



# Theoretical Consideration and Experimental Studies of Bending Deformations of Inhomogeneous Ferroceramic Plates

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**Abstract.** A theoretical analysis of the models of inhomogeneous piezotexture, describing bending deformations in an external electric field is carried out. Preliminary studies of bending quasi-static deformations and bending vibrations, depending on temperature, surface conditions and polarization switching have been performed. Comparisons of theoretical and experimental characteristics are carried out. Estimates of the parameters of surface layers are given.

**Keywords:** Bending deformations · Polarization · Piezoelectric effect · Electrostriction · Unipolarity · Surface layers · Resonance frequency

## 1 Introduction

Bending vibrations of inhomogeneous ferro piezoelectric plates (bimorphs and monomorphs) have attracted the attention of researchers in solving scientific problems (in particular, mathematical modeling) and important practical applications. Mathematical and finite element models of non-uniform polarized piezotransducer are discussed, including implementation in finite-element (FE) package ACELAN. The results of numerical studies of single-layer and multilayered transducers are present. The developed modeling workflow was designed relative to manufacturing process of multilayered transducers. The results were compared with theoretical model of piezoplate and numerical experiments, performed in ANSYS [1]. Non-uniformly polarized piezoceramic materials can be used in effective energy harvesting devices. Axisymmetric and plane models of electric elastic bodies were studied using applied theory and finite element method (FEM). Applied theory for devices made of parts with longitudinal and transverse polarization was developed. It was based on bending of electric elastic plates models [2]. Based on the applied theory of oscillations of a multilayer plate, which takes into account the nonlinear distribution of the electric potential in the piezoelectric layers, a study of the stress-strain state and electric field of the cantilever bimorph was carried out. Such a nonlinear dependence arises when solving problems of finding the natural

resonance frequencies and modes of vibration, or in the case of forced vibrations during mechanical excitation, for some electrical boundary conditions [3].

An applied theory of cylindrical bending vibrations of a bimorph plate is developed, which takes into account the nonlinear distribution of the electric potential in piezoelectric layers. Finite-element analysis of this problem showed that such a distribution takes place by solving the problems of calculating the resonance frequencies and vibration modes or in the case of forced oscillations caused by mechanical excitation, when the electric potentials on the electrodes are zero. The quadratic distribution of the electric potential, adopted in the work, showed good consistency of the results with finite-element calculations for natural and steady-state oscillations for a given potential difference when the electric potential distribution is close to linear [4].

In this study, the presence of piezotexture inhomogeneities is detected by bending deformations, excited by an electric field. In this regard, we have undertaken studies of bending deformations and bending vibrations, excited by an electric field in nominally homogeneous piezoceramic plates. The results of the research are presented in this article. Bending resonant oscillations, depending on temperature, the amplitude of exciting field, the state of surface during polarization switching and aging are studied. Methods for estimating the parameters of surface layers, according to experimental studies, have been developed.

### 1.1 Research Purpose

The results of the study of bending vibrations of initially homogeneous ferroceramic plates under the action of alternating electric fields are presented. The theoretical analysis of models of inhomogeneous piezotexture, describing bending deformations of ferroplates in an external electric field, is carried out. The model [5] is considered in detail, according to which bending deformations are caused by the presence of counter-polarized near-electrode layers in the plate. Such layers can be formed in the area of the rectifying contact by processing the sample with a strong alternating field. The disadvantages of such a model are discussed.

### 1.2 Research Scope

At this study, we consider the following frameworks of the problem:

- (i) description of bending deformations of piezoactive plates;
- (ii) development of a model for describing bending deformations.

## 2 Research Method

### 2.1 Continuous Formulation of the Problem (First Model)

According to [5], the possibility of obtaining bending deformations of initially homogeneous piezoceramic plates is due to their semiconductor properties. The presence of a metal (electrode)—semiconductor contact leads to the application of a potential difference to the electrodes of a ferroceramic plate (monomorph). It causes the appearance

of an inhomogeneous electric field in thickness, the intensity of which increases significantly near one of the electrodes (depending on the sign of the field and the type of conductivity). With sufficiently strong fields, the near-electrode layer is polarized. Obviously, the processing of such a sample with both AC and polarizing DC field leads to the formation of counter-polarized near-electrode layers (an analogue of a bimorph), with which the occurrence of bending deformations is associated.

1. Let us assume that bending deformations under the action of an electric field are due to the inhomogeneous distribution over the thickness of the sample of the piezomodule  $d_{31}$ . In the case of the condition:  $d_{31}(-z) = -d_{31}(z)$ , satisfied for the function  $d_{31}(z)$ , the application of an electric field will lead to a clean bending of the plate (without deformation of the median plane).

Let us consider, in accordance with [6], the case of a long thin rectangular plate. The deformation at an arbitrary point, located at a distance  $z$  from the median plane (due to bending), is equal to.

$$u_{11} = -z \frac{\partial^2 w}{\partial x^2}, \tag{1}$$

where  $w(x)$  is the displacement of the median plane in the direction of the  $z$ -axis. We assume that a potential difference is set on the electrodes of the plate:

$$V = \phi\left(\frac{h}{2}\right) - \phi\left(-\frac{h}{2}\right), \tag{2}$$

where  $h$  is the thickness of the plate.

In this case, the induction and the field in the plate satisfy the equations:

$$\operatorname{div} \vec{D} = 0, \quad \vec{E} = \operatorname{grad} \phi, \tag{3}$$

moreover, for the length of a rectangular plate, the field component  $E_2 = 0$ . We will also neglect the field component  $E_1$ , assuming that the bending vibrations are small and, therefore,  $E_1/E_3 \ll 1$ , (the estimates show this result for an oscillating plate),  $E_1/E_3 \approx h/l$ , where  $h$  and  $l$  are the thickness and length of the plate, respectively. Thus, Eq. (3) take the form:

$$\frac{\partial D_3}{\partial z} = 0; \quad E_3 = -\frac{\partial \phi}{\partial z}. \tag{4}$$

From (4), in particular, it follows:

$$D_3 = D_3(x); \quad \int_{-\frac{h}{2}}^{\frac{h}{2}} E_3 dz = -V. \tag{5}$$

Let us now consider induction, field strength and mechanical moment in the plate using a system of piezoelectric effect equations:

$$\begin{aligned} D_3 &= \varepsilon_{33}^\sigma E_3 + d_{31} \sigma_{11}; \\ u_{11} &= d_{31} E_3 + s_{11}^E \sigma_{11}. \end{aligned} \tag{6}$$

Excluding  $\sigma_{11}$  from the first Eq. (6) and integrating the resulting equation with respect to  $z$ , taking into account (1), (5) we find:

$$D_3(x) = -\frac{V}{s_{11}^E a} - \frac{b}{s_{11}^E a} \frac{\partial^2 w}{\partial x^2}, \tag{7}$$

where the designations are introduced:

$$a = \int_{-h/2}^{h/2} \frac{dz}{\Delta(z)}; \quad b = \int_{-h/2}^{h/2} \frac{z d_{31}(z)}{\Delta(z)} dz; \quad \Delta(z) = \varepsilon_{33}^\sigma s_{11}^E - d_{31}^2(z). \tag{8}$$

The moment in the plate per plate of unit width is defined as  $M_1(x) = \int_{-h/2}^{h/2} z \sigma_{11} dz$ . Using (6) and performing the necessary calculations, we find:

$$M_1(x) = -\varepsilon_{33}^\sigma \lambda \frac{\partial^2 w}{\partial x^2} - b D_3(x), \tag{9}$$

where

$$\lambda = \int_{-h/2}^{h/2} \frac{z^2}{\Delta(z)} dz. \tag{10}$$

Substituting in (9) the expression for  $D_3$  from (7), we finally find the expression for the moment:

$$M_1 = -D^V \frac{\partial^2 w}{\partial x^2} + gV, \tag{11}$$

where the designations are introduced:

$$D^V = \varepsilon_{33}^\sigma \lambda - \frac{b^2}{a s_{11}^E}; \quad g = \frac{b}{a s_{11}^E}. \tag{12}$$

The  $D^V$  coefficient characterizes the cylindrical rigidity of the plate with a fixed potential difference at its electrodes.

