Chapter 6
Solving Portfolio Optimization Using Sine-Cosine Algorithm Embedded Mutation Operations

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1 Introduction

Portfolio optimization is the process in which investors receives appropriate guidance regarding the selection of assets from a variety of other option. The traditional asset location problem is that of an investor who wants to invest money in the stock market in such a way that individual can get a reasonable rate of return while minimizing risk. It is based on modern portfolio theory. MPT, first introduced by Markowitz in 1950, is also known as mean-variance analysis method, and this is a mathematical process which allows the investors to maximize returns for a given risk level. In a study by Zhai et al. [39], hybrid uncertainty, which mixes random returns and uncertain returns, is analysed using the chance theory. We explore the problem of optimizing a portfolio with an unknown random variable, which is the total return.

A new mean risk modal based on this criterion to optimization is proposed by Mehralizade et al. [28], along with a new risk criterion for uncertain random portfolio selection. To solve the portfolio selection problem with uncertain random returns, Ahmadzade et al. [2] used the idea of partial divergence metrics. Mehlawat et al. [27] study uses higher moments to investigate a multi-objective portfolio optimization issue in a chaotic, uncertain setting. We investigate a case with an asset universe, in which some assets have recently been listed assets that lack historical data while others have assets that have historical return data that is sufficient for modelling as random variables. We incorporate skewness (i.e. the third moment) in the portfolio optimization model and use mean absolute semi-deviation as a risk indicator. Ahmadzade and Gao [1] established a mean-variance-entropy model for uncertain random returns using the idea of covariance of uncertain random variables. Huang et al.'s [18] study offers the deterministic equivalents of a

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novel uncertain risk index model with background risk. Experts evaluate the security returns and backdrop asset returns with the assumption that they are uncertain variables. The portfolio problem between a risk-free and a risky asset in the presence of background risk was addressed by Brandtner et al. [6], using the convex shortfall risk measure.

Arhana and Iba [3] proposed a GA-based portfolio optimization method to generate an investment portfolio. Markowitz has used the mean-variance model and correlation variation model to present the expected return and risk of portfolio. This method calculated portfolio value when transaction cost is involved. Bonami and Lejeune [5] proposed portfolio optimization with PSO and solved the two types of risky portfolio, unrestricted and restricted. Ma et al. [24] solved the portfolio optimization problem with cardinally constraint method. Konno and Yamazaki [21] proposed a portfolio optimization model for huge-scale optimization problem on real-time basis. Solved the problem on a linear program as opposed to quadratic programme.

Shiang-Tai-Liu [34] proposed a method to solve the portfolio optimization problem with returns, a mean-absolute deviation risk function, and Zadeen's extension principles are used. Gupta et al. [17] presented the three stages of multiple decision-making portfolio in this study for financial and ethical criteria. GA presented an excellent meta-heuristic approach to solve this portfolio optimization problem [32] invented a interactive genetic algorithm (iGA) has been used to analyzed the nonlinear problem gives better result than GA. Zhang and Liu [37] endorse a hybrid version of fuzzy and genetic algorithm solving the fuzzy problems. It is feasible to solve multigoal issues by remodelling to a single goal. Zhang and Liu [31, 37] proposed a credibility multi-objective mean-semi entropy model with background risk for multi-period portfolio selection.

The importance of hybridization is to unite the benefits and to construct a strong model. Mansini et al. [25] proposed a solution to select a portfolio with fixed transaction cost and mixed integer linear programming model that used semi-deviation model to calculate the risk. Konno and Suzuki [22] proposed a mean-variance-skewness (MVS) portfolio optimization model; in this model, any decreasing utility function allows to maximize the third order approximation of the expected utility. Singh and Dharmendra [35] presented a credit risk optimization model using the l_∞ norm risk measure that is proposed for a portfolio of credit risky bonds.

Because the proposed model is written as a linear programming problem, it is computationally efficient for large portfolios. ZhongFeng [38] proposed a hybrid portfolio optimization and converted it to convex quadratic programming. Ertenlice and Kalayci [10] conducted swarm intelligence research for portfolio optimization, discussing algorithms and applications.

Hu et al. [19] studied the usage of evolutionary computation in the discovery of buying and selling policies in the set of rules of stock buying and selling. They proposed a hybrid technique that mixes the two styles of evaluation demonstrating via simulations that inventory optimization the use of economic indices (derived from essential evaluation) may be used to pick shares the pleasant organizations in phrases of operations with return. De Mighel et.al. [8] provided a general framework for identifying portfolios that perform well out of sample even in the presence of estimated error. This approach uses the sample covariance matrix to solve the standard minimum variance problem. Califore [7] proposed an opportunity to selection trees or pattern paths for multilevel portfolio allocation that results in specific convex confined quadratic programming fashions that may be solved globally and efficiently. The authors expand the multi-duration mean-variance version to cope with competing uncertainty eventualities and advocate a worst-case choice method that mixes a min-max approach with a stochastic optimization set of rules primarily based totally on situation trees. Pinar [30] and Takriti and Ahmed [36] proposed robust optimization in the context of two-stage planning system. An efficient variant of the L-shaped decomposition approach for classical stochastic linear programming can be used to solve a robust optimization model.

Advances in interior-point methods for some classes of nonlinear convex optimization have made heuristics based on repeated solution of a convex optimization problem possible. While these methods date back to the late 1960s (see, e.g. Fiacco and McCormick [11]), Karmarkar's interior-point method for linear programming [20], which was shown to be more efficient than the simplex method in terms of worst-case complexity analysis and in practice, ushered in the modern era.

Sharpe [33] proposed a linear goal programming model for open-end mutual fund portfolios selection. Orito et al. [29] proposed a new technique to initialize the population size using bordered Hessian that solved the problem with GA. Daun [9] proposed the traditional single-goal approach, including the suggest variance method, which solves the trouble by inclusive of one of the optimization goals withinside the goal characteristic and stifling the other. When an investor can promote securities quick in addition to purchase long and while an element and scenario model of covariance is assumed, the study by Levy and Markowitz [23] provides speedy algorithms for calculating mean-variance efficient frontiers.

In this study, an attempt is made to solve the Markowitz's classical mean-variance model using a recently introduced algorithm SCA and five versions of SCA. The result comapred with Laplacian BBO (LX-BBO). A brief literature study on BBO is done by Garg and Deep [15]. An improved variant of BBO called Laplacian BBO (LX-BBO) is developed for solving unconstrained optimization problems and is compared to the unconstrained version of blended BBO [13, 14, 16]. Laplacian BBO has proved its superiority over blended BBO for unconstrained optimization problems. Garg and Deep [12] solved the portfolio optimization problem using the Laplacian biogeography and variant blended biogeography method.

The rest of the paper is organized as follows: Sect. 2 describes the Markowitz model. The test problems, parameter settings, experimental results, and discussions are presented in Sect. 3. Section 4 presents briefly about the standard SCA and proposed approach of SCA. Analysis of result and comparison is presented in Sect. 6. Section 7 gives the conclusion of the present study.

2 Markowitz Model Based on Historical Stock Price Data

Markowitz Mean-Variance Model

Mean-Vriance Analysis technique to that investors choose which financial instruments to invest is based on the level of risk they are willing to take (Risk tolerance). Ideally, investors count on better returns after they spend money on riskier assets. When measuring peril, buyers shouldn't forget the ability deviation (i.e. the volatility of the yield generated through an asset) from that asset's anticipated yield. The evaluation of suggest variance basically checks the suggest variance of the anticipated return on an investment. The mean-variance model embraced with three main elements:

Rate of Return

Capital return is defined as the rate of return over a time interval or given period of time. The following equation is used to calculate capital return mathematically:

$$r_{i,t} = \frac{p_{i,t} - p_{i,t-1} + d_{i,t}}{p_{i,t-1}} \tag{6.1}$$

where $i = 1, 2, 3 - - -$variety of capitals,

$r_{i,\ t}$: returns on the capital over time t
$p_{i,\ t}$: during the time period t closing price i^{th} captial
$d_{i,\ t}$: during the time period t dividend price i^{th} captial

Expected Return

The second factor of mean-variance evaluation is anticipated return. This is the envisioned return that a protection is anticipated to produce, since it's a primarily based totally on historical data. The anticipated of return is not always 100% guaranteed. Mathematically expected return is stated as:

$$r(x_1, x_2, x_3, - - -, x_n) = E\left[\sum_{i=1}^{n} [R_i \middle| x_i]\right] = \sum_{i=1}^{n} E[R_i]x_i = \sum_{i=1}^{n} r_i x_i \tag{6.2}$$

where $[R_i]$ is the expectancy cost of random variable. Past data is used to calculate the value of R_i.

$$r_i = E[R_i] = \frac{1}{T}\sum_{t=1}^{T} r_{i,t} \tag{6.3}$$

Variance

Variance measures how remote or unfold the numbers in a statistics set are from the mean, or average. A massive variance shows that the numbers are in addition unfold out. A small variance shows a small unfold of numbers from the mean. The variance can also be zero, which shows no deviation from the mean. When studying a funding portfolio, variance can display how the returns of a safety are unfold out for the duration of a given period. Mathematically, the variance of the i^{th} assets is stated as follows:

$$\sigma_i^2 = \delta(R_i) = E\left[\left(R_i - E[R_i]\right)^2\right] = E\left[\left(R_i - r_i\right)^2\right] \tag{6.4}$$

The covariance σ_{ij} between asset return R_i and R_j is given as follows:

$$\sigma_{ij} = E\left[(R_i - E[R_i])\left(R_j - E\left[R_j\right]\right)\right] \tag{6.5}$$

Using the archivable data, covariance σ_{ij} is calculated as follows:

$$\sigma_{ij} = \frac{1}{T}\sum_{i=1}^{T} \left(r_{i,t} - r_i\right)\left(r_{i,j} - r_j\right) \tag{6.6}$$

σ_{ij} can also be expressed in terms of correlation coefficient (ρ_{ij}) as follows:

$$\sigma_{ij} = \rho_{ij}\sigma_i\sigma_j \tag{6.7}$$

As a result, the portfolio equation is defined by the equation:

$$\delta(x_1, x_2, - - - - - -, x_n) = \sum_{i=1}^{n}\sum_{j=1}^{n} x_i x_j \sigma_{ij} \tag{6.8}$$

Portfolio Formulation

Markovitz [26] developed the modern portfolio theory as a financial framework via the trader's attempt to take minimum risks and attain most return to a given funding portfolio. The theory emphasizes that a higher return comes with a higher risk and that looking at the expected risk and return of a single asset is insufficient. An individual asset has a higher risk than an asset in a combined portfolio, as long as the risks of the various assets are not directly related.

