledge, as a part of professional knowledge, is characterised, in particular by modelling-specific content. In contrast, self-regulatory skills tend not to contain any modelling-specific aspects and are therefore not considered more closely.

It has already been shown that the pedagogical content knowledge of mathematical modelling as part of the professional competence of pre-service teachers can be promoted through appropriate university seminars. The results of the study show that certain aspects (namely, knowledge of modelling tasks, modelling processes and interventions) have significantly increased (Greefrathet al., 2022).

Teachers' professional competence is composed of cognitive (professional knowledge) and affective (beliefs and motivational orientations) components. In the COACTIV study, teachers' self-efficacy was assigned to motivational orientations and described according to the concept of general self-efficacy (Bandura, 1997), as "a judgement of his or her capabilities to bring about desired outcomes of student engagement and learning, even among those students who may be difficult or unmotivated" (Tschannen-Moran & Woolfolk Hoy, 2001, p. 783).

Accordingly, self-efficacy is considered an empirically founded feature of professional competence (Kunter, 2013) that relates to specific domains. It is thus suitable for understanding perceptions of teachers' own individual capabilities for teaching mathematical modelling. In particular, performance, convictions, and the motivation of trainees are influenced through their self-efficacy (Philippou & Pantziara, 2015). It is thus pivotal for the actions of teachers and goes hand in hand with higher teaching quality, the use of more innovative and effective methods in class, and a higher level of commitment from the teachers (Kunter, 2013).

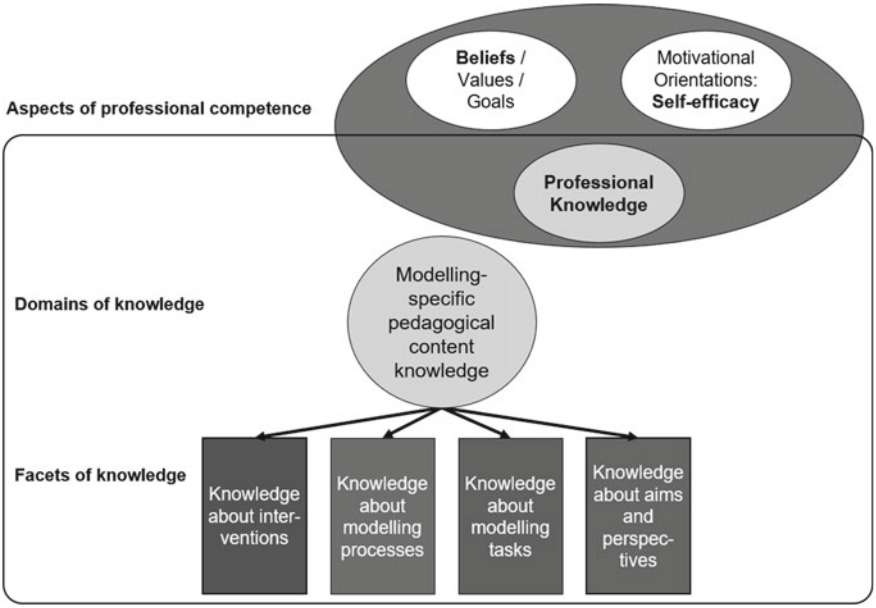

Fig. 16.2 Structural model of professional competence for the teaching of mathematical modelling

It is generally assumed that the different components of professional competence are interrelated and have an impact on teaching practice. It has been shown that teachers' self-efficacy expectancy significantly predicts reported teaching practices (Depaepe & König, 2018). Such self-efficacy varies depending on topic and context and therefore needs to be defined in an appropriately adapted manner (Yoon et al., 2014). For this reason, only limited use can be made of existing instruments (Stohlmann & Yang, 2021).

As already mentioned, knowledge about modelling processes from a theoretical perspective as a diagnostic component of modelling-specific pedagogical content knowledge has a strong influence on students' learning processes (Brunner et al., 2013). Accordingly, it forms a decisive facet of competence for teaching mathematical modelling. For this reason, our structural model operationalises self-efficacy by assessing pre-service teachers' own ability to diagnose the performance potential of learners in the modelling process. We assume that the diagnostic requirements for teachers differ depending on the modelling phase in which their learners work. Thus, the self-efficacy of (pre-service) teachers can also be differentiated according to the phase. Furthermore, scaling analyses indicate that a distinction can be made between phases specific to the modelling process (simplifying, mathematising, interpreting, validating) and unspecific ones (working mathematically) (Wess et al., 2021b).