The expression for  $E_3$  can be derived from (6) by using (7) in the form:

$$E_3(x, z) = -\frac{1}{a \Delta(z)} + \frac{1}{\Delta(z)} \left[ z d_{31}(z) - \frac{b}{a} \frac{\partial^2 w}{\partial x^2} \right]. \tag{13}$$

The coefficients  $D^V$  and  $g$  are the main parameters, characterizing the bending deformations of piezoceramic plates. We give their expressions for some special cases of the function  $d_{31}(z)$ .

2. For a plate with counter-polarized layers and a total thickness  $h_S$ , the function  $d_{31}(Z)$  has the form:

$$d_{31}(z) = \begin{cases} -d_{31}, & -\frac{h}{2} \leq z \leq -\frac{h}{2} + \frac{h_S}{2}; \\ 0, & -\frac{h}{2} + \frac{h_S}{2} \leq z \leq \frac{h}{2} - \frac{h_S}{2}; \\ d_{31}, & \frac{h}{2} - \frac{h_S}{2} \leq z \leq \frac{h}{2}. \end{cases} \tag{14}$$

The expressions for  $D^V$  and  $g$  have the form:

$$D^V = \frac{h^3}{12s_{11}^E(1-K^2)} \left\{ 1 - K^2 \left[ \tau^3 + \frac{3(1-\tau^2)^2}{4(1-K^2\tau)} \right] \right\} \quad (15)$$

$$g = \frac{d_{31}}{s_{11}^E} \cdot \frac{h}{4} \cdot \frac{(1-\tau^2)}{1-K^2\tau}, \quad \text{где } \tau = 1 - \frac{h_S}{h} 4, \quad K^2 = \frac{d_{31}^2}{\varepsilon_{33}^{\sigma} s_{11}^E}.$$

For the bimorph,  $h_S = h$  and from (15), we obtain:

$$D^V = \frac{h^3}{12s_{11}^E} \cdot \frac{1 - \frac{3}{4}K^2}{1 - K^2}; \quad g = \frac{d_{31}}{s_{11}^E} \cdot \frac{h}{4} \quad (16)$$

In the case of thin layers  $h_S \ll h$ ,  $\tau = 1 - \delta$  at  $\delta \ll 1$ , we have:

$$D^V = \frac{h^3}{12s_{11}^E} \cdot \left( 1 + \frac{3\delta K^2}{1 - K^2} \right); \quad g = \frac{d_{31}}{s_{11}^E} \cdot \frac{h}{4} \cdot \frac{2\delta}{1 - K^2} \quad (17)$$

Here and further, it is assumed that the permittivity and elastic compliance do not depend on  $z$ . For the linear dependence:  $d_{31}(z) = (2d_{31}/h)z$ , we get:

$$D^V = \frac{h^3}{4s_{11}^E} \cdot \frac{1}{K^2} \left[ 1 - \frac{2K}{\ln\left(\frac{1+K}{1-K}\right)} \right]; \quad g = \frac{d_{31}}{s_{11}^E} \cdot \frac{h}{2K^2} \left[ 1 - \frac{2K}{\ln\left(\frac{1+K}{1-K}\right)} \right] \quad (18)$$

With weak piezoactivity, when  $K \ll 1$ ,  $D^V = \frac{h^3}{12s_{11}^E}$ ;  $g = \frac{d_{31}}{s_{11}^E} \cdot \frac{h}{6}$ , where  $d_{31}$  is the value of the piezo module near the electrode.

As can be seen from the above expressions, the cylindrical rigidity of the plate depends on the distribution function of the piezo module  $d_{31}$  over the thickness of the sample. At  $K = 0$ , all expressions for  $D^V$  transform to the usual expression for the cylindrical stiffness of a non-piezoactive plate.

3. Consider the static bending of a cantilever plate. We will assume that a constant potential difference is applied to its electrodes  $V$ . The equilibrium equation of the plate has the form:

$$\frac{\partial^2 M_1}{\partial x^2} = 0. \quad (19)$$

We have from (11):

$$\frac{\partial^4 w}{\partial x^4} = 0. \quad (20)$$

Solving this equation together with boundary conditions:

$$w(0) = 0; \quad M_1(l) = 0;$$

$$\frac{\partial w}{\partial x} \Big|_0 = 0; \quad \frac{\partial M_1}{\partial x} \Big|_l = 0, \quad (21)$$

we find an expression for the displacement of the median plane of the plate:

$$w = \frac{gV_0}{2D^V}x^2. \tag{22}$$

From (13) and (22), in particular, it follows that  $E_3$  is a function only of  $z$ , and the condition:  $E_1 = 0$  is exactly fulfilled in the case of static bending. For the free edge of the plate, we have:

$$w = \frac{gl^2}{2D^V}V_0. \tag{23}$$

Thus, for the model under consideration, the displacement of the plate edge is directly proportional to the potential difference applied to the electrodes, provided that  $g$  and  $D^V$  do not depend on  $V_0$ .

4. Let us now consider the bending vibrations of the plate under the action of AC voltage. The equation of plate vibrations has the form:

$$\rho \frac{\partial^2 w}{\partial t^2} - \frac{1}{h} \frac{\partial^2 M_1}{\partial x^2} = 0. \tag{24}$$

Substituting expression (11) in (24) and considering that for steady-state oscillations  $w = w(x)\exp(i\omega t)$ , we find the equation for  $w(x)$ :

$$\frac{\partial^4 w}{\partial x^4} - \kappa^4 w = 0; \quad \kappa^4 = \frac{\rho h}{D^V} \omega^2. \tag{25}$$

Solving (25) together with the boundary conditions (21), we find for  $w(x)$ :

$$w(x) = -\frac{gV}{2\kappa^2 D^V} \frac{(\cos\kappa l + \text{ch}\kappa l)(\cos\alpha l - \text{ch}\kappa x) + (\sin\kappa l - \text{sh}\kappa l)(\sin\kappa x - \text{sh}\kappa x)}{1 + \cos\kappa l \cdot \text{ch}\kappa l} \tag{26}$$

For the plate boundary  $x = l$ , we have:

$$w(x) = -\frac{gV}{\kappa^2 D^V} \cdot \frac{\sin\kappa l \cdot \text{sh}\kappa l}{1 + \cos\kappa l \cdot \text{ch}\kappa l} \tag{27}$$

From (25) and replacing  $\kappa l = \eta$ , we get:

$$w(x) = -\frac{gVl^2}{D^V} \cdot \frac{1}{\eta^2} \cdot \frac{\sin\eta \text{sh}\eta}{1 + \cos\eta \cdot \text{ch}\eta}. \tag{28}$$

The expression for resonance frequencies has the form:

$$2\pi f_n = \omega_n = \frac{\eta_n^2}{l^2} \cdot \left( \frac{D^V}{\rho n} \right), \text{ where } \eta_n \text{ are roots of the equation } \cos\eta \cdot \text{ch}\eta + 1 = 0.$$

For the first four harmonics we obtain:  $\eta_1 = 0.59687\pi$ ,  $\eta_2 = 1.49417\pi$ ,  $\eta_3 = 2.50025\pi$ ,  $\eta_4 = 3.49999\pi$ . Replacing  $DV$  through the resonance frequency, we will finally find:

$$w(l) = \frac{gV}{\rho hl^2 \cdot 4\pi^2 f_n^2} \cdot \frac{\eta_n^4}{\eta^2} \cdot \frac{\sin\eta \cdot \text{sh}\eta}{1 + \cos\eta \cdot \text{ch}\eta}. \tag{29}$$