The modern portfolio theory assumes that a rational investor wants the maximum return for a given level of risk and the least risk for a given level of expected return. As a result, the asset weight vector is the state variable in the asset allocation optimal solution, showing investors how much to invest in each asset in a given portfolio. Weight vector $x = [x_1, x_2, x_3 - - - - x_n]$ with x_i as the weight of asset i is the portfolio. The expected return for each asset in the portfolio is expressed in the vector form $r = [r_1 r_{2,} \_ \_ \_ r_n]$ with r_i as the mean return of assets i . The portfolio expected return is calculated using the weighted average of individual asset returns $= \sum_{i=1}^{n} x_i p_{i.}$

Statement of the Problem

The formulation of mean-variance method can be defined as:

Minimizing

$$\sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j \sigma_{ij} \tag{6.9}$$

Subject to

$$\begin{cases} \sum_{i=1}^{n} r_i = r_0 \\ \sum_{i=1}^{n} x_i = 1, \; x_i \geq 0 \; i = 1, \; 2......10 \end{cases} \tag{6.10}$$

3 Problem Description

The model is implemented using the stock market data obtained from the Indian National Stock Exchange, Mumbai, by selecting ten companies at random. The data is taken from the [12] paper proposed to solve the problem using the LX-BBO and blended BBO method and another variant blended biogeography method. The mean-variance model is shown in Table 6.9. The formulation of the ten-variable constrained optimization problem is as follows:

Problem 1

$$\begin{aligned}\text{Minimize } z = {} & 0.00338x_1^2 + 0.4225x_2^2 + 0.00615x_3^2 + 0.00429x_4^2 \\ & + 0.00686x_5^2 + 0.00260x_6^2 + 0.00275x_7^2 + 0.00224x_8^2 + 0.01036x_9^2 \\ & + 0.00178x_{10}^2 - 0.01584x_1x_2 + 0.00712x_1x_3 + 0.00404x_1x_4 \\ & + 0.00374x_1x_5 + 0.00294x_1x_6 + 0.00610x_1x_7 + 0.00170x_1x_8 \\ & + 0.00384x_1x_9 + 0.00192x_1x_{10} - 0.01350x_2x_3 - 0.00236x_2x_4 \\ & + 0.00614x_2x_5 + 0.00298x_2x_6 + 0.00236x_2x_7 + 0.00622x_2x_8 \\ & + 0.00384x_2x_9 + 0.00192x_2x_{10} + 0.00586x_3x_4 + 0.00456x_3x_5 \\ & + 0.00472x_3x_6 + 0.00182x_3x_7 + 0.00396x_3x_8 + 0.00648x_3x_9 \\ & + 0.00178x_3x_{10} + 0.00884x_4x_5 + 0.00516x_4x_6 + 0.00190x_4x_7 \\ & + 0.00464x_4x_8 + 0.01158x_4x_9 + 0.00288x_4x_{10} + 0.00696x_5x_6 \\ & + 0.00362x_5x_7 + 0.00530x_5x_8 + 0.0124x_5x_9 + 0.00384x_5x_{10} \\ & + 0.0017x_6x_7 + 0.0040x_6x_8 + 0.00766x_6x_9 + 0.00284x_6x_{10} \\ & + 0.00190x_7x_8 + 0.00324x_7x_9 - 0.00082x_7x_{10} + 0.00694x_8x_9 \\ & + 0.00180x_8x_{10} + 0.0054x_9 \end{aligned} \tag{6.11}$$

$$\begin{aligned}\text{Subject to } r_0 = {} & 0.00728x_1 - 0.03613x_2 - 0.02414x_3 + 0.00706x_4 \\ & - 0.00458x_5 + 0.00372x_6 - 0.00461x_7 + 0.00413x_8 - 0.0248x_9 \\ & + 0.00562x_{10} \end{aligned} \tag{6.12}$$

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} = 1 \tag{6.13}$$

$$x_i \geq 0, i = 1, 2, - - - - - - - - - - , 10. \tag{6.14}$$

The above optimization approach is solved using sine-cosine algorithm-based optimization.

Problem 2

The model is implemented using the stock market data (1 April 2020 to 31 March 2021) obtained from the Indian National Stock Exchange, Mumbai, by selecting ten companies at random. Table 6.18 shows the monthly asset return. According to the mean-variance model provided, Table 6.19 shows the expected returns calculated by

Eq. 6.3. Equation 6.4 is used to calculate the variance, and Eq. 6.5 is used to calculate the covariance.

$$\begin{aligned}\text{Minimize } z = {} & 0.087175x_1^2 + 0.03x_2^2 + 0.0061x_3^2 + 0.001x_4^2 + 0.0056x_5^2 \\ & + 0.0056x_6^2 + 0.003841x_7^2 + 0.010814x_8^2 + 0.0169x_9^2 + 0.005133x_{10}^2 \\ & - 0.0038x_1x_2 + 0.0013x_1x_3 + 0.005x_1x_4 + 0.00238x_1x_5 + 0.0022x_1x_6 \\ & + 0.0023x_1x_7 + 0.0041x_1x_8 + 0.00509x_1x_9 - 0.00259x_1x_{10} \\ & + 0.00398\,x_2x_3 - 0.0000000236x_2x_4 - 0.00250x_2x_5 - 0.000601x_2x_6 \\ & - 0.00088x_2x_7 + 0.00142x_2x_8 + 0.000841x_2x_9 + 0.000375x_2x_{10} \\ & - 0.00145x_3x_4 - 0.001x_3x_5 + 0.00114x_3x_6 + 0.00000332x_3x_7 \\ & + 0.00000361x_3x_8 - 0.0017x_3x_9 + 0.00026x_3x_{10} + 0.000202x_4x_5 \\ & + 0.000676x_4x_6 + 0.000706x_4x_7 + 0.002142x_4x_8 + 0.002336x_4x_9 \\ & + 0.00257x_4x_{10} - 0.00022x_5x_6 + 0.000192x_5x_7 + 0.000464x_5x_8 \\ & - 0.00336x_5x_9 - 0.00078x_5x_{10} + 0.000464x_6x_7 + 0.002499x_6x_8 \\ & + 0.007323x_6x_9 - 0.00336x_6x_{10} + 0.008177x_7x_8 + 0.005679x_7x_9 \\ & - 0.00593x_7x_{10} - 0.00418x_8x_9 - 0.00358x_8x_{10} - 0.00418x_9x_{10} \end{aligned} \tag{6.15}$$

$$\begin{aligned}\text{Subject to } r_0 = {} & 0.13036x_1 - 0.0265x_2 - 0.1065x_3 - 0.01833x_4 \\ & - 0.0200x_5 0.0252x_6 - 0.00038x_7 - 0.0153x_8 - 0.0412x_9 - 0.07308x_{10}\end{aligned} \tag{6.16}$$

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} = 1 \tag{6.17}$$

$$x_i \geq 0, i = 1,2, - - - - - - - - - - ,10 \tag{6.18}$$

The above optimization problem is solved using sine-cosine algorithm-based optimization.

4 Sine-Cosine Algorithm

Sine-cosine is constructed on mathematical capabilities of sine-cosine function and discovering new feasible space using the two terms explore and exploit of search space. The SCA method is not usually tormented by the importance and nonlinear nature of the problem and even in other global strategies displays early convergence; the SCA reveals the best solution with more efficaciousness with a quicker convergence rate. The stability among the exploration and exploitation is the gain of this optimization technique. For this purpose, SCA makes use of trigonometric sine and cosine functions. At every step of the calculation, it updates the answers in line with the subsequent equations: The equation are as follows:

$$X_{ij}^{(t+1)} = \begin{cases} X_{ij}^{(t)} + r_1 \cos(r_2)|r_3 P_{ij}^{(t)} - X_{ij}^{(t)}|, & r_4 \geq 0.5 \\ X_{ij}^{(t)} + r_1 \sin(r_2)|r_3 P_{ij}^{(t)} - X_{ij}^{(t)}|, & r_4 \leq 0.5 \end{cases} \tag{6.19}$$

where $X_{ij}^{(t)}$ represents the current individual I at iteration t. $P_{ij}^{(\mathrm{t})}$ shows the best individual position at iteration t, and r_1, r_2, r_3, and r_4 are random parameters.

$$r_1 = a - \frac{ta}{T_{\max}} \tag{6.20}$$

where t denotes the iteration and r_1 is the main parameter that balances the exploration and exploitation phase, decreasing linearly from a constant value a to 0 by each iteration by Eq. 6.10, and r_2 and r_3 are random numbers.

The competency of SCA is different from other metaheuristic technique:

1. SCA works with a group of solution that benefit from the phenomenon of parallel exploration.
2. It simultaneously investigates several regions of solution space for sine and cosine function values outside the range [−1,1].
3. SCA investigates several promising solutions simultaneously during the exploratory process with sine-cosine value in the range [−1,1].
4. The best solution at a given point in the calculations is saved in a variable and becomes the problem's target ensuring that it never gets lost during the optimization phase.
5. The optimization process is convergent in nature (Table 6.1).

Table 6.1 Pseudo code of sine-cosine algorithm

```
Initiate {Evaluate the position X_i(i = 1, 2, − − − − n) and asses the objective function
Set the current best position P_i^t
Set T_max to the maximum number of iterations.
While T< T_max
for i = 1 : n
Update the parameter r_1, r_2,r_3 and r_4
Update X_i using equation (8)
   if   (f(X_i^{t+1}) < f(X_i^t))
   refresh the current best position P_i^t
end if
end for
t = t + 1
end while
Return
the best solution P_i
```

Mutation

Mutation is a vital operator in genetic algorithms (GAs), because it ensures renovation of diversity in the evolving populations of GAs. It performs a crucial position in making the general search efficient. GAs are both simple and powerful in terms of computation, because they make no assumption about the solution space; genetic algorithm is an excellent tool for solving optimization problem.

