



Damage Detection in Structures by Wavelet Transforms: A Review

Yasin Faghieh Larijani¹, Yaser Rostamian¹ (✉), and Samir Khatir²

¹ Department of Mechanical Engineering, Sari Branch, Islamic Azad University, Sari, Iran
yasser.rostamiyan@iausari.ac.ir

² Soete Laboratory, Ghent University, Ghent, Belgium

Abstract. Wavelet transform is a mathematical technique with many applications in signal processing. Anomaly and faults in signals can be detected using wavelet transform due to their sensitivity to local discontinuity and singularity. One of the most important applications in signal processing through wavelet transform is damage detection in structures. In this paper, a comprehensive literature review is performed on notable works about damage detection in structures by wavelet method to introduce various applications of the wavelet transforms for detecting damages in different structures. Depending on the type of signal acquired from the structures, two types of wavelet analysis can be performed: one-dimensional wavelet transform and two-dimensional wavelet transform. This paper describes one-dimensional wavelet transform analyses to clarify how the wavelet analysis may be helpful for damage detection in structures.

Keywords: Damage detection · Wavelet transforms · Signal processing · Anomaly detection · Damaged structures

1 Introduction

Analysis of damaged structure, from nano size to macro size, is an interesting topic for researchers [1–4]. Generally, damaged structures can be analyzed using two approaches: forward analysis and inverse analysis. In forward analysis, the damage is modeled, and modal characteristics of the structure are determined. In inverse analysis, the damage is determined using the modal characteristics of the structure [5, 6]. For many decades, damage detection techniques have been investigated by many researchers [7–11]. The researchers have been used different strategies for damage detection of the structures.

Some strategies are optimization-based methods and an objective function is set to be minimized or maximized in order to localize damage [12–15]. Dinh-Cong et al. [16] employed the Jaya optimization algorithm to localize damages in plates. Gomes et al. [17] used the SFOA algorithm to detect damages in laminate composites. Mishra et al. [18] proposed the ALO algorithm for identifying damages via vibration data. Maity et al. [19] suggested the GA algorithm for detecting damages of structures via shifts in natural frequencies. They demonstrated that the GA algorithm could localize damage with high precision. Also, Alexandrino et al. [20] proposed a multiobjective optimization plan to

identify damages to the plates. This optimization method revealed that using the multi-objective method, artificial neural networks (ANNs) and fuzzy inference systems can act as a proper means to identify damages. In [21], the effectiveness of the PSO and ACO algorithms to localize damages in 3D trusses was considered. The objective function suggested was established using mode shapes and corresponding natural frequencies. Gerist et al. [22] employed the IC optimization Algorithm to localize damage in engineering structures based on a novel objective function. In [23], the DE optimization algorithm was used to determine the damage's location and severity in structures.

Also, many techniques have been created in the inverse analysis in the research. One of the most contemporary inverse methods is the artificial neural network (ANN) for localizing damage's location and severity [24]. ANN is a superior means for localizing damages in the structures. Investigators usually utilized modal characteristics such as natural frequency and mode shape as the input of ANNs to identify the level of damage in various structures [25–28].

Another method of damage detection in structures is signal processing-based methods. In these methods, signals are decomposed to detect the damage's location and level using the existed information in the signal acquired for the damaged structure [29]. Wavelet transform is a signal processing-based method. In this method, a signal is transformed into sub-signals to show the damaged signal's discontinuity [30]. The singularities and discontinuities detected by wavelet transform in signals indicate damages in a specific location or time [31]. Yang et al. utilized two-dimensional wavelets for detecting damage of laminate composites [32]. Cao et al. proposed a new approach named integrated wavelet transforms to detect damages on composite beams. This method combined the continuous wavelet transforms (CWTs) and stationary wavelet transforms (SWTs) to create a powerful wavelet-based damage detection model [33]. Sohn et al. presented a wavelet-based approach for detecting delaminations [34]. Mitra et al. introduced an element-based model combined with wavelet transform to detect damages on composite beams [35]. The reviewed investigations on wavelet-based damage detection methods show that they are the simple and effective methods for damage detection in various structures since they only need signals. In this paper, Damage detection in engineering structures by wavelet transforms for various structures are reviewed.

2 One Dimensional Continuous Wavelet Transforms (1D-CWT)

The signals obtained for cables, bars, beams, and all beam-like structures are one-dimensional; therefore, a one-dimensional wavelet transform is applied to decompose these signals. Wavelet transform can be continuous or discrete. Assume the one-dimensional signal $A(t)$ having a subtle discontinuity or damage. In order to identify this discontinuity, it is required to transform its values into another coordinate system using a wavelet function called $\psi(t)$. The following expression indicates a family of wavelets [23]:

For a signal $f(x)$, , family of wavelets in real space at the Hilbert space $(\psi(x) \in L^2(\mathbb{R}))$ is defined as follows:

$$\psi(x)_{u,s} = \frac{1}{\sqrt{s}} \psi\left(\frac{x-u}{s}\right) \quad (1)$$

where x indicates the space variable related to the original signal $f(x)$, u is the shifting factor, and s denotes the scaling factor. The continuous wavelet coefficients $C(u, s)$ can be calculated as follows [36]:

$$C(u, s) = \langle f, \psi(x)_{u,s} \rangle = \frac{1}{\sqrt{s}} \int_{-\infty}^{+\infty} f(x) \psi\left(\frac{x-u}{s}\right) dx \tag{2}$$

$$C(u, s) = \frac{s^n}{\sqrt{s}} \int_{-\infty}^{+\infty} f(x) \frac{d^n}{dx^n} \theta\left(\frac{-(u-x)}{s}\right) dx = \frac{s^n}{\sqrt{s}} \frac{d^n}{du^n} f * \theta\left(\frac{-u}{s}\right) = s^n \frac{d^n}{du^n} (f * \theta_s)(u) \tag{3}$$

Figure 1 shows an example of a one-dimensional signal obtained from a beam structure and its continuous wavelet transform.

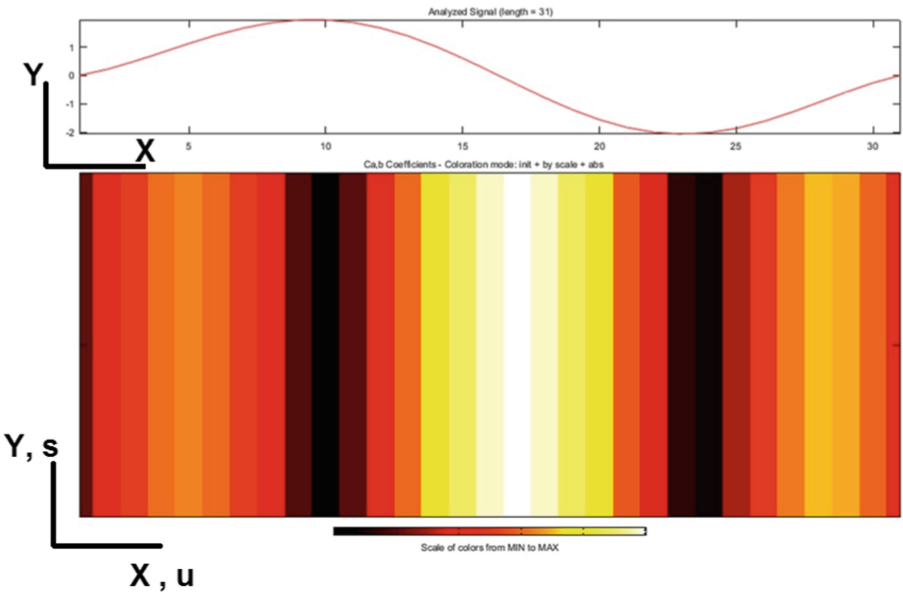


Fig. 1. An example of a one-dimensional signal obtained from a beam structure and its continuous wavelet transform.

In the continuous wavelet transform results, the brightest position horizontal axis of the wavelet coefficients diagram shows the location of the damage. Therefore, the damage is located at $X = 17$.

Janeliukstis et al. [37] conducted an experimental study for damage localization in beam-like structures based on spatial mode shape signals. Montanari et al. [38] used continuous wavelet transform for detecting cracks in beam structures. Rucka et al. [39] investigated the application of one-dimensional wavelet transforms for damage detection of beams.

Figure 1 shows the continuous wavelet transform created from the signal obtained from the finite element method; however, the number of data points in experimental

data is high. Figures 2 and 3 indicate the continuous wavelet transform created from the experimental velocity signals of an intact structure and a damaged structure, respectively. Figures 4 and 5 show the continuous wavelet transform created from the experimental displacement signals of an intact structure and a damaged structure, respectively.

