Chapter 8
Broadening of Gain and Its Bearing on the Laser Subtleties

8.1 Introduction

Spontaneous emission, as we now know, provides the seed photons for the onset of stimulated emission and understandably therefore is expected to have a major say on the emission feature of the laser itself. It is also well known that a fluorescent optical source derives its light from spontaneous emissions and that such a source gives out multicolored light. It is worthwhile at this point to examine the origin of this polychromatic emission. To this end, we revisit the operation of a fluorescent light source elaborated in Chap. 4. Upon undergoing inelastic collisions with the electrons abundantly present in the discharge, the ground state atoms climb to the excited state. Nature has always a preference to preserve a minimum energy configuration, and consequently the excited atoms spontaneously return back to the ground state releasing the excitation energy as a quantum of photons. These photons emitted randomly in all possible directions constitute the output of the fluorescent light. The situation is schematically represented in Fig. 8.1. If the excited level, located at an energy of E_1, were monoenergetic, then the spontaneous light emitted from the transition between this and the ground level would have a fixed frequency E_1/h, called the line center frequency ν_0. Consequently, the resulting fluorescence would have been monochromatic with a corresponding wavelength of c/ν_0! Where lies the catch then? An excited energy level is of course unstable, but that does not mean that an electronically excited atom has to decay at once to the ground state. The excited state, as a matter of fact, is short-lived, and that holds the key to the polychromatic nature of fluorescence. A finite lifetime τ associated with an excited energy state, according to Heisenberg's uncertainty principle, results in its spreading symmetrically on either side of the line center, and the extent of spread is inversely proportional to τ. The ground state being stable has an infinite lifetime and hence would not experience any spread as is also seen in this figure. Clearly, transitions originating from increasingly above E_1 will fluoresce light of frequency ν_0^+ progressively higher

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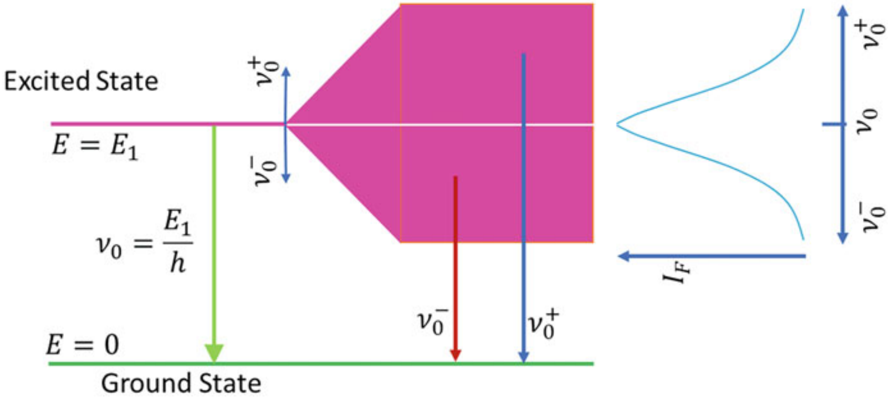

Fig. 8.1 The excited energy state has a finite lifetime, and as a consequence of Heisenberg uncertainty principle, the energy level spreads into a band. This gives rise to a corresponding spread in the frequency or wavelength of the fluorescence light. The intensity of fluorescence (I_F) exhibits a peak at the line center frequency ν_0 and gradually drops off on either side

than ν_0. Likewise, transitions arising from gradually below the central energy will yield light of frequency ν_0^- progressively lower than ν_0. The intensity of fluorescence I_F, if monitored as a function of frequency, will reveal a spectral broadening about the line center frequency ν_0, an irrefutable signature of the polychromatic nature of the fluorescence.

The aforementioned lifetime that is governed by the radiative decay of the excited energy state is denoted as radiative lifetime. There are other parameters, such as the gas pressure and temperature that will also have a strong bearing on the lifetime of an excited state and will be discussed later in this chapter. However, the point that we need to underline here is as to how this broadening of energy level, to which fluorescence owes its polychromatic nature, will impact the emission feature of a laser? As we know, inverting the population between two energy levels is central to the amplification of light by stimulated emission of radiation. Now that the excited energy state is broadened, the population inversion and, in turn, the gain does not occur on a single frequency and instead is distributed over the entire frequency spread. This fact has been qualitatively illustrated in Fig. 8.2, which, for the sake of easy palatability, depicts two situations, namely, the hypothetical case of infinitely long-lived excited state and the real case of an excited state with finite lifetime. In the former case, as shown in the top trace of this figure, the entire population inversion and, in turn, the gain would build up right at the line center frequency, viz., ν_0. In reality, however, as shown in the lower trace, the excited state is short-lived and consequently the energy level is broadened. The population inversion or gain is now distributed over the entire frequency spread across which the excited medium is known to fluoresce. The peak value of the gain, viz., g_0, understandably occurs at ν_0 as the intensity of fluorescence also peaks here. The distribution function depends on the nature of gain broadening and will be described later. Based upon the mechanism that controls the lifetime of the excited state, the broadening of gain can be categorized as either homogeneous or inhomogeneous. Needless to say, both

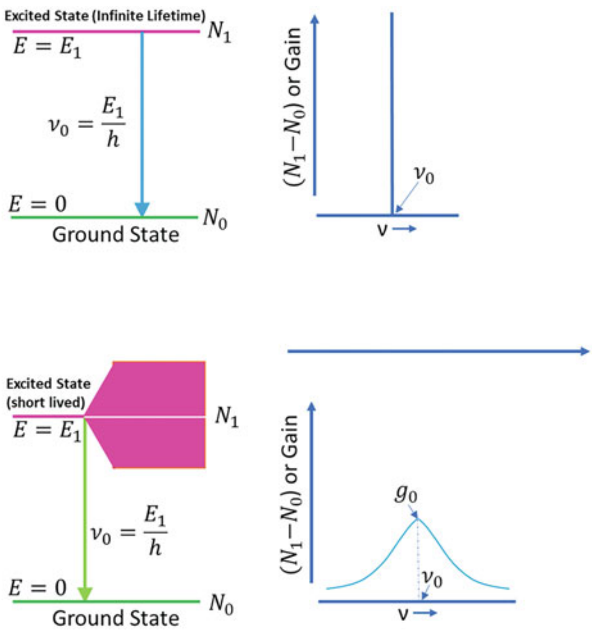

Fig. 8.2 The distribution of gain as a function of frequency of emission. Top trace – hypothetical case of excited state with infinite lifetime – gain exists only at the line center. Bottom trace – in reality, an excited state is short-lived, and gain is broadened in frequency

the broadening of gain and its nature affect the dynamics of a laser in a distinct manner. In addition to providing deeper physical insight into the two kinds of gain broadening, this chapter also reflects the profound impact of it on the operation of lasers. Recounting the gainful exploitation of the indelible trails it leaves on the laser dynamics will be yet another major endeavor of this chapter.

8.2 Nature of Gain Broadening

Broadening of an optical transition in frequency owes its origin to the fact that an excited state cannot exist indefinitely. Put in another way, the excited energy level is not infinitesimally narrow, a prerequisite for emission of light of a unique frequency or wavelength. In the example of the preceding section, the radiative or spontaneous relaxation of the excited atoms to the ground level governed the lifetime of the excited state, aptly termed as the radiative, spontaneous, or natural lifetime. The lifetime here depends on the spectroscopic properties of the energy levels involved in the optical transition and can vary widely from one excited level to another. The active medium of a laser, as we know, can be of all three kinds, viz., solid, liquid, and gas. Depending on the state of the medium, there can be a variety of other effects that can also directly and decisively influence the excited level lifetime and, in turn, the broadening of gain of the corresponding lasing transition. Based on the nature of the gain broadening, it is classified as either homogeneous or inhomogeneous gain broadening.

