Improved ANN for Damage Identification in Laminated Composite Plate

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Abstract. This paper presents an improved Artificial Neural Network (ANN) for structural health monitoring of composite materials. Simply supported three-ply $\left[0^{\circ}\ 90^{\circ}\ 0^{\circ}\right]$ square laminated plate modeled with a 9×9 grid is provided and validated based on the literature review. Modal strain energy change ratio (MSE_{cr}) is used to localize the damaged elements and eliminate the healthy elements. Next, improved ANN using the Arithmetic optimization algorithm (AOA) used for structural quantification. AOA aims to optimize the parameters of ANN for better training. Several scenarios are considered to test the accuracy of the presented approach. The results showed that the approach can localize and quantify the damage correctly.

Keywords: Artificial intelligence · Inverse problem · Damage detection · Structural quantification · Metaheuristic optimization

1 Introduction

All mechanical structures under vibrations are subject to local damage. This is one of the major factors that influence the laws related to maintenance, as it is determinant of the lifetime of a mechanical piece and is indicator for pursuing piece change in a regular fashion. These rules allow us to take a passive role in avoiding danger and to avoid financial risks. Moreover, structural health monitoring (SHM) is the discipline of actively watching the integrity of mechanical structures through sensors at the first place, then modeling and damage detection on the second place. This maintenance approach can be costly and is mainly used for expensive structures. But the rapid development of such methods attracted more adoption in recent years [1–3].

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Based on research literature, there exist three levels of damage identification. The first is to recognize the presence of damages [4]. The second level is to identify its position and the last is to estimate its severity [5–7]. To achieve all three levels, researchers use inverse analysis to compare the actual vibrational response of the structure to several other responses issued from simulation through an optimization algorithm [8–12]. Researchers study various structural responses and build multiple damage indicators based on them [4, 13–17]. On the other hand, the challenge of the ill-posed inverse problem appears in many indicators when one structural response may correspond to complete damage parameters. Metaheuristic optimization algorithms are well placed to solve this problem. They can explore the search space and overcome the local minimums traps, thus finding the correct damage parameters in these cases [5, 6, 8, 9, 18–20].

The inverse analysis can be very demanding computationally, as it requires the simulation of the problem several times in each iteration [4]. Several times here means a value equal to the population size in modern metaheuristic algorithms. So, using a suitable optimization algorithm is critical in terms of computational cost. Moreover, the performance of metaheuristic algorithms is guided by their tuning parameters, like the mutation chance and crossover rate in the genetic algorithm, for example. Each problem requires specific tuning of these parameters to take full advantage of the algorithm potential. But as opposed to most metaheuristic algorithms, the Jaya algorithm does not contain such parameters, which makes it flexible in solving various engineering problems [11].

Significant studies were presented in the field of the structural response of damaged structures, such as truss structures with the use of a flexibility-based approach [21]. The modal analysis of laminated composite with different boundary conditions [9, 14, 22, 23]. The power spectrum and time-frequency analysis are used to identify vibration modes damages in beam-like structures using [7, 24]. And the swept-sine acoustic excitation are used for estimation of natural frequencies in Ref [25]. Rao et al. [26] studied the high-frequency wave characteristic in steel anchor-concrete composite, for damage detection. Non-mechanical crack detection techniques are also investigated in research studies [27, 28]. Hakim and Razak [29] indicated that the methods based on natural frequency can detect global changes, but the method that are based on mode shape data are more accurate for detecting local changes. Researchers created several damage indicators throughout the last two decades. Petrone et al. [30] presented and analyzed various damage identification techniques in different damage scenarios. One of the earliest is the Flexibility Strain Energy-Based Index (FSEBI), Guo [19] proposed a two-stage method based on the Genetic Algorithm (GA) to detect the damage region and the severity.

An improved method for damage identification has been introduced based on assessing the nonlinearity of cracked structures [31]. Another indicator was suggested in [32], called the Response Vector Assurance Criterion (RVAC). Ref [1] investigated the use of the transmissibility technique function instead of FRF in the RVAC. This technique was extended in[33] where it was employed for damage detection in metro tunnel structures. The Residual Force Vector was also found useful for damage identification in truss structure in [34].