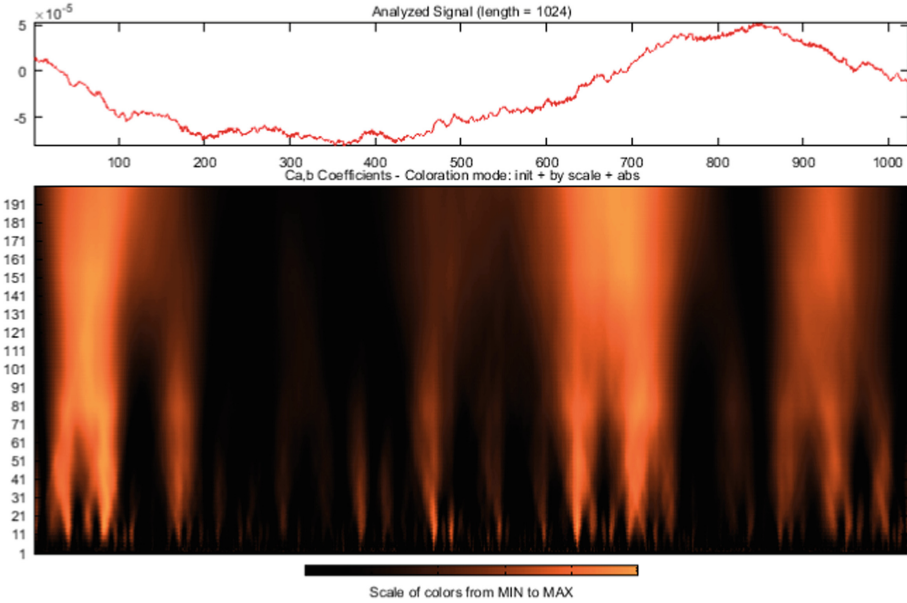


Fig. 2. Experimental velocity signal for an intact structure and its continuous wavelet transform

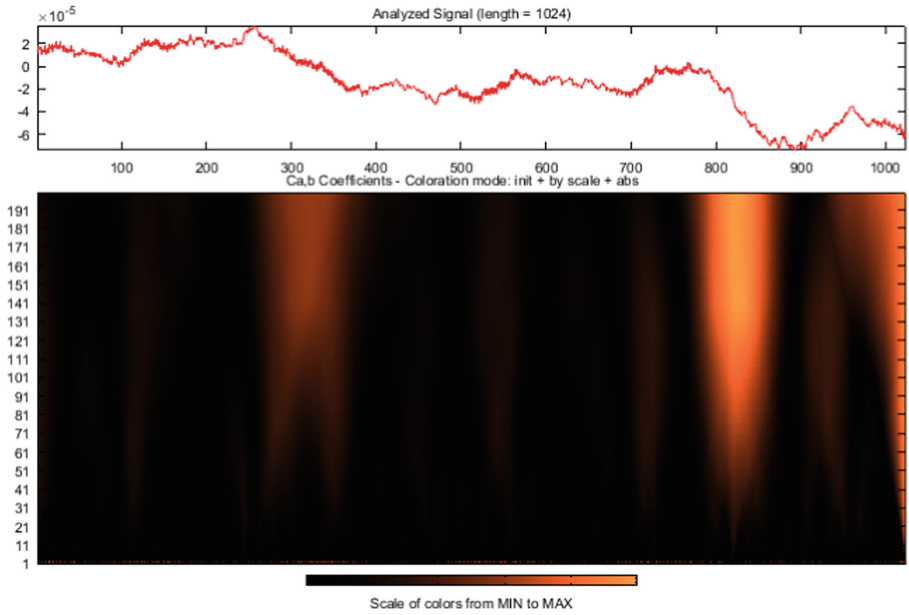


Fig. 3. Experimental velocity signal for a damaged structure and its continuous wavelet transform

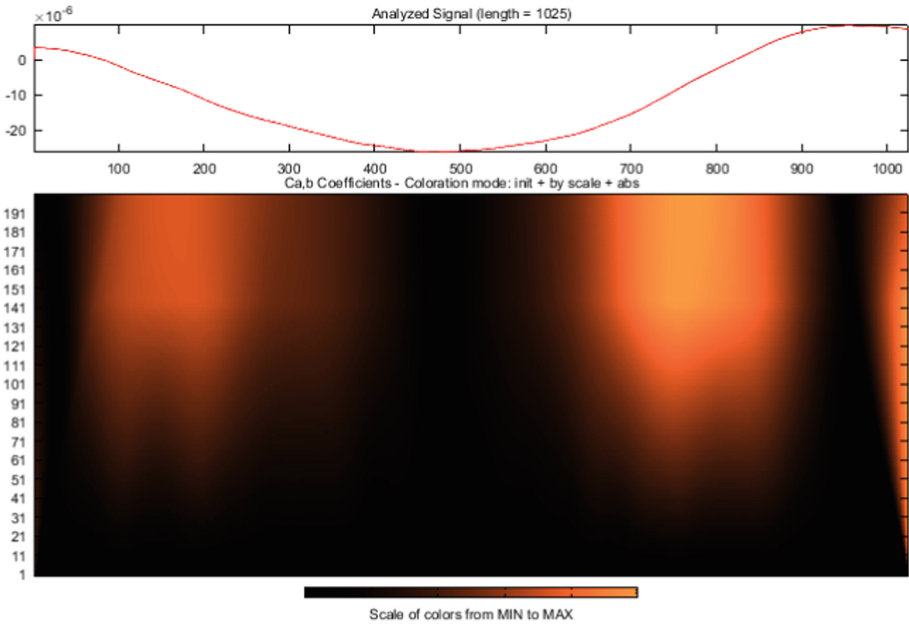


Fig. 4. Experimental displacement signal for an intact structure and its continuous wavelet transform

