



A Study on the Effect of the Grip Coefficient on the Slip Coefficient When Braking the Tractor Semi-trailer on a Straight Road at a Speed of 80 km/h

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Abstract. When a vehicle brakes, slip coefficient occurs at the tire print between the tire and the ground, which affects the braking efficiency and safety of the vehicle. The authors establish a dynamic model of the tractor semi-trailer and use Matlab-Simulink software to simulate the slip coefficient. The results show that when a tractor semi-trailer brakes with the torque $M_B = 80\% M_{Bmax}$ on three types of roads with grip coefficients of $\varphi_{xmax} = [0.8, 0.9, 1.0]$ at a speed of 80 km/h, the slip coefficient is less than 5%, the tractor semi-trailer moves stably and safely. When a tractor semi-trailer brakes on three types of the road with grip coefficients of $\varphi_{xmax} = [0.5, 0.6, 0.7]$, the slip coefficient is 100%, and the tractor semi-trailer moves unstably and unsafely.

Keywords: Grip coefficient · Grip-ability coefficient · Slip coefficient · Tractor semi-trailer

1 Introduction

During a vehicle is driving or braking, the tires often slip, affecting its performance. The slip coefficient of a driving tire and a braking tire is determined by the following formulae (1, 2) [1, 2]:

$$s_d = \frac{R_{dyn}\omega_w - V_x}{R_{dyn}\omega_w} \text{ in accelerating;} \quad (1)$$

$$s_b = \frac{R_{dyn}\omega_w - V_x}{V_x} \text{ in braking,} \quad (2)$$

where s_d is the slip coefficient in accelerating, s_b is the slip coefficient in braking, R_{dyn} is the dynamic radius of the tire, ω_w is the tire's angular velocity, and V_x is the tire's forward velocity. If the vehicle is accelerating then $R_{dyn}\omega_w > V_x$ and $0 < s_d < 1$; while the vehicle is braking, $R_{dyn}\omega_w < V_x$ and $-1 < s_b < 0$.

2 The Dynamics Model

The tractor semi-trailer consists of a three-axle tractor and a three-axle semi-trailer linked together by a fifth wheel. The authors use the method of structural separation of a multibody system (MBS) and Newton-Euler methods to build a three-dimensional (3D) dynamics model of the tractor semi-trailer. The authors hypothesize that i is the axle number of the tractor semi-trailer, $i=1-6$ and j indicates the left wheel ($j=1$) or right wheel ($j=2$) of the tractor semi-trailer. The dynamic model of the tractor and semi-trailer is shown in Fig. 1. Based on the Newton-Euler equation, the authors have written the system of dynamic equations of the tractor and semi-trailer in the Oxy-plane as follows [3-5]:

$$(m_{c1} + \sum_1^3 m_{Ai})(\ddot{x}_{c1} - \dot{\psi}_{c1}\dot{y}_{c1}) = F_{x11}\cos\delta_{11} + F_{x12}\cos\delta_{12} - F_{y11}\sin\delta_{11} - F_{y12}\sin\delta_{12} + (F_{x2j} + F_{x3j}) - F_{wx1} - F_{kx1} \quad (3)$$

$$(m_{c1} + \sum_1^3 m_{Ai})(\ddot{y}_{c1} + \dot{\psi}_{c1}\dot{x}_{c1}) = F_{x11}\sin\delta_{11} - F_{ky1} + F_{x12}\sin\delta_{12} + F_{y11}\cos\delta_{11} + F_{y12}\cos\delta_{12} + (F_{y2j} + F_{y3j}) \quad (4)$$

$$J_{zc1}\ddot{\psi}_{c1} = [F_{x1j}\sin\delta_{1j} + F_{y1j}\cos\delta_{1j}]l_1 + (F_{x12}\cos\delta_{12} - F_{x11}\cos\delta_{11})b_1 + F_{ky1}l_{k1} + (F_{y11}\sin\delta_{11} - F_{y12}\sin\delta_{12} - F_{y2j}l_2 - F_{y3j}l_3 + (F_{x22} - F_{x21})b_2 + (F_{x32} - F_{x31})b_3 \quad (5)$$