Arefi et al. [35] suggested a modified Modal Strain Energy Damage Index (MSEDI) and studied its performance in different structures. And in Ref [36] an indicator based on mode shapes reconstruction was proposed, using an improved reduction system (IRS).

A generalized flexibility matrix change was proposed by for damage identification. And [37] presented the approach of wavelet transform (WT) and Teager Energy Operator (TEO), considering multiple damage identification cases in a composite structure. Ref [38] presented the method of damage detection using sparse sensors installation by System-Equivalent Reduction and Expansion Process (SEREP). Damage identification in beam-like structures using deflections obtained by modal flexibility matrices was presented by [13].

2 AOA-ANN

The Arithmetic algorithm is a population-based metaheuristic search algorithm. It uses Four basic search behaviors, each of which is based on the basic arithmetic operators (division, multiplication addition, and subtraction). The exploitation phase is characterized using the two operators of subtraction and addition, according to the following equations:

$$X_{i,j}(C+1) = \begin{cases} best(X_j) - MOP \times \left[(UB_j - LB_j) \times \mu + LB_j \right] \; if \; rand < 0.5 \\ best(X_j) + MOP \times \left[(UB_j - LB_j) \times \mu + LB_j \right] \; f \; rand > 0.5 \end{cases} \quad (1)$$

where $X_{i,j}(C)$ is the ith solution at the jth position, and $best(X_j)best(X_j)$ is the historical best position found by the jth solution. LB_jLB_j and UB_jUB_j are the lower boundary and the upper boundary of solution j. μ is a tuning parameter for this algorithm, it is set by the user, 0.5 is a commonly used value for multimodal problems. Rand is a random value between 0 and 1 generated at the instance of checking the logical statement. Lastly, $MOP\ MOP$ is the Math Optimizer Probability, calculated at each iteration according to the following expression:

$$MOP = 1 - \frac{C^{1/\alpha}}{M^{1/\alpha}} \quad (2)$$

where C is the current iteration, M represents the maximum number of iterations, and α is the second tuning parameter for this algorithm, called the sensitive parameter and it tracks the search' exploitation accuracy, it is commonly fixed at a value of 5.

The exploration phase of the AOA algorithm is characterized using the two operators of multiplication and division, this behavior is expressed by the following equations, where ϵ is an integer of a small value.

$$X_{i,j}(C+1) = \begin{cases} best(X_j) \div (MOP + \varepsilon) \times \left[(UB_j - LB_j) \times \mu + LB_j \right] \; if \; rand < 0.5 \\ best(X_j) \times MOP \times \left[(UB_j - LB_j) \times \mu + LB_j \right] \quad f \; rand > 0.5 \end{cases} \quad (3)$$

Figure 1. Denotes the structure of the AOA algorithm. Where MOA is a term called Math Optimizer Accelerated calculated at each iteration using the following expression, where min and max are respectively, the minimum and maximum values of the accelerated function.

