

# Granular Soil Relationship Between Angle of Internal Friction and Uniformity Coefficient



Mindaugas Zakarka  and Šarūnas Skuodis 

**Abstract** The strength characteristics of the sand and gravel are influenced by the size of the grains, their distribution and packaging. The theoretical approach states that the sand angle of internal friction decreases if the uniformity coefficient increases. There are insufficient data for gravel correlation between the uniformity coefficient and the angle of internal friction. Consolidated drained triaxial compression tests (CD) were conducted to determine the strength parameters of remolded sand and gravel samples. These samples were classified as sands and gravels. The optimal water content and density were determined by standard Proctor compaction test and used for these samples. Consolidated drained triaxial compression test gives more reliable data that idealize the soil behavior in the real situation. Three different confining pressures of 20, 50 and 70 kPa were applied to restore horizontal stresses for the soil specimens imitating embankment behavior affected with traffic load. The results indicate that the sand angle of internal friction decreases if the uniformity coefficient is increased. The gravel angle of internal friction does not correlate with the uniformity coefficient.

**Keywords** Angle of internal friction · Particle size · Uniformity coefficient · Sand · Gravel · Triaxial compression test

## 1 Introduction

Due to their strength, compressibility and permeability properties, cohesionless soils are widely used for the installation of road structures [14]. Shear strength parameters are very important for the design of a safe and economical road structure. It was observed that the shear strength parameters are affected by the relative density, gradation, particle strength, size, shape, and degree of saturation of the specimen

---

M. Zakarka (✉) · Š. Skuodis

Department of Reinforced Concrete Structures and Geotechnics, Vilnius Gediminas Technical University, Saulėtekio al. 11, LT-0223 Vilnius, Lithuania  
e-mail: [mindaugas.zakarka@vilniustech.lt](mailto:mindaugas.zakarka@vilniustech.lt)

[19, 20]. Particle size has been identified as an important factor in shear strength, and its effects have been studied for the last few decades [2, 14, 15].

The angle of internal friction is specifically emphasized as the most important parameter of sandy soils. The shear strength can be determined using Mohr's circle failure criterion [4]. A soil's angle of internal friction describes the shear resistance of a soil with presence of normal effective stress at which shear failure occurs [13]. Different methods were used to determine the relation of particles size with angle of internal friction: direct shear test [1, 7], triaxial test [8], 2D and 3D discrete element method analysis [6]. Sufficiently contradictory conclusions were obtained. At low sandy soil densities, the angle of internal friction decreases with increasing effective diameter ( $d_{10}$ ) [5]. Sandy soils have a direct relationship between the angle of friction and the coefficient of uniformity [14]. The grain size distribution can greatly affect shear strength characteristics in sandy soils [17]. Although the investigations concluded that there is a relationship between size of the particles and angle of internal friction, the further investigations are needed.

Consolidated drained triaxial compression tests were applied to restore horizontal stresses for the soil specimens, imitating embankment behavior affected with traffic load. This research is oriented to road constructions, therefore three different confining pressures of 20, 50 and 70 kPa were applied [21]. For this study, three sand and three gravel types were analysed, differing from each other in terms of both the mean grain size and uniformity. The objective of this study is to assess granular materials relationship between the size of particle and the angle of internal friction.

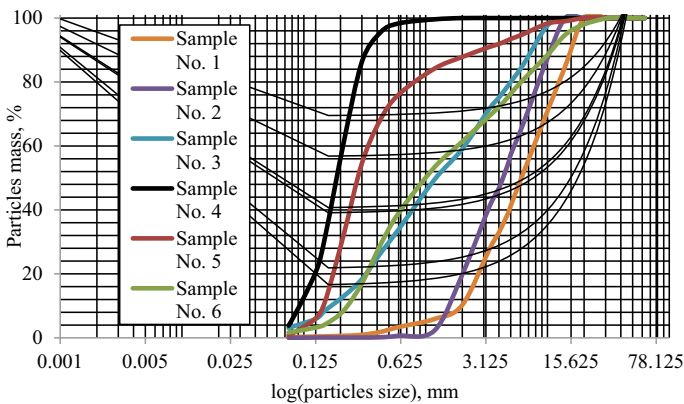
## 2 Sample Preparation and Testing

For this investigation, three different types of sandy and gravelly soils were selected from different locations in Lithuania. A detailed laboratory investigation was carried out to determine the physical properties of the soil samples. Determination of particles size distribution was conducted according to LST EN ISO 17892-4:2017, Proctor compaction was conducted according to LST EN 13,286-2:2015. The physical properties of the soils are presented in Table 1, particle size distribution is presented in Fig. 1. Samples symbols given in Table 1 correspond LST 1331:2015.

