Determination of Optimum Main Design Parameters of a Two-Stage Helical Gearbox for Minimum Gearbox Cross-Section Area

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Abstract. The purpose of this research is to determine the optimum main design parameters for minimizing the cross-section area of a two-stage helical gearbox. In this work, five main design parameters of the gearbox were taken into account to find their optimum values, including the gear ratio of the first stage, the coefficient of wheel face width of stages 1 and 2, and the allowable contact stress of stages 1 and 2. A simulation experiment was designed and implemented by a computer program to achieve the goal. Furthermore, the Minitab R19 software was used to analyze the experimental results. The impact of major design factors on cross-section area was assessed. The optimum values for these parameters were also suggested.

Keywords: Helical gearbox · Main design parameters · Optimum gearbox design · Across-section area

1 Introduction

The most important component of a mechanical drive system is the gearbox. Its purpose is to reduce the speed and torque transmitted from the motor shaft to the working machine shaft. It is widely used in practice due to its simple structure, stable operation, reliability, and low cost in comparison to electric, hydraulic, or pneumatic reduction systems. As a result, the optimal design of the gearbox is a constant concern.

Until now, the optimal design of the gearbox has been carried out on a variety of subjects. The created model is used in [1] to perform vibration and noise analysis on the gearbox housing. The authors in [2] introduced an investigation into the potential for improving energy efficiency and lowering lifecycle costs of electric city buses with multispeed gearboxes. A two-speed dual clutch gearbox and a continuously variable transmission were investigated and compared to a reference fixed gear ratio powertrain in this study. In [3] investigates the optimization of gearbox geometric design parameters

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to minimize rattle noise in an automotive transmission using an empirical model approach. The selection of the coefficient of wheel face width for multi-speed helical gear transmissions was introduced in [4]. However, the coefficient values are not optimal; they were only reasonably chosen based on some advice. In [5], the modal characteristic of gearbox housing with applied load was investigated. The problem of determining optimum gear ratios has received considerable attention. It was done for helical gearboxes with two [6, 7], three [8, 9] or four [10] stages. Although there have been numerous studies on optimizing gearbox parameters, no research has been conducted to identify the optimal design parameters simultaneously.

This paper introduces a study to determine the optimum main design parameters for a two-stage helical gearbox. The objective function of the optimization problem is the minimum gearbox cross-section. To do that, a simulation experiment with the application of the Taguchi method and the Minitab R19 software was conducted. The influence of main design parameters on the objective function was evaluated. Optimal values of main design parameters have been proposed.

2 Methodology

2.1 Determining Gearbox Cross-Section Area

The cross-section area of the gearbox A_{gb} can be calculated by (Fig. 1):

$$A_{gb} = L \cdot B_1 \tag{1}$$

In which, L, and B_1 can be calculated by:

$$L = \frac{d_{w11}}{2} + a_{w1} + a_{w2} + \frac{d_{w22}}{2} + 20 \tag{2}$$

$$B_1 = b_{w1} + 4 \cdot S_G \tag{3}$$

$$S_G = 0.005 \cdot L' + 4.5 \quad [11] \tag{4}$$

In Eqs. (2) and (3), b_{w1} is the gear width of the first stage; d_{w12}, d_{w21}, and d_{w22} are the pitch diameters of the pinion and the gear sets which can be determined by [4]:

$$d_{w21} = 2 \cdot a_{w1} \cdot u_1/(u_1 + 1) \tag{5}$$

$$d_{w12} = 2 \cdot a_{w2}/(u_2 + 1) \tag{6}$$

$$d_{w22} = 2 \cdot a_{w2} \cdot u_2/(u_2 + 1) \tag{7}$$

$$b_{w1} = X_{ba1} \cdot a_{w1} \tag{8}$$

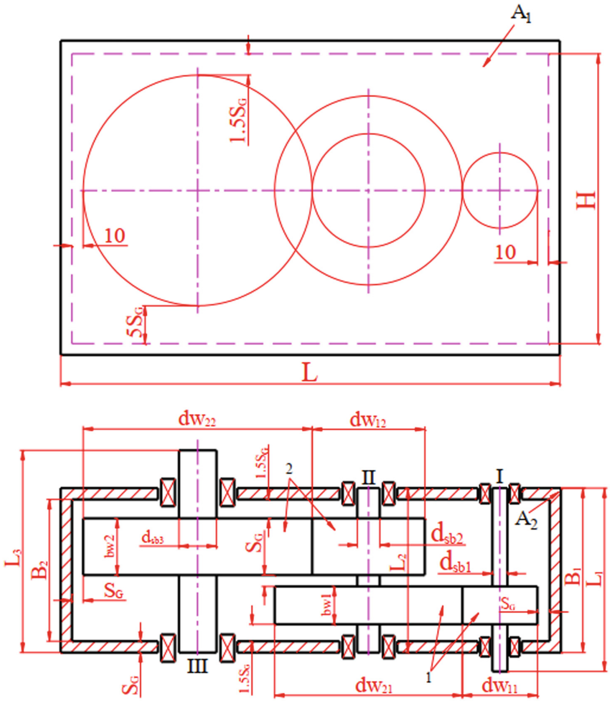

Fig. 1. Schema for determining cross-section gearbox area

In Eqs. (5)–(8), u_1 and u_2 are the gear ratios of the first and the second stages; $u_2 = u_g/u_1$; with u_g is gearbox ratio; a_{w1} and a_{w2} are the center distances of the first and the second stages which are calculated by [4]:

$$a_{w1} = k_a \cdot (u_1 + 1) \cdot \sqrt[3]{T_{11} \cdot k_{H\beta}/\left(\mathrm{AS}_1^2 \cdot u_1 \cdot X_{ba}\right)} \tag{9}$$

$$a_{w2} = k_a \cdot (u_2 + 1) \cdot \sqrt[3]{T_{12} \cdot k_{H\beta 2}/\left(\mathrm{AS}_2^2 \cdot u_2 \cdot X_{ba2}\right)} \tag{10}$$

where, $k_{H\beta}$ is the contacting load ratio for pitting resistance; $k_{H\beta} = 1.05 \div 1.27$ [4] and it was chosen as $k_{H\beta} = 1.16$; AS_1 and AS_2 is the allowable contact stress of the first and the second stages (MPa); k_a is the material coefficient; As the gear material is steel, k_a= 43 [4]; X_{ba1} and X_{ba2} are the wheel face width coefficients of the first and the second stages; T_{11} and T_{12} are the torques on the pinions of the first and the second stages (Nmm):

$$T_{11} = T_{out}/\left(u_g \cdot \eta_{hg}^2 \cdot \eta_b^3\right) \tag{11}$$

$$T_{12} = T_{out}/\left(u_2 \cdot \eta_{hg} \cdot \eta_{be}^2\right) \tag{12}$$

wherein, T_{out} is the system output torque (N.mm); η_{hg} is the efficiency of a helical gear unit ($\eta_{hg} = 0.96 \div 0.98$ [4]; η_b is the efficiency of a rolling bearing pair ($\eta h = 0.99 \div 0.995$ [4]).

2.2 Optimization Problem

Based on the preceding analysis, the optimization problem can be defined as follows:

$$\text{Minimize } A_{gb} \tag{13}$$

With

$$A_{gb} = f(u_1; X_{ba1}; X_{ba2}; AS_1; AS_2) \tag{14}$$

And with the following constraints:

$$1 \leq u_1 \leq 9; \quad 1 \leq u_2 \leq 9 \tag{15}$$

3 Simulation Experiment

A simulation experiment was carried out to investigate the influence of main design paremeters on the objective function (across-section area). Besides, the Minitab R19 software and the Taguchi method were applied for experimental design and analysis the results. In addition, five main design factors including u1, Xba1, Xba2, AS1, and AS2 were investigated. Table 1 describes these parameters and their level.

For the convenience of the survey, the influence of these parameters on the across-section area was evaluated with each value of the gearbox ratios: 5, 10, 15, 20, 25 and 30. For each above value of the gearbox ratio, 25 test runs for the simulation experiment were performed because the 5-level design type of Taguchi design (L25) was selected. Table 2 presents the experimental plan and the output response (the gearbox cross-section area) when the gearbox ratio is 5.

Table 1. Main design parameters and their levels

Factor	Level				
	1	2	3	4	5
Total gearbox ratio u_1	1	3	5	7	9
Coefficient of wheel face width of stage 1 X_{ba1}	0.25	0.27	0.29	0.31	0.33
Coefficient of wheel face width of stage 2 X_{ba2}	0.32	0.34	0.36	0.38	0.4
Allowable contact stress of stage 1 AS_1 (Mpa)	350	368	386	404	420
Allowable contact stress of stage 2 AS_2 (Mpa)	350	368	386	404	420