8.2.1 Homogeneous Gain Broadening

Broadening of the gain is said to be homogeneous when every atom (or molecule or ion, as the case may be) of the active medium has exactly identical fluorescence linewidth. This means that upon excitation, all of them can emit spontaneously at a given frequency within the fluorescence profile with equal probability. This point can be readily understood by referring to Fig. 8.3. We consider an ensemble comprising *N* excited atoms and identify them as atom-1, atom-2, atom-3, and so on. The emission bandwidths of all these atoms are individually depicted in this figure. The probability of emission at a given frequency is the same for all the atoms. This has been shown to hold true for seven randomly chosen frequencies, viz., ν_0, ν_1, ν_2, ν_3, ν_4, ν_5, and ν_6. Clearly therefore the emission bandwidth of the laser when all these atoms are put together will be identical to any of the individual atoms. This

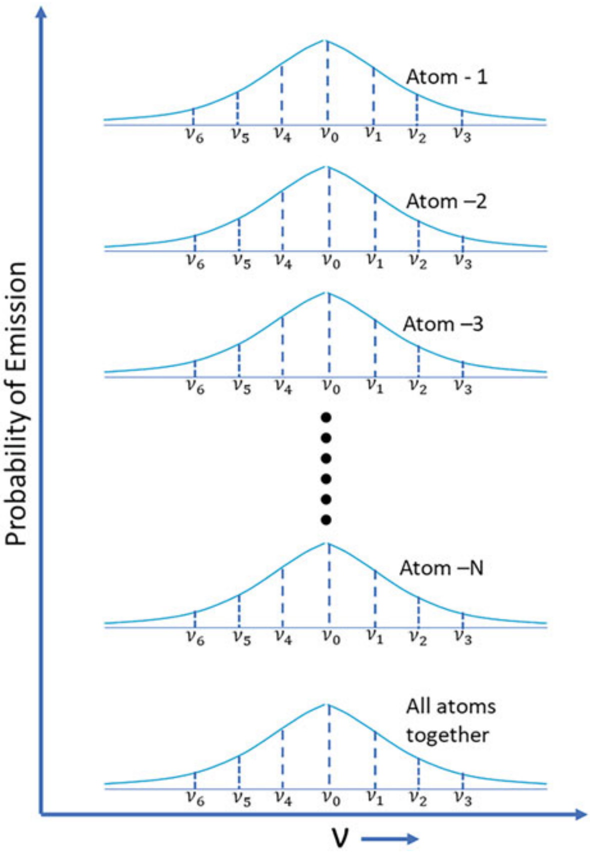

Fig. 8.3 In the case of a laser with homogeneous gain broadening, all the excited atoms emit with equal probability at a given frequency. The combined probability of emission of all the atoms is normalized to fit the scale

means that if a photon of a particular frequency, say ν_1, can interact with any one of the atoms of the ensemble, it is capable of interacting with all of them with identical probability. This fact, which holds true for photons of any frequency lying within the broadened profile, rivetingly manifests itself in the operation of a homogeneously broadened laser influencing its performance in a remarkable manner.

It may be noted here that broadening arising from atomic or molecular collisions is intrinsically homogeneous in nature. To appreciate this fact, it is imperative that we understand how the collisions result in broadening of a spectral line in the first place. To this end, we consider a gaseous atomic system that essentially presents a collisional environment. Here, the time that an atom can spend in an excited state is not governed solely by radiative decay as it can also lose the energy of excitation through collisions. Collision will thus reduce the lifetime of the excited state and consequently enhance the spectral broadening. As the probability of undergoing a collision is the same for all the atoms making up the gain medium, the broadening arising out of collision is therefore essentially an example of homogeneous broadening. Collisional broadening is also called pressure broadening as the rate of collisions increases with increasing gas pressure. As we shall see later, the pressure broadening of the gain plays a dominant role in the operation of a gas laser.

In the case of a solid-state laser, the atoms that constitute the active medium cannot, unlike the gas lasers, move around as each of them is basically tied to the host crystal's[1] corresponding lattice points. Although there is no scope of interatomic collisions, the omnipresent thermal energy invariably sets off vibrational motion of the lattice. The vibration modulates the energy levels of the atoms culminating in the broadening of its emission frequency. Such broadening is termed as thermal broadening as it originates from the thermally induced lattice vibrations. As the lasing atoms are embedded in an ordered manner in the host crystal, each of these atoms is therefore subjected to the same vibration. Consequently, thermal broadening too, like pressure broadening, is homogeneous.

The broadening that arises from the natural lifetime being governed by radiative or spontaneous decay is termed natural broadening. There is no denying of the fact that the radiative decay of the excited atoms (molecules or ions) occurs in an isotropic manner meaning they are indistinguishable as far as spontaneous emission is concerned. Thus, natural broadening, like pressure and thermal broadening, is also homogeneous in nature. It may be noted here that a homogeneously broadened gain profile, regardless of its origin, is always represented by a Lorentzian distribution function [24].

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[1] In a majority of the solid-state lasers, the light emitting atoms are embedded in small proportion into a second species termed as the host material that is transparent to the pump and lasing wavelengths.

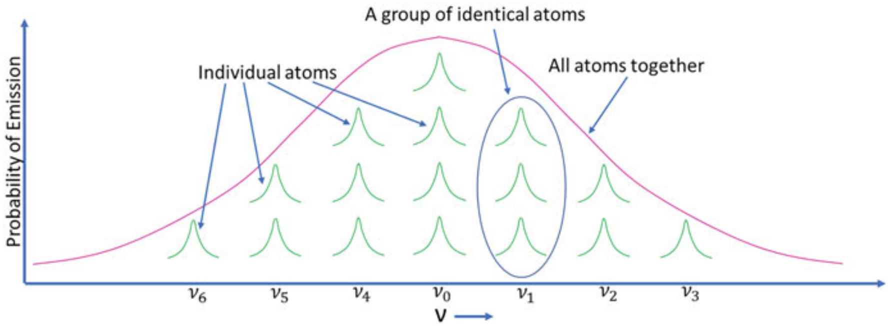

Fig. 8.4 In the case of a laser with inhomogeneously broadened gain, individual atoms emit at different frequencies. The probability of emission of the gain medium as a function of frequency is a convolution of the probability of emission of all the individual atoms putting together

8.2.2 Inhomogeneous Gain Broadening

Broadening of the gain is said to be inhomogeneous when, in total contrast to homogeneous broadening, the frequency of emission differs from atom to atom. For ease of understanding, let us first consider the operation of a gas laser. As far as emission of light is concerned, the gas atoms can be categorized into multiple groups, and each group has its own unique frequency of emission. The number of atoms belonging to any particular group also varies widely. The situation is schematically illustrated in Fig. 8.4, which captures a representative behavior of emission of individual atoms belonging to a few groups as well as the emission profile of all of them putting together. As seen, the number of atoms capable of emitting at ν_0 is maximum, and consequently the intensity of fluorescence at this frequency will also be maximum. The broadening of the fluorescence of individual groups, governed here by the radiative lifetime of the excited atoms, is homogeneous and hence Lorentzian. Clearly, the intensity of fluorescence at any given frequency is proportional to the number of atoms in the group capable of fluorescing at that frequency. The resulting inhomogeneous gain broadening of the laser is the convolution of the homogenous fluorescence broadening of all the individual groups of excited atoms and is also shown in this figure. It is of interest to note here that an inhomogeneously broadened gain profile of a gaseous medium is represented by a Gaussian distribution function [25].

The obvious question that arises here is what causes the frequency of fluorescence of one excited atom to differ from another. The answer to this can be found in the phenomenon of the optical Doppler effect. Named after the Austrian physicist Christian Doppler (1803–1853) who discovered it in 1842, our familiarity with the acoustic Doppler effect dates back to almost the beginning of the nineteenth century when steam powered locomotives began commercial operation. It is common knowledge that to a passenger waiting at a railway station, the whistle of an approaching engine appears high pitched and that of a receding engine seems low

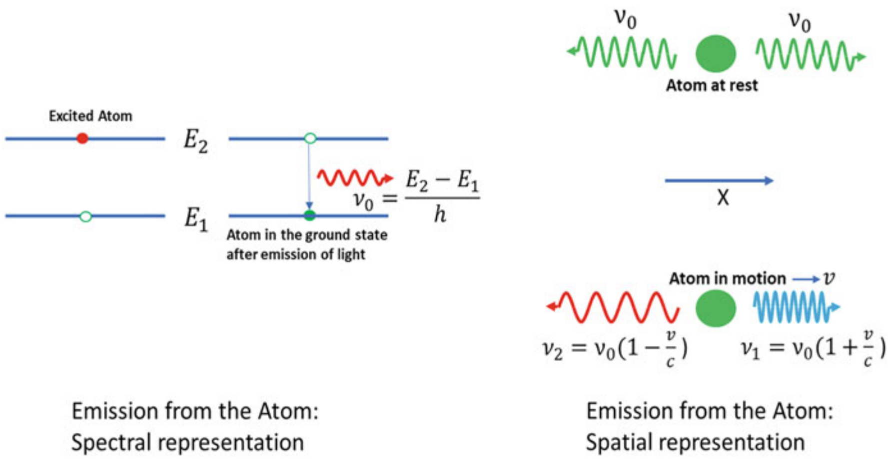

Fig. 8.5 Left: An atom excited to a state of energy E_2 comes down to the ground state of energy E_1 releasing its energy of excitation E_2-E_1 as a photon of frequency ν_0. Right: Emission from the static and moving atoms is shown spatially. The photon can be emitted randomly in any direction. When at rest, the frequency of the emitted photon is always ν_0; emissions along +ive and -ive X direction are shown here. When in motion, the frequency of the photon is blueshifted when emitted in the direction of travel and redshifted in the opposite direction

pitched. This means that when a sound producing source moves toward an observer, the observer perceives the frequency of sound to be higher. If the source, on the other hand, moves away from the observer, its frequency would appear to be lower. The faster the source moves, the greater this frequency shift.

