Finite Element Analysis Characterization of Macro Synthetic Fibre Reinforced Concrete Constitutive Equation

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Abstract. Over the last years, the use of fibre reinforced concrete (FRC) has increased for structural purposes. For the structural design of FRC elements, there was a need of a model that developed the behaviour of the post-cracking response of FRC. In this sense, national and international guidelines have included models to characterise the flexural behaviour of FRC (fib Model Code, EHE-08). These models gather the performance of FRC for serviceability limit state (SLS) and ultimate limit state (ULS) for either steel or macro synthetic polypropylene fibre reinforced concrete (SFRC and MSFRC, respectively). In this regard, the codes and guidelines do not distinguish between FRC comprised of steel or synthetic fibres and establish the FRC ultimate strain in 2.5%. This limitation represents the behaviour of SFRC but limits the full potential of MSFRC for large deformations. Owing to the aspects aforementioned, an extensive experimental programme has been carried out at the Universitat Politècnica de Catalunya (UPC) to characterise the behaviour of MSFRC. This research contribution is focused on an inverse analysis to derive the MSFRC constitutive equations by means of a non-linear finite element simulation. The main goal of this study is to compare the experimental results with those obtained through the simulation using the constitutive equations of the fib MC-2010. The results show a generalised underestimation of MSFRC at ultimate strain and the necessity of adjusting the constitutive equations for SFRC and MSFRC.

Keywords: Macro synthetic fibre reinforced concrete · Non-linear analysis · Constitutive equation

1 Introduction

The addition of fibres into concrete, commonly known as fibre reinforced concrete (FRC), for structural purposes has experienced a huge growth in recent years. Fibres are either used for totally or partially replacing the traditional rebar reinforcement. In the FRC industry, there are different types of fibres such as organic, metallic or synthetic (i.e. glass fibre, steel fibres, polypropylene fibres, respectively) that are used to cover a

J. I. Escalante-Garcia et al. (Eds.): RW 2021, RILEM Bookseries 40, pp. 539–548, 2023.
https://doi.org/10.1007/978-3-031-21735-7_59

wide range of structural typologies: industrial flooring [1, 2], precast concrete segments for tunnel linings [3], elevated flat slabs [4, 5], sewer pipes [6, 7].

Due to the increasing use of this material, codes and guidelines [8–10] have been published as a demand for a reliable design tool for engineers and practitioners. These codes and guidelines developed material models that are able to reproduce the behaviour of FRC. The models consist of constitutive equations, in terms of uniaxial stress-strain (σ-ε) curves (or stress-crack width), that take into account the post-cracking residual strength of FRC given by the pull-out mechanism effect induced by fibres.

In the case of *fib* Model Code—2010 (MC-2010) [8], the post-cracking strength of the material is obtained by carrying out the beam flexural strength test (EN:14651, European Commitee for Standardization, 2005) from which the residual strength for crack mouth opening displacement (CMOD) 0.5 and 2.5 mm are extracted (f_{R1} and f_{R3}, respectively). This results in a bilinear post-cracking constitutive equation where the two main points are f_{Fts} and f_{Ftu}, which correspond to serviceability and ultimate residual strength, being the ultimate crack width (w_{u}) equal to 2.5. Figure 1 depicts the schematic representation of the stress-crack width curve for FRC according to MC-2010, the full curve is obtained as the combination of the post-cracking response of plain concrete (where f_{ctm} and G_{F} stand for mean tensile concrete strength and fracture energy, respectively) and the fibre contribution through the pull-out mechanism. Being the first point $\sigma_1 = f_{\mathrm{ctm}}$ and $w_1 = 0$ mm, the second point σ_2 and w_2 (the intersection between the two curves) and the third one $\sigma_3 = f_{\mathrm{ctm}}$ and $w_3 = 2.5$ mm.

