

Two-Level Algorithms as Part of Digital Logistics Platforms

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Abstract. Transfer of logistics to work in modern concept of digital transport corridors implies optimization of commodity flows processing process. Nodes play an important role in the network structure of logistics chains. From the condition of economic expediency, multi-stage or multi-level distribution subsystems are used for effective work today. The paper presents the results of a study aimed at developing mathematical models of complex multilevel nodes of logistics networks. Algorithms have been obtained that form the basis of the software of decision-making systems as part of digital platforms. Their application makes it possible to more accurately solve the problem of organizing a multi-level system of warehouse and distribution nodes of the logistics chain. Since conventional deterministic models are ineffective in the conditions of market uncertainty characteristic of modern commerce, more complex scientifically based methods are used. Methods of the theory of stochastic processes, simulation modeling, and theoretical optimization methods are used to find a solution. The relevance of the work is due to the formation of a large-scale segment of third-party logistics in the Industry 4.0 concept.

Keywords: Logistics \cdot Digitalization \cdot Algorithm \cdot Commerce \cdot Distribution \cdot Warehouse

1 Introduction

The modern organization of logistics activity reflects the trend towards digitalization. The requirements of the transition to Industry 4.0 platforms quickly led to the formation of a separate segment of third-party logistics. Third Party Logistics providers are the most common. Since, thanks to digitalization, consolidation of various types of business and commerce into network entities is most economically advantageous, a transition to a higher level of logistics outsourcing 4PL is planned, which is integrated at the interface level with cargo owners and service customers [1]. This trend is particularly pronounced within the framework of globalization and among transnational network players in most segments of commerce, trade in goods and services [2, 3]. Starting from the 3PL level, the

successful implementation of digital platforms has slowed down, as a serious problem has appeared caused by the lack of algorithms for managing the activities of logistics network nodes. Conventional deterministic models are ineffective in conditions of market uncertainty. The requirements of the transition to DDT (Demand-driven Techniques) technology focused on the current changeable demand force us to look for a solution based on mathematical models using the theory of stochastic processes. Additionally, it becomes possible to more accurately solve the problem of a two-level system of warehouse and distribution nodes of the logistics chain, characteristic of distribution systems [4]. Also, mathematical models and algorithms make it possible to use for practical purposes the possibilities of widely used methods of the theory of optimal solutions [5]. The criterion in this case is an integral functional, which includes in its composition, in addition to economic indicators, also the time spent on operational logistics activities. It is taking into account the loss of time for transportation, interoperational, loading and unloading, customs procedures, intra-warehouse storage that allows you to balance logistics costs taking into account factors such as expire date and contract terms [6]. Implementation of software products for integration into digital logistics platforms at the management level of optimal distribution systems will combine the capabilities of M2M flow of information about the state of logistics and planning methods based on leading indicators [7, 8].

The goal of the work is the development of mathematical formalisms for modeling multi-stage or multi-level subsystems of distribution chains. First of all, this is caused by the need to plan the activities of transport and logistics hubs. The vast majority of existing logistics systems use a two-level structure. The upper level of DC (distribution center), locations outside large consumption zones, such as settlements, especially megacities, is distinguished. Such placement is due to the economic feasibility [9, 10] of the balance between the cost of the land area and the cost of delivery. The lower or local level SW (Store Warehouse) is located either in the consumer's building itself or on its territory. This is especially true for shopping malls [11] and super/hypermarkets. A distinctive feature of modern network consolidated business entities has become the load on one DC of numerous local SW. The interaction between these levels is subject to the speak-hub topology. In the presented work, the task of forming a scalable algorithm allowing an increase in the number of intermediate levels of movement of goods and cargo through the warehouse nodes of the logistics network is set.

2 Materials and Methods

According to the modern understanding of the Industry 4.0 article, the mathematical model [12] uses data flows about the state of the system, in our case of an economic property. We express them in a set of formalisms, which are the framework of the description. Note that the incoming data must either be completely or as much as possible processed by M2M inter-machine interaction systems. A whole range of devices and technologies has already been developed and widely available for logistics tasks [13] to implement this principle. These are barcode, RFID, QR encodings, glonass/GPS monitoring systems, smart contracts, as well as online payments and other systems [14]. Let 's define the basic formalisms:

w – current load level DC/SW.

v – the volume calculated in quanta of delivery, provided $v \ge 0$;

r = w + v -total value;

z – reflects the market uncertainty of demand also in the supply quanta, $z \ge 0$;

p(z) – the value distribution function obtained from statistical data z;

D – determined by the terms of transactions or contracts, the volume in quanta of delivery in a single planning period;

L – time costs for the execution of the purchase order;

 D_L – the volume of market demand in quanta of supply, can be interpreted as the rationing of the market during the period *L*;

Accordingly, the condition is true: $D_L = D \cdot L$;

