

New Classes of Bent Functions via the Switching Method

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Abstract. The switching method is a powerful method to construct bent functions. In this paper, using this method, we present two generic constructions of piecewise bent functions from known ones, which generalize some earlier works. Further, based on these two generic constructions, we obtain several infinite families of bent functions from quadratic bent functions and the Maiorana-MacFarland class of bent functions by calculating their duals. It is worth noting that our constructions can produce bent functions with the optimal algebraic degree.

Keywords: Bent function · Walsh transform · Switching method

1 Introduction

Boolean bent functions were first introduced by Rothaus in 1976 [\[10](#page-11-0)] as an interesting combinatorial object with maximum Hamming distance to the set of all affine functions. Over the last four decades, bent functions have attracted a lot of research interest due to their important applications in cryptography [\[1\]](#page-10-0), sequences $[8]$ and coding theory $[2,3]$ $[2,3]$ $[2,3]$. Later, Kumar, Scholtz and Welch in $[4]$ $[4]$ generalized the notion of Boolean bent functions to the case of functions over an arbitrary finite field.

Let \mathbb{F}_{p^n} denote the finite field with p^n elements, where p is a prime and n Example integer. Given a function $f(x)$ mapping from \mathbb{F}_{p^n} to \mathbb{F}_p , the Walsh
transform of $f(x)$ is defined by
 $\widehat{f}(b) = \sum_{x \in \mathbb{F}_{p^n}} \omega^{f(x) - \text{Tr}(bx)}$, $b \in \mathbb{F}_{p^n}$, transform of $f(x)$ is defined by

$$
\widehat{f}(b) = \sum\nolimits_{x \in \mathbb{F}_{p^n}} \omega^{f(x) - \text{Tr}(bx)}, \, b \in \mathbb{F}_{p^n},
$$

where $\omega = e^{\frac{2\pi\sqrt{-1}}{p}}$ is a complex primitive p-th root of unity. According to [\[4\]](#page-10-3), $f(x)$ $I(0) = \sum_{x \in \mathbb{F}_{p^n}} \omega^{(x)} \xrightarrow{C(x)} 0 \in \mathbb{F}_{p^n}$,
where $\omega = e^{\frac{2\pi\sqrt{-1}}{p}}$ is a complex primitive *p*-th root of unity. According is called a *p*-ary bent function if all its Walsh coefficients satisfy | $|f(b)| = p^{n/2}.$

This work was supported by the Knowledge Innovation Program of Wuhan-Basic Research under Grant 2022010801010319, the Natural Science Foundation of Hubei Province of China under Grant 2021CFA079 and the National Natural Science Foundation of China under Grant 62072162.

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A *p*-ary bent function $f(x)$ is called regular if $\hat{f}(b) = p^{n/2} \omega^{\tilde{f}(b)}$ holds for some function $\tilde{f}(x)$ mapping \mathbb{F}_{p} to \mathbb{F}_{p} and it is called workly regular if there exists a function $\tilde{f}(x)$ mapping \mathbb{F}_{p^n} to \mathbb{F}_p , and it is called weakly regular if there exists a
complex u boxing unit mognitude such that $\hat{f}(k) = x^{-1} \cdot n/2 \cdot \tilde{f}(b)$ for all $k \in \mathbb{F}$ complex μ having unit magnitude such that $\hat{f}(b) = \mu^{-1} p^{n/2} \omega^{f(b)}$ for all $b \in \mathbb{F}_{p^n}$.
The function $\tilde{f}(a)$ is called the dual of $f(a)$ and it is also bent The function $f(x)$ is called the dual of $f(x)$ and it is also bent.
The switching method is a powerful method to construct

The switching method is a powerful method to construct bent functions. In 2016, Xu et al. $\left[15\right]$ constructed a class of piecewise p-ary bent functions $f(x) = g(x) + c \text{Tr}(x)^{p-1}$ from Gold functions $g(x)$ via the switching method, where p is an odd prime and $c \in \mathbb{F}_p^*$. In 2017, Xu et al. [\[17](#page-11-3)] constructed two
classes of piecewise n-ary bent functions $f(x) = g(x) + Tr(ux)Tr(x)^{p-1}$ from classes of piecewise p-ary bent functions $f(x) = g(x) + \text{Tr}(ux)\text{Tr}(x)^{p-1}$ from p-ary Kasami functions and Sidelnikov functions $g(x)$ [\[6\]](#page-10-4), where p is an odd prime and $u \in \mathbb{F}_{p^n}^*$. Recall that Tang et al. [\[11\]](#page-11-4) constructed bent functions of the form $g(x) + F(T(u,x))$ $Tr(u,x)$... $Tr(u,x)$ where $g(x) : \mathbb{F}_x \to \mathbb{F}_x$ of the form $g(x) + F(\text{Tr}(u_1x), \text{Tr}(u_2x), \cdots, \text{Tr}(u_\tau x))$, where $g(x) : \mathbb{F}_{p^n} \to \mathbb{F}_p$, $F(x_1, \dots, x_\tau) \in \mathbb{F}_p[x_1, \dots, x_\tau]$ and $u_i \in \mathbb{F}_{p^n}$ for $1 \leq i \leq \tau$. Motivated by Xu et al 's and Tang et al 's works, we investigate the bentness of piecewise bent et al.'s and Tang et al.'s works, we investigate the bentness of piecewise bent functions of the forms

$$
f(x) = g(x) + F(\text{Tr}(u_1x), \text{Tr}(u_2x), \cdots, \text{Tr}(u_\tau x)) + c \left(\prod_{i=1}^{\kappa} \text{Tr}(v_i x)\right)^{p-1} \tag{1}
$$

and

$$
(x) = g(x) + F(\text{Tr}(u_1x), \text{Tr}(u_2x), \cdots, \text{Tr}(u_\tau x)) + c\left(\prod_{i=1}^{\infty} \text{Tr}(v_i x)\right)^{p-1}
$$
(1)

$$
f(x) = g(x) + F(\text{Tr}(u_1x), \text{Tr}(u_2x), \cdots, \text{Tr}(u_\tau x))\left(\prod_{i=1}^{\kappa} \text{Tr}(v_i x)\right)^{p-1},
$$
(2)

where $g(x) : \mathbb{F}_{p^n} \to \mathbb{F}_p$, $F(x_1, \dots, x_\tau) \in \mathbb{F}_p[x_1, \dots, x_\tau]$, $c \in \mathbb{F}_p^*$, $u_i \in \mathbb{F}_{p^n}$ for $1 \le i \le \kappa$. In this paper we first present generic $1 \leq i \leq \tau$ and $v_j \in \mathbb{F}_{p^n}^*$ for $1 \leq j \leq \kappa$. In this paper, we first present generic constructions of bent functions with the forms (1) and (2) respectively. Notice constructions of bent functions with the forms [\(1\)](#page-1-0) and [\(2\)](#page-1-1) respectively. Notice that the works of Xu et al. [\[15](#page-11-2),[17\]](#page-11-3) are two special cases of our work for $\kappa = 1$, $F(x_1)=0$ in [\(1\)](#page-1-0) and $\kappa = 1$, $F(x_1) = x_1$ in [\(2\)](#page-1-1). In addition, by calculating the duals of quadratic bent functions and the Maiorana-MacFarland class of bent functions, several infinite families of bent functions are constructed by using these generic constructions.

The rest of this paper is organized as follows. Section [2](#page-1-2) fixes some notation and introduces some preliminaries. Section [3](#page-4-0) proposes two generic constructions of bent functions of the forms [\(1\)](#page-1-0) and [\(2\)](#page-1-1) respectively. Section [4](#page-6-0) constructs several infinite families of bent functions using the two generic constructions given in Sect. [3.](#page-4-0) Finally, Sect. [5](#page-10-5) concludes this paper.

2 Preliminaries

Throughout this paper, let \mathbb{F}_{p^n} denote the finite field with p^n elements, where p is a prime and n is a positive integer. The trace function from \mathbb{F}_{p^n} to its subfield Throughout this paper, let \mathbb{F}_{p^n} denote the finite field with p^n elements, where p
is a prime and n is a positive integer. The trace function from \mathbb{F}_{p^n} to its subfield
 \mathbb{F}_{p^k} is defined by $\text{Tr}_k^n(x) =$ notation Tr(x) instead of Tr₁ⁿ¹(x). A function $F(x_1, \dots, x_n) : \mathbb{F}_p^n \mapsto \mathbb{F}_p$ is often
represented by its algebraic normal form
 $F(x_1, \dots, x_n) = \sum_{e \in \{e_1, \dots, e_n\}} a(e) (\prod_{i=1}^n x_i^{e_i}), \ a(e) \in \mathbb{F}_p.$ (3) represented by its algebraic normal form

$$
F(x_1, \dots, x_n) = \sum_{e = (e_1, \dots, e_n) \in \mathbb{F}_p^n} a(e) (\prod_{i=1}^n x_i^{e_i}), \ a(e) \in \mathbb{F}_p.
$$
 (3)

A polynomial in $\mathbb{F}_p[x_1, \dots, x_n]$ with the form [\(3\)](#page-1-3) is called a reduced polynomial. The algebraic degree of $F(x_1, \dots, x_n)$, denoted by $deg(F)$, is defined as $deg(F)$ = $\max_{e \in \mathbb{F}_p^n} {\sum_{i=1}^n e_i : a(e) \neq 0}, \text{ where } e = (e_1, \dots, e_n) \in \mathbb{F}_p^n.$

Lemma 1. ([\[5,](#page-10-6) Propositions 4.4 and 4.5]) Let $f(x)$ be a bent function from \mathbb{F}_{n^n} *to* \mathbb{F}_p *. Then* deg(*f*) ≤ $\frac{(p-1)n}{2}$ + 1*. Moreover, if* $f(x)$ *is weakly regular bent, then* $deg(f) \leq \frac{(p-1)n}{2}$.

