Classical and Quantum Random Walks to Identify Leaders in Criminal Networks

Annamaria Ficara[1(✉)], Giacomo Fiumara[1], Pasquale De Meo[2], and Salvatore Catanese[1]

[1] MIFT Department, University of Messina, 98166 Messina, Italy
aficara@unime.it, gfiumara@unime.it, scatanese@unime.it
[2] DICAM Department, University of Messina, 98168 Messina, Italy
pdemeo@unime.it

Abstract. Random walks simulate the randomness of objects, and are key instruments in various fields such as computer science, biology and physics. The counter part of classical random walks in quantum mechanics are the quantum walks. Quantum walk algorithms provide an exponential speedup over classical algorithms. Classical and quantum random walks can be applied in social network analysis, and can be used to define specific centrality metrics in terms of node occupation on single-layer and multilayer networks. In this paper, we applied these new centrality measures to three real criminal networks derived from an anti-mafia operation named Montagna and a multilayer network derived from them. Our aim is to (i) identify leaders in our criminal networks, (ii) study the dependence between these centralities and the degree, (iii) compare the results obtained for the real multilayer criminal network with those of a synthetic multilayer network which replicates its structure.

Keywords: Classical random walks · Quantum random walks · Centrality · Criminal networks

1 Introduction

The term "random walk" was firstly introduced in 1905 by Pearson [20]. Random walks can be useful to solve practical problems. In fact, the randomness of objects can be analyzed and simulated through them. Also the correlation among objects can be computed using random walks. They are fast becoming key instruments in various fields such as physics, economics, computer science, biology, or chemistry [25].

A simple random walk model in the mathematical space is a random process on a regular lattice where at each step one point jumps to another position according to a certain probability distribution [25]. Random walks can be applied to a network (or graph) defined as a set of nodes with edges between them. Graphs can be organized into multiple layers, which represent different types of nodes or edges. In this kind of application, the transition probability

H. Cherifi et al. (Eds.): COMPLEX NETWORKS 2016 2022, SCI 1077, pp. 190–201, 2023.
https://doi.org/10.1007/978-3-031-21127-0_16

between nodes is greater when the strength of the association between nodes is stronger. After an appropriate number of steps, a random path able to describe the structure of a network can be obtained. In computer science, the PageRank is the most known algorithm based on random walks [19]. This algorithm randomly walks among web pages to compute their importance.

Aharonov, Davidovich and Zagury were the first to propose quantum walks in 1993 [1]. These walks can be considered as the counter part of classical random walks in quantum mechanics. The main difference between quantum walks and classical random walks is that the former do not converge to some limiting distributions [25]. Quantum interference can cause significantly faster or slower spreading of quantum walks compared to classical random walks. Algorithms based on quantum walks have lower time complexity with respect to the ones based on classical random walks, and they provide an exponential speedup over any classical algorithm [11].

Random walks can find an application in computer vision, semi-supervised learning, network embedding, and social network analysis [17,22]. Some researchers also studied the application of random walks on knowledge discovery, graphs, science of science, and text analysis [2,12,22]. Quantum walks can be used instead to element distinctness, search problems, and decision trees [3,23]. Moreover, classical algorithms are often accelerated by using quantum walks.

Wald and Böttcher [24] at first introduced a framework for classical, quantum, and hybrid random walks with stochastic resetting on networks. Then, Böttcher and Porter [6] used continuous-time classical and quantum random walks to formulate occupation, PageRank, betweenness, and closeness centralities for multilayer networks.

In this paper, we decided to apply classical and quantum occupation centralities on very specific kinds of real networks, i.e. Mafia networks. These networks are different from traditional social networks because first of all they consist entirely of suspected criminals, and then they are considered as organizations of criminal groups and activities. For this reason, research on criminal networks is focused on their structure, efficiency, resilience and capacity to achieve criminal objectives. To the best of our knowledge, classical and quantum walks have never been used to identify leaders in criminal networks. Criminal leaders are strategic brokers within criminal networks [8]. Moreover, to reinforce our finding, we want to compare the results of classical and quantum walks on Mafia networks to the ones obtained on Barabási-Albert (BA) [5] models, which according to our previous work [9], are the one to better replicate criminal networks.

