Chapter 4
Mathematical Modelling as a Stimulus for Curriculum and Instructional Reform in Secondary School Mathematics

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4.1 Mathematical Modelling in the School Curriculum

Curriculum documents often advocate making connections to the real world through the use of mathematical applications and modelling as a means of motivating and engaging students, as well as illustrating the usefulness of mathematics to describe and analyse real-world situations. Although applications and modelling are often coupled together, they have received differing attention in school curricula, with modelling viewed as being more challenging and complex than applications. Applications tasks are typically well structured and demonstrate the relevance of particular mathematical content for solving a problem set in a real-world context. The task situation is fully described; all assumptions about the situation are made explicit; and students know they will normally use all the data provided in order to find the solution. Modelling tasks, on the other hand, are usually more open and require mathematisation of a real-world situation. It is up to the modeller to define the real-world problem, specify assumptions and choose variables, identify relevant mathematical knowledge and tools, formulate and solve the mathematical problem, interpret and validate the solution, and modify the model if necessary. Teaching the modelling process is often viewed as a worthwhile educational goal in itself.

The inclusion of mathematical modelling and applications within school curricula has a history dating back to the early twentieth century. In 1904 the German mathematician Felix Klein developed a new curriculum that placed a larger emphasis on the inclusion of applications in the instruction of secondary mathematical education (Krüger, 2019). Much later, in the 1970s and 1980s, the focus on mathematical modelling and applications came to the fore in many English-speaking countries as large-scale curriculum projects encompassing modelling and

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applications were developed (see Schukajlow et al., 2018, for more on this). This resurgence in the focus on modelling and applications led to the first biennial conference series on the teaching and learning of mathematical modelling and applications in the University of Exeter in 1983, which in 1987 was rebranded as the International Conference on the Teaching of Mathematical Modelling and Applications (ICTMA). A regular working/topic group on mathematical modelling and applications was also included at the quadrennial International Congresses on Mathematical Education (ICMEs). Although they are linked, the distinction between modelling and applications is evident when we realise that when considering mathematical applications, we are looking for ways to use mathematics that has already been chosen, that is, moving from the mathematical world to the real world. In these situations, the necessary mathematical tools and models are already learnt and exist. On the other hand, with modelling we are focusing on the process of finding some mathematics that will help us understand and potentially solve the real-world problem. In this case the model must be constructed through understanding, simplifying, and mathematising the real-world scenario.

While modelling and applications play more significant roles in many countries' curricula and classrooms than in the past, the difficulties of implementing widespread curriculum change represent core barriers to bringing about changes in mathematics teaching and learning (Burkhardt, 2018). Consequently, the inclusion of authentic modelling activities in mathematics classrooms is still rather scarce and sporadic. Recent mathematics curriculum reform in Ireland has promoted a move away from calculation using learned procedures towards engaging students in authentic, challenging tasks. Since 2008 the newly introduced secondary mathematics curriculum has advocated for the use of contexts and applications to develop students' problem-solving abilities and to assist them in seeing the value and relevance of the mathematics being taught (National Council for Curriculum and Assessment [NCCA], n.d.). Additionally, the recent inclusion of a "classroom-based assessment" component at lower secondary school level, which requires students to apply their mathematical knowledge to address a problem of their own choosing, again highlights the emphasis on mathematical modelling and applications in the curriculum.

Although the Irish secondary mathematics curriculum has undergone several major changes in recent years, targeted at making it more applicable and relevant in nature, there is a lack of evidence that this is happening effectively in practice. One possible reason for this is because mathematical modelling is seen to be challenging for both students and teachers. The facilitation of a modelling activity is challenging for teachers as it requires them to mediate a lesson in a manner that they may not have previously received training in nor been sufficiently exposed to as a viable approach to the teaching and learning of mathematics. Moreover, mathematical modelling may not explicitly be listed in the curriculum documentation, and therefore, the incentive to regularly engage in modelling activities in the mathematics classroom may be lacking as they are viewed by many teachers as time consuming and challenging to assess (Blum, 2015). Additionally, in Ireland many teachers still rely heavily on textbooks and use them primarily as their main source of information when it comes to the planning and conducting of lessons (O'Keeffe, 2011; O'Sullivan, 2017). While textbooks may be able to assist teachers in delivering a more standardised curriculum, they offer insufficient advice and ideas regarding the planning and execution of modelling activities and so many teachers may find themselves lacking in confidence and knowledge to properly carry out appropriate mathematical modelling activities (Ang, 2010). Finally, teachers may be reluctant to utilise modelling as a teaching strategy because of the open nature of the tasks and the fact that it is not always clear in advance what mathematical tools and models are available, what assumptions need to be made, and what outcomes can be expected. This lack of certainty can leave teachers feeling underprepared and requires a significant paradigm shift in how teachers view their role in the classroom; moving away from the position of being the authority on the subject knowledge towards acting as a facilitator whose role is to question and query students' approaches and strategies rather than provide answers and guide students towards a single correct solution.

