Parametric Transformation of Fractal Structures

G. M. Kravchenko[1], L. I. Pudanova[1], A. V. Cherpakov[1,2(✉)], and I. A. Parinov[2]

[1] Don State Technical University, Rostov-on-Don, Russia
alex837@yandex.ru
[2] I. I. Vorovich Mathematics, Mechanics and Computer Science Institute, Southern Federal University, Rostov-on-Don, Russia

Abstract. The paper deals with the implementation of unique design solutions using parametric fractal algorithms. The statement is substantiated that natural forms must be used in the design of unique buildings and structures. Fractal geometry allows the development of new biomorphic structures applicable in information design. The evolution of the shaping of flat and volumetric fractal structures is studied. The structures were modeled using specially developed algorithms, including those simulating the Koch fractal. The deterministic approach allows one to control the process of evolution. On the basis of the study, it was found that the obtained fractal structures of older generations are light without loss of stability. Recommendations are given for the use of fractal and parametric structures for solving a wide range of engineering problems: from the creation of load-bearing structures to the development of a road transport network of an urban environment. The introduction of innovative technologies, such as 3D printing, in conjunction with the new paradigm of parametric shaping, allows one to obtain architecturally and structurally more optimized and highly efficient designs. The result is the solution of structural and spatial problems in architecture, and aesthetic improvement of the designed objects.

Keywords: Modeling · Fractal geometry · Parametric architecture · Fractal structures · Finite element method · Shaping · Innovative technologies · 3D printing

1 Introduction

The ideal geometry, based on natural forms, is the natural pattern of the structure. To recreate biomorphic structures, it is advisable to use fractal geometry, which offers new tools for shaping. The resulting structures are subsequently used in the design of structural elements (columns, arches, foundation slabs, etc.), the frame of the structure, the architectural elements of the object, as well as in the development of the urban network of roads and sidewalks around buildings and structures.

For the first time, iterative algorithms were singled out as part of the new fractal science in the late 1970th. Mandelbrot in his works summarized the results of many scientists of the past years (Poincaré A., Fatou P., Julia G. M., Kantor G., Hausdorff F.)

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and created a new theory, namely fractal geometry [1–4]. Modern researchers use the basic provisions of fractal geometry in various fields, from the analysis of culture and musical works to the development of optimal solutions for landing spacecraft on the Moon surface [5–7]. The fractal approach in philosophy is intertwined with the fractal foundations of space sciences, which allows us to speak about the existence of the fractal paradigm for centuries. Thus, the fractal principle has a genetic connection with both scientific and cultural, artistic thinking.

The use of the fractal principle in architecture will allow creating completely new solutions. Fractals in this case are not just borrowed forms, but mathematical abstractions with natural properties. Parametric transformations form unique fractal structures. When considering parametric objects, creativity from an aesthetic viewpoint later develops into engineering calculations. Parametric design has a strong connection with mathematics and is aimed at obtaining a unique computer digital model of an object [8–17]. Simulation models are created for static and dynamic effects. Such studies will ensure the strength, rigidity and stability of high-rise buildings and structures.

2 Description of Model Parameters

The mathematical basis of parametric structures requires the use of flexible and customizable methods that facilitate the implementation of various unique and innovative buildings and structures. When designing unique buildings and structures, a limited number of geometric shapes are often used. With the help of specialized software systems and innovative programs, it is possible to consider the formation of parametric and fractal structures.

The article deals with the transformation of geometric fractal structures. To analyze the evolution of shaping, various types of structures were modeled using specially developed programs. A new paradigm of shaping is proposed, covering flat and volumetric fractal structures.

Modeling of fractal structures by the finite element method is possible only with the use of additional software modules that allow converting mathematical algorithms into a final fixed form. The authors have developed an algorithm for modeling fractal structures, implemented in the innovative program "Iterations of the Fractal Structure" [18]. The program is designed for finite element modeling of a fractal structure by iteration according to a given algorithm. For example, in the zero module of the program, a flat fractal structure is implemented according to the following algorithm: the initial rod is replaced by a rhombus at the first iteration, then all the rods of the rhombus are replaced by rhombuses at the second iteration, and so on. The algorithm is based on the principle of L-system formation. To ensure the flexibility of modeling different types of structures, input and adjustment of the initial parameters subjected to iteration is provided. Consideration of the evolution of the structure is possible by modeling each stage of the iteration (generation), where the results of the calculations are the coordinates of the structure points and a set of rod finite elements that form a unique flat fractal structure. The C# programming language in the Visual Studio software development environment is used to automatically model the structure of an object and transfer data to the SCAD software package.