To simplify the proof of our main result in the sequel, we fix some notation firstly. Let $g(x)$ be a function from \mathbb{F}_{p^n} to \mathbb{F}_p . For $r := (r_1, \dots, r_\kappa) \in \mathbb{F}_p^\kappa$ and $v := (v_1, \dots, v_\kappa) \in \mathbb{F}_{p^n}^* \times \dots \times \mathbb{F}_{p^n}^*$, define
 $\mathbb{T}_{r,v} = \{x \in \mathbb{F}_{p^n} : \text{Tr}(v_i x) = r_i, i = 1, \dots, \kappa\}$ $v := (v_1, \dots, v_\kappa) \in \mathbb{F}_{p^n}^* \times \dots \times \mathbb{F}_{p^n}^*$, define

$$
\mathbb{T}_{r,v} = \{x \in \mathbb{F}_{p^n} : \text{Tr}(v_i x) = r_i, i = 1, \cdots, \kappa\}
$$

and

$$
= \{x \in \mathbb{F}_{p^n} : \text{Tr}(v_i x) = r_i, i = 1, \cdot\}
$$

$$
\mathbb{S}_g(r, v, b) = \sum_{x \in \mathbb{T}_{r, v}} \omega^{g(x) - \text{Tr}(bx)}
$$

for any $b \in \mathbb{F}_{p^n}$. It then can be verified that $\mathbb{F}_{p^n} = \cup_{r \in \mathbb{F}_p^{\kappa}} \mathbb{T}_{r,v}$. Moreover, we have

$$
S_g(r, v, b) = \sum_{x \in \mathbb{T}_{r,v}} \omega^{g(x)} \text{ L}(1, 0)
$$
\n
$$
p^n. \text{ It then can be verified that } \mathbb{F}_{p^n} = \bigcup_{r \in \mathbb{F}_p^{\kappa}} \mathbb{T}_{r,v}. \text{ More}
$$
\n
$$
\widehat{g}(b - \sum_{i=1}^{\kappa} v_i s_i) = \sum_{x \in \mathbb{F}_{p^n}} \omega^{g(x) - \text{Tr}\left((b - \sum_{i=1}^{\kappa} v_i s_i)x\right)}} = \sum_{r \in \mathbb{F}_p^{\kappa}} \sum_{x \in \mathbb{T}_{r,v}} \omega^{g(x) - \text{Tr}(bx) + \sum_{i=1}^{\kappa} r_i s_i}
$$
\n
$$
= \sum_{r \in \mathbb{F}_p^{\kappa}} \mathbb{S}_g(r, v, b) \omega^{\sum_{i=1}^{\kappa} r_i s_i}.
$$

From the inverse Fourier transform we can derive

$$
r \in \mathbb{F}_p^{\kappa}
$$

se Fourier transform we can derive

$$
\mathbb{S}_g(r, v, b) = \frac{1}{p^{\kappa}} \sum_{s \in \mathbb{F}_p^{\kappa}} \omega^{-\sum_{i=1}^{\kappa} r_i s_i} \hat{g}(b - \sum_{i=1}^{\kappa} v_i s_i),
$$
 (4)

where $s := (s_1, \dots, s_\kappa) \in \mathbb{F}_p^\kappa$.
To make the computation

To make the computation of the Walsh transform of the functions investigated in the sequel feasible, we consider a class of weakly regular bent functions
 $g(x) : \mathbb{F}_{p^n} \to \mathbb{F}_p$ whose dual satisfies
 $\widetilde{g}(x - \sum_{i=1}^{\kappa} v_i s_i - \sum_{i=1}^{\tau} u_i t_i) = \widetilde{g}(x - \sum_{i=1}^{\tau} u_i t_i) + \sum_{i=1}^{\kappa} \varphi_{v_i}(x$ $g(x): \mathbb{F}_{p^n} \to \mathbb{F}_p$ whose dual satisfies wash transform of the
 $x = x^{\tau}$ a class of weakly regularity of $(x - \sum_{i=1}^{\tau} u_i t_i) + \sum_{i=1}^{\kappa}$

\n The sequence of the sequence
$$
x
$$
 is a constant, where x is a constant, and x is a constant. The sequence $\widetilde{g}(x) = \sum_{i=1}^{k} v_i s_i - \sum_{i=1}^{n} u_i t_i$, where $\widetilde{g}(x) = \widetilde{g}(x) = \sum_{i=1}^{n} u_i t_i + \sum_{i=1}^{n} \varphi_{v_i}(x) s_i,$ is a constant.\n

where $\varphi_{v_i}(x)$ is a function from \mathbb{F}_{p^n} to \mathbb{F}_p , $u_i \in \mathbb{F}_{p^n}$, $v_j \in \mathbb{F}_{p^n}^*$ and $t_i, s_j \in \mathbb{F}_{p^n}$ for each $1 \le i \le \tau$ and $1 \le i \le \kappa$. Without loss of generality assume that for each $1 \leq i \leq \tau$ and $1 \leq j \leq \kappa$. Without loss of generality, assume that $\hat{\sigma}(b) = \mu^{-1} n^{n/2} \hat{\sigma}(b)$ for any $b \in \mathbb{F}$. Then u
f(*ĝ* There $\varphi_{v_i}(x)$ is a function from \mathbb{F}_{p^n} to \mathbb{F}_p
or each $1 \leq i \leq \tau$ and $1 \leq j \leq \kappa$. With $(b) = \mu^{-1} p^{n/2} \omega^{\widetilde{g}(b)}$ for any $b \in \mathbb{F}_{p^n}$. Then $\begin{aligned} &\mathbf{c} \ \psi_{v_i}(x) \text{ is a function from } \mathbb{E}_{p^n} \text{ to } \mathbb{E}_{p}, \ u_i \in \mathbb{E}_{p^n}, \ v_j \in \mathbb{E}_{p^n} \text{ and } 1 \leq i \leq \tau \text{ and } 1 \leq j \leq \kappa. \text{ Without loss of generality} \\ &= \mu^{-1} p^{n/2} \omega^{\widetilde{g}(b)} \text{ for any } b \in \mathbb{F}_{p^n}. \text{ Then} \\ &\left(b - \sum_{i=1}^{\tau} u_i t_i - \sum_{i=1}^{\kappa} v_i s_i\right) = \mu^{-1} p^{n$ nen andre statistikelse og den statistikelse og den statistikelse og den statistikelse og den statistikelse og
Statistikelse og den statistikelse og den statistikelse og den statistikelse og den statistikelse og den statis

each
$$
1 \leq i \leq 7
$$
 and $1 \leq j \leq \kappa$. Without loss of generating, assume that
\n
$$
\hat{g}(b - \sum_{i=1}^{7} u_i t_i - \sum_{i=1}^{\kappa} v_i s_i) = \mu^{-1} p^{n/2} \omega^{\tilde{g}(b - \sum_{i=1}^{7} u_i t_i) + \sum_{i=1}^{\kappa} \varphi_{v_i}(b) s_i}
$$
\n
$$
= \hat{g}(b - \sum_{i=1}^{7} u_i t_i) \omega^{\sum_{i=1}^{\kappa} \varphi_{v_i}(b) s_i}.
$$
\n(6)

Further we characterize the value of $\mathbb{S}_q(r, v, b)$ as below.

