Measuring Voting Power in Complex Shareholding Structures: A Public Good Index Approach

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1 Introduction

A complex corporate shareholding network consists of direct and indirect ownership relations. For example, company A has a share in company B, company B has a share in company C, company C has a share in company D, and company D has a share in company B. Is there some relation between firm A and company D? One can ask about the control power of company A in company D. This question was posed by many scholars starting from the last century; see Berle and Means (1932) for example. Since then, many researchers have tried to propose methods for measuring such "indirect" control power of a firm in ownership structure. Although it is not our intention to review all the methods proposed so far, let us list a few of those with a cooperative game theory approach and in particular that used power indices to the measurement of the control power of firms in corporate networks: Gambarelli and Owen (1994), Turnovec (1999), Hu and Shapley (2003a, 2003b), Leech (2002), Crama and Leruth (2007, 2013), Karos and Peters (2015), Mercik and Lobos (2016), Levy and Szafarz (2017), Mercik and Stach (2018), Stach et al.(2020), Staudacher et al. (2021a, 2021b), and Stach and Mercik (2021). The applications and comparisons of some of these methods can be found in Bertini et al. (2016), Kołodziej and Stach (2016), Stach (2017), and Mercik and Stach (2018). Not all of the methods proposed can be applied to complex shareholding networks. In corporate

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M. A. Leroch and F. Rupp (eds.), *Power and Responsibility*,
https://doi.org/10.1007/978-3-031-23015-8_9

shareholding structures where circular cross-ownerships exist, many methods fail. Only a few of the methods considered in the literature measure the control power of all firms involved in corporate shareholding structures. Among the methods that deal with the problem of cycles and measuring the control power of all firms in the corporate shareholding networks is one proposed by Karos and Peters in (2015). In this paper, we focus on this method and try to modify it using Holler's (1982) approach to measure players' a priori voting power in simple games.

We propose a new game-theoretical method (which is based on Holler's (1982, 2018) and Karos and Peter's (2015) approaches) to measure a firm's indirect control power as elements of a whole corporate ownership network. We called our approach briefly the *iPGI* index, where "*i*" refers to "indirect control." Our method considers voting rights attached to individual firms (represented by nodes in networks) as input data, i.e., direct and indirect ownership relations. The Karos and Peters method introduced in 2015 is used to create a new method. More precisely, in the Karos and Peters framework, instead of the use of the Shapley and Shubik (1954) index, we propose to use the Public Good Index (PGI for brevity) introduced by Holler in (1982). In this way, we obtain a method to measure the control power of all firms involved in complex shareholding structures (which means investors—firms without shareholdings—and stock companies). Moreover, this method is used to estimate the indirect control power in a theoretical example of a shareholding structure and compared it with the Karos and Peters approach. Finally, we also try to critically discuss the appropriateness of using the PGI index to measure the control power of firms in complex corporate networks.

As mentioned above, in 1982 Manfred Holler proposed a power index based on minimal winning coalitions—the Public Good Index called the Holler index (Holler, 1982, 2018; Holler and Packel, 1983). Holler and Li (1995) generalized this index from simple to cooperative games. Generally speaking, the difference between the PGI index and the very known normalized Banzhaf (1965) index lies in those winning coalitions that are not minimal (Holler & Nurmi, 2013).

A minimal winning coalition includes only those firms needed to obtain a majority position, and a minimal winning coalition consists of the minimal number of firms that can form a majority and control the smallest possible majority of the seats in the boards of companies. Some scholars (Holler, (1982, 1998), Riker, (1986) and others) claim that only minimal winning coalitions are coalitions that can occur and remain stable. Adding superflux players to the minimal winning coalition is costless and time-consuming. "This does not mean that surplus coalition do not form, but they should not be considered when measuring power"—this is cite taken from Holler and Nurmi (2013). This argument seems pertinent in the context of coalition formation in corporate networks. In particular, when we consider the formation of a coalition to a possible takeover of a company.

This paper aims to apply the PGI index to measure the indirect control power. To the authors' knowledge, in Staudacher et al. (2021a, 2021b), the PGI index was first used to measure all firms' indirect control power in complex corporate structures with cycles. In that paper, Staudacher et al. (2021a, 2021b) introduced and analyzed a framework of so-called implicit power indices generalizing the implicit indices introduced by Mercik and Łobos (2016) and then modified in (Mercik & Stach, 2018; Stach et al., 2020; and Stach & Mercik, 2021), by replacing the Johnston (1978) index with several other power indices. Among these indices was the PGI index (Holler, 1982). The implicit power index takes into account not only the power of the individual entities constituting the companies (investors) but also the impact of the companies themselves on implicit relationships. In this paper, we follow the idea of Staudacher et al. (2021a, 2021b) to use the PGI index in the context of indirect control in complex shareholding structures. However, our framework is different as we based it on the Karos and Peters (2015) approach and tried to highlight the pros and cons of the PGI index in this context.

In the literature on power measurement—see Felsenthal and Machover (1995, 1998, 2005), Holler and Owen (2000, 2002a, 2002b), Laruelle and Valenciano (2011), and (Bertini et al., 2013)—there is an ongoing debate on the concept of power in general. For example, the literature discusses what we measure when we apply power measures and what properties an adequate measure of power should satisfy. In the context of indirect control, where we consider a possible acquisition of a company o group of companies, the question about the formation of minimal winning coalitions seems very important.

