On One Approach to Distribution Electrical Networks' State Estimation Under Information Incompleteness Conditions

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Abstract In this chapter one approach to the problem's solving of distribution electrical networks' (ENs) state estimation under telemetered information incompleteness conditions is presented. This approach is focused on the use in the dispatching system and in addition to telemetered EN operational condition parameters involves also using of the results of seasonal control measurements processing and expert fuzzy estimates on the possible boundary values of the loads' active power of the so-called unobservable nodes. Such expert information is formalized in the form of corresponding trapezoidal membership functions. Problem solving results are expert-calculation estimates of EN operational condition parameters. An example of EN scheme's fragment with the rated voltage of 110 kV and corresponding results illustrating the proposed approach application are presented.

Keywords Distribution electrical network · Electrical network dispatching · Observability · State estimation problem · Information incompleteness · Fuzzy expert information

1 Introduction

Some problems of the electrical networks (ENs) dispatching are carried out under operational information incompleteness conditions. That information incompleteness takes place to a greater extent in distribution ENs (with nominal voltage of 110, 35, 20, 10, and 6 kV, such nominal voltage row for distribution ENs is used in Ukraine). Sometimes the factor of operational information incompleteness affects the quality of solving dispatching problems and, accordingly, the adequacy of the results obtained and the efficiency of EN dispatching as a whole. The state of the

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controlled system is determined by the state variables which form its state vector, the classical interpretation of that requires the equality of the number of coordinates of the system's complete mathematical model to the number of independent variables. That is why to determine the EN's state vector operational telemetry information (TI) is required (in general case a lot of TI consists of signals' groups which indicate equipment status, protections relay and EN operational condition parameters' values) which gives possibility to form a corresponding equations to solve the EN state estimation problem. Such system must be at least definite (but it is better to have an excess TI amount to form an overridden system of the equations and to use a TI redundancy to verify information authenticity). In the case of some overridden equations system (provided that there are no gross errors in TI or if such errors already have been eliminated) the problem of EN operational condition parameters estimation can be reduced, as is known, to the problem of so-called EN's operational condition balancing.

A distribution EN is traditionally unobservable (partially observable) due to the large number of connection points and corresponding high costs associated with EN's equipping with telemetry devices. In the case due to the TI deficiency the equations' system turns out to be underdetermined (having a lot of solutions), then without using other available information that compensates TI deficiency it is impossible to correctly estimate EN operational condition parameters. It must be noted that with TI deficiency various methods and means to estimate EN operational condition parameters can be used.

A lot of publications are devoted to the problem of power systems state estimation but, obviously, ones of the fundamental and first ones should be attributed [1–3], although the statement of the problem itself appeared a little earlier (1966) and it was associated with the dispatching creation in the French power system (Electricité de France). Even if we proceed from a lot of number of recent publications devoted to solving this problem, taking into account the different level of technical equipment of ENs with different measuring devices and techniques, for example, [4–6] we can conclude that despite the long period of its existence this problem remains relevant.

2 The Features of Problem's Setting, Formalizing, and Solving

First, let us consider general provisions of the proposed solving approach to the problem of EN state estimation using all available information, including expert one to make up for the TI deficiency. All consumers which are supplied by the lines extending from the buses of electrical substations can be taken into account in the form of *equivalent bus loads* (Fig. 1) or as *equivalent substation loads* (as the sum of such *equivalent bus loads*)—differently for different substations.

That form of loads' accounting also corresponds to the extent of generalization of information obtaining in the ENs during seasonal control measurements. Taking

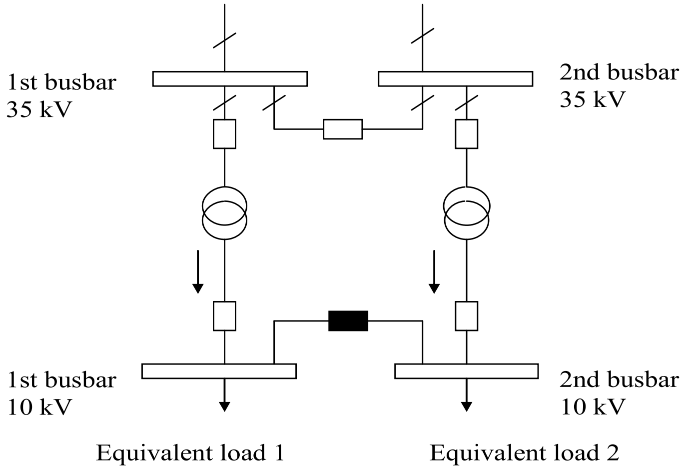

Fig. 1 Electrical substation 35/10 kV scheme's example

into account the information composition that is available in the EN load dispatch centers as a result of EN state estimation the following should be provided:

- a compliance of the *balance* part of EN operational condition parameters' measurements with the estimates (that is, the operational balance of the active power of the EN is taken as given, although, there is certainly some error in its determination);
- the estimates should be of a good quality, in particular, the differences of obtained estimates of EN operational condition parameters and corresponding measurements of the same parameters should be minimal;
- in the case of using the values of the EN operational condition parameters obtained as a result of the EN state estimation the restrictions in the form of EN steady-state condition equations must be satisfied

$$W = W(X, Y) = 0, \qquad (1)$$

where X and Y are the vectors of independent and dependent variables (mode parameters) respectively.

In addition, independent variables and some functions of them $F(X)$ to which $Y(X)$ also belong always have the restrictions due to circumstances of both a physical property (for example, the restrictions due to the thermal resistance limit of electrical lines' wires) and a technological nature (for example, the need to ensure regulated voltage levels on substation buses)

$$X_{\min} \le X \le X_{\max}; \qquad (2)$$

$$F(X) \le 0. \qquad (3)$$

It should be noted that obtained EN operational condition parameters' estimates can be within wider limits when compared with the limits set on the basis of technological requirements because in the real EN operational condition the latter are not always fulfilled if corresponding parameters are not controlled.

A mathematical formulation of the EN state estimation problem can be presented as a mathematical programming problem. At the same time the differences' function (Φ) of the EN operational condition parameters' values (V) those are obtaining during process of problem solving and corresponding measurements of the same parameters

$$\Phi(\Delta V) \to \min .$$

If we neglect the measurement errors' correlation then, as is known, the indicated function $\Phi(\Delta V)$ will be represented by the sum of m scalar functions (φ_i) of scalar variables

$$\Phi(\Delta V) = \sum_{i=1}^{i=m} \rho_i \varphi_i (\Delta v_i),$$

where $\varphi_i(\Delta v_i))$ is a scalar function of the deviation some EN operational condition parameter v_i from its measured value v_{iT}.

In this case a quadratic function is used, that is $\Phi(\Delta V)$ is a function of the least weighted squares method

$$\Phi(\Delta V) = \sum_{i=1}^{i=m} \rho_i \varphi_i (\Delta v_i) = \sum_{i=1}^{i=m} \rho_i (v_i - v_{iT})^2, \tag{4}$$

where ρ_i is the weight coefficient depending on measurement accuracy of corresponding parameters.

Since in the EN under consideration TI incompleteness presents, then, as already noted, the system of independent equations, which contain TI parameters and establish the dependencies between EN operational condition parameters will be underdetermined. With the appearance of the *information pseudo-completeness defect* and corresponding restrictions on the pseudo-TI use in the case of inability to use statistical historical data [7] TI incompleteness can only be filled by using information on the loads obtained on the basis of processing seasonal control measurements and information in the form of expert estimates of unobserved loads consumers (see *equivalent loads* in Fig. 1). Moreover, the experts can judge the values of these loads' active power at different time points of the daily power consumption schedule only in terms of possibility of one or another value with varying degrees of confidence.

Let us give a typical example of the form in which information about the load's active power of some electrical substation can be received from an expert,

"At the indicated time of day a load active power value of the electrical substation will most likely be in the interval from 3 to 4 MW."

