Modern Methods of Searching for the Optimal Assessment of Product Quality

G. Pipiay(✉) and L. Chernenkaya

Peter the Great St.Petersburg Polytechnic University, 29, Polytechnic St, St. Petersburg 194021, Russia
gogpipiy@ya.ru

Abstract. The modern theory of product quality management, and in particular the subject of qualimetry, contains a large list of models and methods for assessing product quality, and over the past 5 years, this list has only expanded. However, the tools used in the processing of heterogeneous information, in the analysis of product quality and decision-making, are mainly focused on the opinions of experts or on mass production, which limits the functionality of the existing theory of product quality management. Authors proposes modern methods of two-level optimization for solving problems of product quality management. The authors give an example of applying the theory of bilevel optimization to find the optimal estimate of product quality. The example contains an algorithm for solving an analytical problem and the results obtained.

Keywords: Qualimetry · Bilevel optimization · Quality monitoring · Quality indicators

1 Introduction

Speaking about the current state of development of the theory and practice of product quality management, it is necessary first of all to mention the main reason for the development of this area, namely production systems and technologies for the production of modern products. Currently, modern production systems and technologies are combined in the concept of "Industry 4.0". The term "Industry 4.0" comprises the following technologies [1]:

1. Smart factory. Fully equipped production with sensors and autonomous systems.
2. Cyber-physical systems. Unification of physical and digital levels. By spanning the production level as well as the product level, systems arise whose physical and digital representation can no longer reasonably be differentiated.
3. Self-organization. Existing production systems are becoming increasingly decentralized. This is accompanied by the decomposition of the classical production hierarchy and the transition to decentralized self-organization.
4. New distribution and purchasing systems: Distribution and purchasing will be more individualized.

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5. New systems in product and service development: product and service development will be more individualized.
6. Adaptation to human needs: new production systems should be designed to follow human needs, and not vice versa.
7. Corporate social responsibility at the heart of which is sustainability and resource efficiency.

In the theory of product quality management, the listed components are embedded in the concept of "Quality 4.0" [2]. Within the framework of this concept, big data technologies and data science are introduced and used, which allows for a comprehensive information accounting of indicators that characterize quality not only from a technical point of view, but also from an economic, technological and social one.

Based on the above descriptions of the concept of "Quality 4.0", it should be said that "Quality 4.0" is a set of modern methods and practices based on the digitalization of production in terms of quality management in the conditions of the fourth industrial revolution [3]. The concept of "Quality 4.0" is implemented through the following principles; Data, Analytics, Interaction, Collaboration, Applications, Scale, Management Systems, Correspondence, Culture, Leadership, Competences.

Today, there are many methods to achieve the requirements of the "Quality 4.0" concept. Among them are statistical methods, expert methods, fuzzy methods, analytical methods, expert-statistical methods, expert-analytical methods and expert-fuzzy methods. There are many articles devoted to these methods and at the moment they are well developed, which cannot be said about the methods of bilevel optimization and decision making.

The development of the theory of quality management based on bilevel optimization methods will make it possible to more effectively fulfill the requirements of the "Quality 4.0" concept and the "Industry 4.0" concept due to the ability to work with a large amount of heterogeneous information and make prompt and optimal decisions regarding the quality of products.

2 Description of the Problem Area

It is known that each company has strategic goals for building long-term plans. Strategic goals encompass all activities of the company, such as defining a goal in economics, goals in talent management, or goals in quality. These goals help companies navigate the process of product release and in the process of building business processes based on the results of decision-making.

The variety of existing factors influencing these goals and the availability of limited resources encourages companies to select priorities and proportionally prioritize their resources to improve the company's performance in a particular area. Hence, the problem of decision-making in the presence of several priorities arises. The need to consider heterogeneous information and decide based on the above factors requires the development of SMOPR for the instrument-making industry.

At present, the mechanisms of work of SMOPR are based on technologies such as "Business intelligence" (BI–systems), which include methods of processing and grouping data, methods of data analysis and analytics, methods of visualization and reporting.