In the presence of internal viscous friction, the oscillation equation will have the form:

$$\rho \frac{\partial^2 w}{\partial t^2} = \frac{1}{h} \frac{\partial^2 M_1}{\partial x^2} - \beta \frac{\partial w}{\partial t}, \quad (30)$$

where  $\beta$  is a coefficient of viscous friction. Substituting the expression for the moment and introducing the notation:  $2\alpha = \beta/\rho$  ( $\alpha$  is an attenuation coefficient), we find the equation:

$$\frac{\partial^2 w}{\partial t^2} - 2\alpha \frac{\partial w}{\partial t} + \frac{D^V}{\rho h} \cdot \frac{\partial^4 w}{\partial x^4} = 0, \quad (31)$$

which must be supplemented with boundary conditions of the form (21). In the steady-state oscillation mode (31), we have:  $w(x, t) = w(x)\exp(i\omega t)$ , where  $\omega$  is the frequency of the exciting field. Substituting expressions for  $w(x, t)$  into (31), we find:

$$\frac{\partial^4 w}{\partial x^4} - \kappa^4 w = 0, \quad (32)$$

where the designation is entered:

$$\kappa^4 = \frac{\rho h \omega^2}{D^V} \left( 1 - i \frac{2\alpha}{\omega} \right). \quad (33)$$

Thus, it is possible to introduce a complex value:

$$\kappa = \kappa_1 - i\kappa_2, \text{ where } \kappa^4 = \frac{\rho h \omega^2}{D^V}, \text{ and } \kappa_2 = \kappa_1 \frac{\alpha}{2\omega}.$$

On the other hand, assuming that the losses in the sample at the resonance of bending vibrations are purely mechanical in nature, it is possible to introduce complex compliance:

$s_{11}^E = (s_{11}^E)^l - i(s_{11}^E)^ll$  or without indices,  $s = s_1 - is_2$ . Then it follows from (25) that  $\kappa_2/\kappa_1 \sim 1/4$ ;  $s_2/s_1 = 1/4Q_M$ , where  $Q_M$  is the mechanical  $Q$  factor. Recall that for the first resonance  $\kappa_1 l \approx 0.6\pi$  at  $Q_M \approx 50$  (for PZT-19),  $\kappa_2/\kappa_1 \sim 1/200$  and  $\kappa_2 l < 0.01$ . Introducing the complex value  $\kappa$  into (27) and taking into account the smallness of  $\kappa_2 l$ , we obtain for the maximum  $w(l)$ :

$$w_M(l) = \frac{0.141gVQ_M}{\rho h l^2 f_1^2}. \quad (34)$$

The mechanical  $Q$  factor can be calculated in the usual way by using the shape of the resonance curve:  $Q_M = f_1/2\Delta f$ , where  $2\Delta f$  is the ‘‘width’’ of the resonance curve at the level  $w = (1/\sqrt{2})w_M$ . Thus, by using the experimental resonance curve, it is possible to determine  $Q_M, f_1, w_M$  with following calculation of the cylindrical stiffness  $D^V$  and the value  $g$ , and according to these values, the distribution of the piezomodule  $d_{31}$  over the plate thickness. The complex conductivity of the plate is determined by the ratio:

$$Y = \frac{I}{V} = \frac{l_1 \int_0^l D_3 dx}{V}, \quad (35)$$

where  $l$  is the width of the plate.

Using (7) and (26), we find:

$$Y = -\frac{i\omega l_1 l}{a s_{11}^E} \cdot \left( 1 + \frac{ag^2 s_{11}^E}{\kappa D^V l l_1} \cdot \frac{\cos \eta \cdot \text{sh} \eta + \sin \eta \cdot \text{ch} \eta}{1 + \cos \eta \cdot \text{ch} \eta} \right) = Y_0 + Y_1. \quad (36)$$

Here  $Y_0$  is the capacitive part of the conductivity,  $Y_1$  is the resonant term. For thin layers

$$Y = Y_0 \left[ 1 + \frac{3K^2 \delta^2}{(1 - K^2) \eta l_1} \cdot \frac{\cos \eta \cdot \text{sh} \eta + \sin \eta \cdot \text{ch} \eta}{1 + \cos \eta \cdot \text{ch} \eta} \right] \quad (37)$$

The resonant term is proportional to the square of the relative thickness of the layer,  $\delta = h_S/h$ . For PZT-19 ferroceramics with electrodes made of burnt silver ( $h = 0.4$  mm,  $l = 40$  mm,  $l_1 = 6$  mm), it was found that  $h_S = 5$   $\mu\text{m}$ . Near resonance:

$$\left| \frac{Y}{Y_0} - 1 \right| = \frac{12Q_M K^2 \delta^2}{(1 - K^2) \eta_1^2} \cdot \frac{\text{tg} \eta_1 + \text{th} \eta_1}{\text{tg} \eta_1 - \text{th} \eta_1}. \quad (38)$$

Assuming  $Q_M \approx 70$  and  $K = 0.4$ , we find  $(Y/Y_0 - 1) \approx 30\delta^2$  or for  $\delta \approx 1/80$ ,  $(Y/Y_0 - 1) \approx 4.7 \times 10^{-3}$ . Such a weak anomaly is characteristic of bending vibrations.

The formation of counter-polarized layers occurs when a strong AC field is applied to the sample, which can be associated, as already stated above, with the presence of a rectifying metal (electrode)—semiconductor (ceramic) contact. However, bending vibrations of ferroceramic plates in weak fields are observed, as a rule, without their pretreatment by a strong AC field. In addition, there is no clear correlation between the amplitude of bending vibrations and the magnitude of the piezomodule that characterizes this type of ceramics. For example, sufficiently large oscillation amplitudes are given by the samples, based on modified lead titanate, for which the piezomodule  $d_{31}$  is practically zero. In this regard, the models describing the contribution of deformations, caused by electrostriction to bending vibrations, were considered [7].

By investigating the dependence of the resonance frequency of bending vibrations on the temperature, the amplitude of the alternating exciting field, its changes by switching the polarization [8], and comparing experimental data with the theoretical models considered, we can make conclusion on a significant contribution to the bending deformation of the oscillatory motion of  $90^\circ$ -domain walls. It opens up opportunities for studying the mobility of domain boundaries by bending deformations, excited by an electric field.

Thus, the study of bending deformations can be a powerful tool for controlling the homogeneity of a ferroelectric sample and the mobility of ferroelastic domain boundaries in it. There are wide opportunities for the construction of ferroelectric monomorphs, based on several complementary phenomena, leading to the bending of a ferroelectric plate in an electric field.

## 2.2 Continuous Formulation of the Problem (Second Model)

As it will be shown later, in all cases, the presence of bending deformations in the external field indicates the existence of inhomogeneities of various types in ferroceramics. Therefore, the development of research methods and theoretical analysis of bending deformations in ferroceramics opens up new possibilities for studying the processes, occurring



in ferroceramic materials and accompanied by the appearance of inhomogeneities. The results of studies of inhomogeneous deformations are also of great practical importance, since they allow optimizing the constant combinations of various transducers using bending deformations.

This paper presents a theoretical consideration of bending deformations and describes the results of experimental studies. The formation of counter-polarized layers occurs, when a strong AC field is applied to the sample and may be due to the presence of a rectifying metal (electrode) – semiconductor (ceramic) contact [9]. However, bending vibrations of ferroceramic plates in weak fields are observed, as a rule, without their pretreatment by a strong belt field. In addition, there is no clear correlation between the amplitude of bending vibrations and the value of the piezoelectric module, which characterizes this type of ceramics. For example, sufficiently large oscillation amplitudes are given by samples based on modified lead titanate, for which the piezomodule  $d_{31}$  is actually zero. In this regard, it is of interest to consider models, describing the contribution to bending vibrations of deformations, caused by electrostriction. We will assume that Schottky-type layers with a thickness  $h_s/2$  and a charge density  $\sigma = eN$  are formed near the electrodes in the plate, where  $e$  is a unit charge,  $N$  is the number of charges per unit volume (the sign of the charge depends on the type of conductivity in ceramics, as well as on the ratio between the electron work functions for ceramics and electrode material). The distribution of the field over the thickness of the plate in this case will have the following form:

$$E_0(z) = \begin{cases} \beta\left(z + \frac{h}{2} - \frac{h_s}{2}\right), & -\frac{h}{2} \leq z \leq -\frac{h}{2} + \frac{h_s}{2}; \\ \beta\left(z - \frac{h}{2} - \frac{h_s}{2}\right), & -\frac{h}{2} + \frac{h_s}{2} \leq z \leq \frac{h}{2} - \frac{h_s}{2}; \\ \frac{h}{2} - \frac{h_s}{2} \leq z \leq \frac{h}{2}. \end{cases} \quad (39)$$