From the student's perspective, mathematical modelling is a demanding activity that requires them to possess a rich and connected mathematical knowledge in parallel with possessing other traits such as perseverance, curiosity, and creativity. The ability to "understand, judge, do, and use mathematics in a variety of intra- and extra-mathematical contexts and situations in which mathematics plays or could play a role" is defined by Niss (2003, p. 7) as mathematical competence. A key element within the development of mathematical competence is the ability to model mathematically, that is, to be able to analyse and build models. For this reason, and others, many countries around the world now accept that the ability of students to model mathematically should be a key component within their school curricula. Additionally, modelling develops within students the ability to solve real-world problems that they may encounter outside of school, in society, or even in their future careers and so is deemed to be a valuable skill to foster and develop within a school curriculum (Mousoulides, 2009).

This brief analysis suggests that there are many elements of curriculum development and implementation that come into play when considering how to introduce mathematical modelling into the school curriculum. These considerations are captured in the analytical framework presented next, which guides our case study investigations.

4.2 Analytical Framework: Curriculum Policy, Design, and Enactment

Remillard and Heck (2014) defined curriculum as "a *plan for the experiences* that learners will encounter, as well as the *actual experiences* they do encounter, that are designed to help them reach specified mathematics objectives" (p. 707, original emphasis). This definition indicates that a curriculum is more than a list of topics or learning objectives, and it points to the distinction between what the curriculum intends and what actually happens in classrooms. Remillard and Heck synthesised and extended existing conceptual frameworks for curriculum that examine relationships between curricular intent and educational outcomes as well as how different actors reformulate curriculum at different levels within an educational system. Their resulting model (shown in Fig. 4.1) delineates the features of a broader curriculum policy, design, and enactment system.

The *official curriculum* is specified by governing authorities and sets out expectations for students' learning. Remillard and Heck (2014) identify three components of the official curriculum: (a) curricular aims and objectives, (b) the content of consequential assessments, and (c) the designated curriculum. In Ireland, the official school curriculum is prepared by the National Council for Curriculum and Assessment (NCCA), and there are separate curriculum specifications for subjects in the junior and senior secondary school. At both these levels, expectations for student learning are expressed as learning outcomes that describe what students should know, understand, and be able to do as a result of having studied mathematics. For example, in every content strand of the senior secondary mathematics curriculum, students are expected to "devise, select and use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions" (NCCA, 2015b, p. 15). Inclusion of the content of consequential assessments in the official curriculum acknowledges the influence of high-stakes assessment on

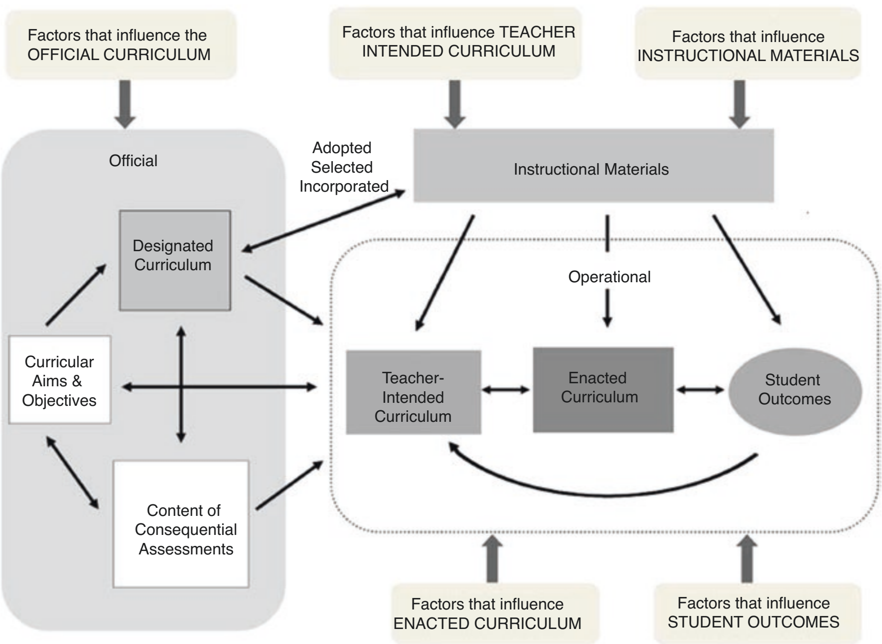


Fig. 4.1 The curriculum policy, design, and enactment system (Remillard & Heck, 2014)

curricular goals. Yet it is common for state mandated assessments to sample only a subset of curriculum goals, especially those goals related to knowledge and skills that are amenable to measurement via a timed written test. In Ireland, the consequential assessment of students' performance in each senior secondary school subject takes place in the final Leaving Certificate examination. The third component of the official curriculum, the designated curriculum, refers to the instructional plans, textbooks, and other materials that might be specified by a ministry of education to offer guidance towards addressing the curriculum's goals. Remillard and Heck note that across educational systems there is variation in the form and specificity of the designated curriculum. In Ireland, the education system does not specify a designated curriculum and schools are free to select from instructional resources produced by commercial publishers, professional associations, or support services within the education system.

In Remillard and Heck's (2014) model, the *operational curriculum* includes (a) the teacher-intended curriculum, (b) the enacted curriculum, and (c) student outcomes. Thus, the operational curriculum represents the transformation of the official curriculum into teachers' personal plans, whether these are in writing or in the teachers' minds, and how these plans play out in the interactions between teachers and students in the classroom.

