# New Classes of Bent Functions via the Switching Method

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**Abstract.** The switching method is a powerful method to construct bent functions. In this paper, using this method, we present two generic constructions of piecewise bent functions from known ones, which generalize some earlier works. Further, based on these two generic constructions, we obtain several infinite families of bent functions from quadratic bent functions and the Maiorana-MacFarland class of bent functions by calculating their duals. It is worth noting that our constructions can produce bent functions with the optimal algebraic degree.

**Keywords:** Bent function · Walsh transform · Switching method

## 1 Introduction

Boolean bent functions were first introduced by Rothaus in 1976 [10] as an interesting combinatorial object with maximum Hamming distance to the set of all affine functions. Over the last four decades, bent functions have attracted a lot of research interest due to their important applications in cryptography [1], sequences [8] and coding theory [2,3]. Later, Kumar, Scholtz and Welch in [4] generalized the notion of Boolean bent functions to the case of functions over an arbitrary finite field.

Let $\mathbb{F}_{p^n}$ denote the finite field with $p^n$ elements, where $p$ is a prime and $n$ is a positive integer. Given a function $f(x)$ mapping from $\mathbb{F}_{p^n}$ to $\mathbb{F}_p$, the Walsh transform of $f(x)$ is defined by

$$\widehat{f}(b) = \sum\nolimits_{x\in\mathbb{F}_{p^n}} \omega^{f(x)-\mathrm{Tr}(bx)},\ b \in \mathbb{F}_{p^n},$$

where $\omega = e^{\frac{2\pi\sqrt{-1}}{p}}$ is a complex primitive $p$-th root of unity. According to [4], $f(x)$ is called a $p$-ary bent function if all its Walsh coefficients satisfy $|\widehat{f}(b)| = p^{n/2}$.

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A $p$-ary bent function $f(x)$ is called regular if $\widehat{f}(b) = p^{n/2}\omega^{\widetilde{f}(b)}$ holds for some function $\widetilde{f}(x)$ mapping $\mathbb{F}_{p^n}$ to $\mathbb{F}_p$, and it is called weakly regular if there exists a complex $\mu$ having unit magnitude such that $\widehat{f}(b) = \mu^{-1}p^{n/2}\omega^{\widetilde{f}(b)}$ for all $b \in \mathbb{F}_{p^n}$. The function $\widetilde{f}(x)$ is called the dual of $f(x)$ and it is also bent.

The switching method is a powerful method to construct bent functions. In 2016, Xu et al. [15] constructed a class of piecewise $p$-ary bent functions $f(x) = g(x) + c\mathrm{Tr}(x)^{p-1}$ from Gold functions $g(x)$ via the switching method, where $p$ is an odd prime and $c \in \mathbb{F}_p^*$. In 2017, Xu et al. [17] constructed two classes of piecewise $p$-ary bent functions $f(x) = g(x) + \mathrm{Tr}(ux)\mathrm{Tr}(x)^{p-1}$ from $p$-ary Kasami functions and Sidelnikov functions $g(x)$ [6], where $p$ is an odd prime and $u \in \mathbb{F}_{p^n}^*$. Recall that Tang et al. [11] constructed bent functions of the form $g(x) + F(\mathrm{Tr}(u_1x), \mathrm{Tr}(u_2x), \cdots, \mathrm{Tr}(u_\tau x))$, where $g(x) : \mathbb{F}_{p^n} \to \mathbb{F}_p$, $F(x_1, \cdots, x_\tau) \in \mathbb{F}_p[x_1, \cdots, x_\tau]$ and $u_i \in \mathbb{F}_{p^n}$ for $1 \le i \le \tau$. Motivated by Xu et al.'s and Tang et al.'s works, we investigate the bentness of piecewise bent functions of the forms

$$f(x) = g(x) + F(\mathrm{Tr}(u_1x), \mathrm{Tr}(u_2x), \cdots, \mathrm{Tr}(u_\tau x)) + c\big(\prod_{i=1}^{\kappa} \mathrm{Tr}(v_ix)\big)^{p-1} \quad (1)$$

and

$$f(x) = g(x) + F(\mathrm{Tr}(u_1x), \mathrm{Tr}(u_2x), \cdots, \mathrm{Tr}(u_\tau x))\big(\prod_{i=1}^{\kappa} \mathrm{Tr}(v_ix)\big)^{p-1}, \quad (2)$$

where $g(x) : \mathbb{F}_{p^n} \to \mathbb{F}_p$, $F(x_1, \cdots, x_\tau) \in \mathbb{F}_p[x_1, \cdots, x_\tau]$, $c \in \mathbb{F}_p^*$, $u_i \in \mathbb{F}_{p^n}$ for $1 \le i \le \tau$ and $v_j \in \mathbb{F}_{p^n}^*$ for $1 \le j \le \kappa$. In this paper, we first present generic constructions of bent functions with the forms (1) and (2) respectively. Notice that the works of Xu et al. [15,17] are two special cases of our work for $\kappa = 1$, $F(x_1) = 0$ in (1) and $\kappa = 1$, $F(x_1) = x_1$ in (2). In addition, by calculating the duals of quadratic bent functions and the Maiorana-MacFarland class of bent functions, several infinite families of bent functions are constructed by using these generic constructions.

The rest of this paper is organized as follows. Section 2 fixes some notation and introduces some preliminaries. Section 3 proposes two generic constructions of bent functions of the forms (1) and (2) respectively. Section 4 constructs several infinite families of bent functions using the two generic constructions given in Sect. 3. Finally, Sect. 5 concludes this paper.

## 2 Preliminaries

Throughout this paper, let $\mathbb{F}_{p^n}$ denote the finite field with $p^n$ elements, where $p$ is a prime and $n$ is a positive integer. The trace function from $\mathbb{F}_{p^n}$ to its subfield $\mathbb{F}_{p^k}$ is defined by $\mathrm{Tr}_k^n(x) = \sum_{i=0}^{n/k-1} x^{p^{ik}}$. In particular, when $k = 1$, we use the notation $\mathrm{Tr}(x)$ instead of $\mathrm{Tr}_1^n(x)$. A function $F(x_1, \cdots, x_n) : \mathbb{F}_p^n \mapsto \mathbb{F}_p$ is often represented by its algebraic normal form

$$F(x_1, \cdots, x_n) = \sum_{e=(e_1,\cdots,e_n)\in\mathbb{F}_p^n} a(e)(\prod_{i=1}^{n} x_i^{e_i}), \;\; a(e) \in \mathbb{F}_p. \quad (3)$$

A polynomial in $\mathbb{F}_p[x_1, \cdots, x_n]$ with the form (3) is called a reduced polynomial. The algebraic degree of $F(x_1, \cdots, x_n)$, denoted by $\deg(F)$, is defined as $\deg(F) = \max_{e \in \mathbb{F}_p^n} \{\sum_{i=1}^n e_i : a(e) \neq 0\}$, where $e = (e_1, \cdots, e_n) \in \mathbb{F}_p^n$.