Here  $\beta = Ne/\varepsilon$ ,  $\varepsilon$  is the permittivity in the layer. We will assume that the charge relaxation time is long, so that a weak external AC field does not lead to the destruction of the layers (their thickness and charge density are preserved). Such layers are similar to chemical layers with a constant charge density in them, and, consequently, a linear dependence of the field on the coordinate inside the layer. When an external field  $E$  is applied, the total field in the plate is  $E_3 = E_0 + E$ . In the absence of residual polarization, the original system of equations has the form:

$$\begin{aligned} D_3 &= \varepsilon_{33}E_3 \\ u_{11} &= s_{11}^E\sigma_{11} + M_{12}E_3^2 \end{aligned} \quad (40)$$

Hence the bending moment in the plate is:

$$M_1 = -D^V \frac{\partial w}{\partial x^2} - \frac{M_{12}}{s_{11}^E} \int_{-\frac{h}{2}}^{\frac{h}{2}} z E_3^2 dz = -D^V \frac{\partial w}{\partial x^2} + g' V, \quad (41)$$

where  $V = E^*h$  is the external voltage,

$$D^V = \frac{h^3}{12s_{11}^E}; \quad g' = \frac{M_{12}}{s_{11}^E} \beta h^2 \delta^2 \left(1 - \frac{1}{3}\delta\right), \quad \delta = \frac{h_s}{h}. \quad (42)$$

In this case, the constant  $g'$  is determined by the magnitude of the electrostrictive modulus  $M_{12}$ , the thickness of the layers, and the charge density in the layer. In the case of non-equivalent layers, the value of  $g'$  in Eq. (41) will be equal to:

$$g' = \frac{M_{12}}{s_{11}^E} \frac{h^2}{8} \left[ \beta_1 \delta_2^1 \left( 1 - \frac{1}{3} \delta_1 \right) + \beta_2 \delta_2^2 \left( 1 - \frac{1}{3} \delta_2 \right) \right]. \quad (43)$$

As follows from the calculation, the striction model for this case gives a displacement value  $w(I)$  proportional to the first power of the applied stress until the parameters of the near-surface layers change (values  $\beta_1$ ,  $\beta_2$ ,  $\delta_1$ ,  $\delta_2$ ). However, in all cases, there will be no anomaly in the conductivity (so far, the dependence  $\varepsilon_{33}(\sigma)$  can be neglected) at the resonance of bending vibrations.

Bending vibrations of a striction nature can also be observed in the case of a nonuniform distribution of the external applied field over the thickness of the sample. Any inhomogeneity of properties in the sample, which almost always takes place, can lead to such a distribution. Significant inhomogeneity of properties (for example, conductivity or polarization) over the thickness of the sample can be caused, for example, by its preliminary treatment with a strong constant field (polarization).

With an inhomogeneous distribution of the external field, the displacement amplitude will be proportional to the square of the applied voltage, and the frequency of mechanical oscillations should be equal to the doubled frequency of the exciting field (frequency doubling effect). In particular, the resonance of such oscillations (with a resonance frequency  $w_1$ ) should be observed when an alternating field of half frequency  $0.5w_1$  is applied to the sample.

Let us assume that another plate (made of the same material) of thickness  $b$  without electrodes is glued to one of the electrodes of the investigated non-polarized plate of thickness  $h$ . Let us consider oscillations of the same combined plate under the assumption that the external voltage is still applied only to the electrodes of the original plate. Obviously, in this way, we artificially simulate a system with an inhomogeneous distribution of the external field  $E$  over the thickness of the sample (the field is zero at thickness  $b$  and different from zero at thickness  $h$  of the combined plate). We will also assume that Schottky-type charge layers with a constant linear field distribution over their thickness have formed near the electrodes of the initial plate.

Let us denote via  $H$  the total thickness of the plate ( $H = h + b$ ) and place the origin of the coordinate system in the center of the plate. Accordingly, the function describing the distribution of the field in the plate will also change (39). Calculate the moment in the plate. Replacing  $h$  by  $H$  and integrating over the thickness of the plate, we find:

$$M_1 = -D^v \frac{\partial^2 w}{\partial x^2} + g_1 V - g_2 V^2, \quad D^v = \frac{H^3}{12s_{11}^E} \quad (44)$$

where

$$g_1 = \frac{M_{12}}{s_{11}^E} \beta H^2 \delta^2 \frac{1 - \frac{2}{3} \delta - \theta}{1 - \theta}$$

$$g_2 = \frac{1}{2} \frac{M_{12}}{s_{11}^E} \frac{\theta}{1 - \theta}, \quad \delta = \frac{h_s}{H}, \quad \theta = \frac{b}{H} \quad (45)$$

The presence of a term proportional to  $V$  in (44) is due to the existence of charge layers in the initial plate, the term proportional to  $V^2$  arises due to the inhomogeneous distribution of the field over the thickness of the combined sample. Solving the oscillation equations separately for each of these terms, we find the displacements:

$$\begin{aligned}\omega_1 &= \frac{g_1}{k^2 D^V} V \frac{\sin kl \cdot \operatorname{sh} kl}{1 + \cos kl \cdot \operatorname{ch} kl}, k^4 = \frac{\rho H}{D^V} \omega^2 \\ \omega_2 &= \frac{g_2}{k^2 D^V} V^2 \frac{\sin kl \cdot \operatorname{sh} kl}{1 + \cos kl \cdot \operatorname{ch} kl}, k^4 = \frac{\rho H}{D^V} (2\omega)^2\end{aligned}\quad (46)$$

where  $\omega$  is the frequency of the incident field, and the resonance of the plate vibrations, caused by the inhomogeneous distribution of the field over the thickness of the sample ( $\omega_2$ ), will be excited at the field frequency  $\omega_2 = 0.5 \omega_1$ . Since the resonance frequencies of mechanical vibrations of both types coincide, it can be expected that the  $Q$ -values for these vibrations will be the same.

The value of the coefficient  $M_{12}$  can be estimated from  $g_1$ , calculated from the experimental values of  $w$ , and  $Q_m$  in resonance at the fundamental frequency or from  $g_2$  at half frequency.

Let us consider a two-layer friction model, according to which there is a polarization  $P_1$  in the layer:  $-h/2 < z < h/2$ , and in the rest of the sample ( $h/2 < z < h/2 + h_s/2$ ;  $-h/2 - h_s/2 < z < -h/2$ ) there is a polarization  $P_2$ . Then for the bending moment of the plate, we have:

$$M_1 = -D^V \frac{\partial^2 w}{\partial x^2} + \frac{h^2 Q_{13}}{2S_{11}^E} \delta(1 - \delta) (P_1^2 - P_2^2) \quad (47)$$

where  $Q_{13}$  is a constant of electrostriction,  $\delta = h_s/h$ .

Let, further, the polarizations  $P_1$  and  $P_2$  contain slowly changing quasi-static parts  $P_{01}$  and  $P_{02}$  excited by a strong repolarizing field  $E_0$ , and a variable part  $\delta E$ . The inhomogeneity in thickness is caused by the difference between  $P_{01}$  and  $P_{02}$  (there should be a volume charge at the boundary of the layers due to the difference  $P_{01} - P_{02}$ ). Assuming  $P_1 = P_{01} + \delta E$  and  $P_2 = P_{02} + \delta E$ , we get:

$$M_1 = -D^V \frac{\partial^2 w}{\partial x^2} + \frac{h^2 Q_{13}}{2S_{11}^E} \delta(1 - \delta) (P_{01}^2 - P_{02}^2) + \frac{1}{S_{11}^E} h Q_{13} \delta(1 - \delta) (P_{01}^2 - P_{02}^2) \delta V \quad (48)$$

Accordingly, the bending deformation of the plate will consist of two parts: quasi-static one,  $w_0$ , and variable (resonant) one,  $w_1$ :

$$w_0 = \frac{h^2 Q_{13}}{4S_{11}^E D^V} \delta(1 - \delta) l^2 (P_{01}^2 - P_{02}^2) \quad (49)$$

it vanishes at  $P_{01} = \pm P_{02}$ , and

$$\omega_1 = \frac{h^2 Q_{13}}{k^2 S_{11}^E D^V} \delta(1 - \delta) (P_{01} - P_{02}) \delta V \frac{\sin kl \cdot \operatorname{sh} kl}{1 + \cos kl \cdot \operatorname{ch} kl} \quad (50)$$

it vanishes at  $P_{01} = P_{02}$ .