TC(v) – total cost as a function with a calculable volume argument;

FC – fixed cost, limited FC > 0, integral indicator of overhead costs;

 $\Phi(r)$ – the function whose argument is the current level, reflects the costs;

 β – coefficient, when β > 0 is equal to the cost of the warehouse maintenance of the supply quantum; γ - coefficient, when γ > 0 is an integral indicator of losses caused by the under-delivery of agreed volumes, including reputational losses and lost profits;

 Ω^* – the critical parameter calculated by the formula $\Omega^* = \frac{\gamma - c}{\gamma + \beta}$, enclosed in the range $0 < \Omega^* < 1$;

S – a characteristic value defined as minimum among the increasing integer values of the summation range in the expression $\Omega(S) = \sum_{z=0}^{S} p(z) \ge \Omega^*$;

 $\varphi(r|w)$ – the function of averaging the costs of the volume of goods r in the presence of volume w at the beginning of the logistics operation.

Since real logistics activities are subject to market uncertainty, an EOQ (Economic Order Quantity) test is needed to assess the sustainability of solutions This will allow you to focus on the EOQ indicators known as Δ^* of calculated parameters [15]. From the formula for calculating the parameter $\Delta^* = \sqrt{\frac{2FC \cdot D}{\beta}}$ it can be concluded that the level of cost impact *FC* is always below the linear trend. In particular, with a doubling or the same decrease in the level of market demand, the indicator *EOQ* ranges up to 40% [16]. Further, analyzing the impact of costs β within the same limits, *EOQ* changes only by 30%. The assessment of the impact of demand uncertainty on the time interval between neighboring restockings T^* is carried out by calculations using the formula:

$$T^* = \frac{EOQ}{D} = \frac{\Delta^*}{D} = \sqrt{\frac{2FC}{\beta D}}.$$

Note that an increase in the standard D will lead to an increase of Δ^* on the one hand, and a reduction of T^* of the other. For practical activities, this means the need to replenish the emergent level at the distribution center more often.

The practical recommendation in this case is to choose a parameter $\widetilde{\Delta}$ as close as possible to the optimal value Δ^* . This is due to the delivery quantum formats, loading regulations, packaging, type of vehicle, restrictions on minimum and maximum delivery. The calculation acceptable for practical implementation is carried out according to the formula:

$$\widetilde{\Delta} = \tau \Delta^* = \tau \sqrt{2 \text{ KM/h}}, \text{ provided } \tau > 0.$$

If we derive a dependency to determine the average costs AC:

$$AC = \frac{FC \cdot D}{\Delta} + cD + \frac{\beta \Delta}{2},$$

then variable costs are calculated [17] in the total composition according to the formula:

$$VC(\tau) = \frac{FC \cdot D}{\Delta^*} + \frac{\beta \cdot \Delta^*}{2} = \frac{FC \cdot D\sqrt{\beta}}{\tau\sqrt{2FC \cdot D}} + \frac{\beta r\sqrt{2FC \cdot D}}{2\sqrt{\beta}}$$

In this expression, Δ^* is understood as a variation of the actual value *EOQ* reduced by volume *cD*, which is equivalent to the parameter Fixed Cost.

Finally, the analysis of the impact of market demand deviation is carried out using the formula: $\xi = VC(\tau)/VC(1) = \frac{(\tau^{-1}+\tau)}{2}$, which we will program on a computer. The result of calculating ξ as a function of τ is shown in Fig. 1.



Fig. 1. Calculation of the dependence of ξ on the deviation of τ .

The analysis shows that there is a nonlinear dependence of ξ on the deviation of τ , which will satisfactorily affect logistics activities with small, up to 25% fluctuations in market demand. Even in the interval 0, $55 \le \tau \le 1$, 6 the spread ξ is below 10%. In the analysis, we introduce the notation and take into account the fact that the value of $\frac{VC(\tau)}{VC(1)}$ is invariant to the statements τ and τ' .

Next, it is necessary to consider an important property [17] of the process of organizing the movement of goods and cargo. This is the discreteness of both the volumes of the supply quantum and the time planning periods. In this formulation, the parameter D in the mathematical model will take the quantitative characteristic of demand as a planning unit. This scale will not violate the generality of the results in any way. Then the logistics costs of the *DT* supply quanta are accompanied by costs C_T , where T = 1, 2, ... This is completely equivalent to C_T , as costs for *T* planning periods. Then the following is true:

$$C_T = FC + cDT + \frac{1}{2}\beta D(T-1)T.$$

Having determined the minimum value of C_T/T we get the optimal mode. Since T = 1, 2, ... This is formalized by finding the value of the expression:

$$\min_{T} \left[\frac{FC}{T} + cD + \frac{1}{2}\beta D(T-1) \right].$$

Having found the solution of the equation with relation to T for the zero value of the first derivative, we denote the result as $T^*: T^* = \sqrt{2FC/\beta D}$. Next, we get the parameter Δ^* necessary for *EOQ* from the calculation: $\Delta^* = D \cdot T^* = \sqrt{2FC \cdot D/\beta}$. We will take into account the discreteness by rounding T^* . We will get an economically significant result.