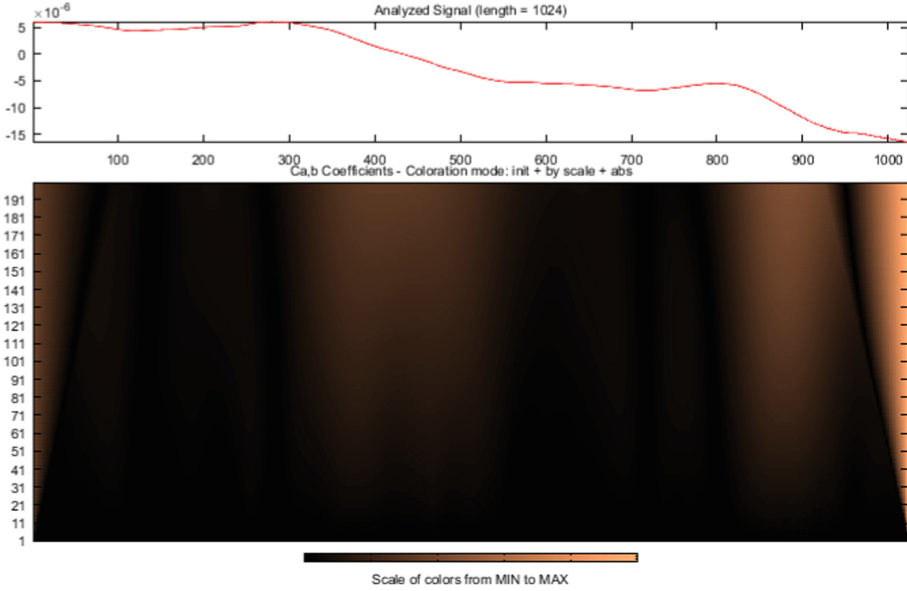


Fig. 5. Experimental displacement signal for a damaged structure and its continuous wavelet transform

3 One Dimensional Discrete Wavelet Transforms (1D-DWT)

Another type of wavelet transform is one-dimensional discrete wavelet transform. The 1D-DWT for a signal $f(x)$ is defined as follows [40]:

$$f(x) = A_j(x) + \sum_{j < J} D_j(x) \tag{4}$$

where A_j are approximation signals at level j , D_j show detail signals at level j .

The approximation signals at level j are calculated as follows:

$$A_j(x) = \sum_{k=-\infty}^{+\infty} cA_{j,k} \phi_{j,k}(x) \tag{5}$$

where $cA_{j,k}$ indicate approximation coefficients at level j . $\phi_{j,k}(x)$ are scaling functions at level j .

The detail signals at level j are expressed as follows:

$$D_j(x) = \sum_{k \in Z} cD_{j,k} \psi_{j,k}(x) \tag{6}$$

where $cD_{j,k}$ denote detail coefficients at level j . $\psi_{j,k}(x)$ indicate wavelet functions.

Figure 2 shows an example of a one-dimensional signal obtained from a delaminated composite beam structure and its discrete wavelet transform. Note that damage causes

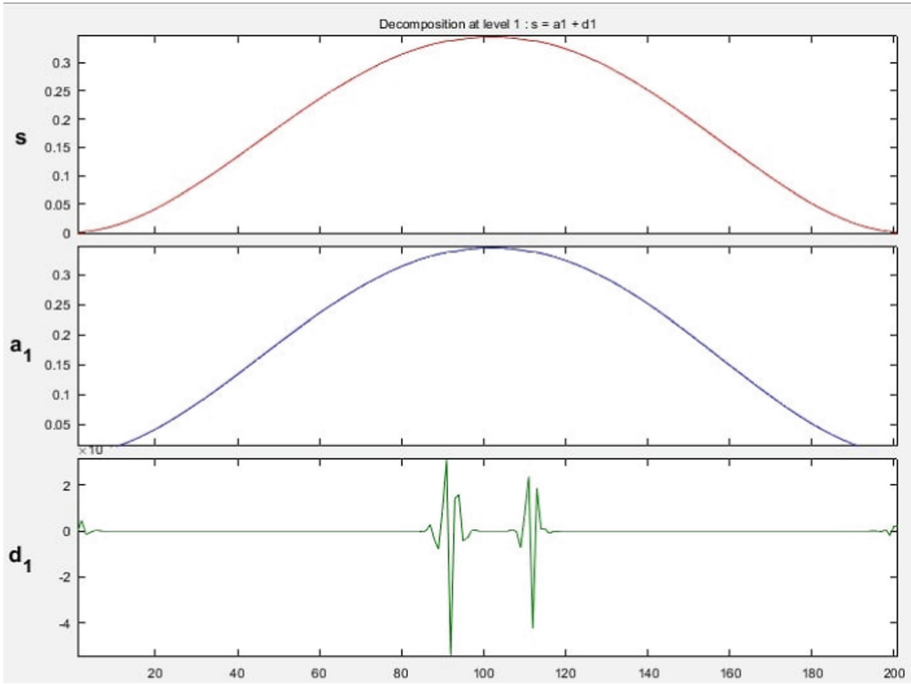


Fig. 6. An example of a one-dimensional signal obtained from a beam structure and its discrete wavelet transform.

a local jump in the signal; therefore, damage can be detected in the detail signal. As shown in Fig. 2, two jumps are observed in the detail signal. These two jumps are the boundaries of the damage delamination in a laminated composite beam.