$$(m_{c2} + \sum_4^6 m_{Ai})(\ddot{x}_{c2} - \dot{\psi}_{c2}\dot{y}_{c2}) = F_{x4j} + F_{x5j} + F_{x6j} + F_{kx2} \quad (6)$$

$$(m_{c2} + \sum_4^6 m_{Ai})(\ddot{y}_{c2} + \dot{\psi}_{c2}\dot{x}_{c2}) = F_{ky2} + F_{y4j} + F_{y5j} + F_{y6j} \quad (7)$$

$$J_{zc2}\ddot{\psi}_{c2} = (F_{x42} - F_{x41})b_4 + (F_{x52} - F_{x51})b_5 + (F_{x62} - F_{x61})b_6 + F_{ky2}l_{k2} - F_{y4j}l_4 - F_{y5j}l_5 - F_{y6j}l_6 \quad (8)$$

The system of dynamic equations of the tractor and semi-trailer in the Oxz-plane is written as follows [3-5]:

$$m_{ci}(\ddot{z}_{ci} - \dot{\varphi}_{ci}\dot{x}_{ci}) = F_{Cij} + F_{Kij} - F_{kz} \quad (9)$$

$$J_{yci}\ddot{\varphi}_{ci} = -(F_{Cij} + F_{Kij})l_i + F'_{xij}(h_{ci} - r_i) + F_{kxi}(h_{ci} - h_{ki}) + F_{kzi}l_{ki} + M_{ij} \quad (10)$$

The dynamic equations of the tractor semi-trailer in the Oyz-plane are written as follows [3-5]:

$$J_{xci}\ddot{\beta}_{ci} = \sum_{i=1}^{i=6} (F_{C2i} + F_{K2i} - F_{C1i} - F_{K1i})w_i + \sum_{i=1}^{i=6} F_i(h_c - h_{Bi}) - M_{kx} \quad (11)$$

$$m_{Ai}(\ddot{z}_{Ai} + \dot{\beta}_{Ai}\dot{y}_{Ai}) = F_{CLij} + F_{KLij} - F_{Cij} - F_{Kij} \quad (12)$$

$$m_{Ai}(\ddot{y}_{Ai} - \dot{\beta}_{Ai}\dot{z}_{Ai}) = F_i + F_{yij} + F_{yij} \quad (13)$$

$$J_{Axi}\ddot{\beta}_{Ai} = (F_{Ci1} + F_{Ki1} - F_{Ci2} - F_{Ki2})w_i + (F_{CLi2} + F_{KLi2} - F_{CLi1} - F_{KLi1})b_i - F_{yij}(r_{ij} + \xi_{Aij}) + F_i(h_{Bi} - r_i) \tag{14}$$

A semi-trailer with six axles is considered to have twelve tires, so the dynamic equations for the twelve tires in the longitudinal plane are written as follows [3–5]:

$$J_{Ayij}\ddot{\varphi}_{ij} = M_{Aij} - M_{Bij} - F_{xij}r_{dij} \tag{15}$$

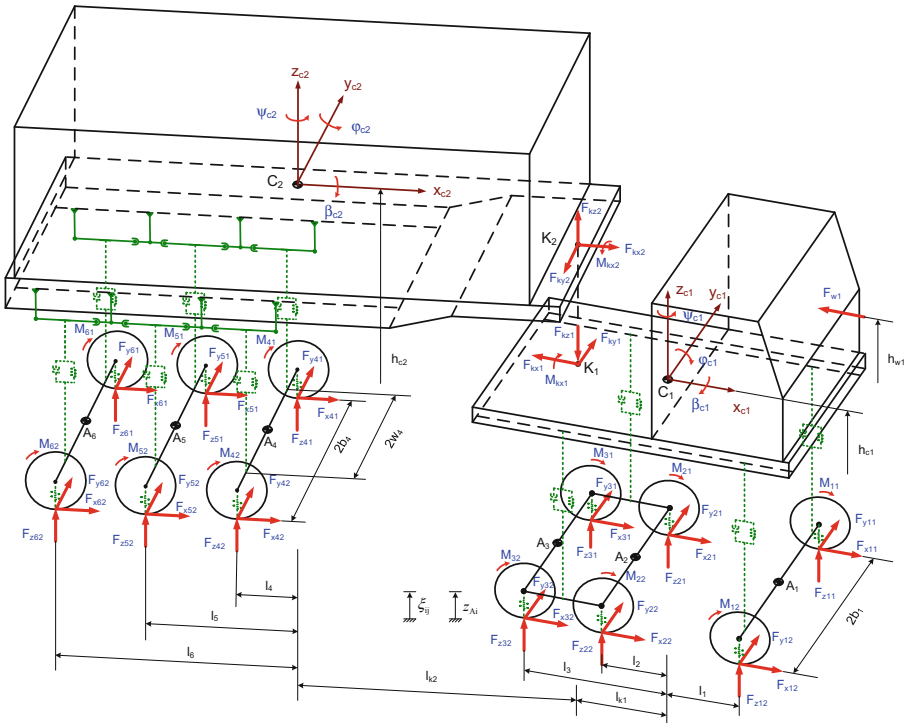


Fig. 1 Dynamics model of the semi-trailer. The list of symbols and abbreviations with necessary explanation is present in Table 1

3 Simulation with Matlab-Simulink Software

Matlab-Simulink software is used to simulate the influence of the grip coefficient on the slip coefficient when braking of the tractor semi-trailer at $V_0 = 80$ km/h. Simulation conditions are as follows. The tractor semi-trailer moves on a straight road at $V_0 = 80$ km/h and then braked with a torque of $M_B = 80\%M_{Bmax} = 80\%F_{z,st}\varphi_{xmax}r_d$. The simulation is performed on six types of roads with different grip coefficients of $\varphi_{xmax} = [0.5, 0.6, 0.7, 0.8, 0.9, 1.0]$. In the graphs, line 1 corresponds to a grip coefficient of 0.5; line 2 corresponds to a grip coefficient of 0.6; line 3 corresponds to a grip coefficient of

0.7; line 4 corresponds to a grip coefficient of 0.8; line 5 corresponds to a grip coefficient of 0.9; line 6 corresponds to a grip coefficient of 1.0. The input function graphs of the simulation model such as driving torque, braking torque of front tires, middle tires, and rear tires are shown in Fig. 2.

Table 1 List of symbols and abbreviations

Symbols	Units	Explain
i		Axle number of the tractor semi-trailer, $i = 1-6$
j		$j = 1$ is left wheel; $j = 2$ is right wheel; $j = 1, 2$
x_c, y_c, z_c	m	Displacement in the $B(Cx_c y_c z_c)$ coordinate system
R_{dyn}	m	Dynamic radius of a tire
δ	Degree	Front-wheel turn angle
l_i	m	Longitudinal distance of axle i from the mass center
h_{c1}, h_{c2}	m	Height of mass center from the ground
b	m	Lateral distance of a wheel from the longitudinal x -axis
φ_{ij}	rad	Rotation angle of the ij -wheel
β, φ, ψ	Degree	Rotation angle of the tractor body around the x -, y -, z -axis
m, mA	kg	Sprung mass and un-sprung mass of the tractor semi-trailer
J_x, J_y, J_z	kg·m ²	Moment of inertia about the x -, y -, z -axis of the sprung mass
J_{Ayij}	kg·m ²	Moment of inertia about the y -axis of the ij -wheel
FC_{ij}	N	Suspension elastic force of the ij -wheel
FK_{ij}	N	Suspension damping force of the ij -wheel
FCL_{ij}	N	Tire elastic force of the ij -wheel
F_{xij}	N	Longitudinal force of the ij -wheel
F_{yij}	N	Lateral force of the ij -wheel
F_{zij}	N	Vertical force of the ij -wheel
F_{Gij}	N	Static weight of the ij -wheel
M_{Aij}, M_{Bij}	N·m	Driving torque and braking torque of the ij -wheel

The braking force of the front, middle and rear tires at 80 km/h are shown in Fig. 3. Looking at Fig. 3, we see that, when the tractor semi-trailer moves on a straight road with $V_0 = 80$ km/h and then brakes with a torque of $M_B = 80\%M_{Bmax}$, the braking force of the front tires is the same, $F_{x11} = 19.5$ kN on six types of roads.

