Chapter 9
Unsteady Longitudinal Mechanodiffusion Vibrations of a Rectangular Plate with Inner Diffusion Flux Relaxation

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Abstract We consider the unsteady problem of elastic diffusion deformations of a rectangular orthotropic plate considering the diffusion fluxes relaxation. External perturbations lay in the plate plane and allow us to use a two-dimensional elastic diffusion model for continuum as a mathematical model. The solution is convolutions of Green's functions with functions defining the boundary perturbations. Green's functions finding method is based on the Laplace transform and double trigonometric Fourier series. We are using residues and tables of operational calculus for transition to the originals of Green's functions. The interaction effects of mechanical and diffusion fields for a three-component material are calculated using the example of a rectangular plate under tensile forces. We have also investigated the influence of relaxation processes on mass transfer kinetics. Calculation results are in analytical and graphical forms.

Keywords Elastic diffusion · Unsteady problems · Coupled problem · Multicomponent continuum · Green's functions

9.1 Introduction

Issues related to the strength study include static/dynamic materials tests and consideration of different physical fields interaction. This is due to various modern structures and different conditions of their operation. It is often necessary to con-

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sider the interaction between mechanical, diffusion, temperature and other fields in applied calculations.

Experimental study of the interaction between mechanical and diffusion fields began in the 20s of the twentieth century. Many scientific theories have been formed that allow describing the coupled mechanical, diffusion and other fields in a solid medium. Among the most recent works are [1–16]. Various aspects of mechanodiffusion processes modelling are considered here: beginning with initial–boundary problem formulation and ending with a description of methods for solving these problems. Here are stationary [4, 11, 12] and unsteady problems [1–3, 5–8, 13–16].

The main complexity is the Laplace inverse transform when solving mechanodiffusion problems analytically. It is often used in solving initial–boundary problems. The Durbin method [2, 3, 7, 8, 14] or similar algorithms based on the fact that the Mellin integral is expressed through the inverse Fourier transform are used here. Special quadrature formulas [1, 5, 15] allow to calculate the integral. These methods have proven themselves in calculating the originals in a certain class of functions. However, these algorithms are not suitable for Green's functions because they belong to the generalized function class, and it is difficult to use numerical integration methods in this case.

Numerical algorithms based on finite difference methods [6, 13] and the finite element methods [16] are an alternative to analytical methods. In addition, numerical methods are the only way to solve the boundary value problem in some cases. A disadvantage of numerical methods is that solution comes to a discrete set of values, which is difficult to analyse and investigate later. A rather complicated mathematical problem is the question of algorithm stability and convergence.

This article proposes an analytical method for solving the unsteady mechanodiffusion problem for a rectangular plate. The method implies the Laplace transform and Fourier series expansion in eigenfunctions of the elastic diffusion operator. This approach has been tested for one-dimensional problems of elastic diffusion and thermoelastic diffusion [17, 18].

9.2 Problem Formulation

We considered a multicomponent rectangular plate under the action of unsteady longitudinal forces. The resulting mass transfer and diffusion flux relaxations are taken into account inside the plate.

The mathematical model describing two-dimensional elastic diffusion processes for a homogeneous orthotropic medium in the rectangular Cartesian coordinate system [19, 20]:

$$\frac{\partial^2 u_1}{\partial x_1^2} + C_{66}\frac{\partial^2 u_1}{\partial x_2^2} + C_0\frac{\partial^2 u_2}{\partial x_1 \partial x_2} = \ddot{u}_1 + \sum_{j=1}^{N}\alpha_1^j\frac{\partial \eta_j}{\partial x_1},$$
$$C_0\frac{\partial^2 u_1}{\partial x_2 \partial x_1} + C_{66}\frac{\partial^2 u_2}{\partial x_1^2} + C_{22}\frac{\partial^2 u_2}{\partial x_2^2} = \ddot{u}_2 + \sum_{j=1}^{N}\alpha_2^j\frac{\partial \eta_j}{\partial x_2},$$

$$D_1^q\frac{\partial^2 \eta_q}{\partial x_1^2} + D_2^q\frac{\partial^2 \eta_q}{\partial x_2^2} = \dot{\eta}_q + \tau_q\ddot{\eta}_q + \Lambda_{11}^q\frac{\partial^3 u_1}{\partial x_1^3} + \Lambda_{21}^q\frac{\partial^3 u_1}{\partial x_2^2 \partial x_1} +$$
$$+\Lambda_{12}^q\frac{\partial^3 u_2}{\partial x_1^2 \partial x_2} + \Lambda_{22}^q\frac{\partial^3 u_2}{\partial x_2^3}, \quad \eta_{N+1} = -\sum_{j=1}^{N}\eta_j \ \left(q = \overline{1, N}\right); \tag{9.1}$$

$$u_2|_{x_1=0} = f_{111}(x_2, \tau), \quad \eta_q\big|_{x_1=0} = f_{q+2,11}(x_2, \tau),$$
$$u_1|_{x_2=0} = f_{121}(x_1, \tau), \quad \eta_q\big|_{x_2=0} = f_{q+2,21}(x_1, \tau),$$
$$\left.\left(\frac{\partial u_1}{\partial x_1} + C_{12}\frac{\partial u_2}{\partial x_2} - \sum_{j=1}^{N}\alpha_1^j\eta_j\right)\right|_{x_1=0} = f_{211}(x_2, \tau), \tag{9.2}$$
$$\left.\left(C_{12}\frac{\partial u_1}{\partial x_1} + C_{22}\frac{\partial u_2}{\partial x_2} - \sum_{j=1}^{N}\alpha_2^j\eta_j\right)\right|_{x_2=0} = f_{221}(x_1, \tau),$$

$$u_2|_{x_1=l_1} = f_{112}(x_2, \tau), \quad \eta_q\big|_{x_1=l_1} = f_{q+2,12}(x_2, \tau),$$
$$u_1|_{x_2=l_2} = f_{122}(x_1, \tau), \quad \eta_q\big|_{x_2=l_2} = f_{q+2,22}(x_1, \tau),$$
$$\left.\left(\frac{\partial u_1}{\partial x_1} + C_{12}\frac{\partial u_2}{\partial x_2} - \sum_{j=1}^{N}\alpha_1^j\eta_j\right)\right|_{x_1=l_1} = f_{212}(x_2, \tau),$$
$$\left.\left(C_{12}\frac{\partial u_1}{\partial x_1} + C_{22}\frac{\partial u_2}{\partial x_2} - \sum_{j=1}^{N}\alpha_2^j\eta_j\right)\right|_{x_2=l_2} = f_{222}(x_1, \tau).$$