It is obvious that such information is subjective and fuzzy (the information provided by a more experienced expert, as a rule, is also more certain). If for the above expert judgment we use the method of the membership function constructing based on expert estimates described, then the membership function of the fuzzy set of load's active power values of the electrical substation (μ_P) "The value of the load's active power will most likely be in the interval from 3 to 4 MW" we obtain based on the membership functions' construction for two point expert estimates (for 3 and 4 MW in this example) each of which is represented by the expressions:

$$\mu_3(P) = e^{-\alpha_3(3-P)^2}; \quad \mu_4(P) = e^{-\alpha_4(4-P)^2},$$

where $\mu_3(P)$ and $\mu_4(P)$ are the membership functions of fuzzy set of loads' active power values which approximately equal to 3 MW and 4 MW respectively, and $\alpha_3 = -4\ln 0.5 / \beta_3^2$, $\alpha_4 = -4\ln 0.5 / \beta_4^2$ where β_3, β_4 are the distances between transition points for $\mu_3(P)$ and $\mu_4(P)$ respectively, that is the points at which the functions $\mu_3(P)$ and $\mu_4(P)$ take the value 0.5.

Thus, in the case under consideration, the task of constructing $\mu_{P_{L2}}$ is reduced to β_3, α_3, β_4, α_4 determining and $\mu_3(P)$ and $\mu_4(P)$ constructing; their left and right halves together with the upper segment connecting their vertices form μ_{P_L} (it is shown in Fig. 2 by a solid line).

The membership function μ_P can be interpreted as "expert's confidence distribution that the load's active power will take an appropriate value in the indicated interval of possible values."

It should be noted that both normal and logarithmically normal distribution of expert estimations are possible (this allows use quartile characteristics for expert survey results' processing, because their calculation and use does not require knowledge of the law of expert estimates distribution [8]).

In the most practical cases trapezoidal membership functions are used. The form of such membership function is shown in Fig. 3 where P^b (bottom) and P^u (upper) are the boundaries of the interval (point estimates) to which in the considered example the values of 3 and 4 MW correspond.

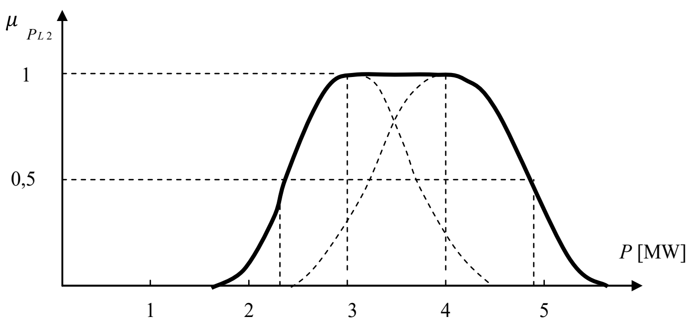

Fig. 2 $\mu_{P_{L2}}$-construction taking into account some expert estimation

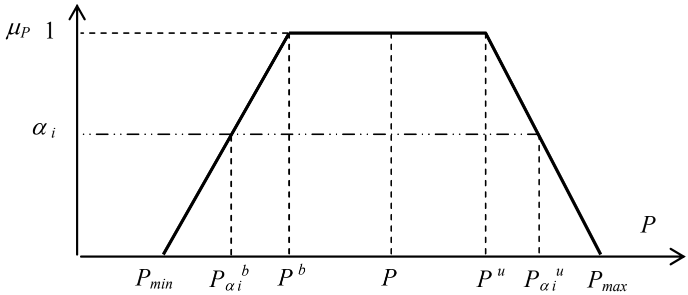

Fig. 3 Trapezoidal membership function

Such representation is acceptable if we take into account the nature of available information and a need to take into account clear boundary values (P_{min} and P_{max} in Fig. 3) beyond which the considered value cannot (physically or technologically) go. In support of such representation the circumstance can also be used that in the presence of information only about the range of variation of a random variable the hypothesis on the law of uniform probability distribution density is always accepted as the most cautious hypothesis.

Since the direct consideration of the *clear* boundary values of the loads' active power of unobserved EN nodes when using membership functions of such form as in Fig. 2 is problematic, it is quite natural to take into account indicated boundary values by constructing the combined membership function, the upper part of which (corresponding to the values $0.5 \leq \mu \leq 1$) has the same form as in Fig. 2, and the lower one consists on the linear sections allowing to take into account the boundary values of the load's active power.