Mafia networks are in fact a good example of real-world networks with respect to the geometry of connections [16]. Indeed, these connections are the building blocks of the entire Mafia, and more generally of organized crime. The Sicilian Mafia is one of the major native mafia-like organizations. It is composed by loose confederations of about one hundred families (or cosche). Since at least the 19th century, the social and economic life especially in Southern Italy is affected by these families [18]. Social network analysis and graph theory can be applied to Mafia networks to describe their structure and functioning, to identify

leaders [15], to evaluate police interventions aimed at disrupting them [10], or to construct crime prevention systems [7].

2 Classical and Quantum Random Walk Centrality Measures

2.1 Classical Random Walks

Random walks on graphs are sequences of nodes which start from a source node by selecting an edge, crossing the edge to reach a new node, and repeating the process. An equivalent concept can be found in the Markov chain which is a stochastic process that assumes values in a discrete set, where the next state of the chain only depends on the current state and not on the past states [21]. The Markov chain can be viewed as a graph where nodes are represented by the states and edges by the possible next states, which are randomly determined. The set of states is discrete, but their evolution in terms of time can be discrete or continuous.

In a discrete-time Markov chain, each step is associated with a probability distribution, i.e. a set of probabilities that a walker is on a node or in a state. This probability distribution can be described with a vector, after choosing an order for the states. Given a graph $G(V, E)$ with set of nodes $V = \{v_1, \ldots, v_n\}$ ($|V| = n$) and set of edges E, the probability distribution is described by the vector $[p_1(t) \cdots p_n(t)]$, where $p_1(t)$ is the probability that a walker is on node v_1 at time t and so on. If the process begins with the walker on the first node, $p_1(0) = 1$ and $p_i(0) = 0$ for $i = 2, \ldots, n$.

The future position of the walker cannot be precisely identified, but it is possible to determine a probability distribution knowing the transition matrix M. By knowing the probability distribution at the time t, the distribution at time $t + 1$ can be obtained by the following formula:

$$p_i(t+1) = \sum_{j=1}^{n} M_{ij} p_j(t)\,. \tag{1}$$

In vector form, we have $\vec{p}(t+1) = Mp(t)$.

The entry M_{ij} of the transition matrix is the probability of the walker, who is on node v_j, to go to node v_i. The transition matrix is related to the adjacency matrix A of the graph, and it can be defined as:

$$M_{ij} = \frac{A_{ij}}{k_j}\,, \tag{2}$$

where k_j is the degree of node v_j, $A_{ij} = 1$ when two nodes v_i and v_j are connected or $A_{ij} = 0$ if they are not. If there is no edge from v_j to v_i, the walker goes to one of the adjacent nodes and transition probability is the same for all of them.

When time is a continuous variable, the problem of a walker who can go from node v_j to a neighbour v_i at any time can be seen as a liquid seeping from v_j to v_i. At the beginning, it is likely that the walker is in the node v_j. As time goes by, the probability that the walker is in v_j decreases, but the probability he is in one of its neighbors increases. The transition between neighbors occurs with a probability γ per unit time, which is a transition rate presumably constant for all times and for all nodes. An infinitesimal time interval ϵ is generally used to set up and solve the differential equation of the problem with continuous variables. Then, the probability that a walker goes from node v_j to v_i is given by $\gamma\epsilon$. If node v_j has a degree k_j, it means it has k_j neighbors. Therefore, the probability that the the walker is in one of the neighbors of v_j after time ϵ is $k_j\gamma\epsilon$, while the probability of finding him in v_j is $1 - k_j\gamma\epsilon$. At this point, the entry $M_{ij}(t)$ of the transition matrix at time t is defined as the probability that the walker on node v_j goes to node v_i in the time interval t:

$$M_{ij}(\epsilon) = \begin{cases} 1 - k_j\gamma\epsilon + O(\epsilon^2), & \text{if } i = j; \\ \gamma\epsilon + O(\epsilon^2), & \text{if } i \neq j. \end{cases} \tag{3}$$

An auxiliary matrix called generating matrix H, is also defined as:

$$H_{ij}(\epsilon) = \begin{cases} k_j\gamma, & \text{if } i = j; \\ -\gamma, & \text{if } i \neq j \text{ and adjacent;} \\ 0, & \text{if } i \neq j \text{ and non-adjacent.} \end{cases} \tag{4}$$

The next state of a Markov chain only depends on the current configuration of the chain. The transition matrix can be multiplied at different times as follows:

$$M_{ij}(t + \epsilon) = \sum_k M_{ik}(t) M_{kj}(\epsilon)\,. \tag{5}$$

The index k runs over all the neighbours of node v_j. In fact, if the walker is in node v_j, the probability that he goes to v_k in the time interval is $M_{kj}(\epsilon)$, independently of the value of k. If there is no edge between v_j and v_k for a specific k, $M_{kj}(\epsilon) = 0$.

