CIFD: A Distance for Complex Intuitionistic Fuzzy Set

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Abstract. Intuitionistic fuzzy set (IFS) has attracted much attention because it can deal with fuzziness and uncertainty more flexibly than traditional fuzzy set. Complex intuitionistic fuzzy set (CIFS) extends intuitionistic fuzzy to the complex plane, which can better express and process uncertain information. In this paper, a novel distance measure complex intuitionistic fuzzy distance (CIFD) is proposed for CIFSs. Firstly, inspired by Tanimoto coefficient, a new similarity measure between CIFSs is proposed. Then, based on the similarity measure, the CIFD is proposed and its non-negativity, non-degeneracy, and symmetry are analyzed. Moreover, when CIFS degenerates into classical IFS, CIFD is also applicable to measure the differences between IFSs. Finally, to illustrate the effectiveness of CIFD, an example is given at the end.

Keywords: Complex intuitionistic fuzzy set · Similarity measure · Complex intuitionistic fuzzy distance

1 Introduction

In information fusion, it is inevitable to deal with uncertain information [1–3]. Therefore, many methods are proposed to deal with uncertain information, including Dempster-Shafer (D-S) evidence theory [4–6], evidential reasoning [7–9], entropy-based [10], and fuzzy sets [11]. These methods have been widely used in numerous fields, including clustering [12], classification [13–15], complex network [16–18], uncertainty-based multidisciplinary design optimization [19], parrondo effect [20,21], and decision-making [22,23].

As is well known, fuzzy sets can handle fuzzy and uncertain information well. Therefore, fuzzy sets have been widely studied and extended [24–26]. As one of the extensions of fuzzy sets, intuitionistic fuzzy set (IFS) contains membership

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function, non-membership function and hesitation function, which can better express uncertainty [27–29].

In a recent work, a novel complex intuitionistic fuzzy set (CIFS) was proposed, which extends IFS to the complex plane [30]. In CIFS, membership function, non-membership function, and hesitation function are all represented by complex numbers, including amplitude term and phase term. Due to the potentially beneficial property of complex numbers in expressing uncertain information, complex-valued models have been extensively studied [31]. Xiao [32] extended the D-S evidence theory to the complex plane, and proposed complex evidence theory (CET), which has been applied to target recognition, classification, decision-making, and other fields. In addition, since both CET and quantum mechanics are based on complex numbers [33,34], CET can be combined with quantum mechanics to deal with uncertain information [35]. As an extension of IFS, CIFS will degenerate into IFS when the membership function, non-membership function, and hesitation function degenerate from complex numbers to real numbers. Therefore, CIFS provides a more promising framework for dealing with uncertain information.

In IFS, there exist a host of distances that measure the difference between IFSs [36]. However, no existing method can measure the difference between CIFSs in [30]. Therefore, inspired by the Tanimoto coefficient, this paper proposes a novel distance measure complex intuitionistic fuzzy distance (CIFD) for CIFS. When CIFS degenerates into classical IFS, CIFD is also applicable to measure the differences between IFSs. Before proposing CIFD, a novel method to measure the similarity between CIFSs was proposed and its properties were analyzed, including non-negativity, non-degeneracy, and symmetry. Then, based on the novel similarity measure, we propose the complex intuitionistic fuzzy distance, which can measure the differences between CIFSs. CIFD is proved to satisfy non-negativity, non-degeneracy and symmetry. Specifically, to illustrate the effectiveness of CIFD, an example is given at the end.

In Sect. 2, the definition of CIFS is introduced. In Sect. 3, a novel similarity measure is proposed, and then CIFD between CIFS is proposed based on the similarity measure. In Sect. 4, an example is given to show the effectiveness of CIFD. Section 5 gives a conclusion of this paper.

2 Complex Intuitionistic Fuzzy Set

As an extension of fuzzy sets, IFS takes into account the information of membership, non-membership and hesitation. It can better express and process uncertain information. Therefore, IFS is widely studied and extended. A recent work extended IFS to the complex plane and proposed a novel CIFS [30]. The specific definition is below.