Lemma 2. Let $g(x): \mathbb{F}_{p^n} \to \mathbb{F}_p$ be a weakly regular bent function whose dual **Lemma 2.** Let $g(x)$: $\mathbb{F}_{p^n} \to \mathbb{F}_p$ be a weakly regular bent function whose dual satisfies [\(5\)](#page-2-0). For any $b \in \mathbb{F}_{p^n}$, $\mathbb{S}_g(r, v, b) = \hat{g}(b)$ if $r_i = \varphi_{v_i}(b)$ for any $1 \leq i \leq \kappa$ and otherwise $\mathbb{S}_g(r, v, b) =$ $i \leq \kappa$ *and otherwise*, $\mathbb{S}_g(r, v, b) = 0$ *, where* $r = (r_1, \dots, r_\kappa) \in \mathbb{F}_p^{\kappa}$ *and* $v = (v_1, \dots, v_n) \in \mathbb{F}_{p}^{\kappa}$ *s* $\cdots \times \mathbb{F}_{p}^{\kappa}$ $(v_1, \dots, v_\kappa) \in \mathbb{F}_{p^n}^* \times \dots \times \mathbb{F}_{p^n}^*.$

Proof. For any $b \in \mathbb{F}_{p^n}$, $r = (r_1, \dots, r_{\kappa}) \in \mathbb{F}_p^{\kappa}$ and $v = (v_1, \dots, v_{\kappa}) \in \mathbb{F}_{p^n}^{\kappa} \times \dots \times$
 $\mathbb{F}_{p^n}^{\kappa}$, (4) yields
 $\mathbb{S}_g(r, v, b) = \frac{1}{p^{\kappa}} \sum_{e \in \mathbb{F}^{\kappa}} \omega^{-\sum_{i=1}^{\kappa} r_i s_i} \widehat{g}(b - \sum_{i=1}^$ $\mathbb{F}_{p^n}^*$, [\(4\)](#page-2-1) yields

$$
\mathbb{S}_g(r, v, b) = \frac{1}{p^{\kappa}} \sum_{s \in \mathbb{F}_p^{\kappa}} \omega^{-\sum_{i=1}^{\kappa} r_i s_i} \widehat{g}(b - \sum_{i=1}^{\kappa} v_i s_i),
$$

where $s = (s_1, \dots, s_\kappa) \in \mathbb{F}_p^\kappa$. By using [\(6\)](#page-2-2), one gives $(\frac{6}{2})$

$$
,s_{\kappa}) \in \mathbb{F}_{p}^{\kappa}. \text{ By using (6), one gives}
$$

$$
\mathbb{S}_{g}(r, v, b) = \frac{1}{p^{\kappa}} \widehat{g}(b) \sum_{s \in \mathbb{F}_{p}^{\kappa}} \omega^{\sum_{i=1}^{\kappa} (\varphi_{v_{i}}(b) - r_{i})s_{i}}.
$$

 $\mathbb{S}_g(r, v, b) = \frac{1}{p^{\kappa}} \widehat{g}(b) \sum_{s \in \mathbb{F}_p^{\kappa}} \omega^{\sum_{i=1}^{\kappa} (\varphi_{v_i}(b)-r_i)s_i}$.
Further the desired result follows from the fact that $\sum_{s_i \in \mathbb{F}_p} \omega^{(\varphi_{v_i}(b)-r_i)s_i} = p$ if $r_i = \omega$, (b) and 0 otherwise for any $1 \leq i$ $r_i = \varphi_{v_i}(b)$ and 0 otherwise for any $1 \leq i \leq \kappa$. This completes the proof.

Let $h: \mathbb{F}_{p^n} \to \mathbb{F}_p$ be defined as in

$$
h(x) = g(x) + F(\text{Tr}(u_1x), \text{Tr}(u_2x), \cdots, \text{Tr}(u_\tau x)),
$$
\n(7)

where $g(x) : \mathbb{F}_{p^n} \to \mathbb{F}_p$, $F(x_1, \dots, x_{\tau})$ is an arbitrary reduced polynomial in $\mathbb{F}_p[x_1, \dots, x_{\tau}], u_i \in \mathbb{F}_{p^n}$ for $1 \leq i \leq \tau$. The Walsh transform of $h(x)$ can be given as follows given as follows.

Lemma 3. ([13, Theorem 1]) Let
$$
h(x)
$$
 be defined as in (7). Then for any $b \in \mathbb{F}_{p^n}$,
\n
$$
\widehat{h}(b) = \frac{1}{p^{\tau}} \sum_{(t_1, \dots, t_{\tau}) \in \mathbb{F}_p^{\tau}} \widehat{F}(t_1, \dots, t_{\tau}) \widehat{g}(b - \sum_{i=1}^{\tau} u_i t_i).
$$

Next we characterize the value of $\mathcal{S}_h(r, v, b)$ for a class of weakly regular bent functions $g(x)$.

Lemma 4. Let $h(x)$ be defined as in [\(7\)](#page-3-0) and $g(x): \mathbb{F}_{p^n} \to \mathbb{F}_p$ be a weakly regular **functions** $g(x)$ **.**
Lemma 4. Let $h(x)$ be defined as in (7) and $g(x): \mathbb{F}_{p^n} \to \mathbb{F}_p$ be a weakly regular function whose dual satisfies [\(5\)](#page-2-0). Then for any $b \in \mathbb{F}_{p^n}$, $\mathbb{S}_h(r, v, b) = \hat{h}$ if $r_1 - (a, b)$ for any $1 \$ $\frac{h(b)}{r}$ *if* $r_i = \varphi_{v_i}(b)$ *for any* $1 \leq i \leq \kappa$ *and otherwise*, $\mathbb{S}_h(r, v, b) = 0$ *, where* $r =$ $(r_1, \dots, r_\kappa) \in \mathbb{F}_p^\kappa$ and $v = (v_1, \dots, v_\kappa) \in \mathbb{F}_{p^n}^* \times \dots \times \mathbb{F}_{p^n}^*$.

Proof. For any $b \in \mathbb{F}_{p^n}$, $r = (r_1, \dots, r_\kappa) \in \mathbb{F}_p^\kappa$ and $v = (v_1, \dots, v_\kappa) \in \mathbb{F}_{p^n}^* \times \dots \times$
 $\mathbb{F}_{p^n}^*$, (4) gives
 $\mathbb{S}_h(r, v, b) = \frac{1}{p^\kappa} \sum_{e \in \mathbb{R}^\kappa} \omega^{-\sum_{i=1}^\kappa r_i s_i} \hat{h}(b - \sum_{i=1}^\kappa v_i s_i),$ (8) $\mathbb{F}_{p^n}^*$, [\(4\)](#page-2-1) gives

$$
\mathbb{S}_h(r, v, b) = \frac{1}{p^{\kappa}} \sum_{s \in \mathbb{F}_p^{\kappa}} \omega^{-\sum_{i=1}^{\kappa} r_i s_i} \widehat{h}\left(b - \sum_{i=1}^{\kappa} v_i s_i\right),\tag{8}
$$

where
$$
s = (s_1, \dots, s_\kappa) \in \mathbb{F}_p^{\kappa}
$$
. Note that Lemma 3 yields
\n
$$
\hat{h}(b - \sum_{i=1}^{\kappa} v_i s_i) = \frac{1}{p^{\tau}} \sum_{\mathbf{t} \in \mathbb{F}_p^{\tau}} \hat{F}(t_1, \dots, t_\tau) \hat{g}(b - \sum_{i=1}^{\tau} u_i t_i - \sum_{i=1}^{\kappa} v_i s_i)
$$
\n
$$
= \frac{1}{p^{\tau}} \omega^{\sum_{i=1}^{\kappa} \varphi_{v_i}(b)s_i} \sum_{t \in \mathbb{F}_p^{\tau}} \hat{F}(t_1, \dots, t_\tau) \hat{g}(b - \sum_{i=1}^{\tau} u_i t_i),
$$

where $t := (t_1, \dots, t_\tau)$. The last equality follows from [\(6\)](#page-2-2). Then from Lemma [3,](#page-3-1)
one obtains $\hat{h}(b - \sum_{i=1}^{\kappa} v_i s_i) = \omega^{\sum_{i=1}^{\kappa} \varphi_{v_i}(b)s_i} \hat{h}(b)$. (9)

$$
\widehat{h}\left(b - \sum_{i=1}^{k} v_i s_i\right) = \omega^{\sum_{i=1}^{k} \varphi_{v_i}(b)s_i} \widehat{h}(b).
$$
\n(9)

Substituting (9) into (8) gives

nto (8) gives
\n
$$
\mathbb{S}_h(r, v, b) = \frac{1}{p^{\kappa}} \sum_{s \in \mathbb{F}_p^{\kappa}} \omega^{\sum_{i=1}^{\kappa} (\varphi_{v_i}(b) - r_i)s_i} \widehat{h}(b).
$$

 $\mathbb{S}_h(r, v, b) = \frac{1}{p^{\kappa}} \sum_{s \in \mathbb{F}_p^{\kappa}} \omega^{\sum_{i=1}^{\kappa} (\varphi_{v_i}(b) - r_i)s_i} \hat{h}(b).$
Further the desired result follows from the fact $\sum_{s_i \in \mathbb{F}_p} \omega^{(\varphi_{v_i}(b) - r_i)s_i} = p$ if $r_i =$
 $\omega^{(b)}$ and 0 otherwise for any $1 \leq i \leq$ $\varphi_{v_i}(b)$ and 0 otherwise for any $1 \leq i \leq \kappa$. This completes the proof.

3 The Generic Constructions of Bent Functions

In this section, we will present two generic constructions of bent functions from known ones with certain properties.

3.1 The First New Class of Bent Functions

A class of non-quadratic p-ary bent functions $f(x) = \text{Tr}(\lambda x^{p^k+1}) + c\text{Tr}(x)^{p-1}$ with $\deg(f) = p - 1$ was presented in [\[15](#page-11-2)], where p is an odd prime and $n/\gcd(k, n)$ is odd. Based on Xu et al.'s work [\[15](#page-11-2)], we present the first construction.