By isolating the term $k = j$, using the Eqs. 3 and 4, and then by moving the first term on the right-hand side to the left-hand side and dividing it by ϵ, the following differential equation is obtained:

$$\frac{dM_{ij}(t)}{dt} = -\sum_k H_{kj} M_{ik}(t). \tag{6}$$

The solution of Eq. 6 with initial condition $M_{ij}(0) = \delta_{ij}$ is:

$$M(t) = e^{-Ht}\,. \tag{7}$$

After the definition of the transition matrix, the probability distribution at time t can be easily obtained. If the initial distribution is $\vec{p}(0)$, $\vec{p}(t) = M(t)\vec{p}(0)$.

2.2 Quantum Random Walks

The electron spin is described by quantum mechanics as a unit vector in the Hilbert space $\mathbb{C}^2$ [21]. The spin up and the spin down are respectively described by the vectors $|0\rangle$ and $|1\rangle$:

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \qquad |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \tag{8}$$

The notions of spin up and spin down refer to $\mathbb{R}^3$. Quantum mechanics describes in fact the behavior of the electron before entering the magnetic field. In this case and if the electron is somehow isolated from the macroscopic environment, its spin state is described by a linear combination of vectors $|0\rangle$ and $|1\rangle$. Therefore, $|\psi\rangle = a_0|0\rangle + a_1|1\rangle$, where the coefficients a_0 and a_1 are complex numbers which satisfy the constraint $|a_0|^2 + |a_1|^2 = 1$.

The time evolution of an isolated quantum system is described by a unitary transformation. If the state of the quantum system at time t_1 is described by vector $|\psi_1\rangle$, the system state $|\psi_2\rangle$ at time t_2 is obtained from $|\psi_1\rangle$ by a unitary transformation U, which depends only on t_1 and t_2. Therefore, we have $|\psi_2\rangle = U|\psi_1\rangle$.

The evolution postulate is to be written in the form of a differential equation which provides a method to obtain operator U once given the physical context. It is called Schrödinger equation and it is as follows:

$$\frac{d|\psi\rangle}{dt} = -iH|\psi\rangle\,. \tag{9}$$

In the passage from the continuous-time classical walk model to the continuous-time quantum walk model, the transition matrix is simply converted to an equivalent unitary operator, and the vector which describes the probability distribution to a state vector. Matrix H, given by Eq. 4 is Hermitian, and therefore matrix M given by Eq. 7 is not unitary. M can be easily made unitary within the context of Hilbert spaces by multipling h with the imaginary unit, that is by replacing H with iH. Then, the evolution operator of the continuous-time quantum walk can be defined as follows:

$$U(t) = e^{-iHt}\,. \tag{10}$$

If the initial condition is $|\psi(0)\rangle$, the quantum state at time t is:

$$|\psi(t)\rangle = U(t)|\psi(0)\rangle\,, \tag{11}$$

and the probability distribution is:

$$p_k = |\langle k|\psi(t)\rangle|^2\,, \tag{12}$$

where k runs over all nodes of the graph (or states of the Markov chain) and $|k\rangle$ is the state of the computational basis which correspond to the node v_k. The computational basis of space $\mathbb{C}^2$ is the set $\{|0\rangle, |1\rangle\}$.

2.3 Random Walk Occupation Centralities

Recently continuous-time classical and quantum random walk centrality measures have been defined for single-layer and multilayer networks [6,24].

Starting from Eq. 7, the explicit Euler integration scheme can be used to simulate a classical random walker:

$$p_{t+1} = 1 - H_c \, \Delta t \, p_t \,, \tag{13}$$

where H_c is the classical Hamiltonian defined as $H_c = LD^{-1}$, where $L = D - A$ is the Laplacian matrix and D is the degree matrix with $D_{ii} = k_i$ if $i = j$ and $D_{ij} = 0$ if $i \neq j$.