The affinity of GAs is one of their advantages. GA uses a population of individual to search a solution space, making it less likely for them to become stuck in the local optimum. This comes at a price, which is the computational time. The longer runtime of Gas, on the other hand, can be reduced by terminating the evaluation earlier in order to obtain a satisfactory solution. Banerjee and Garg [4], incorporated five mutation operators power mutation, Polynomial mutation, Random mutation, Cauchy mutation & Gaussian mutation in SCA and presented a new version of SCA where cauchy & Random mutation performed better with constraint and unconstrained problems.

Power Mutation

Power mutation is a new form of SCA that incorporates the power mutation reported in (Banerjee and Garg). The power mutation p is set for 0.25 and $p = 0.50$. The mutation's strength is determined by the mutation's index (p). The smaller value of p should result in less fluctuation in the solution, while the larger value of p should result in more diversity. The mutation operator that has been proposed is based on power distribution. It's known as power mutation. Its distribution function is defined as follows:

$$f(x) = px^{p-1}, 0 \le x \le 1 \tag{6.21}$$

And the density function is presented by:

$$F(x) = x^p, 0 \le x \le 1 \tag{6.22}$$

The index of the distribution is denoted by p. The PM is used to generate a solution y near a parent solution z that follows the previously mentioned distribution. The mutated solution is then created using this formula below.

$$y = \begin{cases} x - z(x - x_l) & \text{if } r < t \\ x - z(x_u - x) & \text{if } r \ge t \end{cases} \tag{6.23}$$

Polynomial Mutation

Polynomial muatation (Banerjee & Garg 2022) presented incorporated in SCA. A new version of SCA in called Poly-SCA. To confound a solution in the neighbourhood of a parent, a polynomial probability distribution is used; the mutation operator adjusted the probability distribution to the left and right of a variable value so that no value outside the specific range $[a, b]$ is created. For a given parent solution $x\epsilon[x_l, x_u]$, mutated solution x' is constructed.

$$x' = \begin{cases} x+\delta_i(x-x_l) & \text{if} \quad u \leq 0.5 \\ x+\delta_i(x_u-x) & \text{if} \quad u > 0.5 \end{cases}, \tag{6.24}$$

Where $u\epsilon[0, 1]$ is a random number. The values δ_l and δ_r are computed as given by the formula.

$$\delta_l = (2u)^{\frac{1}{1+\rho}} - 1 \ u \leq 0.5$$

$$\delta_r = 1 - (2(1-u))^{\frac{1}{1+\rho}} \ u > 0.5$$

where $\rho_m \in [20, 100]$ is the user-defined parameter.

Random Mutation

Random mutation is incorporated in SCA.A new version of SCA is called Rand-SCA. Suppose x is any given solution, then a rand mutation operator is used as $x \in [x_l,x_u]$, and a random solution h is created using a neighbourhood of the replaced solution.

$$h = x_l + (x_u - x_l) * \text{rand} \tag{6.25}$$

where rand $\in[0, 1]$ represents a uniform distribution.

Gaussian Mutation

Gaussian mutation causes a small random change in the population. A random number from Gaussian distribution N (0,1) with parameter 0 as a mean and 1 as std. dev. is generated. $X(i,j)$ is chosen; then, find a new generated position.

$$z = X(i, j) + N_i(0, 1) \tag{6.26}$$

Cauchy Mutation

Cauchy mutation is defined in SCA as the same way as G-SCA. Suppose a random number is generated from the Cauchy distribution and defined by $\delta_i(t)$. The scale parameter is represented by t, where $t > 0$. Consider the value $t = 1$ as used in SCA-Cauchy. $X(i, j)$ is chosen; then, find a new generated position

$$z = X(i, j) + \delta_i(1) \tag{6.27}$$

5 Numerical Analysis of Results Obtained by the Proposed Version of SCA

Problem 1

The optimization problem described below is solved in two stages. The goal of the first stage is to figure out what the value of an unknown r_0 is in the restriction, given in Eq. 6.16. r_0 values are calculated for the upper and lower bounds, and this r_0 range is used in stage 2. The goal of stage 2 is to find the most cost-effective solution to the optimization problem. Different portfolios are created by considering various r_o values. These portfolios are then used to find the most cost-effective solution to the portfolio optimization problem.

Using SCA and variant of SCA, the value of undetermined r_0 is calculated in segment 1. The optimization problem is solved by removing the equality constraint in Eq. 6.17. r_0 min calculates the minimum value. The upper bound of r_0, denoted by $r_{\max}$, is calculated by investing all of one's money in the highest-returning asset. The r_0 values obtained are shown in Tables 6.2, 6.3, 6.4, 6.5, 6.6, and 6.7 for five the versions of SCA (Table 6.8).

In segment 2, five distinct portfolios are considered, namely, Portfolio 1, Portfolio 2, Portfolio 3, Portfolio 4, and Portfolio 5, in the same way that five different values r_0 are considered. These values of r_0 have to lie withinside the range ($r_{\min,}r_{\max}$) acquired in segment 1. The solution of those optimization problem using SCA and the variant of SCA is acquired with populace size 30, 50, and 100. Result obtained by the algorithms are tabulated in Tables 6.9, 6.10, 6.11, 6.12, 6.13, 6.14, 6.15, 6.16, and 6.17.

Table 6.8 depicts the expected rate of return is calculated over a specified range for different population sizes 30, 50, and100 and applied five versions of sine-cosine algorithm, compared the values with LX-BBO.

Result analysis for population size 30:

Figure 6.1 shows the Gaussian version of SCA gives an optimal portfolio with min risk 0.0012 for population size 30. The graph depicts that risk increases as return also increases while the difference between the maximum and minimum average

Table 6.2 For various population sizes, the range of r_0

The size of the population for SCA	risk_{min}	r_{min}	r_{max}
30	0.001428	0.000213	0.00612
50	0.0008	0.000312	0.00612
100	0.00033	0.00000003	0.0878

Table 6.3 For various population sizes, the range of r_0

The size of the population for PMSCA	risk_{min}	r_{min}	r_{max}
30	0.0012	0.0000000567	0.909
50	0.0006	0.00000343	0.00989
100	0.00055	0.001528	0.909

Table 6.4 For various population sizes, the range of r_0

The size of the population for Cauchy SCA	risk_{min}	r_{min}	r_{max}
30	0.0001256	0.00009	0.909
50	0.00065	0.000676	0.8889
100	0.00076	0.00009	0.89876

Table 6.5 For various population sizes, the range of r_0

The size of the population for Poly SCA	risk_{min}	r_{min}	r_{max}
30	0.001278	0.00009	0.909
50	0.00057	0.000676	0.8889
100	0.00039	0.00009	0.89876

Table 6.6 For various population sizes, the range of r_0

The size of the population for Gaussian SCA	risk_{min}	r_{min}	r_{max}
30	0.0012	0.0000765	0.9998
50	0.00055	0.000098	0.565
100	0.00058	0.000089	0.8988

Table 6.7 For various population sizes, the range of r_0

The size of the population for RM SCA	risk_{min}	r_{min}	r_{max}
30	0.0015	0.0000011	0.007
50	0.00082	0.000121	0.6789
100	0.0004	0.0000343	0.00564

Table 6.8 Comparison with other NIA

Algorithm	Population size	r_{min}	r_{max}
LX-BBO	30	0.000221	0.00728
	50	0.00518	0.00728
	100	−9E-06	0.0072
SCA	30	0.00213	0.00612
	50	0.000312	0.00612
	100	0.0000003	0.0878
PM-SCA	30	0.00000567	0.909
	50	0.00000343	0.00989
	100	0.001528	0.909
R-SCA	30	0.0000011	0.007
	50	0.000121	0.6789
	100	0.0000343	0.00564
C-SCA	30	0.0009	0.909
	50	0.00676	0.8809
	100	0.00009	0.89876
G-SCA	30	0.000765	0.9998
	50	0.000098	0.565
	100	0.000089	0.8988
Poly-SCA	30	0.00009	0.909
	50	0.000676	0.8989
	100	0.00009	0.89876

annual returns of the portfolio set decreases. The risk-reward trade-off is a trading principle that connects the high risk and high return. The best risk-return trade-off is determined by a number of factors, including the investor's risk tolerance and the ability to replace lost funds.

Result analysis for population size 50:

Figure 6.2 depicts SCA gives the best result with min risk 0.0005; it gives a set of optimal portfolios to strike a balance between an investment's expected return and its defined level of risk.

Result analysis for population size 100:

Poly-SCA variant of SCA anticipated range of expected return of different portfolio gives a good return with min risk 0.0013 for population size 100. Investing your money across a range of asset classes and securities to lower the portfolio's overall risk (Fig. 6.3 and Tables 6.18, 6.19, and 6.20).