Similar to sound waves, light waves also undergo a similar effect. To gain insight into the optical Doppler effect, we replace the sound emitting engine with a light emitting atom and examine how the frequency of light given out by a mobile atom will differ from that emitted by it when at rest. The situation is represented in the traces of Fig. 8.5. The emission of photons based on the energy level diagram is indicated in the left of this figure, while its right is a spatial representation of the emissions in the two cases, viz., when the atom is at rest and in motion. As we know, the photon can be emitted by the atom in any direction around it. However, when the atom is at rest, the frequency of the photon ν_0 is independent of its direction of travel and essentially carries the excitation energy of the atom. Thus,

$$\nu_0 = \frac{E_2 - E_1}{h} \tag{8.1}$$

where E_2 and E_1 are the energies of the excited and ground states, respectively, and h is Planck's constant. When the atom is in motion, the phenomenon of the Doppler effect comes into being, and consequently the frequency of the emitted photon now depends on the direction of emission. For the sake of simplicity, the emissions have been indicated only in two directions, along and opposite to the direction of travel of the atom. This does not mean that the emissions in the remaining directions are not important, of course they are, and as a matter of fact, we shall study the optical Doppler effect more exhaustively later in this chapter. The photon's frequency increases along the direction of atomic motion and reduces in the opposite direction and is consequently termed as blue- and redshifts of light,[2] respectively. The blue ν_1 and red ν_2 shifted frequencies can be expressed as

$$\nu_1 = \nu_0\left(1 + {}^{v}\!/_{c}\right) \tag{8.2}$$

and

$$\nu_2 = \nu_0\left(1 - {}^{v}\!/_{c}\right) \tag{8.3}$$

where ν_0 is the frequency of the photon that the atom emits when at rest, v is its velocity with which the atom is moving, and c is the velocity of light.

Upon acquiring this rudimentary knowledge on the optical Doppler effect, we are now in a position to take in hand the question, posed earlier, pertaining to the origin of inhomogeneous gain broadening in a gas laser. It is well known from the kinetic theory of gases that gas atoms/molecules move randomly with wide ranging velocities. The gas species are distributed within this velocity spread satisfying Maxwell's velocity distribution [26], which is critically dependent on the gas temperature. Thus, individual atoms, in the case of an atomic gas laser, move about at random velocities and emit light of a frequency, Doppler shifted by an amount dependent on the velocity. Taking another look at Fig. 8.4, some of these Doppler shifted frequencies can be readily associated with emissions at ν_6, ν_5, ν_4, ν_0, ν_1, ν_2, ν_3. As the velocity spread in the Maxwellian distribution is continuous, the Doppler shift when accounted for each individual excited atom would, in entirety, result in an inhomogeneous broadening of the laser gain. With increasing temperature, as the gas gets hotter, the faster the atoms move on average and the wider the laser gain broadening is. Doppler broadening, which is intrinsically inhomogeneous in nature, therefore increases with temperature.

It is pretty clear now that atoms belonging to any particular group of Fig. 8.4, capable of fluorescing identically at a point of time, do so as they possess the same velocity at that instant. It does therefore make sense to redraw Fig. 8.4 by portraying the fluorescence of each individual group rather than that of its constituent atoms. Figure 8.6 depicts qualitatively this new representation. As seen, the height of the fluorescence peaks that represent the intensity of fluorescence differs from group to group in line with the changing number of their constituent atoms. The linewidth of emission, however, is invariant for the following reason. Although the atoms belonging to different velocity groups fluoresce at different frequencies owing to the Doppler shift, their emission width that is governed by the excited level lifetime

[2]Red- and blueshift of light is the phenomenon wherein light undergoes a shift in its wavelength. The shift is red when the wavelength increases and it is blue when the wavelength decreases.

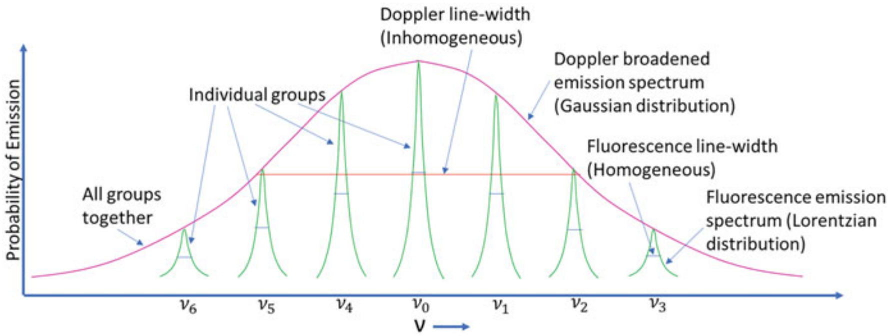

Fig. 8.6 Figure 8.4 redrawn depicting the fluorescence of each individual groups. Although the height of the fluorescence peaks differs from group to group, their linewidths remain unchanged

of the atom will remain unchanged. The convolution of the line broadening of all the individual groups represents the overall broadening of the lasing transition, which indeed is inhomogeneous and of Doppler origin. The lifetime induced broadening, as we know now, is homogeneous in nature. As long as the gas pressure is low and we conveniently ignore collisional broadening, the lifetime and the corresponding natural linewidth of an individual excited atom will be decided basically by its radiative lifetime, which is smaller than the Doppler broadening unless the gas is cooled to reduce it. At low operating pressures, the gain of a gas laser is thus predominantly inhomogeneously broadened. With increasing gas pressure, collisions begin to reduce the lifetime, and at high enough pressure, the homogeneous component of the overall broadening may outweigh the inhomogeneous part. It is of interest to note here that inhomogeneous gain broadening may also arise in the operation of certain solid-state lasers due to the presence of imperfections in the host material such as the case of an amorphous host like glass.

8.3 Gain Clamping

Gain clamping is a phenomenon that occurs when the laser gain becomes equal to the optical cavity loss and gets locked to it. Any attempt to make the gain overtake the loss by increasing the pump power would prove futile as the gain stays firmly tied with the loss line at the point of cavity mode frequency where lasing has set in. Although this may seem to contradict what has been stated earlier in many of the chapters with regard to the onset of lasing, the ensuing discussion will nevertheless help resolve the issue. For this, let us consider the *cw* operation of a homogeneously broadened laser. One obvious way to increase the gain of the laser is to increase the strength of pumping such as the magnitude of the current in the case of electrical pumping or the density of pump photons in the case of optical pumping. We, for the sake of convenience, shall confine ourselves to the case of electrical

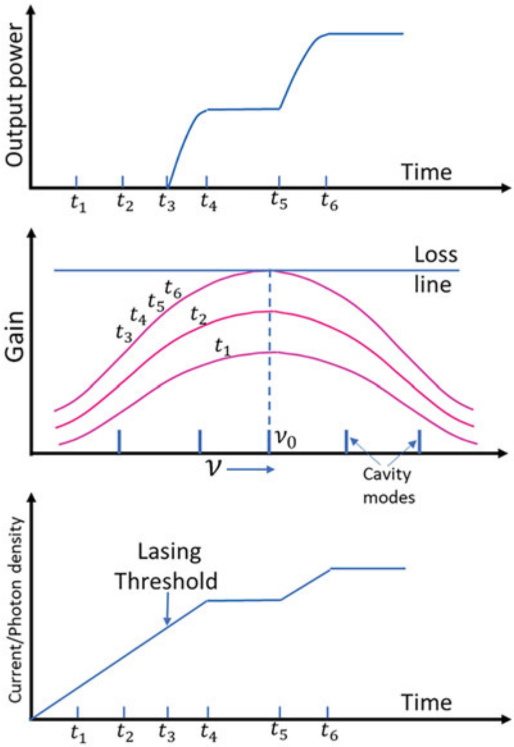

Fig. 8.7 A typical variation of the pump current or photon density in the operation of a cw laser is shown (bottom trace). Gain as a function of frequency at few specific times beginning from the initiation of the process of pumping is shown in the middle trace where the location of the loss line is also indicated. The laser output as a function of time is displayed in the top trace

pumping alone hereafter in this chapter. Raising the current over time (Fig. 8.7, bottom trace) will also steadily increase the population inversion and, in turn, the gain. Gain, as we know, does not occur on a single frequency but instead spreads across the entire broadened, considered here to be homogeneous, lasing transition. The variation in gain with frequency is also captured in this figure (middle trace) at a few specific times beginning from the initiation of the pump. The peak value of the gain, as we know, always occurs at the line center frequency ν_0 of the lasing transition. The cavity modes that happen to lie within the gain profile have also been indicated here. For the sake of convenience, frequency of one of the cavity modes is shown to be ν_0. Cavity loss is usually independent of the frequency and thus has been represented here as a line parallel to the frequency axis. In the beginning (time t_1), the pump current is low, and consequently the gain lies much below the loss line and the process of lasing cannot obviously begin. At time t_2, the gain has risen to some extent in accordance with the increased value of the current but still lies below the loss line, and so there is no lasing yet. After a time t_3, the gain becomes high enough to touch the loss line at the line center frequency ν_0 causing the onset of lasing at this cavity mode frequency. This point has been indicated in the current vs. time curve (bottom trace) as the “lasing threshold.” The plot thickens here