Definition 1. *(Complex Intuitionistic Fuzyy Set): Let a non-empty set X be a universe of discourse. A complex intuitionistic fuzzy set (CIFS) $\mathbb{A} \in X$ is defined as:*

$$\mathbb{A} = \{\langle x, \mu^c_{\mathbb{A}}(x), \nu^c_{\mathbb{A}}(x)\rangle | x \in X\}, \tag{1}$$

in which

$$\mu_{\mathbb{A}}^c(x) = \mu_{\mathbb{A}}(x)e^{i\theta_{\mu_{\mathbb{A}}}(x)} : X \to \{\mu_{\mathbb{A}}^c(x)|\mu_{\mathbb{A}}^c(x) \in C, |\mu_{\mathbb{A}}^c(x)| \leqslant 1\}, \tag{2}$$

$$\nu_{\mathbb{A}}^c(x) = \nu_{\mathbb{A}}(x)e^{i\theta_{\nu_{\mathbb{A}}}(x)} : X \to \{\nu_{\mathbb{A}}^c(x)|\nu_{\mathbb{A}}^c(x) \in C, |\nu_{\mathbb{A}}^c(x)| \leqslant 1\}, \tag{3}$$

where $i = \sqrt{-1}$, $\mu_{\mathbb{A}}(x) \in [0,1]$ *and* $\nu_{\mathbb{A}}(x) \in [0,1]$ *represent the magnitude of* $\mu_{\mathbb{A}}^c(x)$ *and* $\nu_{\mathbb{A}}^c$. $\theta_{\mu\mathbb{A}}(x) \in [-\pi, \pi]$ *and* $\theta_{\nu\mathbb{A}}(x) \in [-\pi, \pi]$ *are the phase term. The functions* $\mu_{\mathbb{A}}^c(x)$ *and* $\nu_{\mathbb{A}}^c(x)$ *are the degree of membership and degree of non-membership of the element* $x \in X$ *to the set* X*, respectively.*

In addition to the membership function and the non-membership function, the remaining uncertain membership is assigned to the hesitation function $\pi_{\mathbb{A}}^c(x)$*:*

$$\pi_{\mathbb{A}}^c(x) = 1 - \mu_{\mathbb{A}}^c(x) - \nu_{\mathbb{A}}^c(x). \tag{4}$$

A CIFS can also be written in the Cartesian form, denoted by:

$$\mu_{\mathbb{A}}^c(x) = a_\mu(x) + b_\mu(x)i, \tag{5}$$

$$\nu_{\mathbb{A}}^c(x) = a_\nu(x) + b_\nu(x)i, \tag{6}$$

$$\pi_{\mathbb{A}}^c(x) = a_\pi(x) + b_\pi(x)i, \tag{7}$$

with

$$0 \leqslant \sqrt{a_\mu(x)^2 + b_\mu(x)^2}, \sqrt{a_\nu(x)^2 + b_\nu(x)^2}, \sqrt{a_\pi(x)^2 + b_\pi(x)^2} \leqslant 1. \tag{8}$$

Through Euler's formula, Eq.(2) and Eq.(5) have the following relationship:

$$\mu_{\mathbb{A}}(x) = \sqrt{a_\mu(x)^2 + b_\mu(x)^2}, \tag{9}$$

$$\theta_{\mu_{\mathbb{A}}}(x) = arctan\frac{b_\mu(x)}{a_\mu(x)}, \tag{10}$$

where $a_\mu(x) = \mu_{\mathbb{A}}(x)cos(\theta_{\mu_{\mathbb{A}}}(x))$ and $b_\mu(x) = \mu_{\mathbb{A}}(x)sin(\theta_{\mu_{\mathbb{A}}}(x))$. The absolute value of $\mu_{\mathbb{A}}^c(x)$ is calculated by:

$$|\mu_{\mathbb{A}}^c(x)| = \sqrt{\mu_{\mathbb{A}}^c(x)\bar{\mu}_{\mathbb{A}}^c(x)} = \sqrt{a_\mu(x)^2 + b_\mu(x)^2}, \tag{11}$$

where $\bar{\mu}_{\mathbb{A}}^c(x)$ is the complex conjugate of $\mu_{\mathbb{A}}^c(x)$ denoted by $\bar{\mu}_{\mathbb{A}}^c(x) = a_\mu(x) - b_\mu(x)i$. From the above relationship, we can get:

$$\mu_{\mathbb{A}}(x) = |\mu_{\mathbb{A}}^c(x)|, \tag{12}$$

$$\theta_{\mu_{\mathbb{A}}}(x) = \angle\mu_{\mathbb{A}}^c(x). \tag{13}$$

In addition, non-membership function $\nu_{\mathbb{A}}(x)$ and hesitation function $\pi_{\mathbb{A}}(x)$ also have the above corresponding transformation formula.