16.2.3 Test Instrument for Self-Efficacy

A test instrument for pre-service teachers has been developed and evaluated based on the theoretical model mentioned above (Wess et al., 2021a). Wess et al. (2021b) confirmed the construct validity of the whole test instrument with the help of a structural equation analysis. Further they checked the one-dimensionality of the scales of the constructs by means of both confirmatory factor and Rasch analyses. Two scales were used to capture teachers' self-efficacy. All items were assigned a five-point Likert scale (from 1 = "Strongly disagree" to 5 = "Strongly agree"), and both scales exhibited a good Cronbach's α (see Table 16.2).

16.3 Research Question

There are findings that provide a differentiated insight for changes in pre-service teachers' self-efficacy during the study (Bilali, 2013; Schüle et al., 2017). Most of them reconstruct a u-shaped progression of self-efficacy throughout the course of studies, which is explained by excessive expectations at the start of them, the reduction in individuals' own evaluation benchmark due to first practical experiences (Tschannen-Moran & Woolfolk Hoy, 2007) and then an increase due to successful experiences in internship. Accordingly, both successful self-performed and observed successful actions, together with positive emotions, contribute to an enhancement (Bandura, 1977). Since there is most probably an increase in self-efficacy in connection with reflective practice, a positive development in both facets of self-efficacy for mathematical modelling can be assumed. Thus, the following research question is of interest:

> Can self-efficacy of mathematics pre-service teachers for mathematical modelling be meaningfully and significantly increased through a teaching–learning laboratory?

Table 16.2 Scales for self-efficacy

Scale	Item number	Example item	Cronbach's α
Self-efficacy for working mathematically	8	It is easy for me to recognise the different abilities of the students using their handling of the mathematical symbols and operators used in modelling	0.84
Self-efficacy for modelling	13	It is easy for me to recognise the different abilities of students using their translation of mathematical results into reality	0.88

16.4 Research Design

The quasi-experimental study was conducted in a pre-post design to measure the self-efficacy expectations of the participating pre-service teachers. The treatment consisted of a 12-session teaching–learning-laboratory-seminar for pre-service teachers in one semester. This seminar on teaching mathematical modelling with integrated practical elements was designed in two variants for this study (task experimental group and intervention experimental group).

The seminar for the task experimental group comprises 12 sessions and additional blended learning formats. In this treatment, there is a special focus on the conception of own modelling tasks. The seminar consists of a theory-based preparation phase, a practical phase, and a reflection phase. The structure of the seminar for the intervention experimental group is similar to that of the task experimental group. The differences are, on the one hand, that students work in teams of two on given, selectable complex modelling tasks. The results are then discussed in plenary and potential solutions and difficulties of the students are anticipated. Another difference is the focus on interventions in mathematical modelling processes. In addition, there was a baseline group without thematic reference to mathematical modelling. The pre-service teachers completed the same test instrument before and after the treatment.

After piloting in the 2017 summer semester, the treatments were integrated into the regular seminar of mathematics pre-service teachers across three consecutive semesters (winter 2017/2018, summer 2018, winter 2018/2019)—see Fig. 16.3.