Solving the problem of assessing and improving the quality of instrument-making products includes the following stages:

1. Determination of the scale for the assessment of single indicators of the quality of instrument-making products.
2. Development of a methodology for processing and differential assessment of single indicators of the quality of instrument-making products.
3. Formalization of target functions for assessing the quality of instrument-making products.
4. Approbation of the results obtained.

3 Overview of Bilevel Optimization Methods

Bilevel decision-making methods have been developed to address trade-offs between two interactive decision makers that are distributed throughout a hierarchical organization. In a bilevel decision-making process, decision makers (DMs) at the upper and lower levels are named leader and follower, respectively, and make their individual decisions sequentially with the goal of optimizing their respective goals. The original form of the bilevel process was proposed by Bracken and McGill [4]. A wide range of research in this area has been carried out using the notations of bilevel programming, bilevel optimization, and bilevel decision making. Bilevel decision making is basically a bilevel optimization problem and mainly uses bilevel programming techniques. It appears frequently in many decentralized control problems, and this has prompted a number of researchers to work on bilevel decision models [5], decision approaches [6], and applications [7].

Bilevel optimization problems are a tool for modeling decision-making processes in complex economic, environmental, and financial systems, which have a hierarchical structure. The analysis of such problems does not fit into the framework of the usual optimization theory and requires the development of new mathematical methods and approaches. Despite the external simplicity of the formulations, the solution to these problems is very difficult and a significant part of the research in bilevel programming is reduced to the selection of separate classes of problems that can be solved efficiently.

The development of research into bilevel decision problems can be traced from two sources. The first relates to the field of game theory: when studying oligopolistic market models, Stackelberg used the hierarchy of players to build descriptive decision-making models and establish game-theoretic equilibria. The second source refers to the field of mathematical programming, where bilevel optimization problems have arisen that contain nested internal optimization problems as constraints on the external optimization problem. Since that time, there has been a significant body of literature on bilevel optimization problems, which have been actively studied over the past few decades.

The general formulation of the bilevel optimization problem is formed as follows. We define two vectors of variables $x = [x_1, x_2, ..., x_n]$ and $y = [y_1, y_2, ..., y_n]$ where x- the leader, a y- the follower, wherein $x \in X$, $y \in Y$. For $x \in X \subset R^n$, $y \in Y \subset R^m$,

$F : X \times Y \to R$ и $f : X \times Y \to R$ linear programming is given as follows:

$$\begin{aligned} &\min_{x \in X} F(x, y) = c_1 x + d_1 y \\ &A_1 x + B_1 y \le b_1 \\ &\min_{y \in Y} f(x, y) = c_2 x + d_2 y \\ &A_2 x + B_2 y \le b_2 \end{aligned} \tag{1}$$

where $c_1, c_2 \in R^n$, $d_1, d_2 \in R^m$, $b_1 \in R^p$, $b_2 \in R^q$, $A_1 \in R^{p \times n}$, $B_1 \in R^{p \times m}$, $A_2 \in R^{q \times n}$, $B_2 \in R^{q \times m}$.

The problem of bilevel optimization is solved by finding the optimal point on the polyhedral set S, built from a set of constraints imposed on two levels of optimization. The admissible area for solving the problem of bilevel optimization is defined as follows:

$S = \{(x, y) | x \in X, y \in Y, A_1 x + B_1 y \le b_1, A_2 x + B_2 y \le b_2\}$.

As mentioned above, the follower makes decisions based on the leader's constraints, so the range of feasible decisions of the follower is limited by the set S(x):

$S(x) = \{y \in Y | B_2 y \le b_2 - A_2 x\}$.

The projection S on the decision space of the leader is given as follows:

$S(X) = \{x \in X | \exists y \in Y, A_1 x + B_1 y \le b_1, A_2 x + B_2 y \le b_2\}$.

Hence the rational reaction of the follower to optimize his objective function for $x \in S(X)$:

$P(x) = \{y \in Y | y \in \arg\min[f(x, \hat{y}) | \hat{y} \in S(x)]\}$,

where $\arg\min[f(x, \hat{y} | \hat{y} \in S(x)]$.

Permissible decision area for the leader after the follower's turn:

$IR = \{(x, y) | (x, y) \in S, y \in P(x)\}$.