Here, all quantities are dimensionless. Their relation to their dimensional counterparts (when written the same way, they are indicated with a dash) is defined as follows:

$$x_i = \frac{x_i'}{L},\ u_i = \frac{u_i'}{L},\ \tau = \frac{Ct}{L},\ C_{ij} = \frac{C_{ij}'}{C_{11}'},\ C^2 = \frac{C_{11}'}{\rho},\ \alpha_\alpha^q = \frac{\alpha_{\alpha\alpha}^{(q)}}{C_{11}'},$$
$$D_\alpha^q = \frac{D_{\alpha\alpha}^{(q)}}{CL},\ \Lambda_{\alpha\beta}^q = \frac{m^{(q)}D_{\alpha\alpha}^{(qq)}\alpha_{\beta\beta}^{(q)}n_0^{(q)}}{\rho RT_0CL},\ l_i = \frac{l_i^*}{L},\ \tau_q = \frac{C\tau^{(q)}}{L}, \tag{9.3}$$

$$C_0 = C_{66} + C_{12},\ C_{11}' = C_{1111},\ C_{22}' = C_{2222},\ C_{66}' = C_{1212},\ C_{12}' = C_{1122},$$
$$f_{1kl}(x_1, \tau) = \frac{f_{1kl}^*(x_1, t)}{L},\ f_{q+2,kl}(x_1, \tau) = f_{q+2,kl}^*(x_1, t),\ f_{2kl}(x_1, \tau) = \frac{f_{2kl}^*(x_1, \tau)}{C_{1111}},$$

where t is a time parametr; x_i^* is a rectangular Cartesian coordinate; u_i^* is a displacement vector component; L is the diagonal size of the plate having dimensions $l_1^* \times l_2^*$; η_q is the concentration increment of "q"th component in $N+1$-component medium; $n_0^{(q)}$ is the initial concentration of "q"th component; C_{ijkl} is the component of elastic constants tensor; ρ is a mass density; $\alpha_{ij}^{(q)}$ is a coefficient characterizing the volume change of the medium due to diffusion; $D_{ij}^{(q)}$ is a diffusion coefficient; R is the universal gas constant; T_0 is an average temperature; $m^{(q)}$ is a molar mass of "q"th component, f_{klm}^* is an external perturbation; $\tau^{(q)}$ is the relaxation time of the diffusion flux.

We assume the initial conditions are zero.

9.3 Integral Representation of the Solution

The solution to the problem is sought in the integral form as follows:

$$\begin{gathered}
\begin{Bmatrix} u_1(x_1,x_2,\tau) \\ u_2(x_1,x_2,\tau) \\ \eta_q(x_1,x_2,\tau) \end{Bmatrix} = \sum_{k=1}^{N+2} \int_0^\tau \int_0^{l_2} \begin{Bmatrix} G_{1k1}(x_1,x_2,\zeta,t) \\ G_{2k1}(x_1,x_2,\zeta,t) \\ G_{q+2,k1}(x_1,x_2,\zeta,t) \end{Bmatrix} f_{k11}(\zeta,\tau-t)\,d\zeta\,dt + \\
+\sum_{k=1}^{N+2} \int_0^\tau \int_0^{l_2} \begin{Bmatrix} -G_{1k1}(l_1-x_1,x_2,\zeta,t) \\ G_{2k1}(l_1-x_1,x_2,\zeta,t) \\ G_{q+2,k1}(l_1-x_1,x_2,\zeta,t) \end{Bmatrix} f_{k12}(\zeta,\tau-t)\,d\zeta\,dt + \\
+\sum_{k=1}^{N+2} \int_0^\tau \int_0^{l_1} \begin{Bmatrix} G_{1k2}(x_1,x_2,\xi,t) \\ G_{2k2}(x_1,x_2,\xi,t) \\ G_{q+2,k2}(x_1,x_2,\xi,t) \end{Bmatrix} f_{k21}(\xi,\tau-t)\,d\xi\,dt + \\
+\sum_{k=1}^{N+2} \int_0^\tau \int_0^{l_1} \begin{Bmatrix} G_{1k2}(x_1,l_2-x_2,\xi,t) \\ -G_{2k2}(x_1,l_2-x_2,\xi,t) \\ G_{q+2,k2}(x_1,l_2-x_2,\xi,t) \end{Bmatrix} f_{k22}(\xi,\tau-t)\,d\xi\,dt,
\end{gathered} \tag{9.4}$$

where G_{mkl} are Green's functions, which satisfy the following initial–boundary value problem (initial conditions are zero):

$$\begin{gathered}
\frac{\partial^2 G_{1kl}}{\partial x_1^2} + C_{66}\frac{\partial^2 G_{1kl}}{\partial x_2^2} + C_0\frac{\partial^2 G_{2kl}}{\partial x_1 \partial x_2} = \ddot{G}_{1kl} + \sum_{j=1}^{N} \alpha_1^j \frac{\partial G_{j+2,kl}}{\partial x_1}, \\
C_0\frac{\partial^2 G_{1kl}}{\partial x_2 \partial x_1} + C_{66}\frac{\partial^2 G_{2kl}}{\partial x_1^2} + C_{22}\frac{\partial^2 G_{2kl}}{\partial x_2^2} = \ddot{G}_{2kl} + \sum_{j=1}^{N} \alpha_2^j \frac{\partial G_{j+2,kl}}{\partial x_2}, \\
D_1^q \frac{\partial^2 G_{q+2,kl}}{\partial x_1^2} + D_2^q \frac{\partial^2 G_{q+2,kl}}{\partial x_2^2} = \dot{G}_{q+2,kl} + \tau_q \ddot{G}_{q+2,kl} + \Lambda_{11}^q \frac{\partial^3 G_{1kl}}{\partial x_1^3} + \\
+\Lambda_{21}^q \frac{\partial^3 G_{1kl}}{\partial x_2^2 \partial x_1} + \Lambda_{12}^q \frac{\partial^3 G_{2kl}}{\partial x_1^2 \partial x_2} + \Lambda_{22}^q \frac{\partial^3 G_{2kl}}{\partial x_2^3} \;\left(q = \overline{1,N}\right);
\end{gathered} \tag{9.5}$$