Table 6.9 Efficient solution of portfolio optimization with population size 30 & 50 by SCA

	r_0	Risk	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
Portfolio 1	0.00023	0.001428	0	0	0.302	0.0256	0.250	0.002	0.102	0.003	0.210	0.100
Portfolio 2	0.00244	0.0017	0.05	0.012	0	0	0.235	0.0045	0.120	0.230	0.140	0.120
Portfolio 3	0.00449	0.00185	0.003	0.210	0.120	0.215	0.145	0	0	0.102	0.110	0.005
Portfolio 4	0.00551	0.00198	0	0.0543	0.4246	0.2405	0	0	0.0710	0	0.1230	0
Portfolio 5	0.00612	0.00620	0.00520	0.0020	0.001	0.102	0.203	0.403	0	0.006	0	0
Portfolio 1	0.00023	0.0008	0.005	0	0	0.208	0.310	0.008	0	0.025	0.30	0
Portfolio 2	0.00244	0.0012	0.0012	0.114	0.030	0.005	0.502	0	0	0.020	0.20	0
Portfolio 3	0.00449	0.0018	0.502	0.002	0	0.062	0.206	0.003	0.001	0.102	0	0.100
Portfolio 4	0.00551	0.002	0.302	0.133	0.123	0.007	0.159	0	0	0.162	0.0060	0
Portfolio 5	0.00620	0.00520	0.0020	0.001	0.102	0.203	0.403	0	0.0001	0.03	0.004	0

Table 6.10 Efficient solution of portfolio optimization with population 100 by SCA and 30 by PMSCA

	r_0	Risk	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
Portfolio 1	0.000748	0.0045	0.001	0.203	0.502	0.0002	0.310	0	0	0.020	0.30	0.003
Portfolio 2	0.00482	0.0055	0.402	0.004	0.0020	0.005	0	0	0.0023	0.010	0.200	0.004
Portfolio 3	0.006	0.06121	0	0.006	0.004	0.062	0.010	0.07	0.003	0	0.050	0.0020
Portfolio 4	0.0067	0.0912	1	0	0	0.007	0	0	0	0	0	0
Portfolio 5	0.8878	0.8878	0.09	0.01	0.502	0.203	0.001	0	0.05	0.002	0.01	0.004
Portfolio 1	0.0000343	0.0047	0.00528	0.003	0.502	0.007	0.008	0.009	0.001	0.002	0.055	0.101
Portfolio 2	0.00268	0.0052	0.00702	0.0004	0.00302	0.00702	0.0102	0.009	0.101	0.002	0.004	0.0404
Portfolio 3	0.0125	0.0056	0.0065	0.0054	0.00123	0.034	0.0045	0.0654	0.0006	0.0054	0.034	0.0054
Portfolio 4	0.100	0.0067	NAN	–	–	–	–	–	–	–	–	–
Portfolio 5	0.9298	0.9298	NAN	–	–	–	–	–	–	–	–	–

Table 6.11 Efficient solution of portfolio optimization with population size 50 and 100 by PMSCA

	r_0	Risk	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
Portfolio 1	0.00023	0.0006	0.001	0.203	0.502	0.0002	0.310	0	0	0.020	0.30	0.003
Portfolio 2	0.00244	0.001212	0.402	0.004	0.0020	0.005	0	0	0.0023	0.010	0.200	0.004
Portfolio 3	0.00449	0.001315	0	0.006	0.004	0.062	0.010	0.07	0.003	0	0.050	0.0020
Portfolio 4	0.00551	0.001517	1	0	0	0.007	0	0	0	0	0	0
Portfolio 5	0.00620	0.0052	0.09	0.01	0.502	0.203	0.001	0	0.05	0.002	0.01	0.004
Portfolio 1	0.001428	0.00055	0.00528	0.003	0.502	0.007	0.008	0.009	0.001	0.002	0.055	0.101
Portfolio 2	0.001202	0.00095	0.00702	0.0004	0.00302	0.00702	0.0102	0.009	0.101	0.002	0.004	0.0404
Portfolio 3	0.001109	0.0011	0.0065	0.0054	0.00123	0.034	0.0045	0.0654	0.0006	0.0054	0.034	0.0054
Portfolio 4	0.001025	0.00193	NAN	–	–	–	–	–	–	–	–	–
Portfolio 5	0.909	0.0015	NAN	–	–	–	–	–	–	–	–	–

Table 6.12 Efficient solution of portfolio optimization with population size 30 & 50 by RM- SCA

	r_0	Risk	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
Portfolio 1	0.00000011	0.0015	0.005	0.03	0.004	0.01	0.601	0.002	0.101	0.002	0.001	0.1
Portfolio 2	0.00230	0.0019	0.003	0.02	0.202	0.501	0.002	0.105	0	0.102	0	0
Portfolio 3	0.0250	0.0021	0.002	0.03	0.04	0.002	0.201	0.006	0.70	0	0	0.101
Portfolio 4	0.350	0.0022	NAN	–	–	–	–	–	–	–	–	–
Portfolio 5	0.70	0.004	NAN	–	–	–	–	–	–	–	–	–
Portfolio 1	0.000121	0.00082	0.003	0.04	0.05	0.05	0.001	0.045	0.501	0.001	0.30	0.001
Portfolio 2	0.00230	0.0015	0.04	0.005	0.06	0.06	0.01	0.05	0.001	0.02	0.04	0.5
Portfolio 3	0.00450	0.001567	0.003	0.05	0.7	0.7	0.001	0.05	0.001	0.20	0.02	0.001
Portfolio 4	0.00501	0.0016	0.04	0.05	0.07	0.7	0.001	0.1	0.002	0.20	0.40	0.001
Portfolio 5	0.6789	0.098	NAN	–	–	–	–	–	–	–	–	–

Table 6.13 Efficient solution of portfolio optimization with population size 100 by RM-SCA & 30 by Poly- SCA

	r_0	Risk	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
Portfolio 1	0.0000343	0.0004	0.005	0.03	0.004	0.01	0.601	0.002	0.101	0.002	0.001	0.1
Portfolio 2	0.000250	0.00085	0.003	0.02	0.202	0.501	0.002	0.105	0	0.102	0	0
Portfolio 3	0.00160	0.001167	0.002	0.03	0.04	0.002	0.201	0.006	0.70	0	0	0.101
Portfolio 4	0.00350	0.0015	NAN	–	–	–	–	–	–	–	–	–
Portfolio 5	0.00564	0.0020	NAN	–	–	–	–	–	–	–	–	–
Portfolio 1	0.00009	0.001278	0.003	0.001278	0.05	0.05	0.001	0.045	0.1	0.002	0.004	0
Portfolio 2	0.000890	0.001456	0.04	0.001456	0.06	0.06	0.01	0.005	0	0	0.065	0.043
Portfolio 3	0.000750	0.001628	0.003	0.001628	0.7	0.7	0.001	0	0.767	0.043	0	0
Portfolio 4	0.00501	0.001978	0.04	0.001978	0.07	0.7	0.001	0.004	0.0012	0.0023	0.0045	0
Portfolio 5	0.909	0.056	NAN	–	–	–	–	–	–	–	–	–

Table 6.14 Efficient solution of portfolio optimization with population size 50, 100 by Poly-SCA

	r_0	Risk	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
Portfolio 1	0.000676	0.00057	0.007	0.003	0.003	0.43	0.03	0.004	0.005	0.065	0.78	0
Portfolio 2	0.000350	0.00078	0.04	0.043	0	0.0032	0.005	0.002	0.87	0.006	0	0
Portfolio 3	0.00550	0.00098	NAN	–	–	–	–	–	–	–	–	–
Portfolio 4	0.0250	0.0015	NAN	–	–	–	–	–	–	–	–	–
Portfolio 5	0.8889	0.0022	NAN	–	–	–	–	–	–	–	–	–
Portfolio 1	0.00009	0.00039	0.001	0.0006	0.0054	0.0076	0.0043	0.03	0.43	0.34	0	0
Portfolio 2	0.00750	0.00078	0.43	0.0043	0.005	0.0054	0.00043	0.54	0.12	0	0.44	0
Portfolio 3	0.00550	0.001456	NAN	–	–	–	–	–	–	–	–	–
Portfolio 4	0.075	0.0018	NAN	–	–	–	–	–	–	–	–	–
Portfolio 5	0.89876	0.00165	NAN	–	–	–	–	–	–	–	–	–

Table 6.15 Efficient solution of portfolio optimization with population size 30 & 50 by Cauchy- SCA

	r_0	Risk	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
Portfolio 1	0.00009	0.001256	0.02	0.03	0.040	0.50	0.001	0.020	0.010	0.001	0.020	0
Portfolio 2	0.00150	0.001628	0.05	0.02	0	0	0.30	0.20	0.001	0.07	0	0.08
Portfolio 3	0.0080	0.001834	0.06	0.03	0.70	0.1	0.20	0.40	0.005	0	0.001	0.070
Portfolio 4	0.070	0.001928	0.1	0.01	0.10	0.02	0.20	0.050	0.070	0.050	0.021	0.10
Portfolio 5	0.909	0.0020	NAN	–	–	–	–	–	–	–	–	–
Portfolio 1	0.000676	0.00065	0.003	0.50	0.001	0.30	0.20	0.60	0.001	0.20	0.10	0.001
Portfolio 2	0.00249	0.00095	0.02	0.30	0.01	0.50	0.30	0.001	0.01	0.20	0.001	0.03
Portfolio 3	0.0060	0.001125	NAN	–	–	–	–	–	–	–	–	–
Portfolio 4	0.050	0.001356	NAN	–	–	–	–	–	–	–	–	–
Portfolio 5	0.889	0.0700	NAN	–	–	–	–	–	–	–	–	–

Table 6.16 Efficient solution of portfolio optimization with population size 100 by Cauchy-SCA & 30 by GA- SCA

	r_0	Risk	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
Portfolio 1	0.00009	0.00076	0.005	0.004	0.00056	0.004	0.005	0.065	0.076	0.0034	0.008	0.0034
Portfolio 2	0.00175	0.001267	0.03	0.5	0.006	0.0087	0.00654	0.05	0.005	0.006	0.03	0.03
Portfolio 3	0.0020	0.001834	0.05	023	0.008	0.65	0.9	0.087	0.65	0.56	0.004	0.04
Portfolio 4	0.040	0.001928	0.067	0.06	0.004	0.008	0.003	0.065	0.003	0.007	0.035	0.003
Portfolio 5	0.89876	0.0698	0.002	0.09	0.0087	0.05	0.8	0.004	0.004	0.0034	0.045	0.005
Portfolio 1	0.0000765	0.0012	0.0012	0.004	0.45	0.0043	0.0004	0.0065	0.87	0.0002	0.0067	0.0056
Portfolio 2	0.000560	0.0017	0.0017	0.0065	0.3	0.0056	0.0065	0.09	0.0054	0.34	0.23	0.07
Portfolio 3	0.00350	0.002123	0.002123	0.00054	0.1	0.0076	0.006	0.2	0.004	0.56	0.34	0.006
Portfolio 4	0.075	0.0022	0.0022	0.0006	0.076	0.87	0.112	0.45	0.54	0	0.002	0.001
Portfolio 5	0.9998	0.0076	0.878	NAN	–	–	–	–	–	–	–	–