With the help of the proposed tools, a large number of geometric fractals are realized. Several modules have been developed for modeling flat fractal structures, based on various algorithms, including those simulating the Koch fractal. To consider the transformation of fractal structures, modules have been developed for obtaining volumetric fractal structures. The approach to shaping remains the same, but the consideration and growth of the structure is already taking place in three-dimensional space.

3 Results and Discussion

Consideration of structures in a plane is at the heart of fractal geometry. Initially, scientists considered algorithms for the implementation of fractals on a plane [19, 20]. This made it possible to determine the main patterns and methodologies, subsequently applicable to real volumetric objects. Within the framework of this study, the transformation of flat fractal structures, based on various deterministic algorithms, including those reflecting the Koch and Sierpinski sequences, is considered. At the same time, the structures are comparable to classical truss structures, especially in the early generations. The older generations of such structures will already be recursive, which leads to the presence of a large number of elements and an openwork structure.

To analyze the evolution of shaping, it is proposed to use the following terminology. Axiom is the zero generation of the structure, the basis for iteration. In all the models under study, a simple rod is taken as an axiom. The rule of a replacement structure, serves to transform the structure into the next generation. The rule is repeated at each iteration. In this case, a deterministic approach is used, that is, at each iteration, one rule is applied to the elements of the structure. Each next iteration is a new generation of the structure. The older the generation, the more elements of the structure it contains. An example of the evolution of shaping based on the classical algorithm is shown in Fig. 1.

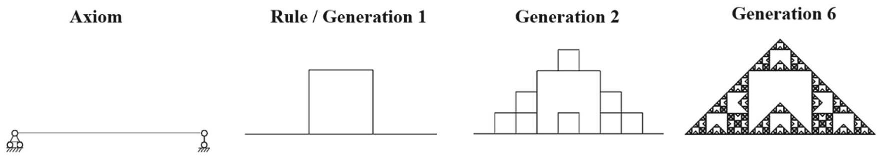

Fig. 1 Algorithm for shaping a flat fractal structure

Finding a rule and selecting parameters for recursive structures applicable in construction is a non-trivial task. Incorrect initial parameters can lead to a structure that has a large number of intersecting elements, preventing the structure to work efficiently and complicating the manufacturing process. The selection of initial parameters and analysis of the evolution of the formation of various structures is a laborious stage in the study of fractal structures.

The first options for modeling a flat fractal structure are presented in [21]. A study was made on the evolution of shaping, an analysis of the forces arising in fractal structures of different generations, and a study of the influence of the construction material used.

In this paper, we propose to consider other structures obtained by changing the initial simulation parameters. An analysis of the obtained structures makes it possible to conditionally divide the forms into two groups. The first group should include fractal structures used as load-bearing structures, and also, if necessary, for solving a wide range of engineering problems. The second group includes fractal structures applicable for solving a wide range of engineering problems, including the reinforcement of solid structures, the design of a pile field, the development of a road transport network, etc.

One of the structures of the first group is the structure obtained on the base of the rule for transforming the axiom into a triangle with a connecting rod (see Fig. 2). For identity with the classical structure of the segment truss in early generations and to increase stability, the connecting rod inside the triangle is made not iterable element. By scaling the structure becomes more detailed and has almost nothing to do with the initial triangle. The structures of the older generations are openwork and light. The fractal structure of the fourth generation can already be used as a supporting structure of the new generation. Fractal modeling was carried out using the finite element method. Initially, a flat fractal structure based on the triangular rule of the first generation consists of 5 rods, the second generation includes 19 rods, and the third generation has 67 rods.

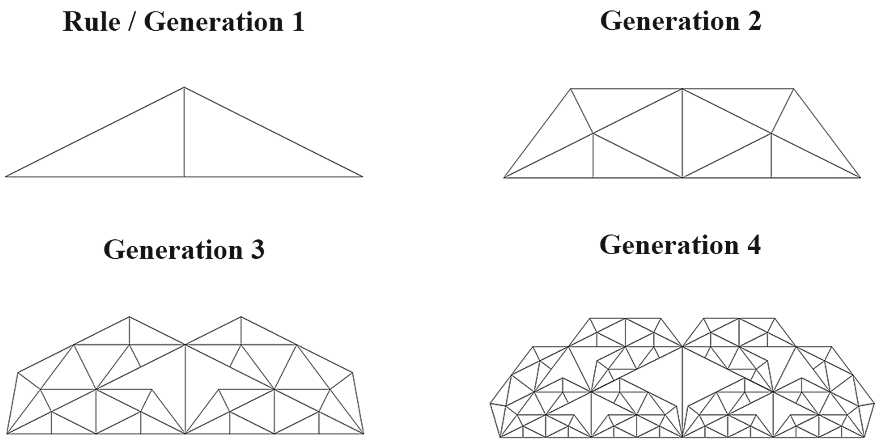

Fig. 2 Flat fractal structure based on the triangular rule

With regard to fractal structures, based on the triangular rule, a numerical experiment was carried out. The goal is to determine the forces that arise in the elements from the action of their own weight and the load of 1 kN/m distributed over the upper chord. The total weight of the structures is taken as a constant parameter for structures of different generations, which makes it possible to analyze the displacements and moments of forces, occurring in the elements of the structures. Adaptation with constant weight is a simple and understandable assumption for analyzing the formation of fractal structures. The main results are present in Table 1.