The classical random walks approaches for connected networks and sufficiently long times a stationary probability distribution p^* such that:

$$p_i^* = \sum_{j=1}^{n} \frac{A_{ij}}{k_j} p_j^* = \frac{k_i}{\sum\limits_{i=1}^{n} k_i} \,. \tag{14}$$

The stochastic vector p_i^* indicates the stationary classic random walk occupation probability OP_c for a node v_i, and it can be used to define a random walk centrality measure for graphs.

The Schrödinger equation (see Eq. 9) can be instead solved using the Crank-Nicholson integration scheme [4,24]. Expanding the evolution operator in Eq. 10 into a Taylor series, we have:

$$|\psi_{t+1}\rangle = -2i\Delta t H_q |\psi_t\rangle + |\psi_{t-1}\rangle \,, \tag{15}$$

where H_q is the Hermitian quantum Hamiltonian defined as $H_q = \mathcal{L}$, where $\mathcal{L} = D^{-\frac{1}{2}} L D^{-\frac{1}{2}}$ is the normalized version of the Laplacian matrix $\mathcal{L}$.

For continuous-time quantum walks, unitary time evolution does not lead to a stationary state. Instead, the long-time behavior of a quantum walk is characterized by its long-time mean:

$$q_i^* = \lim_{T \to \infty} \frac{1}{T} \int_0^T \langle i || \psi(t) \rangle \langle \psi(t) || i \rangle dt. \tag{16}$$

where dt is an infinitesimal time step, $|\psi(t)\rangle\langle\psi(t)|$ is a density operator, and $|i\rangle \in \mathbb{C}^{\mathcal{N}}$ is an orthonormal basis vector.

Even in this case, q_i^* denotes the continuous-time quantum random walk occupation probability OP_q of a node v_i, and therefore it gives a type of random walk centrality.

3 Methods and Results

We used the classical and quantum occupation centralities described in Sect. 2.3 to study the importance of nodes in three real single-layer criminal networks

(Meetings, Phone Calls and Crimes) related to an anti-mafia operation called Montagna [7,10,13], from which we derived a fourth network in the form of a flattened multilayer network. The Montagna operation focused on the Mistretta and Batanesi clans who monopolized the sector of public contracts in the Tyrrhenian strip and in the nebroidal district of the province of Messina, by working with some entrepreneurs associated to the Sicilian Mafia. Meetings contains 256 physical meetings between 101 suspected criminals observed during the police physical surveillance. Phone Calls contains 124 phone calls between 100 suspects wiretapped during the police audio surveillance. Crimes contains 74 connections between 25 suspected criminals who committed crimes together. The three networks share 20 nodes. We also created an undirected and weighted multilayer network with 226 actors, 454 intralayer edges and 3 layers corresponding to the three single-layer criminal networks Meetings, Phone Calls and Crimes. In this work, a single-layer network (flattened) is created by merging the three layers of the multilayer network. In the flattened network, there is one node for each actor of the starting multilayer network and an edge between two nodes if the corresponding actors are linked in one layer. Meeting, wiretap and co-offending networks provide fundamentally different pictures of the criminal organization, and therefore are all particularly useful in finding leaders. Also the flattened multilayer network can be useful to have a more complete picture of the organization. Figure 1 shows Meetings, Phone Calls, Crimes and the flattened multilayer network.