For the tires of the middle axle, when the tractor semi-trailer brakes on four types of roads with grip coefficients of $\varphi_{xmax} = [0.7, 0.8, 0.9, 1.0]$, the braking force of the middle tires is $F_{x31} = 27.5$ kN. When the tractor semi-trailer brakes on the road with a grip coefficient of $\varphi_{xmax} = 0.6$, the braking force of the middle tires is $F_{x31} = 20.5$ kN. When the tractor semi-trailer brakes on the road with a grip coefficient of $\varphi_{xmax} = 0.5$, the braking force of the middle tires is $F_{x31} = 16.5$ kN.

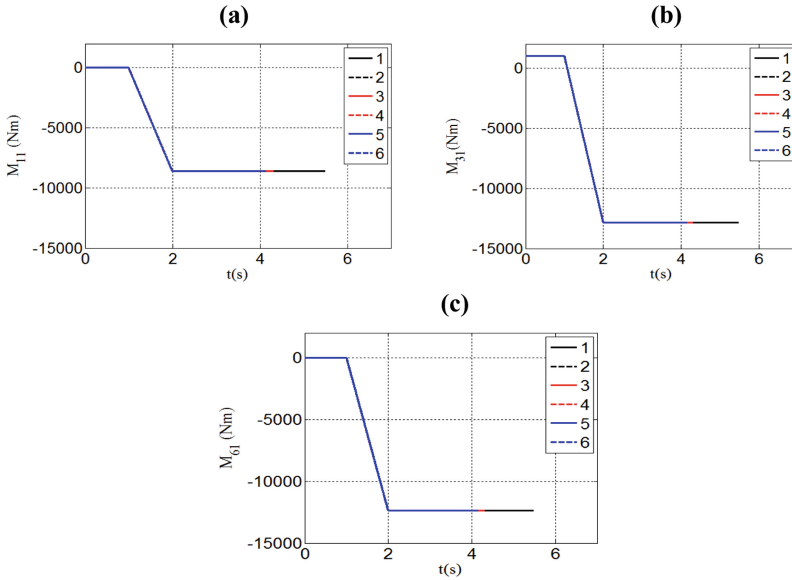


Fig.2 Driving and braking torque of the front (a), middle (b) and rear (c) tires

The same situation is for rear axle tires, when the tractor semi-trailer brakes on two types of roads with grip coefficients of $\varphi_{xmax} = [0.9, 1.0]$, the braking force of the rear tires is $F_{x61} = 25\text{kN}$. When the tractor semi-trailer brakes on two types of roads with grip coefficients of $\varphi_{xmax} = [0.7, 0.8]$, the braking force of the rear tires is $F_{x61} = 20.5\text{ kN}$. When the tractor semi-trailer brakes on the road with grip coefficients of 0.5 and 0.6, the braking forces of the rear tires are 14.5 kN and 17.5 kN, respectively.

The slip coefficients of the front, middle and rear tires after the tractor semi-trailer brakes with the torque $M_B = 80\%M_{Bmax}$, on six types of roads at a speed of 80 km/h are shown in Fig. 4. For the front axle, when the tractor semi-trailer brakes on six types of roads, the slip coefficient of the front tires is always less than 5%, the tractor semi-trailer moves stably and safely.

With the middle axle, when the tractor semi-trailer brakes on four types of roads with grip coefficients of $\varphi_{xmax} = [0.7, 0.8, 0.9, 1.0]$, the slip coefficient is about 5%, the tractor semi-trailer moves stably, and safely. If two types of roads with grip coefficients of $\varphi_{xmax} = [0.5, 0.6]$, the slip coefficient is 100%, the tires are completely locked, the tractor semi-trailer moves unstably and unsafely.

The same, for the rear axle, when the tractor semi-trailer is braked on three types of roads with grip coefficients of $\varphi_{xmax} = [0.8, 0.9, 1.0]$, the slip coefficient is less than 5%, the tractor semi-trailer moves stably, and safely. If three types of the road with grip coefficients of $\varphi_{xmax} = [0.5, 0.6, 0.7]$, the slip coefficient is 100%, the tires are completely locked, the tractor semi-trailer moves unstably and unsafely.

