George J. Boole

A Nineteenth Century Man for the Modern Digital Era

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Abstract. It is entirely justified to use the attribute digital in describing contemporary era due to omnipresence of digital technologies and devices based on them and their strong influence to almost all aspects of human activities. The present paper is a tribute to a man whose work in logic and mathematics, leading to the mathematical logic, set theoretical foundations for the development and establishing of digital era.

Keywords: Mathematical logic · Boolean functions · Boolean algebra

1 Introduction

The current epoch in the development and evolution of humanity can with full justification be called the digital era due to the omnipresence of various kinds of digital devices ranging from many essentially important and extremely useful to the enormous number of various gadgets. Whatever aimed at deep space or micro cosmos exploring, or intended to simplify and facilitate performing of everyday tasks, they are all based on the same principles.

We are in the era characterised by the laboratory-on-chip, network-on-chip, Internet-of-Things, etc., and all this is possible just because we, humans, have learned to represent data and information encoded in them by binary sequences, and then we have learned the basic laws how to manipulate this knowledge in order to detect relationships, interconnections, similarities or differences, and afterwards derive conclusions, make decisions, and act accordingly. In other words, we have learned how to represent, model, and interpret, after some simplification and approximation, the basic laws of human thinking in terms of binary sequences and operations over them. Exactly this was the subject of study of George J. Boole already in the mid of nineteenth century.

This paper is a yet another tribute to the work of George Boole. We first shortly present his professional biography to set the circumstances under which he was working and understand where from his motivations and selection of research subject were coming. Then, we briefly discuss his three principal works [1–3] concerning the subject of the

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present paper. We also discuss his communication with contemporary scholars, notably with Augustus De Morgan based on the collected 90 letters that they exchanged and which are still preserved [22], which highlights a bit the style of work and the attitude towards the research subjects of these two and some other scholars at that time [9]. The intention is to provide a piece of information that can possibly be useful for understanding how the mathematical foundations of modern digital era have been formulated and established by starting from the work of George Boole and then many others which have followed the same ideas.

2 Personal Education of George J. Boole

Life circumstances forced Boole towards a hard and tedious, but interesting way of personal schooling and education.

George Boole was oriented towards an academic education and development by his father John Boole, who, although being a professional shoemaker, have had a passionate interest in science and mathematics, as well as for making scientific and optical instruments. It is recorded that the father and the son together built cameras, kaleidoscopes, microscopes, telescopes, and a sundial. As a very young kid, Boole joined first a school for the children of tradesmen, then a commercial school, and after that, in the age of seven, a primary school, where it was noticed his talent for languages. Due to that, his father arranged additional lectures in Latin for him given by William Brooke, a book seller and printer in Lincoln where the family Boole lived at that time. After mastering Latin, Boole continued to teach himself Greek and, at the age of 14, translated a poem by Meleager, entitled “Ode to the Spring”, which translation his proud father had published. The translation was so deep, mature, and profound that some scholars refused to believe this is the work of a 14 years old man. The word teenager” had not yet been coined.

In 1828, Boole joined Bainbridge Commercial Academy in Fish Hill in Lincoln, but continued his study in Latin, Greek, and algebra, as well as thought himself French, German, and later Italian.

Since in 1831, the business of his father was ruined, George Boole become the main provider for his family. To achieve the task, he has to abandon the wish of becoming a clergyman, and accepted the position of an assistant teacher first in Doncaster and then Liverpool. This was the beginning of his career as an outstanding teacher.

Together with teaching work, Boole carried out an ambitious self-education program in mathematics, by starting reading the book *Calcul Différentiel* by Lacroix in French. Boole continued his self-education by studying the work of Lagrange, Laplace, and Newton including reading of *Principia.* It can be observed that Boole followed a specific approach in his self-learning by taking advantage of his broad knowledge of languages. Boole used to persistently read a work in the original language many times, until he finally completely understood and mastered the contents.

3 Professional Biography of Boole

As noticed above, the professional career of Boole started already in his young days in 1831 while accepting the position of a teaching assistant. Then, in 1833, Boole was working at the Hall Academy at Waddington, near Lincoln, and then in 1834 opened his own school in Free School Lane, Lincoln. In 1838, Boole worked in the Waddington Academy and in 1840 opened this own Boarding School for Young Gentlemen at Pottergate, Lincoln.

In 1849, Boole joined the Queen's College in Cork, Ireland, as the first professor in mathematics, thanks to testimonials in support by leading mathematicians at that time including Augustus De Morgan, Philip Kelland, Arthur Cayley, and William Thomson.