Among the bilevel optimization methods, one can single out fuzzy bilevel optimization methods, bilevel optimization methods based on artificial intelligence, or meta-heuristic methods of bilevel optimization.

One of the problems of solving problems at two levels is that various uncertainties (including fuzziness and randomness) naturally appear in the decision-making process. Since the problem of uncertainties can arise both in the process of determining the parameters of the model and in the process of making a decision, the information associated with making decisions becomes very inaccurate and ambiguous, especially in this age of big data.

To solve bilevel decision-making problems under uncertainty, fuzzy sets and fuzzy systems were used in terms of both decision modeling and decision approaches formed by fuzzy bilevel decision-making methods.

The difference between the fuzzy method of bilevel optimization and the conventional method lies in the fuzzy definition of the parameters c_i, d_i, A_i, B_i и b_i. The fuzzy bilevel optimization model is given as follows:

$$\begin{aligned} &\min_{x \in X} F(x, y) = \tilde{c}_1 x + d_1 \tilde{y} \\ &\tilde{A}_1 x + \tilde{B}_1 y \le \tilde{b}_1 \\ &\min_{y \in Y} f(x, y) = \tilde{c}_2 x + \tilde{d}_2 y \\ &\tilde{A}_2 x + \tilde{B}_2 y \le \tilde{b}_2 \end{aligned} \tag{2}$$

In the category of meta-heuristics, Mathieu et al. [8] were the first to propose a bilevel programming algorithm based on a genetic algorithm (GABBA) for solving the DLP. Later Hejazi et al. [9] presented a method based on a genetic algorithm (GA). It is determined by comparing the results with the method proposed in [10]. Odugawa and Roy [11] developed a bilevel genetic algorithm, which is an elitist optimization algorithm designed to induce limited asymmetric interaction between two players to solve different types of DLPs in a single framework. Yin [12] has also proposed a GA-based approach to solve two PDLPs: road pricing and reserve capacity of a signal-controlled road network. Wang et al. [13] proposed an evolutionary algorithm for solving the non-linear bilevel programming problem. Recently, Wang et al. [14] developed a GA in which some methods were adopted to ensure that not only the original chromosomes are possible, but the chromosomes generated by the algorithm. Shahin and Kirik [15] presented an algorithm based on simulated annealing.

4 Decentralized Approach to Product Quality Management

Decentralized management refers to the process of distributing management functions to elements that are lower in the hierarchical structure. To avoid negative factors in the application of a decentralized approach, which undoubtedly exist, for example, loss of control over departments, increased time for making strategic decisions, the risk of violating the principles of total quality management in accordance with ISO 9000 and ISO 9001.

To avoid or reduce the risk of problem situations when application of decentralized management, it is necessary to determine the list of management functions or decision-making functions that will be transferred to lower levels of management. This problem is solved by solving the following tasks:

1. Determination of the dimension of the decision-making area, i.e. it is necessary to determine in respect of which elements of production and product quality the decision will be made.
2. Determination of the list of factors influencing product quality and sources of information for quantitative or qualitative registration of these factors.
3. Determination of structural divisions, whose information will be taken into account when making decisions and assessing product quality.
4. Delegation to certain structural divisions of management functions in relation to the collection of information, processing, coordination and intermediate assessment of product quality, for its part.

5 An Example of Applying a Bilevel Optimization Model to Find the Optimal Estimate of Product Quality

To demonstrate the possibility of using the bilevel optimization technique for product quality assessment tasks, we present a specific example on calculation.

Example. Product N consists of five blocks, production includes seven cost categories and requires three suppliers who supply three types of components. The input data for quality indicators are presented in Table 1.

Table 1. Input data.

No. of the block	Block quality indicators				
	x_1	y_2	y_3	z_2	z_3
E1	0.8	0.5	0.39	0.5	0.6
E2	0.6	0.42	0.39	0.48	0.7
E3	0.64	0.25	0.32	0.51	0.25
E4	0.72	0.36	0.15	0.7	0.39
Quality cost group No	Quality cost indicators				
	x_1	y_2	y_3	-	-
E1	0.1	0	0	-	-
E2	0	0.5	0	-	-
E3	0	1	0	-	-
E4	0	0	1	-	-
E5	0	0	0.9	-	-
E6	0.9	0.7	0	-	-
Supplier's No	Supplier's quality indicators				
	x_1	z_2	z_3	-	-
E1	0.3	0.1	0.6	-	-
E2	0.5	0.2	0.4	-	-
E3	1	1	1	-	-

Optimization of the bilevel model with two sequences will be carried out using the Kuhn-Tucker conditions and the simplex method.