The rest of the paper is organized as follows. Section 2 provides the necessary background on simple games and power indices. Section 3 focuses on the key issue, i.e., the definition of a measure of the power of firms in complex corporate shareholding structures. This measure, which we call the *iPGI* index, is a modification of the Karos and Peters (2015) approach and is based on the index proposed by Holler (1982)—the Public Good Index. In this section, we also briefly discuss some properties, among other things, that axiomatically characterized the Karos and Peters index, and we try to check which of these are possessed by the *iPGI*. Section 4 provides a theoretical example of corporate shareholding structure in which we illustrate how the *iPGI* index can be applied in complex shareholding networks. We also compare the results thus obtained with the Karos and Peters approach. Finally, conclusions and some further developments are presented in Sect. 5.

2 Preliminaries and Notation on Simple Games and Power Indices

A *simple game* is a pair (N, v) consisting of a non-empty and finite set of *players* $N = \{1, 2, ..., n\}$ and a binary-valued function $v : 2^N \to \{0, 1\}$ domained on the set of all subsets of N—2^N, satisfying the following condition: $v(\emptyset) = 0$, $v(N) = 1$, and $v(S) \leq v(T)$ for all $S \subseteq T \subseteq N$. Any subset $S \in 2^N$ is called a *coalition*, and N is called the *grand coalition*. If $v(S) = 1$, then S is called a *winning coalition*; otherwise, $(v(S) = 0)$ it is called a *losing coalition*. By W and W_i we denote the set of all winning coalitions and the set of all winning coalitions containing player i, respectively, in a simple game (N, v). A player i is called a *critical* player in a winning coalition S if $v(S\backslash\{i\}) = 0$. The set of all winning coalitions in which player i is a critical player is denoted by $\eta_i(v) = \{S \in W_i : v(S\backslash\{i\}) = 0\}$. A winning coalition S is called a *minimal winning coalition* if $v(S\backslash\{i\}) = 0$ for each $i \in S$. This implies that each proper subset of a minimal winning coalition is losing. By W^m and W_i^m we denote the set of all minimal winning coalitions and the set of all minimal winning coalitions containing player i, respectively, in a simple game (N, v). A player i is called a *null* player if there exists no coalition $S \in 2^N$ in which i is critical.

A *weighted game* $[q; w_1, ..., w_n]$ is a simple game (N, v) consisting of a non-negative vector of the weights of players $(w_1, ..., w_n)$, $\sum_{i \in N} w_i = 1$, and a majority quota $q \sum_{i \in N} w_i \geq q \geq \sum_{i \in N} \frac{w_i}{2}$ such that $v(S) = 1$ if and only if $\sum_{i \in S} w_i > q$. The weighted majority games are often used to model the voting situations in the stock companies.

2.1 Power Indices

For the purpose of measuring a priori the voting power of players in simple games, various one-point solutions called power indices have been proposed by different scholars. Generally, a *power index* f is a function that assigns a unique real-valued vector $f(v) = (f_1(v), f_2(v), \ldots, f_n(v))$ to each simple game (N, v). A component $f_i(v)$ assesses the power of player i in a simple game (N, v) for each $i \in N$. One of the best-known and frequently applied power indices was introduced by Shapley and Shubik (1954). The Shapley and Shubik index for a simple game (N, v) and each $i \in N$ is defined as follows

$$\sigma_i(v) = \sum_{S \in \eta_i} \frac{(s-1)!(n-s)!}{n!} \tag{1}$$

where $s = |S|$ denotes the cardinality of S. For more information on σ, see (Shapley & Shubik, 1954) or (Stach, 2011), for example.

The Public Good Index (PGI), also called the Holler (1982, 2018) index, for a simple game (N, v), and each $i \in N$ is given as follows

$$h_i(v) = \frac{|W_i^m|}{\sum_{j \in N} |W_j^m|} \tag{2}$$

The PGI index is also known as the Holler–Packel index, thanks to an axiomatization by Holler and Packel (1983) and then completed by Napel (1999).

2.2 Some Properties of Power Indices in Simple Games

In this section, we provide definitions of some desirable postulates of power indices in simple games. In particular, we quote only: bloc, efficiency, local monotonicity, null player, null player removable property, and symmetry properties. In Sect. 2.3, we will compare the power indices defined in Sect. 2.1 (σ and h) by taking these properties into consideration. The reason to discuss these properties and the σ and h power indices is to prepare a background to compare Φ and *iPGI*. As in Sect. 3.2, any difference in satisfying the equivalent properties in corporate networks observed in Φ and *iPGI* is the result of a difference in σ and h.

It is said that a power index f satisfies the following:

- The *bloc* property if, for all games (N', v') arising from the weighted game (N, v), $v = [q; w_1, ..., w_n]$ by removing two players $i, j \in N$ and introducing a new player representing the bloc $i\&j$ with $w_{i\&j} = w_i + w_j$, the following inequality holds $f_{i\&j}(v') \geq f_i(v)$;
- The *efficiency property* if, for all simple games (N, v), $\sum_{i \in N} f_i(v) = 1$;
- The *local monotonicity* (*dominance*) *property* if, for all weighted games $[q; w_1, ..., w_n]$ and any two distinct players $i, j \in N$, inequality $w_i \geq w_j$ implies $f_i(v) \geq f_j(v)$;
- The *null player property* if $f_i(v) = 0$ for all simple games (N, v) and each null player $i \in N$;
- The *null player removable property* if, for all simple games (N', v') arising from (N, v) by eliminating the null players, $f_i(v') = f_i(v)$ holds for each non-null player $i \in N$ ($i \in N'$ and $i \notin N \backslash N'$);
- The *symmetry property* if, for all simple games (N, v), each player $i \in N$, and every permutation $\pi : N \to N$, the following condition holds: $f_i(v) = f_{\pi(i)}(\pi(v))$, where $(\pi(v))(S) = v(\pi^{-1}(S))$;
- The *transfer property* if, for all pairs of simple games (N, v_1), (N, v_2) and each player $i \in N$, the following equation $f_i(v_1 \wedge v_2) + f_i(v_1 \vee v_2) = f_i(v_1) + f_i(v_2)$ holds, where $(v_1 \wedge v_2)$ and $(v_1 \vee v_2)$ are defined by the following sets of winning coalitions: $W(v_1 \wedge v_2) = \{S \in 2^N : S \in W(v_1) \text{ and } S \in W(v_2)\}$, $W(v_1 \vee v_2) = \{S \in 2^N : S \in W(v_1) \text{ or } S \in W(v_2)\}$.