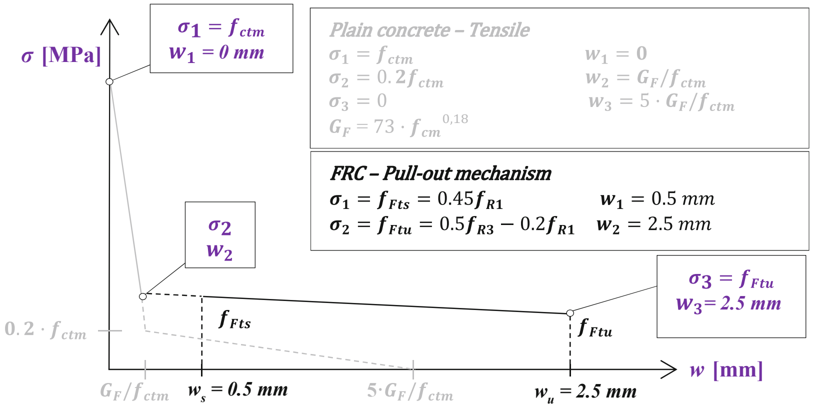

Fig. 1. MC-2010 FRC post-cracking curve

However, the aforementioned guidelines do not distinguish between different types of fibres and w_{u} considered in terms of ductility or durability may not be adjusted for all type of fibres. This is especially important in beam flexural strength test of macro synthetic polypropylene fibre reinforced concrete (MSFRC) where, after the sudden drop due to cracking, a hardening response is observed up to failure and the peak of the residual strength is reached beyond 2.5 mm (w_{u}).

In view of this, in order to assess the behaviour of MSFRC, a wide experimental programme considering a broad range of compressive concrete strength along with a representative fibre dosage used for actual applications was performed at the Universitat Politècnica de Catalunya. The concrete mixes tested in this experimental research had strength classes of C30/37 (with fibre amount of 2.5, 3.5 and 5.0 kg/m^3), C40/50 (with 5.0, 7.5 and 10.0 kg/m^3) and C50/60 (with 5.0, 7.5 and 10 kg/m^3).

The results of this experimental research were used for deriving the constitutive equations (MC-2010) and were compared with the ones obtained performing back analysis (BA) by means of a FE software. The aim of this conference proceeding is twofold: (1) compare the equations derived using the MC-2010 equations with the ones obtained by the inverse analysis and (2) propose changes in the equation in order to take into account the full potential of the MSFRC concrete. The outcome of this research work proved that the current MC-2010 constitutive equation do not take into account the full potential of MSFRC.

2 Experimental Results

In the experimental campaign, residual flexural strength and compressive tests were carried out. The mechanical and geometrical properties of macro synthetic fibres made of polypropylene (PPMSF) for structural purposes used are gathered in Table 1. In order to have sufficient representativeness of the MSFRC behaviour, nine beam tests were carried out per each concrete mix (a total amount of 81). The average curves of the beam flexural strength test of each concrete mix are depicted in Fig. 2. In some cases the tests were carried out beyond the standard CMOD for research purposes, to see any possible fibre failure. As can be seen from Fig. 2, the maximum post-cracking strength registered is beyond 2.5 mm for every individual mix, which is w_u for the constitutive equation of the MC-2010. These values are gathered in Table 2.

Table 1. Characteristics of PPMSF fibre

Material	Anchorage	Length	Young's modulus	Tensile strength	Number fibres/kg
Virgin polypropylene	Continuous embossing	48 mm	12 GPa	640 MPa	59500

Table 2. CMOD at maximum post-cracking strength

	C30/37	C40/50	C50/60
2.5 kg/m^3	2.76 mm	–	–
3.5 kg/m^3	2.69 mm	–	–
5.0 kg/m^3	2.91 mm	2.70 mm	2.80 mm
7.5 kg/m^3		2.51 mm	2.69 mm
10.0 kg/m^3	–	2.71 mm	2.68 mm