Figure 4 shows the combined membership function of the fuzzy set of load's active power values corresponding to the interval estimate "most likely to be in the values' interval from 3 to 4 MW" where dotted lines 1 and 2 are drawn from points *c* and *d*, as well the dashed line 3 that is drawn from point *a* correspond to the examples of different clear restrictions.

However, to solve the state estimation problem of considered ENs it is necessary to take into account practical aspects of constructing and using membership functions of a fuzzy set of active power values of the loads. Even in the case of taken into account consumers' loads in the form of equivalent loads their number can be significant. It must be noted that combined membership functions construction (such function is shown in Fig. 4) is too laborious process. In addition, unjustified computational costs also arise when calculating the boundary interval values of the active power of such loads ($P_{\alpha i}^{b}$ and $P_{\alpha i}^{u}$ in Fig. 4), corresponding to certain given values of the membership function (α_i in Fig. 4), which will be discussed below. The construction of trapezoidal membership functions analytically represented by expressions (5) is a very real task for the practical implementation.

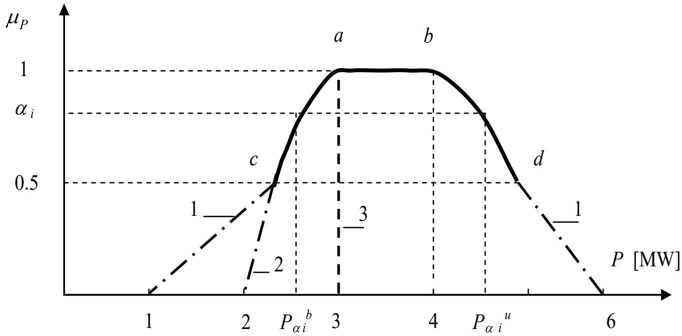

Fig. 4 Combined membership function

$$\mu(P) = \begin{cases} \frac{P-P_{\min}}{P^b-P_{\min}} & \text{if } P_{\min} < P < P^b; \\ 1 & \text{if } P^b \le P \le P^u; \\ \frac{P_{\max}-P}{P_{\max}-P^u} & \text{if } P^u < P < P_{\max}; \\ 0 & \text{if } P \le P_{\min} \text{ or } P \ge P_{\max}. \end{cases} \quad (5)$$

In addition, boundary interval values of the loads' active powers ($P^b_{\alpha i}$ and $P^u_{\alpha i}$ in Fig. 3) corresponding to certain values of membership functions (α_i in Fig. 3) are easily determined by using the first and third equalities-conditions (5): first, the values of P are determined and then using the obtained values of P (respectively, we denote P_1 and P_3) $P^b_{\alpha i} = P_1$ and $P^u_{\alpha i} = P_3$ are taken.

Let us now consider the issue of information using on the loads determined on the basis of processing data obtained during seasonal control measurements, when power system operational condition parameters are measured every hour on the appointed day by all available means. In the absence of other (more accurate) information about the loads of unobservable nodes (except the information obtained from the data of seasonal control measurements) as the most cautious hypothesis about the load's power graphs in such nodes may be accepted the hypothesis about maintaining their similarity to the corresponding graphs obtained from seasonal control measurements. Then in accordance with this assumption the current value of the load's active power of the unobserved i-th node is determined as follows:

$$P_i = \left(P_{\sum} - P_{c\sum} - \Delta P\right) P_i^{sm} / P_{\sum}^{sm}, \quad (6)$$

where $P_{\sum}$ is the current (determined based on telemetry data) total value of active power entering to the NE; $P_{c\sum}$ is the current total value of active power in the observed load nodes (these include both nodes, the load power value of which can be determined by telemetering using, and the nodes the load graphs of which are a priori known); ΔP is the average (for corresponding EN operational conditions) value of total active power losses in the EN (more precisely the value ΔP can be determined using polynomial dependence ΔP from $P_{\sum}$ and several observed

EN operational condition parameters (such polynomial dependence can be obtained based on a series of EN steady state operational conditions calculations); P_i^{sm} is the value of the load active power in the *i*-th unobserved node at the hour of the day of the control measurements, corresponding to the current hour (in the EN load dispatch centers there are corresponding load graphs based on the data of the day of control measurements); $P_{\sum}^{sm}$ is the total value of the loads' active power of unobserved nodes at the hour of the day of control measurements corresponding to the current hour.