Table 1 Main numerical results

Generation	Own weight		Own weight and load on upper belt 1 kN	
	Maximum total displacement, mm	Maximum force, kN	Maximum total displacement, mm	Maximum force, kN
1	17.157	0.947	92.166	5.581
2	2.301	0.125	26.49	0.724
3	0.376	0.020	7.931	0.345
4	0.018	0.003	0.328	0.114

The structures of the first and second generations are similar to classical truss structures, while the structures of the third and fourth generations are already recursive and complicated. However, the maximum forces arising in structural elements significantly decrease with the growth of generations. In fact, the use of the same material parameters for structures of the second and fourth generations is not the optimal solution. For thin and complex structures, it is necessary to provide completely different materials that allow rational implementation of fractal forms. The use of innovative digital technologies, including modernized software systems, will allow competently simulating the operating conditions of unique structures.

Structures of the second type have an unusual shape from the viewpoint of classical ideas. The development of the structure in all directions determines its openwork and subtlety in older generations. As an example, a structure is present, the rule of which is based on the transformation of the axiom into a rhombus growing from the center of the rod and occupying half of the entire length of the initial rod (see Fig. 3). All sides of the rhombus are equal; additional rods inside the rhombus are not supposed. This approach is an interpretation of the Koch curve.

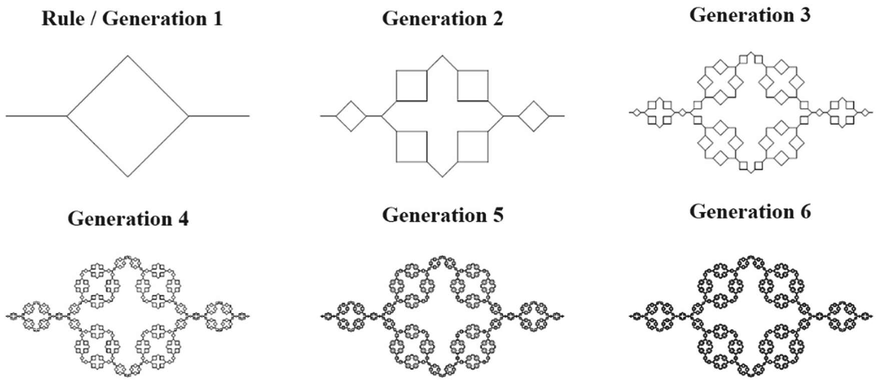

Fig. 3 Flat fractal structure based on the rhombus rule

The first generation is a reflection of the rule. The symmetry of the structure is clearly visible already in the second and third generations. Each element is iterated without any exceptions. The older generations of the structure differ from the structure of the third generation in higher detail. The general form is preserved. This construction is a reflection of the fractal structure growth paradigm. At the same time, the use of such a structure as a supporting structure is complicated by the unevenness of the edges, which requires the use of a special type of coating. This kind of structure can be useful when developing, for example, a pile field or forming a reinforcement scheme for flat structures. On the other hand, a flat fractal structure, based on the rhombus rule, can be used as a road network. The main properties of fractal structures in urban planning are hierarchy, the ability to develop and continuous movement, continuity, belonging to chaos and order at the same time. The development of the urban road transport network according to the fractal scheme is a natural process. The fractal structure under consideration corresponds to the position of the city in the settlement system, functional and compositional tasks, sign-information conditions.

Assuming that a flat fractal is a projection of a three-dimensional structure, the transformation of a fractal is considered based on a geometric algorithm using simple classical methodologies. For the analysis of geometric volumetric structures, the terminology used for flat fractal structures is applied. The deterministic approach allows one to control the process of evolution. The bulky complex structure, used as a rule, is selected depending on the tasks set. Options for constructing volumetric fractal structures, implemented by the finite element method, are shown in Fig. 4. Due to the significant complexity of the structures of older generations and the complexity of displaying the volumetric structure, projections onto the *XOZ*-plane are presented. It should be noted that, in contrast to flat fractal structures, in this case it is necessary to use a more complex rule, which will contribute to the stability of the structure. At the same time, already in the early generations, the structure becomes openwork, which is due to the presence of a large number of elements in the three-dimensional iteration rule. Given the significant complexity of the structure already in the first generation, the second generation consists of 1,552 rods. When iterating, it is assumed that some framing rods are not iterable, which allows for future 3D printing of the structure. The third-generation structure consists of 49,660 rods.