2.3 Comparison of the Shapley and Shubik and PGI Indices

Let us consider an example of the voting system [51; 35, 21, 14, 15, 15] to compare the Shapley and Shubik index with the PGI index. In this system, there are five players, and the approval of a decision requires at least 51 votes of the total 100. The corresponding weights of players are: thirty-five votes for player 1, twenty-one votes for player 2, fourteen votes for player 3, and 15 votes each for players 4 and 5.

For each player i ($i = 1, 2, \ldots, 5$), Table 1 presents the set of minimal winning coalitions with player i, the set of winning coalitions in which player i is a critical player (η_i), and the distributions of power calculated by the Shapley and Shubik and PGI indices.

Table 1 Coalitions with critical players and distribution of power in the game [51; 35, 21, 14, 15, 15]

Player	W_i^m (set of minimal winning coalitions with player i critical)	η_i (set of winning coalitions with player i critical)	σ	h
1	$\{1, 2\}, \{1, 3, 4\}, \{1, 3, 5\}, \{1, 4, 5\}$	$W_1^m \cup \{\{1, 2, 3\}, \{1, 2, 4\}, \{1, 2, 5\}, \{1, 2, 3, 4\}, \{1, 2, 3, 5\}, \{1, 3, 4, 5\}\}$	48/120	4/14
2	$\{1, 2\}, \{2, 4, 5\}$	$W_2^m \cup \{\{1, 2, 3\}, \{1, 2, 4\}, \{1, 2, 5\}, \{2, 3, 4, 5\}\}$	28/120	2/14
3	$\{1, 3, 4\}, \{1, 3, 5\}$	W_3^m	8/120	2/14
4	$\{1, 3, 4\}, \{1, 4, 5\}, \{2, 4, 5\}$	$W_4^m \cup \{2, 3, 4, 5\}$	18/120	3/14
5	$\{1, 3, 5\}, \{1, 4, 5\}, \{2, 4, 5\}$	$W_5^m \cup \{2, 3, 4, 5\}$	18/120	3/14

In this game set of the minimal winning coalitions consists of five coalitions ($W^m = \{\{1,2\}, \{1,3,4\}, \{1,3,5\}, \{1,4,5\}, \{2,4,5\}\}$). So, $|W_1^m| = 4$, $|W_2^m| = |W_3^m| = 2$, $|W_4^m| = |W_5^m| = 3$, and $\sum\limits_{j \in \{1,\ldots,5\}} |W_j^m| = 14$. Then, the distribution of power by the PGI index immediately follows from formula (2), see Table 1.

Both indices are based on the concept of criticality of the player. However, the Shapley and Shubik index takes into account also all $n!$ possible orders to form the grand coalition. Therefore, in the formula of σ, *we* have the coefficient based on the cardinality of a coalition in which player i is critical. The PGI index considers only the number of minimal winning coalitions with player i. How a grand coalition is formed is not important in the PGI index. Player 2 is critical in one winning coalition of cardinality two, four winning coalitions of cardinality three, and one coalition of cardinality of size four. Thus, from formula (1), we have $\sigma_2(v) = \frac{3!}{5!} + 4 \cdot \frac{2!2!}{5!} + \frac{3!}{5!} = \frac{28}{120}$. Player 3 is critical only in two coalitions of cardinality three, so $\sigma_3(v) = 2 \cdot \frac{2!2!}{5!} = \frac{8}{120}$. Similarly we can calculate the power of players 4 and 5 by the Shapley and Shubik index: $\sigma_4(v) = \sigma_5(v) = 3 \cdot \frac{2!2!}{5!} + \frac{3!}{5!} = \frac{18}{120}$. As σ satisfies the efficiency property (see Shapley and Shubik (1954) and Sect. 2.2), then the power of player 1 is equal to $\sigma_1(v) = 1 - \sum\limits_{i=2}^{5} \sigma_i(v) = 1 - \frac{52}{120} = \frac{48}{120}$.

Player 1 belongs to four of five minimal winning coalitions. Moreover, player 1 is critical in ten winning coalitions $|\eta_1| = 10$, the greatest value in this game. Therefore, there is nothing surprising in the fact that he obtains the greatest power by both σ and h indices, see Table 1. Players 2 and 3 belong only to only two minimal winning coalitions. So, according to the PGI index, they receive equal and the least power (2/14). The Shapley and Shubik index assigns player 2 much more power than player 3. Player 2 is critical in six winning coalitions, whereas player 3 is critical in only two.

The Shapley and Shubik index satisfies all properties mentioned in Sect. 2.2, whereas the PGI index does not satisfy the bloc, local monotonicity, and transfer properties (see Felsenthal and Machover (1995, 1998), Freixas and Gambarelli (1997), and Bertini et al. (2013), for example.

A violation of the local monotonicity property can be also observed in the above example. In particular, player 2, with more votes than player 4 (and player 5), obtains less power according to the PGI index, see Table 1.