**Lemma 1.** ([5, Propositions 4.4 and 4.5]) *Let $f(x)$ be a bent function from $\mathbb{F}_{p^n}$ to $\mathbb{F}_p$. Then $\deg(f) \leq \frac{(p-1)n}{2} + 1$. Moreover, if $f(x)$ is weakly regular bent, then $\deg(f) \leq \frac{(p-1)n}{2}$.*

To simplify the proof of our main result in the sequel, we fix some notation firstly. Let $g(x)$ be a function from $\mathbb{F}_{p^n}$ to $\mathbb{F}_p$. For $r := (r_1, \cdots, r_\kappa) \in \mathbb{F}_p^\kappa$ and $v := (v_1, \cdots, v_\kappa) \in \mathbb{F}_{p^n}^* \times \cdots \times \mathbb{F}_{p^n}^*$, define

$$\mathbb{T}_{r,v} = \{x \in \mathbb{F}_{p^n} : \mathrm{Tr}(v_i x) = r_i,\ i = 1, \cdots, \kappa\}$$

and

$$\mathbb{S}_g(r, v, b) = \sum_{x \in \mathbb{T}_{r,v}} \omega^{g(x) - \mathrm{Tr}(bx)}$$

for any $b \in \mathbb{F}_{p^n}$. It then can be verified that $\mathbb{F}_{p^n} = \cup_{r \in \mathbb{F}_p^\kappa} \mathbb{T}_{r,v}$. Moreover, we have

$$\begin{aligned}\widehat{g}\big(b - \sum_{i=1}^{\kappa} v_i s_i\big) &= \sum_{x \in \mathbb{F}_{p^n}} \omega^{g(x) - \mathrm{Tr}\left((b - \sum_{i=1}^{\kappa} v_i s_i)x\right)} \\ &= \sum_{r \in \mathbb{F}_p^\kappa} \sum_{x \in \mathbb{T}_{r,v}} \omega^{g(x) - \mathrm{Tr}(bx) + \sum_{i=1}^{\kappa} r_i s_i} \\ &= \sum_{r \in \mathbb{F}_p^\kappa} \mathbb{S}_g(r, v, b) \omega^{\sum_{i=1}^{\kappa} r_i s_i}.\end{aligned}$$

From the inverse Fourier transform we can derive

$$\mathbb{S}_g(r, v, b) = \frac{1}{p^\kappa} \sum_{s \in \mathbb{F}_p^\kappa} \omega^{-\sum_{i=1}^{\kappa} r_i s_i} \widehat{g}\big(b - \sum_{i=1}^{\kappa} v_i s_i\big), \tag{4}$$

where $s := (s_1, \cdots, s_\kappa) \in \mathbb{F}_p^\kappa$.

To make the computation of the Walsh transform of the functions investigated in the sequel feasible, we consider a class of weakly regular bent functions $g(x) : \mathbb{F}_{p^n} \to \mathbb{F}_p$ whose dual satisfies

$$\widetilde{g}\big(x - \sum_{i=1}^{\kappa} v_i s_i - \sum_{i=1}^{\tau} u_i t_i\big) = \widetilde{g}\big(x - \sum_{i=1}^{\tau} u_i t_i\big) + \sum_{i=1}^{\kappa} \varphi_{v_i}(x) s_i, \tag{5}$$

where $\varphi_{v_i}(x)$ is a function from $\mathbb{F}_{p^n}$ to $\mathbb{F}_p$, $u_i \in \mathbb{F}_{p^n}$, $v_j \in \mathbb{F}_{p^n}^*$ and $t_i, s_j \in \mathbb{F}_p$ for each $1 \leq i \leq \tau$ and $1 \leq j \leq \kappa$. Without loss of generality, assume that $\widehat{g}(b) = \mu^{-1} p^{n/2} \omega^{\widetilde{g}(b)}$ for any $b \in \mathbb{F}_{p^n}$. Then

$$\begin{aligned}\widehat{g}\big(b - \sum_{i=1}^{\tau} u_i t_i - \sum_{i=1}^{\kappa} v_i s_i\big) &= \mu^{-1} p^{n/2} \omega^{\widetilde{g}\left(b - \sum_{i=1}^{\tau} u_i t_i\right) + \sum_{i=1}^{\kappa} \varphi_{v_i}(b) s_i} \\ &= \widehat{g}\big(b - \sum_{i=1}^{\tau} u_i t_i\big) \omega^{\sum_{i=1}^{\kappa} \varphi_{v_i}(b) s_i}.\end{aligned} \tag{6}$$

Further we characterize the value of $\mathbb{S}_g(r, v, b)$ as below.

**Lemma 2.** *Let $g(x) : \mathbb{F}_{p^n} \rightarrow \mathbb{F}_p$ be a weakly regular bent function whose dual satisfies* (5)*. For any $b \in \mathbb{F}_{p^n}$, $\mathbb{S}_g(r, v, b) = \widehat{g}(b)$ if $r_i = \varphi_{v_i}(b)$ for any $1 \leq i \leq \kappa$ and otherwise, $\mathbb{S}_g(r, v, b) = 0$, where $r = (r_1, \cdots, r_\kappa) \in \mathbb{F}_p^\kappa$ and $v = (v_1, \cdots, v_\kappa) \in \mathbb{F}_{p^n}^* \times \cdots \times \mathbb{F}_{p^n}^*$.*