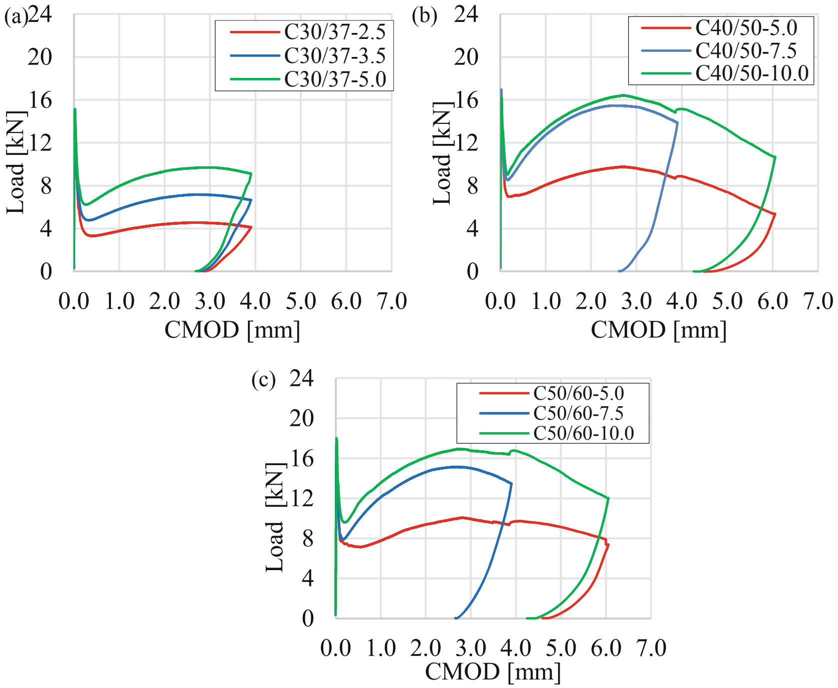

Fig. 2. Average curves of beam flexural strength test for (a) C30/37 (b) C40/50 and (c) C50/60.

In view of these results, it is highly unlikely that the σ–w constitutive curve is able to capture properly the late post-cracking behaviour of the MSFRC. In the following sections, a FE model is presented to carry out a back analysis (BA) and derive the constitutive curve suitable to reproduce the behaviour of MSFRC.

3 FE Analysis

In order to perform the BA, a non-linear 2D plain strain model was created in the FE software ABAQUS. The Concrete Damage Plasticity (CDP) model available in ABAQUS [12] was selected. This software presents a versatile tool to model a broad range of phenomena of structural concrete behaviour. The model assumes that the main two failure mechanisms for concrete are tensile cracking and compressive crushing. To model the concrete behaviour, the input data required are uniaxial σ–ε curves for compression and tension. In this study, to overcome mesh dependence due to different mesh size, the σ–w tensile curve was used instead of σ–ε.

In Fig. 3 are presented the geometry, boundary conditions and loading for the beam flexural strength test configuration. The boundary conditions were imposed so that in the vertical axis $u_y = 0$ in both supports and $u_x = 0$ in one of those. The load was applied by displacement control in order to guarantee proper convergence in case of flexural-softening response is detected. The mesh comprised of 1361 nodes and 2560 triangular linear elements (CPE3) with a mesh size of 10 mm, refined in the mid-section with 5 mm size elements. The post-cracking behaviour of FRC was captured by means of ABAQUS Explicit Dynamic algorithm (quasi-static analysis).

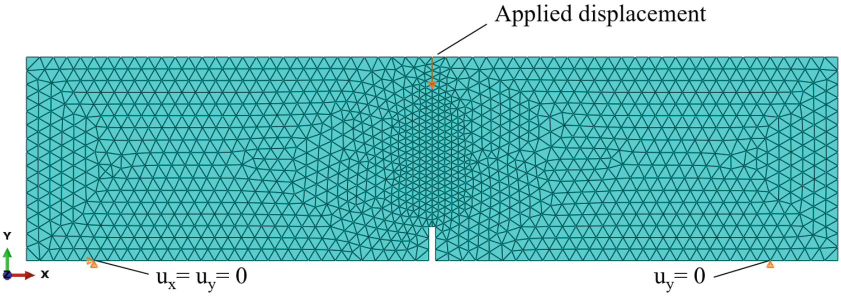

Fig. 3. 2D FEM model adopted: mesh and boundary conditions considered

4 Results

To obtain the constitutive relationships, a generalised method of back analysis was used [13] considering an iterative trial and error to fit the experimental curves with the model results. The MC-2010 constitutive equations were obtained using the results presented in Fig. 2.