Construction 1: Let u_1, \dots, u_τ be $\tau \geq 1$ elements in $\mathbb{F}_{p^n}, v_1, \dots, v_\kappa$ be $\kappa \geq 1$ elements in $\mathbb{F}_{p^n}^*$ and $c \in \mathbb{F}_p^*$. Let $g(x)$ be a weakly regular bent function over
 \mathbb{F}_p , whose dual satisfies (5) and $F(x_1, \ldots, x_n)$ be any reduced polynomial in \mathbb{F}_{p^n} whose dual satisfies [\(5\)](#page-2-0) and $F(x_1, \dots, x_\tau)$ be any reduced polynomial in $\mathbb{F}[x_1, \dots, x_\tau]$ Generate the function $f(x) \cdot \mathbb{F}_{x_\tau} \to \mathbb{F}$ from a and F as in (1) $\mathbb{F}_p[x_1, \dots, x_\tau]$. Generate the function $f(x) : \mathbb{F}_{p^n} \to \mathbb{F}_p$ from g and F as in [\(1\)](#page-1-0).
Then our first main result is stated as follows Then our first main result is stated as follows.

Theorem [1.](#page-4-2) Let $f(x): \mathbb{F}_{p^n} \to \mathbb{F}_p$ be the function generated by Construction 1. *Then* $f(x)$ *is a bent function if* $h(x)$ *given by* [\(7\)](#page-3-0) *is weakly regular bent.*

Proof. For any $b \in \mathbb{F}_{p^n}$, the Walsh transform of $f(x)$ defined by [\(1\)](#page-1-0) is $\overline{ }$

x) is a bent function if
$$
h(x)
$$
 given by (7) is weakly regular bent
For any $b \in \mathbb{F}_{p^n}$, the Walsh transform of $f(x)$ defined by (1) is

$$
\widehat{f}(b) = \sum_{\prod_{i=1}^{\kappa} \text{Tr}(v_i x) = 0} \omega^{h(x) - \text{Tr}(bx)} + \sum_{\prod_{i=1}^{\kappa} \text{Tr}(v_i x) \neq 0} \omega^{h(x) - \text{Tr}(bx) + c}.
$$

Note that

$$
\Pi_{i=1}^{\kappa} \operatorname{Tr}(v_i x) = 0 \qquad \qquad \Pi_{i=1}^{\kappa} \operatorname{Tr}(v_i x) \neq 0
$$
\nthat

\n
$$
\left\{ x \in \mathbb{F}_{p^n} : \prod_{i=1}^{\kappa} \operatorname{Tr}(v_i x) = 0 \right\} = \bigcup_{r \in \mathbb{F}_p^{\kappa}} \left\{ x \in \mathbb{T}_{r,v} : \prod_{i=1}^{\kappa} r_i = 0 \right\},
$$

where $r := (r_1, \dots, r_\kappa)$. Then Δ

$$
\hat{f}(b) = \sum_{\substack{\prod_{i=1}^c r_i = 0 \\ r_i \in \mathbb{F}_p}} \sum_{x \in \mathbb{T}_{r,v}} (1 - \omega^c) \omega^{h(x) - \text{Tr}(bx)} + \omega^c \hat{h}(b)
$$

$$
= \sum_{\substack{\prod_{i=1}^c r_i = 0 \\ r_i \in \mathbb{F}_p}} (1 - \omega^c) \mathbb{S}_h(r, v, b) + \omega^c \hat{h}(b).
$$

From Lemma [4](#page-3-3) we can deduce that

n deduce that
\n
$$
\widehat{f}(b) = (1 - \omega^c)\widehat{h}(b) + \omega^c \widehat{h}(b) = \widehat{h}(b)
$$

From Echina 4 we can deduce that
 $\hat{f}(b) = (1 - \omega^c)$

if $\prod_{i=1}^{\kappa} \varphi_{v_i}(b) = 0$ and $\hat{f}(b) = \omega^c \hat{h}$

conclusion follows $\hat{h}(b) + \omega^c \hat{h}(b) = \hat{h}(b)$
 $\hat{h}(b)$ if $\prod_{i=1}^{\kappa} \varphi_{v_i}(b) \neq 0$. Hence the desired conclusion follows.

3.2 The Second New Class of Bent Functions

Based on the work in [\[15](#page-11-2)], Xu et al. [\[17\]](#page-11-3) further constructed two classes of piecewise p-ary bent functions $f(x) = g(x) + \text{Tr}(ux)\text{Tr}(x)^{p-1}$ from the p-ary Kasami functions and Sidelnikov functions. Inspired by the idea coined in [\[15\]](#page-11-2) and [\[17\]](#page-11-3), we present the second construction.

Construction 2: Let u_1, \dots, u_τ be $\tau \geq 1$ elements in $\mathbb{F}_{p^n}, v_1, \dots, v_\kappa$ be $\kappa \geq 1$ elements in $\mathbb{F}_{p^n}^*$ and $c \in \mathbb{F}_p^*$. Let $g(x)$ be a weakly regular bent function over
 \mathbb{F}_p , whose dual satisfies (5) and $F(x_1, \ldots, x_n)$ be any reduced polynomial in \mathbb{F}_{p^n} whose dual satisfies [\(5\)](#page-2-0) and $F(x_1, \dots, x_\tau)$ be any reduced polynomial in
 $\mathbb{F}[x_1, \dots, x_\tau]$ Generate the function $f(x) \cdot \mathbb{F}_x \to \mathbb{F}$ from a and E as in (2) $\mathbb{F}_p[x_1, \dots, x_\tau]$. Generate the function $f(x) : \mathbb{F}_{p^n} \to \mathbb{F}_p$ from g and F as in [\(2\)](#page-1-1).

Note that $f(x)$ can be rewritten as
 $f(x) = \begin{cases} g(x), & \text{if } \prod_{i=1}^{\kappa} \text{Tr}(v_i x) = 0, \\ h(x), & \text{if } \prod_{i=1}^{\kappa} \text{Tr}(v_i x) \neq 0, \end{cases}$ Note that $f(x)$ can be rewritten as

$$
f(x) = \begin{cases} g(x), & \text{if } \prod_{i=1}^{\kappa} \text{Tr}(v_i x) = 0, \\ h(x), & \text{if } \prod_{i=1}^{\kappa} \text{Tr}(v_i x) \neq 0, \end{cases}
$$

where $h(x)$ is defined as in [\(7\)](#page-3-0).

Then our second main result is stated as follows.

Theorem [2.](#page-5-0) Let $f(x): \mathbb{F}_{p^n} \to \mathbb{F}_p$ be the function generated by Construction 2. *Then* $f(x)$ *is a bent function if* $h(x)$ *given by* [\(7\)](#page-3-0) *is bent.*

Proof. For any $b \in \mathbb{F}_{p^n}$, the Walsh transform of $f(x)$ defined by [\(2\)](#page-1-1) is \overline{a} $\ddot{}$

$$
x) \text{ is a bent function if } h(x) \text{ given by (1) is bent.}
$$
\n
$$
\text{or any } b \in \mathbb{F}_{p^n}, \text{ the Walsh transform of } f(x) \text{ defined by (2) is}
$$
\n
$$
\hat{f}(b) = \sum_{\prod_{i=1}^{\kappa} \text{Tr}(v_i x) = 0} \omega^{g(x) - \text{Tr}(bx)} + \sum_{\prod_{i=1}^{\kappa} \text{Tr}(v_i x) \neq 0} \omega^{h(x) - \text{Tr}(bx)}
$$

Note that

$$
\Pi_{i=1}^{\kappa} \operatorname{Tr}(v_i x) = 0 \qquad \Pi_{i=1}^{\kappa} \operatorname{Tr}(v_i x) \neq 0
$$
\nhat\n
$$
\left\{ x \in \mathbb{F}_{p^n} : \prod_{i=1}^{\kappa} \operatorname{Tr}(v_i x) = 0 \right\} = \bigcup_{r \in \mathbb{F}_p^{\kappa}} \left\{ x \in \mathbb{T}_{r,v} : \prod_{i=1}^{\kappa} r_i = 0 \right\},
$$

where $r := (r_1, \dots, r_\kappa)$. Then

$$
(r_1, \dots, r_\kappa). \text{ Then}
$$
\n
$$
\widehat{f}(b) = \sum_{\substack{\prod_{i=1}^{\kappa} r_i = 0 \\ r_i \in \mathbb{F}_p}} \sum_{x \in \mathbb{T}_{r,v}} (\omega^{g(x) - \text{Tr}(bx)} - \omega^{h(x) - \text{Tr}(bx)}) + \widehat{h}(b)
$$
\n
$$
= \sum_{\substack{\prod_{i=1}^{\kappa} r_i = 0 \\ r_i \in \mathbb{F}_p}} (\mathbb{S}_g(r, v, b) - \mathbb{S}_h(r, v, b)) + \widehat{h}(b),
$$

where $v = (v_1, \dots, v_\kappa)$. Together with Lemmas [2](#page-2-3) and [4,](#page-3-3) we can conclude that

^p
Together with Lemmas 2 and 4,

$$
\hat{f}(b) = \hat{g}(b) + \hat{h}(b) - \hat{h}(b) = \hat{g}(b)
$$

where $v = (v_1, \dots, v_\kappa)$. Togethermultiple
 $\widehat{f}(b) = \widehat{f}$

if $\prod_{i=1}^{\kappa} \varphi_{v_i}(b) = 0$ and $\widehat{f}(b) = \widehat{h}$

sion follows therefore with Lemmas 2 and 4, we can conclude that
 $\hat{g}(b) + \hat{h}(b) - \hat{h}(b) = \hat{g}(b)$
 $\hat{h}(b)$ if $\prod_{i=1}^{\kappa} \varphi_{v_i}(b) \neq 0$. Hence the desired conclusion follows.