$$\left.\left(\frac{\partial G_{1kl}}{\partial x_1}+C_{12}\frac{\partial G_{2kl}}{\partial x_2}-\sum_{j=1}^{N}\alpha_1^j G_{j+2,kl}\right)\right|_{x_1=0}=\delta_{1k}\delta_{1l}\delta(x_2-\zeta)\delta(\tau),$$
$$\left.\left(C_{12}\frac{\partial G_{1kl}}{\partial x_1}+C_{22}\frac{\partial G_{2kl}}{\partial x_2}-\sum_{j=1}^{N}\alpha_2^j G_{j+2,kl}\right)\right|_{x_2=0}=\delta_{2k}\delta_{2l}\delta(x_1-\xi)\delta(\tau),$$
$$\left.\left(\frac{\partial G_{1kl}}{\partial x_1}+C_{12}\frac{\partial G_{2kl}}{\partial x_2}-\sum_{j=1}^{N}\alpha_1^j G_{j+2,kl}\right)\right|_{x_1=l_1}=0, \tag{9.6}$$
$$\left.\left(C_{12}\frac{\partial G_{1kl}}{\partial x_1}+C_{22}\frac{\partial G_{2kl}}{\partial x_2}-\sum_{j=1}^{N}\alpha_2^j G_{j+2,kl}\right)\right|_{x_2=l_2}=0,$$
$$G_{2kl}|_{x_1=0}=\delta_{2k}\delta_{1l}\delta(x_2-\zeta)\delta(\tau),\quad G_{q+2,kl}\big|_{x_1=0}=\delta_{q+2,k}\delta_{1l}\delta(x_2-\zeta)\delta(\tau),$$
$$G_{1kl}|_{x_2=0}=\delta_{1k}\delta_{2l}\delta(x_1-\xi)\delta(\tau),\quad G_{q+2,kl}\big|_{x_2=0}=\delta_{q+2,k}\delta_{2l}\delta(x_1-\xi)\delta(\tau),$$
$$G_{2kl}|_{x_1=l_1}=0,\quad G_{q+2,kl}\big|_{x_1=l_1}=0,\quad G_{1kl}|_{x_2=l_2}=0,\quad G_{q+2,kl}\big|_{x_2=l_2}=0.$$

9.4 Algorithm for Green's Functions

To find the Green's functions, we apply the Laplace transform to (9.5) and (9.6) and then represent the unknown functions as a double Fourier series. Let's multiply the first equation in (9.5) by $\cos\mu_m x_1 \sin\lambda_n x_2$, the second by $\sin\mu_m x_1 \cos\lambda_n x_2$, and the rest by $\sin\lambda_n x_1 \sin\mu_n x_2$ ($\mu_m=\pi m/l_1$, $\lambda_n=\pi n/l_2$). Then we integrate into the rectangle $[0, l_1]\times[0, l_2]$ and obtain the following system of linear algebraic equations:

$$\begin{gathered}
k_{1mn}(s)\,G^L_{1klmn}(\xi,\zeta,s)+C_0\lambda_n\mu_m G^L_{2klmn}(\xi,\zeta,s)+\\
+\mu_m\sum_{j=1}^{N}\alpha_1^j G^L_{j+2,klmn}(\xi,\zeta,s)=F_{1mn}(\xi,\zeta,s),\\
C_0\mu_m\lambda_n G^L_{1klmn}(\xi,\zeta,s)+k_{2mn}(s)\,G^L_{2klmn}(\xi,\zeta,s)+\\
+\lambda_n\sum_{j=1}^{N}\alpha_2^j G^L_{j+2,klmn}(\xi,\zeta,s)=F_{2mn}(\xi,\zeta,s),\\
-\mu_m K_{qmn}G^L_{1klmn}(\xi,\zeta,s)-\lambda_n M_{qmn}G^L_{2klmn}(\xi,\zeta,s)+\\
+k_{q+2,mn}(s)\,G^L_{q+2,klmn}(\xi,\zeta,s)=F_{q+2,mn}(\xi,\zeta,s);
\end{gathered} \tag{9.7}$$

$$F_{1mn}(\xi,\zeta,s)=\frac{4}{l_1l_2}\left[C_{66}\lambda_n\delta_{1k}\left(\delta_{1l}\cos\lambda_n\zeta+\delta_{2l}\cos\mu_m\xi\right)-\delta_{2k}\delta_{1l}\sin\lambda_n\zeta\right],$$
$$F_{2mn}(\xi,\zeta,s)=\frac{4}{l_1l_2}\left[\mu_mC_{66}\delta_{1k}\left(\delta_{1l}\cos\lambda_n\zeta+\delta_{2l}\cos\mu_m\xi\right)-\delta_{2k}\delta_{2l}\sin\mu_m\xi\right],$$
$$F_{q+2,mn}(\xi,\zeta,s)=\frac{4}{l_1l_2}\lambda_n\mu_m\delta_{1k}\left(\delta_{2l}\Lambda_{2q}\cos\mu_m\xi+\delta_{1l}\Lambda_{1q}\cos\lambda_n\zeta\right)+$$
$$+\frac{4}{l_1l_2}\mu_m\delta_{1l}\left(D_1^q\delta_{q+2,k}-\Lambda_{11}^q\sum_{j=1}^N\alpha_1^j\delta_{j+2,k}-\Lambda_{11}^q\delta_{2k}\right)\sin\lambda_n\zeta+$$
$$+\frac{4}{l_1l_2}\lambda_n\delta_{2l}\left(D_2^q\delta_{q+2,k}-\frac{\Lambda_{22}^q}{C_{22}}\sum_{j=1}^N\alpha_2^j\delta_{j+2,k}-\frac{\Lambda_{22}^q}{C_{22}}\delta_{2k}\right)\sin\mu_m\xi,$$
$$k_{1mn}(s)=s^2+\mu_m^2+C_{66}\lambda_n^2,\ \ k_{2mn}(s)=s^2+C_{66}\mu_m^2+C_{22}\lambda_n^2,$$
$$k_{q+2,mn}(s)=s+\tau_qs^2+D_1^q\mu_m^2+D_2^q\lambda_n^2,\ \ \Lambda_{1q}=\Lambda_{12}^q-\Lambda_{11}^qC_{12},$$
$$\Lambda_{2q}=\Lambda_{21}^q-\frac{\Lambda_{22}^q}{C_{22}}C_{12},\ \ K_{qmn}=\Lambda_{11}^q\mu_m^2+\Lambda_{21}^q\lambda_n^2,\ \ M_{qmn}=\Lambda_{12}^q\mu_m^2+\Lambda_{22}^q\lambda_n^2; \tag{9.8}$$

$$G_{rkl}^L(x_1,x_2,\xi,\zeta,s)=\int_0^\infty G_{rkl}(x_1,x_2,\xi,\zeta,\tau)\,e^{-s\tau}d\tau,$$

$$G_{1kl}^L(x_1,x_2,\xi,\zeta,s)=\sum_{m=0}^\infty\sum_{n=1}^\infty G_{1klmn}^L(\xi,\zeta,s)\cos\mu_mx_1\sin\lambda_nx_2,$$
$$G_{2kl}^L(x_1,x_2,\xi,\zeta,s)=\sum_{m=1}^\infty\sum_{n=0}^\infty G_{2klmn}^L(\xi,\zeta,s)\sin\mu_mx_1\cos\lambda_nx_2, \tag{9.9}$$
$$G_{q+2,kl}^L(x_1,x_2,\xi,\zeta,s)=\sum_{m=1}^\infty\sum_{n=1}^\infty G_{q+2,klmn}^L(\xi,\zeta,s)\sin\mu_mx_1\sin\lambda_nx_2,$$