From Figs. 4, 5, 6, 7, 8, 9, 10, 11 to 12 are depicted the Load—CMOD curves for the experimental results and three FE model simulations (1) using the MC-2010 constitutive curves, (2) using the constitutive curve obtained by BA and (3) an obtained simplified trilinear curve (based on the back analysis results). Additionally, the figures with stress-crack width present the constitutive curves used for the FE simulations.

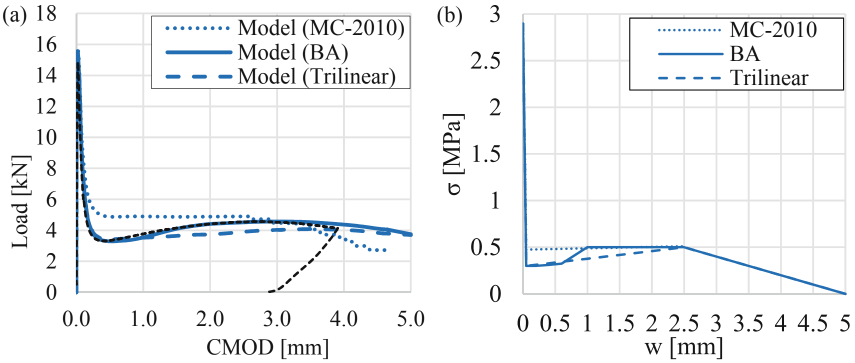

Fig. 4. C30/37 2.5 kg/m^3 **a** Load—CMOD curves **b** Constitutive equations.

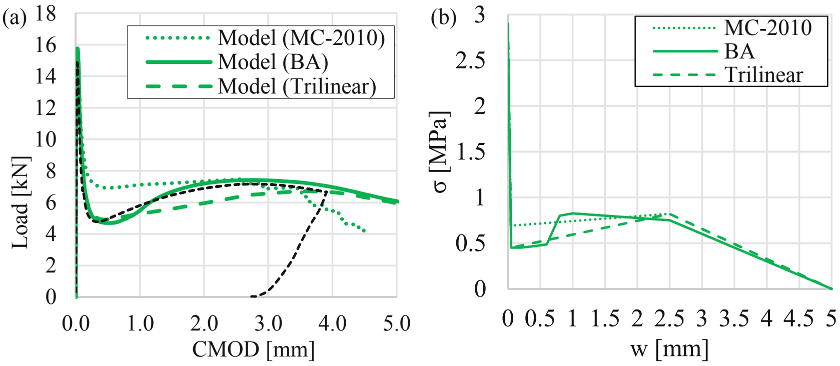

Fig. 5. C30/37 3.5 kg/m^3 **a** Load—CMOD curves **b** Constitutive equations.

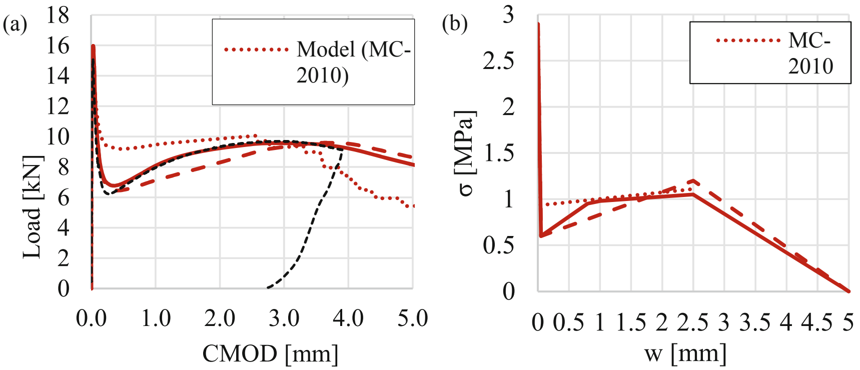

Fig. 6. C30/37 5.0 kg/m^3 **a** Load—CMOD curves **b** Constitutive equations.