CIFS extends the intuitionistic fuzzy set to the complex plane, when $\theta_{\mu\mathbb{A}}(x) = \theta_{\nu\mathbb{A}}(x) = \theta_{\pi\mathbb{A}}(x) = 0$, CIFS degenerates into intuitionistic fuzzy set, so it can better express and deal with the uncertainty in the process of the information fusion. However, there is no existing method to measure the differences between CIFSs. Therefore, inspired by the Tanimoto coefficient, a novel distance is proposed in the next section.

3 New Distance for Complex Intuitionistic Fuzzy Sets

In this section, we first propose a novel similarity measure between CIFSs. Then, based on the novel similarity measure, the complex intuitionistic fuzzy distance (CIFD) is proposed for CIFS. In addition, the properties of CIFD are analyzed and proved.

3.1 Similarity Measure Between CIFSs

Tanimoto coefficient is widely used in many fields for similarity measurement [37]. The specific definition of the Tanimoto coefficient is as follows.

Definition 2. *Let $R = \{r_1, r_2, ..., r_n\}$ and $S = \{s_1, s_2, ..., s_n\}$ be two probability distribution. The Tanimoto coefficient between R and S is defined as follows [37]:*

$$T(R,S) = \frac{\sum_{i=1}^{n} r_i s_i}{\sum_{i=1}^{n} r_i^2 + \sum_{i=1}^{n} s_i^2 - \sum_{i=1}^{n} r_i s_i}. \tag{14}$$

Inspired by the Tanimoto coefficient, we extend it to the complex plane and propose a novel similarity measure between CIFSs.

Definition 3. *(Similarity Measure Between CIFSs): Let $\mathbb{A}$ and $\mathbb{B}$ be two CIFSs on a universe of discourse $X = \{x_1, x_2, ..., x_n\}$ denoted by:*

$$\mathbb{A} = \{\langle x, \mu_{\mathbb{A}}^c(x), \nu_{\mathbb{A}}^c(x), \pi_{\mathbb{A}}^c(x)\rangle | x \in X\},$$

$$\mathbb{B} = \{\langle x, \mu_{\mathbb{B}}^c(x), \nu_{\mathbb{B}}^c(x), \pi_{\mathbb{B}}^c(x)\rangle | x \in X\}.$$

The similarity between CIFSs $\mathbb{A}$ and $\mathbb{B}$ is defined as:

$$Sim(\mathbb{A},\mathbb{B}) = \frac{1}{n}\sum_{i=1}^{n} \frac{|\sum_{j=1}^{3} \mathbb{A}_j(x_i)\bar{\mathbb{B}}_j(x_i)|}{|\sum_{j=1}^{3} |\mathbb{A}_j(x_i)|^2 + |\mathbb{B}_j(x_i)|^2 - \mathbb{A}_j(x_i)\bar{\mathbb{B}}_j(x_i)|}, \tag{15}$$

in which $\mathbb{A}_1(x)$, $\mathbb{A}_2(x)$ and $\mathbb{A}_3(x)$ represent $\mu_{\mathbb{A}}^c(x)$, $\nu_{\mathbb{A}}^c(x)$ and $\pi_{\mathbb{A}}^c(x)$, respectively. $\bar{\mathbb{B}}_j(x)$ is the complex conjugate of $\mathbb{B}_j(x)$, and $|\cdot|$ represents the absolute value operation.

For the denominator in Eq. (15), the following equation can be obtained.