On May 30, 1851, Boole was elected Dean of the Science Division of the Faculty of Arts, and was re-elected for the next mandate. In the same year, Boole was awarded an honorary LLD by the University of Dublin, most probably by the suggestion from his friend Reverend Charles Graves, who was at that time a Professor of Mathematics at Trinity College Dublin.

Boole was renown as a devoted teacher with a lot of patience and understanding for all his students expressing great willingness to help them in learning. This aspect of his personality is further highlighted by pointing out his engagement and activity in the Cuverian Society for the Cultivation of the Sciences, the goal of which was to provide public education in the sciences in the city of Cork. Boole was elected to full membership of the Society on November 6, 1850, then on September 19, 1851, to the Council, and further in the same year to the Sectional Committee on Statistics and Political Economy. The same year, Boole joined the Dublin Statistical Society. The following year, Boole become a Vice President of the Cuverian Society, and on May 24, 1854, the President of the Cuverian Society. It is important to notice that on June 11, 1857, Boole was awarded the great honour of membership of the highly prestigious Royal Society of London.

The biography of Boole is presented and discussed in an excellent way in the first book devoted to this subject by MacHale [14] published in 1985. The second edition of the book is published in 2014 under the title *The Life and Work of George Boole - A Prelude to the Digital Age* [15].

In the book [14], Boole is presented as a reserved and somber person but warm human. Boole expressed a strong sense of purpose and duty regarding institutional and civic levels. From the respect of religion, Boole shared elements of Unitarianism and Judaism, and in time became inclined towards agnosticism.

In the review of the book by MacHale, the reviewer Jongsma wrote [12] *Early employment as a schoolteacher, his development into an independent research mathematician and logician, and his sometimes-turbulent career as a conscientious and well-respected professor of mathematics at Queen's College in Cork, Ireland (now University College, where MacHale used to teach mathematics, and presently he is a professeur emeritus) are all fleshed out in detail unavailable anywhere else.*

4 Boolean Algebra

Major scientific contribution by George J. Boole, which provided for him such a prominent position in history of sciences, is certainly the mathematical concept that is presently called the Boolean algebra. Besides its importance as a mathematical object, it served as the key concept, which transformed the design process of switching circuits from an art to a science, based on the idea of describing both the functions performed and the circuits themselves realizing them in terms of the Boolean algebra. These fundamental observations were proposed by C. E. Shannon first in his master thesis [20], and then in a related very influential publication [21]. It is worth noticing that before discussing the problem of relay and contact switching circuits synthesis, Shannon attended at the Michigan University a course in mathematic where the Boolean algebra was among the topics.

In March 1941, the Japanesse engineer and scholar, Akira Nakashima, concluded that the algebra he has been developing from 1935 [17], through a thorough analysis of many examples of relay circuits and networks is identical to the Boolean algebra and put the reference to the work by Boole [18]. For further details, see [23].

It can be observed that already in 1910, Paul Ehrenfest [8] in a review of the book *Algebra of Logic* by Louis Couturat [6], wrote *Is it right, that regardless of the existence of the already elaborated algebra of logic, the specific algebra of switching networks should be considered as a utopia?*, see [24].

In former USSR, Gellius Nikolaevich Povarov, pointed out the remark by Ehrenfest and suitability of Boolean algebra for solving such tasks to V. I. Shestakov who defended a PhD thesis in the physic-mathematical sciences on September 28, 1938 at the State University Lomonosov, Moscow [25] where the references to the work of Soviet logicians Glivenko, Zhegalkin, and Sludskaja were given. For more details on this topic, see [24].

Table 1. A correspondence between logical and algebraic expressions.

Logical	Algebraic
Every X is Y	$x(1-y)=0$
No X is Y	$xy=0$
Some X is Y	$xy \neq 06$
Some X is not Y	$x(1-y) \neq 06$

The main idea of Boole which led to the definition of the Boolean algebra, can be shortly formulated as developing a symbolizing scheme for symbolizing logical relationships as algebraic relationships in a way allowing that logical deductions could be achieved by algebraic manipulations. Thus, in practice, the approach of Boole consists of the three steps

1. Express the logical data as equations in terms of suitably defined operations,
2. Solve these equations by algebraic techniques,
3. Translate the solution, if possible, into the original logical language.

Table 1 shows examples illustrating a correspondence between logical an algebraic expressions in the context of the Boolean algebra.

The related mathematical work by Boole is reported in his three important publications [1–3], and it was favorably estimated by many scholars. For instance, Tarski [26] wrote *The development of mathematical logic began at the time when Boole published* *his works on logic. Laws of Thought is Boole's principal work.* In [13], Lewis and Langford said *The work of Boole is the basis of the whole development [of mathematical logic]*. More recently, Corcoran [5] stated *Boole did the first mathematical treatment of logic*. Similar statements can be found at many places in the literature.