Regarding the bloc property, for example, Bertini et al., (2013) provided a description of a failure of this property by the PGH index in the seven-players weighted game [6; 4, 1, 1, 1, 1, 1, 1] when a bloc between two players with weights 1 is forced.

A failure of the transfer property for the PGI index can be observed in the following pair of games: $v_1 = [4;3, 1, 1, 1]$ and $v_2 = [4;2, 2, 1, 1]$, see Bertini et al., (2013).

3 Indices for Measurement Indirect Control

In Sect. 3.1, we present the method introduced by Karos and Peters (2015) for measuring the indirect control of firms in the corporate shareholding networks. Then, in Sect. 3.2, we propose a modification of the Karos and Peters approach using the Public Good Index (Holler, 1982).

3.1 Karos and Peters Approach

Karos and Peters (2015) provide a method for measuring the power control of all firms in a corporate network (investors and stock companies as well). Namely, Karos and Peters propose the Φ index modelling the indirect relations among firms by so-called invariant mutual structures.

Let N be a set of all firms involved in a corporate shareholding structure. The invariant mutual control structure C is a function that assigns to each coalition $S \in 2^N$ the set of all firms controlled by S, such that satisfies:

- $C(\emptyset) = \emptyset$,
- The monotonicity property (i.e., $C(S) \subseteq C(T)$) for all coalitions $S \subseteq T \subseteq N$), and
- The indirect control condition (i.e., $\forall R, S, T \in 2^N$ with $S \subseteq C(T)$ and $R \subseteq C(S \cup T)$ we have $R \subseteq C(T)$).

Let denote by $\overline{C}$ the set of all invariant mutual control structures based on N. For every invariant mutual control structure C in $\overline{C}$ there is defined a vector of simple games $v^C = (v_1^C, ..., v_n^C)$. Each v_k^C indicates who controls firm $k \in N$ for C, and $v_k^C(S) = 1$ if k is controlled by S; otherwise, $v_k^C(S) = 0$. Thus, for every firm $i \in N$ in a shareholding structure, there is a simple game whose winning coalitions are exactly those that control i. The Φ index is defined as follows:

$$\Phi_i(C) = \sum_{k \in N} \sigma_i(v_k^C) - v_i^C(N) \; for \; every \; i \in N \; and \; C \in \overline{C} \tag{3}$$

where σ is the Shapley and Shubik (1954) index. In particular, $v_i^C(N) = 0$ for each null firm *i*, i.e., a firm that does not belong to any winning coalition to exercise its control, and *i* is also not controlled by any coalition.

The Karos and Peters method to measure indirect control in corporate shareholding networks is an axiomatic approach. They started from the set of properties that, in their opinion, should characterize a good measure and, as a consequence, came up with an index defined by formula (1). Namely, the Φ index satisfies five axioms: null player, constant sum, anonymity, transfer, and controlled player.

In particular, Karos and Peters null player axiom states that the power of null players is equal to zero.

The second axiom—the constant sum property—states that the sum of all assigned powers is the same over $\overline{C}$. The first (null player axiom) and second axiom imply that this sum is equal to zero.

The third axiom—anonymity—states that the names of the players should not matter.

The fourth axiom—transfer property—states that for each player, the change in power when enlarging a mutual control structure *X* to *X'* should be equal to the change in power when enlarging a mutual control structure *Y* to *Y'*, assuming that the same control relations are added going from *X* to *X'* as when going from *Y* to *Y'*. The name is not casual, as this axiom is related to the transfer axiom used by Dubey (1975) to characterize the Shapley value and the Shapley-Shubik index (Shapley, 1953; Shapley & Shubik, 1954).

The fifth axiom—controlled player axiom—states that if firm *i* is a "controlled player", it means., controlled by at least one coalition and, as a consequence, by grand coalition *N*, but does not control any firm, then the power of firm *i* is set at -1. Subsequently, if firm *j* is an uncontrolled player, it means, controlled by no coalition at all, but firms *i* and *j* exert the same marginal control with respect to any coalition, then their difference in power is set at 1, i.e., firm *j* obtains 1 more than firm *i*.

For a precise definition of the Φ index and its properties, see Karos and Peters (2015).

3.2 Holler-Based Estimation of Firm's Control Power

What happens if we change the Shapley and Shubik (1954) index in the definition of the Φ index given by formula (3)? In the Karos and Peters (2015) framework, instead of the Shapley-Shubik index, we propose to use the Public Good Index (Holler, 1982). In this way, we obtain a modification of the Karos and Peters index—Holler-based estimation—for measuring firms' power control in corporate structures. In particular, this new index, *iPGI*, is defined by the following formula:

$$iPGI_i(C) = \sum_{k \in N} h_i(v_k^C) - v_i^C(N) \; for \; every \; i \in N \; and \; C \in \overline{C} \tag{4}$$

where C, $\overline{C}$, and $v_i^C(N)$ are defined as in Sect. 3.1.

Generally, formula (4) implies that each player i obtains the sum of all his Public Good Index values in the games in which he contributes to controlling the other players, minus the sum of all Public Good Index values of the other players in the game describing the control undergone by player i.

Of course, if we change the σ index with the h index in formula (3), we have to take into account that some of the properties satisfied by the Φ index will not be met by the new index—*iPGI*. The question is which of the properties will be preserved.

The PGI index, as the Shapley and Shubik index, satisfies the null player property in simple games, thus also the Karos and Peters null player property is satisfied by the *iPGI* index.