$$G_{1klmn}^L(\xi,\zeta,s)=\frac{4}{l_1l_2}\int_0^{l_1}\cos\mu_mx_1dx_1\int_0^{l_2}G_{1kl}^L(x_1,x_2,\xi,\zeta,s)\sin\lambda_nx_2dx_2,$$
$$G_{1kl0n}^L(\xi,\zeta,s)=\frac{2}{l_1l_2}\int_0^{l_1}dx_1\int_0^{l_2}G_{1kl}^L(x_1,x_2,\xi,\zeta,s)\sin\lambda_nx_2dx_2,$$
$$G_{2klmn}^L(\xi,\zeta,s)=\frac{4}{l_1l_2}\int_0^{l_1}\sin\mu_mx_1dx_1\int_0^{l_2}G_{2kl}^L(x_1,x_2,\xi,\zeta,s)\cos\lambda_nx_2dx_2,$$
$$G_{2klm0}^L(\xi,\zeta,s)=\frac{2}{l_1l_2}\int_0^{l_1}\sin\mu_mx_1dx_1\int_0^{l_2}G_{2kl}^L(x_1,x_2,\xi,\zeta,s)\,dx_2,$$
$$G_{q+2,klmn}^L(\xi,\zeta,s)=\frac{4}{l_1l_2}\int_0^{l_1}\sin\mu_mx_1dx_1\int_0^{l_2}G_{q+2,kl}^L(x_1,x_2,\xi,\zeta,s)\sin\lambda_nx_2dx_2.$$

Below are the systems of equations for the zero harmonics:

$$
\begin{gathered}
k_{10n}(s)\, G^L_{1kl0n}(\xi,\zeta,s) = F_{10n}(\xi,\zeta,s),\\
k_{2m0}(s)\, G^L_{2klm0}(\xi,\zeta,s) = F_{2m0}(\xi,\zeta,s),\\
F_{10n}(\xi,\zeta,s) = \frac{2}{l_1 l_2}\left[C_{66}\lambda_n\delta_{1k}\left(\delta_{1l}\cos\lambda_n\zeta + \delta_{2l}\right) - \delta_{2k}\delta_{1l}\sin\lambda_n\zeta\right],\\
F_{2m0}(\xi,\zeta,s) = \frac{2}{l_1 l_2}\left[\mu_m C_{66}\delta_{1k}\left(\delta_{1l} + \delta_{2l}\cos\mu_m\xi\right] - \delta_{2k}\delta_{2l}\sin\mu_m\xi\right].
\end{gathered} \tag{9.10}
$$

The solution of the systems (9.7) and (9.10) has the following form:

$$
\begin{gathered}
G^L_{1kl0n}(\xi,\zeta,s) = \frac{2}{l_1 l_2}\frac{C_{66}\lambda_n\delta_{1k}\left(\delta_{1l}\cos\lambda_n\zeta + \delta_{2l}\right) - \delta_{2k}\delta_{1l}\sin\lambda_n\zeta}{k_1(0,\lambda_n,s)},\\
G^L_{2klm0}(\xi,\zeta,s) = \frac{2}{l_1 l_2}\frac{\mu_m C_{66}\delta_{1k}\left(\delta_{1l} + \delta_{2l}\cos\mu_m\xi\right) - \delta_{2k}\delta_{2l}\sin\mu_m\xi}{k_2(\mu_m,0,s)};
\end{gathered} \tag{9.11}
$$

$$
\begin{gathered}
G^L_{iklmn}(\xi,\zeta,s) = \frac{P_{iklmn}(\xi,\zeta,s)}{P_{mn}(s)}\ \ (i = 1,2),\\
G^L_{q+2,klmn}(\xi,\zeta,s) = \hat{G}^L_{q+2,klmn}(\xi,\zeta,s) + \frac{P_{q+2,klmn}(\xi,\zeta,s)}{Q_{qmn}(s)};
\end{gathered} \tag{9.12}
$$

$$
\begin{gathered}
\hat{G}^L_{q+2,21mn}(\xi,\zeta,s) = -\frac{4}{l_1 l_2}\frac{\Lambda^q_{11}\mu_m}{k_{q+2,mn}(s)}\sin\lambda_n\zeta,\\
\hat{G}^L_{q+2,12mn}(\xi,\zeta,s) = \frac{4}{l_1 l_2}\frac{\Lambda_{2q}\lambda_n\mu_m}{k_{q+2,mn}(s)}\cos\mu_m\xi,\\
\hat{G}^L_{q+2,11mn}(\xi,\zeta,s) = \frac{4}{l_1 l_2}\frac{\Lambda_{1q}\lambda_n\mu_m}{k_{q+2,mn}(s)}\cos\lambda_n\zeta,\\
\hat{G}^L_{q+2,22mn}(\xi,\zeta,s) = -\frac{4}{l_1 l_2 C_{22}}\frac{\Lambda^q_{22}\lambda_n}{k_{q+2,mn}(s)}\sin\mu_m\xi,\\
\hat{G}^L_{q+2,p+2,1mn}(\xi,\zeta,s) = \frac{4D_{1jq}\mu_m}{l_1 l_2}\frac{\sin\lambda_n\zeta}{k_{q+2,mn}(s)},\\
\hat{G}^L_{q+2,p+2,2mn}(\xi,\zeta,s) = \frac{4D_{2qp}\lambda_n}{l_1 l_2}\frac{\sin\mu_m\xi}{k_{q+2,mn}(s)},
\end{gathered} \tag{9.13}
$$

where

$$
\begin{gathered}
P_{mn}(s) = \left[k_{1mn}(s)\,k_{2mn}(s) - \mu_m^2\lambda_n^2 C_0^2\right]\Pi_{mn}(s) -\\
-\lambda_n^2\sum_{j=1}^{N} S_{1mn}(s)\,\Pi_{jmn}(s)\,M_{jmn} - \mu_m^2\sum_{j=1}^{N} S_{2mn}(s)\,\Pi_{jmn}(s)\,K_{jmn} +\\
+\mu_m^2\lambda_n^2\sum_{i=1}^{N}\sum_{j=1}^{N} \mathrm{A}_{ij} K_{imn} M_{jmn}\Pi_{ijmn}(s),\\
Q_{qmn}(s) = k_{q+2,mn}(s)\,P_{mn}(s),
\end{gathered} \tag{9.14}
$$