5 The Boole - De Morgan Correspondence

The correspondence between these two mathematicians provides a good insight into their work in logic, but also highlights some other aspects of their personalities, various personal interests, and mutual topics to discuss. The correspondence started in late 1842 after the Boole published his first works in logic, and De Morgan noticed and commented. The book [22] contains 90 letters that are still preserved, 64 letters written by Boole and 26 by De Morgan, with a draft of a letter by De Morgan, the parts of which were not included in the corresponding letter to Boole.

The mathematical concepts discussed are certain topics of calculus, differential equations, mathematical logic, and probability. Among social and personal topics, they discussed homeopathic medicine, the plight of the Jews, psychic phenomena and theories (spiritualism), family matters, etc.

In the book by [22] the letters are arranged first chronologically, and then grouped into periods related by general themes. At the beginning of each group of letters, the author provided a brief summary of the contents and at some places the transitional material were used and shortly commented in order to provide necessary explanatory comments which is very useful for the readers. Some of the remarks and conclusions by G. C. Smith are commented and slightly corrected in the reviews of the book, as for example by Hailperin [10], and Jongsma [11].

Boole and De Morgan were in friendly personal relations as can be seen from their mutual correspondence where they besides scientific considerations, comments, and thought, also exchanged personal and family matters and general thoughts about certain contemporary affairs, literature, etc.

They worked in the same field and on the identical subjects and then discussed them. For instance, the first works in logic by Boole [1] and De Morgan [7] were published in November 1847. In his letter dated on November 28, 1847, the letter 12 in [22], De Morgan points *remarkable similarities* in these their works by adding also that he did not use the algebraic notation in his system which *employed mechanical modes of making transitions.*

An insight into the Boole De Morgan correspondence raises some interesting questions about their attitude towards particular concepts. We briefly point out three of them.

In a letter dated on February 24, 1845, Boole is answering to De Morgan to acknowledge receiving of his memoir containing already at the first page a discussion of associativity of triples that de Morgan invented by an analogy with quaternions defined by W. R. Hamilton. Thus, Boole was aware of the associativity, and it is an interesting question why he did not even mention this mathematical property in connection with either logical addition or multiplication, discussed in his work.

As pointed out in [10], by referring to the letter by De Morgan dated on April 3, 1849, letter 16 in [22], as well as a comment at the page 149 in [7], both Boole and De

Morgan allowed the possibility of a three-valued logic. In [10], it is quoted a statement at the page 149 in [7], where De Morgan states *But we should be led to extend our system if we consider propositions under three points of view, as true, false, or inapplicable. We may confine ourselves to single alternatives either by introducing not-true (including both false and inapplicable) as the recognized contrary of true: or else by confining our results to universes in which there is always applicability, so that true or false holds in every case. The latter hypothesis will best suit my present purpose.*

Continuing the discussion of the subject, on the side of Boole, Hailperin [10] quotes the following statement at page 51 in [3].

Now if the equation in question [i.e., $x^2 = x$] *had been of the third degree, still admitting of interpretation as such* [in a footnote on the preceding page Boole argues against being able to interpret $x _ x^3 = x(l - x)\,(l + x) = 0$ in logic], *the mental division must have been threefold in character, and we must have proceeded by a species of trichotomy, the real nature of which it is impossible for us, with our existing faculties, adequately to conceive, but the laws of which we might still investigate as an object of intellectual speculation.*

Hailperin concludes it is curious that Boole makes no mention of De Morgan in that respect.

Another interesting discussion is related to the question if Boole and De Morgan considered the concepts as Nothing and Universe. Nothing is represented by Boole by the symbol 0 and Universe by 1, and they are considered as classes. De Morgan however follows the traditional syllogistic forms and excludes the extreme names but recognizes that these can be given formal treatment. The letters enumerated by 79 and 80 in [22] explain the difference between their understanding of these concepts. See also a brief discussion of this subject in [10].

6 Instead of Conclusions

There are several reasons supporting the presented point of view that George J. Boole can be viewed as a nineteenth century man for the modern digital era. We point out two of them

1. In contemporary computer science and engineering practice there are several fundamental concepts bearing the name of George J. Boole. For instance, the following concepts are widely used *Boolean algebra, Boolean ring, Boolean variables, Boolean functions, Boolean circuits, Boolean networks, Boolean difference, Boolean Operators, Boolean filtering, Boolean data type*, etc.
2. The way of learning, the self-education programs that Boole created forhimself, selection of topics to learn, manner of teaching others, which fits well with the present way of teaching and learning taking into account various modes of e-learning, distance learning, on-line courses, and other forms of self-education in the digital era.

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