$$\begin{aligned}
&|\mathbb{A}_1\bar{\mathbb{A}}_1 + \mathbb{A}_2\bar{\mathbb{A}}_2 + \mathbb{A}_3\bar{\mathbb{A}}_3 + \mathbb{B}_1\bar{\mathbb{B}}_1 + \mathbb{B}_2\bar{\mathbb{B}}_2 + \mathbb{B}_3\bar{\mathbb{B}}_3 - \mathbb{A}_1\bar{\mathbb{B}}_1 - \mathbb{A}_2\bar{\mathbb{B}}_2 - \mathbb{A}_2\bar{\mathbb{B}}_2| \\
&= |(\mathbb{A}_1 - \mathbb{B}_1)(\bar{\mathbb{A}}_1 - \bar{\mathbb{B}}_1) + (\mathbb{A}_2 - \mathbb{B}_2)(\bar{\mathbb{A}}_2 - \bar{\mathbb{B}}_2) + (\mathbb{A}_3 - \mathbb{B}_3)(\bar{\mathbb{A}}_3 - \bar{\mathbb{B}}_3) \\
&\quad + \mathbb{B}_1\bar{\mathbb{A}}_1 + \mathbb{B}_2\bar{\mathbb{A}}_2 + \mathbb{B}_3\bar{\mathbb{A}}_3| \\
&= |(\mathbb{A}_1 - \mathbb{B}_1)(\bar{\mathbb{A}}_1 - \bar{\mathbb{B}}_1) + (\mathbb{A}_2 - \mathbb{B}_2)(\bar{\mathbb{A}}_2 - \bar{\mathbb{B}}_2) + (\mathbb{A}_3 - \mathbb{B}_3)(\bar{\mathbb{A}}_3 - \bar{\mathbb{B}}_3) \\
&\quad + \mathbb{A}_1\bar{\mathbb{B}}_1 + \mathbb{A}_2\bar{\mathbb{B}}_2 + \mathbb{A}_3\bar{\mathbb{B}}_3|
\end{aligned} \tag{16}$$

Therefore, Eq. (15) can also be written as:

$$Sim(\mathbb{A},\mathbb{B}) = \frac{1}{n}\sum_{i=1}^{n} \frac{|\sum_{j=1}^{3} \mathbb{A}_j(x_i)\bar{\mathbb{B}}_j(x_i)|}{|\sum_{j=1}^{3} |\mathbb{A}_j(x_i) - \mathbb{B}_j(x_i)|^2 + \mathbb{A}_j(x_i)\bar{\mathbb{B}}_j(x_i)|}. \tag{17}$$

The similarity measure between two CIFSs, $Sim(\mathbb{A},\mathbb{B})$, has the following properties.

Property 1. Suppose $\mathbb{A}$ and $\mathbb{B}$ are two CIFSs on a universe of discourse X. The properties of $Sim(\mathbb{A},\mathbb{B})$ are given as follows:

$P1.1: 0 \leqslant Sim(\mathbb{A},\mathbb{B}) \leqslant 1$.

$P1.2: Sim(\mathbb{A},\mathbb{B}) = 1$ if and only if $\mathbb{A} = \mathbb{B}$.

$P1.3: Sim(\mathbb{A},\mathbb{B}) = Sim(\mathbb{B},\mathbb{A})$.

For the above properties, the specific proof is as below.

Proof. (1) Through Eq. (17), since $|\cdot|$ represents the absolute value operation, $Sim(\mathbb{A},\mathbb{B}) \geqslant 0$. $|\mathbb{A}_j(x) - \mathbb{B}_j(x)|$ is a real number greater that 0 and is equal to 0 when $\mathbb{A}_j(x) = \mathbb{B}_j(x)$, therefore, $|\sum_{j=1}^{3} |\mathbb{A}_j(x_i) - \mathbb{B}_j(x_i)|^2 + \mathbb{A}_j(x_i)\bar{\mathbb{B}}_j(x_i)| \geqslant |\sum_{j=1}^{3} \mathbb{A}_j(x_i)\bar{\mathbb{B}}_j(x_i)|$ indicating that $Sim(\mathbb{A},\mathbb{B}) \leqslant 1$. As a consequence, $P1.1$ and $P1.2$ have been proved.

(2) For two complex number C and $\bar{C}$ that are conjugate to each other, $|C| = |\bar{C}|$. $\mathbb{A}_j(x)\bar{\mathbb{B}}_j(x)$ and $\mathbb{B}_j(x)\bar{\mathbb{A}}_j(x)$ are conjugate to each other, therefore,

$$\begin{aligned} Sim(\mathbb{A},\mathbb{B}) &= \frac{1}{n}\sum_{i=1}^{n} \frac{|\sum_{j=1}^{3} \mathbb{A}_j(x_i)\bar{\mathbb{B}}_j(x_i)|}{|\sum_{j=1}^{3} |\mathbb{A}_j(x_i)|^2 + |\mathbb{B}_j(x_i)|^2 - \mathbb{A}_j(x_i)\bar{\mathbb{B}}_j(x_i)|} \\ &= \frac{1}{n}\sum_{i=1}^{n} \frac{|\sum_{j=1}^{3} \mathbb{B}_j(x_i)\bar{\mathbb{A}}_j(x_i)|}{|\sum_{j=1}^{3} |\mathbb{B}_j(x_i)|^2 + |\mathbb{A}_j(x_i)|^2 - \mathbb{B}_j(x_i)\bar{\mathbb{A}}_j(x_i)|} \\ &= Sim(\mathbb{B},\mathbb{A}). \end{aligned} \tag{18}$$