Soil specimens for triaxial test apparatus were remolded with water content determined by standard Proctor compaction test. To each different soil, three sets of cylinder samples were prepared, the diameter of which 10.0 cm, height—20.0 cm. The tests were chosen to run under unsaturated conditions. Samples were prepared at Proctor water content without additional saturation. Three different cell pressures were used for consolidation: 20, 50, 70 kPa. The water can drain during the test. All of these conditions were selected according to LST EN ISO 17892-9:2017. All samples were consolidated for 30 min. The vertical strain velocity 0.950 %/min was accepted, based on consolidation time according to LST EN ISO 17892-9:2017. The tests were conducted by deforming the specimen until 15% of the vertical deformation. This research is oriented to road constructions, therefore the experiments used

**Table 1** Physical properties of the soils

Sample no.	Sample symbol	Soil type	Water content determined by standard proctor compaction test $w, \%$	Density of specimen determined by standard proctor compaction test $\rho, \text{g/cm}^3$
1	GrP	Poorly graded gravel	3.0	1.90
2	GrP	Poorly graded gravel	3.5	1.83
3	GrM	Medium graded gravel	7.5	2.08
4	SaP	Poorly graded sand	14.3	1.65
5	SaP	Poorly graded sand	8.0	1.75
6	SaM	Medium graded sand	8.8	1.98



**Fig. 1** Particle size distribution curves

the following cell pressures:  $\sigma_3 = 20, 50, 70 \text{ kPa}$ . The Mohr–Coulomb criterion  $\tau = \sigma' \tan\phi' + c'$  was applied for results interpretation [3].

### 3 Results

Triaxial compression test results of different soil samples are presented in Table 2. The samples failure shape presented in Fig. 2. For samples Nos. 1–2, it is impossible to take photographs after the test without a membrane because the soils are too sensitive and collapse immediately. To make the results comparable, gravels (Nos. 1–3) and sands (Nos. 4–6) were separated.

As observed from test results, sands angle of internal friction ranges from  $41.92^\circ$  to  $43.93^\circ$ . The angle of internal friction of the sandy soil increases as the coefficient of uniformity of the samples decreases (Fig. 3). The same dependency was obtained

**Table 2** Physical and mechanical properties of the soils

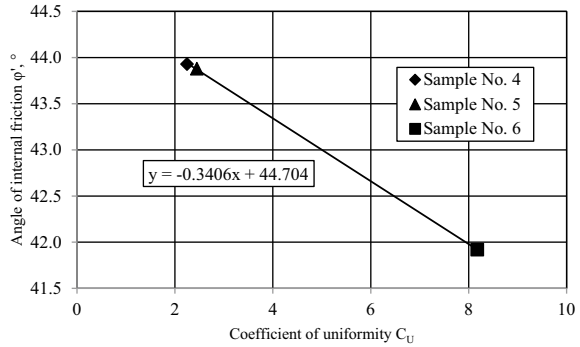
Sample no.	Sample symbol	$d_{10}$	$d_{30}$	$d_{50}$	$d_{60}$	$C_U$	$C_C$	$\varphi'$ , °	$c'$ , kPa
1	GrP	1.95	3.62	6.13	7.94	4.08	0.85	44.95	10.7
2	GrP	1.54	2.64	4.31	5.37	3.48	0.84	43.16	3.1
3	GrM	0.17	0.51	1.23	2.10	12.35	0.73	42.35	14.5
4	SaP	0.09	0.15	0.19	0.21	2.25	1.08	43.93	0
5	SaP	0.14	0.21	0.28	0.35	2.45	0.85	43.88	0
6	SaM	0.23	0.44	1.05	1.86	8.17	0.46	41.92	0

**Fig. 2** The shape of failure from the left: sample No. 1; sample No. 2; sample No. 3; sample No. 4; sample No. 5; sample No. 6

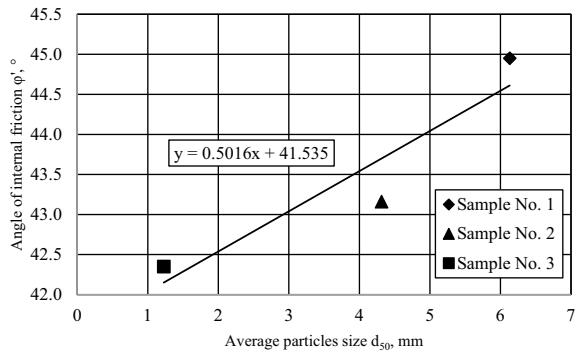
with the average particles size  $d_{50}$ , which is the opposite of what is found in the literature [16, 18].

Gravels' angle of internal friction varies from  $42.35^\circ$  to  $44.95^\circ$ . Figure 4 shows that angle of internal friction increased with the increasing average size of the particles  $d_{50}$ .

**Fig. 3** The relationship between the coefficient of uniformity and the angle of internal friction for sands



**Fig. 4** The relationship between the average size of particles and the angle of internal friction for gravels



For investigated gravels there is no relationship between friction angle and coefficient of uniformity.

Correlation that can be established between coefficient of uniformity and angle of internal friction in sandy soils is graphically presented in Fig. 3. Another correlation established between average particle size  $d_{50}$  and angle of internal friction in gravelly soils is graphically presented in Fig. 4.