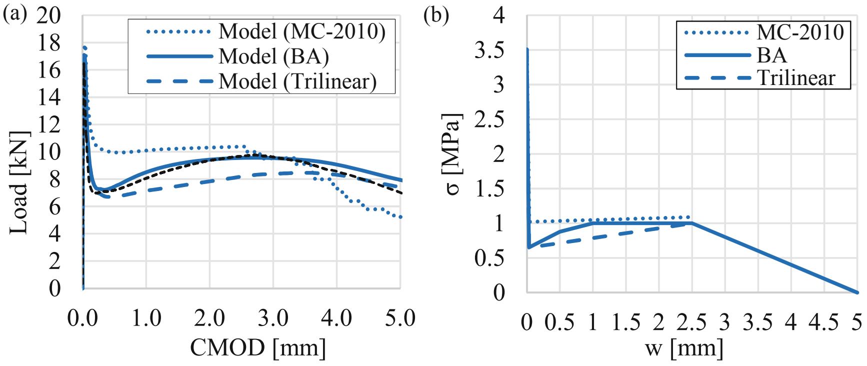

Fig. 7. C40/50 5.0 kg/m^3 **a** Load—CMOD curves **b** Constitutive equations.

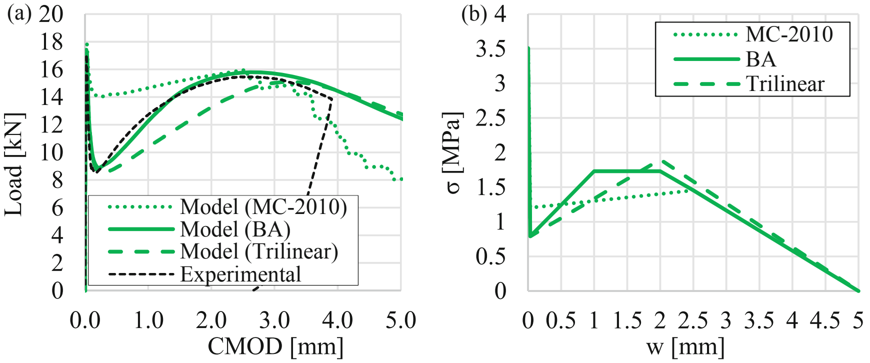

Fig. 8. C40/50 7.5 kg/m^3 **a** Load—CMOD curves **b** Constitutive equations.

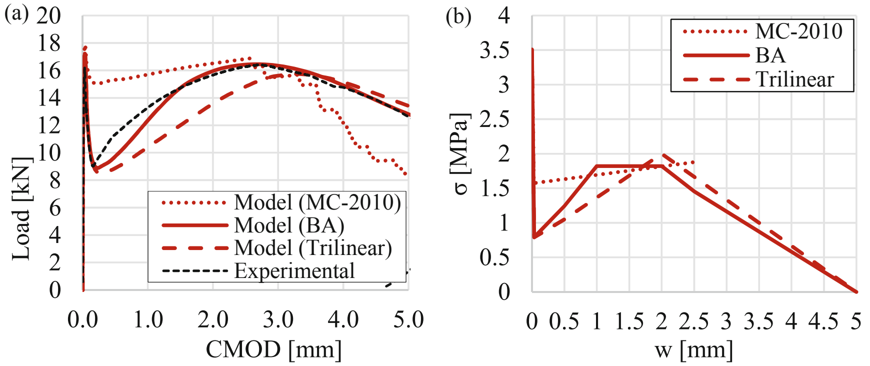

Fig. 9. C40/50 10.0 kg/m^3 **a** Load—CMOD curves **b** Constitutive equations.