$$P_{121mn}(\xi,\zeta,s) = -\frac{4}{l_1 l_2} k_{2mn}(s)\,\Pi_{mn}(s)\sin\lambda_n\zeta -$$
$$-\frac{4}{l_1 l_2}\sum_{j=1}^{N}\left[\mu_m^2 \Lambda_{11}^j S_{2mn}(s) + \alpha_2^j \lambda_n^2 M_{jmn}\right]\Pi_{jmn}(s)\sin\lambda_n\zeta +$$
$$+\frac{4}{l_1 l_2}\mu_m^2\lambda_n^2\sum_{j=1}^{N}\Lambda_{11}^j\sum_{i=1}^{N}\mathrm{A}_{ji}M_{imn}\Pi_{ijmn}(s)\sin\lambda_n\zeta,$$

$$P_{112mn}(\xi,\zeta,s) = \frac{4}{l_1 l_2}C_{66}\lambda_n\left(k_{2mn}(s) - \mu_m^2 C_0\right)\Pi_{mn}(s)\cos\mu_m\xi +$$
$$+\frac{4}{l_1 l_2}\lambda_n\sum_{j=1}^{N}\left[C_{66}B_{jmn}M_{jmn} + \mu_m^2\Lambda_{2j}S_{2mn}(s)\right]\Pi_{jmn}(s)\cos\mu_m\xi -$$
$$-\frac{4}{l_1 l_2}\lambda_n^3\mu_m^2\sum_{j=1}^{N}\Lambda_{2j}\sum_{i=1}^{N}\mathrm{A}_{ji}M_{imn}\Pi_{ijmn}(s)\cos\mu_m\xi,$$

$$P_{111mn}(\xi,\zeta,s) = \frac{4}{l_1 l_2}C_{66}\lambda_n\left(k_{2mn}(s) - C_0\mu_m^2\right)\Pi_{mn}(s)\cos\lambda_n\zeta +$$
$$+\frac{4}{l_1 l_2}\lambda_n\sum_{j=1}^{N}\left[C_{66}B_{jmn}M_{jmn} + \mu_m^2\Lambda_{1j}S_{2mn}(s)\right]\Pi_{jmn}(s)\cos\lambda_n\zeta -$$
$$-\frac{4}{l_1 l_2}\lambda_n^3\mu_m^2\sum_{j=1}^{N}\Lambda_{1j}\sum_{i=1}^{N}\mathrm{A}_{ji}M_{imn}\Pi_{ijmn}(s)\cos\lambda_n\zeta,$$

$$P_{122mn}(\xi,\zeta,s) = \frac{4}{l_1 l_2}\lambda_n\mu_m C_0\Pi_{mn}(s)\sin\mu_m\xi +$$
$$+\frac{4}{l_1 l_2}\mu_m\lambda_n\sum_{j=1}^{N}\left[\alpha_1^j M_{jmn}(s) - \frac{\Lambda_{22}^j}{C_{22}}S_{2mn}(s)\right]\Pi_{jmn}(s)\sin\mu_m\xi +$$
$$+\frac{4}{l_1 l_2}\mu_m\lambda_n^3\sum_{j=1}^{N}\frac{\Lambda_{22}^j}{C_{22}}\sum_{i=1}^{N}\mathrm{A}_{ji}M_{imn}\Pi_{ijmn}(s)\sin\mu_m\xi,$$

$$P_{221mn}(\xi,\zeta,s) = \frac{4}{l_1 l_2}\mu_m\lambda_n C_0\Pi_{mn}(s)\sin\mu_m\zeta +$$
$$+\frac{4}{l_1 l_2}\mu_m\lambda_n\sum_{j=1}^{N}\left[\alpha_2^j K_{jmn} - \Lambda_{11}^j S_{1mn}(s)\right]\Pi_{jmn}(s)\sin\lambda_n\zeta +$$
$$+\frac{4}{l_1 l_2}\mu_m^3\lambda_n\sum_{j=1}^{N}\Lambda_{11}^j\sum_{i=1}^{N}\mathrm{A}_{ji}K_{imn}\Pi_{ijmn}(s)\sin\lambda_n\zeta,$$

$$
\begin{gathered}
P_{212mn}(\xi,\zeta,s)=\frac{4}{l_1l_2}C_{66}\mu_m\left(k_{1mn}-C_0\lambda_n^2\right)\Pi_{mn}(s)\cos\mu_m\xi+\\
+\frac{4}{l_1l_2}\mu_m\sum_{j=1}^{N}\left[-C_{66}B_{jmn}K_{jmn}+\lambda_n^2\Lambda_{2j}S_{1mn}(s)\right]\Pi_{jmn}(s)\cos\mu_m\xi-\\
-\frac{4}{l_1l_2}\lambda_n^2\mu_m^3\sum_{j=1}^{N}\Lambda_{2j}\sum_{i=1}^{N}\mathrm{A}_{ji}K_{imn}\Pi_{ijmn}(s)\cos\mu_m\xi,
\end{gathered}
$$

$$
\begin{gathered}
P_{211mn}(\xi,\zeta,s)=\frac{4}{l_1l_2}C_{66}\mu_m\left(k_{1mn}(s)-C_0\lambda_n^2\right)\Pi_{mn}(s)\cos\lambda_n\zeta+\\
+\frac{4}{l_1l_2}\mu_m\sum_{j=1}^{N}\left[-C_{66}B_{jmn}K_{jmn}+\lambda_n^2\Lambda_{1j}S_{1mn}(s)\right]\Pi_{jmn}(s)\cos\lambda_n\zeta-\\
-\frac{4}{l_1l_2}\lambda_n^2\mu_m^3\sum_{j=1}^{N}\Lambda_{1j}\sum_{i=1}^{N}\mathrm{A}_{ji}K_{imn}\Pi_{ijmn}(s)\cos\lambda_n\zeta,
\end{gathered}
$$

$$
\begin{gathered}
P_{222mn}(\xi,\zeta,s)=-\frac{4}{l_1l_2}k_{1mn}(s)\Pi_{mn}(s)\sin\lambda_n\xi-\\
-\frac{4}{l_1l_2}\sum_{j=1}^{N}\left[\frac{\Lambda_{22}^{j}}{C_{22}}\lambda_n^2S_{1mn}(s)+\mu_m^2\alpha_1^jK_{jmn}\right]\Pi_{jmn}(s)\sin\mu_m\xi+\\
+\frac{4}{l_1l_2}\mu_m^2\lambda_n^2\sum_{j=1}^{N}\frac{\Lambda_{22}^{j}}{C_{22}}\sum_{i=1}^{N}\mathrm{A}_{ji}K_{imn}\Pi_{ijmn}(s)\sin\mu_m\xi,
\end{gathered}
$$