A fractal structure of this type can replace both a classic column, suggesting the rejection of reinforced concrete structures, and can be used as a supporting horizontal structure, namely a bulk beam or truss. Another option is to use older generations of volumetric fractal structures as outrigger floor structures.

The use of a three-dimensional fractal structure instead of a solid material, such as a reinforced concrete beam, is possible only with the introduction of innovative materials and 3D printing technology. The high detail and subtlety of the constituent elements do not allow the structure to be realized manually in modern conditions. An assumption is made about the use of industrial robots, combined with a welding machine and developed software. This combination makes it possible to implement 3D printing of 3D rigid and complex structures from various metals or their alloys. The advantage of using innovative technologies for printing structures is the flexibility, scalability and economy

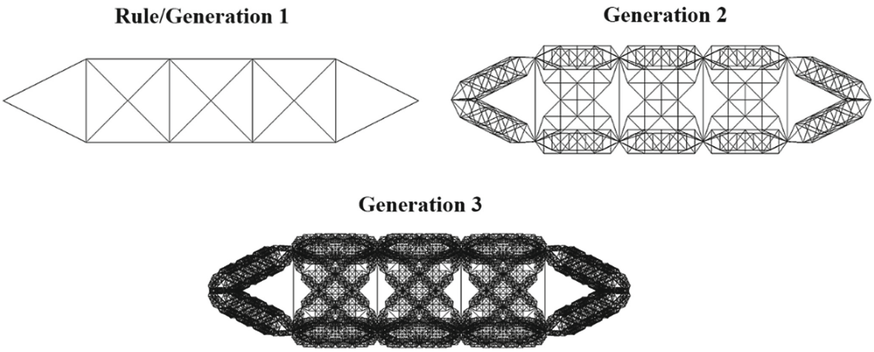

Fig. 4 Evolution of volumetric geometric fractal structures

of the material used [22–24]. Creative robotic production solutions will allow not only modeling fractal structures of any complexity, but also using them in the construction and operation of unique buildings and structures.

4 Discussion

The method of designing buildings and structures is significantly changing with the help of mathematics, algorithms and digital technologies. The use of parametric foundations and the paradigm of fractal shaping allows one to create completely new architectural objects that are distinguished by aesthetics, ergonomics and lightness, without losing stability.

The use of recursive planar and volumetric structures represents a new stage of research in scientific theory, which includes the rapid development of areas adjacent to structural mechanics. When modeling, special attention should be paid to the analysis of mathematical patterns and algorithmization of the design process. The balance between mathematics, nature and modern mechanisms allows us to create completely new efficient and reliable hierarchical structures.

5 Conclusion

The principle of hierarchical or recursive design is applied to improve the mechanical efficiency of systems. Parametric modeling, using the finite element method, makes it possible to implement fractal principles in new types of structures. The design solutions proposed in this paper are non-standard and unique.

It has been established that a multi-component volumetric structure allows carrying a significant load and being stable, while having a relatively low weight. It is recommended to use innovative materials, including those based on carbon fiber, in order to be able to implement structures of older generations.

The main conclusions, obtained in the analysis of the formation of fractal structures, implemented on the basis of geometric algorithms are the following:

(i) the older generations of the structure differ from the structures of the younger generation in higher detail and complexity;
(ii) maintaining the general mechanism of shaping, with each iteration the structure grows and becomes openwork, aesthetically lighter;
(iii) an increase in the number of elements in the development of the iteration rule contributes to the formation of light and stable structures already in the early generations;
(iv) adding not iterable elements to the rule allows one to adjust the structure of future generations and change the stability indicators of the structure;
(v) fractal structures of various types can be used in construction for various purposes: as load-bearing horizontal and vertical structures, when developing the structure of a pile field, forming a reinforcement scheme for horizontal structures, etc.

Variation of parameters and analysis of the stressstrain state of structures of different generations and shapes, performed to solve specific problems, leads to a more efficient design.

The collaboration of parametric structures, digital technologies and innovative materials makes it possible to implement new ideas when creating unique buildings and structures. It is recommended to introduce modern 3D printing technologies for the implementation of biomorphic structures that facilitate the building and reduce the load on the base. At the same time, fractal and parametric structures should be used to solve a wide range of engineering problems: from the creation of load-bearing structures to the development of a road transport network in an urban environment.

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