Table 6.17 Efficient solution of portfolio optimization with population 50 & 100 by Gaussian- SCA

	r_0	Risk	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
Portfolio 1	0.000098	0.00055	0.003	0	0.004	0.007	0.0065	0.54	0.064	0.06	0	0
Portfolio 2	0.000878	0.00085	0.008	0	0.10	0.03	0.008	0.5	0.05	0	0	0
Portfolio 3	0.00780	0.0011	0.02	0.006	0.002	0.03	0.007	0	0.0045	0	0.007	0.0076
Portfolio 4	0.0850	0.0013	0.02	0.05	0.003	0.002	0.006	0.004	0	0	0.034	0.0054
Portfolio 5	0.565	0.0098	0.003	0.006	0.02	0.003	0.005	0	0		0	0.0056
Portfolio 1	0.000089	0.00058	0.007	0.05	0	0	0.10	0.03	0.008	0.5	0.05	0
Portfolio 2	0.00077	0.001176	0.0065	0.0045	0	0.006	0.002	0.03	0.007	0	0.0045	0
Portfolio 3	0.00617	0.001345	0.56	0	0	0.05	0.003	0.002	0.006	0.004	0	0
Portfolio 4	0.035	0.0016	NAN	–	–	–	–	–	–	–		–
Portfolio 5	0.8988	0.0020	NAN	–	–	–	–	–	–	–	–	–

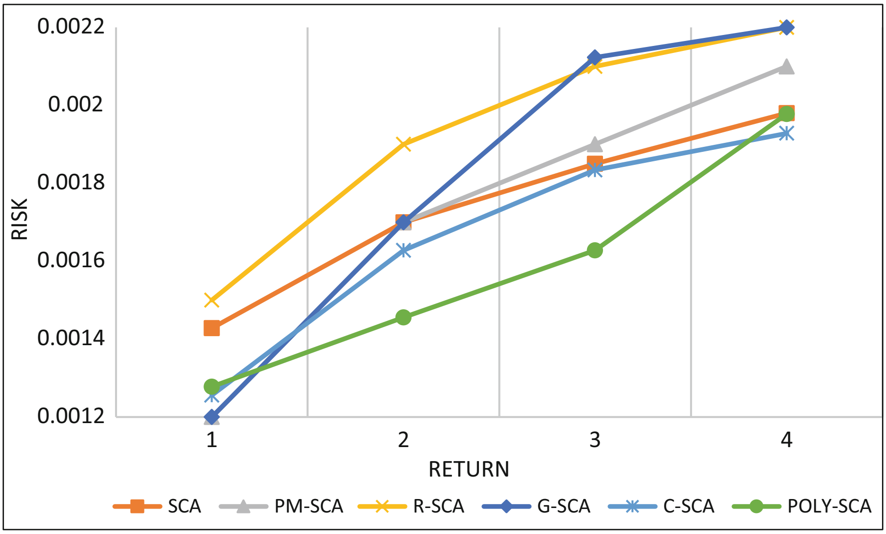

Fig. 6.1 Optimal portfolio for population size 30

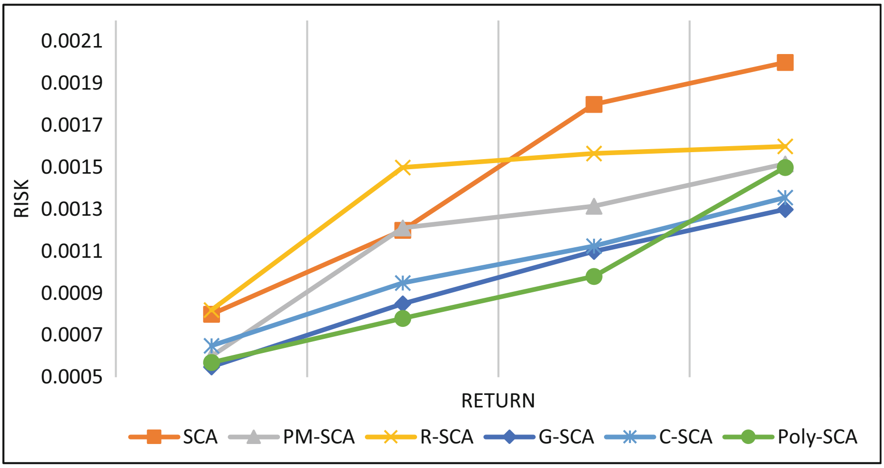

Fig. 6.2 Optimal portfolio for population size 50

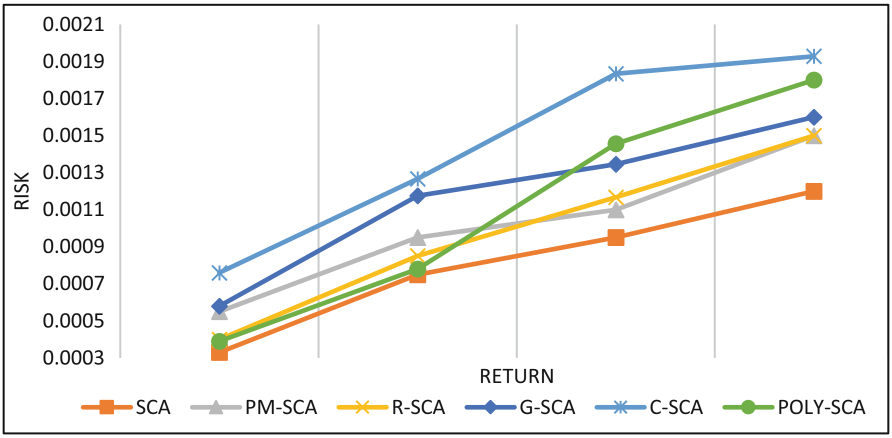

Fig. 6.3 Optimal portfolio for population size 100

Problem 2

The numerical analysis of the result is described similarly as in Sect. 5. Using SCA and the variant of SCA, the value of undetermined r_0 is calculated in segment 1. The optimization problem is solved by removing the equality constraint in Eq. 6.16. r_0 min calculates the minimum value. The upper bound of r_0, denoted by r_{max}, is calculated by investing all of one's money in the highest-returning asset. The r_0 values obtained are shown in Tables 6.21, 6.22, 6.23, 6.24, 6.25, and 6.26 for the five versions of SCA.

In segment 2, five distinct portfolios are considered, namely, Portfolio 1, Portfolio 2, Portfolio 3, Portfolio 4, and Portfolio 5, in the same way that five different values r_0 are considered. These values of r_0 have to lie withinside the range (r_{min},r_{max}) acquired in segment I. The solution of those optimization problem using SCA and the variant of SCA is acquired with populace size 30, 50, and 100. The results obtained by the algorithms are tabulated in Tables 6.27, 6.28, 6.29, 6.30, 6.31, 6.32, 6.33, 6.34, 6.35, and 6.36.

Table 6.37 depicts the expected rate of return that is calculated over a specified range for different population sizes 30, 50, and100 and applied five versions of sine-cosine algorithm, which compared the values with LX-BBO.

Result analysis for population size 30:

PM-SCA and poly-SCA give the best convergence graph with min risk 0.0013 and 0.0022 for population size 30, which is shown in Fig. 6.4.

Result analysis for population size 50:

Table 6.18 Monthly assets return data from 1st April 2020 to 31st march 2021 of 10 companies

Security name	Apr_20	May_20	June_20	July_20	Aug_20	Sep_20	Oct_20	Nov_20	Dec_20	Jan_21	Feb_21	Mar_21
Asian Paints Ltd	1.055326	0.04299	0.002585	0.016623	0.107024	0.045969	0.113321	0.001718	0.247912	–0.12919	–0.05406	0.11426
Bajaj Auto Ltd	–0.03217	0.033241	–0.03217	–0.08645	0.029815	–0.00201	–0.09032	–0.07854	–0.14023	0.054515	0.034899	–0.04256
Cipla Ltd	–0.9033	0.012339	–0.11095	0.00925	–0.07893	0.026773	0.011937	–0.09068	–0.0072	0.0493	–0.03441	–0.0463
Grasim Industries Ltd	0.03	0.03	–0.02	–0.06	–0.09	–0.04	–0.011	–0.06	–0.012	–0.012	–0.017	0.03
Ambuja Cement Ltd	–0.10389	0.01059	–0.1204	0.04537	0.02972	–0.16442	–0.00517	0.049027	0.022601	0.11024	–0.11446	–0.00065
HDFC Bank Ltd	0.052698	–0.10714	0.032	–0.07443	0.034536	–0.08867	–0.17858	0.003168	0.032938	–0.09378	0.027282	0.05760
Hindustn Unilever Ltd	0.066906	–0.05626	–0.01353	0.04371	0.02374	–0.00147	–0.03129	–0.10737	0.058086	0.061842	–0.12315	0.03303
Kotak Mahindra Bank Ltd	0.108824	–0.1003	–0.00388	–0.0254	0.104991	–0.18043	–0.18861	–0.04435	0.165008	–0.03786	0.015602	0.00240
State Bank Ltd	0.181029	–0.09611	–0.0679	–0.0969	0.1434	–0.02034	–0.22518	–0.11166	–0.02535	–0.2769	0.07095	0.03055
Wipro Ltd	–0.10268	–0.03119	–0.21819	0.035569	–0.134	–0.07989	–0.02796	–0.09256	–0.07574	0.0185	–0.0093	–0.15951

Table 6.19 Expected return of individual stocks

Security name	Expected return
BAL	0.13036
CL	−0.0265
APL	−0.1065
GIL	−0.01833
ACL	−0.0200
HDFC	0.0252
HUL	−0.003811
KMBL	−0.01533
SBI	−0.0412
WL	−0.07308

Figure 6.5 depicts Cauchy-SCA gives the best result for population size 50.

Result analysis for population size 100:

This poly-SCA gives the best result with 100 population size in the year 2015–2016 and 2020–2021. As a result, portfolio optimization performed effectively with 100 population size (Fig. 6.6).

6 Result Analysis

A sensitivity analysis, performed with population size 30, 50, 100 and an algorithm, is applied in five different versions of SCA. Five different portfolios are presumed in the numerical problem for two data set year 2015–2016 and year 2020–2021. The convergence graphs for all of the cases derived with different population sizes. It is seen that portfolio theory attitude depends entirely on the size of the population. Because when the size of the population achieves 30, the risk goes up at the very same speed as the rates of return. Whenever the population size is placed to 50, the risk increases as the rates of return rise, but at a varying rates. When the size of the population reaches 100, the risk would be almost consistent as the rates of return enhance.