$$MOA(C) = \min + C \times \left(\frac{\max - \min}{M} \right) \quad (4)$$

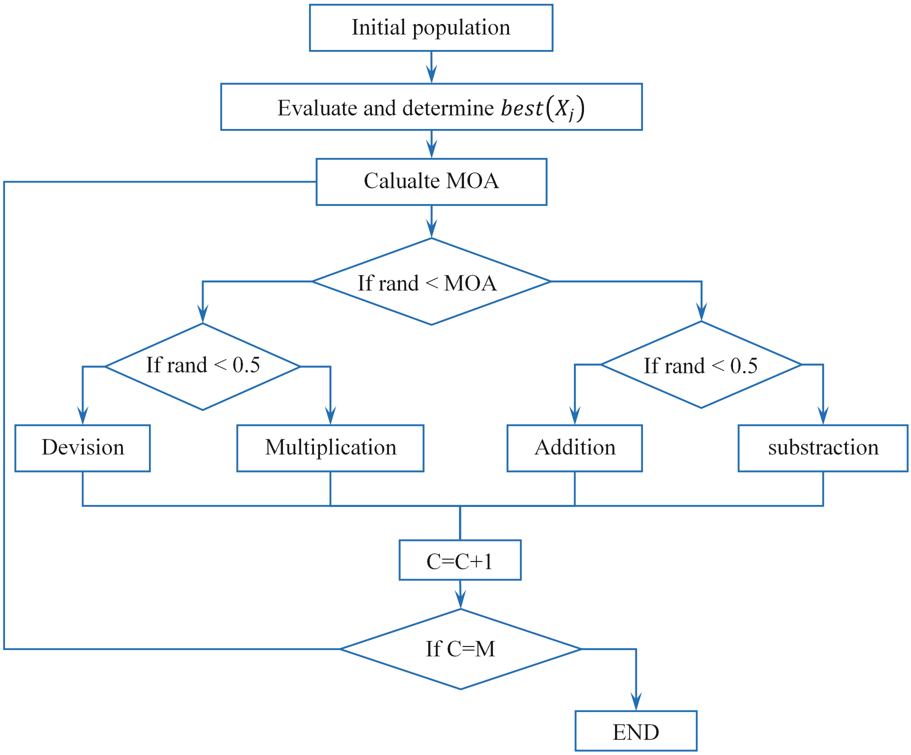

Fig. 1. AOA algorithm structure.

3 Theoretical Background

In this section, we describe the preliminaries and essential definitions.

3.1 Modal Strain Energy Change Ratio

Nel is the number of elements with reduced stiffness. The damage parameter $\delta_i (i = 1, 2, \ldots, nel)$ is presented in the following equation:

$$K_d = \sum_{i=1}^{nel} (1 - \delta_i) k_i^e \tag{4}$$

The matrix K_d represents the damaged stiffness, where k_i^e is the stiffness of the i^{th} element and δ_i is a damage parameter with a value between 0 and 1; i.e., 0 for intact and 1 is fully damaged structures.

The modal strain energy (*MSE*) for undamaged and damaged structures are presented in the following formulation:

$$MSE_{ij}^{h} = \frac{1}{2}\left(\phi_i^h\right)^T K_j \phi_i^h \; ; \; MSE_{ij}^{d} = \frac{1}{2}\left(\phi_i^d\right)^T K_j \phi_i^d \tag{5}$$

i^{th} is the mode number and j^{th} is the index number of element. K_j presents the stiffness matrix. h and d represents the healthy and damaged systems, respectively, and ϕ_i is mode shape, T is the vector transpose. ($MSEcr$) is the modal strain energy change ratio, it is proposed in this paper to predict the exact location of the damage. It can be expressed by the total energy in the structure, as the sum of MSE's of all elements:

$$MSEcr_j = \frac{1}{m}\sum_{j=1}^{m}\frac{MSEcr_{ij}}{MSEcr_{ij}^{\max}} \tag{6}$$

where

$$MSEcr_{ij} = \frac{\left|MSE_{ij}^{d} - MSE_{ij}^{h}\right|}{MSE_{ij}^{h}}\ ;\ MSEcr_{ij}^{\max} = \max_k\{MSEcr_{ik}\} \tag{7}$$

The damage identification experiment is performed on a three-ply [0° 90° 0°] composite plate, with square dimensions, and under the simply supported boundary conditions. We assume that all layers of the laminate are made of the same linearly elastic composite material, have the same thickness, and the same density. With the following characteristics: $E_1/E_2 = 40$, $G_{12} = G_{13} = 0.6E_2$; $G_{23} = 0.5E_2$; $\upsilon_{12} = 0.25$, where the index 1 and 2 are for the directions parallel and perpendicular to the fibre orientation. The plate is modeled in two methods. First, in the Finite Element Method, with three discretization levels for each square side, 9 elements. 15 elements and 20 elements. Figure 1 shows the considered composite and the 9 × 9 meshing. And in the Isogeometric Analysis method, with the discretization of 9 and 14 for each square side. The choice of the number of elements is made based on the related research by Ritz [39] and Reddy [40], to be able to compare our simulation results for the same conditions. We compare the vibrational modes in the undamaged structure in Table 1.