Such simplified approach to distribution the part of total active power entering to the EN among unobserved nodes, of course, does not allow to take into account many factors' influence and is very approximate, but it allows to determine initial approximations of active power of unobserved nodes (these approximations are subject to clarification about which will be discussed below), because more accurate methods and meaningful mathematical loads' models use (decomposition into characteristic components, decomposition into harmonic components etc.) requires information which is very problematic to obtain under existing conditions. It should be noted that initial approximations of the loads' active power values of unobserved nodes obtained in this way in most cases fall into trapezoidal membership function cores $\left[P^b, P^u\right]$; trapezoidal membership functions previously are constructed for the corresponding time points. However, the cases are not excluded when the values of the loads' active powers of some unobserved nodes are outside of corresponding intervals $\left[P^b, P^u\right]$ which is not consistent with expert estimates. For preliminary coordination of obtained values with expert estimates, the procedures of *pulling up* the loads' active powers of unobserved nodes to the expert estimates and *compensating* the resulting power imbalances are performed. The implementation of the *pull-up* and *compensation* procedures is an attempt to find a certain compromise between the values of the loads' active power obtained with a rough distribution in accordance with the expression (6) and the experts' knowledge about these loads. Moreover, one can try to perform the *pull-up* either in full (until the membership degree of the value of the load's active power of each node obtained as a result of the *pull-up* becomes equal to one) or limit oneself to an attempt to *pull up* active powers to the boundaries of corresponding intervals (membership function cores) $\left[P^b, P^u\right]$.

In the first case a clear preference is given to expert estimates and there is a greater difference between obtained values and the values determined using expression (6). The second approach which provides for *pulling-up* expert estimates (values) to the boundaries of the intervals (membership function cores) $\left[P^b, P^u\right]$) seems to be more balanced, although it is also aimed at achieving a compromise between expert estimates and the values of active power which are determined using expression (6). Each of the approaches is based on the same type of procedures. To clarify the features of these procedures, let consider the second approach using the same Fig. 3.

Let for a certain unobservable *k*-th node using (6) the power value was obtained which turned out to be equal to $P_{\alpha i}^b$ or $P_{\alpha i}^u$ (see Fig. 3), that is, it went beyond the boundaries of the interval $\left[P^b, P^u\right]$. To *pull- up* the obtained value of the *k*-th node's active power to the expert estimate it is necessary to bring it to the boundary of the interval $\left[P^b, P^u\right]$.

There are two possible options to do this:

- if the power value turned out to be less than P^b then obtained active power value should be changed by the value $\Delta P_{Uk} = P^b - P^b_{\alpha i}$,
- if the power value turned out to be more than P^u then obtained active power value should be changed by the value $\Delta P_{Uk} = P^u - P^u_{\alpha i}$.

In the general case there can be n_L such nodes, and the total value of the active power imbalance of unobserved nodes ($\Delta P_{U\sum}$) which will arise as a result of the considered *pulling up* the power of all n_L nodes is calculated by algebraic summation of ΔP_{Uk} over all such *k*-th nodes.

To eliminate the emerging imbalance the value of $\Delta P_{U\sum}$ must be compensated by changing the power of remaining unobserved nodes by the same value $\Delta P_{U\sum}$ but of the opposite sign. For this purpose the nodes are used the values of the active power of the loads of which obtained according to the expression (6) turn out to be in their intervals $\left[P^b, P^u\right]$ as for example, P_i in Fig. 3.

Some remoteness of the active power value of such *j*-th node from the boundaries of the specified interval (a kind of adjustment range) in the direction of increasing and decreasing is $\Delta P_{Uj} = P^u - P_j$ and $\Delta P_{Dj} = P_j - P^b$ respectively.