Around the perimeter of Fig. 4.1, Remillard and Heck (2014) pointed to factors that influence elements of the official curriculum and the operational curriculum. They noted that these factors "may be social, political, cultural, structural, or cognitive" (p. 714). Drawing on existing research, they identified influencing factors such as societal needs, values, expectations, and beliefs; views of individuals and groups wielding power; research on learning, teaching, and assessment; teacher knowledge, beliefs and practices; teachers' access to resources and support; contextual opportunities and constraints; and a range of student characteristics and cultural resources. These factors interact with each other in complex ways, and their degree of influence on the curriculum system may be either direct or subtle. Remillard and Heck also commented that further research is needed in order to explore and elaborate on the ways in which these factors exercise influence.

In this chapter, we draw on Remillard and Heck's (2014) curriculum system model to address the following research question:

What factors support or hinder the implementation of modelling as an exemplar of mathematical challenge in the school curriculum?

To answer this question we present two case studies illustrating how modelling is being introduced into the secondary school mathematics curriculum in Ireland. The first case study explores contested attempts to infuse a modelling focus into the specialist Applied Mathematics curriculum at senior secondary level. The second discusses a university-led professional development project that exploited the Transition Year – a non-academic year between the junior and senior secondary school examination cycles – as an opportunity to introduce teachers to modelling tasks and pedagogical strategies. Each case study begins with an account of the curriculum and educational context. This is followed by an analysis of curriculum change in terms of factors that are influencing either the official or operational curriculum.

4.3 Case Study 1: Modelling as a Stimulus for Mathematics Curriculum Reform

4.3.1 Background to Applied Mathematics Curriculum Reform

In Ireland, senior secondary students typically study between six and eight subjects for the final Leaving Certificate examination (O'Meara & Prendergast, 2017). Mathematics is considered a core subject that can be taken at Ordinary or Higher level, but without being compulsory. However, it is treated as such by schools due to the fact that mathematics is a gatekeeper for the vast majority of tertiary courses (Prendergast et al., 2020). On the other hand, Applied Mathematics is an additional, optional subject which is only available to students in a small number of secondary schools. Even in these schools, Applied Mathematics is not usually offered as part of the daily timetable. Instead, students take the subject either "off timetable", that is, with lessons before or after school, or with a private teacher outside school hours.

Applied Mathematics is viewed as a subject which "mirror(s) a section of the Leaving Certificate Physics syllabus" (NCCA, 2014, p. 1). Its subject matter differs from that of the mainstream Mathematics subject at senior secondary level, which focuses on statistics, probability, geometry, trigonometry, number, algebra, functions, and calculus. Applied Mathematics instead deals with topics from the domain of physics known as mechanics, including laws of motion, projectiles, and statics (State Examinations Commission, 2018). First introduced in Ireland over 40 years ago, the Applied Mathematics syllabus has undergone very few changes in the intervening years. The syllabus lacks an explicit aim or rationale and consists only of the list of topics to be examined. However, in late 2014 the National Council for Curriculum and Assessment undertook a review process with the ultimate aim of revising this very dated curriculum. There were a multitude of concerns which led to the review of the Applied Mathematics curriculum and shaped the revised curriculum.

Firstly, there were concerns about the low numbers of students choosing Applied Mathematics. Figure 4.2 summarises data collected by the State Examinations Commission between 2014 and 2019, which shows the number of students who sat the Leaving Certificate examination across a range of different science subjects.

Figure 4.2 clearly highlights how, in the period from 2014 to 2019, Applied Mathematics was the least popular of all the science subjects among Leaving Certificate students. Data collected by the State Examinations Commission (2018) show that, from 2014 to 2018, Applied Mathematics candidates comprised around 3.2% to 3.8% of the full Leaving Certificate cohort.

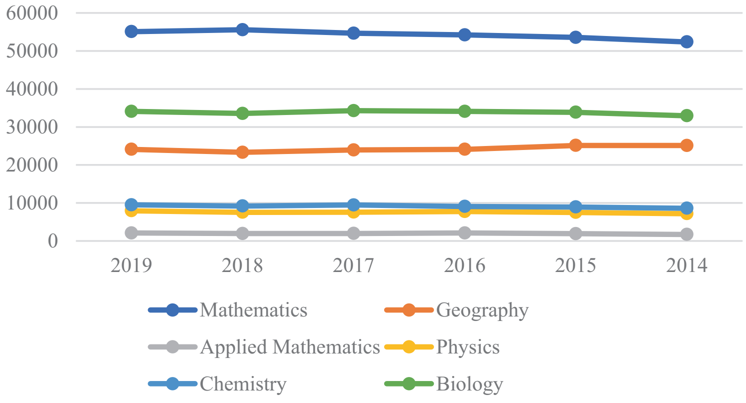


Fig. 4.2 Number of candidates in Leaving Certificate science subjects from 2014 to 2019