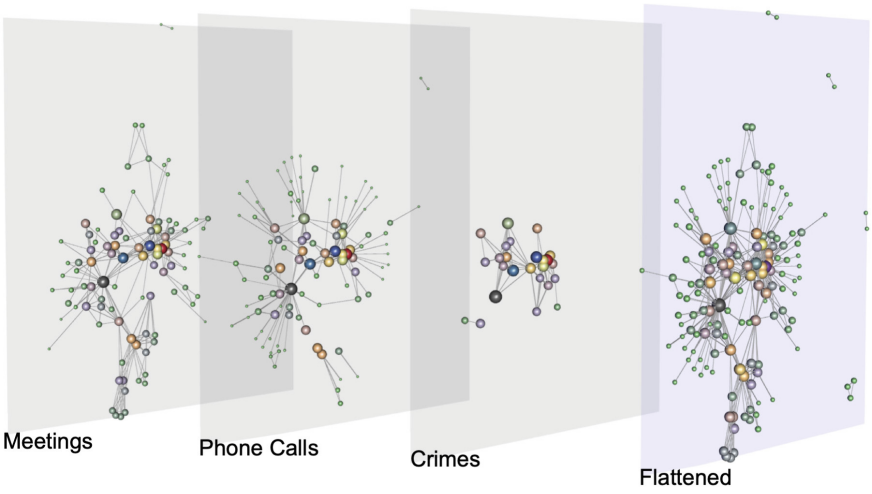

Fig. 1. Meetings, Phone Calls and Crimes and the flattened multilayer network from the Montagna anti-mafia operation.

Our analysis was conducted using the mathematical framework[1] of continuous-time classical and quantum centrality measures created by Böttcher and Porter [6].

The top 20 nodes ranked by their classical and quantum occupation probabilities are shown in Fig. 2. In the analysis of central nodes in a Mafia network, we have to consider the hierarchical structure of a Mafia family in which individuals can be categorized as boss, underboss, consigliere (i.e., an advisor to the boss), messaggero (i.e., a messenger among families), caporegime (i.e., who heads a local crew of soldiers), soldier (*i.e.*, average type criminal) and associate (e.g., entrepreneurs, pharmacists, police officers, politicians etc.). The results for the Meetings and Phone Calls networks confirm the one obtained through the application of the degree as we did in [15], revealing that the most important actors are nodes 18 and 47. These nodes are respectively Caporegime of the Mistretta family and deputy Caporegime of the Batanesi family. The application of OP_c and OP_q on the Crimes network reveals instead different central nodes who are not key members of the Mafia hierarchy but are associates such as entrepreneurs (i.e., nodes 54 and 64), construction workers (i.e., node 72) or soldiers (i.e., nodes 48 and 49). Differently from the application of degree on the multilayer network, in this case, we obtain different central nodes such as 103, 136 and 111. Nodes 103 and 111 were intercepted because they were in phone contact respectively with a Caporegime of the Mistretta family (i.e., node 18) and a non-existent entrepreneur, created for the purpose of fraud (i.e., node 109). Node 136 is instead a cohabitant of a Caporegime of the Batanesi family (i.e., node 27). The role of these figures in the mafia business in unclear but their centrality can direct investigations and lead to deeper investigation of them. Classical and quantum occupation probabilities identify in most cases the same leaders in criminal networks. Therefore, when a micro approach is adopted in SNA application to organized crime focusing, as in our case, on one organization or network with a relatively low number of nodes, there is no clear advantage to using quantum random walks instead of classical ones. However, quantum walks can be useful to speed up the investigations, and to find leaders when a macro approach is adopted to study crime through SNA. In this case, in fact, larger networks focusing on specific national or regional criminal markets or offender categories are analyzed.

In Fig. 3, we show the probabilities that a node with a certain degree is occupied by a classical and quantum random walker. As explained in [6], we can observe a linear dependence between OP_c and OP_q which is due to the fact that quantum walks satisfy Eq. 14 because they do not approach a stationary state. Their long-time behavior is instead characterized by the long-time mean as shown in Eq. 16. Quantum walks are not proportional to the degree which is not able to capture node occupation properties of this kind of walks. These properties, in fact, also depend on other structural features of the underlying networks. The minimum and maximum degrees of the Meetings, Phone Calls, Crimes networks are respectively 1 and 24, 1 and 25, 1 and 13, as shown in the

[1] Available at: https://gitlab.com/ComputationalScience/ctqw_centrality.