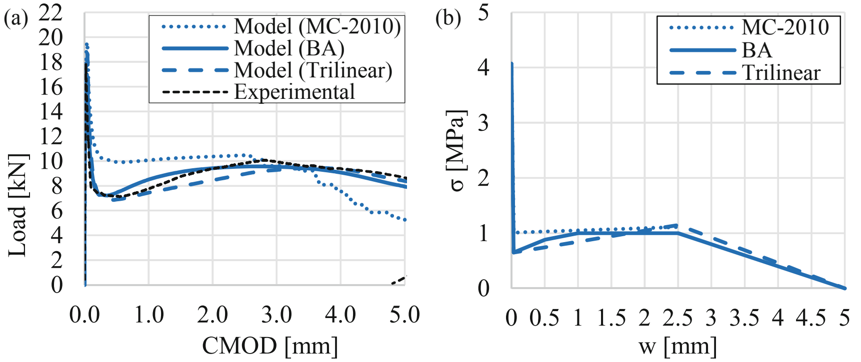

Fig. 10. C50/60 5.0 kg/m^3 **a** Load—CMOD curves **b** Constitutive equations.

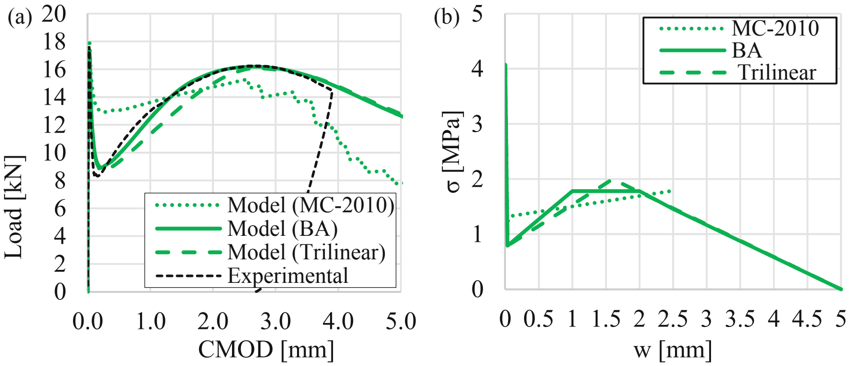

Fig. 11. C50/60 7.5 kg/m^3 **a** Load—CMOD curves **b** Constitutive equations.

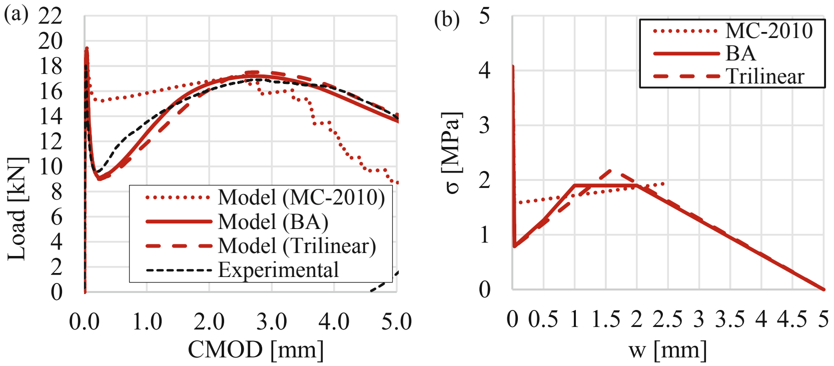

Fig. 12. C50/60 10.0 kg/m^3 **a** Load—CMOD curves **b** Constitutive equations.

The results show how the MC-2010 constitutive equation overestimates the mechanical performance for CMOD $<$ 1.0 mm, i.e., the loss of strength is larger in the Experimental test than in the Model (MC-2010). This is of great importance, suggesting that the MC-2010 curve tends to show higher strength for serviceability limit state (f_{Fts}). Although the MC-2010 bilinear curve captures well the maximum post-cracking strength, it is reached at smaller CMOD as compared to the experimental results. Further, when the maximum post-cracking strength for MSFRC is expected (CMOD $>$ 2.5 mm), the Load-CMOD curve starts decreasing, which does not correspond with the behaviour observed in the actual tests. This is even more evident in tests with high hardening behaviour (e.g. C40/50–10 and C50/60–10). In view of these results, two main issues can be highlighted with the MC-2010 approach: overestimates the residual strength for serviceability limit state (SLS) and it is not representative of the full potential of MSFRC reached at larger CMOD.