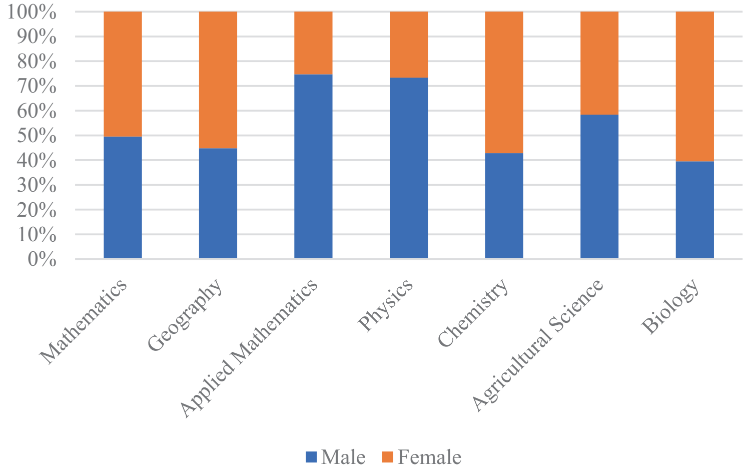


Fig. 4.3 Gender participation in Leaving Certificate science subjects 2019

In addition to poor uptake, concerns were also expressed regarding the gender imbalance in participation in Applied Mathematics. As demonstrated in Fig. 4.3, significantly more males than females opted for Applied Mathematics in 2019. The statistics revealed that 25.3% of those who sat the Applied Mathematics examination were female. Corresponding figures for physics, chemistry, and biology were 26.6%, 57.2%, and 60.5%, respectively. Similar statistics were reported by the State Examinations Commission in previous years also.

A third concern in relation to the dated Applied Mathematics curriculum was connected to the misalignment between the Leaving Certificate Mathematics and Applied Mathematics curricula. In 2006 efforts began to completely overhaul the senior secondary Mathematics curriculum in Ireland with a new curriculum, known locally as Project Maths, being introduced nationally in 2010. As discussed previously, the new curriculum promoted a fundamental shift in the teaching and learning of mathematics in Ireland. The emphasis changed from examination-driven teaching that promoted memorisation and practice of learned procedures to student-centred teaching that promoted conceptual understanding (Department of Education and Skills, 2010). However, this change to the Mathematics curriculum did not occur in tandem with a change to the Applied Mathematics curriculum and resulted in the differences between the two curricula becoming even more stark. For example, while problem solving and modelling were at the heart of the new Mathematics curriculum, Applied Mathematics continued to focus on particular types of problems in mechanics that were "quite disconnected from 'applications'" (O'Reilly, 2002, p. 1). Furthermore, the change to the Mathematics curriculum resulted in examination questions becoming much less predictable (Shiel & Kelleher, 2017). However, this was not the case with Applied Mathematics. In fact, many teachers involved in the consultation process for Applied Mathematics were of the opinion that "...you can train people to do well in Applied Mathematics because the exam is so predictable" (NCCA, 2015a, p. 7).

Due to these concerns a consultation process began in 2014 with the publication of the *Background Paper for the Review of Applied Mathematics* (NCCA, 2014). The consultation process subsequently led to the publication of a *Consultation Report* in December 2015 (NCCA, 2015a) and following this the Mathematics Development Group agreed that the revised Leaving Certificate Applied Mathematics subject would aim to develop the learner's capacity to use mathematics to solve real-world, twenty-first century problems. The new Applied Mathematics syllabus is still under development, but from draft versions it is clear that it will focus on all aspects of the problem-solving cycle. In doing so it is envisaged that learners will see beyond calculating procedures and gain experience in asking appropriate questions, formulating mathematical representation of problems and interpreting and verifying results. Based on the consultation process it was also decided that the new Applied Mathematics specification would place a strong emphasis on mathematical modelling, as this was seen to be "...at the heart of modern applications of mathematics" (NCCA, 2015a, p. 18).

The draft Applied Mathematics syllabus resulting from the consultation and review process describes the subject as involving the use of the language of mathematics to study and solve real-world problems. It introduces mathematical modelling as the process through which real-world phenomena are represented, analysed, and understood. The syllabus aims and objectives are centred on the modelling process: formulating a real-world problem, translating the problem into a mathematical representation, solving the mathematical problem, and interpreting the solution in the original real-world context. The syllabus is organised around four strands:

- Mathematical modelling
- Mathematical modelling with networks and graphs
- Mathematically modelling the physical world
- Mathematically modelling a changing world

This organisation foregrounds the modelling process as a means by which to explore mathematical content.

4.3.2 Factors That Support or Hinder the Implementation of Modelling in the Applied Mathematics Curriculum

We analysed the two key documents that initiated the Applied Mathematics curriculum reform (the *Background Paper*) and collected stakeholder responses (the *Consultation Report*), looking for evidence of the influencing factors on the official curriculum that were identified by Remillard and Heck (2014).

The first factor acknowledges the *perceived and expressed needs of society*, and this was a source of tension in the consultation process. In the *Background Paper*, the context for curriculum reform was framed in terms of economic needs and the STEM (science, technology, engineering, and mathematics) agenda, with STEM education underpinned by mathematical knowledge and skills seen as "help[ing] Ireland to generate the capable and flexible workforce needed to compete in a global marketplace" (NCCA, 2014, p. 5). However, participants in the consultation process "who were critical of the background paper claimed it had been overly influenced by the needs of third level and industry and that it was prepared in response to PISA" (NCCA, 2015a, p. 7).