Having information about the frequency of receipt of indicators of logistics parameters DC, we will combine [18] data with the costs of placing goods and cargo In reality, DC operates with a contractual lag. The time difference is equal to $-s/\Delta$, where $s \le 0$ is allowed, i.e. the critical level is below the demand The volume for a given period (-s/2) of the delivery quantum. From here, the probable loss of the volume of market demand is calculated, for one period equal to $-s^2/2\Delta$.

Parameter γ (> 0), an integral indicator of losses caused by under-delivery of agreed volumes, including reputational losses and lost profits. Then we can write the formula for \widetilde{AC} – the volume of average costs:

$$\widetilde{AC} = \frac{FC \cdot D}{\Delta} + cD + \frac{(\beta + \gamma)S^2}{2\Delta} - \gamma S + \frac{\gamma}{2}\Delta$$

Acting similarly, with the difference that we differentiate \widetilde{AC} by S. As a result, we calculate the optimal value for S^* :

$$S^* = \left(\frac{\gamma}{\beta + \gamma}\right)\Delta.$$

Then substituting S^* into the calculation of the optimal EOQ, we get a set of formulas acceptable for use in machine calculation algorithms Δ^* , S, s:

$$\Delta^* = \sqrt{2FC \cdot D\left(\frac{1}{\beta} + \frac{1}{\gamma}\right)}, S = \sqrt{2FC \cdot D\left(\frac{\gamma}{\beta(\beta + \gamma)}\right)}, S = -\sqrt{2FC \cdot D\left(\frac{\beta}{\gamma(\beta + \gamma)}\right)}$$

3 Results and Discussion

In logistics, the work of an outsourcing operator takes place within the framework of a very voluminous assortment matrix [19]. It should be taken into account that the costs

FC, as well as the costs of maintaining current stocks, are not deterministic parameters of the model. The value *L* of the time spent on delivery always corresponds to the inequality L > 0. We also additionally define z_L of the real demand for *L*. It's clear that the value z_L is stochastic. The ratio $z_L > s$ characterizes the presence of delayed delivery. The distribution function $p_L(z_L)$ fully describes the parameters of a random process. Since the statements about stationarity, ordinariness, as well as the absence of aftereffect are introduced a priori, $p_L(z_L)$, on the planning horizon *T*, can be formalized by an exponential law:

Note that in real commercial activity, the standard deviation may not be equal to the mathematical expectation. For a more correct mathematical description [20], an arbitrary character $p_L(z_L)$ is easily embedded in software applications, which increases the accuracy of management decisions.

Next, we define the components of the objective function. To do this, we will introduce implementation $\cos \theta_S$. Then by definition EOQ it is fair: $\theta_S = FC \cdot D/\Delta + cD$. We will also introduce into consideration the execution option $z_L < S$ and the opposite If we apply them to the previous arguments, then using a well-known function $p_L(z_L)$ we write:

$$=\sum_{z_L=0}^{s} \frac{1}{2} [s + (s - z_L)] p_L(z_L) + \sum_{z_L>s} \frac{1}{2} [s + 0] p_L(z_L) = \frac{1}{2} \left[s + \sum_{z_L=0}^{s} (s - z_L) p_L(z_L) \right]$$

 M_L – Mathematical expectation M_L of the distribution $p_L(z_L)$ gives the average load level of terminals and DC:

$$\frac{1}{2}[(s - M_L + \Delta) + s] = \frac{1}{2}(2s - M_L + \Delta)$$

We will also add weight coefficients to the quality function. For the predicted DC load level, this will be the $\mu = M_L/\Delta$ parameter. We will take the average load with a coefficient $(1 - M_L/\Delta)$, which together with μ gives a complete set of data. As a result, we write down the calculation equation \tilde{r} - the average load level relative to a single time unit:

$$\widetilde{r} = \frac{M_L}{2\Delta} \left[-2s + M_L - \Delta + s + \sum_{z_L=0}^s (s - z_L) p_L(z_L) \right] + \frac{1}{2} (2s - M_L + \Delta)$$

Since for any kind of $p_L(z_L)$ it is fulfilled:

 $\sum_{z_L=0}^{s} (s-z_L)p_L(z_L) = s - M_L - \sum_{z_L>s} (s-z_L)p_L(z_L), \text{ then using } p_L(z_L) \text{ we formalize}$ the situation of a possible deficit: $\sum_{z_L>s} (s-z_L)p_L(z_L), \text{ and we use the coefficient } D/\Delta$ accordingly. This set of formalisms has led to an important result, namely, it is possible to put into the program the calculation of the costs of $\tilde{\Phi}$ with work *DC* per unit of time:

$$\widetilde{\Phi} = \frac{FC \cdot D}{\Delta} + cD + \beta \left(\frac{\Delta}{2} - M_L + s\right) + \left(\frac{\beta M_L}{2\Delta} + \frac{D\gamma}{\Delta}\right) \sum_{z_L > s} (z_L - s) p_L(z_L)$$

After that, we apply the procedure for finding the extremum of $\tilde{\Phi}$ by equating the partial derivative to zero $\frac{\partial E[AC]}{\partial \Delta} = 0$. Solving this equation for Δ , we get the result in the form of quadratures:

$$\Delta^* = \sqrt{\frac{2FC \cdot D}{\beta} + \left(M_L + \frac{2D\gamma}{\beta}\right)} \sum_{z_L > s} (z_L - s) p_L(z_L)$$

The integer condition is implemented by a discrete form of the function $\Omega_L(r) = \sum_{z_L=0}^{r} p_L(z_L)$. The result will be the optimal value s^* ($s^* > 0$) as an integer satisfying the inequality:

$$\Omega_L(s^*) > \Omega^*$$
, where $\Omega^* = 1 - \frac{\beta \Delta}{\beta M_L/2 + D\gamma}$,

At the same time, $s^* = \inf_{\Omega_L(s^*) > \Omega^*} s^*$, which corresponds to the critical level for *DC*. The minimum cost as an economically significant indicator is calculated as:

$$\min \widetilde{\Phi} = cD + \sqrt{2\beta \left[FC \cdot D + \left(\frac{\beta M_L}{2} + D\gamma\right) \sum_{z_L > s} (z_L - s^*) p_L(z_L)\right]} + \beta(s^* - M_L).$$

The set of obtained formulas is used in forecasting the economic feasibility of placing reserves on *DC* for each of the positions. The assortment matrix *DC* is usually very large and it's physically difficult to maintain the entire nomenclature. Considering the random nature of *L* is necessary for a number of reasons: it is the delivery of goods from various sites, cross-border deliveries, customs and control delays. Thanks to $p_L(z_L)$ it is possible to optimize not only storage costs, but also capital construction costs.

3.1 Example of Calculation

Within the framework of cooperation with the business incubator Universitat de Barcelona, the calculation was carried out according to the proposed mathematical model. The data is taken from the documents of The financial statements of The Barcelona Port Authority.

The port of Barcelona is the most important element of Spain's business infrastructure. It is the largest logistics hub receiving more than 10,000 ships and handles more than 53 million tons of cargo of all formats. The calculation according to the above model was carried out for the warehouse division of the port engaged in servicing incoming ships, both commercial and cruise. At the same time, before the pandemic, the share of cruise liners was about 12%, with an additional 4% of the volume accounted for by large ferry ships. The warehouse division provides a full set of services: refueling, loading of water, hygiene materials, medicines, provision of compressed air, electricity, steam, replacement of batteries, refueling with technical and chemical consumables, including maintenance of refrigerators, refueling with all kinds of lubricants, spare parts, etc., in total more than 20 thousand names and articles.





Fig. 2. Analysis of the distribution function.



The results of calculating the dynamics of the movement of a monoproduct according to the presented mathematical model are shown in Fig. 2. For four articles in Fig. 3.

The analysis reflects the possible presence of a negative level as a result of the calculation. This situation is typical for reserving for deferred demand. It is explained by the time delay between the request for the cargo and the time of delivery. This is the main reserve for online or smart contact interaction, which provides operational opportunities to optimize the economic indicators of the distribution center, as well as reduce the costs of SW, both operational and capital.

4 Conclusions

The work of all levels of logistics has undergone a serious transformation over the past year in the conditions of COVID19. The load on the distribution centers of seaports did not decrease, but the resulting instability required a deeper scientific analysis of flows in such transport hubs. Such tests as traffic surges caused by the Ever given accident in the Suez Canal led to a one-time delay of more than 150 large and especially large vessels, including over 20 million barrels of oil. There were more than 20,000 FEU (Fourty-foot Equivalent Unit) on the stuck container ship alone. The analysis of the stability of mathematical models [21, 22] is devoted to this. The formation of digital transport corridors begins with optimizing the operation of logistics hubs, as the most capital-intensive and serving to dampen the instability of flows in logistics networks. Calculations based on the presented model allow us to identify resources for optimizing their performance indicators. It is the use of formalized mathematical models that provides the basis for the formation of programmatically implemented algorithms for managerial decision-making. At the same time, there is a natural transition to the next stage of attracting the capabilities of artificial intelligence in the management of such a complex business object as a distribution warehouse complex of a seaport.

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