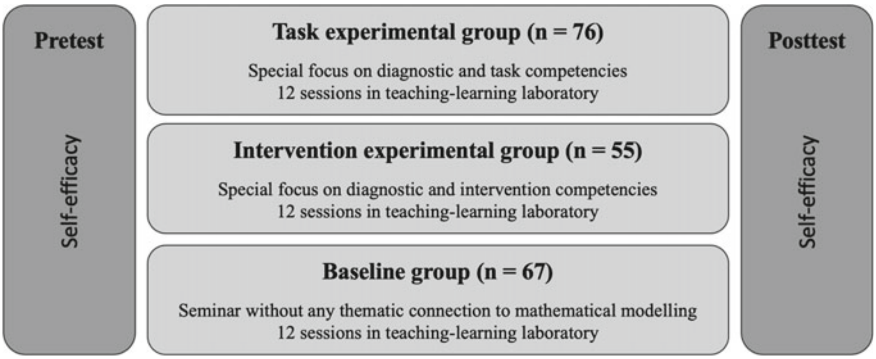

Fig. 16.3 Study design

16.4.1 Treatment Design: Teaching–Learning Laboratories

A teaching–learning laboratory encompasses a seminar with 12 seminar sessions for the pre-service teachers (see Fig. 16.3). It is comprised of a theory-based preparatory phase, a practical phase, and a reflection phase. Modelling processes form the core content of all phases in the experimental groups at the universities of Koblenz-Landau and Münster in Germany. The preparatory phase of the seminars, starting with an introduction to the fundamental notions, includes selected didactic and theoretical backgrounds of mathematical modelling through to pre-service teachers' own modelling and the associated assessment of individual modelling routes (Borromeo Ferri, 2018). It is not always easy to select or develop the right modelling task. As an indication, characteristics may be specified of what a modelling task should fulfil (see Sect. 16.2.3).

With respect to the focus on modelling activity, sub-competencies of modelling are observed closely. As regards the relation to reality, the relevance and authenticity of the context are also examined closely. An example of a modelling task used in the seminar is illustrated in Fig. 16.4.

Criteria and indicators were created for the set modelling sub-processes, to be able to observe and diagnose the learning processes of the schoolchildren in the

Fig. 16.4 Hot air balloon task: "How many litres of air are in this hot air balloon?"

project sessions. During these sessions, a team of three pre-service teachers (Master of Education) supports a small group of grade nine students with the processing of the modelling tasks. The teams monitor the competencies of mathematical modelling in a targeted manner and record these in the previously created monitoring sheet. The grade nine students work on content that would enhance the curriculum in motivating project contexts. This interlacing of theory and practice in the context of diagnostic actions and tasks represents the practical promotion of modelling-specific diagnostic and task-based competence.

While the task experimental group created the tasks used in the practical sessions themselves, the intervention experimental group used predefined tasks and focused on adaptive interventions. In the reflection phase, the project sessions were first discussed in the form of written reflection discussions so that pre-service teachers could benefit from the experiences of other seminar participants. Cross-task, theory-based group reflections on the respective areas of focus of the monitoring were carried out, considering in particular the heterogeneity aspects of the learning groups monitored. The pre-service teachers added to their diagnostic assessments the feedback from their colleagues. The knowledge obtained was then used to professionalise the participants' own teaching activities and evaluate the modelling tasks they had created. The pre-service teachers also reflected on and, where necessary, adapted the modelling tasks in light of the criteria for good modelling tasks drawn up in the preparatory phase. The experience and knowledge gained were summarised in a reflection report.

16.4.2 Data Acquisition and Analysis

To answer the question posed, a paper–pencil questionnaire in pre-post design was used to collect data from 198 pre-service teachers at grammar/comprehensive schools by the universities of Koblenz-Landau and Münster. In addition to the task experimental group in Münster (4 courses, $N = 76$) and the intervention experimental group in Koblenz (3 courses, $N = 55$), a baseline group in Münster (5 courses, $N = 67$) was also recorded. Since the students were reached via participation in seminars, no randomised assignment of the subjects to the treatments was possible. All students took part in both the pre-test and the post-test. The gender, age, subject-semester, and Abitur grade of the students examined were recorded (cf. Table 16.3). The differences in the subject-semester can primarily be attributed to the different structures of the subject teacher training programme at the two locations. This must be taken into account when interpreting the results, as must the differences in the average Abitur grades.

Paired t-tests were used to ascertain gains within each group. To investigate differences in the developments of self-efficacy between the experimental groups repeated measures analysis of variance (ANOVA) were used.