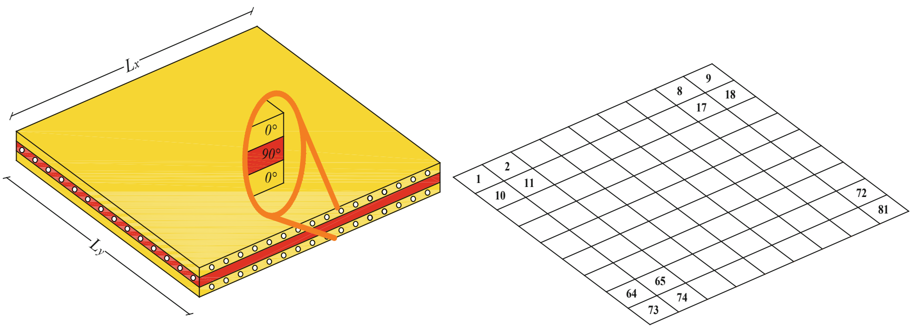

Fig. 2. Simply supported three-ply $\left[0° \;\; 90° \;\; 0°\right]$ square laminated plate modeled with a grid of 9 × 9.

The simulation results show that IGA simulation is more close to reference results than the Finite Element Method (FEM), in terms of precision, the vibrational modes error is within the uncertainty margin and within the difference between the Ritz results in [39] and Reddy results in [40]. In terms of discretization levels, the vibrational modes

Table 1. Natural frequencies of the undamaged plate structure.

		Grid	Mode 1							
			1	2	3	4	5	6	7	8
Intact	FEM	9×9	10.16	27.23	32.62	41.55	69.58	69.58	74.28	78.85
		14×14	7.44	17.28	24.98	29.85	44.76	44.76	55.40	58.09
		20×20	6.62	14.00	22.85	26.41	37.25	37.25	50.38	51.31
	IGA	9×9	6.60	9.42	16.23	24.78	27.25	27.25	30.00	37.90
		14×14	6.60	9.42	16.15	24.77	26.55	26.55	29.96	37.40
	Liew (p-Ritz) [39]	9×9	6.63	9.45	16.21	25.11	26.69	26.69	30.32	37.81
		14×14	6.63	9.45	16.21	25.11	26.66	26.66	31.31	37.79
	Reddy [40]	9×9	6.62	9.44	16.20	25.11	26.65	26.65	30.31	37.78
Damaged	Case 1	9×9	6.60	9.41	16.19	24.76	27.16	27.16	29.84	37.57
	Case 2	9×9	6.54	9.30	16.17	24.27	27.16	27.16	29.84	37.71
	Case 3	9×9	6.40	9.42	15.87	24.72	27.14	27.14	29.94	37.83
	Case 4	9×9	6.59	9.41	16.22	24.72	27.23	27.23	29.84	37.75
	Case 5	9×9	6.60	9.41	16.22	24.74	27.23	27.23	29.88	37.78

are found equivalent, with the same number of elements, Ie. 9 and 14. However, the FEM errors are very high in the same discretization level and could reach equivalent vibrational modes 1 and 4 with the discretization of 20×20 elements. The rest modes cannot be considered correct, as they are very far from the references. The FEM requires a higher number of elements, however, in the study of damage identification by damage indicators, the number of elements is very important, as the damage is simulated as the change in the rigidity of selected elements.

Table 1. Also, show the vibrational modes of the plate in the presence of five different damage scenarios using IGA. These damage scenarios vary in terms of positioning from corner edge like in scenario 1 (element 8) to center in scenario 3 (element 41) to side edge like scenario 4 (element 64). They also vary in severity, between 10% reduction of rigidity to 30%. As shown in Table 2. These choices are made to create to test the ability of the suggested method in identifying variable damage cases in terms of their position and also in terms of their similarity of vibrational mode responses, like cases 4 and 5, which have very close modes, and case 1 and 5, that have close first four modes. And cases 1 and 2 have the same 6^{th} and 7^{th} modes.