## 4 Conclusion

Since all the specimens were tested under the same experimental conditions (optimal water content, cell pressure, etc.), grain size distribution was the only physical parameter varying from one test to another. After the tests and analysis of tests results such conclusions can be made:

1. Increasing the coefficient of uniformity in sandy soil decreases the angle of internal friction;
2. Increasing the average particles size  $d_{50}$  in gravelly soil increases the angle of internal friction;

3. There is no direct relationship between the size of particles  $d_{50}$  and the angle of internal friction in investigated sandy soils;
4. There is no direct relationship between the coefficient of uniformity and the angle of internal friction in investigated gravelly soils.

The relationship between the particle size distribution and the angle of internal friction for granular materials is still debatable.

## References

1. Afrazi M, Yazdani M (2021) Determination of the effect of soil particle size distribution on the shear behavior of sand. *J Adv Eng Computation* 5(2):125–134
2. Deng Y, Yilmaz Y, Gokce A, Chang CS (2021) Influence of particle size on the drained shear behavior of a dense fluvial sand. *Acta Geotech* 16:2071–2088
3. Dirgėlienė N, Skuodis Š, Vasys E (2022) The behaviour of stress variation in sandy soil. *Open Geosci.* 14(1):13–23
4. Heyman J (1972) *Coulomb's memoir on statics*. Cambridge University Press, London
5. Islam M, Badhon FF, Abedin MZ (2017) Relation between effective particle size and angle of internal friction of cohesionless soil. In: *Proceedings of international conference on planning, architecture and civil engineering*
6. Islam M, Siddika A, Hossain B, Rahman AN, Asad MA (2011) Effect of particle size on the shear strength behavior of sands. *Australian Geomechanics.* 46(3):85–95
7. Kara EM, Meghachou M, Aboubekr N (2013) Contribution of particles size ranges to sand friction. *Eng, Technol & Appl Sci Res* 3(4):497–501
8. Kirkpatrick WM (1965) Effects of grain size and grading on the shearing behaviour of granular materials. In: *Proceedings of the sixth International conference on soil mechanics and foundation engineering*, vol 1. Canada, pp. 273–277
9. LST 1331:2015 (2015) *Gruntai skirti keliams ir jų statiniams. Klasifikacija*. Vilnius, Lietuvos standartizacijos departamentas
10. LST EN 13286-2:2015 (2015) *Unbound and hydraulically bound mixtures—Part 2: test methods for laboratory reference density and water content—Proctor compaction*. Vilnius, Lietuvos standartizacijos departamentas
11. LST EN ISO 17892-4:2017 (2017) *Geotechnical investigation and testing—Laboratory testing of soil—part 4: Determination of particle size distribution*. Vilnius, Lietuvos standartizacijos departamentas
12. LST EN ISO 17892-9:2017 (2017) *Geotechnical investigation and testing—Laboratory testing of soil—part 9: Consolidated triaxial compression tests on water saturated soils*. Vilnius, Lietuvos standartizacijos departamentas
13. Mitchell JK, Houston WN, Scott RF, Costes NC, Carrier WD, Bromwell LG (1972) Mechanical properties of lunar soil: Density, porosity, cohesion and angle of internal friction. In: *Proceedings of the lunar science conference*, pp. 3235–3253
14. Rasti A, Adarmanabadi HR, Pineda M, Reinikainen J (2021) Evaluating the effect of soil particle characterization on internal friction angle. *Am J Eng Appl Sci* 14(1):129–138
15. Sharma V, Kumar A, Priyadarshree A, Chhotu AK (2019) Prediction of shear strength parameter from the particle size distribution and relative density of granular soil. In: Agnihotri A, Reddy K, Bansal A (eds) *Environmental geotechnology. Lecture notes in civil engineering*, vol 31. Springer, Singapore
16. Vangla P, Latha GM (2015) Influence of particle size on the friction and interfacial shear strength of sands of similar morphology. *Int J Geosynth Ground Eng* 1(6)

17. Viggiani G, Kuntz M, Desrues JJM (2001) An experimental investigation of the relationships between grain size distribution and shear banding in sand. In: Continuous and discontinuous modelling of cohesive-frictional materials. Lecture notes in physics, vol 568. Springer, Berlin, pp 111–127
18. Wang JJ, Zhang HP, Tang SC, Liang Y (2013) Effects of particle size distribution on shear strength of accumulation soil. *J Geotech Geoenvironmental Engi-Neering* 139(11):1994–1997
19. Whalley WB (1979) Discussion on ‘Effect of sand grain shape on interparticle friction’’. *Geotechnics* 29(3):341–350
20. Yu X, Ji S, Janoyan KD (2006) Direct shear testing of rockfill material in soil and rock behavior and modeling. Geotechnical Special Publication, American Society of Civil Engineers, pp 149–155
21. Zakarka M (2022) Analysis of the traffic load—induced stresses of embankment. *Mokslas—Lietuvos ateitis* 14: 1–7