The grip-ability coefficients of the front, middle and rear tires, when the tractor semi-trailer brakes with the torque $M_B = 80\%M_{Bmax}$, on six types of roads at a speed of 80 km/h are shown in Fig. 5. For the front tires, when the tractor semi-trailer brakes

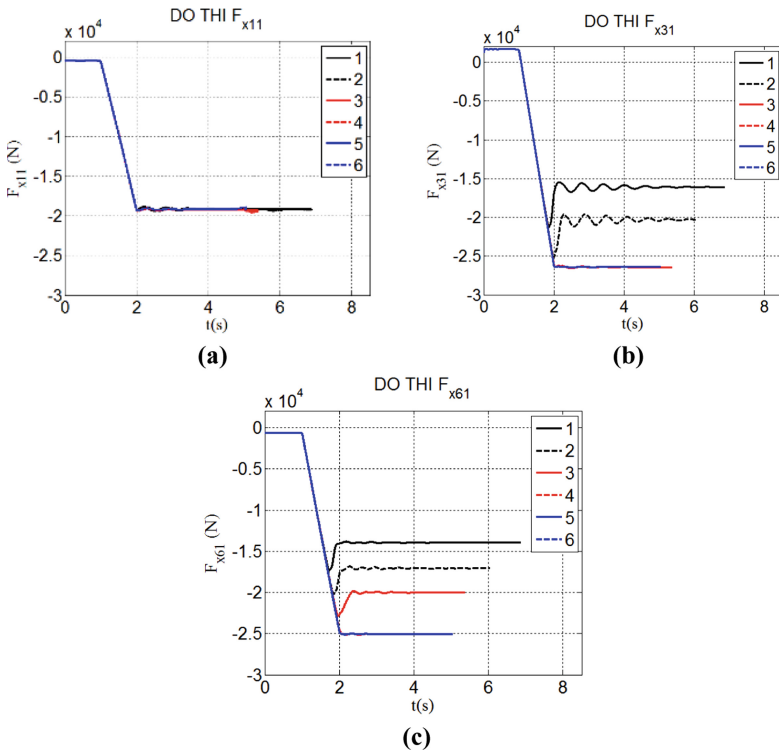


Fig. 3 Braking force of the front (a), middle (b) and rear (c) tires

with $M_B = 80\%M_{Bmax}$ on six types of roads, the grip-ability coefficient is 40–90%. We can take advantage of the tire’s grip-ability to increase the braking force to increase the braking efficiency of the vehicle. For the middle tires, when the tractor semi-trailer brakes with $M_B = 80\%M_{Bmax}$ on two types of roads with grip coefficients of $\varphi_{xmax} = [0.5, 0.6]$, the braking force is greater than the grip force, the tires are slipped, and the grip-ability coefficient reduces from 100 to 80%. If two types of roads with $\varphi_{xmax} = [0.7, 0.8]$, the braking force is almost equal to the grip force, the grip-ability coefficient is 80–95%. When the tractor semi-trailer is braked on two types of roads with $\varphi_{xmax} = [0.9, 1.0]$, the braking force is less than the grip force, the grip-ability coefficient is 60–70%, we can increase the braking force to increase the braking efficiency of the vehicle.

For the rear tires, when the tractor semi-trailer is braked with $M_B = 80\%M_{Bmax}$ on three types of roads with grip coefficients of $\varphi_{xmax} = [0.5, 0.6, 0.7]$, the braking force is greater than the grip force, the tires are slipped, the grip-ability coefficient is reduced from 100 to 80%. If three types of roads with grip coefficient are $\varphi_{xmax} = [0.8, 0.9, 1.0]$, the braking force is almost equal to the grip force, the grip-ability coefficient is 80–98%.