The constitutive curve obtained by means of BA fits well the behaviour of the actual test. In this regard, seven points (up to nine in the case of C30/37–3.5) were necessary to match the curve. The loss of strength and the mechanical performance for CMOD

< 1.0 mm was well captured, which highlights the necessity of decreasing the value of σ_2 for the MC-2010 constitutive equation. For high values of CMOD ($>$2.5 mm), an additional point was required to capture well the behaviour ($\sigma = 0$ MPa, $w = 5$ mm), decreasing the residual strength with a smooth slope down to zero.

Although the BA curve worked perfectly, in terms of structural design it is not a quick process due to tedious trial and error process. In view of this, a trilinear curve (four points) was proposed, so the MSFRC behaviour was better reproduced than using the MC-2010 constitutive curve. To this end, two changes in the MC-2010 are proposed to improve the results. First, a reduction in σ_2 was necessary so the loss of strength after cracking was similar to the experimental test. Moreover, the f_{Fts} was closer to the values of the experimental test being on the safe side for serviceability requirements. The second change regards the late post-cracking response (CMOD $>$ 2.5 mm), which was sorted out adding an additional point with zero strength and $w = 5.0$ mm. In cases where a huge hardening behaviour was observed (i.e. C40/50–7.5, C40/50–10, C50/60–7.5 and C50/60–10), the third point of the trilinear equation was set for $w < 2.5$ mm to better fit the experimental behaviour.

The results using the trilinear constitutive curve showed a good agreement with the Experimental curve and a huge improvement compared to using the MC-2010 curves. However, implementing the trilinear curve there is a region (1.0 mm $<$ CMOD $<$ 3.0 mm) in which the Model does not fit the experimental curve (being this easily solved by setting an additional point in the σ–w curve) although the proposed trilinear approach is on the safe side for design purposes. The results evidenced that the trilinear curve proposed (based on the BA) considerably improved the results compared to adopting the MC-2010 curve.

5 Conclusions

Numerical simulations of beam flexural post-cracking strength tests have been carried out by means of a non-linear FE model. Based on the outcomes of an experimental research work, the model was used for assessing the MC-2010 FRC constitutive curve with macro synthetic fibres made of polypropylene. Moreover, a BA was performed and a trilinear curved was proposed to better fit the experimental behaviour observed.

Based on the results presented in this research contribution, the following conclusions can be drawn:

- The MC-2010 constitutive curve does not fully reproduce the potential of MSFRC. The maximum post-cracking strength for MSFRC appears for CMOD $>$ 2.5 mm, and this is not captured for the MC-2010 constitutive curve since it is limited to $w_{\text{u}} = 2.5$ mm.
- The MC-2010 constitutive curve slightly overestimates the performance of MSFRC for CMOD $<$ 1.0 mm as well as the loss of strength after cracking.
- Two modifications in the tri-linear constitutive curve induced a huge improvement in order to better represent the post-cracking behaviour of MSFRC: a reduction in σ_2, so the loss of strength after cracking reproduces the drop in strength in the actual behaviour, and progressively reducing the strength of the constitutive equation beyond $w_{\text{u}} = 2.5$ mm up to $w = 5.0$ mm and $\sigma = 0$ MPa.

- The non-corrosive behaviour of the PP fibres justifies an opening of the crack width limitations of MC-2010. The constitutive model developed and presented here accurately represents the flexural material behaviour of MSFRC used in this experimental research. Its deviation from the MC-2010 model enables a significantly better exploitation of the favourable material response, especially at larger CMOD.

Acknowledgements. The first author acknowledges the Spanish Ministry of Science, Innovation and University for providing support through the PhD Industrial Fellowship (DI-17–09390) in collaboration with Smart Engineering Ltd. (UPC's Spin-Off). Funding for the experimental research work in this study was provided by BarChip Inc.

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