The PGI index, as the Shapley-Shubik index, satisfies the efficiency property in simple games, i.e., the sum of power assigned to all players equals 1. Thus, this property, together with the null player axiom, makes that the sum of all assigned powers is the same over $\overline{C}$ is equal to zero. So, the *iPGI* index satisfies the constant sum property.

The third axiom—anonymity—is also satisfied by the *iPGI* index as the PGI index satisfies this property in simple games.

It is difficult for the fourth axiom (transfer property) to be satisfied by the *iPGI* index as the PGI index does not satisfy this property in simple games, see (Bertini et al., 2013} or Sects. 2.2–2.3 for example.

The five axiom—controlled player axiom—is satisfied by the *iPGI* index by the construction of the formula (4), and that h satisfies the null player property.

In the context of indirect control, the null player removable property—which states that after removing null players from a simple game the power assigned to non-null players remains the same—is one of the desirable properties that was highlighted first in (Mercik & Stach, 2018) and next in (Staudacher et al., 2021a, 2021b). Still, (Staudacher et al., 2021a, 2021b) offers a bit more cautious "null investor removable property" for corporate shareholding networks with distinguishable investors and companies. Namely, let's cite here this property: "After removing the null investors, i.e., the investors whose voting rights cannot transform any losing coalition into a winning one, from a corporate shareholding network with distinguishable investors and companies, the non-null firm's measures of power should remain unchanged. Equivalently, the value of any firm in a corporate shareholding network is unchanged if the network is extended by adding a new null investor." It was just noted in (Staudacher et al., 2021a, 2021b) that the Φ index fulfils the null investor removable property. The PGI index satisfies the null player removable property in simple games, and as a consequence also the *iPGI* satisfies the null investor removable property for corporate shareholding networks with distinguishable investors and companies.

In a weighted game, we say that a power index satisfies local monotonicity if a firm that controls a large share of the total weight vote does not have less power than a firm with a smaller voting weight. Felsenthal and Machover (1998) state that any power index that does not satisfy the local monotonicity property is "pathological" and should be disqualified as a valid measure of power. Holler and Napel in (2005) claim the following: "Power indices that detect rather than postulate monotonicity can also be of help for a more abstract analysis of decision situations with respect to power." Holler and Napel in (2005), see the violation of this property as an advantage in some sense; see also Freixas and Kurz (2016). By the construction of formula (2), we see that any violation of the local monotonicity for the Public Good index (see Holler, 1982; Holler and Packel, 1983) implies a violation of the *iPGI* index. It is well-known that the Shapley and Shubik (1954) index satisfy the local monotonicity, see (Bertini et al., 2013), and also the index Φ possess this property. Moreover, the Φ index satisfies the monotonicity property proposed by Karos and Peters (2015) in the context of indirect control. Namely, their monotonicity postulate states that for two invariant mutual control structures X and Y: if a firm i is at least as much controlled in Y as he is in X, and his marginal control with respect to each coalition S is in X at least as large as in Y, then this firm should be assigned at least as much power in X as in Y.

The bloc property is also worth being mentioned, see (Mercik & Stach, 2018). The bloc property requires that the power of the merged entity $\{i\&j\}$—a bloc formed by player i with player j—will be larger than the power of player i if player j is not a null player. The bloc between players i and j, $\{i\&j\}$, may be regarded as a result of a takeover, in which player i, having annexed *j's* voting rights, now trades under the new name $\{i\&j\}$. Intuitively, it seems reasonable that a player should not lose in annexing the voting rights of another player who is not null. The PGI index in simple games does not satisfy this property, see (Bertini et al., 2013). On the other hand, the measure that does not satisfy this postulate can be used to reveal this information. Let us cite Holler and Napel, (2005): "Obviously, (…) the bloc principle presuppose that votes are transferable, at least, to some extent. However, if vote transfers are voluntarily, then, in fact, we do not need these principles (in the form of axioms) because i will not form a bloc with j if the power of $\{i\&j\}$ is smaller than the power of $\{i\}$, unless i wants to give up power. But we need a measure that tells player i that he should not merge with player j in this case, what is a desirable property if a power measure can point out this 'dilemma'." The Banzhaf index (1965) violates the bloc postulate, but not the local monotonicity property. While the Shapley and Shubik (1954) index obeys both, as already mentioned in Sect. 2.

4 An Example

Let us consider an example of a corporate shareholding structure with 13 firms—five stock companies (Companies: 1, 2, 3, 4, 5) and eight investors, i.e., firms without shareholdings (Firms 6, 7, 8, 9, 10, 11, 12, 13), see Fig. 1.

Figure 1 shows direct and indirect ownership in a theoretical example of corporate shareholding structure already studied in (Stach, 2017; Mercik & Stach, 2018; and Stach & Mercik, 2021). Percentages of ownership are indicated next to the links (direct arrows). For example, Company 5 has 25 per cent of direct ownership (i.e., we regard this value—25% —as a percentage of own voting rights) in Company 2. Through Company 2, it also has an indirect ownership in Companies 1 and 3. This

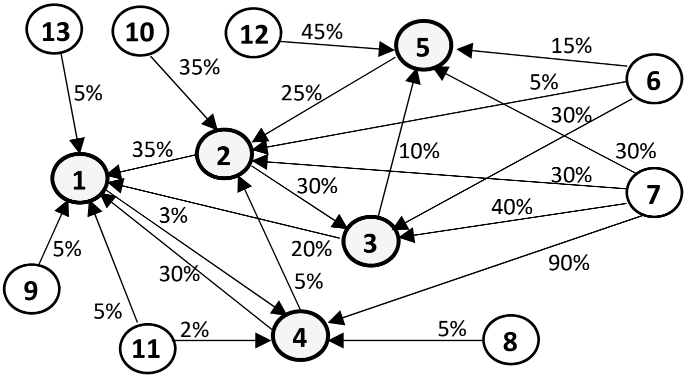

Fig. 1 Corporate shareholding network with 13 firms. *Source*: (Mercik & Stach, 2018)

theoretical ownership structure is not free of cycles, i.e., Company 5 owns 25% of Company 2, Company 2 owns 30% of Company 3, and Company 3 has 10% of Company 5, for example.