Remark 1. Here we show that our constructions of bent functions are different from those in the previous works. Observe that bent functions of the forms [\(1\)](#page-1-0) and [\(2\)](#page-1-1) given by this paper are of the form $g(x) + F(\text{Tr}(\alpha_1 x), \cdots, \text{Tr}(\alpha_\lambda x)),$ where $g(x)$ is a weakly regular bent function from \mathbb{F}_{p^n} to \mathbb{F}_p , F is a reduced polynomial given by [\(3\)](#page-1-3) and $\alpha_i \in \mathbb{F}_{p^n}$ for $1 \leq i \leq \lambda$. Recently, some attempts have been made to construct bent functions of the above general form, see [\[7](#page-11-6), [9](#page-11-7)[,11](#page-11-4)[–16](#page-11-8),[18\]](#page-11-9). It should be noted that all these known results in this direction
depended on the dual of weakly regular bent function $g(x)$, that is,
 $\tilde{g}(x - \sum_{i=1}^{\lambda} \alpha_i t_i) = \tilde{g}(x) + \sum_{1 \leq i \leq j \leq \lambda} A_{ij} t_i t_j + \sum_{i=1}^{\lambda$ depended on the dual of weakly regular bent function $g(x)$, that is,

$$
\widetilde{g}(x - \sum_{i=1}^{\lambda} \alpha_i t_i) = \widetilde{g}(x) + \sum_{1 \le i \le j \le \lambda} A_{ij} t_i t_j + \sum_{i=1}^{\lambda} g_i(x) t_i, \qquad (10)
$$

where $A_{ij} \in \mathbb{F}_p$ for $1 \leq i \leq j \leq \lambda$, $g_i(x) : \mathbb{F}_{p^n} \to \mathbb{F}_p$ and $t_i \in \mathbb{F}_p$ for $1 \leq i \leq \lambda$. Denote the number of nonzero elements in $\{A_{ij}: 1 \le i \le j \le \lambda\}$ by N. Notice that these known results of the above general form were given for $N \leq 3$ [\[7,](#page-11-6)[9,](#page-11-7)[11](#page-11-4)– [16](#page-11-8)[,18](#page-11-9)] expect a class of bent functions with $\deg(F)=2$ and $N > 3$ [\[13](#page-11-5)] when p is an odd prime. However, although our results also depend on the dual of $g(x)$, bent functions with $\deg(F) > 2$ and $N > 3$ can be constructed by this paper for odd p, see Examples 1–4 for details.

Remark 2. As pointed out in [\[11](#page-11-4), Lemma 2.1], for $p = 2$, the algebraic degree of $F(\text{Tr}(u_1x), \dots, \text{Tr}(u_\tau x))$ is the same as that of $F(x_1, \dots, x_\tau)$ for some u_i , and it can be generalized to any characteristic directly, where $u_i \in \mathbb{F}_{p^n}$ for $1 \leq i \leq \tau$. Hence there is no doubt that the generic constructions given in Theorems [1](#page-4-3) and [2](#page-5-1) can produce bent functions with the maximal algebraic degree given in Lemma [1.](#page-2-4)

4 Specific Constructions of Infinite Families of Bent Functions

In this section, by using constructions [1](#page-4-2) and [2,](#page-5-0) we shall introduce specific constructions of bent functions from some known ones whose duals satisfy [\(5\)](#page-2-0).

4.1 New Classes of Bent Functions from Quadratic Bent Functions

It is well-known that a homogeneous quadratic bent function is weakly regular and its dual is also a homogenous quadratic bent function [\[6](#page-10-4)]. Let $g(x)$ be a
homogeneous quadratic bent function with the dual $\tilde{\epsilon}(x) = \sum^{n-1} \text{Tr}(a, x^{p^k+1})$ **4.1** New Classes of Bent Functions from Quadratic Bent Functions
It is well-known that a homogeneous quadratic bent function is weakly regular
and its dual is also a homogeneous quadratic bent function [6]. Let $g(x)$ be It is well-known that a homogeneous quadratic bent function is weakly regular
and its dual is also a homogenous quadratic bent function [6]. Let $g(x)$ be a
homogeneous quadratic bent function with the dual $\tilde{g}(x) = \sum_{k=$ $\sum_{i=1}^{7} u_i t_i$ is equal to and its dual is also a homogenous quadratic bent function [6]. Let $g(x)$ be a

$$
a_k \in \mathbb{F}_{p^n}^*
$$
. Through some calculation, we can obtain that $\overline{\widetilde{g}}(x - \sum_{i=1}^{\kappa} v_i s)$

$$
\sum_{i=1}^{\tau} u_i t_i
$$
 is equal to

$$
\widetilde{g}(x - \sum_{i=1}^{\tau} u_i t_i) + \sum_{i=1}^{\kappa} \varphi_{v_i}(x) s_i + \sum_{i=1}^{\kappa} \sum_{j=1}^{\tau} \varphi_{v_i}(u_j) s_i t_j + \sum_{1 \le i \le j \le \kappa} G(v_i, v_j) s_i s_j
$$

for all $x \in \mathbb{F}_{p^n}$, $u_i \in \mathbb{F}_{p^n}$, $v_j \in \mathbb{F}_{p^n}^*$ and $s_i, t_j \in \mathbb{F}_p$, $1 \le i \le \tau$, $1 \le j \le \kappa$, where $G(v_i, v_j) = \varphi_{v_i}(v_j)$ if $i < j$ and $S_i, t_j \in \mathbb{F}_p$, $1 \leq i \leq j \leq \kappa$, where
 $G(v_i, v_j) = \varphi_{v_i}(v_j)$ if $i < j$ and $G(v_i, v_i) = \sum_{k=0}^{n-1} \text{Tr}(a_k v_i^{p^k+1})$ if $i = j$. Here for all $x \in \mathbb{F}_{p^n}$,
 $G(v_i, v_j) = \varphi_{v_i}$
 $\varphi_{v_i}(x) := \sum_{k=0}^{n-1}$ $\sum_{k=0}^{n-1} \text{Tr}(a_k(v_i x^{p^k} + v_i^{p^k} x))$. Obviously, when $\varphi_{v_i}(u_j) = 0$ for all $1 \leq$ $G(v_i, v_j) = \varphi_{v_i}(v_j)$ if $i < j$ and $G(v_i, v_i) = \sum_{k=0}^{n-1} \text{Tr}(a_k v_i^{p^k + \varphi_{v_i}})$,
 $\varphi_{v_i}(x) := \sum_{k=0}^{n-1} \text{Tr}(a_k (v_i x^{p^k} + v_i^{p^k} x))$. Obviously, when $\varphi_{v_i}(u_j)$
 $i \leq \kappa, 1 \leq j \leq \tau, \varphi_{v_i}(v_j) = 0$ for all $1 \leq i < j \leq \kappa$ $\sum_{k=0}^{n-1} \text{Tr}(a_k v_i^{p^k+1}) = 0$ for all $1 \leq i \leq \kappa$, the dual of $g(x)$ satisfies [\(5\)](#page-2-0).

The specific construction of bent functions from quadratic bent functions can be simply introduced by using the Sidelnikov function. From [\[6](#page-10-4)], we know that $g(x) = \text{Tr}(ax^2)$ is weakly regular bent and its dual is $\tilde{g}(x) = -\text{Tr}(x^2/(4a))$, where $a \in \mathbb{F}_{p^n}^*$ and p is an odd prime. Then the following two theorems are directly obtained from Theorem 1 and Theorem 2 respectively directly obtained from Theorem [1](#page-4-3) and Theorem [2](#page-5-1) respectively.