$$
\begin{gathered}
P_{1,q+2,1mn}(\xi,\zeta,s)=\frac{4}{l_1l_2}\mu_m^2\sum_{j=1}^{N}D_{1jq}S_{2mn}(s)\Pi_{jmn}(s)\sin\lambda_n\zeta-\\
-\frac{4}{l_1l_2}\mu_m^2\lambda_n^2\sum_{j=1}^{N}D_{1jq}\sum_{r=1}^{N}\mathrm{A}_{jr}M_{rmn}\Pi_{rjmn}(s)\sin\lambda_n\zeta,\\
P_{1,q+2,2mn}(\xi,\zeta,s)=\frac{4}{l_1l_2}\mu_m\lambda_n\sum_{j=1}^{N}D_{2jq}S_{2mn}(s)\Pi_{jmn}(s)\sin\mu_m\xi-\\
-\frac{4}{l_1l_2}\mu_m\lambda_n^3\sum_{j=1}^{N}D_{2jq}\sum_{r=1}^{N}\mathrm{A}_{jr}M_{rmn}\Pi_{rjmn}(s)\sin\mu_m\xi,
\end{gathered}
$$

$$
\begin{gathered}
P_{2,q+2,1mn}(\xi,\zeta,s)=\frac{4}{l_1l_2}\lambda_n\mu_m\sum_{j=1}^{N}D_{1jq}S_{1mn}(s)\Pi_{jmn}(s)\sin\lambda_n\zeta-\\
-\frac{4}{l_1l_2}\mu_m^3\lambda_n\sum_{j=1}^{N}D_{1jq}\sum_{r=1}^{N}\mathrm{A}_{jr}K_{rmn}\Pi_{rjmn}(s)\sin\lambda_n\zeta,
\end{gathered}
$$

$$P_{2,q+2,2mn}(\xi,\zeta,s)=\frac{4}{l_1l_2}\lambda_n^2\sum_{j=1}^{N}D_{2jq}S_{1mn}(s)\,\Pi_{jmn}(s)\sin\mu_m\xi-$$
$$-\frac{4}{l_1l_2}\mu_m^2\lambda_n^2\sum_{j=1}^{N}D_{2jq}\sum_{r=1}^{N}\mathrm{A}_{jr}K_{rmn}\Pi_{rjmn}(s)\sin\mu_m\xi,$$

$$P_{q+2,klmn}(\xi,\zeta,s)=\mu_mK_{qmn}P_{1klmn}(\xi,\zeta,s)+\lambda_nM_{qmn}P_{2klmn}(\xi,\zeta,s),$$

$$\Pi_{mn}(s)=\prod_{j=1}^{N}k_{j+2,mn}(s),\ \ \Pi_{qmn}(s)=\prod_{j=1,j\neq q}^{N}k_{j+2,mn}(s),\ \ \Pi_{qpmn}(s)=\prod_{j=1,j\neq q,p}^{N}k_{j+2,mn}(s),$$
$$S_{1mn}(s)=\alpha_1^jC_0\mu_m^2-\alpha_2^jk_{1mn}(s),\ \ S_{2mn}(s)=\alpha_2^jC_0\lambda_n^2-\alpha_1^jk_{2mn}(s),$$
$$\mathrm{A}_{ji}=\alpha_1^j\alpha_2^i-\alpha_2^j\alpha_1^i,\ \ B_{jmn}=\alpha_2^j\lambda_n^2-\mu_m^2\alpha_1^j,$$
$$D_{1jq}=D_1^j\delta_{qj}-\Lambda_{11}^j\alpha_1^q,\ \ D_{2jq}=D_2^j\delta_{qj}-\frac{\Lambda_{22}^j}{C_{22}}\alpha_2^q.$$

The transition to the original domain is done by residues and tables of operational calculus as follows [21]:

$$G_{1kl0n}(\xi,\zeta,\tau)=\frac{2\sqrt{C_{66}}}{l_1l_2}\left[\delta_{1k}(\delta_{1l}+\delta_{2l})-\frac{\delta_{2k}\delta_{1l}\sin\lambda_n\zeta}{\lambda_nC_{66}}\right]\sin\left(\lambda_n\sqrt{C_{66}}\tau\right),$$
$$G_{2klm0}^{sc}(\xi,\zeta,\tau)=\frac{2\sqrt{C_{66}}}{l_1l_2}\left[\delta_{1k}(\delta_{1l}+\delta_{2l})-\frac{\delta_{2k}\delta_{2l}\sin\mu_m\xi}{\mu_mC_{66}}\right]\sin\left(\mu_m\sqrt{C_{66}}\tau\right);\tag{9.15}$$

$$G_{iklmn}(\xi,\zeta,\tau)=\sum_{j=1}^{2N+4}A_{iklmn}^{(j)}(\xi,\zeta)\,e^{s_{jmn}\tau}\ \ (i=1,2),$$
$$G_{q+2,klmn}(\xi,\zeta,\tau)=\hat{G}_{q+2,klmn}(\xi,\zeta,\tau)+\sum_{j=1}^{2N+6}A_{q+2,klmn}^{(j)}(\xi,\zeta)e^{s_{jmn}\tau},\tag{9.16}$$
$$A_{iklmn}^{(j)}(\xi,\zeta)=\frac{P_{iklmn}\left(\xi,\zeta,s_{jmn}\right)}{P'_{mn}\left(s_{jmn}\right)},\ \ A_{q+2,klmn}^{(j)}(\xi,\zeta)=\frac{P_{q+2,klmn}\left(\xi,\zeta,s_{jmn}\right)}{Q'_{qmn}\left(s_{jmn}\right)},$$

$$\hat{G}_{q+2,21mn}(\xi,\zeta,\tau)=-\frac{4}{l_1l_2}\Lambda_{11}^q\mu_m\sin\lambda_n\zeta\sum_{j=1}^{2}\frac{e^{\chi_{jmn}\tau}}{k'_{q+2,mn}\left(\chi_{jmn}\right)},$$
$$\hat{G}_{q+2,12mn}(\xi,\zeta,\tau)=\frac{4}{l_1l_2}\Lambda_{2q}\lambda_n\mu_m\cos\mu_m\xi\sum_{j=1}^{2}\frac{e^{\chi_{jmn}\tau}}{k'_{q+2,mn}\left(\chi_{jmn}\right)},$$
$$\hat{G}_{q+2,11mn}(\xi,\zeta,\tau)=\frac{4}{l_1l_2}\Lambda_{1q}\lambda_n\mu_m\cos\lambda_n\zeta\sum_{j=1}^{2}\frac{e^{\chi_{jmn}\tau}}{k'_{q+2,mn}\left(\chi_{jmn}\right)},$$
$$\hat{G}_{q+2,22mn}(\xi,\zeta,\tau)=-\frac{4}{l_1l_2}\frac{\Lambda_{22}^q\lambda_n}{C_{22}}\sin\mu_m\xi\sum_{j=1}^{2}\frac{e^{\chi_{jmn}\tau}}{k'_{q+2,mn}\left(\chi_{jmn}\right)}.$$