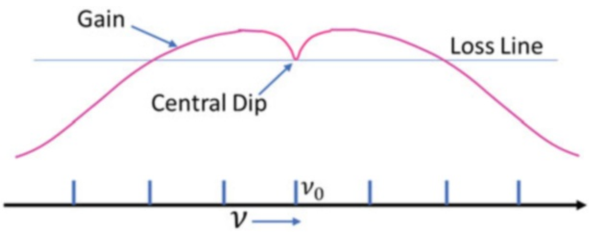

Fig. 8.8 If the cavity mode with the maximum gain, on which lasing invariably sets in, were incapable of burning the gain linked to the other frequencies, a spectral hole on the gain with a dip at the center will be formed inevitably

as the rising current attempts to push the gain beyond the loss, and the stimulated emission (SE), on the other hand, tries to pull it down.[3] With the growing density of stimulating photons inside the cavity, the rate of SE builds up so rapidly that in just a flash, the rate at which the gain builds up due to pumping equals the rate at which it is burnt out by the SE. It is equivalent to saying that the gain stays clamped to the loss line from the moment the amplification of light sets in the process of lasing. Raising the pumping strength beyond this threshold point will allow the gain to rise momentarily above the loss, and the increased rate of SE, as a consequence of the increased gain, will bring it down to the loss line practically at that moment. This means that a new equilibrium is established at a correspondingly increased power of intracavity light commensurate to the increased level of pumping. As a fraction of the intracavity light emerges as the laser beam, during the time interval t_3 to t_4 when the pump current continues to rise (lower trace of Fig. 8.7), the laser output will also exhibit a monotonic rise (upper trace of Fig. 8.7). In the operation of a laser, upon reaching the required output power, the current remains unchanged (during the interval t_4 to t_5) unless there is a requirement to boost the output. This calls for raising accordingly the magnitude of the current, and the laser output will reflect the changing pattern of current as long as the heat dissipated into the lasing medium as a consequence of lasing (and elaborated in the preceding chapter) is rapidly removed. The gain, needless to say, will remain clamped to the loss all the while.

Clearly thus, the onset of lasing at the cavity mode frequency ν_0, where the homogeneously broadened gain touches the loss line first, triggers the phenomenon of gain clamping. One may begin to wonder at this point as to what prevents gain from rising on the neighboring frequencies where it is still below the loss, thereby establishing a dip at ν_0 as illustrated in Fig. 8.8. The answer to this basically lies in the conduct of a homogeneously broadened gain medium vis-à-vis the emission of light. We know that every excited atom of a homogeneously broadened gain medium can emit at any frequency that lies within the domain of broadening. Returning to the situation described by the middle trace of Fig. 8.7, the gain, which exhibits a

[3]That each stimulated emission adds one photon to the intracavity light and reduces the population inversion by two has been touched upon in quite a few preceding chapters. Clearly, therefore, the stimulated emission causes amplification of light at the expense of the gain.

maximal behavior at ν_0, will surely touch the loss line first at this frequency setting off the process of lasing here. Being a homogeneously broadened system, gain belonging to every other frequency will also contribute to the growth of light at ν_0 ruling out any possibility of the gain touching the loss line at any frequency other than ν_0 let alone crossing it. The appearance of a central dip on a homogeneously broadened gain profile, therefore, cannot happen. (As we shall determine in a latter section of this chapter, appearance of such a dip on the gain profile, often referred to as spectral hole, is, however, inevitable in the operation of an inhomogeneously broadened laser.) It therefore appears that in the case of a homogeneously broadened laser, the mode at which lasing begins will continue to lase by burning the entire gain of the active medium ruling out the possibility of lasing on another mode. This is equivalent to saying that a homogeneously broadened laser will always lase on a single mode, a statement that, as we shall find out in the following section, is not always true.

8.4 Homogeneously Broadened Laser and Spatial Hole Burning

We have thus far confined ourselves to the spectral broadening of gain and have arrived at a conclusion that the phenomenon of gain clamping should force a homogeneously broadened laser to lase only on one mode. We intend now to delve into the possibility of a spatial effect on gain, invariably present in a linear cavity such as Fabry-Perot (FP), profoundly impacting the emission feature of such a laser, and the same is underlined in the traces of Fig. 8.9. Let us begin by considering a homogeneously broadened gain medium placed inside an FP cavity of length l. For the sake of convenience, both mirrors forming the cavity have been assumed to be fully reflective although in reality one of them will be partly leaky allowing extraction of the laser beam. The spectral profile of gain, the loss line, and a few relevant cavity modes have been indicated in trace a. As seen, the cavity mode of wavelength λ_1 experiences the highest gain, and consequently lasing sets in at this wavelength. It is imperative at this point to examine as to how the onset of lasing will spatially modify the gain that initially is invariant across the entire length of the active medium (trace b). The forward and backward waves will interfere in an FP cavity to produce a standing wave[4] characterized by the presence of nodes and antinodes (trace d). The nodal planes are devoid of any radiation flux, while the antinodal planes have the maximum share of it. Clearly, the population in the antinodal regions will be eaten up by this mode, while the same will remain practically unutilized across the nodal planes. This is equivalent to saying that spatial holes will be burnt on the population inversion and, in turn, gain. These spatial holes,

[4]When two counterpropagating waves are superimposed, they interfere to form a standing wave interference pattern, the period of which is half the wavelength of the waves.

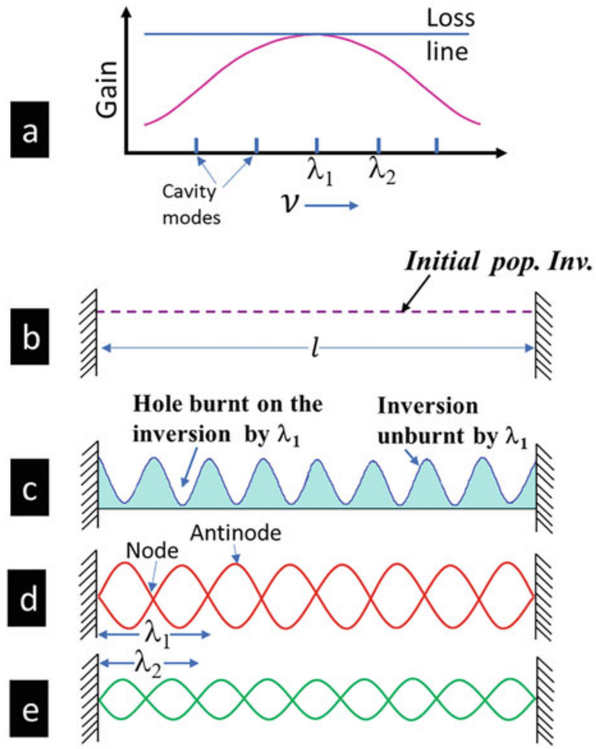

Fig. 8.9 The cavity mode of wavelength λ_1 has the highest gain (trace a). Upon placing the active medium inside a Fabry-Perot cavity of length *l* (trace b), lasing sets in on this mode that forms a stationary wave (trace d). The population inversion is burnt at the antinodal planes and remains unutilized across the nodal planes (trace c). An adjacent mode of wavelength λ_2 with lower gain can manage to survive by taking advantage of this unburnt gain (trace e)

occurring periodically on the population inversion (trace c), enrich the dynamics of a homogeneously broadened laser, as we shall below, in a remarkable manner. Let us consider the case of an adjacent mode of wavelength λ_2 that by virtue of possessing lower spectral gain (trace a) would, under normal circumstances, not be able to lase. However, as the standing wave pattern of one wave will have a spatial shift with respect to the other (trace d and e), the weaker mode can break into lasing by using up the gain that remained unutilized by the stronger mode. The amplitude of the standing wave for λ_2 is deliberately kept smaller to emphasize its feebleness. It should be noted that the weaker mode is not required to compete with the stronger mode for its survival as it basically feeds on the gain unseen by the other. In a nutshell, in an FP cavity, where the formation of a standing wave pattern is inevitable, a homogeneously broadened laser can therefore lase on multiple cavity modes by exploiting the phenomenon of spatial hole burning. It would be intriguing in this context to examine the case of a ring cavity that is capable of supporting a traveling wave.