*Proof.* For any $b \in \mathbb{F}_{p^n}$, $r = (r_1, \cdots, r_\kappa) \in \mathbb{F}_p^\kappa$ and $v = (v_1, \cdots, v_\kappa) \in \mathbb{F}_{p^n}^* \times \cdots \times \mathbb{F}_{p^n}^*$, (4) yields

$$\mathbb{S}_g(r, v, b) = \frac{1}{p^\kappa} \sum_{s \in \mathbb{F}_p^\kappa} \omega^{-\sum_{i=1}^{\kappa} r_i s_i} \widehat{g}\big(b - \sum_{i=1}^{\kappa} v_i s_i\big),$$

where $s = (s_1, \cdots, s_\kappa) \in \mathbb{F}_p^\kappa$. By using (6), one gives

$$\mathbb{S}_g\big(r, v, b\big) = \frac{1}{p^\kappa} \widehat{g}\big(b\big) \sum_{s \in \mathbb{F}_p^\kappa} \omega^{\sum_{i=1}^{\kappa} (\varphi_{v_i}(b) - r_i) s_i}.$$

Further the desired result follows from the fact that $\sum_{s_i \in \mathbb{F}_p} \omega^{(\varphi_{v_i}(b) - r_i) s_i} = p$ if $r_i = \varphi_{v_i}(b)$ and 0 otherwise for any $1 \leq i \leq \kappa$. This completes the proof.

Let $h : \mathbb{F}_{p^n} \rightarrow \mathbb{F}_p$ be defined as in

$$h(x) = g(x) + F(\mathrm{Tr}(u_1 x), \mathrm{Tr}(u_2 x), \cdots, \mathrm{Tr}(u_\tau x)), \tag{7}$$

where $g(x) : \mathbb{F}_{p^n} \rightarrow \mathbb{F}_p$, $F(x_1, \cdots, x_\tau)$ is an arbitrary reduced polynomial in $\mathbb{F}_p[x_1, \cdots, x_\tau]$, $u_i \in \mathbb{F}_{p^n}$ for $1 \leq i \leq \tau$. The Walsh transform of $h(x)$ can be given as follows.

**Lemma 3.** ([13, Theorem 1]) *Let $h(x)$ be defined as in* (7)*. Then for any $b \in \mathbb{F}_{p^n}$,*

$$\widehat{h}(b) = \frac{1}{p^\tau} \sum_{(t_1, \cdots, t_\tau) \in \mathbb{F}_p^\tau} \widehat{F}(t_1, \cdots, t_\tau) \widehat{g}(b - \sum_{i=1}^{\tau} u_i t_i).$$

Next we characterize the value of $\mathbb{S}_h(r, v, b)$ for a class of weakly regular bent functions $g(x)$.

**Lemma 4.** *Let $h(x)$ be defined as in* (7) *and $g(x) : \mathbb{F}_{p^n} \rightarrow \mathbb{F}_p$ be a weakly regular bent function whose dual satisfies* (5)*. Then for any $b \in \mathbb{F}_{p^n}$, $\mathbb{S}_h(r, v, b) = \widehat{h}(b)$ if $r_i = \varphi_{v_i}(b)$ for any $1 \leq i \leq \kappa$ and otherwise, $\mathbb{S}_h(r, v, b) = 0$, where $r = (r_1, \cdots, r_\kappa) \in \mathbb{F}_p^\kappa$ and $v = (v_1, \cdots, v_\kappa) \in \mathbb{F}_{p^n}^* \times \cdots \times \mathbb{F}_{p^n}^*$.*

*Proof.* For any $b \in \mathbb{F}_{p^n}$, $r = (r_1, \cdots, r_\kappa) \in \mathbb{F}_p^\kappa$ and $v = (v_1, \cdots, v_\kappa) \in \mathbb{F}_{p^n}^* \times \cdots \times \mathbb{F}_{p^n}^*$, (4) gives

$$\mathbb{S}_h(r, v, b) = \frac{1}{p^\kappa} \sum_{s \in \mathbb{F}_p^\kappa} \omega^{-\sum_{i=1}^{\kappa} r_i s_i} \widehat{h}\big(b - \sum_{i=1}^{\kappa} v_i s_i\big), \tag{8}$$

where $s = (s_1, \cdots, s_\kappa) \in \mathbb{F}_p^\kappa$. Note that Lemma 3 yields

$$\begin{aligned}\widehat{h}(b - \sum_{i=1}^{\kappa} v_i s_i) &= \frac{1}{p^\tau} \sum_{\mathbf{t} \in \mathbb{F}_p^\tau} \widehat{F}(t_1, \cdots, t_\tau) \widehat{g}(b - \sum_{i=1}^{\tau} u_i t_i - \sum_{i=1}^{\kappa} v_i s_i) \\ &= \frac{1}{p^\tau} \omega^{\sum_{i=1}^{\kappa} \varphi_{v_i}(b) s_i} \sum_{t \in \mathbb{F}_p^\tau} \widehat{F}(t_1, \cdots, t_\tau) \widehat{g}(b - \sum_{i=1}^{\tau} u_i t_i),\end{aligned}$$

where $t := (t_1, \cdots, t_\tau)$. The last equality follows from (6). Then from Lemma 3, one obtains

$$\widehat{h}\big(b - \sum_{i=1}^{\kappa} v_i s_i\big) = \omega^{\sum_{i=1}^{\kappa} \varphi_{v_i}(b) s_i} \widehat{h}(b). \tag{9}$$

Substituting (9) into (8) gives

$$\mathbb{S}_h(r, v,\, b) = \frac{1}{p^\kappa} \sum_{s \in \mathbb{F}_p^\kappa} \omega^{\sum_{i=1}^{\kappa} (\varphi_{v_i}(b) - r_i) s_i} \widehat{h}(b).$$

Further the desired result follows from the fact $\sum_{s_i \in \mathbb{F}_p} \omega^{(\varphi_{v_i}(b) - r_i) s_i} = p$ if $r_i = \varphi_{v_i}(b)$ and 0 otherwise for any $1 \leq i \leq \kappa$. This completes the proof.

## 3 The Generic Constructions of Bent Functions

In this section, we will present two generic constructions of bent functions from known ones with certain properties.

### 3.1 The First New Class of Bent Functions

A class of non-quadratic $p$-ary bent functions $f(x) = \mathrm{Tr}(\lambda x^{p^k+1}) + c\mathrm{Tr}(x)^{p-1}$ with $\deg(f) = p - 1$ was presented in [15], where $p$ is an odd prime and $n/\gcd(k, n)$ is odd. Based on Xu et al.'s work [15], we present the first construction.