Along with this, there are many types of research; for instance, Yang et al. [41] identified a crack in aluminum beams using discrete wavelet transform. The experimental findings showed that the suggested discrete wavelet transform could effectively determine the damage locations and be applied to more complicated structures.

Figure 6 shows the discrete wavelet transform created from the signal obtained from the finite element method; however, as mentioned, the number of data points in experimental data is high. Figures 7 and 8 indicate the discrete wavelet transform created from the experimental velocity signals of an intact structure and a damaged structure, respectively. Figures 9 and 10 show the discrete wavelet transform created from the experimental displacement signals of an intact structure and a damaged structure, respectively.

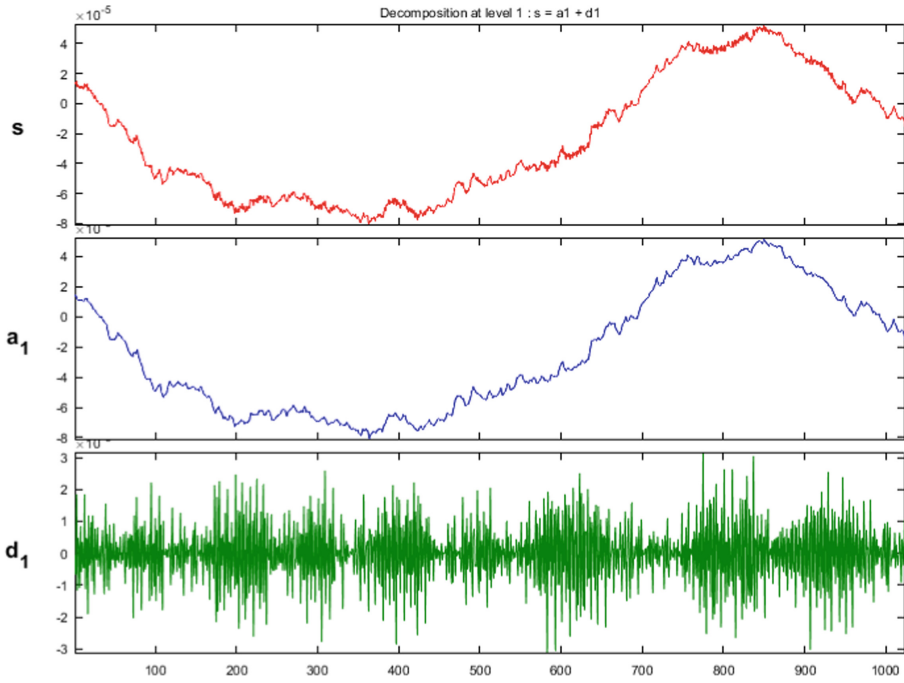


Fig. 7. Experimental velocity signal for an intact structure and its discrete wavelet transform

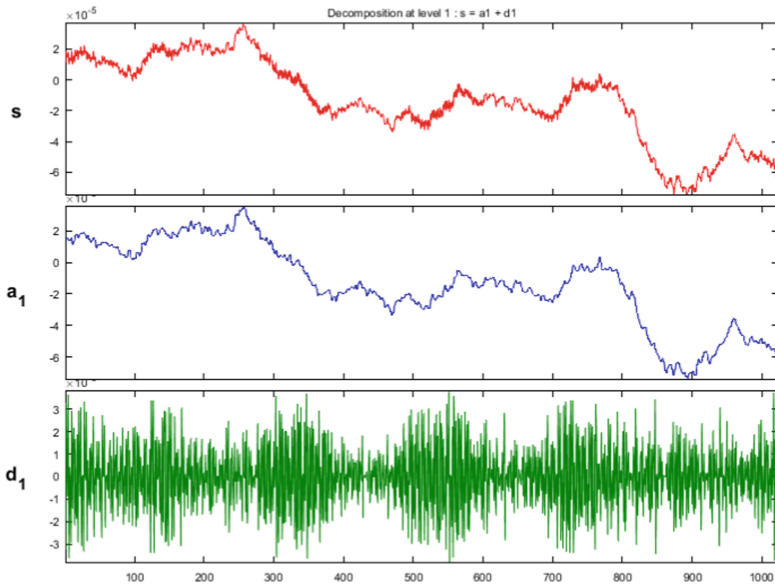


Fig. 8. Experimental velocity signal for a damaged structure and its discrete wavelet transform