Theorem 3. Let p be an odd prime, $a \in \mathbb{F}_{p^n}^*$, $c \in \mathbb{F}_p^*$, u_1, \dots, u_τ be $\tau \geq 1$ elements in \mathbb{F}_p , and v_1, \dots, v_n be $\kappa \geq 1$ elements in \mathbb{F}_p^* satisfying $\text{Tr}(v_1, v_1)/(4a)$ *ments in* \mathbb{F}_{p^n} *and* v_1, \dots, v_{κ} *be* $\kappa \geq 1$ *elements in* $\mathbb{F}_{p^n}^*$ *satisfying* $\text{Tr}(v_i v_j/(4a)) = 0$ *for all* $1 \leq i \leq \tau, 1 \leq i \leq \kappa$. *Then* 0 *for all* $1 \leq i \leq j \leq \kappa$ *and* $\text{Tr}(u_i v_j/(4a)) = 0$ *for all* $1 \leq i \leq \tau, 1 \leq j \leq \kappa$. Then u_{τ} u_{τ} u_{τ} u_{τ} \leq

$$
f(x) = \text{Tr}(ax^2) + F(\text{Tr}(u_1x), \text{Tr}(u_2x), \cdots, \text{Tr}(u_\tau x)) + c\left(\prod_{i=1}^{\kappa} \text{Tr}(v_i x)\right)^{p-1}
$$

is a bent function if h(x) *given by* [\(7\)](#page-3-0) *is weakly regular bent.*

Example 1. Let $p = 3$, $n = 5$, $a = 1$, $c = 1$, $\tau = 4$, $\kappa = 2$, $F(x_1, x_2, x_3, x_4) =$ $x_1x_2 + x_3x_4$ and ξ be a primitive element of \mathbb{F}_3 . Take $u_1 = 1$, $u_2 = \xi^{212}$, $u_3 = 2$, $u_4 = \xi^{11}, v_1 = \xi^{341}$ and $v_2 = \xi^{705}$. Then $\text{Tr}(u_1^2) = \text{Tr}(u_3^2) = \text{Tr}(u_4^2) = \text{Tr}(u_1u_2) =$
 -1 $\text{Tr}(u_2^2) = \text{Tr}(u_3u_2) = \text{Tr}(u_3u_3) = 1$ $\text{Tr}(u_1u_4) = \text{Tr}(u_2u_4) = \text{Tr}(u_3u_5) = 0$ -1 , $\text{Tr}(u_2^2) = \text{Tr}(u_1u_3) = \text{Tr}(u_2u_3) = 1$, $\text{Tr}(u_1u_4) = \text{Tr}(u_2u_4) = \text{Tr}(u_3u_4) = 0$,
 $\text{Tr}(u_2^2) = \text{Tr}(u_3u_2) = \text{Tr}(u_2^2) = 0$ and $\text{Tr}(u_3u_3) = 0$ for all $1 \le i \le 4$, $i = 1, 2$. $\text{Tr}(v_1^2) = \text{Tr}(v_1v_2) = \text{Tr}(v_2^2) = 0$ and $\text{Tr}(u_iv_j) = 0$ for all $1 \le i \le 4$, $j = 1, 2$. It
can be verified that $h(x) = \text{Tr}(x^2) + \text{Tr}(x)\text{Tr}(x^{212}x) + \text{Tr}(2x)\text{Tr}(x^{11}x)$ is bent by can be verified that $h(x) = \text{Tr}(x^2) + \text{Tr}(x)\text{Tr}(\xi^{212}x) + \text{Tr}(2x)\text{Tr}(\xi^{11}x)$ is bent by the discussion of Case II of [\[13](#page-11-5)]. Theorem [3](#page-7-0) now establishes that

$$
f(x) = \text{Tr}(x^2) + \text{Tr}(x)\text{Tr}(\xi^{212}x) + \text{Tr}(2x)\text{Tr}(\xi^{11}x) + (\text{Tr}(\xi^{341}x)\text{Tr}(\xi^{705}x))^2
$$

is a bent function over \mathbb{F}_{3^5} . Moreover, it can be checked that $N = 7$ and $\deg(F) =$ ⁴, where N is defined as in Remark [1.](#page-6-1)

Theorem 4. Let p be an odd prime, $a \in \mathbb{F}_{p^n}^*$, u_1, \dots, u_τ be $\tau \geq 1$ elements in \mathbb{F}_{p^n} and v_1, \dots, v_n be $\tau \geq 1$ elements in $\mathbb{F}_{p^n}^*$ estisting $Tr(v_1, v_1/(da)) = 0$ for all \mathbb{F}_{p^n} and v_1, \dots, v_{κ} be $\kappa \geq 1$ elements in $\mathbb{F}_{p^n}^{*}$ satisfying $\text{Tr}(v_i v_j/(4a)) = 0$ for all
 $1 \leq i \leq \kappa$ and $\text{Tr}(v_i v_j/(4a)) = 0$ for all $1 \leq i \leq \pi$, $1 \leq i \leq \kappa$. Then $1 \leq i \leq j \leq \kappa$ and $\text{Tr}(u_i v_j/(4a)) = 0$ for all $1 \leq i \leq \tau, 1 \leq j \leq \kappa$. Then form 4. Let p be an odd prime, $a \in \mathbb{F}_{p^n}$, u_1, \dots, u_τ be

and v_1, \dots, v_κ be $\kappa \ge 1$ elements in $\mathbb{F}_{p^n}^*$ satisfying $\text{Tr}(v_i\iota$
 $i \le j \le \kappa$ and $\text{Tr}(u_iv_j/(4a)) = 0$ for all $1 \le i \le \tau$, $1 \le j$
 $f(x) = \text{Tr}(ax^2$

$$
f(x) = \text{Tr}(ax^2) + F(\text{Tr}(u_1x), \text{Tr}(u_2x), \cdots, \text{Tr}(u_\tau x)) \left(\prod_{i=1}^{\kappa} \text{Tr}(v_i x)\right)^{p-1}
$$

is a bent function if $h(x)$ *given by* [\(7\)](#page-3-0) *is bent.*

Example 2. Let $p = 3, n = 6, a = 1, \tau = 4, \kappa = 1, F(x_1, x_2, x_3, x_4) = x_1x_2+x_3x_4$ and ξ be a primitive element of \mathbb{F}_{3^6} . Take $u_1 = \xi^2$, $u_2 = \xi$, $u_3 = \xi^{26}$, $u_4 = \xi^{118}$ and $v_1 = \xi^7$. Then $\text{Tr}(u_1^2) = \text{Tr}(u_3^2) = 1$, $\text{Tr}(u_2^2) = \text{Tr}(u_4^2) = 2$, $\text{Tr}(u_i u_j) = 0$ for others $1 \le i \le 4$. Tr $(v_1^2) = 0$ and $\text{Tr}(u_1 v_1) = 0$ for all $1 \le i \le 4$. It can be others $1 \leq i \leq j \leq 4$, $\text{Tr}(v_1^2) = 0$ and $\text{Tr}(u_i v_1) = 0$ for all $1 \leq i \leq 4$. It can be verified that $h(r) = \text{Tr}(r^2) + \text{Tr}(f(r) \text{Tr}(f^2 r) + \text{Tr}(f^{26} r) \text{Tr}(f^{118} r)$ is bent by [13] verified that $h(x) = \text{Tr}(x^2) + \text{Tr}(\xi x) \text{Tr}(\xi^2 x) + \text{Tr}(\xi^{26} x) \text{Tr}(\xi^{118} x)$ is bent by [\[13](#page-11-5), Theorem 5]. Theorem [4](#page-7-1) now establishes that

$$
f(x) = \text{Tr}(x^2) + (\text{Tr}(\xi x)\text{Tr}(\xi^2 x) + \text{Tr}(\xi^{26} x)\text{Tr}(\xi^{118} x))\text{Tr}(\xi^7 x)^2
$$

is a bent function over \mathbb{F}_{36} . Moreover, it can be checked that $N = 4$ and $\deg(F) =$ ⁴, where N is defined as in Remark [1.](#page-6-1)

4.2 New Classes of Bent Functions from the Maiorana-MacFarland Class of Bent Functions

Let $n = 2m$ be a positive integer. By identifying an element $x \in \mathbb{F}_{p^n}$ with a vector $(y, z) \in \mathbb{F}_{p^m} \times \mathbb{F}_{p^m}$, the Maiorana-MacFarland class of bent functions on \mathbb{F}_{p^n} can be expressed as

$$
g(x) = g(y, z) = \text{Tr}_{1}^{m}(y\pi(z)) + h(z), y, z \in \mathbb{F}_{p^{m}},
$$
\n(11)

where $\pi : \mathbb{F}_{p^m} \to \mathbb{F}_{p^m}$ is a permutation and h is a function from \mathbb{F}_{p^m} to \mathbb{F}_p . Such class of bent functions is regular and its dual [4] is equal to class of bent functions is regular and its dual [\[4\]](#page-10-3) is equal to \mathbb{F}_{p^m} is a permutation
tions is regular and
 $\widetilde{g}(x) = \widetilde{g}(y, z) = \text{Tr}_1^m$

$$
\widetilde{g}(x) = \widetilde{g}(y, z) = \text{Tr}_{1}^{m} (z \pi^{-1}(y)) + h (\pi^{-1}(y)),
$$