The similarity measure $Sim(\mathbb{A},\mathbb{B})$ can effectively measure the similarity between CIFSs. The range of $Sim(\mathbb{A},\mathbb{B})$ is 0 to 1. The larger the value of $Sim(\mathbb{A},\mathbb{B})$, the more similar the two CIFSs are. In addition, when CIFS degenerates into classical IFS, $Sim(\mathbb{A},\mathbb{B})$ can also measure the similarity between IFSs.

3.2 The Novel Distance Between CIFSs

Based on the similarity measure between CIFSs proposed above, the novel distance between CIFSs is defined below.

Definition 4. *(Complex Intuitionistic Fuzzy Distance): Let $\mathbb{A}$ and $\mathbb{B}$ be two CIFSs on a universe of discourse X. The CIFD between $\mathbb{A}$ and $\mathbb{B}$ is defined as:*

$$d_{CIFS}(\mathbb{A},\mathbb{B}) = \sqrt{1 - Sim(\mathbb{A},\mathbb{B})}. \tag{19}$$

CIFD can measure the differences between CIFSs and has the following properties. The smaller the value of CIFD, the smaller the difference between CIFSs.

Property 2. Let $\mathbb{A}$ and $\mathbb{B}$ be two CIFSs on a universe of discourse X. The CIFD holds the following properties.

$P2.1$ $(Nonnegativity): 0 \leqslant d_{CIFS}(\mathbb{A},\mathbb{B}) \leqslant 1$.

$P2.2$ $(Nondegeneracy): d_{CIFS}(\mathbb{A},\mathbb{B}) = 0$ if and only if $\mathbb{A} = \mathbb{B}$.

$P2.3$ $(Symmetry): d_{CIFS}(\mathbb{A},\mathbb{B}) = d_{CIFS}(\mathbb{B},\mathbb{A})$.

As previously proved by the similarity measure $Sim(\mathbb{A}, \mathbb{B})$, the above properties of CIFD can be easily proved. Moreover, when CIFS degenerates into classical IFS, CIFD is still applicable to measure the distance between IFSs.

4 Numerical Example

In this section, an example is given to verify the effectiveness and properties of CIFD, including non-negativity, non-degeneracy and symmetry.

Example 1. Let $\mathbb{A}$ and $\mathbb{B}$ be two CIFSs on a universe of discourse X given by:

$$\mathbb{A} = \{\langle x + yi, 1 - x - yi \rangle\},$$

$$\mathbb{B} = \{\langle 1 - x + yi, x - yi \rangle\}.$$

First, we set $y = 0$, and x changes from 0 to 1. In this case, CIFSs $\mathbb{A}$ and $\mathbb{B}$ degenerate to classical IFSs. From Fig. 1, CIFD is also applicable to classical IFS. When $x = 0.5$, in this case $\mathbb{A} = \mathbb{B}$, the distance between them is 0. When x is 0 or 1, the maximum distance of 1 is obtained. As x increases from 0 to 1, the distance decreases from 1 to 0 and then increases from 0 to 1.

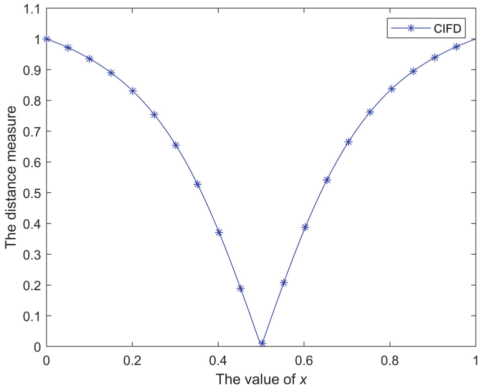

Fig. 1. The result of CIFD in Example 1 under variations in x

Then, to study the relationship between the variation of CIFD and the real and imaginary parts of CIFS, we set $x \in [0, 1]$ and $y \in [-1, 1]$, satisfying $(x^2 + y^2)^{1/2} \leqslant 1$ and $((1 - x)^2 + y^2)^{1/2} \leqslant 1$. Under these conditions, the values of CIFD between $\mathbb{A}$ and $\mathbb{B}$ are shown in Fig. 2.