Table 2. Experimental plan and output response (A_{gb}) when $u_{gb} = 5$

Trial	u_1	X_{ba1}	X_{ba2}	AS_1	AS_2	A_{gb} (mm^2)
1	1.8	0.25	0.32	350	350	70885.64
2	1.8	0.27	0.34	368	368	64257.65
3	1.8	0.29	0.36	386	386	58581.72
4	1.8	0.31	0.38	404	404	53680.36
5	1.8	0.33	0.4	420	420	49692.26
6	1.9	0.25	0.34	386	404	57562
…						
24	2.2	0.31	0.36	368	350	60129.7
25	2.2	0.33	0.38	386	368	54857.51

4 Results Discussion

The Analysis of Variance (ANOVA) method is used with Minitab R19 software to evaluate the influence of the main design parameters when $u_{gb} = 5$ on A_{gb}. The signal-to-noise ratio, or S/N number, must be calculated for each experiment to determine the effect of each parameter on the output. The calculated S/N number will determine the best parameters and which parameter has the most influence on the outcome. The following S/N ratio Eq. (16) should be used to minimize performance characteristics:

$$S/N = -10\log_{10}\left(\frac{1}{n}\sum_{i=1}^{n} y_i^2\right) \tag{16}$$

After calculating the SN ratio for each experiment, the average SN ratio for each parameter and level is calculated. The effect of main design factors on A_{gb} was described in Table 3 and Fig. 2. From Fig. 2 it is easy to see that A_{gb} is proportional to all five main design parameters. Besides, Table 3 shows that AS_2 has the greatest influence on A_{gb} (59.19%); followed by X_{ba2} (22.46 percent) and u_1 (10.86%). AS_1 and X_{ba1} have little effect on A_{gb} (4.62% and 2.81%). Table 4 shows the order of the main parameters' influence on gearbox across-section area.

To obtain the smallest gearbox cross-section area using the objective function in Eq. (13), the S/N value is maximized for each main design parameter. As a result of the analysis of the effect of the parameters on the S/N ratio in the chart in Fig. 3, the optimum main design parameters were discovered as follows: $u_1 = 2.22$; $X_{ba1} = 0.33$; Xba2 = 0.4; $AS_1 = 420$; $AS_2 = 420$.

For other values of u_{gb} including 10, 15, 20, 25 and 30, proceed in the same way as above. Table 5 shows the optimal values of main design parameters. From Table 5, the following observations were reported:

Table 3. Analysis of variance for means

Analysis of variance for means							
Source	DF	Seq SS	Adj SS	Adj MS	F	P	C (%)
u1	4	56879084	56879084	14219771	208.26	0	10.86
Xba1	4	14732884	14732884	3683221	53.94	0.001	2.81
Xba2	4	117683210	117683210	29420803	430.9	0	22.46
AS1	4	24220444	24220444	6055111	88.68	0	4.62
AS2	4	310092572	310092572	77523143	1135.41	0	59.19
Residual error	4	273111	273111	68278			0.05
Total	24	523881305					
Model summary							
S	*R-Sq*	*R-Sq(adj)*					
261.3001	99.95%	99.69%					

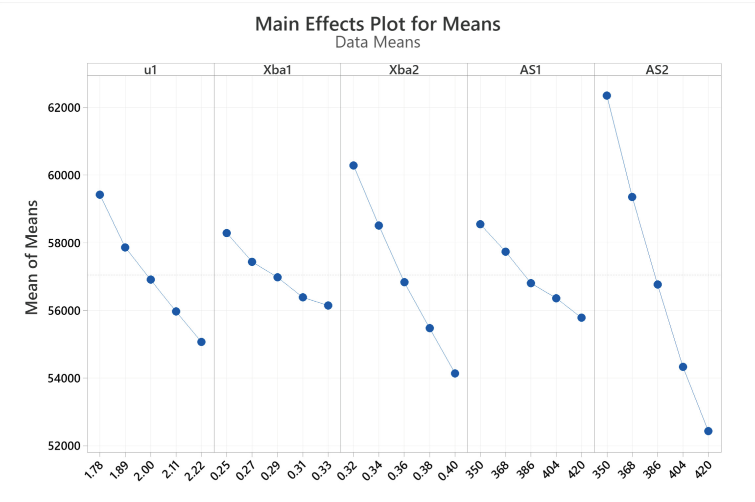

Fig. 2. Effect of main design factors on A_{gb}

- Both X_{ba1} and X_{ba2} optimal values are their maximum values. The reason for this is that in order to have a small gearbox cross-section, the coefficients Xba1 and Xba2 must be as large as possible in order to reduce the center distance of the gear sets, resulting in a smaller size L (Eq. (2) and A_{gb} (formula (1)).
- The optimal values of AS_1 and AS_2 also take their maximum values. The reason is the same as above, in order to have the minimum gearbox cross-section, the AS_1 and AS_2 values need to be taken as large as possible to reduce the axial distance of the gear transmissions, leading to a reduction in the size L (formula (2)) and Agb ((formula (1)).