The implementation of Kuhn-Tucker conditions eliminates the ambiguous determination problem of the main level (leader) function variables in the acceptable domain of the sublevel (follower), by replacing the problem of finding the optimal follower function with the Kuhn-Tucker conditions. The classical bilevel optimization task, after replacing the follower' task with Kuhn-Tucker conditions, is expressed by the model:

$$\begin{aligned} &\min_{x \in X}\{Q(x, y) = c_1 x + d_1 y\} \\ &A_1 x + B_1 y \leq b_1 \end{aligned} \tag{3}$$

$$A_2x + B_2y \leq b_2 \tag{4}$$

$$uB_2 - v \leq -d_2 \tag{5}$$

$$u(b_2 - A_2x - B_2y) + vy = 0 \tag{6}$$

$$x \geq 0, y \geq 0, u \geq 0, v \geq 0. \tag{7}$$

The analytical model of the bilevel technical product quality assessment' optimization will take the following form:

$$\begin{aligned}
&\min_{x \in X}\{Q(x, y, z) = x_1 - y_2 - y_3 + z_2 + z_3\\
&0.8x_1 - 0.5y_2 - 0.39y_3 + 0.5z_2 + 0.6z_3 \geq 0.22\\
&0.6x_1 - 0.42y_2 - 0.39y_3 + 0.48z_2 + 0.7z_3 \geq 0.19\\
&0.64x_1 - 0.25y_2 - 0.32y_3 + 0.51z_2 + 0.25z_3 \geq 0.17\\
&0.72x_1 - 0.36y_2 - 0.15y_3 + 0.7z_2 + 0.39z_3 \geq 0.26\\
&x_1 + y_2 + y_3 + z_2 + z_3 = 1\\
&\min_{y_i \in Y}\{F_1(x, y) = (1 - x_1) + y_2 + y_3\}\\
&0.1x_1 \leq 0.1\\
&0.5y_2 \leq 0.5\\
&y_2 \leq 1\\
&y_3 \leq 1\\
&0.9y_3 \leq 0.9\\
&0.9x_1 + 0.7y_2 \geq 0.8\\
&\min_{z_i \in Z}\{F_2(x, z) = x_1 + z_2 + z_3\}\\
&0.3x_1 + 0.1z_2 + 0.6z_3 \geq 0.33\\
&0.5x_1 + 0.2z_2 + 0.4z_3 \geq 0.37\\
&x_1 + z_2 + z_3 \leq 1.
\end{aligned}$$

The algorithm for solving the bilevel optimization task consists of 13 steps:

1. Formulation of the task for the bilevel product quality optimization.
2. Transformation of linear constraints for the objective function of the sequence (4) to functions of the form $g(x, y) \geq 0$.
3. Setting Lagrange multipliers for the function and equality (5).
4. Setting the complementary non-rigidity (6).
5. The solution of the task by the simplex method without taking into account the condition (6), and in the case of finding a solution, fixed it as the initial solution $Q(x, y)_0$.

6. Checking the fulfillment of conditions (6) at the initial u_i (where i is the number of the founded Lagrange coefficients' vector). If the conditions are not met, go to step 7.
7. Changing values of previous coefficients u_i.
8. Checking the fulfillment of conditions (5)–(7) with new u_i.
9. If conditions (5)–(7) are not met, repeat steps 7 and 8. If the conditions (5)–(1.4) are met, go to step 10.
10. Changing the inequalities of linear constraints (4) to equality conditions under which coefficients u_i are nonzero.
11. Solving a new task with modified constraints by the simplex method. If $Q(x, y)_i \geq Q(x, y)_{i+1}$ and $Q(x, y)_{i+1} \geq F(x, y)_{i+1}$ (where i is the number of the objective function founded solution), repeat the iteration of steps 7–11, while preserving the values, consider the founded numerical value $Q(x, y)_{i+1}$ as optimal.
12. When $Q(x, y)_i \geq Q(x, y)_{i+1}$ and $Q(x, y)_{i+1} < F(x, y)_{i+1}$, repeat steps 7–11 until the conditions $Q(x, y)_i \geq Q(x, y)_{i+1}$, $Q(x, y)_{i+1} \geq F(x, y)_{i+1}$ are met.
13. When the values are found, repeat the iteration of steps 7–11, while saving the values, consider the founded numerical value $Q(x, y)_2$ as optimal.