**Construction 1:** Let $u_1, \cdots, u_\tau$ be $\tau \geq 1$ elements in $\mathbb{F}_{p^n}$, $v_1, \cdots, v_\kappa$ be $\kappa \geq 1$ elements in $\mathbb{F}_{p^n}^*$ and $c \in \mathbb{F}_p^*$. Let $g(x)$ be a weakly regular bent function over $\mathbb{F}_{p^n}$ whose dual satisfies (5) and $F(x_1, \cdots, x_\tau)$ be any reduced polynomial in $\mathbb{F}_p[x_1, \cdots, x_\tau]$. Generate the function $f(x) : \mathbb{F}_{p^n} \to \mathbb{F}_p$ from $g$ and $F$ as in (1).

Then our first main result is stated as follows.

**Theorem 1.** *Let $f(x) : \mathbb{F}_{p^n} \to \mathbb{F}_p$ be the function generated by Construction 1. Then $f(x)$ is a bent function if $h(x)$ given by (7) is weakly regular bent.*

*Proof.* For any $b \in \mathbb{F}_{p^n}$, the Walsh transform of $f(x)$ defined by (1) is

$$\widehat{f}(b) = \sum_{\prod_{i=1}^{\kappa} \mathrm{Tr}(v_i x) = 0} \omega^{h(x) - \mathrm{Tr}(bx)} + \sum_{\prod_{i=1}^{\kappa} \mathrm{Tr}(v_i x) \neq 0} \omega^{h(x) - \mathrm{Tr}(bx) + c}.$$

Note that

$$\big\{x \in \mathbb{F}_{p^n} : \prod_{i=1}^{\kappa} \mathrm{Tr}(v_i x) = 0\big\} = \bigcup_{r \in \mathbb{F}_p^\kappa} \big\{x \in \mathbb{T}_{r,v} : \prod_{i=1}^{\kappa} r_i = 0\big\},$$

where $r := (r_1, \cdots, r_\kappa)$. Then

$$\begin{aligned}\widehat{f}(b) &= \sum_{\substack{\prod_{i=1}^{\kappa} r_i=0 \\ r_i \in \mathbb{F}_p}} \sum_{x \in \mathbb{T}_{r,v}} (1-\omega^c)\omega^{h(x)-\mathrm{Tr}(bx)} + \omega^c \widehat{h}(b) \\ &= \sum_{\substack{\prod_{i=1}^{\kappa} r_i=0 \\ r_i \in \mathbb{F}_p}} (1-\omega^c)\mathbb{S}_h(r,v,b) + \omega^c \widehat{h}(b).\end{aligned}$$

From Lemma 4 we can deduce that

$$\widehat{f}(b) = (1-\omega^c)\widehat{h}(b) + \omega^c\widehat{h}(b) = \widehat{h}(b)$$

if $\prod_{i=1}^{\kappa} \varphi_{v_i}(b) = 0$ and $\widehat{f}(b) = \omega^c \widehat{h}(b)$ if $\prod_{i=1}^{\kappa} \varphi_{v_i}(b) \neq 0$. Hence the desired conclusion follows.

### 3.2 The Second New Class of Bent Functions

Based on the work in [15], Xu et al. [17] further constructed two classes of piecewise $p$-ary bent functions $f(x) = g(x) + \mathrm{Tr}(ux)\mathrm{Tr}(x)^{p-1}$ from the $p$-ary Kasami functions and Sidelnikov functions. Inspired by the idea coined in [15] and [17], we present the second construction.

**Construction 2:** Let $u_1, \cdots, u_\tau$ be $\tau \geq 1$ elements in $\mathbb{F}_{p^n}$, $v_1, \cdots, v_\kappa$ be $\kappa \geq 1$ elements in $\mathbb{F}_{p^n}^*$ and $c \in \mathbb{F}_p^*$. Let $g(x)$ be a weakly regular bent function over $\mathbb{F}_{p^n}$ whose dual satisfies (5) and $F(x_1, \cdots, x_\tau)$ be any reduced polynomial in $\mathbb{F}_p[x_1, \cdots, x_\tau]$. Generate the function $f(x) : \mathbb{F}_{p^n} \to \mathbb{F}_p$ from $g$ and $F$ as in (2). Note that $f(x)$ can be rewritten as

$$f(x) = \begin{cases} g(x), \text{ if } \prod_{i=1}^{\kappa} \mathrm{Tr}(v_i x) = 0, \\ h(x), \text{ if } \prod_{i=1}^{\kappa} \mathrm{Tr}(v_i x) \neq 0, \end{cases}$$

where $h(x)$ is defined as in (7).

Then our second main result is stated as follows.

**Theorem 2.** *Let $f(x) : \mathbb{F}_{p^n} \to \mathbb{F}_p$ be the function generated by Construction 2. Then $f(x)$ is a bent function if $h(x)$ given by (7) is bent.*

*Proof.* For any $b \in \mathbb{F}_{p^n}$, the Walsh transform of $f(x)$ defined by (2) is

$$\widehat{f}(b) = \sum_{\prod_{i=1}^{\kappa} \mathrm{Tr}(v_i x)=0} \omega^{g(x)-\mathrm{Tr}(bx)} + \sum_{\prod_{i=1}^{\kappa} \mathrm{Tr}(v_i x)\neq 0} \omega^{h(x)-\mathrm{Tr}(bx)}.$$

Note that

$$\left\{x \in \mathbb{F}_{p^n} : \prod\nolimits_{i=1}^{\kappa} \mathrm{Tr}(v_i x) = 0\right\} = \bigcup\nolimits_{r \in \mathbb{F}_p^\kappa} \left\{x \in \mathbb{T}_{r,v} : \prod\nolimits_{i=1}^{\kappa} r_i = 0\right\},$$

where $r := (r_1, \cdots, r_\kappa)$. Then

$$\begin{aligned}\widehat{f}(b) &= \sum_{\substack{\prod_{i=1}^{\kappa} r_i = 0 \\ r_i \in \mathbb{F}_p}} \sum_{x \in \mathbb{T}_{r,v}} (\omega^{g(x)-\mathrm{Tr}(bx)} - \omega^{h(x)-\mathrm{Tr}(bx)}) + \widehat{h}(b) \\ &= \sum_{\substack{\prod_{i=1}^{\kappa} r_i = 0 \\ r_i \in \mathbb{F}_p}} \big(\mathbb{S}_g(r, v, b) - \mathbb{S}_h(r, v, b)\big) + \widehat{h}(b),\end{aligned}$$

where $v = (v_1, \cdots, v_\kappa)$. Together with Lemmas 2 and 4, we can conclude that

$$\widehat{f}(b) = \widehat{g}(b) + \widehat{h}(b) - \widehat{h}(b) = \widehat{g}(b)$$

if $\prod_{i=1}^{\kappa} \varphi_{v_i}(b) = 0$ and $\widehat{f}(b) = \widehat{h}(b)$ if $\prod_{i=1}^{\kappa} \varphi_{v_i}(b) \neq 0$. Hence the desired conclusion follows.