Table 2. Damage scenarios for laminated composite plate structure.

	Case 1	Case 2	Case 3	Case 4	Case 5
Damaged element	8	21	41	64	72
% Reduction in stiffness	30%	20%	10%	15%	25%

The damage identification results by strain energy change ratio are shown in Fig. 2, along with 9 × 9 meshing of the composite plate, where the actual damaged elements are marked in red. The result shows that the indicator successfully predicts the region of the damage. However, the neighboring elements are assigned a damage value in all cases, even though they are not damaged. And we notice that the central element is the least affected by this error (3 false damages). And the most affected in case 2. Where the damage is near the edge of the plate, with 15 false damages, the other edge cases have a relatively similar number of false damaged elements (4 to 5). On the other hand, the predicted damage severity error of this method is very high.

3.2 Damage quantification using AOA-ANN

ANN in this paper is used to model the vibrational characteristics of the undamaged composite plate and the presence of damages in various positions. It can learn such characteristics from a learning dataset by optimizing the weights and biases of the network nodes. Because the vibration output can be very close for different damages, it is critical to distinguish the right damages corresponding to each response. We investigated the network training using the AOA algorithm. For its higher ability to reach the global optimum than classical training algorithms. The objective is to minimize the Root-Mean-Square Error of the network, which is expressed by Eq. 5. With $l_i l\,_i$ denotes actual output as considered in the target set. $O_i\ O\,_i$ is the output corresponding to i^{th} i^{th} data point in the training set, and d is the number of data points considered in the training dataset.

$$Error = \sqrt{\frac{\sum_{i=1}^{n} (O\,_i - l\,_i)^2}{d}} \tag{8}$$

Figure 3. Shows the regression results, comparing the real and estimated responses in the five testing cases of damaged plates, indicating that the suggested technique can reproduce the vibrational response in an accurate manner. With lowerest R-value equal to 0.998 in the first testing case. Notice that the estimated responses in close response are accurately distinguishable. Like cases 4 and 5. And cases 1 and 2.