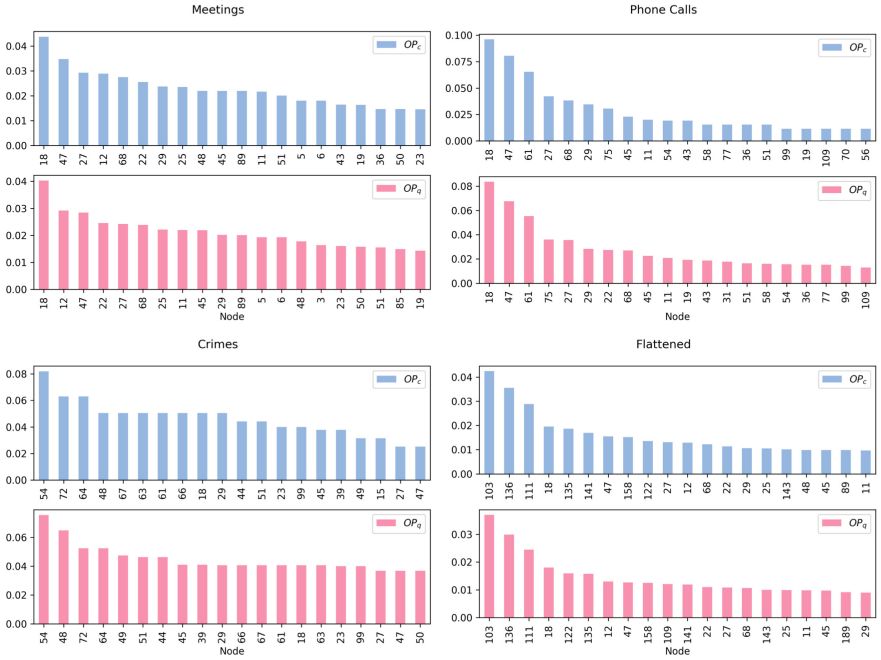

Fig. 2. 20 top ranked nodes computed with classical OP_c and quantum OP_q random walk occupation probabilities in the single layer networks meetings, phone calls and crimes, and the flattened Montagna multilayer network.

plot of the degree distribution in [14]. The differences between these classical and quantum centralities seem to be greater in the flattened multilayer network.

In [6], the authors asserted that the differences between the classical and quantum occupation centralities were (i) larger in BA single-layer networks [24] or in synthetic multilayer networks which include BA layers, and (ii) smaller for empirical networks. The second results is not valuable for real criminal networks, and this can be explained just through the first finding. In our previous work [9], we used some popular network models such as Erdös-Rényi, Watts-Strogatz and different configurations of BA to replicate the topology of a criminal network. Our experiments identified the BA model as the one which better represents a criminal network. To prove our thesis, we decided to build two flattened multilayer networks with three BA layers. Each layer is created with the same number of nodes of the Meetings, Phone Calls, Crimes networks (i.e., 101, 100, and 25, respectively) and a number of edges as close as possible to that of these networks (i.e., 256, 124, and 74, respectively). BA models are built based on a specific parameter m, which indicates the number of edges that are preferentially attached to existing nodes with high degree. For the first layer, we tried two different configurations of the BA graphs with $m = 2$ (obtaining 198 edges) and $m = 3$ (obtaining 294 edges). For the second layer, we used $m = 1$ (obtaining

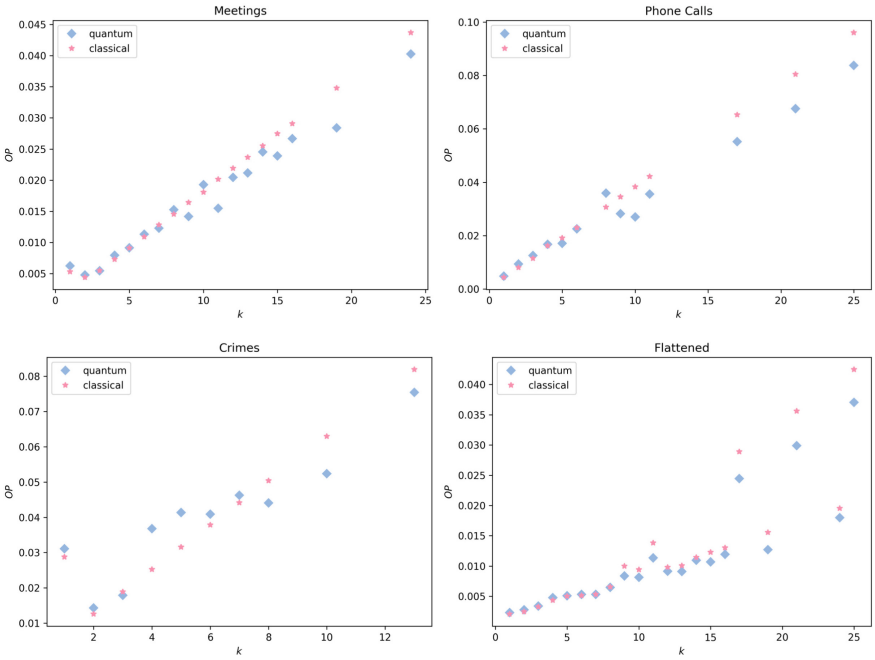

Fig. 3. Classical and quantum random walk occupation probabilities for the single layer networks meetings, phone calls and crimes, and the flattened Montagna multilayer network that a node of degree k is occupied by a classical and quantum random walker, respectively.