Table 16.3 Description of groups

	Number	Gender	Age		Semester		Abitur-grade	
		m/w	M	SD	M	SD	M	SD
Task experimental group	76	37/39	22.99	1.70	7.58	2.47	1.82	0.48
Intervention experimental group	55	25/30	22.87	2.91	5.69	2.59	2.40	0.63
Baseline-group	67	22/45	22.88	1.79	7.33	2.11	1.72	0.37
Total	198	84/114	22.91	2.12	6.97	2.50	1.94	0.57

16.5 Results

The self-efficacy of mathematics pre-service teachers for the diagnosis of performance potential for working mathematically ($t = -7.058, p < 0.001; 1 - \beta = 0.99; d = 0.53; n = 131$), as well as for modelling ($t = -7.251, p < 0.001; 1 - \beta = 0.99; d = 0.55; n = 131$), can be meaningfully and significantly increased through a teaching–learning laboratory. The pre-service teachers assessed their own capabilities for the diagnosis of performance potential as significantly higher after the treatment. In the baseline group, as expected, there were no significant changes ($t = 0.465, p = 0.644; t = -0.655, p = 0.514; n = 67$).

In the seminar of a repeated measures analysis of variance, it can also be ascertained that differences in the development of the self-efficacy for working mathematically ($F(1,128) = 11.007, p < 0.001; 1 - \beta = 0.93; \eta^2 = 0.079; n = 131$), as well as for modelling ($F(1,128) = 6.436, p < 0.05; 1 - \beta = 0.89; \eta^2 = 0.049; n = 131$), existed between the two experimental groups. These manifested themselves in significant interactions, which is why the group affiliation of the pre-service teachers had a clear and meaningful influence on the changes in their self-efficacy from the first to the second time of measurement. Thus, the metrics for the diagnosis of performance potential for mathematical modelling or for working mathematically considered here could each be significantly and more effectively increased in a teaching–learning laboratory in which the modelling tasks for use with students are created by pre-service teachers themselves (task experimental group) than was the case in a teaching–learning laboratory in which predefined tasks were used (intervention experimental group).

16.6 Discussion

The results of the study provide a first impression of the contribution that teaching–learning laboratories can make to the professionalisation of pre-service teachers. In particular, it is apparent that such laboratories for mathematical modelling represent a beneficial learning environment. Self-efficacy for mathematical modelling as part of professional competence could be increased and differences in the development could be seen. These will be investigated in further studies. Furthermore, it can be assumed that intensive involvement in modelling tasks facilitates a significant increase in self-efficacy in this area. It is also possible that there are correlations between the development of self-efficacy and the other components of professional competence (Depaepe & König, 2018; Kunter, 2013). Pedagogical content knowledge also developed slightly differently between the two groups, even though it increased overall (Greefrath et al., 2022). Therefore, there may be correlations here that should be investigated further.

The above is in line with findings from professional research that systematic and reflected practice experiences represent profitable opportunities for the development of affective-motivational components of (modelling-specific) teacher professionalism (Tschannen-Moran & Woolfolk Hoy, 2001).

Accordingly, also in the case of self-efficacy, the necessary theoretical-formal foundation, the integration of experiential knowledge, the systematic reflection of experiences from practice, and the university coaching in authentic teaching–learning arrangements may prove to be conducive to competence in their respective modelling-specific design as well as in their concrete implementation. Furthermore, it is conceivable that the characteristics of the motivational orientations of the pre-service teachers will increase because of working with students in the teaching–learning laboratory.

However, due to the low proportion of subject didactics in the study programmes considered and the associated high failure rates, follow-up testing had to be dispensed. Consequently, no statements can be made regarding the sustainability of the teaching formats regarding the affective-motivational aspects. It would also be desirable to monitor the competence acquisition of the grade nine students in the teaching–learning laboratory; however, due to the short interventions in this study, this was not done. There may also be connections here to the professional competence of teachers, especially to their self-efficacy.

Based on a common concept of competence and an established structural model of professional competence, a test instrument focused on the teaching of mathematical modelling was successfully applied to pre-service teachers in teaching–learning laboratories. It should be noted that self-efficacy was only measured at two points in time and that the measurement was done with the help of a questionnaire. In this way, not all areas of professional competence could be measured. Nevertheless, it is very useful that another measurement instrument for modelling-specific professional competence, including self-efficacy expectations, is now available.

Overall, the study provides a well-founded insight into the development of the professional knowledge of prospective teachers for a special sub-area of mathematics didactics. This is done within the framework of an approach in which theory and practice phases are interlocked in such a way that the promotion of professional competence in teaching mathematical modelling is made possible.

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