Let's proceed to the definition of constraints for linear equations systems. If partial values of the i-th equation are differed from 1, then the average of these values is inserted in the right part with the sign $\geq$. If all partial values of the i-th equation are equal to 1, then the number 1 with the sign $\leq$ is inserted in the right side. If the left part of inequality consists of a single term, then numerical value of the left term is written in the right part the with the sign $\leq$.

The formulated task of product quality assessment is illustrated by Table 2.

Table 2. Example.

Q(x, y, z)	x_1	y_2	y_3	z_2	z_3	*b*
E1	0.8	0.5	0.39	0.5	0,6	0.8
E2	0.6	0.42	0.39	0.48	0.7	0.6
E3	0.64	0.25	0.32	0.51	0.25	0.64
E4	0.72	0.36	0.15	0.7	0.39	0.72
E5	0.8	0.5	0.39	0.5	0.6	0.8
$F_1(x, y)$	x_1	y_2	y_3		-	b'
E1	0.1	0	0	-	-	$\leq$ 0.1
E2	0	0.5	0	-	-	$\leq$ 0.5
E3	0	1	0	-	-	$\leq$ 1.0
E4	0	0	1	-	-	$\leq$ 1.0
E5	0	0	0.9	-	-	$\leq$ 0.9
E6	0.9	0.7	0	-	-	$\geq$ 0.8

(*continued*)

Table 2. (*continued*)

$Q(x, y, z)$	x_1	y_2	y_3	z_2	z_3	b
$F_2(x,z)$	x_1	-	-	z_2	z_3	$b^{//}$
E1	0.3	-	-	0.1	0.6	≥ 0.33
E2	0.5	-	-	0.2	0.4	≥ 0.37
E3	1	-	-	1	1	≤ 1

The numerical partial values of objective functions (quality indicators) are presented in Table 3, numerical values of all the functions are presented in Table 4.

Table 3. Output data.

No. of the block	Block quality indicators				
	x_1	$x_2(y_2)$	$x_2(y_3)$	$x_3(z_2)$	$x_3(z_3)$
E1	0.7048	0.0045	0	0	0.0552
E2	0.5286	0.00378	0	0	0.0644
E3	0.56384	0.00225	0	0	0.023
E4	0.63432	0.00324	0	0	0.03588
Quality cost group No	Quality cost indicators				
	$y_1(x_1)$	y_2	y_3		
E1	0.0881	0	0	-	-
E2	0	0.0045	0	-	-
E3	0	0.009	0	-	-
E4	0	0	0	-	-
E5	0	0	0	-	-
E6	0.7929	0.0063	0	-	-
Supplier's No	Supplier's quality indicators				
	$z_1(x_1)$	z_2	z_3		
E1	0.2643	0	0.0552	-	-
E2	0.4405	0	0.0368	-	-
E3	0.881	0	0.092	-	-

Numerical values of all the functions are presented in Table 4.

Table 4. Numerical values on objective functions.

Designation of objective functions	Numerical value of objective functions
$Q(x, y, z)$	0.881
$F_1(x, y)$	0.119
$F_2(x, z)$	0.776

6 Conclusion

The presented methods of bilevel optimization are a powerful tool for product quality management due to the possibility of their use when taking into account a large amount of heterogeneous information and searching for optimal ways to improve or ensure the quality of products.

Adapting these methods to the tasks of assessing product quality and making subsequent decisions will allow us to develop and expand the theory of product quality management.