In this example, we regard a threshold of 50%—a simple majority. If a firm (or a coalition of firms) has ownership exceeding 50%, it has full control (100%), and the others have none (0%). In other words, with each stock company, we connect a weighted game with a simple majority.

Taking into consideration the direct and indirect ownership/control in this example, we can find the sets of all minimal winning coalitions for all companies (see Table 1), which facilitates to calculate the Φ and *iPGI* indices in this example (see Table 2).

In order to explain the result presented in Table 2, let's consider Company 1, for example. Taking into account only direct ownership, Company 2 with Company 3 have in total 35% + 20% = 55% voting rights in Company 1, which gives coalition {2, 3} full control over Company 1. Similarly, coalition {2, 4} having 65% of voting rights can exert total control over Company 1. Next, Company 2 and Firm 6 have 60% voting rights in Company 3. Thus, coalition {2, 6} controls Company 3. This implies that coalition {2, 6} indirectly controls, via Company 3, Company 1. Similarly,

Table 2 Minimal winning coalitions in the example

Company	Minimal winning coalitions considering direct and indirect control
Co. 1	{2, 3}, {2, 4}, {2, 6}, {2, 7}, {3, 7, 9}, {3, 7, 11}, {3, 7, 13}, {5, 7}, {6, 7}, {7, 10}, {7, 12}, {3, 10, 12}, {3, 5, 10}, {3, 4, 9}, {3, 4, 11}, {3, 4, 13}, {4, 5, 10}, {5, 6, 10}, {6, 10, 12}
Co. 2	{5, 7}, {5, 10}, {6, 7}, {7, 10}, {7, 12}, {3, 10, 12}, {6, 10, 12}
Co. 3	{2, 6}, {2, 7}, {5, 7}, {6, 7}, {7, 10}, {7, 12}, {5, 6, 10}, {6, 10, 12}
Co. 4	{7}
Co. 5	{3, 12}, {6, 12}, {6, 7}, {7, 12}

considering indirect control, coalition {2, 7} controls Company 1. Firm 7, with 90% voting rights in Company 4, controls Company 4 totally. Thus, coalition {2, 7} controls Company 1. Then, coalition {3, 4, 9} has 55% voting rights in Company 1. As Firm 7 totally controls Company 4, then coalition {3, 7, 9} controls indirectly (via Company 4) Company 1 as well. Continuing consideration about direct and indirect ownership in Company 1, we have nineteen minimal winning coalitions that control Company 1.

Stach and Mercik in (2021) calculated the Φ index in this example. However, for clarity and to give the possibility to compare Φ with a new proposed index—*iPGI*, we present these calculations in Table 3.

The results in Table 3 are obtained by calculating the σ index for each company i in the weighted game of this company v_i, $i = 1, 2, \ldots, 13$. Let C be an invariant mutual control structure, based on $N = \{1, 2, \ldots 13\}$, defined by the vector of weighted games $(v_{11}^C, \cdots v_{13}^C)$. The set of all winning coalitions of each game v_i^C $(i \in N)$ indicates coalitions that control i. To calculate the σ index by formula (1) the set of all winning coalitions is needed, but this can be easy find when the set of all minimal winning coalitions in Table 2. For example, in Company 5, we have four non-null players (3, 6, 7, and 12) and four minimal winning coalitions: {3, 12}, {6, 12}, {6, 7}, {7, 12}, see Table 2. As the Shapley and Shubik index satisfies the null player property, then $\sigma_i(v_5^C) = 0$ for each player $i = 1, 2, 4, 5, 8, 9, 10$, and 11. The σ index satisfies the null player removable property as well, so we can calculate the power of non-null players considering a simple game with only these four players. The set of all winning coalitions consists of the four minimal winning

Table 3 Calculations of Φ index in the example

	Power distribution in accordance with σ index in simple game v_i							
Firm	$i = 1$	$i = 2$	$i = 3$	$i = 4$	$i = 5$	$i = 6, \ldots, 13$	Total	Φ
1	0.000	0.000	0.000	0.000	0.000	0.000	0.000	–1.000
2	0.196	0.000	0.133	0.000	0.000	0.000	0.329	–0.671
3	0.121	0.017	0.000	0.000	0.083	0.000	0.221	–0.779
4	0.098	0.000	0.000	0.000	0.000	0.000	0.098	–0.902
5	0.056	0.150	0.050	0.000	0.000	0.000	0.256	–0.744
6	0.096	0.067	0.250	0.000	0.250	0.000	0.662	0.662
7	0.265	0.400	0.433	1.000	0.250	0.000	2.348	2.348
8	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
9	0.010	0.000	0.000	0.000	0.000	0.000	0.010	0.010
10	0.092	0.267	0.083	0.000	0.000	0.000	0.442	0.442
11	0.010	0.000	0.000	0.000	0.000	0.000	0.010	0.010
12	0.047	0.100	0.050	0.000	0.417	0.000	0.613	0.613
13	0.010	0.000	0.000	0.000	0.000	0.000	0.010	0.010
Total	1	1	1	1	1	0	5	0