*Remark 1.* Here we show that our constructions of bent functions are different from those in the previous works. Observe that bent functions of the forms (1) and (2) given by this paper are of the form $g(x) + F(\mathrm{Tr}(\alpha_1 x), \cdots, \mathrm{Tr}(\alpha_\lambda x))$, where $g(x)$ is a weakly regular bent function from $\mathbb{F}_{p^n}$ to $\mathbb{F}_p$, $F$ is a reduced polynomial given by (3) and $\alpha_i \in \mathbb{F}_{p^n}$ for $1 \le i \le \lambda$. Recently, some attempts have been made to construct bent functions of the above general form, see [7, 9, 11–16, 18]. It should be noted that all these known results in this direction depended on the dual of weakly regular bent function $g(x)$, that is,

$$\widetilde{g}(x - \sum\nolimits_{i=1}^{\lambda} \alpha_i t_i) = \widetilde{g}(x) + \sum\nolimits_{1 \le i \le j \le \lambda} A_{ij} t_i t_j + \sum\nolimits_{i=1}^{\lambda} g_i(x) t_i, \qquad (10)$$

where $A_{ij} \in \mathbb{F}_p$ for $1 \le i \le j \le \lambda$, $g_i(x) : \mathbb{F}_{p^n} \to \mathbb{F}_p$ and $t_i \in \mathbb{F}_p$ for $1 \le i \le \lambda$. Denote the number of nonzero elements in $\{A_{ij} : 1 \le i \le j \le \lambda\}$ by $N$. Notice that these known results of the above general form were given for $N \le 3$ [7, 9, 11–16, 18] expect a class of bent functions with $\deg(F) = 2$ and $N > 3$ [13] when $p$ is an odd prime. However, although our results also depend on the dual of $g(x)$, bent functions with $\deg(F) > 2$ and $N > 3$ can be constructed by this paper for odd $p$, see Examples 1–4 for details.

*Remark 2.* As pointed out in [11, Lemma 2.1], for $p = 2$, the algebraic degree of $F(\mathrm{Tr}(u_1 x), \cdots, \mathrm{Tr}(u_\tau x))$ is the same as that of $F(x_1, \cdots, x_\tau)$ for some $u_i$, and it can be generalized to any characteristic directly, where $u_i \in \mathbb{F}_{p^n}$ for $1 \le i \le \tau$. Hence there is no doubt that the generic constructions given in Theorems 1 and 2 can produce bent functions with the maximal algebraic degree given in Lemma 1.

## 4 Specific Constructions of Infinite Families of Bent Functions

In this section, by using constructions 1 and 2, we shall introduce specific constructions of bent functions from some known ones whose duals satisfy (5).

### 4.1 New Classes of Bent Functions from Quadratic Bent Functions

It is well-known that a homogeneous quadratic bent function is weakly regular and its dual is also a homogenous quadratic bent function [6]. Let $g(x)$ be a homogeneous quadratic bent function with the dual $\widetilde{g}(x) = \sum_{k=0}^{n-1} \mathrm{Tr}(a_k x^{p^k+1})$, $a_k \in \mathbb{F}_{p^n}^*$. Through some calculation, we can obtain that $\widetilde{g}\big(x - \sum_{i=1}^{\kappa} v_i s_i - \sum_{i=1}^{\tau} u_i t_i\big)$ is equal to

$$\widetilde{g}(x - \sum_{i=1}^{\tau} u_i t_i) + \sum_{i=1}^{\kappa} \varphi_{v_i}(x) s_i + \sum_{i=1}^{\kappa}\sum_{j=1}^{\tau} \varphi_{v_i}(u_j) s_i t_j + \sum_{1\le i\le j\le \kappa} G(v_i, v_j) s_i s_j$$

for all $x \in \mathbb{F}_{p^n}$, $u_i \in \mathbb{F}_{p^n}$, $v_j \in \mathbb{F}_{p^n}^*$ and $s_i, t_j \in \mathbb{F}_p$, $1 \le i \le \tau$, $1 \le j \le \kappa$, where $G(v_i, v_j) = \varphi_{v_i}(v_j)$ if $i < j$ and $G(v_i, v_i) = \sum_{k=0}^{n-1} \mathrm{Tr}(a_k v_i^{p^k+1})$ if $i = j$. Here $\varphi_{v_i}(x) := \sum_{k=0}^{n-1} \mathrm{Tr}(a_k(v_i x^{p^k} + v_i^{p^k} x))$. Obviously, when $\varphi_{v_i}(u_j) = 0$ for all $1 \le i \le \kappa, 1 \le j \le \tau$, $\varphi_{v_i}(v_j) = 0$ for all $1 \le i < j \le \kappa$ and $\sum_{k=0}^{n-1} \mathrm{Tr}(a_k v_i^{p^k+1}) = 0$ for all $1 \le i \le \kappa$, the dual of $g(x)$ satisfies (5).

The specific construction of bent functions from quadratic bent functions can be simply introduced by using the Sidelnikov function. From [6], we know that $g(x) = \mathrm{Tr}(ax^2)$ is weakly regular bent and its dual is $\widetilde{g}(x) = -\mathrm{Tr}(x^2/(4a))$, where $a \in \mathbb{F}_{p^n}^*$ and $p$ is an odd prime. Then the following two theorems are directly obtained from Theorem 1 and Theorem 2 respectively.