It follows from (49) and (50) that if the static deformation is determined by the difference of the polarization squares in the sample volume and layer, the variable (resonant) deformation depends on the difference of the first degrees ( $P_{01} - P_{02}$ ). When value of ( $P_{01} - P_{02}$ ) changes, for example, under the action of a DC polarizing field, it will remain to be unchanged (with unchanged values of  $S_{11}^E$ ,  $Q_M$ ,  $V$ ), until the volume charge at the boundary of the layer, determined by the difference ( $P_{01} - P_{02}$ ), does not change. At the same time, the static deformation in this case will vary proportionally to  $P_{01} + P_{02}$  over a wide range and vanish at  $P_{01} = -P_{02}$ . In general case, when both  $P_{01}$  and  $P_{02}$  change, the value of  $w_0$  passes through zero twice (at  $P_{01} = \pm P_{02}$ ), and  $w_1$  – only once, when  $P_{01} = P_{02}$ . At the same time, due to the relaxation of the volume charge, relatively slow changes in  $w_1$  and  $w_0$  will be observed.

The above examples indicate a fairly wide class of phenomena leading to bending deformations of ferroelectric plates. All of them are somehow related to the inhomogeneity of the ferroelectric state in thickness. Below, some results of an experimental study of bending deformations in ferroceramic plates are present.

According to the above models, the mechanisms of such a destruction can be:

- (i) a decrease in the piezoactivity of the surface layers due to the decay of the polarized state (it is known that piezomodules of polarized ceramics begin to decrease at temperatures  $T < T_C$ );
- (ii) reduction of spontaneous polarization and its magnitude in the surface layer;
- (iii) changing the parameters of Schottky layers with temperature.

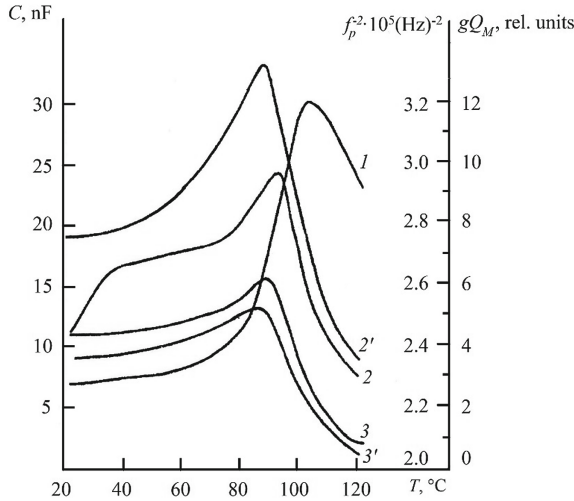
### 2.3 Technology of Piezoelement Preparation and Polarization

Piezoceramic plates of two standard sizes were studied:  $31.5 \times 6.0 \times 0.4 \text{ mm}^3$  and  $46.4 \times 3.0 \times 0.4 \text{ mm}^3$ . Bending deformations were studied for PCR-1, PCR-8, PCR-7M, PZT-19 and TBC-3 ceramics. The audio frequency voltage was applied to the sample from a G3–109 generator, the voltage value  $V$  (in the range of 5 – 200 V) was measured with a digital voltmeter V7–40, and the frequency  $f$  was measured with a frequency meter CH-64. The influences of temperature (20 – 210 °C) on the oscillation amplitude (at the fundamental frequency)  $w_M$  and resonance frequency  $f_P$  for ceramics TBC-3 ( $T_C \sim 105 \text{ °C}$ ) and PCR-7M ( $T_C \sim 170 \text{ °C}$ ) were studied. The dependences of  $w_M$  and  $f_P$  on the amplitude of the excitation voltage  $V$  are studied, too. Changes in  $w_0$ ,  $w_M$  and  $f_P$  were studied for PZT-19 ceramics during repolarization of samples in a stepwise changing electric field  $E_0$  (in the range from – 600 to + 600 V). For the same ceramics, preliminary studies of the surface state and electrode material were carried out.

## 3 Results and Discussion

An increase in the ferroceramic plate temperature is accompanied by an increase in the oscillation amplitude  $w_M$  and a decrease in the resonance frequency  $f_P$ . The oscillation amplitude reaches its maximum at temperatures  $T_m$  on 15–20 °C below  $T_C$ , corresponding to the maximum permittivity. After passing through the maximum  $w_M$ ,

it decreases sharply even at temperatures, exceeding  $T_C$  on 15–20 °C, not fixed by the device ( $w_M = 1.5 \mu\text{m}$ ). The resonance frequency passes through a minimum at  $T_2 < T_m$  and then increases again. As the excitation voltage amplitudes increase  $T_m$  and  $T_2$  decrease slightly. Figure 1 shows the temperature dependences of the capacity  $C$  of the TBC-3 sample at a frequency of 1 kHz (curve 1), the inverse square of the resonant frequency  $f_p^{-2}$  (curves 2 and 2'), and the bending efficiency  $gQ_M$  (curves 3 and 3') for two values of the applied voltage ( $V = 46.5 \text{ V}$ , corresponding to curves 2 and 3;  $V = 98 \text{ V}$ , corresponding to curves 2' and 3').



**Fig. 1** Temperature dependence of the parameters of bending vibrations

The inverse square of the resonant frequency according to [10] is proportional to the compliance of ceramics  $s_{11}^E$ . As the temperature increases, this value increases and passes through a maximum in the region of rapid increase in the sample capacity; this region is known to be characterized by the maximum mobility of domain boundaries. The possible contribution of the movement of domain boundaries to  $s_{11}^E$  is also indicated by an increase of  $s_{11}^E$  with an increase in the amplitude of the exciting voltage, and this increase is most pronounced in the vicinity of  $s_{11}^E$ . With a further increase in temperature, the value of  $s_{11}^E$  decreases rather sharply and at the same time the dependence of  $s_{11}^E$  on the exciting voltage is significantly weakened. The value of  $gQ_M$  also has a maximum in the region of increasing capacitance; the subsequent decrease in this value with temperature can be caused by two reasons: (i) an increase in mechanical losses with increased mobility of 90°-domain boundaries and (ii) the destruction of the inhomogeneous structure responsible for bending vibrations. According to the considered models [7, 10], the mechanisms of such destruction can be:

- (i) Reduction of piezoactivity of surface layers due to the decay of the polarized state (it is known that piezoelectric modules of polarized ceramics begin to decrease at temperatures  $T < T_C$  [12]);

- (ii) Reduction of spontaneous polarization and its value in the surface layer;
- (iii) Changing the parameters of Schottky layers with temperature.

The effect of the excitation voltage amplitude was studied for PCR-1, PCR-7M, PZT-19, and TBC-3 ceramics. In all cases considered, the dependence  $w_M(V)$  was close to linear. At large oscillation amplitudes ( $w_M > 100 \mu\text{m}$ ), a transition to saturation was observed. The resonance frequency  $f_P$  in all cases decreased with increasing  $V$ . In the voltage range of  $V = 5 - 150 \text{ V}$ , the relative change  $\Delta f_P/f_P$  ranged from 8 to 15%. The value of  $gQ_M$ , as a rule, decreases with increasing  $V$ . For pre-aged ceramics that were not exposed to external influences and for ceramics aged after polarization for 7 months, the value of  $gQ_M$  was practically independent of the field. A decrease  $f_P$  with an increase  $V$  can be caused by two main mechanisms:

- (i) For models of inhomogeneous piezoactivity [10],  $f_P$  can decrease due to a decrease in the electromechanical coupling coefficient of the surface layer ( $K$ ), or due to a decrease in the layer thickness  $h_s$ . However, the maximum relative changes in  $f_P$  in this case (from a bimorph to an unpolarized sample) for the studied ceramics do not exceed 4%;
- (ii) The second reason may be an increase  $s_{11}^E$  in the variable field due to the contribution of domain walls separating ferroelastic domains (for example,  $90^\circ$ -domains in the tetragonal phase).