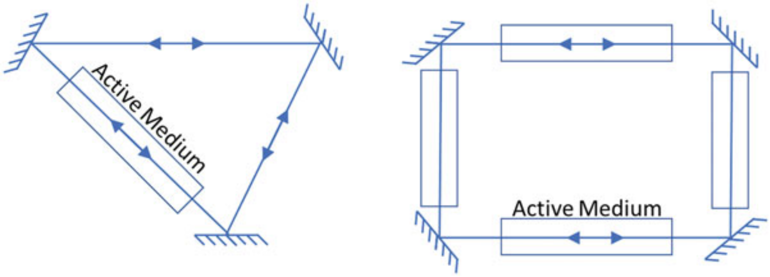

Fig. 8.10 Schematic representation of three- (left trace) and four-mirror (right trace) bidirectional ring cavities. The four-mirror ring is shown to contain active medium in all its arms basically to underline the fact that a ring cavity has a built-in ability to accommodate gain medium of longer length

8.5 Homogeneously Broadened Ring Cavity Laser

Unlike an FP cavity that is basically straight or linear, a ring cavity is not straight and is formed by at least three mirrors or more. Examples of ring cavities comprising three and four mirrors, which are most common, are depicted in Fig. 8.10. The active medium can be placed in any one of the arms of the cavity, as is shown in the case of the three-mirror ring cavity. The ring configuration, however, allows boosting the laser power by placing multiple gain cells in its different arms, as depicted in this figure. When the mirrors forming the cavity are perfectly aligned, the light will be amplified as it goes round and round inside it. A simple ring cavity, such as the ones shown here, will allow waves to travel in both forward and backward directions. The counterpropagating waves thus will once again interfere, as in the case of a linear FP cavity, with each other to produce a standing wave pattern. The phenomenon of spatial hole burning will therefore also be present here, and consequently, a homogeneously broadened laser will also lase here, like an FP cavity, on multiple modes, the only difference being the forward and backward waves will result in bidirectional emission from the ring laser. In the case of a four-mirror ring cavity, the two output beams will emerge orthogonally through the partially transparent mirror, as shown in Fig. 8.11. Understandably, therefore, to capitalize on the advantage offered by the ring cavity, one of the counterpropagating waves must be repressed. The ring cavity then essentially becomes a traveling wave cavity generating a single output beam. An optical valve that lets the passage of, say, the clockwise light and blocks the counterclockwise beam, much the same way as a diode blocks flow of electric current in one direction, will essentially effect operation of a unidirectional ring cavity (Fig. 8.12). An example of such an optical valve is a Faraday isolator, a device that rotates the polarization state of light by exploiting the magneto-optic effect[5] which is both expensive and delicate. A rudimentary optical apparatus that is capable

[5]The magneto-optic effect refers to the modulation of the properties of light when it interacts with magnetic field.

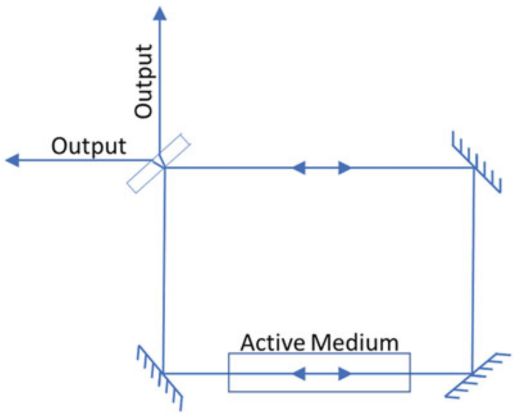

Fig. 8.11 A laser based on a simple ring cavity that intrinsically allows oscillation of both clockwise and counterclockwise waves will yield two output beams. In case of a four-mirror cavity, the two beams will emerge orthogonally

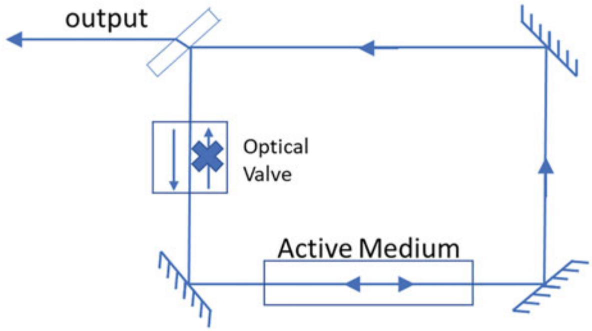

Fig. 8.12 Insertion of an optical valve, which blocks light moving in one direction and allows its passage in the opposite direction, will essentially make it a travelling wave cavity

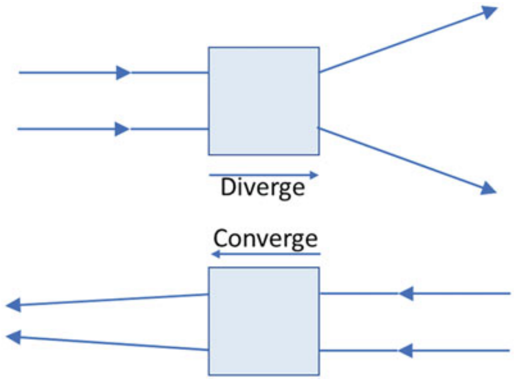

Fig. 8.13 A telescope is an optical apparatus that would converge a beam of light in one direction and diverge it in the opposite direction

of providing selective loss to light in one direction can also prevent formation of standing waves when introduced inside the ring. A simple telescope that diverges light in one direction and converges in the other will also fit the bill as an unsophisticated optical valve here. The same is illustrated in Fig. 8.13. The converging light, by virtue of possessing a higher density of photons, will grow so rapidly by bleaching the gain that the diverging light will simply disappear in practically no time. In such a unidirectional or traveling wave ring cavity, where the formation of standing waves is clearly unfeasible, a homogeneously broadened laser will always lase on only one cavity mode that has the highest spectral gain. A homogeneously broadened unidirectional ring laser thus has the built-in capability of manufacturing coherent light with color of extreme purity. Although purity of color is compromised in the operation of a bidirectional ring laser, it, however, presents a situation conducive for a host of interesting applications. Most notable is the ring laser gyroscope that operates on the principle of the Sagnac effect [27] which is widely used for navigation in moving vessels like airplanes, ships, submarines, automobiles, and missiles. The ginormous ring lasers built specifically for LIGO (Laser Interferometer Gravitational-wave Observatory) experiments designed to detect gravitational waves are yet another widely known application of such a bidirectional laser. The laser gyroscope and LIGO system will be elaborated in detail in the second volume of this book dealing with the impact of lasers in science and humanity.

8.6 Inhomogeneously Broadened Laser and Spectral Hole Burning

We have up till now restricted ourselves to the manifestation of the phenomenon of gain clamping in the operation of only homogeneously broadened FP and ring cavity lasers. It is now time to examine as to how gain clamping will affect the operation of an inhomogeneously broadened laser. As the presence of inhomogeneous broadening is inevitable in a gas laser, we consider here a gaseous active medium placed inside an FP cavity of length l. The broadening of the gas laser, as a matter of fact, is an admixture of inhomogeneous and homogeneous components. Clearly the inhomogeneous broadening that originates from the velocity distribution of the gas atoms (or molecules) will increase with the gas temperature. The gas pressure, on the other hand, determines the homogeneous part of the broadening. For low or moderate gas pressures, the dominating mechanism of gain broadening will therefore be inhomogeneous, and the same applies to the present situation. With increasing pumping current, the population inversion and, in turn, the gain progressively increases. We pick a case when the gain has momentarily risen above the loss line setting off the process of lasing at the cavity mode that for the sake of convenience has been assumed to coincide with the line center frequency ν_0. The situation is qualitatively represented in Fig. 8.14. The group of atoms that can emit at ν_0 will only contribute to the process of lasing here as the gain stays clamped at this frequency. The gain at the frequencies neighboring to ν_0, being inhomogeneous, cannot participate in the process of lasing at this frequency and will remain unburnt. Consequently, a spectral hole will be burnt on the gain symmetrically about ν_0, and the width of the hole will understandably be homogeneous, governed basically by the gas pressure. As the

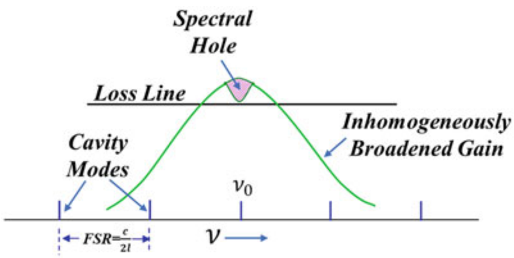

Fig. 8.14 Formation of spectral hole in the operation of an inhomogeneously broadened laser