**Theorem 3.** *Let $p$ be an odd prime, $a \in \mathbb{F}_{p^n}^*$, $c \in \mathbb{F}_p^*$, $u_1, \cdots, u_\tau$ be $\tau \ge 1$ elements in $\mathbb{F}_{p^n}$ and $v_1, \cdots, v_\kappa$ be $\kappa \ge 1$ elements in $\mathbb{F}_{p^n}^*$ satisfying $\mathrm{Tr}(v_i v_j/(4a)) = 0$ for all $1 \le i \le j \le \kappa$ and $\mathrm{Tr}(u_i v_j/(4a)) = 0$ for all $1 \le i \le \tau$, $1 \le j \le \kappa$. Then*

$$f(x) = \mathrm{Tr}(ax^2) + F(\mathrm{Tr}(u_1 x), \mathrm{Tr}(u_2 x), \cdots, \mathrm{Tr}(u_\tau x)) + c\Big(\prod_{i=1}^{\kappa} \mathrm{Tr}(v_i x)\Big)^{p-1}$$

*is a bent function if $h(x)$ given by (7) is weakly regular bent.*

*Example 1.* Let $p = 3$, $n = 5$, $a = 1$, $c = 1$, $\tau = 4$, $\kappa = 2$, $F(x_1, x_2, x_3, x_4) = x_1 x_2 + x_3 x_4$ and $\xi$ be a primitive element of $\mathbb{F}_{3^5}$. Take $u_1 = 1$, $u_2 = \xi^{212}$, $u_3 = 2$, $u_4 = \xi^{11}$, $v_1 = \xi^{341}$ and $v_2 = \xi^{705}$. Then $\mathrm{Tr}(u_1^2) = \mathrm{Tr}(u_3^2) = \mathrm{Tr}(u_4^2) = \mathrm{Tr}(u_1 u_2) = -1$, $\mathrm{Tr}(u_2^2) = \mathrm{Tr}(u_1 u_3) = \mathrm{Tr}(u_2 u_3) = 1$, $\mathrm{Tr}(u_1 u_4) = \mathrm{Tr}(u_2 u_4) = \mathrm{Tr}(u_3 u_4) = 0$, $\mathrm{Tr}(v_1^2) = \mathrm{Tr}(v_1 v_2) = \mathrm{Tr}(v_2^2) = 0$ and $\mathrm{Tr}(u_i v_j) = 0$ for all $1 \le i \le 4$, $j = 1, 2$. It can be verified that $h(x) = \mathrm{Tr}(x^2) + \mathrm{Tr}(x)\mathrm{Tr}(\xi^{212} x) + \mathrm{Tr}(2x)\mathrm{Tr}(\xi^{11} x)$ is bent by the discussion of Case II of [13]. Theorem 3 now establishes that

$$f(x) = \mathrm{Tr}(x^2) + \mathrm{Tr}(x)\mathrm{Tr}(\xi^{212} x) + \mathrm{Tr}(2x)\mathrm{Tr}(\xi^{11} x) + (\mathrm{Tr}(\xi^{341} x)\mathrm{Tr}(\xi^{705} x))^2$$

is a bent function over $\mathbb{F}_{3^5}$. Moreover, it can be checked that $N = 7$ and $\deg(F) = 4$, where $N$ is defined as in Remark 1.

**Theorem 4.** *Let $p$ be an odd prime, $a \in \mathbb{F}_{p^n}^*$, $u_1, \cdots, u_\tau$ be $\tau \ge 1$ elements in $\mathbb{F}_{p^n}$ and $v_1, \cdots, v_\kappa$ be $\kappa \ge 1$ elements in $\mathbb{F}_{p^n}^*$ satisfying $\mathrm{Tr}(v_i v_j/(4a)) = 0$ for all $1 \le i \le j \le \kappa$ and $\mathrm{Tr}(u_i v_j/(4a)) = 0$ for all $1 \le i \le \tau$, $1 \le j \le \kappa$. Then*

$$f(x) = \mathrm{Tr}(ax^2) + F(\mathrm{Tr}(u_1 x), \mathrm{Tr}(u_2 x), \cdots, \mathrm{Tr}(u_\tau x))\Big(\prod_{i=1}^{\kappa} \mathrm{Tr}(v_i x)\Big)^{p-1}$$

*is a bent function if $h(x)$ given by (7) is bent.*

*Example 2.* Let $p=3, n=6, a=1, \tau=4, \kappa=1, F(x_1,x_2,x_3,x_4)=x_1x_2+x_3x_4$ and $\xi$ be a primitive element of $\mathbb{F}_{3^6}$. Take $u_1=\xi^2$, $u_2=\xi$, $u_3=\xi^{26}$, $u_4=\xi^{118}$ and $v_1=\xi^7$. Then $\mathrm{Tr}(u_1^2)=\mathrm{Tr}(u_3^2)=1$, $\mathrm{Tr}(u_2^2)=\mathrm{Tr}(u_4^2)=2$, $\mathrm{Tr}(u_iu_j)=0$ for others $1\leq i<j\leq 4$, $\mathrm{Tr}(v_1^2)=0$ and $\mathrm{Tr}(u_iv_1)=0$ for all $1\leq i\leq 4$. It can be verified that $h(x)=\mathrm{Tr}(x^2)+\mathrm{Tr}(\xi x)\mathrm{Tr}(\xi^2x)+\mathrm{Tr}(\xi^{26}x)\mathrm{Tr}(\xi^{118}x)$ is bent by [13, Theorem 5]. Theorem 4 now establishes that

$$f(x)=\mathrm{Tr}(x^2)+(\mathrm{Tr}(\xi x)\mathrm{Tr}(\xi^2x)+\mathrm{Tr}(\xi^{26}x)\mathrm{Tr}(\xi^{118}x))\mathrm{Tr}(\xi^7x)^2$$

is a bent function over $\mathbb{F}_{3^6}$. Moreover, it can be checked that $N=4$ and $\deg(F)=4$, where $N$ is defined as in Remark 1.