99 edges) and $m = 2$ (obtaining 196 edges). For the third layer, we choose $m = 3$ (obtaining 66 edges) and $m = 4$ (obtaining 84 edges). Figure 4 shows classical and quantum occupation centralities versus the node degrees on the described synthetic multilayer networks.

4 Conclusions and Discussion

In this paper, we used classical and quantum continuous-time random walk centrality measures to identify leaders in (i) three single-layer criminal networks related to an anti-Mafia operation called Montagna, and (ii) a flattened multilayer network created from them.

In the formulation of these measures, different Hamiltonians describe the evolution of classical and quantum walks. Other classical and quantum continuous-time random walk centralities, such as random walk betweenness or closeness, can be derived by modifying these Hamiltonians, and tracking different properties of absorbing random walks. In our future works, we want to apply these other measures on our criminal networks to identify leaders, and to define a new disruption framework based on random walks, comparing the targeted attacks

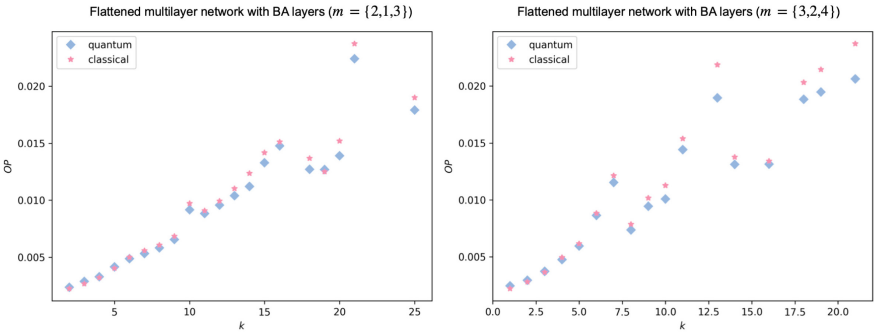

Fig. 4. Classical and quantum random walk occupation probabilities for a flattened multilayer network composed by three Barabási-Albert layers with 101 nodes and 198 edges, 100 nodes and 99 edges, 25 nodes and 66 edges (left column); 101 nodes and 294 edges, 100 nodes and 196 edges, 25 nodes and 84 edges (right column).

based on the classical centrality measures with those based on their random walk versions.

Moreover, we are planning to extend the mathematical framework by Böttcher and Porter [6] for whole multilayer networks considering the layered structure, and not just the flattening function. In fact, we can obtain a complete picture of the activities and the structure of criminal organizations only considering the various layers not as single-layer networks but as a whole multilayer network [14]. Then, we want to explore the possibility of developing a disruption framework for multilayer networks.

References

1. Aharonov, Y., Davidovich, L., Zagury, N.: Quantum random walks. Phys. Rev. A **48**(2), 1687 (1993)
2. Amancio, D.R.: Comparing the topological properties of real and artificially generated scientific manuscripts. Scientometrics **105**(3), 1763–1779 (2015)
3. Ambainis, A.: Quantum walk algorithm for element distinctness. SIAM J. Comput. **37**(1), 210–239 (2007)
4. Askar, A., Cakmak, A.S.: Explicit integration method for the time-dependent Schrodinger equation for collision problems. J. Chem. Phys. **68**(6), 2794–2798 (1978)
5. Barabási, A.L., Albert, R.: Emergence of scaling in random networks. Science **286**(5439), 509–512 (1999)
6. Böttcher, L., Porter, M.A.: Classical and quantum random-walk centrality measures in multilayer networks. SIAM J. Appl. Math. **81**(6), 2704–2724 (2021)
7. Calderoni, F., Catanese, S., De Meo, P., Ficara, A., Fiumara, G.: Robust link prediction in criminal networks: a case study of the Sicilian Mafia. Expert Syst. Appl. **161**, 113666 (2020)
8. Calderoni, F., Superchi, E.: The nature of organized crime leadership: criminal leaders in meeting and wiretap networks. Crime Law Social Change **72**(4), 419–444 (2019)