9.5 Calculation Example

Let us take a three-component material ($N = 2$, independent components, zinc and copper, diffusing into aluminium) with the following characteristics [22]:

$$C'_{12} = 5.11 \cdot 10^{10}\,\frac{\text{N}}{\text{m}^2},\ C'_{66} = 2.63 \cdot 10^{10}\,\frac{\text{N}}{\text{m}^2},\ T_0 = 700\ K,\ \rho = 2700\,\frac{\text{kg}}{\text{m}^3},$$
$$\alpha_{11}^{(1)} = \alpha_{22}^{(1)} = 6.55 \cdot 10^7\,\frac{\text{J}}{\text{kg}},\ \alpha_{11}^{(2)} = \alpha_{22}^{(2)} = 6.14 \cdot 10^7\,\frac{\text{J}}{\text{kg}},\ l = 10^{-2}\ \text{m},$$

$$D_{11}^{(1)} = D_{22}^{(1)} = 2.62 \cdot 10^{-12}\,\frac{\text{m}^2}{\text{s}},\ D_{11}^{(2)} = D_{22}^{(2)} = 2.89 \cdot 10^{-15}\,\frac{\text{m}^2}{\text{s}},$$
$$n_0^{(1)} = 0.01,\ n_0^{(2)} = 0.045,\ m^{(1)} = 0.065\ \frac{\text{kg}}{\text{mol}},\ m^{(2)} = 0.064\ \frac{\text{kg}}{\text{mol}}.$$

Assume that the plate is under the action of tensile forces applied to the boundaries $x_1 = 0$ and $x_1 = l_1$:

$$f_{211}(x_2, \tau) = f_{212}(x_2, \tau) = H(\tau) \sin \frac{\pi x_2}{l_2},$$

where $H(\tau)$ is the Heaviside function.

All other load parameters in the boundary conditions (9.2) are assumed to be zero. Calculating the convolutions (9.4), we obtain

$$\begin{aligned} u_1(x_1, x_2, \tau) &= \int_0^{l_2} G_{121}(x_1, x_2, \zeta, \tau) * f_{211}(\zeta, \tau)\, d\zeta - \\ &- \int_0^{l_2} G_{121}(l_1 - x_1, x_2, \zeta, \tau) * f_{212}(\zeta, \tau)\, d\zeta = \\ &= l_2 \sin \lambda_1 x_2 \sum_{m=1}^{\infty} (-1)^{m+1} \sin\left[\mu_m\left(\frac{l_1}{2} - x_1\right)\right] \sum_{j=1}^{2N+4} \tilde{A}_{121m1}^{(j)} \frac{e^{s_{jm1}\tau} - 1}{s_{jm1}}, \end{aligned}$$

$$\begin{aligned} u_2(x_1, x_2, \tau) &= \int_0^{l_2} G_{221}(x_1, x_2, \zeta, \tau) * f_{211}(\zeta, \tau)\, d\zeta + \\ &+ \int_0^{l_2} G_{221}(l_1 - x_1, x_2, \zeta, \tau) * f_{212}(\zeta, \tau)\, d\zeta = \\ &= l_2 \cos \lambda_1 x_2 \sum_{m=1}^{\infty} (-1)^{m+1} \cos\left[\mu_m\left(\frac{l_1}{2} - x_1\right)\right] \sum_{j=1}^{2N+4} \tilde{A}_{221m1}^{(j)} \frac{e^{s_{jm1}\tau} - 1}{s_{jm1}}, \end{aligned}$$

$$\begin{aligned} \eta_q(x_1, x_2, \tau) &= \int_0^{l_2} G_{q+2,21}(x_1, x_2, \zeta, \tau) * f_{211}(\zeta, \tau)\, d\zeta + \\ &+ \int_0^{l_2} G_{q+2,21}(l_1 - x_1, x_2, \zeta, \tau) * f_{212}(\zeta, \tau)\, d\zeta = \\ &= \frac{4}{l_1} \sin \lambda_1 x_2 \sum_{m=1}^{\infty} (-1)^{m+1} \cos\left[\mu_m\left(\frac{l_1}{2} - x_1\right)\right] \sum_{j=1}^{2} \frac{\Lambda_{1q}\lambda_1\mu_m}{k'_{q+2,m1}(\chi_{jm1})} \frac{e^{\chi_{jm1}\tau} - 1}{\chi_{jm1}} + \\ &+ l_2 \sin \lambda_1 x_2 \sum_{m=1}^{\infty} (-1)^{m+1} \cos\left[\mu_m\left(\frac{l_1}{2} - x_1\right)\right] \sum_{j=1}^{2N+6} \tilde{A}_{q+2,21m1}^{(j)} \frac{e^{s_{jm1}\tau} - 1}{s_{jm1}}, \end{aligned}$$

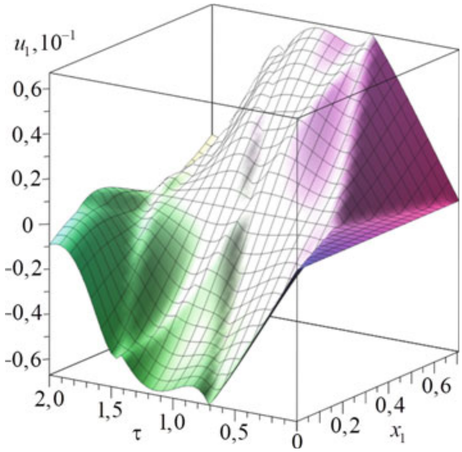

Fig. 9.1 Displacement $u_1(x_1, x_2, \tau)$ at $x_2 = l_2/2$

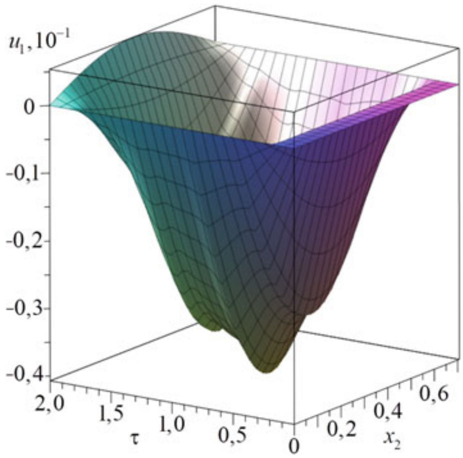

Fig. 9.2 Displacement $u_1(x_1, x_2, \tau)$ at $x_1 = l_1/4$

$$\tilde{A}^{(j)}_{k21mn} = \frac{A^{(j)}_{k21mn}(\zeta)}{\sin \lambda_1 \zeta}.$$

The calculation results are shown in the Figs. 9.1, 9.2, 9.3, 9.4, 9.5, 9.6, 9.7 and 9.8. Figures 9.1, 9.2, 9.3 and 9.4 show dependences of longitudinal displacements u_i on time and coordinates. Calculations show that diffusion processes at the initial stages of deformation do not affect the displacement field. At initial times, the elastic and elastic diffusion displacements coincide.