### 4.2 New Classes of Bent Functions from the Maiorana-MacFarland Class of Bent Functions

Let $n=2m$ be a positive integer. By identifying an element $x\in\mathbb{F}_{p^n}$ with a vector $(y,z)\in\mathbb{F}_{p^m}\times\mathbb{F}_{p^m}$, the Maiorana-MacFarland class of bent functions on $\mathbb{F}_{p^n}$ can be expressed as

$$g(x)=g(y,z)=\mathrm{Tr}_1^m(y\pi(z))+h(z),\ y,z\in\mathbb{F}_{p^m}, \tag{11}$$

where $\pi:\mathbb{F}_{p^m}\rightarrow\mathbb{F}_{p^m}$ is a permutation and $h$ is a function from $\mathbb{F}_{p^m}$ to $\mathbb{F}_p$. Such class of bent functions is regular and its dual [4] is equal to

$$\widetilde{g}(x)=\widetilde{g}(y,z)=\mathrm{Tr}_1^m\left(z\pi^{-1}(y)\right)+h\left(\pi^{-1}(y)\right),$$

where $\pi^{-1}$ is the inverse permutation of $\pi$. There are several $\pi$ and $h$ such that the dual of $g(y,z)$ satisfies (5). In the following, we give the calculation of $\widetilde{g}(y,z)$ for the case while $\pi$ is a linear permutation and $h(z)=0$. In the same manner, it can be verified that $g(x)$ with some other $\pi$ and $h$ is suitable for Construction 1 and Construction 2. When $\pi$ is a linear permutation and $h(z)=0$, one has

$$\pi^{-1}\Big(y-\sum_{i=1}^{\kappa}v_{i,1}s_i-\sum_{i=1}^{\tau}u_{i,1}t_i\Big)=\pi^{-1}(y-\sum_{i=1}^{\tau}u_{i,1}t_i)-\sum_{i=1}^{\kappa}\pi^{-1}(v_{i,1})s_i,$$

where $u_i=(u_{i,1},u_{i,2})\in\mathbb{F}_{p^m}\times\mathbb{F}_{p^m}$, $v_j=(v_{j,1},v_{j,2})\in\mathbb{F}_{p^m}^*\times\mathbb{F}_{p^m}^*$ and $t_i,s_j\in\mathbb{F}_p$, $1\leq i\leq\tau$, $1\leq j\leq\kappa$. Then by a direct calculation, we can derive that

$$\widetilde{g}\Big(y-\sum_{i=1}^{\kappa}v_{i,1}s_i-\sum_{i=1}^{\tau}u_{i,1}t_i, z-\sum_{i=1}^{\kappa}v_{i,2}s_i-\sum_{i=1}^{\tau}u_{i,2}t_i\Big)$$

equals to

$$\widetilde{g}(y-\sum_{i=1}^{\tau}u_{i,1}t_i, z-\sum_{i=1}^{\tau}u_{i,2}t_i)+\sum_{i=1}^{\kappa}\varphi_{v_{i,1},v_{i,2}}(y,z)s_i+\mathcal{G}$$

with

$$\mathcal{G} = \sum_{i=1}^{\kappa}\sum_{j=1}^{\tau} \varphi_{v_{i,1},v_{i,2}}(u_{j,1},u_{j,2})s_i t_j + \sum_{1\le i\le j\le \kappa} G(v_{i,1},v_{i,2},v_{j,1},v_{j,2})s_i s_j$$

where $G(v_{i,1},v_{i,2},v_{j,1},v_{j,2}) = \varphi_{v_{i,1},v_{i,2}}(v_{j,1},v_{j,2})$ if $i<j$ and $G(v_{i,1},v_{i,2},v_{i,1},v_{i,2}) = \mathrm{Tr}_1^m(\pi^{-1}(v_{i,1})v_{i,2})$ if $i=j$. Here $\varphi_{v_{i,1},v_{i,2}}(y,z) := \mathrm{Tr}_1^m(\pi^{-1}(v_{i,1})z + v_{i,2}\pi^{-1}(y))$. If $\varphi_{v_{i,1},v_{i,2}}(u_{j,1},u_{j,2}) = 0$ for all $1\le i\le\kappa, 1\le j\le\tau$, $\varphi_{v_{i,1},v_{i,2}}(v_{j,1}, v_{j,2}) = 0$ for all $1\le i<j\le\kappa$ and $\mathrm{Tr}_1^m(\pi^{-1}(v_{i,1})v_{i,2}) = 0$ for all $1\le i\le\kappa$, the dual of $g(x)$ satisfies (5). Then the following two theorems are directly obtained from Theorem 1 and Theorem 2 respectively.

**Theorem 5.** *Let $c\in\mathbb{F}_p^*$, $u_1,\cdots,u_\tau$ be $\tau\ge 1$ elements in $\mathbb{F}_{p^{2m}}$ and $v_1,\cdots,v_\kappa$ be $\kappa\ge 1$ elements in $\mathbb{F}_{p^{2m}}^*$, where $p$ is a prime. Write $u_i$ as $(u_{i,1},u_{i,2})\in\mathbb{F}_{p^m}\times\mathbb{F}_{p^m}$ for each $1\le i\le\tau$ and $v_j$ as $(v_{j,1},v_{j,2})\in\mathbb{F}_{p^m}\times\mathbb{F}_{p^m}$ for each $1\le j\le\kappa$. Let $g(y,z)=\mathrm{Tr}_1^m(y\pi(z))$, where $\pi$ is a linear permutation over $\mathbb{F}_{p^m}$. If $\mathrm{Tr}_1^m(\pi^{-1}(v_{i,1})u_{j,2}+v_{i,2}\pi^{-1}(u_{j,1}))=0$ for all $1\le i\le\kappa, 1\le j\le\tau$, $\mathrm{Tr}_1^m(\pi^{-1}(v_{i,1})v_{j,2}+v_{i,2}\pi^{-1}(v_{j,1}))=0$ for all $1\le i<j\le\kappa$ and $\mathrm{Tr}_1^m(\pi^{-1}(v_{i,1})v_{i,2})=0$ for all $1\le i\le\kappa$. Then*

$$f(y,z) = h(y,z) + c\big(\prod_{i=1}^{\kappa}\mathrm{Tr}_1^m(v_{i,1}y+v_{i,2}z)\big)^{p-1}$$

*is a bent function if*

$$h(y,z) = \mathrm{Tr}_1^m(y\pi(z)) + F(\mathrm{Tr}_1^m(u_{1,1}y+u_{1,2}z),\cdots,\mathrm{Tr}_1^m(u_{\tau,1}y+u_{\tau,2}z)) \quad (12)$$