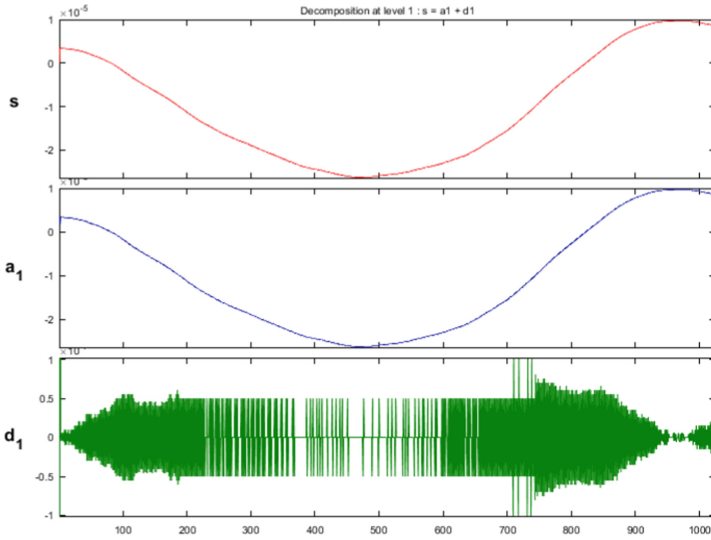


Fig. 9. Experimental displacement signal for an intact structure and its discrete wavelet transform

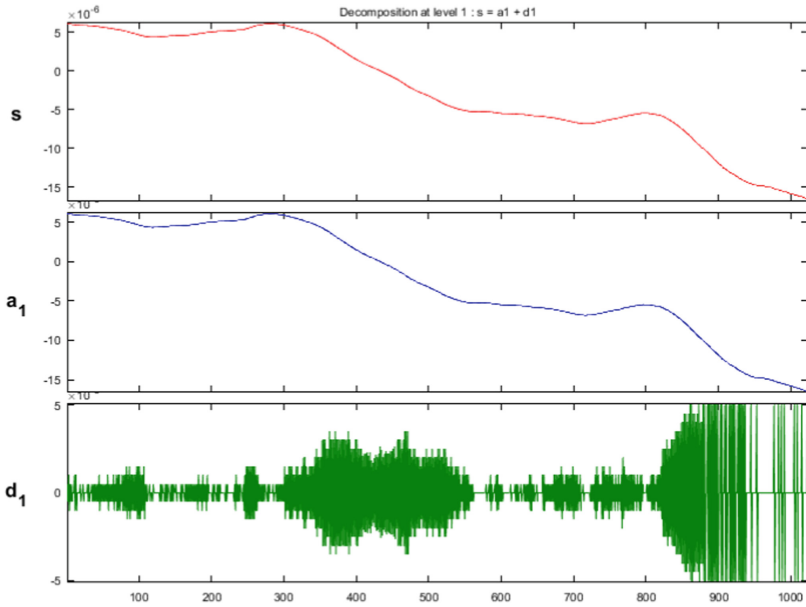


Fig. 10. Experimental displacement signal for a damaged structure and its discrete wavelet transform

4 Discussion

Damage detection using one-dimensional wavelet transform is a powerful and proper method based on the structure's signal. There is no need for having other modal characteristics such as natural frequencies. Signals from structures such as cables, bars, and beams are one-dimensional signals. Thus, these structures can be detected by the one-dimensional wavelet transforms. The one-dimensional continuous wavelet transforms (1D-CWT) and one-dimensional discrete wavelet transforms are two main types of one-dimensional wavelet transforms. These two different methods decompose the original signals using two different ways. The 1D-CWT shifts and scales the original signal at various continuous levels, and the 1D-DWT decomposes the original signals into approximation and detail sub-signals at various discrete levels. Both methods are powerful tools for detecting damages and faults in one-dimensional signals.

5 Conclusions

In this study, a review of damage detection methods in structures is performed. The emphasis of the research is on the introduction of one-dimensional damage detection methods using wavelet transform. Two common types of one-dimensional wavelet transform are introduced. Their formulation and how to evaluate the results of these two continuous and discrete types of damage detection based on one-dimensional wavelet transform are discussed. It has been proven that damage detection by the one-dimensional wavelet transform method can be a powerful tool for locating damage in cable structures, rods, isotropic beams, and laminated composite beams.

Acknowledgement. The first three authors acknowledge the funding support of Babol Noshirvani University of Technology through Grant program No. BNUT/965919012/99.

References

1. Mofidian, R., Barati, A., Jahanshahi, M., Shahavi, M.H.: Fabrication of novel Agarose–Nickel Bilayer composite for purification of protein nanoparticles in expanded bed adsorption column. *Chem. Eng. Res. Des.* **159**, 291–299 (2020)
2. Shahavi, M.H., Selakjani, P.P., Abatari, M.N., Antov, P., Savov, V.: Novel biodegradable poly (lactic acid)/wood leachate composites: investigation of antibacterial, mechanical, morphological, and thermal properties. *Polymers* **14**(6), 1227 (2022)
3. JafariTalookolaei, R.A., Abedi, M., Kargarnovin, M.H., Ahmadian, M.T.: Dynamic analysis of generally laminated composite beam with a delamination based on a higher-order shear deformable theory. *J. Compos. Mater.* **49**(2), 141–162 (2015)
4. Khalili, M., Razmjou, A., Shafiei, R., Shahavi, M.H., Li, M.C., Orooji, Y.: High durability of food due to the flow cytometry proved antibacterial and antifouling properties of TiO₂ decorated nanocomposite films. *Food Chem. Toxicol.* **168**, 113291 (2022)
5. Jafari-Talookolaei, R.A., Abedi, M.: Analytical solution for the free vibration analysis of delaminated Timoshenko beams. *Sci. World J.* (2014)