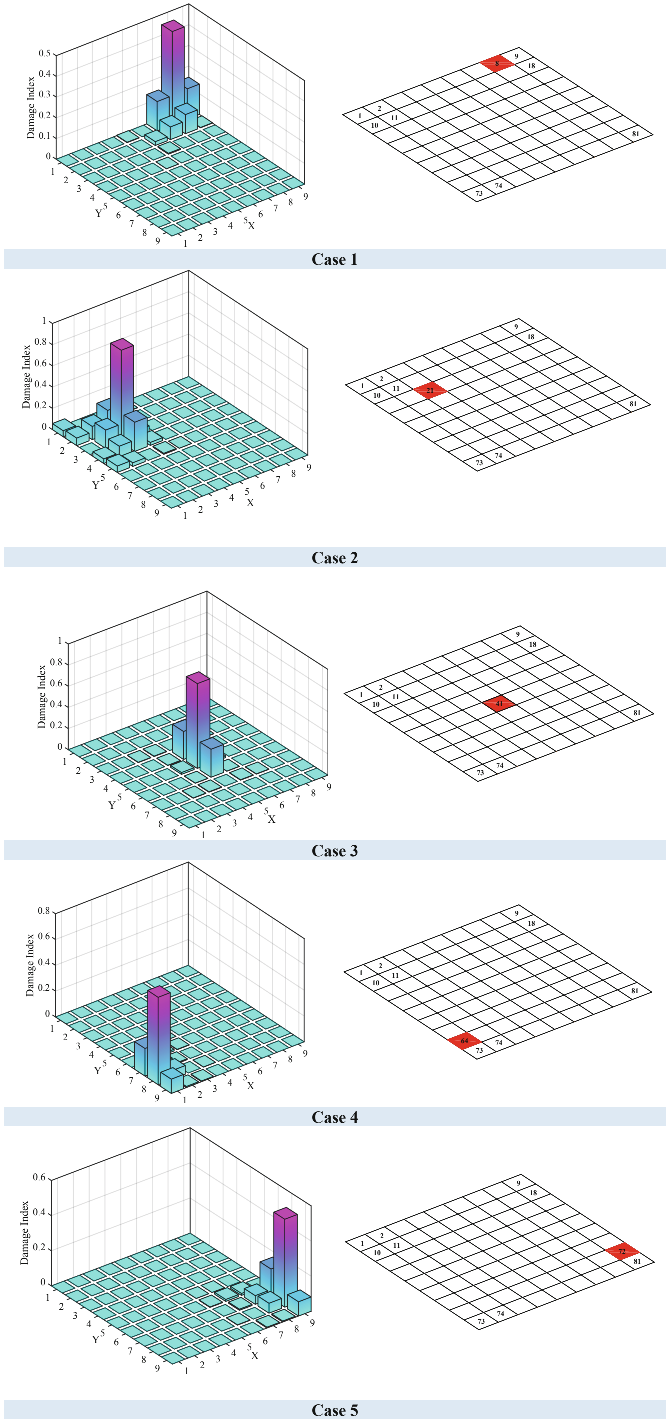

Fig. 3. Identification of damage for different damage scenarios using IGA-MSEcr.