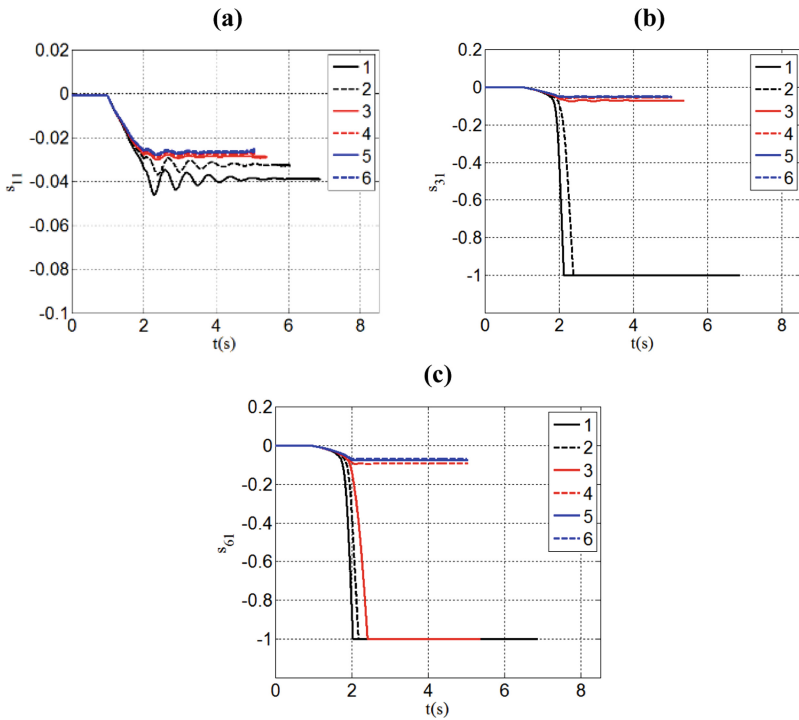


Fig. 4 Slip coefficient of the front (a), middle (b) and rear (c) tires

4 Conclusions

The tractor semi-trailer brakes with the torque $M_B = 80\%M_{Bmax}$ on three types of roads with grip coefficients are $\varphi_{xmax} = [0.8, 0.9, 1.0]$ at a speed of 80 km/h, the braking force is almost equal to the grip force, the grip-ability coefficient is 80–98%, the slip ratio is less than 5%, the tractor semi-trailer moves stably and safely. When the tractor semi-trailer brakes on three types of the road with a grip coefficient of $\varphi_{xmax} = [0.5, 0.6, 0.7]$, the braking force is greater than the grip force, the tires are completely locked, the slip coefficient is 100%, the grip-ability coefficient is reduced by about 20%, the tractor semi-trailer moves unstably and unsafely.

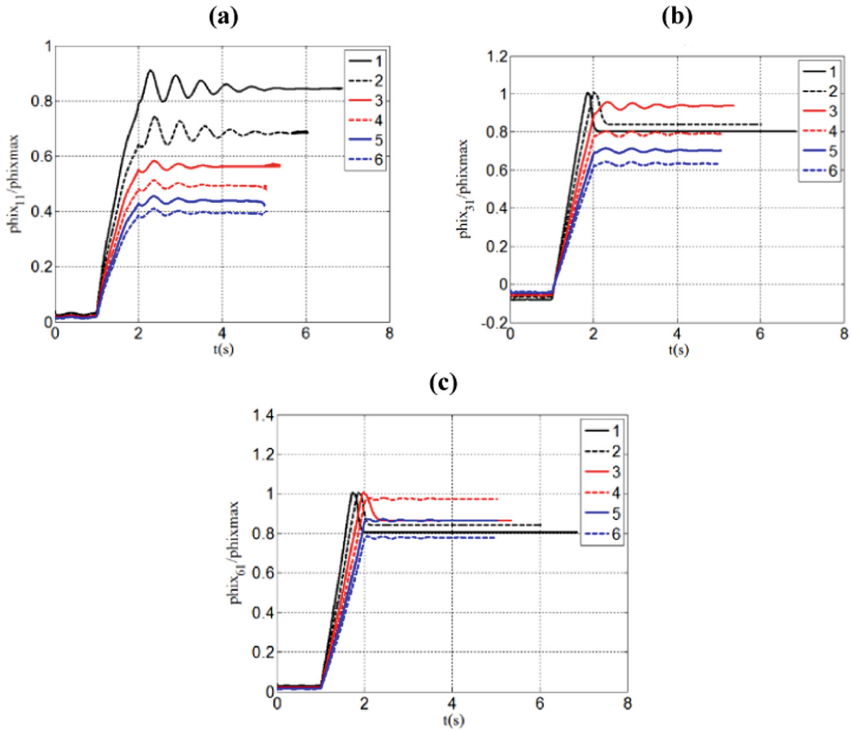


Fig. 5 Grip-ability coefficient of the front (a), middle (b) and rear (c) tires

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