6. Saadatmorad, M., Siavashi, M., JafariTalookolaei, R.-A., Pashaei, M.H., Khatir, S., Thanh, C.-L.: Genetic and particle swarm optimization algorithms for damage detection of beam-like structures using residual force method. In: Bui, T.Q., Cuong, Le Thanh, Khatir, Samir (eds.) *Structural Health Monitoring and Engineering Structures*. LNCE, vol. 148, pp. 143–157. Springer, Singapore (2021). https://doi.org/10.1007/978-981-16-0945-9_12
7. Wahab, M.A., De Roeck, G.: Damage detection in bridges using modal curvatures: application to a real damage scenario. *J. Sound Vib.* **226**(2), 217–235 (1999)
8. Saadatmorad, M., JafariTalookolaei, R.A., Pashaei, M.H., Khatir, S.: Damage detection on rectangular laminated composite plates using wavelet based convolutional neural network technique. *Compos. Struct.* **278**, 114656 (2021)
9. Khatir, S., Tiachacht, S., Le Thanh, C., Ghandourah, E., Mirjalili, S., Wahab, M.A.: An improved artificial neural network using arithmetic optimization algorithm for damage assessment in FGM composite plates. *Compos. Struct.* **273**, 114287 (2021)
10. Saadatmorad, M., Talookolaei, R.A.J., Pashaei, M.H., Khatir, S., Wahab, M.A.: Pearson correlation and discrete wavelet transform for crack identification in steel beams. *Mathematics* **10**(15), 2689 (2022)
11. Tiachacht, S., Khatir, S., Le Thanh, C., Rao, R.V., Mirjalili, S., Wahab, M.A.: Inverse problem for dynamic structural health monitoring based on slime mould algorithm. *Eng. Comput.* 1–24 (2021)
12. Behtani, A., Bouazzouni, A., Khatir, S., Tiachacht, S., Zhou, Y.L., Abdel Wahab, M.: Damage localization and quantification of composite beam structures using residual force and optimization. *J. Vibroeng.* **19**(7), 4977–4988 (2017)
13. Khatir, S., Wahab, M.A., Benaissa, B., Köppen, M.: Crack identification using eXtended IsoGeometric analysis and particle swarm optimization. In: Abdel Wahab, M. (ed.) *FFW 2018*. LNME, pp. 210–222. Springer, Singapore (2019). https://doi.org/10.1007/978-981-13-0411-8_21
14. Kang, F., Li, J.J., Xu, Q.: Damage detection based on improved particle swarm optimization using vibration data. *Appl. Soft Comput.* **12**(8), 2329–2335 (2012)
15. Wei, Z., Liu, J., Lu, Z.: Structural damage detection using improved particle swarm optimization. *Inverse Probl. Sci. Eng.* **26**(6), 792–810 (2018)
16. DinhCong, D., VoDuy, T., HoHuu, V., Nguyen-Thoi, T.: Damage assessment in plate-like structures using a two-stage method based on modal strain energy change and Jaya algorithm. *Inverse Probl. Sci. Eng.* **27**(2), 166–189 (2019)
17. Gomes, G.F., da Cunha, S.S., Ancelotti, A.C.: A sunflower optimization (SFO) algorithm applied to damage identification on laminated composite plates. *Eng. Comput.* **35**(2), 619–626 (2018). <https://doi.org/10.1007/s00366-018-0620-8>
18. Mishra, M., Barman, S.K., Maity, D., Maiti, D.K.: Ant lion optimisation algorithm for structural damage detection using vibration data. *J. Civ. Struct. Heal. Monit.* **9**(1), 117–136 (2019)
19. Maity, D., Tripathy, R.R.: Damage assessment of structures from changes in natural frequencies using genetic algorithm. *Struct. Eng. Mech.* **19**(1), 21–42 (2005)
20. Alexandrino, P.D.S.L., Gomes, G.F., Cunha Jr., S.S.: A robust optimization for damage detection using multiobjective genetic algorithm, neural network and fuzzy decision making. *Inverse Probl. Sci. Eng.* 1–26 (2019)
21. Barman, S.K., Maiti, D.K., Maity, D.: Damage detection of truss employing swarm-based optimization techniques: a comparison. In: Venkata Rao, R., Taler, J. (eds.) *AEOTIT 2018*. AISC, vol. 949, pp. 21–37. Springer, Singapore (2020). https://doi.org/10.1007/978-981-13-8196-6_3
22. Gerist, S., Maheri, M.R.: Structural damage detection using imperialist competitive algorithm and damage function. *Appl. Soft Comput.* **77**, 1–23 (2019)