where π^{-1} is the inverse permutation of π . There are several π and h such that the dual of $a(u, z)$ satisfies (5). In the following, we give the calculation of $\tilde{a}(u, z)$ $\tilde{g}(x) = \tilde{g}(y, z) = \text{Tr}_{1}^{m} (z\pi^{-1}(y)) + h (\pi^{-1}(y)),$
where π^{-1} is the inverse permutation of π . There are several π and h such that
the dual of $g(y, z)$ satisfies [\(5\)](#page-2-0). In the following, we give the calculation of for the case while π is a linear permutation and $h(z)=0$. In the same manner, [1](#page-4-2) and Construction [2.](#page-5-0) When π is a linear permutation and $h(z)=0$, one has

it can be verified that
$$
g(x)
$$
 with some other π and h is suitable for Construction 1 and Construction 2. When π is a linear permutation and $h(z) = 0$, one has\n
$$
\pi^{-1}\left(y - \sum_{i=1}^{k} v_{i,1} s_i - \sum_{i=1}^{T} u_{i,1} t_i\right) = \pi^{-1}\left(y - \sum_{i=1}^{T} u_{i,1} t_i\right) - \sum_{i=1}^{K} \pi^{-1}(v_{i,1}) s_i,
$$

where $u_i = (u_{i,1}, u_{i,2}) \in \mathbb{F}_{p^m} \times \mathbb{F}_{p^m}, v_j = (v_{j,1}, v_{j,2}) \in \mathbb{F}_{p^m}^* \times \mathbb{F}_{p^m}^*$ and $t_i, s_j \in \mathbb{F}_p$,
 $1 \le i \le \tau$, $1 \le j \le \kappa$. Then by a direct calculation, we can derive that
 $\widetilde{g}\left(y - \sum_{i=1}^{\kappa} v_{i,1} s$ $1 \leq i \leq \tau, 1 \leq j \leq \kappa$. Then by a direct calculation, we can derive that

$$
\leq j \leq \kappa. \text{ Then by a direct calculation, we can derive the following property:}
$$
\n
$$
\widetilde{g}\left(y - \sum_{i=1}^{k} v_{i,1} s_i - \sum_{i=1}^{r} u_{i,1} t_i, z - \sum_{i=1}^{k} v_{i,2} s_i - \sum_{i=1}^{r} u_{i,2} t_i\right)
$$

equals to

$$
\widetilde{g}(y - \sum_{i=1}^{\tau} u_{i,1}t_i, z - \sum_{i=1}^{\tau} u_{i,2}t_i) + \sum_{i=1}^{\kappa} \varphi_{v_{i,1}, v_{i,2}}(y, z)s_i + \mathcal{G}
$$

with

$$
\mathcal{G} = \sum_{i=1}^{\kappa} \sum_{j=1}^{\tau} \varphi_{v_{i,1}, v_{i,2}}(u_{j,1}, u_{j,2}) s_i t_j + \sum_{1 \le i \le j \le \kappa} G(v_{i,1}, v_{i,2}, v_{j,1}, v_{j,2}) s_i s_j
$$

where $G(v_{i,1}, v_{i,2}, v_{j,1}, v_{j,2}) = \varphi_{v_{i,1}, v_{i,2}}(v_{j,1}, v_{j,2})$ if $i < j$ and $G(v_{i,1}, v_{i,2}, v_{i,1}, v_{i,2})$ $=\operatorname{Tr}_{1}^{m}(\pi^{-1}(v_{i,1})v_{i,2})$ if $i = j$. Here $\varphi_{v_{i,1},v_{i,2}}(y,z) := \operatorname{Tr}_{1}^{m}(\pi^{-1}(v_{i,1})z + v_{i,2})$
 $\pi^{-1}(u)$ If $(\varphi_{v_{i,1},v_{i,2}}(y,z)) = 0$ for all $1 \leq i \leq \kappa$ $1 \leq i \leq \pi$ (e.g. $(v_{i,1})$ $\pi^{-1}(y)$). If $\varphi_{v_{i-1},v_{i-2}}(u_{i,1}, u_{i,2})=0$ for all $1 \leq i \leq \kappa, 1 \leq j \leq \tau, \varphi_{v_{i-1},v_{i-2}}(v_{i,1}, u_{i,2})$ $v_{j,2}$ = 0 for all $1 \le i < j \le \kappa$ and $Tr_1^m(\pi^{-1}(v_{i,1})v_{i,2})=0$ for all $1 \le i \le \kappa$, the dual of $g(x)$ satisfies (5) Then the following two theorems are directly obtained dual of $g(x)$ satisfies [\(5\)](#page-2-0). Then the following two theorems are directly obtained from Theorem [1](#page-4-3) and Theorem [2](#page-5-1) respectively.

Theorem 5. Let $c \in \mathbb{F}_p^*$, u_1, \dots, u_τ be $\tau \geq 1$ elements in $\mathbb{F}_{p^{2m}}$ and v_1, \dots, v_k
be $\kappa > 1$ elements in \mathbb{F}_p^* , where n is a prime. Write $u_1, a_2 \in \mathbb{F}_p$ *be* $\kappa \geq 1$ *elements* in $\mathbb{F}_{p^{2m}}^*$, where *p* is a prime. Write u_i as $(u_{i,1}, u_{i,2}) \in \mathbb{F}_{p^{2m}}$ for each $1 \leq i \leq \pi$ and v_i as $(v_i, v_i) \in \mathbb{F}_{p^{2m}} \times \mathbb{F}_{p^{2m}}$ $\mathbb{F}_{p^m} \times \mathbb{F}_{p^m}$ *for each* $1 \leq i \leq \tau$ *and* v_j *as* $(v_{j,1}, v_{j,2}) \in \mathbb{F}_{p^m} \times \mathbb{F}_{p^m}$ *for each* $1 \leq i \leq \kappa$ *Let* $a(u, z) = \text{Tr}^m(u\pi(z))$ where π is a linear permutation *each* $1 \leq j \leq \kappa$ *. Let* $g(y, z) = \text{Tr}_1^m(y\pi(z))$ *, where* π *is a linear permutation* $\text{over } \mathbb{F}_m$ *If* $\text{Tr}_1^m(\pi^{-1}(y, \cdot, y) \cdot \mathbb{F}_m + y \cdot \pi^{-1}(y, \cdot, y)) = 0$ for all $1 \leq i \leq \kappa, 1 \leq \kappa$ *over* \mathbb{F}_{p^m} . If $\text{Tr}_{1}^{m}(\pi^{-1}(v_{i,1})u_{j,2} + v_{i,2}\pi^{-1}(u_{j,1})) = 0$ *for all* $1 \leq i \leq \kappa, 1 \leq i \leq \pi$. Tr_m($\pi^{-1}(v_{i,1})v_{i,2} + v_{i,2}\pi^{-1}(v_{i,1}) = 0$ *for all* $1 \leq i \leq \kappa$ and $j \leq \tau$, $Tr_1^m(\pi^{-1}(v_{i,1})v_{j,2} + v_{i,2}\pi^{-1}(v_{j,1})) = 0$ *for all* $1 \leq i \leq j \leq \kappa$ *and*
 $Tr_1^m(\pi^{-1}(v_{i,1})v_{j,2} + v_{i,2}\pi^{-1}(v_{j,1})) = 0$ *for all* $1 \leq i \leq j \leq \kappa$ *and*
 $Tr_1^m(\pi^{-1}(v_{i,1})v_{i,2}) = 0$ *for all* $1 \leq i \leq \kappa$. $\text{Tr}_{1}^{m}(\pi^{-1}(v_{i,1})v_{i,2})=0$ *for all* $1 \leq i \leq \kappa$. *Then*

$$
f(y, z) = h(y, z) + c \left(\prod_{i=1}^{k} \text{Tr}_1^m(v_{i,1}y + v_{i,2}z) \right)^{p-1}
$$

is a bent function if

$$
h(y, z) = \text{Tr}_1^m(y\pi(z)) + F(\text{Tr}_1^m(u_{1,1}y + u_{1,2}z), \cdots, \text{Tr}_1^m(u_{\tau,1}y + u_{\tau,2}z)) \tag{12}
$$

is weakly regular bent.