From Fig. 2(a), when $x = 0.5$, no matter what the value of y is, CIFD always takes the minimum value of 0. This is reasonable since $\mathbb{A} = \mathbb{B}$ in this case. The variation of CIFD between $\mathbb{A}$ and $\mathbb{B}$ with variables x and y is shown in Fig. 2(b). When $x = 1$ and $y = 0$, we have $\mathbb{A} = \{\langle 1, 0 \rangle\}$ and $\mathbb{B} = \{\langle 0, 1 \rangle\}$, and when $x = 0$ and $y = 0$, we have $\mathbb{A} = \{\langle 0, 1 \rangle\}$ and $\mathbb{B} = \{\langle 1, 0 \rangle\}$. In both two cases, $\mathbb{A}$ and

$\mathbb{B}$ are completely different, so CIFD between $\mathbb{A}$ and $\mathbb{B}$ gets the maximum value of 1.

Figures 2(c) and (d) show the CIFD under variables x and y, respectively. CIFD decreases when x changes from 0 to 0.5 and increases when x changes from 0.5 to 1. This is because when $x = 0.5$, in this case $\mathbb{A} = \mathbb{B}$, and as it moves away from 0.5, the distance between them becomes larger and larger. Furthermore, as y varies from -1 to 1, the CIFD value gradually increases form 0 to 1 and then decreases from 1 to 0.

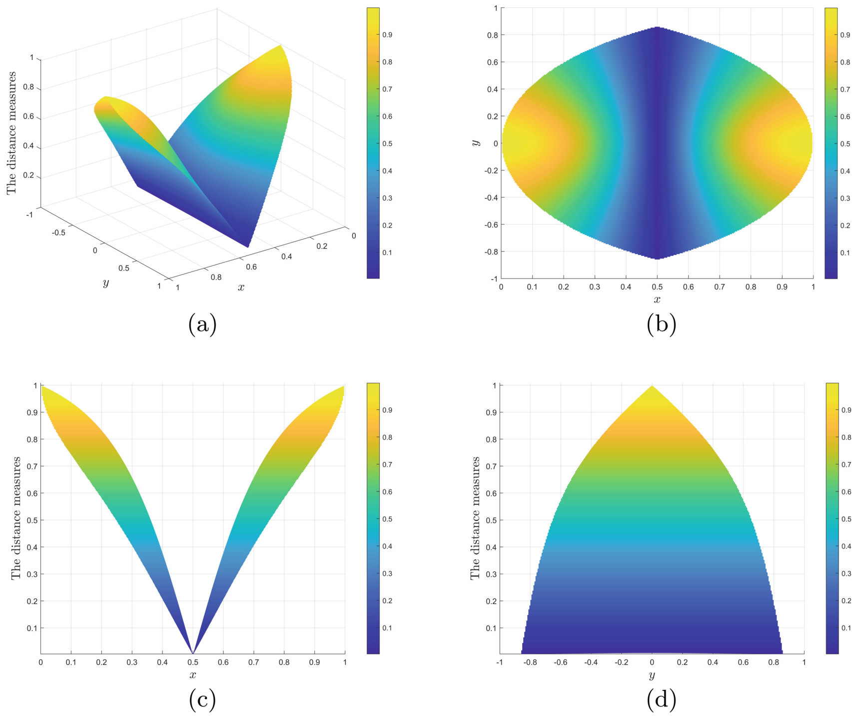

Fig. 2. The value of CIFD varies with the real and imaginary parts of the CIFSs in Example 1. (a) CIFD values. (b) Variations in x and y. (c) CIFD under x variations. (d) CIFD under y variations.

As can be seen from the above analysis, this example also verifies the non-negativity, non-degeneracy and symmetry of CIFD.

5 Conclusion

In this paper, we proposed a novel distance measure CIFD for CIFS. Before giving the definition of CIFD, inspired by Tanimoto coefficient, we proposed a novel similarity measure between CIFS. Then CIFD was proposed based on the novel similarity measure. Moreover, CIFD has been verified to be non-negative, non-degeneracy and symmetry. When CIFS degenerates into IFS, CIFD is also applicable to measuring the distance between IFSs. Specifically, to illustrate the effectiveness of CIFD, an example was given at the end.

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