The novelty of the improved methods and algorithms of bilevel optimization for the problems of product quality management is as follows:

1. Refinement of existing optimization methods and algorithms in terms of concepts and theory of quality management.
2. Designing the problem of bilevel optimization on the Pareto optimal area, taking into account the cooperation or relationship between followers.
3. Extension of the class of solvable applied problems of bilevel optimization, taking into account heterogeneous information from various sources.
4. Projection of the solution of the problem of bilevel optimization of product quality assessment with one leader and many followers on the Pareto area by using dialogue decision cards.

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References

1. Lasi, H., Fettke, P., Kemper, H.-G., Feld, T., Hoffmann, M.: Industry 4.0. Bus. Inf. Syst. Eng. **6**(4), 239–242 (2014). https://doi.org/10.1007/s12599-014-0334-4
2. Arsovski S (2019). Social oriented quality: from quality 4.0 towards quailty 5.0. In 13th International Quality Conference 13:397–404
3. Sony M, Antony J, Douglas JA (2020) Essential ingredients for the implementation of Quality 4.0: A narrative review of literature and future directions for research. The TQM Journal 32(4):779–793. https://doi.org/10.1108/TQM-12-2019-0275
4. Bracken, J., McGill, J.T.: Mathematical programs with optimization problems in the constraints. Oper. Res. **21**(1), 37–44 (1973)

5. Lu, J., Shi, C., Zhang, G.: On bilevel multi-follower decision making: General framework and solutions. Inf. Sci. **176**(11), 1607–1627 (2006). https://doi.org/10.1016/j.ins.2005.04.010
6. Han, J., Lu, J., Hu, Y., Zhang, G.: Tri-level decision-making with multiple followers: Model, algorithm and case study. Inf. Sci. **311**, 182–204 (2015). https://doi.org/10.1016/j.ins.2015.03.043
7. Kalashnikov, V.V., Dempe, S., Pérez-Valdés, G.A., Kalashnykova, N.I., Camacho-Vallejo, J.F.: Bilevel programming and applications. Math. Probl. Eng. **2015**, 16 (2015). https://doi.org/10.1155/2015/310301
8. Mathieu, R., Pittard, L., Anandalingam, G.: Genetic algorithm based approach to bi-level linear programming. RAIRO-Operations Research-Recherche Opérationnelle **28**(1), 1–21 (1994)
9. Hejazi, S.R., Memariani, A., Jahanshahloo, G., Sepehri, M.M.: Linear bilevel programming solution by genetic algorithm. Comput. Oper. Res. **29**(13), 191–1925 (2002). https://doi.org/10.1016/S0305-0548(01)00066-1
10. Gendreau, M., Marcotte, P., Savard, G.: A hybrid tabu-ascent algorithm for the linear bilevel programming problem. J. Global Optim. **8**(3), 217–233 (1996). https://doi.org/10.1007/BF00121266
11. Oduguwa V, Roy R (2002) Bi-level optimisation using genetic algorithm. In Proceedings 2002 IEEE International Conference on Artificial Intelligence Systems (ICAIS 2002), pp. 322–327. IEEE. https://doi.org/10.1109/ICAIS.2002.1048121
12. Yin, Y.: Genetic-algorithms-based approach for bilevel programming models. J. Transp. Eng. **126**(2), 115–120 (2000). https://doi.org/10.1061/(ASCE)0733-947X(2000)126:2(115)
13. Wang Y, Jiao YC, Li H (2005) An evolutionary algorithm for solving nonlinear bilevel programming based on a new constraint-handling scheme. IEEE Transactions on Systems, Man, and Cybernetics, Part C (Applications and Reviews) 35(2):221–232. https://doi.org/10.1109/TSMCC.2004.841908
14. Guang-Min W, Zhong-Ping W, Xian-Jia W, Ya-Lin C (2005) Genetic algorithms for solving linear bilevel programming. In Sixth International Conference on Parallel and Distributed Computing Applications and Technologies (PDCAT'05), pp. 920–924. IEEE. https://doi.org/10.1109/PDCAT.2005.145
15. Sahin, K.H., Ciric, A.R.: A dual temperature simulated annealing approach for solving bilevel programming problems. Comput. Chem. Eng. **23**(1), 11–25 (1998). https://doi.org/10.1016/S0098-1354(98)00267-1