7 Conclusion

In this paper, we presented a novel attempt to solve the model of portfolio optimization for five variants of SCA. Portfolio diversification is one of the most important tenets of investing and is essential for risk management. Diversification has numerous advantages. It must, however, be done with caution. Modern investors do not concentrate their wealth in a single security or a single type of security; instead, they

Table 6.20 Covariance and variance for the month-to-month asset returns

Security name	APL	BAL	CL	GIL	ACL	HDFC	HUL	KMBL	SBI	WL
APL	0.087175	0.0003435	−0.07	0.004	−0.0004	0.0006	0.009911	0.009911	0.018284	−0.00269
BAL	0.0003435	0.003	−0.03	0.003	0.0010	0.0001	−0.00022	−0.00022	0.002033	0.00022
CL	−0.07	−0.003	0.0061	−0.003	0.0060	−0.08	−0.00442	−0.00442	−0.01932	0.005223
GIL	0.004	0.003	−0.003	0.001	−0.0005	0.0000046	0.000286	0.000286	4.8E-05	−0.00013
ACL	−0.0004	0.0010	0.006	−0.0005	0.0056	0.0013	0.0011	0.0011	−0.00516	0.002236
HDFC	0.0006	0.0001	−0.008	0.0000046	0.0013	0.0056	0.000464	0.000464	0.007322	−0.00336
HUL	0.009911	−0.00022	−0.00442	0.000286	0.0011	0.000464	0.003841	0.00234	0.000705	0.0003
KMBL	0.01039	−0.00022	−0.0111	−7.6E-05	0.00105	0.006361	0.002499	0.010814	0.008176	−0.00213
SBI	0.018284	0.002033	−0.01932	4.8E-05	−0.00516	0.007322	0.000705	0.008176	0.016923	−0.00417
WL	−0.00269	0.00022	0.005223	−0.00013	0.002236	−0.00336	−0.0003	−0.00213	−0.00417	0.005133

Table 6.21 For various population sizes, the range of r_0

The size of the population for SCA	$risk_{min}$	r_{min}	r_{max}
30	0.0005	0.000323	0.323
50	0.0017	0.0000545	0.8878
100	0.0112	0.0000897	0.8878

Table 6.22 For various population sizes, the range of r_0

The size of the population for PMSCA	$risk_{min}$	r_{min}	r_{max}
30	0.0013	0.0000343	0.9298
50	0.0013	0.000032	0.333
100	0.0025	0.0000434	0.5656

Table 6.23 For various population sizes, the range of r_0

The size of the population Cauchy SCA	$risk_{min}$	r_{min}	r_{max}
30	0.0022	0.00011	0.7773
50	0.00085	0.0000343	0.323
100	0.0013	0.000088	0.7766

Table 6.24 For various population sizes, the range of r_0

The size of the population for Poly SCA	$risk_{min}$	r_{min}	r_{max}
30	0.0022	0.000211	0.676
50	0.0011	0.00232	0.576
100	0.0013	0.0000656	0.576

Table 6.25 For various population sizes, the range of r_0

The size of the population for Gaussian SCA	$risk_{min}$	r_{min}	r_{max}
30	0.0056	0.0000332	0.9998
50	0.0012	0.0011	0.4434
100	0.0012	0.00011	0.8988

Table 6.26 For various population sizes, the range of r_0

The size of the population for RM SCA	$risk_{min}$	r_{min}	r_{max}
30	0.0037	0.000111	0.777
50	0.0056	0.000343	0.323
100	0.0012	0.00011	0.7766

Table 6.27 Efficient solution of portfolio optimization with population 30 & 50 by SCA

	r_0	Risk	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
Portfolio 1	0.00323	0.0026	0	0	0.302	0.0134	0.250	0.001	0.102	0.004	0.210	0.100
Portfolio 2	0.0050	0.0031	0.05	0.012	0	0	0.235	0.0045	0.120	0.230	0.140	0.120
Portfolio 3	0.0030	0.00332	0.003	0.210	0.120	0.215	0.145	0	0	0.102	0.110	0.005
Portfolio 4	0.0211	0.0041	0	0.0543	0.4246	0.2405	0	0	0.0710	0	0.1230	0
Portfolio 5	0.323	0.323	0.0070	0.212	0.0053	0.0213	0.423	0	0	0.006	0	0
Portfolio 1	0.000641	0.0049	0.005	0	0	0.208	0.310	0.008	0	0.025	0.30	0
Portfolio 2	0.00324	0.0055	0.0012	0.114	0.030	0.005	0.502	0	0	0.020	0.20	0
Portfolio 3	0.0125	0.0058	0.502	0.002	0	0.062	0.206	0.003	0.001	0.102	0	0.100
Portfolio 4	0.0100	0.0066	0.302	0.133	0.123	0.007	0.159	0	0	0.162	0.0060	0
Portfolio 5	0.8878	0.8878	0.0020	0.001	0.102	0.203	0.403	0	0.0001	0.03	0.004	0

Table 6.28 Efficient solution of portfolio optimization with population 100 by SCA and 30 by PMSCA

	r_0	Risk	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
Portfolio 1	0.000748	0.0045	0.001	0.203	0.502	0.0002	0.310	0	0	0.020	0.30	0.003
Portfolio 2	0.00482	0.0055	0.402	0.004	0.0020	0.005	0	0	0.0023	0.010	0.200	0.004
Portfolio 3	0.006	0.06121	0	0.006	0.004	0.062	0.010	0.07	0.003	0	0.050	0.0020
Portfolio 4	0.0067	0.0912	1	0	0	0.007	0	0	0	0	0	0
Portfolio 5	0.8878	0.8878	0.09	0.01	0.502	0.203	0.001	0	0.05	0.002	0.01	0.004
Portfolio 1	0.0000343	0.0047	0.00528	0.003	0.502	0.007	0.008	0.009	0.001	0.002	0.055	0.101
Portfolio 2	0.00268	0.0052	0.00702	0.0004	0.00302	0.00702	0.0102	0.009	0.101	0.002	0.004	0.0404
Portfolio 3	0.0125	0.0056	0.0065	0.0054	0.00123	0.034	0.0045	0.0654	0.0006	0.0054	0.034	0.0054
Portfolio 4	0.100	0.0067	NAN	–	–	–	–	–	–	–	–	–
Portfolio 5	0.9298	0.9298	NAN	–	–	–	–	–	–	–	–	–

Table 6.29 Efficient solution of portfolio optimization with population size 50 & 100 by PMSCA

	r_0	Risk	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
Portfolio 1	0.000032	0.00013	0.001	0.203	0.502	0.0002	0.310	0	0	0.020	0.30	0.003
Portfolio 2	0.000172	0.0015	0.402	0.004	0.0020	0.005	0	0	0.0023	0.010	0.200	0.004
Portfolio 3	0.00115	0.0019	0	0.006	0.004	0.062	0.010	0.07	0.003	0	0.050	0.0020
Portfolio 4	0.0105	0.002	1	0	0	0.007	0	0	0	0	0	0
Portfolio 5	0.332	0.332	0.09	0.01	0.502	0.203	0.001	0	0.05	0.002	0.01	0.004
Portfolio 1	0.0000434	0.0043	0.00528	0.003	0.502	0.007	0.008	0.009	0.001	0.002	0.055	0.101
Portfolio 2	0.000265	0.0056	0.00702	0.0004	0.00302	0.00702	0.0102	0.009	0.101	0.002	0.004	0.0404
Portfolio 3	0.00154	0.0059	0.0065	0.0054	0.00123	0.034	0.0045	0.0654	0.0006	0.0054	0.034	0.0054
Portfolio 4	0.065	0.0064	NAN	–	–	–	–	–	–	–	–	–
Portfolio 5	0.5656	0.5656	NAN	–	–	–	–	–	–	–	–	–

Table 6.30 Efficient solution of portfolio optimization with population 50 & 100 by PMSCA

	r_0	Risk	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
Portfolio 1	0.000032	0.00013	0.001	0.203	0.502	0.0002	0.310	0	0	0.020	0.30	0.003
Portfolio 2	0.000172	0.0015	0.402	0.004	0.0020	0.005	0	0	0.0023	0.010	0.200	0.004
Portfolio 3	0.00115	0.0019	0	0.006	0.004	0.062	0.010	0.07	0.003	0	0.050	0.0020
Portfolio 4	0.0105	0.002	1	0	0	0.007	0	0	0	0	0	0
Portfolio 5	0.332	0.332	0.09	0.01	0.502	0.203	0.001	0	0.05	0.002	0.01	0.004
Portfolio 1	0.0000434	0.0043	0.00528	0.003	0.502	0.007	0.008	0.009	0.001	0.002	0.055	0.101
Portfolio 2	0.000265	0.0056	0.00702	0.0004	0.00302	0.00702	0.0102	0.009	0.101	0.002	0.004	0.0404
Portfolio 3	0.00154	0.0059	0.0065	0.0054	0.00123	0.034	0.0045	0.0654	0.0006	0.0054	0.034	0.0054
Portfolio 4	0.065	0.0064	NAN	–	–	–	–	–	–	–	–	–
Portfolio 5	0.5656	0.5656	NAN	–	–	–	–	–	–	–	–	–

Table 6.31 Efficient solution of portfolio optimization with population size 30 & 50 by RM- SCA

	r_0	Risk	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
Portfolio 1	0.00011	0.0028	0.005	0.03	0.004	0.01	0.601	0.002	0.101	0.002	0.001	0.1
Portfolio 2	0.00562	0.00338	0.003	0.02	0.202	0.501	0.002	0.105	0	0.102	0	0
Portfolio 3	0.00268	0.0039	0.002	0.03	0.04	0.002	0.201	0.006	0.70	0	0	0.101
Portfolio 4	0.0142	0.0044	NAN	–	–	–	–	–	–	–	–	–
Portfolio 5	0.77	0.77	NAN	–	–	–	–	–	–	–	–	–
Portfolio 1	0.000343	0.005	0.003	0.04	0.05	0.05	0.001	0.045	0.501	0.001	0.30	0.001
Portfolio 2	0.00478	0.0053	0.04	0.005	0.06	0.06	0.01	0.05	0.001	0.02	0.04	0.5
Portfolio 3	0.00261	0.0058	0.003	0.05	0.7	0.7	0.001	0.05	0.001	0.20	0.02	0.001
Portfolio 4	0.0128	0.0066	0.04	0.05	0.07	0.7	0.001	0.1	0.002	0.20	0.40	0.001
Portfolio 5	0.323	0.323	NAN	–	–	–	–	–	–	–	–	–