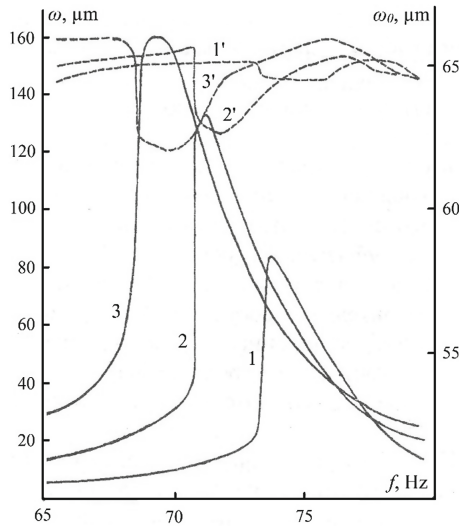
In Table 1, data on changes in the frequency of resonant vibrations ( $\Delta f = f_0 - f_{\min}$ ) and the amplitude  $w_m$  at  $V = 150 \text{ V}$  are presented for a sample of PZT-19 ferroceramics with electrodes, obtained by burning silver paste.

**Table 1** Characteristics of the studied samples

Sample	Value	Initial state	After polarization	After 14 days	After 2 months	After 7 months
PZT-19	$f_0$ , Hz	140.1	141.1	141.8	138.1	104.6
	$f_{\min}$ , Hz	120.0	125.9	127.4	123.6	20.5
	$\Delta f/f_0$ , %	14.3	10.8	10.2	10.5	13.5
	$w_m$ , $\mu\text{m}$	30	21.4	41	42.4	26.7

Calculation of the value of  $gQ_M$  for the sample from the table as a function of  $V$  shows that this value, slightly decreasing with increasing  $V$  for  $V \leq 40 \text{ V}$ , for higher values,  $V$  remains virtually constant. The dependence  $gQ_M(V)$  has a similar form for the same sample, aged after polarization for 7 months. However, in both cases, the decrease in  $f_P$  with increasing  $V$  remains significant. Therefore, taking into account the decrease of  $Q_m$  with increasing  $V$ , which will be discussed below, we must recognize that the decrease of  $f_P$  with increasing  $V$  is not determined by the change in the parameters of the surface layers.

After polarization ( $T_P = 20\text{ }^\circ\text{C}$ ,  $\tau_P = 1\text{ h}$ ,  $E_0 = 17.5\text{ kV/cm}$ ), the value of  $w_M$  decreases. These data show that if we assume that the main mechanism leading to the change  $f_P$  is the movement of  $90^\circ$ -domain boundaries, then it should be recognized that their contribution to the bending moment is probably not decisive, and the contribution of  $s_{11}^E$  is not determined by the amplitude of vibrations of the sample as a whole. The value of  $g$ , according to [10], includes the ratio  $d_{31}/s_{11}^E$ , so the mobility contributions of  $90^\circ$ -domain boundaries to  $d_{31}$  and  $s_{11}^E$  can both increase and decrease  $g$ . Their contribution to  $w_M$  remains small. It also follows from Table 1 that in the aging process after polarization,  $\Delta f/f_P$  first decreases, and then begins to increase. Apparently, there are two competitive processes acting here. One of them is undoubtedly the general stabilization of the domain structure, which leads to a decrease in the contribution of domain boundaries to  $s_{11}^E$ . The increase in this contribution is probably related to the rearrangement of the residual mechanical stresses after polarization.



**Fig. 2** Temperature dependence of the parameters of bending deformations

Then the resonance curves of flexural vibrations of ferroelectric plates were studied (Fig. 2). The  $Q$ -factor calculation from the dependence  $w(f)$  shows that the  $Q_M$  value decreases significantly. For the curves in Fig. 2, the following values are obtained: 39.2; 36.0 and 25.0 for the value  $V$  of 30, 60 and 90 V (curves 1, 2, 3), respectively. In the resonance region, the electric field exerts a certain formative effect on the surface layers, as evidenced by the static deflection of samples in the resonance region shown in Fig. 2. However, as the figure shows, it is reversible. Attention is drawn to the strong asymmetry of the resonance curves, which reflects an increase of  $s_{11}^E$  with an increase in the oscillation amplitude. At relatively weak amplitudes, the asymmetry is not detected, but  $w_m$  increases it with the growth; for  $w_M = 80 - 100\text{ }\mu\text{m}$ , it is most pronounced, then it weakens with the blurring of the resonance curves. For  $V$  from 2 V to 9 V,  $Q_M$  values from 84 to 67 were obtained using the capacitive probe method. In general, taking

into account changes in  $Q_M$ , we find that the parameter  $g$  after 60 days aging shows a tendency to increase with increasing  $V$ , and the constancy of value of  $gQ_M$  after 7 months of aging indicates an increase in this trend.

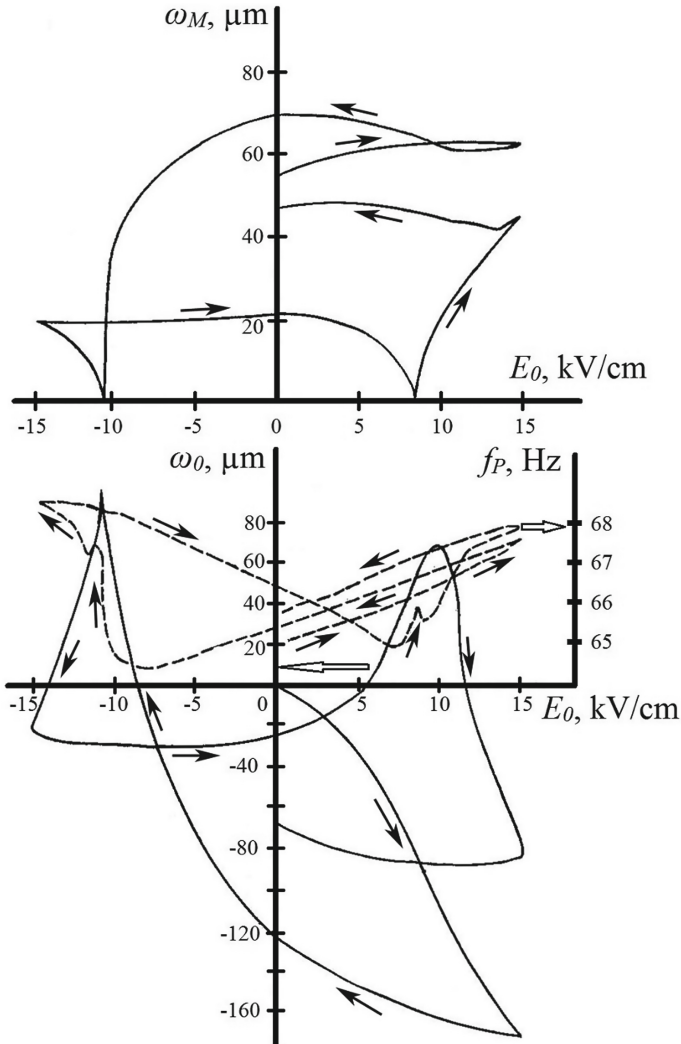
To assess the effect of the surface state on bending deformations, a comparison of samples of PZT-19 with one-sided polishing, with two-sided polishing and with two-sided polishing and a layer of  $\text{PbTiO}_3$  on one of the sides was carried out. The electrodes in all cases were graphite (Iacosage). It is established that the amplitude of vibrations increases during one-sided polishing; the static displacement  $w_0$  with an increase in the amplitude of the alternating voltage  $V$  is insignificant (within  $5 \mu\text{m}$ , for polished surfaces within  $1 \mu\text{m}$ ). The amplitude of vibrations increases sharply (by more than 2 times) for a sample with both surfaces polished. At the same time, the static offset also becomes noticeable (it is  $36 \mu\text{m}$  at  $150 \text{ V}$ ). However, applying a thin (several microns of thickness)  $\text{PbTiO}_3$  film to one of the surfaces leads to 2.5 times decrease in the oscillation amplitude and a sharp increase in the static displacement ( $330 \mu\text{m}$ ), for example, directed towards the surface not covered with  $\text{PbTiO}_3$ . Pyroelectric probing revealed that the sample was polarized ( $\vec{P}_r$  is directed towards the  $\text{PbTiO}_3$ -coated surface). Thus, in this case, there is a rectifying contact in the layer with  $\text{PbTiO}_3$ . The value  $(f_0/f_{\min})^2$ , which is proportional  $s_{11}^E$ , increases by 30% for a polished sample, by 36% for one-side polishing, and by 61% for two-side polishing, but by 32% for a sample with a  $\text{PbTiO}_3$  sublayer, which also indicates a decrease in the contribution of domain boundaries with increasing constant voltage due to the rectification effect.