A second influencing factor is related to *advancements in the fields of mathematics, learning, educational practice, and technology*. As we have previously indicated, there had been many advancements to mathematics education in Ireland since the introduction of the original Applied Mathematics curriculum. In particular, the revised senior secondary Mathematics curriculum, launched in 2010, seemed to act as a catalyst for the reform of the Applied Mathematics curriculum. The *Consultation Report* indicated that many stakeholders believed there was a need for better alignment between these two subjects. Similar to curriculum reforms internationally, the revised Mathematics curriculum in Ireland placed increased emphasis on problem solving, and so there was now scope for the Applied Mathematics curriculum to continue to further develop students' problem-solving skills through the lens of mathematical modelling. Giving more weight to problem solving led to removal of some content from the Mathematics syllabus, and this too led to calls for a reimagining of the content of Applied Mathematics. For example, the *Background Paper* raised the possibility of shifting some of this excluded content, such as vectors and matrices, into the Applied Mathematics curriculum. However, this proposal was not welcomed by some participants in the consultation process who claimed that the content "is there for political reasons because it was left out of Project Maths and this was seen as a deficit and will be lumped into the new spec [i.e. into the Applied Mathematics subject] to say it is there" (NCCA, 2015a, p. 7).

The *Background Paper* and consultation process gave explicit attention to the potential impact of *advancements in technology* on the new Applied Mathematics curriculum. Technology in mathematics education was now seen as “a tool with the potential to change how concepts are demonstrated, projects assigned and progress assessed” (NCCA, 2014, p. 22). The *Consultation Report* indicated all stakeholders agreed that technology had the potential to facilitate a shift towards mathematical modelling, but emphasised that technology should support, and not compromise, the development of skills. One challenge that arises here involves providing teachers with professional development opportunities. A substantial amount of research on technology in mathematics education has found that developing teachers’ technology skills in isolation is insufficient; instead, professional development must seek to simultaneously develop teachers’ technological and pedagogical knowledge (e.g. Li et al., 2019). A second challenge in the Irish context is the need for investment in technology resources to complement the revised Applied Mathematics curriculum. Research has shown that the textbook is the primary resource used in mathematics classrooms in Ireland and internationally (Nathan et al., 2002; O’Meara et al., 2020). However, textbooks alone will not facilitate the use of technology in the Applied Mathematics classroom and will not promote student-centred learning experiences in the area of mathematical modelling.

A third factor influencing the official curriculum is the *values and beliefs about mathematics and the goals of education held publicly and by individuals and groups wielding power*. The values and beliefs of different stakeholders played an important role in the instigation of reform to Advanced Mathematics, but led to some challenges in the framing/design of the revised curriculum. In particular, values and beliefs relating to what constituted problem solving and how problem solving and content knowledge could co-exist within a curriculum document with a strong emphasis on mathematical modelling came to the fore in both documents, with different stakeholders, at times, holding contrasting views. For example, the *Consultation Report* indicated that proponents of the old Applied Mathematics syllabus believed that problem solving was already central to that curriculum. They feared that any additional content included in a revised curriculum would lead to a dilution of the problem-solving aspects of the course. Hence, calls to include content omitted from the revised mathematics curriculum were not welcomed by all, with some expressing the view that “...those involved in teaching Applied Mathematics do not believe that they should have to teach mathematics. Instead they see their focus as being on teaching problem solving skills” (NCCA, 2015a, p. 12). On the other hand, the *Background Document* suggested that “with its [the old curriculum] emphasis on content as opposed to the development of skills and mathematical reasoning students are not problem solving per se” (NCCA, 2014, p. 3).

These contrasting views and beliefs present their own set of challenges for this curriculum reform effort as they will have a significant impact on the content that is included in the syllabus, as well as the skills and dispositions that the curriculum promotes. Such differences could potentially lead to an overcrowded curriculum that lacks depth and causes practical difficulties with timetabling and the allocation of class time. Similar issues have plagued previous mathematics curriculum reform in Ireland (see O’Meara & Prendergast, 2017). However, another view expressed in the *Consultation Document* in relation to the content versus problem-solving debate could offer a potential solution. One respondent suggested that Applied Mathematics should be viewed as a "subject [that] draws upon concepts and methods of mathematics from the fields of application and in turn, brings ideas, techniques and scientific knowledge back to influence the development of mathematics" (NCCA, 2015a, p. 12). Adopting this outlook would help overcome the "either/or" debate and instead mathematical content, problem solving, and mathematical understanding could be seen as key components which complement each other when engaging in the mathematical modelling process.

4.4 Case Study 2: Modelling as a Stimulus for Mathematics Instructional Reform

4.4.1 Background to the Young Modellers Transition Year Project

Transition Year (TY), an optional non-academic school year between the junior and senior secondary examination cycles, is unique to Ireland (Clerkin, 2012; Jeffers, 2007; Smyth & Calvert, 2011). It was introduced as a pilot scheme in three schools in 1974 and was mainstreamed in secondary schools by 1994. Currently, it is offered in 75% of secondary schools in Ireland (Jeffers, 2011) with just over half of the potential student cohort participating (Clerkin, 2012). The guidelines distributed to schools emphasise that TY is to be neither viewed nor utilised as an extra year to prepare students for the Leaving Certificate examination. Indeed,

> Where Leaving Certificate material is chosen for study it should be done so on the clear understanding that it is to be explored in an original and stimulating way that is significantly different from the way in which it would have been treated in the two years to Leaving Certificate. (Department of Education, 1993, p. 4).