Table 6.32 Efficient solution of portfolio optimization with population size 100 by RM-SCA & 30 by Poly- SCA

	r_0	Risk	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
Portfolio 1	0.00011	0.0044	0.005	0.03	0.004	0.01	0.601	0.002	0.101	0.002	0.001	0.1
Portfolio 2	0.00562	0.0048	0.003	0.02	0.202	0.501	0.002	0.105	0	0.102	0	0
Portfolio 3	0.00362	0.0053	0.002	0.03	0.04	0.002	0.201	0.006	0.70	0	0	0.101
Portfolio 4	0.0135	0.0058	NAN	–	–	–	–	–	–	–	–	–
Portfolio 5	0.7766	0.7766	NAN	–	–	–	–	–	–	–	–	–
Portfolio 1	0.000211	0.0033	0.003	0.04	0.05	0.05	0.001	0.045	0.1	0.002	0.004	0
Portfolio 2	0.00565	0.0037	0.04	0.005	0.06	0.06	0.01	0.005	0	0	0.065	0.043
Portfolio 3	0.0268	0.0044	0.003	0.05	0.7	0.7	0.001	0	0.767	0.043	0	0
Portfolio 4	0.0128	0.0048	0.04	0.05	0.07	0.7	0.001	0.004	0.0012	0.0023	0.0045	0
Portfolio 5	0.676	0.676	NAN	–	–	–	–	–	–	–	–	

Table 6.33 Efficient solution of portfolio optimization with population size 50 and 100 by Poly-SCA

	r_0	Risk	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
Portfolio 1	0.00232	0.0049	0.007	0.003	0.003	0.43	0.03	0.004	0.005	0.065	0.78	0
Portfolio 2	0.0468	0.0057	0.04	0.043	0	0.0032	0.005	0.002	0.87	0.006	0	0
Portfolio 3	0.0267	0.0068	NAN	–	–	–	–	–	–	–	–	–
Portfolio 4	0.0120	0.0071	NAN	–	–	–	–	–	–	–	–	–
Portfolio 5	0.576	0.576	NAN	–	–	–	–	–	–	–	–	–
Portfolio 1	0.0000656	0.0045	0.001	0.0006	0.0054	0.0076	0.0043	0.03	0.43	0.34	0	0
Portfolio 2	0.000568	0.0057	0.43	0.0043	0.005	0.0054	0.00043	0.54	0.12	0	0.44	0
Portfolio 3	0.00278	0.0071	NAN	–	–	–	–	–	–	–	–	–
Portfolio 4	0.0178	0.0078	NAN	–	–	–	–	–	–	–	–	–
Portfolio 5	0.576	0.576	NAN	–	–	–	–	–	–	–	–	–

Table 6.34 Efficient solution of portfolio optimization with population size 30 & 50 by Cauchy- SCA

	r_0	Risk	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
Portfolio 1	0.00011	0.0032	0.02	0.03	0.040	0.50	0.001	0.020	0.010	0.001	0.020	0
Portfolio 2	0.00782	0.0037	0.05	0.02	0	0	0.30	0.20	0.001	0.07	0	0.08
Portfolio 3	0.00684	0.0042	0.06	0.03	0.70	0.1	0.20	0.40	0.005	0	0.001	0.070
Portfolio 4	0.077	0.0046	0.1	0.01	0.10	0.02	0.20	0.050	0.070	0.050	0.021	0.10
Portfolio 5	0.7773	0.7773	NAN	–	–	–	–	–	–	–	–	–
Portfolio 1	0.0000343	0.0055	0.003	0.50	0.001	0.30	0.20	0.60	0.001	0.20	0.10	0.001
Portfolio 2	0.000436	0.0067	0.02	0.30	0.01	0.50	0.30	0.001	0.01	0.20	0.001	0.03
Portfolio 3	0.00320	0.0068	NAN	–	–	–	–	–	–	–	–	–
Portfolio 4	0.0105	0.0076	NAN	–	–	–	–	–	–	–	–	–
Portfolio 5	0.323	0.323	NAN	–	–	–	–	–	–	–	–	–

Table 6.35 Efficient solution of portfolio optimization with population size 100 by Cauchy-SCA & 30 by GA- SCA

	r_0	Risk	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
Portfolio 1	0.0000088	0.0044	0.005	0.004	0.00056	0.004	0.005	0.065	0.076	0.0034	0.008	0.0034
Portfolio 2	0.000728	0.0005	0.03	0.5	0.006	0.0087	0.00654	0.05	0.005	0.006	0.03	0.03
Portfolio 3	0.00620	0.0056	0.05	023	0.008	0.65	0.9	0.087	0.65	0.56	0.004	0.04
Portfolio 4	0.0158	0.0068	0.067	0.06	0.004	0.008	0.003	0.065	0.003	0.007	0.035	0.003
Portfolio 5	0.7766	0.7766	0.002	0.09	0.0087	0.05	0.8	0.004	0.004	0.0034	0.045	0.005
Portfolio 1	0.0000765	0.0027	0.007	0.004	0.45	0.0043	0.0004	0.0065	0.87	0.0002	0.0067	0.0056
Portfolio 2	0.000650	0.0032	0.07	0.0065	0.3	0.0056	0.0065	0.09	0.0054	0.34	0.23	0.07
Portfolio 3	0.00560	0.0037	0.45	0.00054	0.1	0.0076	0.006	0.2	0.004	0.56	0.34	0.006
Portfolio 4	0.0340	0.0046	0.006	0.0006	0.076	0.87	0.112	0.45	0.54	0	0.002	0.001
Portfolio 5	0.998	0.998	0.878	NAN	–	–	–	–	–	–	–	–

Table 6.36 Efficient solution of portfolio optimization with population size 50 & 100 by Gaussian- SCA

	r_0	Risk	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
Portfolio 1	0.000098	0.0048	0.003	0	0.004	0.007	0.0065	0.54	0.064	0.06	0	0
Portfolio 2	0.00068	0.0058	0.008	0	0.10	0.03	0.008	0.5	0.05	0	0	0
Portfolio 3	0.0052	0.0066	0.02	0.006	0.002	0.03	0.007	0	0.0045	0	0.007	0.0076
Portfolio 4	0.028	0.0071	0.02	0.05	0.003	0.002	0.006	0.004	0	0	0.034	0.0054
Portfolio 5	0.565	0.565	0.003	0.006	0.02	0.003	0.005	0	0		0	0.0056
Portfolio 1	0.000089	0.0047	0.007	0.05	0	0	0.10	0.03	0.008	0.5	0.05	0
Portfolio 2	0.000780	0.0005	0.0065	0.0045	0	0.006	0.002	0.03	0.007	0	0.0045	0
Portfolio 3	0.00640	0.0062	0.56	0	0	0.05	0.003	0.002	0.006	0.004	0	0
Portfolio 4	0.0520	0.0007	NAN	–	–	–	–	–	–	–		–
Portfolio 5	0.8988	0.8998	NAN	–	–	–	–	–	–	–	–	–

Table 6.37 Comparison with other NIA – 2020–21

Algorithm	Population size	r_{min}	r_{max}
LX-BBO	30	0.000221	0.00728
	50	0.00518	0.00728
	100	-9E-06	0.0072
SCA	30	0.000343	0.9298
	50	0.00032	0.333
	100	0.0000434	0.5656
PM-SCA	30	0.0000343	0.9298
	50	0.000032	0.333
	100	0.0000434	0.5656
R-SCA	30	0.00011	0.7777
	50	0.000343	0.323
	100	0.00011	0.7766
C-SCA	30	0.00011	0.7773
	50	0.00000343	0.323
	100	0.000088	0.7766
G-SCA	30	0.0000332	0.9998
	50	0.0011	0.4434
	100	0.00011	0.8988
Poly-SCA	30	0.000211	0.676
	50	0.00232	0.576
	100	0.0000656	0.576