coalitions and {3, 6, 7}, {3, 6, 12}, {3, 7, 12}, {6, 7, 12}, and {3, 6, 7, 12}. Player 3 is critical in only one two-person winning coalition {3, 12}, so from formula (1) we have $\sigma_3(v_5^C) = \frac{(2-1)!(4-2)!}{4!} = \frac{1}{12} \approx 0.083$. Both players 6 and 7 are critical in two two-person minimal winning coalitions and one three-person coalition. Thus, $\sigma_6(v_5^C) = \sigma_7(v_5^C) = 2 \cdot \frac{(2-1)!(4-2)!}{4!} + \frac{(3-1)!(4-3)!}{4!} = \frac{3}{12} = 0.25$. The σ index satisfies the efficiency property, so $\sigma_{12}(v_5^C) = 1 - \frac{1}{12} - 2 \cdot \frac{3}{12} = \frac{5}{12} \approx 0.417$, see Table 3.

Having σ for each simple game (v_i^C, $i = 1, \ldots, 13$) the results, in Table 3, are immediately obtained by formula (3). Consider Company 1, for example. Company 1 is controlled by grand coalitions, so $v_1^C(N) = 1$. Moreover, Company 1 does not have voting rights in any of the other companies. Thus $\sigma_1(v_i^C) = 0$, for each $i = 1, \ldots, 13$. So, $\Phi_1 = 0 – 1 = –1$. Each investor is not controlled by any firm, so $v_i^C(N) = 0$ for $i = 6, 7\ldots, 13$. Investor 6 takes part in winning coalitions in Companies 1, 2, 3, and 5. Thus his voting power in these companies calculated by σ is greater than zero. By summing up these values and subtracting 0 we obtain $\Phi_6 = 0.096 + 0.067 + 0.250 + 0.250 – 0 = 0.662$.

In order to calculate the power control of each firm in the theoretical example in accordance with the *iPGI* index, it is necessary to calculate first the power distributions of the Public Good Index (the h index) in all companies, which is provided in Table 4.

The results in Table 4 are obtained by calculating the h index for each company i in the weighted game of this company v_i, $i = 1, 2, \ldots, 13$. As the PGI index satisfies both null player and null player removable properties, we can consider each game v_i, $i = 1, 2, \ldots, 13$ as a game that consists of only those players that are members of minimal winning coalitions, which makes them non-null players. Consider Company 5, for example. Game v_5 is a four-person game with four minimal winning coalitions, see Table 2. Player 3 is critical in minimal winning coalition {3, 12}; player 6 is critical in minimal winning coalitions {6, 7} and {6, 12}; player 7 is critical in {6, 7} and {7, 12}; and player 12 in {6, 12} and {7, 12}. Thus, from formula (2), we immediately have: $h_3(v_5) = \frac{1}{8} = 0.125$, $h_6(v_5) = h_7(v_5) = \frac{2}{8} = 0.25$, $h_{12}(v_5) = \frac{3}{8} = 0.375$. Having h for each simple game (v_i, $i = 1, \ldots, 13$) the *iPGI* index is immediately obtained by formula (4), see Table 4.

The *iPGI* index of thirteen firms in the example is (–1, –0.8073, –0.6288, –0.8980, –0.6823, 0.6788, 1.9411, 0.0000, 0.0408, 0.5391, 0.0408, 0.7348, 0.0408), whereas the Φ index is equal to (–1, –0.671, –0.779, –0.902, –0.744, 0.662, 2.348, 0, 0.01, 0.442, 0.01, 0.613, 0.01).

In the considered example, we observe some similarities and differences between the *iPGI* and Karos-Peters (Φ) indices in assessing the power control of firms in the corporate shareholding network. Table 5 presents rankings of the stock companies and investors separately in accordance with both indices. Namely, considering the ranking of companies, the difference is only in Companies 2 and 3. The positions of the remaining companies estimated by Φ and *iPGI* are the same. The *iPGI* index gives more control power to Company 3 than to Company 5 and 2, classifying Company 3 in the first position. The Φ index classifies Company 2 in the first position, next

Table 4 Calculations of *iPGI* in the example

Firm	Power distribution in accordance with *h* index in company *i*						
	1	2	3	4	5	Total	*iPGI*
1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	– 1.0000
2	0.0816	0.0000	0.1111	0.0000	0.0000	0.1927	– 0.8073
3	0.1837	0.0625	0.0000	0.0000	0.1250	0.3712	– 0.6288
4	0.1020	0.0000	0.0000	0.0000	0.0000	0.1020	– 0.8980
5	0.0816	0.1250	0.1111	0.0000	0.0000	0.3177	– 0.6823
6	0.0816	0.1250	0.2222	0.0000	0.2500	0.6788	0.6788
7	0.1633	0.2500	0.2778	1.0000	0.2500	1.9411	1.9411
8	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
9	0.0408	0.0000	0.0000	0.0000	0.0000	0.0408	0.0408
10	0.1224	0.2500	0.1667	0.0000	0.0000	0.5391	0.5391
11	0.0408	0.0000	0.0000	0.0000	0.0000	0.0408	0.0408
12	0.0612	0.1875	0.1111	0.0000	0.3750	0.7348	0.7348
13	0.0408	0.0000	0.0000	0.0000	0.0000	0.0408	0.0408
Total	1	1	1	1	1	5	0

Table 5 Rankings of firms by Φ and *iPGI* indices in the example

Position	Power distribution in accordance with			
	Φ for stock companies	Φ for investors	*iPGI* for stock companies	*iPGI* for investors
1	Company 2	Firm 7	Company 3	Firm 7
2	Company 5	Firm 6	Company 5	Firm 12
3	Company 3	Firm 12	Company 2	Firm 6
4	Company 4	Firm 10	Company 4	Firm 10
5	Company 1	Firms 9, 11, 13	Company 1	Firms 9, 11, 13
6		Firm 8		Firm 8

Company 5, and in the third position Company 3. Other companies have the same position in rankings of control power in accordance with both indices.