The purpose of TY, therefore, is to replace formal study with a broad range of non-academic educational and vocational experiences in the absence of examination pressure (Department of Education, 1993; Smyth & Calvert, 2011). Teachers have great flexibility, and indeed are strongly encouraged, to create a TY programme to suit their students' needs (Clerkin, 2012; Smyth & Calvert, 2011). As such, it offers an ideal period to implement innovative educational interventions. The Young Modellers project was one such intervention that aimed to introduce mathematically challenging tasks to teachers and students.

The significance of the Young Modellers project needs to be understood in the context of the high-stakes summative assessment environment in Irish secondary schools, and teachers' perception of their role, "at least in part, as that of exam coach" committed to "covering" in class all question types that might be asked in the Leaving Certificate examination (NCCA, 2014, p. 16). Although the Leaving Certificate Mathematics curriculum purports to develop students' ability to solve mathematical problems in familiar and unfamiliar contexts, there is little evidence that this actually happens. For example, the most recent Chief Examiner's Report on Leaving Certificate Mathematics stated that, in the Ordinary level examination, "the majority of candidates seemed unable to deal with problems presented in an unfamiliar context" (State Examinations Commission, 2015, p. 23). It was observed that students were more inclined to abandon their work than persevere when difficulties arose. Students taking the Higher level examination also struggled with problem solving, and applying knowledge in unfamiliar contexts. The report concluded by recommending that teachers should

> provide students with opportunities to practise solving problems involving real-life applications of mathematics, and to get used to dealing with "messy data" in such problems. Students should also be encouraged to construct algebraic expressions or equations to model these situations, and / or to draw diagrams to represent them. (p. 30)

Young Modellers was a 10-week teaching and learning module implemented in 15 secondary schools in Ireland that served to address many of the issues referred to in the Chief Examiner's report mentioned above. The programme was designed and delivered by university-based mathematicians and mathematics educators, who have significant experience in both mathematical modelling and teacher professional development. The purpose of the initiative was to challenge students and teachers to exploit problem-solving skills to solve real-world problems that appear in science, engineering, technology, and industry, using mathematical techniques. Young Modellers aimed to engage students in how to use mathematics in realistic problems providing them with an insight into mathematics in action. It was hoped that participation in the Young Modellers programme would help develop perseverance skills and encourage different ways of approaching a problem. The Young Modellers development team wanted to encourage teachers and students to appreciate the links between mathematical concepts and skills acquired at school with the utility of mathematics in the real world by applying the mathematics that they learn at school to solve real-world problems, giving them a first-hand experience of mathematical and statistical modelling.

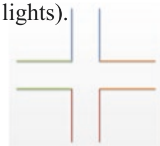
TY teachers were provided with a 2-day professional development programme on how to move from a problem formulated in non-mathematical terms to developing a mathematical solution. Throughout the programme the use of collaborative and communication skills was emphasised and encouraged. Emphasis was placed on how to formulate a problem, represent it in mathematical terms, investigate various different methods for solving the problem, and interpret that solution in terms of the real-world problem. Figure 4.4 shows an example of a modelling task from the professional development programme that illustrated important modelling strategies, such as using an appropriate representation, making simplifying assumptions, developing a simulation, specialising and generalising, and considering extreme cases. A package of teaching and learning materials was also developed for use in the classroom. This package consisted of two parts. Part 1 comprised problem-solving tasks, some of which introduced students, in a structured way, to strategies which are also useful in modelling (using appropriate representations and identifying assumptions). Part 2 provided real problems which required students to engage

Example of a modelling problem:

At which kind of intersection (a roundabout or a crossroads with traffic lights) can more cars pass a crossing?

Strategies :

- Use an appropriate representation.
- Make simplifying assumptions (e.g. number of cars/hour; direction of travel at intersection; car length, acceleration, separation; timing of traffic lights).
- Make use of symmetry.



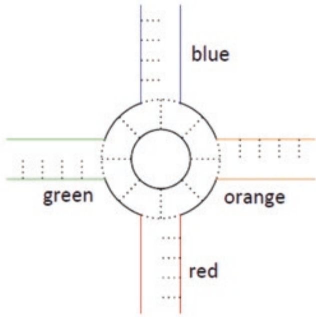


Fig. 4.4 Task from the Young Modellers professional development programme (Stender, 2019, p. 202)

in mathematical modelling. Figures 4.5 and 4.6 show two tasks from Part 1 of the package of teaching and learning materials.

Teachers then implemented Young Modellers in the classroom over a 10-week period. They were supported by members of the Young Modellers development team and received three school visits from a team member along with two PhD students during the project. Participating teachers first introduced mathematical modelling to their students and explained how modelling fits into everyday life using simple examples. Students then were presented with a selection of real problems (e.g. modelling disease spread in a population) from Part 2 of the package of teaching and learning materials. Students selected one of these real problems to work on with a small group of their peers for the period. Throughout the school visits, the research team supported students in selecting parameters, identifying assumptions, producing a simple model, and eventually finalising and testing their models. Students were required to create a presentation of their work and present it as a team in front of other school participants, and answer questions from a panel of judges at a showcase event in the host university.