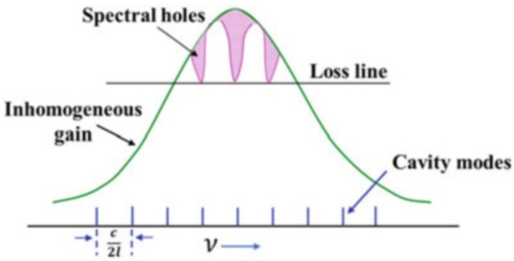

Fig. 8.15 Formation of spectral holes in the operation of an inhomogeneously broadened laser facilitates multimode operation of such lasers

FSR of the cavity in this example exceeds the lasing bandwidth, the lasing will be restricted only to the central mode that possesses the highest spectral gain. It is imperative at this point to examine the possibility of multimode lasing by considering a situation where the lasing bandwidth exceeds the FSR, and the same has been qualitatively illustrated in Fig. 8.15. Three cavity modes can be seen in this example to lie within the window of lasing. As, unlike in the case of homogeneous broadening, different groups of excited atoms are capable of emitting on these three different cavity mode frequencies, spectral holes will now burn on all three modes. This would facilitate the operation of the laser in multimode of which the line center mode possessing the highest spectral gain will be the dominant one. The spatial hole burning effect that is omnipresent in FP or bidirectional ring cavities will further aid the process of lasing on multiple modes just the way it happens for a homogeneously broadened laser. Removal of spatial effects by going for a unidirectional ring cavity cannot prevent lasing on multiple modes, as the prevailing spectral burning of holes in the gain will ensure their survival. Not surprisingly therefore, as far as purity of emitted light is concerned, an inhomogeneously broadened laser is prone to falling behind a laser with homogeneously broadened gain.

8.7 Spectral Hole Burning and Lamb Dip

Willis E. Lamb (1913–2008), regarded as a theoretician turned experimental physicists, predicted in 1964, a decade after winning Nobel Prize for discovering the famed Lamb shift, the occurrence of a dip at the line center in the output of a single mode Doppler broadened laser. Popular now as “Lamb dip,” this is basically a manifestation of the optical Doppler effect. This dip occurring in lasers and its inverse counterpart in absorptive media enabled not only the accurate determination of the line center of an atomic (or molecular) transition but also the creation of light of mind-boggling spectral purity. That the unwrapping of physics enshrouding the "Lamb dip" has led to the addition of a new dimension to the science of spectroscopy, viz., Doppler free spectroscopy, offering unprecedented spectral resolution, is now a history. We shall study Doppler free spectroscopy in greater detail later in volume II of this book and determine how the advent of it has made possible to take a look where no one has looked before.

Toward gaining insight into the origin of the "Lamb dip," we take a closer look into the operation of a tunable single mode Doppler broadened laser. Tunability in the present context implies that the frequency of the oscillating mode can be scanned across the lasing bandwidth. Let us begin by considering a gaseous active medium with Doppler (inhomogeneous) broadened gain that is placed inside an FP cavity of length l (Fig. 8.16). Figure 8.17 depicts the gain as a function of frequency, the loss line, and a few pertinent cavity modes. In order to maintain a condition conducive for single mode lasing, the gain and loss are chosen here to ensure that the FSR of the cavity exceeds the lasing bandwidth. This will guarantee that at any point in time, only one mode can lie within the frequency domain $(\nu_B - \nu_A)$ of the gain intercepted by the loss line. Our endeavor here is to monitor the output power of the laser as a function of the frequency of the lasing mode by moving it across this lasing window from ν_A, the point of onset of lasing, to ν_B, the point of its termination. We know that the cavity mode frequency ν and the length l are related through the following cavity resonance equation:

$$\nu_n = \frac{nc}{2l}, \text{ where } n \text{ is the cavity mode index and } c \text{ is the velocity of light} \quad (8.4)$$

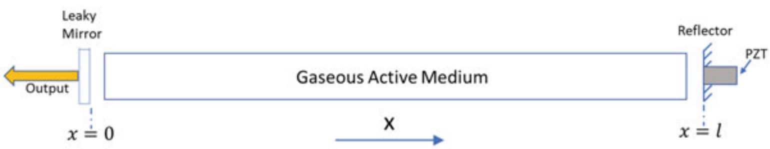

Fig. 8.16 Schematic of a single mode laser with provision of PZT driven cavity length tuning

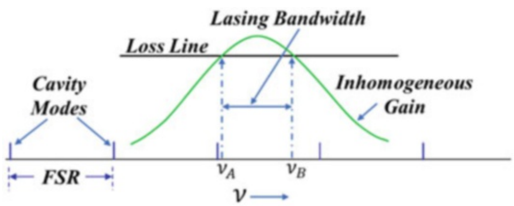

Fig. 8.17 This depicts the gain, the loss line, and a few cavity modes lying within and in the vicinity of the gain to essentially reflect the single mode character of this inhomogeneously broadened laser

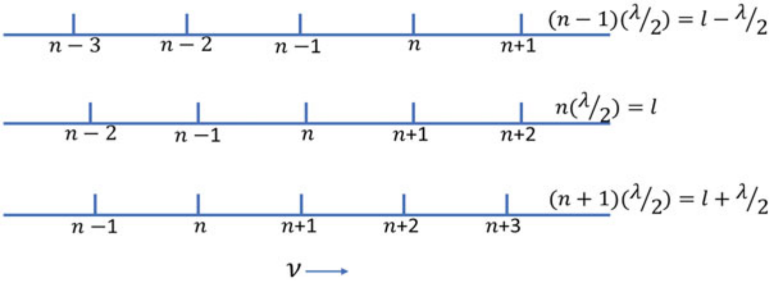

Fig. 8.18 This illustrates that the cavity mode belonging to a certain wavelength sweeps through the entire FSR if the cavity length is either increased or decreased by half that wavelength

Changing the cavity length is a way to shift the mode frequency, and this can be readily accomplished by translating any one of the mirrors forming the cavity. The obvious question that arises here is through what distance the mirror must be moved to ensure scanning of the entire lasing window by the lasing mode. To answer this question, we rewrite the resonance equation replacing frequency with wavelength as follows:

$$n \times \lambda/2 = l \tag{8.5}$$

$$\rightarrow n \times \lambda/2 \pm \frac{\lambda}{2} = l \pm \frac{\lambda}{2} \tag{8.6}$$

$$\rightarrow (n \pm 1)\frac{\lambda}{2} = l \pm \frac{\lambda}{2} \tag{8.7}$$

A comparison of Eqs. 8.7 and 8.5 helps us to conclude that if the cavity length is increased or decreased by half of the wavelength, then the mode belonging to this wavelength sweeps through the entire FSR of the cavity. This fact has been amply illustrated in the traces of Fig. 8.18. Thus, discerning the distance through which the mirror must move becomes straightforward once we know the fraction of FSR over which lasing persists. For instance, if the lasing bandwidth is half of the FSR, then any one of the cavity mirrors must move through $\lambda/4$ toward or away from the other. Considering the case of a visible laser that operates, say, at 500 nm wavelength, $\lambda/4$ measures to be 125 nm, a distance too small to be realized by the translation of a cavity mirror in a traditional manner. To this end, the piezoelectric transducers[6] (PZT), which exhibit extremely minute changes (to the tune of submicron level) in the longitudinal dimension upon application of a very high transverse electric field,

[6]A piezoelectric transducer (PZT) is a device that exhibits a minute change in length upon application of a voltage across its side. The cavity length of a laser can be controlled in a precise manner by mounting one of its cavity mirrors on a PZT.

come in handy. As illustrated in Fig. 8.16, one of the cavity mirrors, preferably the fully reflective one, when mounted on a PZT will allow a change in cavity length in a precise and controllable manner upon varying the magnitude of the voltage applied to it. We are thus now in a position to accomplish the task of monitoring the output power of this single mode Doppler broadened laser as a function of its emission frequency.

The condition prevailing here has been qualitatively described in Fig. 8.17. It essentially presents a case of single mode lasing as the cavity FSR exceeds the lasing bandwidth $\nu_B - \nu_A$. A situation has been captured where none of the cavity modes lie within the domain of lasing and the mode on which the onset of lasing is most likely to occur is located just at the left of ν_A. Upon application of an appropriate voltage across the PZT, this mode can be made to progressively move to the right. Lasing that sets in at the point of the mode crossing ν_A will continue until it moves past ν_B. The objective of this study is essentially to monitor the output of this single mode laser as the mode sweeps through the lasing bandwidth. Doppler broadening is inhomogeneous, and only a specific group of atoms contributes to lasing for each position of the cavity mode underneath the gain profile. The output power is thus expected to bear the same functional relationship with frequency as the gain. In reality, however, as shown in Fig. 8.19, the laser power exhibits a dip at the line center right where the gain actually peaks. The occurrence of such a line center dip on the power output of a single mode Doppler broadened laser was first theorized by Lamb in 1964 [28]. It is imperative at this point to gain physical insight into the origin of the Lamb dip that has a strong bearing on laser physics and applications.