9. Cavallaro, L., Ficara, A., Curreri, F., Fiumara, G., De Meo, P., Bagdasar, O., Liotta, A.: Graph comparison and artificial models for simulating real criminal networks. In: Benito, R.M., Cherifi, C., Cherifi, H., Moro, E., Rocha, L.M., Sales-Pardo, M. (eds.) Complex Networks and Their Applications IX, pp. 286–297. Springer, Cham (2021)
10. Cavallaro, L., Ficara, A., De Meo, P., Fiumara, G., Catanese, S., Bagdasar, O., Song, W., Liotta, A.: Disrupting resilient criminal networks through data analysis: the case of Sicilian Mafia. PLOS ONE **15**(8), 1–22 (2020)
11. Childs, A.M., Cleve, R., Deotto, E., Farhi, E., Gutmann, S., Spielman, D.A.: Exponential algorithmic speedup by a quantum walk. In: Proceedings of the Thirty-fifth Annual ACM Symposium on Theory of Computing, pp. 59–68 (2003)
12. de Arruda, H.F., Silva, F.N., da F. Costa, L., Amancio, D.R.: Knowledge acquisition: a complex networks approach. Inf. Sci. **421**, 154–166 (2017)
13. Ficara, A., Cavallaro, L., Curreri, F., Fiumara, G., De Meo, P., Bagdasar, O., Song, W., Liotta, A.: Criminal networks analysis in missing data scenarios through graph distances. PLOS ONE **16**(8), 1–18 (2021)
14. Ficara, A., Fiumara, G., Catanese, S., De Meo, P., Liu, X.: The whole is greater than the sum of the parts: a multilayer approach on criminal networks. Future Internet **14**(5), 123 (2022)
15. Ficara, A., Fiumara, G., De Meo, P., Catanese, S.: Multilayer network analysis: the identification of key actors in a Sicilian Mafia operation. In: Perakovic, D., Knapcikova, L. (eds.) Future Access Enablers for Ubiquitous and Intelligent Infrastructures, pp. 120–134. Springer, Cham (2021)
16. Ficara, A., Saitta, R., Fiumara, G., De Meo, P., Liotta, A.: Game of Thieves and WERW-Kpath: two novel measures of node and edge centrality for Mafia networks. In: Teixeira, A.S., Pacheco, D., Oliveira, M., Barbosa, H., Gonçalves, B., Menezes, R. (eds.) Complex Networks XII, pp. 12–23. Springer, Cham (2021)
17. Li, X., Han, Z., Wang, L., Lu, H.: Visual tracking via random walks on graph model. IEEE Trans. Cybern. **46**(9), 2144–2155 (2016)
18. Paoli, L.: Mafia Brotherhoods: Organized Crime, Italian style. OUP (2008)
19. Paparo, G.D., Müller, M., Comellas, F., Martin-Delgado, M.A.: Quantum google in a complex network. Sci. Rep. **3**, 2773 (2013)
20. Pearson, K.: The problem of the random walk. Nature **72**, 294 (1905)
21. Portugal, R.: Quantum Walks and Search Algorithms. Quantum Science and Technology, Springer, New York (2013)
22. Sarkar, P., Moore, A.W.: Random walks in social networks and their applications: a survey. In: Aggarwal, C. (ed.) Social Network Data Analytics, pp. 43–77. Springer, Boston, MA (2011)
23. Shenvi, N., Kempe, J., Whaley, K.B.: Quantum random-walk search algorithm. Phys. Rev. A **67**(5), 052307 (2003)
24. Wald, S., Böttcher, L.: From classical to quantum walks with stochastic resetting on networks. Phys. Rev. E **103**(1), 012122 (2021)
25. Xia, F., Liu, J., Nie, H., Fu, Y., Wan, L., Kong, X.: Random walks: a review of algorithms and applications. IEEE Trans. Emerg. Topics Comput. Intel. **4**(2), 95–107 (2020)