The influence of a repolarizing, stepwise changing field  $E_0$  on the parameters of bending deformations of the PZT-19 in an AC field of constant amplitude is studied. The field  $E_0$  was changed in increments of  $1.25 \text{ kV/cm}$  in the range from  $-15$  to  $+15 \text{ kV/cm}$ . In  $E_0 \approx E_k$ , the increment was reduced to  $0.25 \text{ kV/cm}$ . Exposure time at each point was up to 10 min. The amplitude of vibrations in the resonance at the fundamental frequency ( $w_M$ ), the resonance frequency ( $f_P$ ), and the static displacement ( $w_0$ ) were measured. Measurements were carried out for three values of AC voltage of  $0.75$ ,  $1.5$  and  $3.0 \text{ kV/cm}$ .

Figure 3 shows the curves  $w_M(E_0)$ ,  $w_0(E_0)$  for a sample whose polished surfaces are coated with Ag electrodes, obtained by burning silver paste. The dependence  $f_P(E_0)$  is represented by a dashed line. It follows from the figure that with a primary increase of  $E_0$ , the values of  $w_M$  and  $f_P$  increase, and the static bend goes towards the cathode. The subsequent decrease of  $E_0$  is accompanied by a further increase  $w_M$ , but  $f_P$  and  $w_0$  decrease, and at  $E_0 = 0$ , the static bending remains quite significant. An increase of  $E_0$  in the opposite direction is accompanied by a decrease of  $w_M$ , which is initially slow, then faster. In the region of repolarization,  $w_M$  changes in time at an almost constant  $E_0$  and passes through a sharp minimum ( $w_M \approx 0$ ), and then increases again. The resonance frequency also passes through a minimum at  $E_0 < E_k$ . In the region of the minimum of  $w_M$ , it increases sharply, passes through two closely located, weakly expressed extrema and then gradually increases. The static deflection  $w_0$  decreases, passes through zero, changes its sign, reaches a maximum at  $E_0 \approx E_k$ , then decreases, and passes through zero again to the initial voltage. With a subsequent decrease in the field  $E_0$ ,  $w_M$  practically remains constant,  $f_P$  decreases almost linearly with  $E_0$ ,  $w_m$  initially remains constant, then (at  $|E_0| < 5 \text{ kV/cm}$ ) it begins to slowly decrease. When changing the sign and



repolarizing to the original direction,  $w_M$  again passes through zero at  $E_0 \approx E_k, f_P$  has a minimum at  $E_0 < E_k$ , and the static deflection  $w_0$  passes through zero twice. The unipolarity of the characteristics is noteworthy. Since the Ag electrodes were burned into the ground surfaces, it can be expected that the unipolarity is determined here by the primary polarization.



**Fig. 3** Field dependencies of the  $w_M, w_0$  and  $f_P$  (Ag)

An increase of  $V$  by half is accompanied by a decrease of ratio  $w_M/V$ , the minima of  $w_M$  become more blurred, the dependences  $w_M(E_0), w_0(E_0)$  and  $f_P(E_0)$  become more symmetric with respect to the sign of  $E_0$ . The static deflection in strong fields decreases, and the maxima of  $w_0$  in the region of the repolarizations are becoming more pronounced.

Additional maxima on the dependence  $f_P(E_0)$  become more clearly defined. At  $V = 3.0$  kV/cm, there are a further decrease  $w_M/V$ , a further blurring of the minima  $w_M$ , a slight decrease  $E_k$ , weakening of the unipolarity of  $w_m(E_0)$  and the extremes on the curve  $f_P(E_0)$  are blurred. The unipolarity of the curves  $f_P(E_0)$ ,  $w_0(E_0)$  is preserved.

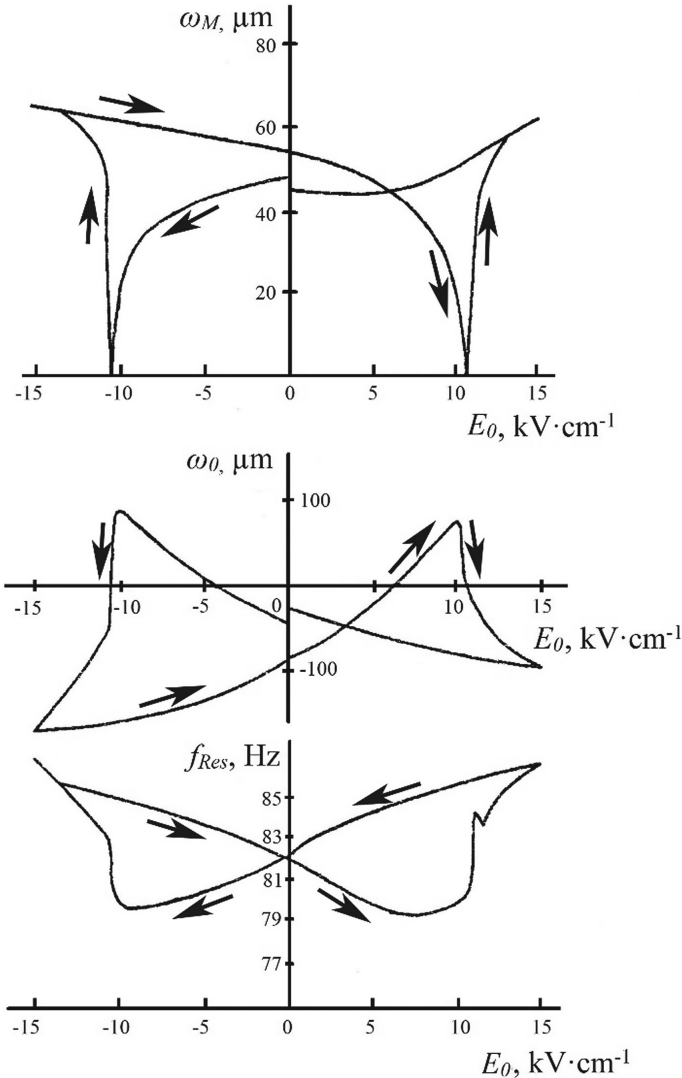
It is noteworthy that the main minimum of  $f_P$ , corresponding to the maximum of  $s_{11}^E$ , always precedes the minimum of  $w_M$ . The fact that  $w_M$  vanishes at the minimum, associated with repolarization, is confirmed by the study of the dynamic pyroelectric effect in these samples in a similar mode of the change of  $E_0$  that leads to that for  $E_0 - E_k$  there is a maximum of temporary changes in the parameters.