We know from the optical Doppler effect, introduced earlier in this chapter (Sect. 8.2.2), that an excited atom, when at rest, emits light at the line center frequency and, when in motion, emits blueshifted light in the direction of its travel and redshifted light in the opposite direction. The emission from the single mode Doppler broadened laser vis-à-vis these three situations is illustrated in Fig. 8.20. Let us assume that the FP cavity of length l lies along the X-axis. In the active gaseous medium, the atoms move randomly in all possible directions and thus follow a Maxwellian velocity distribution. We, however, need to consider only the component of their velocity in the X direction, v_x, as the intracavity light travels, to and fro, along this direction alone. Let us first consider the case when the laser is tuned to ν_+, slightly

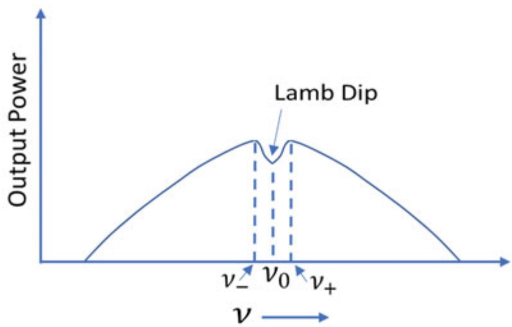

Fig. 8.19 As the frequency of a single mode gas laser is scanned across the Doppler broadened gain profile, its output shows a dip at the line center. The laser power attains maximum value at frequencies ν_+ and ν_- symmetrically located on either side of ν_0

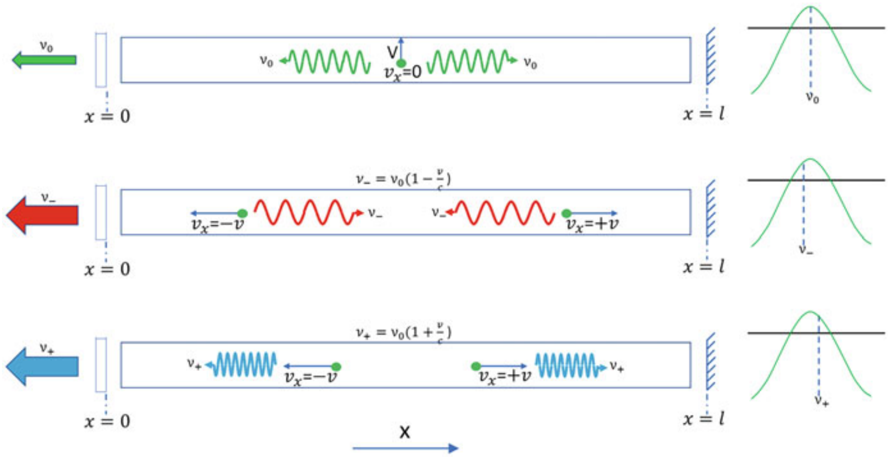

Fig. 8.20 Depending on the frequency at which the single mode laser has been tuned, different groups of atoms contribute to the process of lasing. Two velocity groups of atoms participate in lasing when the cavity mode is tuned above or below the line center, while only one group of atoms with zero velocity component along the resonator cavity contributes to lasing at the line center. Participation of two groups of atoms yields higher power, and the same is depicted in the lower and middle traces with wider arrows as the output. Participation of fewer atoms for lasing at the line center, on the other hand, results in occurrence of a dip in the power output here and consequently reduced output (indicated by a thinner arrow in the upper trace)

above the central frequency ν_0. This is therefore a case of a blue Doppler shift, meaning that the photons emitted along the direction of travel of the atoms will only be supported by the cavity. The group of excited atoms traveling with a longitudinal velocity v such that

$$\nu_+ = \nu_0\left(1 + \frac{v}{c}\right) \tag{8.8}$$

can only contribute to the output of the laser at this frequency. As shown in the lower trace of Fig. 8.20, there are basically two groups of atoms that participate here in the process of amplification of light through stimulated emissions; atoms moving with the velocity v toward the output mirror will be stimulated by light moving from right to left, and atoms moving with this velocity away from the output mirror will be stimulated by light going from left to right. Atoms moving with a velocity other than v cannot be stimulated to emit at ν_+.

In the second case, the laser has been tuned to ν_- slightly below the central frequency ν_0. This thus represents a case of red Doppler shift meaning that the photons emitted opposite to the direction of travel of the atoms will only be supported by the cavity (middle trace, Fig. 8.20). As in the previous case, there will again be two groups of atoms that contribute to the process of lasing here. However, the atoms moving toward the output mirror will now be stimulated by intracavity light traveling from left to right, and the atoms moving away from the output mirror will be stimulated by light going from right to left.

Finally, the laser is tuned to ν_0, the line center of the transition (top trace of Fig. 8.20). The atoms that can contribute to lasing at this frequency are obviously the ones that do not have any component of velocity along the X-axis, i.e., they are orthogonal to the laser cavity. Intracavity light circulating in both directions will succeed in stimulating only one group of atoms to yield output at ν_0. Thus, there are now fewer atoms available to manufacture light at the line center compared to cases when the laser is tuned slightly away from ν_0 on either side. With increasing detuning of the cavity from ν_0 on either side, atoms with progressively higher velocity components along the X-axis contribute to the process of lasing. As the number of atoms, with increasing velocity components along the laser cavity, monotonically reduces, the laser power will eventually drop with detuning after showing an initial rise in the vicinity of ν_0 on either side. Thus, when the frequency of a single mode laser is tuned across its Doppler broadened gain profile, the laser output will exhibit a dip at the line center. Clearly, this dip owes its origin to the ability of the two counterpropagating waves, which exist both in an FP and bidirectional ring cavity, to interact with atoms of two different velocity groups, under the detuned condition. It is not surprising that a unidirectional single mode ring laser will not exhibit the occurrence of a Lamb dip.

8.8 Lamb Dip and Frequency Stabilization of Gas Lasers

We have seen before (Sect. 6.6) that the fluctuation of the cavity length, inescapable in the operation of a laser, can easily mar the extreme spectral purity of the laser emission that basically stems from the ever-present spectral narrowing effect. In many applications of a laser, stability of emission frequency is an essential prerequisite. To develop an understanding of the dependence of the cavity length of a single mode laser on its frequency stability, we revisit the resonance equation of a cavity, namely,

$$\nu = \frac{nc}{2l} \tag{8.9}$$

where ν is the frequency of the n^{th} mode of the laser cavity of length l and c is the velocity of light. In order to find out as to how the change in cavity length affects the frequency of the oscillating mode, we arrive at the following equation upon deriving Eq. 8.9:

$$d\nu = -\frac{nc}{2l^2} dl$$

$$\rightarrow \frac{d\nu}{\nu} = -\frac{dl}{l} \tag{8.10}$$

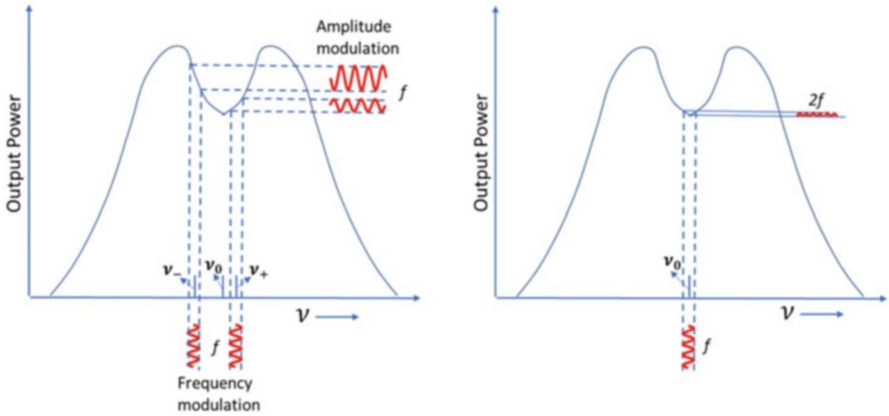

Fig. 8.21 Principle of "Lamb dip"-based frequency stabilization of a single mode Doppler broadened laser. The amplitude modulation of the laser output when the mode is tuned above and below ν_0 arising from the imposed frequency modulation is exactly out of phase (left trace). When the cavity mode is tuned to the line center, amplitude modulation vanishes at f and a weak modulation appears at 2f instead (right trace)

Clearly, any change in the cavity length is reflected in the frequency instability of the laser. Consequently, minimizing the fluctuation in the cavity length of the laser is the key to stabilizing its frequency. Although, as we have seen, the cavity length can be influenced by a number of factors, two common causes that affect it most detrimentally are temperature fluctuations and ground borne vibrations. Mounting the cavity optics on spacers made of invar[7] and the use of vibration dampers have been proven to be quite effective for stabilizing the laser cavity in a passive manner. Although some degree of stability is achievable through passive stabilization, it cannot safeguard against long-term instability. It is desirable to develop an alternative more rugged and reliable technique that would offer improved frequency stability in a persistent manner. As we shall find below, the occurrence of the line center dip on the output power of a single mode laser presents an ideal condition to develop precisely such a scheme of frequency stabilization.