*is weakly regular bent.*

*Example 3.* Let $p=5$, $m=3$, $\tau=4$, $\kappa=1$, $F(x_1,x_2,x_3,x_4) = x_1x_2+x_3x_4$, $c=1$ and $\xi$ be a primitive element of $\mathbb{F}_{5^3}$. Take $\pi(z)=z^5$, $(u_{1,1},u_{1,2})=(\xi^{26},\xi^{50})$, $(u_{2,1},u_{2,2})=(\xi^{32},\xi^{51})$, $(u_{3,1},u_{3,2})=(\xi^{119},\xi^{8})$, $(u_{4,1},u_{4,2})=(\xi^{63},\xi^{5})$ and $(v_{1,1},v_{1,2})=(\xi^{36},\xi^{114})$. Then $\pi^{-1}(z)=z^{25}$, $\mathrm{Tr}_1^3((u_{1,1})^{25}u_{1,2})=\mathrm{Tr}_1^3((u_{3,1})^{25}u_{3,2})=1$, $\mathrm{Tr}_1^3((u_{2,1})^{25}u_{2,2})=\mathrm{Tr}_1^3((u_{4,1})^{25}u_{4,2})=2$, $\mathrm{Tr}_1^3((u_{i,1})^{25}u_{j,2}+u_{i,2}(u_{j,1})^{25})=0$ for $1\le i<j\le 4$, $\mathrm{Tr}_1^3((v_{1,1})^{25}u_{j,2}+v_{1,2}(u_{j,1})^{25})=0$ for all $1\le j\le 4$ and $\mathrm{Tr}_1^3((v_{1,1})^{25}v_{1,2})=0$. It can be verified that $h(y,z)=\mathrm{Tr}_1^3(yz^5)+F(\mathrm{Tr}_1^3(u_{1,1}y+u_{1,2}z),\cdots,\mathrm{Tr}_1^3(u_{4,1}y+u_{4,2}z))$ is bent by [13, Theorem 5]. Theorem 5 now establishes that

$$f(y,z)=\mathrm{Tr}_1^3(yz^5)+F(\mathrm{Tr}_1^m(u_{1,1}y+u_{1,2}z),\cdots,\mathrm{Tr}_1^m(u_{4,1}y+u_{4,2}z))+\mathrm{Tr}_1^m(\xi y+2z)^4$$

is a bent function over $\mathbb{F}_{5^6}$. Moreover, it can be checked that $N=4$ and $\deg(F)=4$, where $N$ is defined as in Remark 1.

**Theorem 6.** *Let $u_i=(u_{i,1},u_{i,2})(1\le i\le\tau)$ be $\tau$ elements in $\mathbb{F}_{p^m}\times\mathbb{F}_{p^m}$ and $v_j=(v_{j,1},v_{j,2})(1\le j\le\kappa)$ be $\kappa$ elements in $\mathbb{F}_{p^m}^*\times\mathbb{F}_{p^m}^*$, where $p$ is a prime. Let $g(y,z)=\mathrm{Tr}_1^m(y\pi(z))$, where $\pi$ is a linear permutation over $\mathbb{F}_{p^m}$. If $\mathrm{Tr}(\pi^{-1}(v_{i,1})u_{j,2}+v_{i,2}\pi^{-1}(u_{j,1}))=0$ for all $1\le i\le\kappa, 1\le j\le\tau$, $\mathrm{Tr}(\pi^{-1}(v_{i,1})v_{j,2}$*

$+v_{i,2}\pi^{-1}(v_{j,1})) = 0$ *for all* $1 \leq i < j \leq \kappa$ *and* $\mathrm{Tr}(\pi^{-1}(v_{i,1})v_{i,2}) = 0$ *for all* $1 \leq i \leq \kappa$. *Then*

$$\widetilde{g}(y,z) + F(\mathrm{Tr}(u_{1,1}y + u_{1,2}z), \cdots, \mathrm{Tr}(u_{\tau,1}y + u_{\tau,2}z))\big(\prod_{i=1}^{\kappa} \mathrm{Tr}(v_{i,1}y + v_{i,2}z)\big)^{p-1}$$

*is a bent function if* $h(x)$ *given by* (12) *is bent.*

*Example 4.* Let $p = 3$, $m = 3$, $\tau = 4$, $\kappa = 1$, $F(x_1, x_2, x_3, x_4) = x_1x_2 + x_3x_4$ and $\xi$ be a primitive element of $\mathbb{F}_{3^3}$. Take $\pi(z) = z^3$, $(u_{1,1}, u_{1,2}) = (\xi^6, \xi^{21})$, $(u_{2,1}, u_{2,2}) = (\xi^{25}, \xi^{15})$, $(u_{3,1}, u_{3,2}) = (\xi^{11}, \xi^{24})$, $(u_{4,1}, u_{4,2}) = (\xi^5, \xi^{12})$ and $(v_{1,1}, v_{1,2}) = (\xi, 2)$. Then $\pi^{-1}(z) = z^9$, $\mathrm{Tr}_1^3((u_{1,1})^9u_{1,2}) = \mathrm{Tr}_1^3((u_{3,1})^9u_{3,2}) = 1$, $\mathrm{Tr}_1^3((u_{2,1})^9u_{2,2}) = \mathrm{Tr}_1^3((u_{4,1})^9u_{4,2}) = 2$, $\mathrm{Tr}_1^3((u_{i,1})^9u_{j,2} + u_{i,2}(u_{j,1})^9) = 0$ for $1 \leq i < j \leq 4$, $\mathrm{Tr}_1^3((v_{1,1})^9u_{j,2} + v_{1,2}(u_{j,1})^9) = 0$ for all $1 \leq j \leq 4$ and $\mathrm{Tr}_1^3((v_{1,1})^9v_{1,2}) = 0$. It can be verified that $h(y,z) = \mathrm{Tr}_1^3(yz^3) + F(\mathrm{Tr}_1^3(u_{1,1}y + u_{1,2}z), \cdots, \mathrm{Tr}_1^3(u_{4,1}y + u_{4,2}z))$ is bent by [13, Theorem 5]. Theorem 6 now establishes that

$$f(y,z) = \mathrm{Tr}_1^3(yz^3) + F(\mathrm{Tr}_1^m(u_{1,1}y + u_{1,2}z), \cdots, \mathrm{Tr}_1^m(u_{4,1}y + u_{4,2}z))\mathrm{Tr}_1^m(\xi y + 2z)^2$$

is a bent function over $\mathbb{F}_{3^6}$. Moreover, it can be checked that $N = 4$ and $\deg(F) = 4$, where $N$ is defined as in Remark 1.

## 5 Conclusions

In this paper, we proposed two generic constructions of bent functions with the forms (1) and (2), which generalized some previous works [15,17]. Moreover, based on our constructions, several infinite families of bent functions can be obtained from quadratic bent functions and the Maiorana-MacFarland class of bent functions by calculating their duals. In addition, it was shown that bent functions with the maximal algebraic degree can be obtained from our constructions.

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