It is known that in  $c$ -domain crystals of BaTiO<sub>3</sub>, containing single  $a$ -domains, the latter, when the crystals are repolarized, reach the maximum volume in the fields preceding the switching of the  $c$ -domain regions. Therefore, it is logical to associate the minimum of  $f_P$  with an increase of  $s_{11}^E$  due to the mobility of 90°-domain walls. Additional maxima on the curves  $f_P(E_0)$  also seem to reflect the irregular nature of the movement of the domain walls.

Figure 4 shows the switching characteristics for a PZT-19 plate with aquadag electrodes (at a voltage of 0.75 and 3.0 kV/cm). In contrast to the sample with reburned silver electrodes,  $w_M$  was more pronounced here. A sharp increase in  $w_M$  after switching polarization is noteworthy. Zero value of  $w_M$  coincides with zero  $w_0$ ; a sharp increase  $w_M$  coincides with a sharp increase  $f_P$  and a sharp change of  $w_0$ . All curves have a sharp unipolarity and additional maxima  $f_P(E_0)$  are weakly expressed. As the amplitude increases, a decrease  $w_M/V$  is observed, the switching remains quite sharp, and additional maxima are detected on the curve  $f_P(E_0)$ ; however, the main maxima are strongly blurred. When  $E_k$  decreases noticeably, the AC voltage significantly affects the repolarization in field  $E_0$ .

It should be noted that these repolarization characteristics are well described by the friction model of a polarized surface layer [11]:  $w_0$  passes through zero twice; in a weaker field, when  $P_{01} = -P_{02}$  and the difference  $P_{01} - P_{02}$  is preserved,  $w_0 = 0$ , but when the oscillation amplitude  $w_M \neq 0$ , and the volume charge at the layer boundary does not relax, the temporary changes are practically absent. At  $E_0 \approx E_k$ ,  $P_{01} = -P_{02}$ , which corresponds to  $w_M = w_0 = 0$ , the volume charge at the layer boundary changes sign. It shows that strong temporary changes in the parameters of bending deformations are observed.

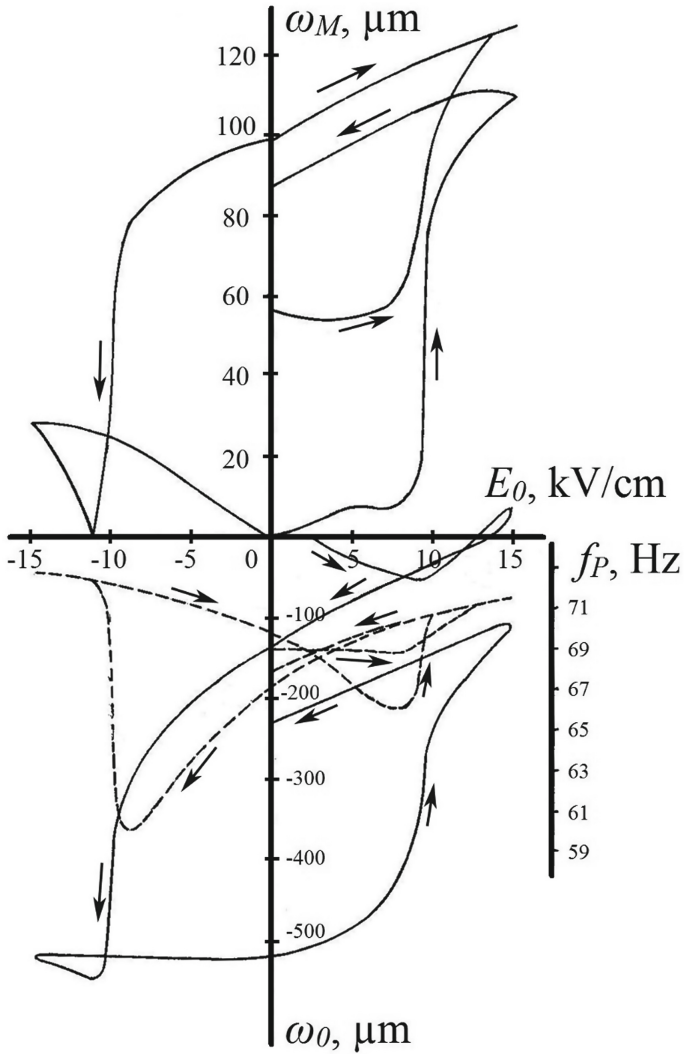
Figure 5 shows the switching characteristics for the PZT-19 sample, one side of which was polished and the other side was polished (the electrodes were painted). The dashed lines show the dependence  $f_P(E_0)$ . The positive direction  $E_0$  corresponds to a plus on a polished surface. The switching characteristics show strong unipolarity: switching in the “forward” direction occurs at smaller fields than in the reverse direction. Then  $w_M$  for the “forward” direction is significantly larger than for the reverse direction, static bending in the opposite direction is stronger and decreases for the forward direction. The unipolarity of the curves  $f_P(E_0)$  is noteworthy: minimum of  $f_P$  for the previous repolarization turns out is much deeper for the reverse direction, than for the “forward” one. If we assume that the minimum of  $f_P$  reflects the maximum of  $s_{11}^E$ , due to the contribution of the movement of 90°-domain boundaries, then it follows that the “inhibition” of



**Fig. 4** Field dependencies (aquadag)

$180^\circ$ -domain switching in the opposite direction is accompanied by an increase in  $90^\circ$ -domain switching. Increasing  $V$  from  $0.75 \text{ kV/cm}$  to  $3.0 \text{ kV/cm}$  slightly weakens the unipolarity of the  $w_M(E_0)$ , while at the same time, it increases the unipolarity of  $w_0(E_0)$  and  $f_P(E_0)$ . The hysteresis decreases sharply with changes of  $E_0$  in the forward direction; the difference between the minima of  $f_P$  for the forward and reverse directions increases.

An experimental study of bending deformations in nominally homogeneous ferro-ceramic plates allowed us to establish the following. By studying the dependence of the resonance frequency of bending vibrations on the temperature and amplitude of the



**Fig. 5** Field dependences for unipolar sample

AC exciting field, as well as its changes during polarization switching, we can conclude that the oscillatory motion of  $90^\circ$ -domain walls significantly contributes to the bending deformation, which opens up opportunities for studying the mobility of domain boundaries by the method of bending deformations, excited by an electric field. This can be a powerful tool for monitoring the homogeneity of a ferroelectric sample and the mobility of ferroelastic domain boundaries in it. This opens up a wide range of possibilities for constructing ferroelectric monomorphs, based on several complementary phenomena that lead to bending of a ferroelectric plate in an electric field.

## 4 Conclusion

In this paper, devoted to the study of bending deformations in nominally homogeneous ferroceramic plates, we have for the first time carried out theoretical modeling of such processes, based on the presence of thickness inhomogeneities (piezomodule  $d_{31}$ , polarization  $P_3$ , contact fields (Schottky-type layer). It is clearly shown that in the case of thin layers sufficient, however, to excite noticeable resonances of bending vibrations, their contribution to admittance is vanishingly small, which sometimes allows such resonances defined in the literature as purely mechanical.

Investigating the dependence of the resonance frequency of bending vibrations on temperature, the amplitude of the AC exciting field, and its changes during switching of polarization, we came to the conclusion about a significant contribution to the bending deformations of the oscillatory motion of  $90^\circ$ -domain walls, which opens up opportunities for studying the mobility of domain boundaries by the method of bending deformations, excited by an electric field.

Thus, the study of bending deformations can be a powerful tool for controlling the homogeneity of a ferroelectric sample and the mobility of ferroelastic domain boundaries in it.

There are wide opportunities for the construction of ferroelectric monomorphs, based on several complementary phenomena leading to the bending of a ferroelectric plate in an electric field.

**Acknowledgement.** The study was carried out with the financial support of the Ministry of Science and Higher Education of the Russian Federation (State task in the field of scientific activity, scientific project No. 0852–2020-0032), (BAZ0110/20–3-07IF). Work was performed using the equipment of the Center for Collective Use “Electromagnetic, Electromechanical and Thermal Properties of Solids”, Research Institute of Physics, Southern Federal University.

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