Figures 9.5 and 9.6 show the zinc and copper concentration increments, respectively, caused by longitudinal deformations.

The influence of relaxation effects on the mass transfer kinetics is shown in Figs. 9.7 and 9.8. Here, different lines show zinc concentration increments for models with a finite and infinite speed of diffusion fluxes. The relaxation effects manifest themselves at some finite interval of time and then disappear. Thus, in Fig. 9.8, corresponding to the time $\tau = 10^9$, both curves already coincide.

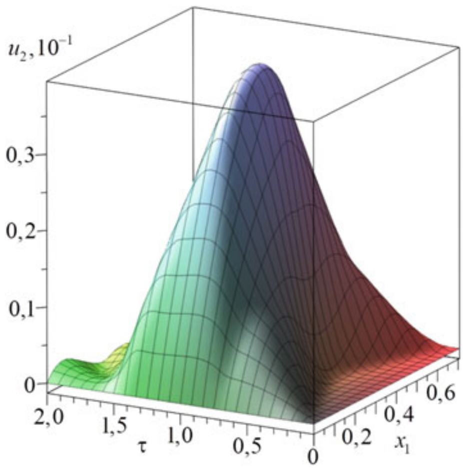

Fig. 9.3 Displacement $u_2\,(x_1, x_2, \tau)$ at $x_2 = l_2/4$

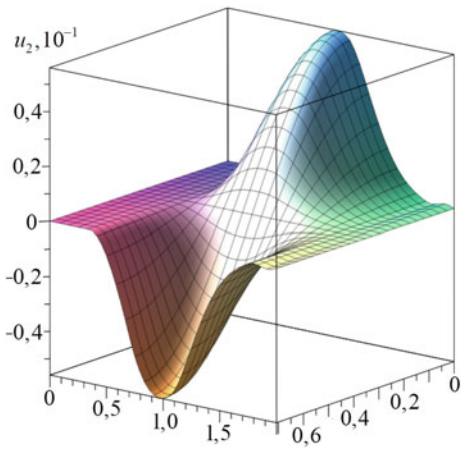

Fig. 9.4 Displacements $u_2\,(x_1, x_2, \tau)$ at $x_1 = l_1/2$

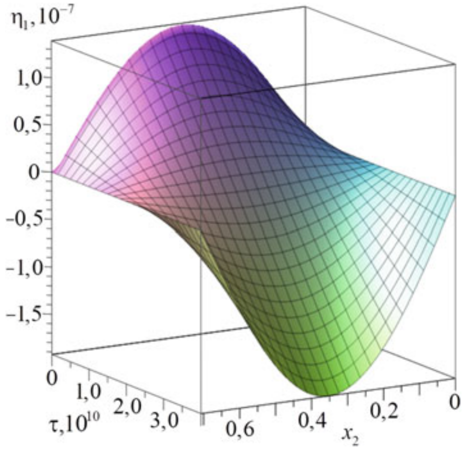

Fig. 9.5 Zinc concentration increment $\eta_1\,(x_1, x_2, \tau)$ at $x_1 = l_1/2$

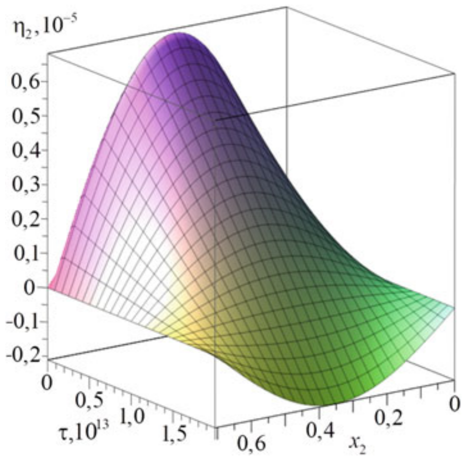

Fig. 9.6 Copper concentration increment $\eta_2\,(x_1, x_2, \tau)$ at $x_1 = l_1/2$

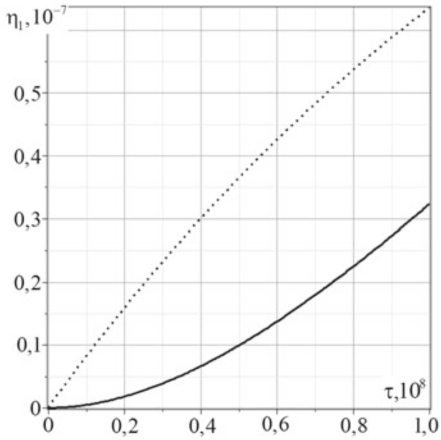

Fig. 9.7 Zinc concentration increment $\eta_1\,(x_1, x_2, \tau)$. The solid line corresponds to the time $\tau^{(q)} = 200\,\mathrm{s}$, the dotted line to $\tau^{(q)} = 0$

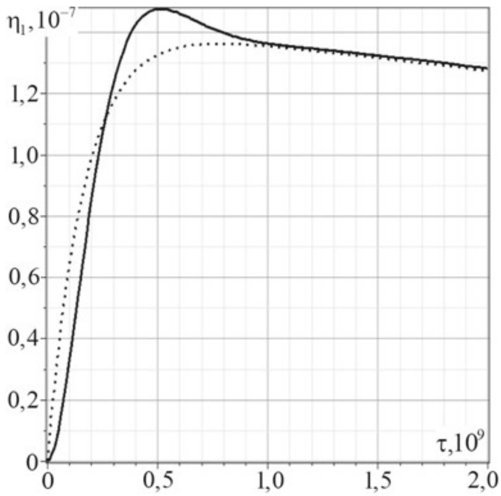

Fig. 9.8 Zinc concentration increment $\eta_1\,(x_1, x_2, \tau)$. The solid line corresponds to the time $\tau^{(q)} = 200\,\mathrm{s}$, the dotted line to $\tau^{(q)} = 0$

9.6 Conclusions

We propose a mathematical model describing the effects of the interaction between mechanical and diffusion fields during a rectangular orthotropic plate deformation. The algorithm is developed for finding Green's functions based on Laplace transform and decomposition into trigonometric Fourier series. It allows us to reduce the general problem of Laplace transform inversion to the problem of rational function inversion. Originals of Green's functions are found using residues and tables of operational calculus. As a result, it is possible to obtain the solution in analytical form, which provides ample opportunities for various kinds of numerical experiments.

The effect of interaction between the mechanical and diffusion fields is demonstrated by the example of a rectangular plate under the action of tensile forces. Unsteady loads initiate the process of mass transfer. At the same time, the relaxation diffusion processes reduce with time. The kinetics of mass transfer at significant times can be described by classical mechanodiffusion with the infinite speed of diffusion fluxes. All results are presented in analytical and graphical forms.

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