How can we explain this? The PGI index is based on the minimal winning coalitions, see Sect. 2.1 or Holler (1982). If we consider direct and indirect ownership and participation of a firm in all minimal winning coalitions presented in Table 2, we see that Company 3 participates in the greatest number of minimum winning coalitions of all firms involved in the example under consideration, see Table 6. Precisely, it takes part in 10 minimal winning coalitions, while Company 2 belongs to only four minimal winning coalitions and Company 5 belongs to only five minimal winning coalitions. Moreover, Company 3, through the minimal winning coalitions to which it belongs, controls three companies: Company 1, 2, and 5. While Company 2 controls only two Companies: 1 and 3, and Company 5 controls three companies: 1, 2, and 3; see Tables 2 and 6. On the other hand, the Φ index is based on the Shapley and Shubik index, which in turn is based on the concept of a player's average "criticality" to all winning coalitions with him, see formula (3) and the formula (1) of the Shapley and Shubik index in Sect. 2.1. So, even Company 3 belongs to more minimal winning coalitions than Company 2 in the weighted game corresponding to Company 1; Company 2 belongs to more winning coalitions in which it is critical. Namely, the direct ownership of Company 2 in Company 1 (i.e., 35%) is greater than the direct ownership of Company 3 in Company 1 (i.e., 20%); see Fig. 1. Players 2 and 3 form a minimal winning coalition. However, with the enlargement of the coalition by players 4, 6, or 7, for example, player 3 is no longer a critical player, whereas player 2 is still critical. Thus, $|\eta_2| > |\eta_3|$ (the number of winning coalitions in which player 2 is critical is greater than the number of winning coalitions in which player 3 is critical) in Company 1. As a consequence, the power assigned to Company 2 is greater than the power assigned to Company 3 by the σ index in the simple game corresponding to Company 1. Then, the total power assigned to Company 2 in simple games corresponding to Companies 1 and 3 is greater than the power assigned to Company 3 in simple games corresponding to Companies 1, 2, and 5. Eventually, the Φ index classifies Company 2 in the higher post than Company 3.

When it comes to investors' ranking, the *iPGI* index gives more control power to Company 12 than to Company 6, as opposed to the Φ index. The power of Firm 12 in Co. 5 is really strong ($h_{12}(v_5) = 0.375$) and this decides about its total power in the whole network that is greater than the power of Firm 6 calculated by *iPGI*.

Table 6 Firm's control and participation in minimal winning coalitions in the example

Firm i	Number of minimal winning coalitions containing Firm i	Number of companies controlled by minimal winning coalitions with Firm i (companies' names)
1	0	0
2	4	2 (Companies 1 and 3)
3	10	3 (Companies 1, 2, and 5)
4	5	1 (Company 1)
5	5	3 (Companies 1, 2, and 3)
6	5	4 (Companies 1, 2, 3, and 5)
7	9	4 (Companies 1, 3, 4, and 5)
8	0	0
9	2	1 (Company 1)
10	7	3 (Companies 1, 2, and 3)
11	2	1 (Company 1)
12	5	4 (Companies 1, 2, 3, and 5)
13	2	1 (Company 1)

Both indices rank the control power of Company 7 as first and assign much more power to Firm 7 than to other investors. Namely, the *iPGI* index gives Company 7 48% of all the power assigned to investors, and the Φ index allocates even more: 57%; see Tables 3 and 4.

The Public Good Index (Holler, 1982) and the Shapley and Shubik (1954) index satisfy the null player property (it means that null players obtain zero power), so it is not strange that both *iPGI* and Φ indices classify Firm 8 on the last position with control power equal zero. Firm 8 does not belong to any minimal winning coalition (see Table 2), so Firm 8 obtains null power in each company (it means in the weighted game related to each company). So, the difference between both indices (*iPGI* and Φ) is only in the second and third position in investors' control power ranking, see Table 5.

5 Concluding Remarks

This paper has drawn up a discussion about the possible use of the PGI index (Holler, 1982) to measure the power of firms in corporate shareholding networks in terms of properties that this index possesses and fails. We proposed an approach based on the modification of the Karos and Peters (2015) method and the PGI index, see Sect. 3.

A justification for selecting the PGI index to assess player control power in complex corporate networks and using it in the framework proposed by Karos and Peters (2015) was that the index is based on minimal winning coalitions. In its non-normalized version, i.e., the raw measure, it counts the number of times that a player belongs to a minimal winning coalition. Therefore, the values assigned to companies according to the index may reflect the power of firms to form such coalitions. In the context of a possible takeover and the speed of companies' actions, it seems plausible to regard this kind of coalition at first. From this point of view, it is interesting to consider other indices based on minimal winning coalitions like the shift index–proposed by Alonso Meijide and Friexas (2010), and the Deegan and Packel (1978) index in the scheme proposed by Karos and Peters (2015).

Further development can refer to the application of the *iPGI* index to the estimation of company value in a complex market seen as a network of firms, see (Mercik et al., 2021), (Gładysz et al., 2019), and (Forlicz et al., 2018).

Acknowledgements This paper was funded under subvention funds for the AGH University of Science and Technology in Krakow, Poland; from WSB University in Wroclaw, Poland; and from the University of Bergamo, Italy.

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