Table 4. The order of influence of the parameters on gearbox across-section area

Response table for means					
Level	u1	Xba1	Xba2	AS1	AS2
1	59420	58287	60281	58545	62340
2	57863	57434	58505	57735	59353
3	56905	56979	56829	56806	56765
4	55971	56387	55472	56362	54335
5	55072	56142	54144	55782	52438
Delta	4348	2145	6137	2763	9903
Rank	3	5	2	4	1

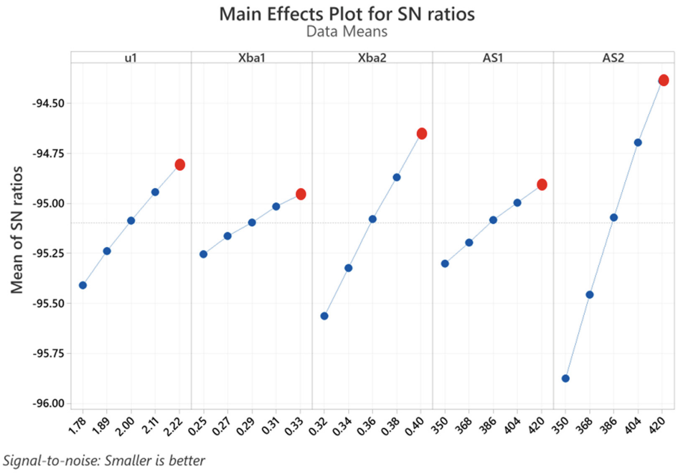

Fig. 3. Main effects plot for S/N ratios

Table 5. Optimum values of main design parameters

No	u_{gb}					
	5	10	15	20	25	30
u_1	2.22	3.72	5.22	6.19	7.5	8.56
X_{ba1}	0.33	0.33	0.33	0.33	0.33	0.33
X_{ba2}	0.4	0.4	0.4	0.4	0.4	0.4
AS_1	420	420	420	420	420	420
AS_2	420	420	420	420	420	420

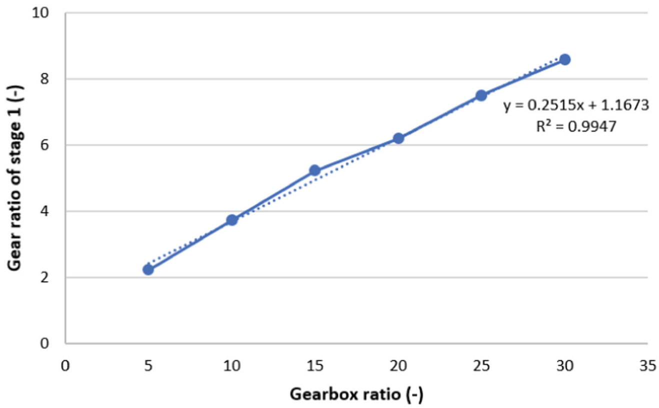

Fig. 4. Gear ratio of the first stage versus gearbox ratio

– The gear ratios of the first stage and the gearbox have a first-order relationship (Fig. 4). From this relationship, the following regression formula (with $R^2 = 0.9947$) is proposed to determine the optimum values of u_1:

$$u_1 = 0.2515 \cdot u_{gb} + 1.1673 \tag{17}$$

After having u_1, the optimum values of u_2 can be easily found by $u_2 = u_{gb}/u_1$.

5 Conclusions

The results of a study on optimization of a two-stage helical gearbox to get the smallest gearbox cross-section are presented in this paper. In this study, the gear ratio of the first stage, the coefficient of wheel face width of stages 1 and 2, and the allowable contact stress of stages 1 and 2 were the five main design parameters of the gearbox that were optimized. A simulation experiment with Taguchi L25 type was designed and carried out to solve this work. The effect of main design factors on gearbox cross-section area was also investigated. Furthermore, the optimum values of the main gearbox parameters and a regression model to calculate the optimum gear ratios of the first stage have been proposed as follows: $X_{ba1} = 0.33$; $X_{ba2} = 0.4$; $AS_1 = 420$ (MPa); $AS_2 = 420$ (MPa); u_1 is calculated by Eq. (17).

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