4.4.2 Factors Supporting or Hindering the Implementation of Modelling in the Young Modellers Transition Year Project

The Young Modellers project tried to address several factors among those identified by Remillard and Heck (2014) as influencing both the teacher-intended and the enacted components of the operational curriculum. We discuss the nature and role

On December 28th 2015, the average price for unleaded petrol was € 1.50 per litre.

A driver in Cork plans to drive to Donegal and back, a distance of approximately 850km return in one day.

The driver owns a BMW with fuel consumption of 10km per litre.

Her tz Rent a Car, Ltd., offers a Golf car rental in Cork at €45 per day.

It is estimated that the car has fuel consumption of 25km per litre.

(i) Should the driver rent a Golf to drive to Donegal or should he drive the BMW to Donegal?

(ii) Solve this problem in a different way from how you did for (i) and compare both solutions. Is your solution correct?

(iii) From your solutions:

- For what range of distances is it more expensive to rent the Golf?
- For what range of distances is it more expensive to drive the BMW?
- Illustrate your answers graphically if you have not already done so.

Fig. 4.5 Car hire task from the Young Modellers teaching and learning package (Guerin, 2017)

The back of a truck passes the exit sign for Shannon airport, travelling at 42km/hr. One and a half hours later, the back of a car passes the same exit sign for Shannon airport traveling at a speed of 63km/hr. The length of the truck is three and a half times the length of the car.

It is 3am and there is no traffic.

(i) At what distance from that exit sign, will the back of the car be in line with the back of the truck?

(ii) State all assumptions you are making in solving this problem.

(iii) Under what conditions will it be possible that the car will catch up with the truck?

(iv) Explain a few scenarios where it might be not possible for the car to catch up with the truck.

Fig. 4.6 Motorway task from the Young Modellers teaching and learning package (Guerin, 2017)

of each of these factors as they played out in the Young Modellers project. To do so we draw on relevant literature as well as three sources of data from the project:

teacher surveys completed at the project's beginning and end, teacher reflective diary entries collected from week 4 and week 8 of the classroom implementation period, and an interview conducted via email with the project leader.

The first influencing factor comprised *contextual opportunities and constraints* that create expectations about the value and feasibility of implementing a modelling focus. The decision to situate the project in the Transition Year took advantage of the contextual opportunities afforded by this unique curriculum context and avoided the constraints experienced by teachers and students in the senior secondary years leading to the Leaving Certificate examination. Firstly, teachers report experiencing tremendous pressure to complete the senior secondary mathematics syllabus, particularly at Higher level, within the 2 year time frame (O'Meara & Prendergast, 2017). That, added to a heavy emphasis on performance in the Leaving Certificate examination, lends little opportunity to spend time on content that will not appear in examination papers (Gill, 2006). In contrast, in Transition Year there is a lack of examination pressure and teachers have much more flexibility in terms of what will be taught. Furthermore, activities which promote the application of mathematical skills and concepts to real-life problems and "problem-solving using interpretation, approximation, model making" are strongly endorsed in the TY Guidelines (Department of Education, 1993, p.13).

Teacher knowledge, beliefs, and practices in relation to mathematics and mathematical modelling also influence how teachers interpret the generalised aims and objectives of the official curriculum, and then plan, adapt, and enact instruction with the students in their classes. All teachers who participated in the Young Modellers project were new to modelling and they faced challenges to their conceptions of mathematics and mathematics teaching. In mathematical modelling students must figure out and formulate their own responses to problems and so a more student-centred approach is warranted in these situations. Dealing with these non-routine problems, from real-world situations, in a more student-centred classroom environment places the majority of mathematics teachers in a situation which is not routine for them and outside of their past experiences of teaching mathematics (Burkhardt, 2013). Facilitating mathematical modelling requires a broader range of teaching approaches than most teachers currently use (Burkhardt, 2006), such as knowing when and how to help, orchestrating student discussion, and providing a wide range of authentic, non-routine tasks. Thus, teacher knowledge, beliefs, and practices were recognised and explicitly addressed as potential implementation barriers in the Young Modellers project.

Some teachers in the Young Modellers project reported feeling challenged because they were no longer in the position of "expert" knower in the classroom:

> There were times I felt out of my depth as a teacher because one particular group found this interesting formula they wanted to use but there were elements of that formula that were very complex for them and I was unable to explain it to them. (Teacher KML3AHTU diary, week 4)

> I felt out of my depth at times when students asking about coastal heights of areas across Ireland – where could they get answers to certain questions. (Teacher BRL6NOHG diary, week 8)

There were also mixed responses from teacher surveys regarding perceived changes in their self-reported levels of anxiety when engaging their students with modelling tasks, and levels of confidence in their mathematical content knowledge and mathematical pedagogical knowledge. While it might be unrealistic to expect significant change in teaching practice in the relatively short time frame of the project, some teachers did report changes in their questioning practice by “asking more higher order questions, getting students to lead their own learning” (Teacher TBR2RIYU, post-survey). Others described change in terms of what they learned about students, especially in relation to their desire to arrive at a correct answer:

> [I learned] that students are too used to getting a definite answer in maths and can’t deal with a general answer problem. (Teacher IOG0IAMU, post-survey)