The underlying principle operative here can be readily understood by referring to the illustrations of Fig. 8.21. As the cavity length of the single mode laser depicted in Fig. 8.16 is piezoelectrically tuned, making the mode frequency ν traverse across the lasing bandwidth, the generated power tuning curve will, as we know now, exhibit a dip at the line center, ν_0. We now impose a low frequency (f) sinusoidal modulation '$a \sin ft$' on the cavity length l such that $a \gg l$ and examine its impact on the laser

[7] Invar has very low thermal expansion coefficient, and therefore usage of invar spacers will make cavity length less sensitive to the temperature fluctuations.

power as a function of ν. Two situations, namely, when the cavity mode is tuned above ν_+ and below ν_- the line center, are illustrated in the left trace of Fig. 8.21. As seen, the modulation of power, which also occurs at *f*, is in phase with the length modulation when the cavity mode is located above ν_0 and out of phase when located below ν_0. However, when the cavity mode is tuned toward the line center from either side, the steady flattening of the curve manifests in a gradual reduction of the intensity of the power modulation dropping eventually to zero at ν_0. As seen on the right trace of this figure, while the modulation on power disappears at *f* at the line center, a weak modulation instead develops here at *2f*. The magnitude of the modulation of laser power therefore conveys information on the extent of detuning of the laser frequency from the line center, while its phase relative to the applied modulation indicates to which side of the dip the mode lies. The bottom line of "Lamb dip stabilization" is clearly therefore to monitor the modulation depth of the laser output and compare its phase with the imposed modulation on the cavity length all the while. An electronic servocontrol system, which forms the crux of this stabilization scheme, is shown in Fig. 8.22. The servomechanism essentially comprises an oscillator to impose PZT driven cavity length modulation, a photodetector to probe the modulated output, and a phase sensitive detection (PSD) system to compare the phases of input and output modulation and generation of the correction signal. Every time the cavity mode is drifted away from the line center, the PSD will generate a correction signal of appropriate magnitude and polarity that upon application to the PZT will pull the mode back to the line center. In the illustration of Fig. 8.22, both mirrors of the laser are PZT driven, and while modulation is imposed on the PZT holding the output mirror, the correction signal is fed to the other PZT.

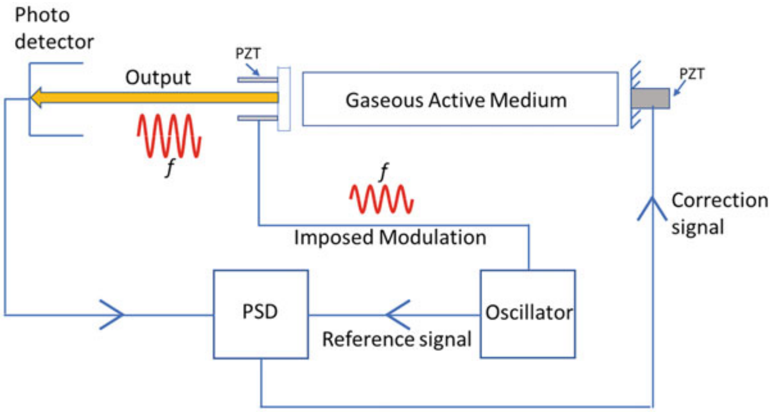

Fig. 8.22 A servomechanism upon integration with the piezo driven laser cavity generates the correction signal for application to the PZT to ensure locking of the lasing cavity mode at the line center

It is also possible to integrate the servocontrol mechanism into a laser driven by a single PZT. The popularity of Lamb dip frequency stabilization owes primarily to its inherent simplicity, and no wonder it is at the heart of a host of national standard laboratories spread across the globe like the erstwhile National Bureau of Standard (renamed National Institute of Standard Technology), USA; National Physical Laboratory, UK; and National Metrology Institute, Germany.

8.9 Inverse Lamb Dip and Stabilizing a Laser Away from the Line Center

Although "Lamb dip" offers extreme stability in the emission frequency of a laser, it however suffers from a major limitation as stabilization is possible only at the central frequency of the lasing transition. This drawback can be overcome by adding to the laser cavity, as shown in Fig. 8.23, a second cell containing another gas at very low pressure with an absorption frequency (ν_a) falling within the lasing bandwidth. This arrangement presents the possibility of using the absorption center of this gas as the new reference frequency in place of the line center frequency of the lasing transition. As we shall see below, this gaseous medium offers a ready prospect of stabilizing the laser emission on a frequency other than the laser line center. Just as it happens at the line center of the active medium, in the absorber, the two counterpropagating waves in the FP cavity will also undergo absorption by only one velocity group of atoms at the center of absorption. This will result in a narrow dip at the center of the absorption ν_a as depicted in the middle trace of Fig. 8.24. The spectral gain of the laser along with the lasing bandwidth is shown in the top trace. The convolution of

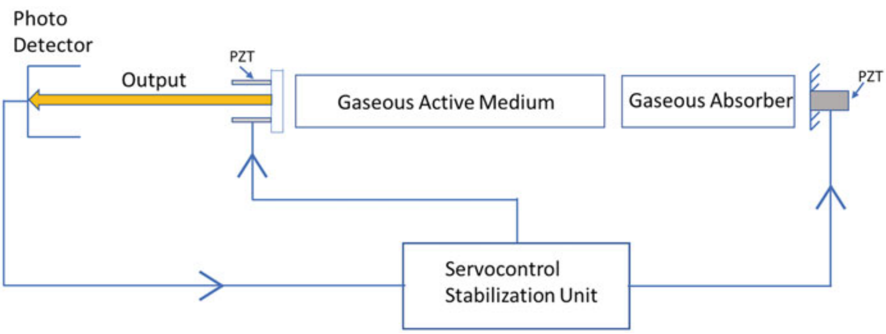

Fig. 8.23 Schematic of an "inverse Lamb dip"-based laser frequency stabilization setup. Usage of a gaseous absorbing medium in tandem with the gaseous active medium allows tuning of the cavity mode away from the line center for stabilization

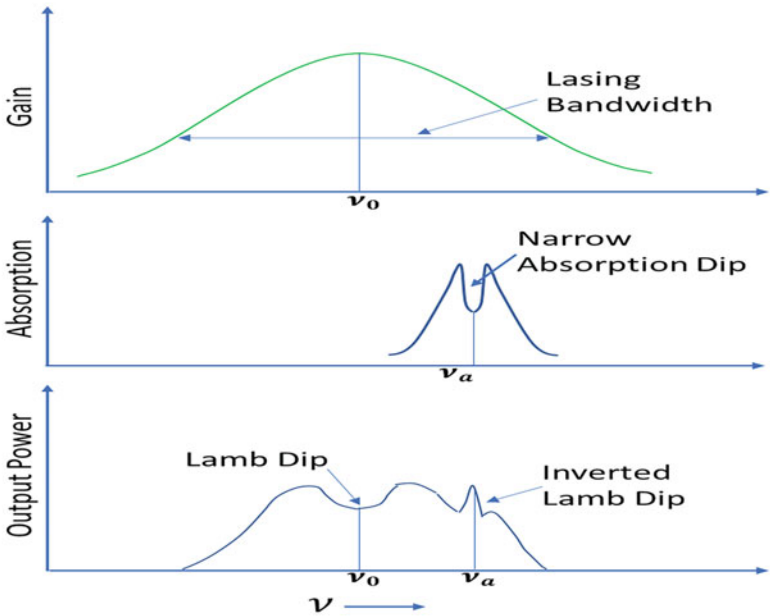

Fig. 8.24 Amalgamation of the gain in the active medium (upper trace) and absorption in the absorbing cell (middle trace) result in the occurrence of the inverted Lamb dip on the output power as the cavity mode is scanned across the lasing bandwidth of the single mode laser (bottom trace)

the two curves will manifest to produce a narrow "inverted Lamb dip" as the frequency of the single mode gas laser is tuned across its Doppler broadened gain profile (bottom trace). The inverted Lamb dip can be exploited to stabilize the laser frequency at ν_a, the absorption center of the gaseous species contained in the second cell just as the "Lamb dip" stabilizes the laser at its line center ν_0. It also offers a straightforward technique of locating ν_a and, in turn, provides a means to determine the exact center frequency of an optical transition.