23. Kim, N.I., Kim, S., Lee, J.: Vibration-based damage detection of planar and space trusses using differential evolution algorithm. *Appl. Acoust.* **148**, 308–321 (2019)
24. Bakhary, N., Hao, H., Deeks, A.J.: Damage detection using artificial neural network with consideration of uncertainties. *Eng. Struct.* **29**(11), 2806–2815 (2007)
25. Mehrjoo, M., Khaji, N., Moharrami, H., Bahreininejad, A.: Damage detection of truss bridge joints using artificial neural networks. *Expert Syst. Appl.* **35**(3), 1122–1131 (2008)
26. Rosales, M.B., Filipich, C.P., Buezas, F.S.: Crack detection in beam-like structures. *Eng. Struct.* **31**(10), 2257–2264 (2009)
27. Saadatmorad, M., JafariTalookolaei, R.-A., Pashaei, M.-H., Khatir, S., Abdel Wahab, M.: Adaptive network-based fuzzy inference for damage detection in rectangular laminated composite plates using vibrational data. In: Abdel Wahab, M. (ed.) *SDMA 2021. LNCE*, vol. 204, pp. 179–196. Springer, Singapore (2022). https://doi.org/10.1007/978-981-16-7216-3_14
28. Saadatmorad, M., JafariTalookolaei, R.-A., Pashaei, M.-H., Khatir, S., Abdel Wahab, M.: Application of multilayer perceptron neural network for damage detection in rectangular laminated composite plates based on vibrational analysis. In: Abdel Wahab, M. (ed.) *SDMA 2021. LNCE*, vol. 204, pp. 163–178. Springer, Singapore (2022). https://doi.org/10.1007/978-981-16-7216-3_13
29. Staszewski, W.J.: Intelligent signal processing for damage detection in composite materials. *Compos. Sci. Technol.* **62**(7–8), 941–950 (2002)
30. Kim, H., Melhem, H.: Damage detection of structures by wavelet analysis. *Eng. Struct.* **26**(3), 347–362 (2004)
31. Hou, Z., Noori, M., Amand, R.S.: Wavelet-based approach for structural damage detection. *J. Eng. Mech.* **126**(7), 677–683 (2000)
32. Yang, C., Oyadji, S.O.: Delamination detection in composite laminate plates using 2D wavelet analysis of modal frequency surface. *Comput. Struct.* **179**, 109–126 (2017)
33. Cao, M., Qiao, P.: Damage detection of laminated composite beams with progressive wavelet transforms. In: *Nondestructive Characterization for Composite Materials, Aerospace Engineering, Civil Infrastructure, and Homeland Security 2008*, vol. 6934, p. 693402. International Society for Optics and Photonics, March 2008
34. Sohn, H., Park, G., Wait, J.R., Limback, N.P., Farrar, C.R.: Wavelet-based active sensing for delamination detection in composite structures. *Smart Mater. Struct.* **13**(1), 153 (2003)
35. Mitra, M., Gopalakrishnan, S.: Wavelet based spectral finite element modelling and detection of de-lamination in composite beams. *Proc. Roy. Soc. A Math. Phys. Eng. Sci.* **462**(2070), 1721–1740 (2006)
36. Rucka, M., Wilde, K.: Application of continuous wavelet transform in vibration based damage detection method for beams and plates. *J. Sound Vib.* **297**(3–5), 536–550 (2006)
37. Janeliukstis, R., Rucevskis, S., Wesolowski, M., Chate, A.: Experimental structural damage localization in beam structure using spatial continuous wavelet transform and mode shape curvature methods. *Measurement* **102**, 253–270 (2017)
38. Montanari, L., Spagnoli, A., Basu, B., Broderick, B.: On the effect of spatial sampling in damage detection of cracked beams by continuous wavelet transform. *J. Sound Vib.* **345**, 233–249 (2015)
39. Rucka, M.: Damage detection in beams using wavelet transform on higher vibration modes. *J. Theor. Appl. Mech.* **49**(2), 399–417 (2011)
40. Puchala, D., Stokfiszewski, K.: Highly effective GPU realization of discrete wavelet transform for big-data problems. In: Paszynski, M., Kranzlmüller, D., Krzhizhanovskaya, V.V., Dongarra, J.J., Sliot, P.M.A. (eds.) *ICCS 2021. LNCS*, vol. 12742, pp. 213–227. Springer, Cham (2021). https://doi.org/10.1007/978-3-030-77961-0_19
41. Yang, J.M., Hwang, C.N., Yang, B.L.: Crack identification in beams and plates by discrete wavelet transform method. *Chuan Bo Li Xue/J. Ship Mech.* **12**(3), 464–472 (2008)

42. Guminiak, M., Knitter-Piątkowska, A.: Selected problems of damage detection in internally supported plates using one-dimensional Discrete Wavelet Transform. *J. Theor. Appl. Mech.* **56** (2018)