> [I learned} that there is always more than one correct answer. Allow students to explore all answers. (Teacher TBR2RIYU, post-survey)

These comments from teachers show that *student knowledge, beliefs, and practices* in relation to mathematics can also influence the implementation of a modelling focus. The Young Modellers project leader, a research mathematician and modeller with many years of experience in working with school teachers, commented:

> The ability and willingness to try and fail is crucial to good modelling. We find that most students’ concept of mathematics is heavily influenced by the notion of “there is one correct answer”, but good mathematical models cannot be created without trial and error. We emphasised to the teachers, and repeated on visits to schools, that the modelling process is an iterative one: we start with a very crude and probably inaccurate model, and only by solving this simple model do we gain insight into how it can be improved and made more accurate. Students who are not willing to have “wrong” models find this process very difficult to accept; indeed, it is the source of the “this isn’t mathematics!” comments that we sometimes get in feedback! Indeed, it is not mathematics as these students have learned it in the traditional classroom, but of course it very much *is* mathematics as it is applied in real-world industry and research applications!

Perseverance in the face of challenge and failure is an essential trait that students need to develop if they are to be successful at modelling. Many teachers were pleasantly surprised by the sustained engagement of their students in modelling tasks.

> There is a lot more resilience. They don’t throw in the towel so quickly. (Teacher EBU0IANU diary, week 4)

> Students seem more invested in the amount of time they are spending to solve a problem. They are committed to solving it and they are not giving up on the problem straight away. (Teacher KML3AHTU diary, week 4)

> I noticed a huge difference in the time students were willing to spend to solve a problem. (Teacher HJR8GAAG diary, week 8)

Others commented on the increased levels of independence demonstrated by students:

> It was good to see them working with problems on their own without the teacher dictating the approach or pace. (Teacher EBU0IANU diary, week 4)

> One group decided to carry out experimental work and did so independently. They didn't look for prompts as how to design and implement the experiment and they worked in a very efficient and conscientious manner. I was hugely impressed with this as I felt it was a huge departure from the typical scenarios where students usually would need to be heavily coached when it comes to experimental work. (Teacher SMP diary, week 8)

A further factor that influences implementation of modelling in the operational curriculum is teachers' *access to resources and support*. We have already pointed to Irish mathematics teachers' strong reliance on textbooks (O'Keeffe, 2011; O'Sullivan, 2017) for planning and instruction; however, the Young Modellers project viewed textbooks as being inadequate for teaching modelling, and especially for developing the ability and willingness to try and fail that is crucial to a modelling disposition. Instead, a range of modelling tasks and resources was created or accessed from other sources, such as COMAP's Mathematical Contest in Modelling (Consortium for Mathematics and Its Applications, n.d.). Teachers who were unfamiliar with modelling and accustomed to having textbooks were keen to have access to these exemplar tasks and strategies.

4.5 Discussion

The aim of this chapter was to explore strategies for promoting mathematical modelling as a stimulus for curriculum and instructional reform, where modelling is considered to exemplify a kind of mathematical challenge that is still rarely found in secondary school curricula and classrooms. Remillard and Heck's (2014) model of the curriculum policy, design, and enactment system provided the framework for analysing factors that influence implementation of modelling in the official curriculum and the operational curriculum in Irish secondary schools. While much research attention has been directed at the distinction between what the official curriculum endorses and how teachers translate this curriculum into classroom practice, there are other factors that mediate curriculum enactment. In this chapter we showed how a range of institutional constraints; needs, values, and beliefs expressed by various stakeholders; advances in mathematics, technology, and educational research and practice; and access to resources and support interacted to influence curriculum development and enactment.

There is no doubt that it is challenging to introduce teachers and students to the mathematically challenging tasks and ways of working that characterise modelling. We think of this as the *meta-challenge* of institutionalising mathematical challenge in the school curriculum. Nevertheless, the two case studies presented in this chapter illustrate some ways in which the meta-challenge can be addressed. Common to both is a strategy for taking advantage of contextual opportunities which, while unique to Ireland, might find application in other curriculum contexts. In the first case, a dated curriculum for a mathematical physics subject, currently taken by very small numbers of students, is being overhauled by infusing a modelling focus. While this subject is offered in a high-stakes assessment environment that might otherwise act as a constraint to curriculum and instructional reform, its "niche" character provides a small-scale and potentially less risky context for innovation than the mainstream Mathematics subject that is taken by almost all senior secondary students. In the second case, a low-stakes curriculum and assessment environment in the form of Transition Year presents ideal opportunities for small-scale innovation and experimentation with modelling, away from the pressures of external examinations. In both cases, there is potential for teachers to become more comfortable with modelling, gradually building confidence and expertise without the expectation of implementing a full-scale modelling focus in an examinable mathematics subject taken by nearly all senior secondary students across the country. While the numbers of teachers involved in these initiatives is relatively small, their participation creates an existence proof for implementing modelling that, over time, might encourage others to try this approach.

Good modelling requires the ability and willingness to try and fail, and these requirements apply just as much to teachers as to students. We would argue that, in addition, curriculum authorities and education systems need to embrace "trying and failing", by taking a long-term view of the time and support that teachers need in order to meet the meta-challenge of embedding modelling into the school mathematics curriculum.

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