Example 3. Let $p = 5$, $m = 3$, $\tau = 4$, $\kappa = 1$, $F(x_1, x_2, x_3, x_4) =$ $x_1x_2 + x_3x_4$, $c = 1$ and ξ be a primitive element of \mathbb{F}_5 ³. Take $\pi(z) = z^5$, $(u_{1,1}, u_{1,2}) = (\xi^{26}, \xi^{50})$, $(u_{2,1}, u_{2,2}) = (\xi^{32}, \xi^{51})$, $(u_{3,1}, u_{3,2}) =$ $\pi(z) = z^5$, $(u_{1,1}, u_{1,2}) = (\xi^{26}, \xi^{50})$, $(u_{2,1}, u_{2,2}) = (\xi^{32}, \xi^{51})$, $(u_{3,1}, u_{3,2}) = (\xi^{119}, \xi^{8})$, $(u_{1,1}, u_{1,2}) = (\xi^{63}, \xi^{5})$ and $(u_{1,1}, u_{1,2}) = (\xi^{36}, \xi^{114})$. Then (ξ^{119}, ξ^8) , $(u_{4,1}, u_{4,2}) = (\xi^{63}, \xi^5)$ and $(v_{1,1}, v_{1,2}) = (\xi^{36}, \xi^{114})$. Then
 $\pi^{-1}(z) = z^{25} \operatorname{Tr}^3((u_{1,1})^{25}u_{1,2}) = \operatorname{Tr}^3((u_{2,1})^{25}u_{2,2}) = 1 \operatorname{Tr}^3((u_{2,1})^{25}u_{2,2}) =$ $\pi^{-1}(z) = z^{25}, \ \text{Tr}_{1}^{3}((u_{1,1})^{25}u_{1,2}) = \text{Tr}_{1}^{3}((u_{3,1})^{25}u_{3,2}) = 1, \ \text{Tr}_{1}^{3}((u_{2,1})^{25}u_{2,2}) = \text{Tr}_{1}^{3}((u_{1,1})^{25}u_{1,2}) = 2, \ \text{Tr}_{1}^{3}((u_{1,1})^{25}u_{1,2}) = 4, \ \text{Tr}_{1}^{3}((u_{1,1})^{25}u_{1,2}) = 0, \text{ for } 1 \leq i \leq j \leq 4$ $\text{Tr}_{1}^{3}((u_{4,1})^{25}u_{4,2})=2, \text{Tr}_{1}^{3}$
 $\text{Tr}_{3}^{3}((v_{1,1})^{25}u_{1,2}+v_{1,3}(u_{1,3}))$ $\text{Tr}^3_1((u_{4,1})^{25}u_{4,2}) = 2$, $\text{Tr}^3_1((u_{i,1})^{25}u_{j,2} + u_{i,2}(u_{j,1})^{25}) = 0$ for $1 \leq i \leq j \leq 4$,
 $\text{Tr}^3_1((v_{1,1})^{25}u_{j,2} + v_{1,2}(u_{j,1})^{25}) = 0$ for all $1 \leq j \leq 4$ and $\text{Tr}^3_1((v_{1,1})^{25}v_{1,2}) = 0$. It
can be can be verified that $h(y, z) = \text{Tr}_1^3(yz^5) + F(\text{Tr}_1^3(u_{1,1}y + u_{1,2}z), \cdots, \text{Tr}_1^3(u_{4,1}y + u_{1,2}z))$ is bent by [13] Theorem 5. Theorem 5. now establishes that $u_{4,2}z$) is bent by [\[13](#page-11-5), Theorem [5](#page-9-0)]. Theorem 5 now establishes that

$$
f(y,z) = \text{Tr}_1^3(yz^5) + F(\text{Tr}_1^m(u_{1,1}y + u_{1,2}z), \cdots, \text{Tr}_1^m(u_{4,1}y + u_{4,2}z)) + \text{Tr}_1^m(\xi y + 2z)^4
$$

is a bent function over \mathbb{F}_{56} . Moreover, it can be checked that $N = 4$ and $\deg(F) =$ ⁴, where N is defined as in Remark [1.](#page-6-1)

Theorem 6. Let $u_i = (u_{i,1}, u_{i,2})(1 \leq i \leq \tau)$ be τ elements in $\mathbb{F}_{p^m} \times \mathbb{F}_{p^m}$ *and* $v_j = (v_{j,1}, v_{j,2})(1 \leq j \leq \kappa)$ *be* κ *elements in* $\mathbb{F}_{p^m}^* \times \mathbb{F}_{p^m}^*$, where *p is a* prime. Let $g(u, x) = \text{Tr}^m(u\pi(x))$, where π *is a linear permutation over* \mathbb{F}_{p^m} . If *prime. Let* $g(y, z) = \text{Tr}_1^m(y\pi(z))$, where π *is a linear permutation over* \mathbb{F}_{p^m} . If $\text{Tr}(\pi^{-1}(y, \cdot, y)$ $\pi^{-1}(y, \cdot, y) = 0$ for all $1 \leq i \leq \kappa$, $1 \leq i \leq \pi$, $\text{Tr}(\pi^{-1}(y, \cdot, y))$ $\text{Tr}(\pi^{-1}(v_{i,1})u_{i,2}+v_{i,2}\pi^{-1}(u_{j,1}))=0$ for all $1 \leq i \leq \kappa, 1 \leq j \leq \tau$, $\text{Tr}(\pi^{-1}(v_{i,1})v_{j,2})$ +v_{i,2}π⁻¹(v_{j,1})) = 0 *for all* 1 ≤ *i* < *j* ≤ κ *and* Tr(π⁻¹(v_{i,1})v_{i,2}) = 0 *for all*

1 ≤ *i* ≤ κ. Then
 $\tilde{g}(y, z) + F(\text{Tr}(u_{1,1}y + u_{1,2}z), \cdots, \text{Tr}(u_{7,1}y + u_{7,2}z))(\prod_{i=1}^{\kappa} \text{Tr}(v_{i,1}y + v_{i,2}z))^{p-1}$ $1 \leq i \leq \kappa$. Then

$$
\widetilde{g}(y, z) + F(\text{Tr}(u_{1,1}y + u_{1,2}z), \cdots, \text{Tr}(u_{\tau,1}y + u_{\tau,2}z)) \left(\prod_{i=1}^{\kappa} \text{Tr}(v_{i,1}y + v_{i,2}z)\right)^{p-1}
$$

is a bent function if $h(x)$ *given by* [\(12\)](#page-9-1) *is bent.*

Example 4. Let $p = 3$, $m = 3$, $\tau = 4$, $\kappa = 1$, $F(x_1, x_2, x_3, x_4) = x_1x_2 + x_3x_4$
and ξ be a primitive element of \mathbb{F}_{33} . Take $\pi(z) = z^3$, $(y_{1,1}, y_{1,2}) = (z^6, z^{21})$ and ξ be a primitive element of \mathbb{F}_{33} . Take $\pi(z) = z^3$, $(u_{1,1}, u_{1,2}) = (\xi^6, \xi^{21})$,
 $(u_{1,1}, u_{1,2}) = (\xi^2 \xi^{15})$, $(u_{1,1}, u_{1,2}) = (\xi^{11} \xi^{24})$, $(u_{1,1}, u_{1,2}) = (\xi^5 \xi^{12})$ and $(u_{2,1}, u_{2,2}) = (\xi^{25}, \xi^{15}), (u_{3,1}, u_{3,2}) = (\xi^{11}, \xi^{24}), (u_{4,1}, u_{4,2}) = (\xi^{5}, \xi^{12})$ and
 $(v_{1,1}, v_{1,0}) = (\xi^{25}, \xi^{15})$. Then $\pi^{-1}(z) = z^9$. Tr³ $((u_{1,1})^9 u_{1,0}) = \text{Tr}^3((u_{2,1})^9 u_{2,0}) = 1$. $(v_{1,1}, v_{1,2}) = (\xi, 2)$. Then $\pi^{-1}(z) = z^9$, $\text{Tr}_1^3((u_{1,1})^9 u_{1,2}) = \text{Tr}_1^3((u_{3,1})^9 u_{3,2}) = 1$,
 $\text{Tr}_1^3((u_{2,1})^9 u_{2,2}) = \text{Tr}_1^3((u_{4,1})^9 u_{4,2}) = 2$, $\text{Tr}_1^3((u_{i,1})^9 u_{j,2} + u_{i,2}(u_{j,1})^9) = 0$ for
 $1 \le i \le 4$, Tr_2 $1 \leq i < j \leq 4$, $\text{Tr}_{1}^{3}((v_{1,1})^{9}v_{j,2} + v_{1,2}(u_{j,1})^{9}) = 0$ for all $1 \leq j \leq 4$ and $\text{Tr}_{1}^{3}((v_{1,1})^{9}v_{1,2}) = 0$. It can be verified that $h(y, z) = \text{Tr}_{1}^{3}(yz^{3}) + F(\text{Tr}_{1}^{3}(u_{1,1}y + u_{1,2}z))$ is bent by [13] Theorem $u_{1,2}z), \cdots, \text{Tr}_{1}^{3}(u_{4,1}y+u_{4,2}z))$ is bent by [\[13](#page-11-5), Theorem 5]. Theorem [6](#page-9-2) now establishes that lishes that

$$
f(y, z) = \text{Tr}_1^3(yz^3) + F(\text{Tr}_1^m(u_{1,1}y + u_{1,2}z), \cdots, \text{Tr}_1^m(u_{4,1}y + u_{4,2}z)) \text{Tr}_1^m(\xi y + 2z)^2
$$

is a bent function over \mathbb{F}_{3^6} . Moreover, it can be checked that $N = 4$ and $\deg(F) =$ ⁴, where N is defined as in Remark [1.](#page-6-1)

5 Conclusions

In this paper, we proposed two generic constructions of bent functions with the forms [\(1\)](#page-1-0) and [\(2\)](#page-1-1), which generalized some previous works [\[15](#page-11-2),[17\]](#page-11-3). Moreover, based on our constructions, several infinite families of bent functions can be obtained from quadratic bent functions and the Maiorana-MacFarland class of bent functions by calculating their duals. In addition, it was shown that bent functions with the maximal algebraic degree can be obtained from our constructions.

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