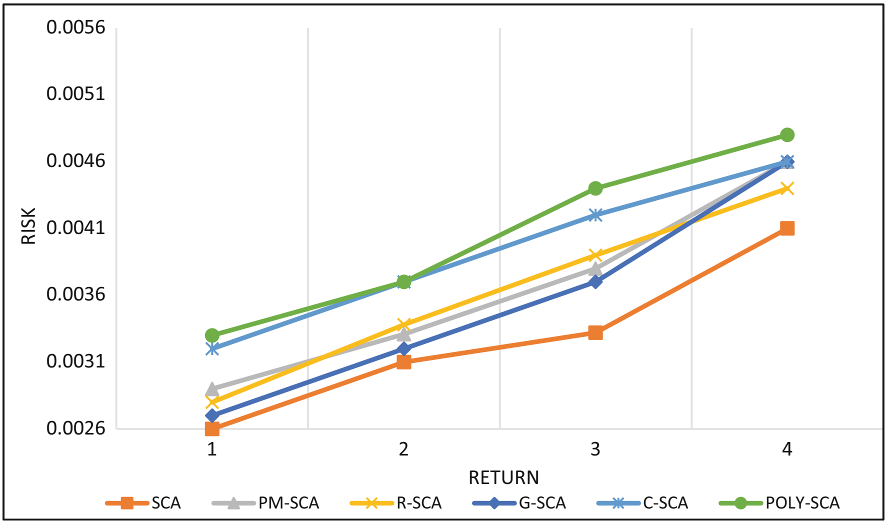

Fig. 6.4 Optimal portfolio for population size 30

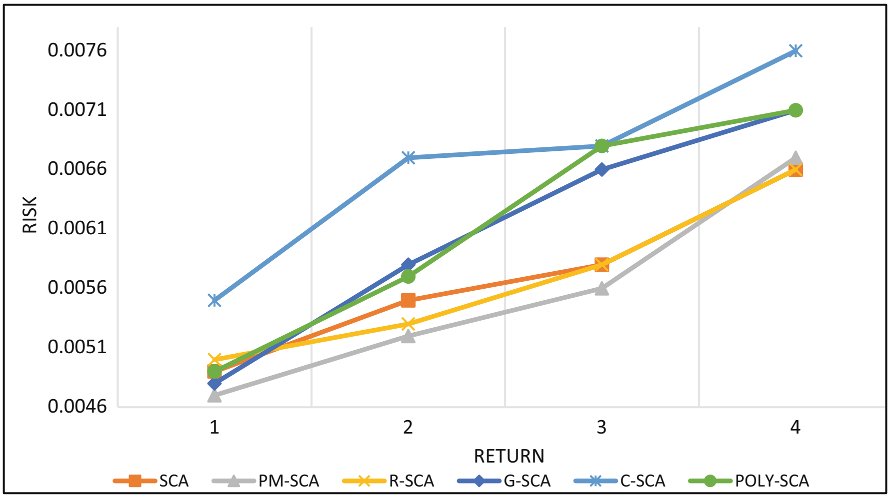

Fig. 6.5 Optimal portfolio for population size 50

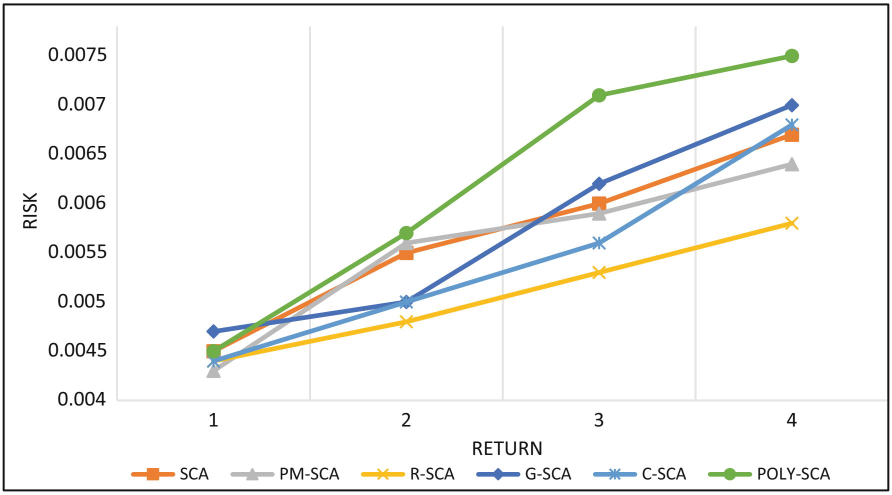

Fig. 6.6 Optimal portfolio for population size 100

diversify their portfolio by investing in a variety of securities. Portfolio's variance can be reduced by proper diversification for a given level of return. Diversification's benefits in terms of maintaining a portfolio's expected return (while reducing portfolio risk at the same time) can be seen when assets with low or even negative correlation are combined. The sensitivity analysis on five algorithms for different population sizes concludes that poly-SCA performed better than another variant of SCA for portfolio-based optimization.

References

1. Ahmadzade, H., Gao, R.: Covariance of uncertain random variables and its application to portfolio optimization. J. Ambient. Intell. Humaniz. Comput. **11**(4) (2019). https://doi.org/10.1007/s12652-019-01323-0
2. Ahmadzade, H., Gao, R., Dehghan, M.H., Ahmadi, R.: Partial triangular entropy of uncertain random variables and its application. J. Ambient. Intell. Humaniz. Comput. **9**, 1455–1464 (2018)
3. Aranha, C., Iba, H.: Modelling cost into a genetic algorithm-based portfolio optimization system by seeding and objective sharing. In: Proceedings of IEEE Congress on Evolutionary Computation, Singapore, pp. 196–203 (2007)
4. Banerjee, M., Garg, V. (communicated): Solving structural and reliability optimization problems by investing efficient mutation strategies embedded in Sine-Cosine Algorithm. Int. J. Syst. Assur. Eng. Manag. (2022)
5. Bonami, P., Lejeune, M.A.: An exact solution approach for portfolio optimization problems under stochastic and integer constraints. Oper. Res. **57**, 650–670 (2009)
6. Brandtner, M., Wolfgang, K., Rischau, R.: Entropic risk measures and their comparative statics in portfolio selection: coherence vs convexity. Eur. J. Oper. Res. **264**, 707–716 (2018)
7. Califore, G.C.: Multi-period portfolio optimization with linear control policies. Automatica. **44**(10), 2463–2473 (2008)
8. Daun,Y.C.,: A Multi-objective Approach to portfolio optimization. **8**, 1–12 (2007)
9. DeMiguel, V., Garlappi, L., Nogales, F.J., Uppal, R.: A generalized approach to portfolio optimization: improving performance by constraining portfolio norms. Manag. Sci. **55**, 798–812 (2009)
10. Ertenlice, O., Kalayci, C.B.: A survey of swarm intelligence for portfolio optimization: algorithms and applications. Swarm Evol. Comput. **39**, 36–52 (2018)
11. Fiacco, A., Cormick, M.C.G.: Nonlinear programming: sequential unconstrained minimization techniques. Comput. J. **12**, 207 (1968)
12. Garg, V., Deep, K.: Portfolio optimization using Laplacian biogeography-based optimization. Springer. **56**, 1117–1141 (2019)
13. Garg, V., Deep, K.: Efficient mutation strategies embedded in Laplacian-biogeography-based optimization algorithm for unconstrained function minimization. Int. J. Appl. Swarm Intell. **7**, 12–44 (2016a)
14. Garg, V., Deep, K.: Performance of Laplacian biogeography-based optimization algorithm on CEC 2014 continuous optimization benchmarks and camera calibration problem. Swarm Evol. Comput. **27**, 132–144 (2016b)
15. Garg, V., Deep, K.: A state-of-the-art review of biogeography-based optimization. Adv. Intell. Syst. Comput. **336**, 533–549 (2015)
16. Garg, V., Deep, K.: Constrained Laplacian biogeography-based optimization. Int. J. Syst. Assur. Eng. Manag. **8**, 867–885 (2016)
17. Gupta, P., Mehlawat, M.K., Inuiguchi, M., Chandra, S.: Fuzzy portfolio optimization. In: Studies in Fuzzy Items & Soft Computings, vol. 316. Springer, Berlin/Heidelberg (2014)
18. Huang, X., Jiang, G., Gupta, P., Mehlawat, M.K.: A risk index model for uncertain portfolio selection with background risk. Comput. Oper. Res. **132**, 1–15 (2021)
19. Hu, Y., Liu, K., Zhang, X., Su, L., Ngai, E.W.T., Liu, M.: Application of evolutionary computation for rule discovery in stock algorithmic trading: a literature review. Appl. Soft Comput. **36**, 534–551 (2015)
20. Karmarkar, N.A.: New polynomial-time algorithm for linear programming. Combinatorica. **4**, 373–395 (1984)
21. Konno, H., Yamazaki, H.: Mean-absolute deviation portfolio optimization model and its application to Tokyo stock market. Manag. Sci. **37**, 519–531 (1991)
22. Konno, H., Suzuki, K.: A mean-variance-skewness portfolio optimization model. J. Oper. Res. Soc. Jpn. **38**, 173–187 (1995)

23. Levy, N.K., Markowitz, M.H.: Portfolio optimization with factors, scenarios, and realistic short positions. Oper. Res. **53**, 586–559 (2005)
24. Ma, X., Gao, Y., Wang, B.: Portfolio optimization with cardinality constraints band on hybrid differential evolution. Comput. Intell. Bioinfo. (2012)
25. Markowitz, H.: Portfolio selection. J. Financ. **7**, 77–91 (1952)
26. Mansini, R., Seperanza, M.: Heuristic algorithm for the portfolio selection problem with minimum transaction lots. Eur. J. Oper. Res. **114**, 219–233 (2003)
27. Mahawat, M.K., Gupta, P., Khan, A.Z.: Portfolio optimization using higher moments in an uncertain random environment. Inf. Sci. **567**, 348–374 (2021)
28. Mehralizade, R., Mohammad, A., Gildeh, B.S., Ahmadzade, H.: Uncertain random portfolio selection based on risk curve. Soft. Comput. **25**, 9789–9810 (2020)
29. Orito, Y., Hanada, Y., Shibata, S., Yamamoto, H.: A new population initialization approach based on bordered hessian for portfolio optimization problems. In: Proceedings of IEEE International Conference on Systems, Man, and Cybernetics (SMC), Manchester, England, pp. 1341–1346 (2013)
30. Pinar, M.: Robust scenario optimization based on downside-risk measure for multi-period portfolio selection. OR Spectr. **29**, 295–309 (2007)
31. Rubio, A., Bermúdez, J.D., Vercher, E.: Forecasting portfolio returns using weighted fuzzy time series methods. Int. J. Approx. Reason. **75**, 1–12 (2016)
32. Sasaki, M., Laamrani, A., Yamashiro, M., Aiehegn, C.: Portfolio optimization by fuzzy interactive genetic algorithm journal of advanced. Manag. Sci. **6**, 125–131 (2018)
33. Sharpe, W.F.: A linear programming algorithm for mutual funds portfolio selection. Manag. Sci. **13**, 499–510 (1967)
34. Shiang-Tai-Liu: Solving portfolio optimization problem based on extension principle. In: Conference on Industrial Engineering and Other Application of Applied Intelligent System, pp. 164–174 (2010)
35. Singh, A., Dharmaraja, S.: A portfolio optimisation model for credit risky bonds with Markov model credit rating dynamics. Int. J. Financial Mark. Deriv. **6**, 102–119 (2017)
36. Takriti, S., Ahmed, S.: On robust optimization of two-stage systems. Math. Program. Ser. **99**, 109–126 (2004)
37. Zhang, W.G., Liu, Y.J.: Credibility mean-variance model for multi-period portfolio selection problem with risk control. OR Spectr. **36**, 113–132 (2015)
38. Zhongfeng, Q.: Mean-variance model for portfolio optimization problem in the simultaneous presence of random and uncertain return. Eur. J. Oper. Res. **245**, 480–488 (2015)
39. Zhai, J., Bai, M., Hao, J.: Uncertain random mean–variance–skewness models for the portfolio optimization problem. J. Math. Program. Oper. Res., 2–24 (2021)