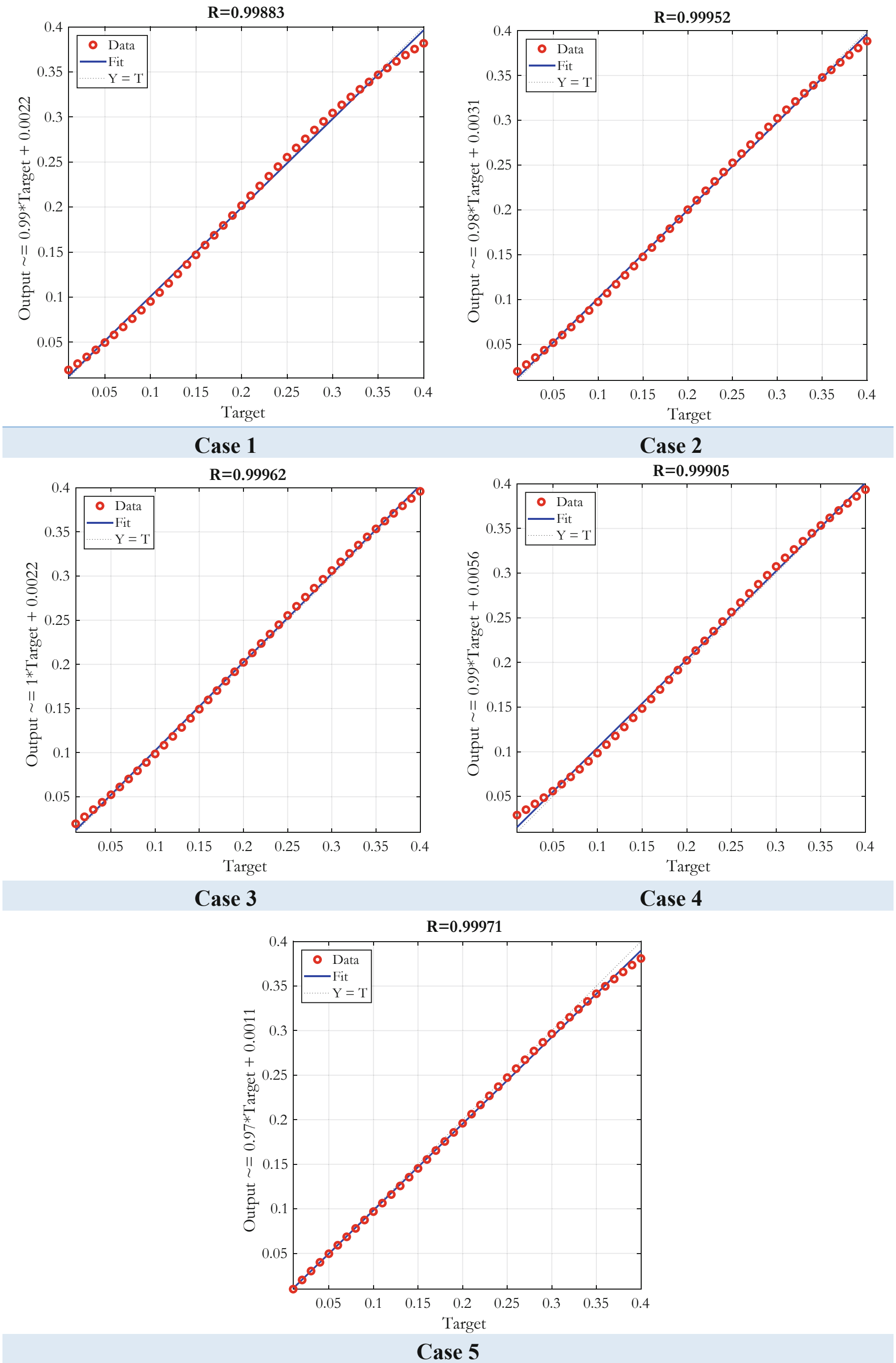

Fig. 4. The regression plot for each damage case.

Table 3. Damage scenarios.

	The best solution fitness		**CPU Time** [s]
	At iteration 50	At iteration 100	
Case 1	0.00005496800	0.00003356700	68.94327
Case 2	0.00005604700	0.00001602000	69.52839
Case 3	0.00005124800	0.00001628000	69.81313
Case 4	0.00003781200	0.00003781200	68.41801
Case 5	0.00003981600	0.00003981600	69.1692

After establishing the improved ANN model, we use it to identify the damage properties in the testing cases. And the fitness convergence results are shown in Table 3. Indicating the estimated damages fitness value reached a low error value, with most of the progress being made within 50 iterations. CPU time indicates equivalent results in all cases. Figure 4. Compares the damage characteristics corresponding to these error values, summarizing the five test cases in one graph. All the damages are identified accurately, With the first case corresponding with the highest error between real damage severity and estimated damage severity equal to 4.4%. Indicating that the accuracy in this problem in terms of fitness value should be very high, as a fitness value of 0.00003 can be equivalent to a 5% error in damage estimation. Table 4. Provides the details of the estimated damages.

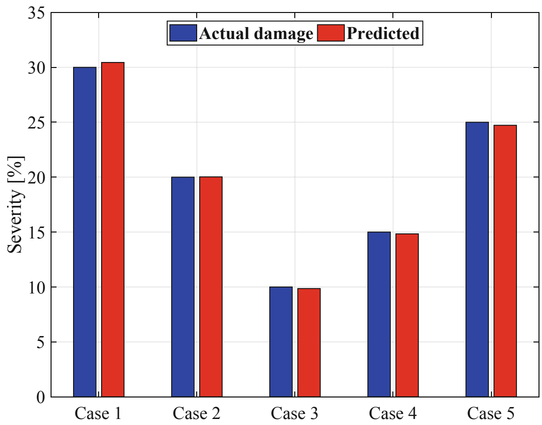

Fig. 5. Actual and predicted damage for all cases.

Table 4. Damage scenarios.

	Case 1	Case 2	Case 3	Case 4	Case 5
Actual Damage	30%	20%	10%	15%	25%
Predicted	30.444%	20.027%	9.855%	14.837%	24.723%

4 Conclusion

This paper investigates damage identification in the laminated composite plate using an improved Artificial Neural Network with the AOA algorithm. The composite plate is simulated using the FEM method and validated against other methods from litterature. In the first section, we examined the performance of the Modal Strain Energy Change Ratio indicator, where we found its good performance in estimating the area in which the damage can be located, but it has limitations in terms of precise damage severity estimation. In the next stage, we suggested using the ANN to improve the damage identification results. The network is trained using the AOA algorithm, and its results showed a significant improvement in estimation quality. The suggested method can distinguish between close vibrational responses corresponding to various damages. The results showed that our method overcomes the challenge by predicting the exact damaged element with maximum error of about 4.4% of the actual damage.

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