The number of such *j*-th nodes is equal to the total number of unobserved nodes minus n_L, therefore their total *compensation* powers (in each of the $\Delta P_{U\Sigma}$ and $\Delta P_{D\Sigma}$ directions) are determined by the corresponding summation of the values ΔP_{Uj} and ΔP_{Dj} for all such *j*-th nodes. If it turns out that the value of $\Delta P_{U\Sigma}$ can be compensated by $\Delta P_{U\Sigma}$ or $\Delta P_{U\Sigma}$ (depending on $\Delta P_{U\Sigma}$ sign), then a full *compensation* procedure is performed, otherwise, it is performed only to the extent that corresponds to available *compensation* possibilities ($\Delta P_{U\Sigma}$ or $\Delta P_{D\Sigma}$ choice depends on $\Delta P_{U\Sigma}$ sign).

It should be noted, if the *compensation* powers are sufficient to perform full *compensation*, then the sequence in which the selection and *pull-up* of the power of unobserved nodes is performed does not matter, otherwise, the *pull-up* should be performed in such way as to increase the degree certainty of active power values of unobserved nodes. In this case different approaches are possible both to the implementation of the *compensation* procedure and to the implementation of the *pull-up* procedure.

When the *compensation* procedure is carried out at least two main approaches to changing the power of *compensation* nodes (the nodes by certain changes of load powers of which the indicated *compensation* is carried out) are practically applicable. At the first of two indicated approaches the weighted distribution of the total value of the *compensation* power is carry out (depending on the power and the available adjustment range of each *compensation* node). At the second approach an attempt to ensure (as a result of the implementation of the *compensation* procedure) a shift of power values of *compensation* nodes to the centers of their power intervals $\left[P^b, P^u\right]$ is carry out.

The first approach is quite simple and universal while the second one is limited in use and explicitly applicable when the *compensation* powers significantly exceed $\Delta P_{U\Sigma}$ (so much so that they can provide *compensation* $\Delta P_{U\Sigma}$ limiting themselves practically only to those nodes whose power values will be shifted to the centers of their intervals $\left[P^b, P^u\right]$ and then will not go far from them).

The first of these approaches is focused on the maximum preservation of the proportions of the power distribution obtained in accordance with (6) while the second one leads to more their change because expert estimates are preferred (when implementing the second approach the first one can be used as its component-procedure). Note that as a result of performing the *compensation* procedure in the first approach the values of *compensated* powers also do not leave their intervals $\left[P^b, P^u\right]$, therefore, given its greater universality and weightedness this approach is used as the main one in determining initial approximations of the load powers of unobserved nodes.

Let us consider **main stages of the implementation of the *compensation* procedure** in accordance with this approach.

STAGE 1. Determining the amount of *compensated* power $\Delta P_{C\Sigma}$. It is determined by the value $\Delta P_{U\Sigma}$ (with full *compensation*), or (if the *compensation* possibilities are limited) by one of the values $\Delta P_{U\Sigma}$ or $\Delta P_{D\Sigma}$ (depending on the sign of $\Delta P_{U\Sigma}$).

STAGE 2. The coefficients (K_{Pj}) of active power ($\Delta P_{C\Sigma}$) distribution determination between *compensation* (*j*-th) nodes:

$$K_{P_j} = \beta_j \gamma_j \Big/ \sum \beta_j \gamma_j,$$

where $\beta_j = P_j \big/ \sum P_j$; P_j is the value of the *j*-th *compensation* node's active power obtained in accordance with expression (6); $\gamma_j = \Delta P_{adjj} \big/ \Delta P_{C\sum}$; ΔP_{adjj} is an adjustment range of the *j*-th *compensation* node's active power (in the direction determined by the sign $\Delta P_{U\Sigma}$).

STAGE 3. The power changing of each *j*-th *compensation* node by the value $\Delta P_{C\sum} K_{P_j}$.

Let us consider **the features of the *pull-up* procedure implementation** in accordance with this approach.

Two cases are of our interest:

1. when the value $\Delta P_{U\Sigma}$ cannot be *compensated* completely;
2. when compensatory capabilities of the nodes are generally absent.

In implementation terms both of above cases come down to the same procedure. The purpose of *pulling up* is to achieve the maximum possible value (the same for all) of membership functions of fuzzy sets of active power values of the loads of all or part of unobserved nodes of the EN (in the latter case, this applies to those nodes where the values of the indicated membership functions are less than a certain value) without decreasing original values of membership functions. To achieve this goal the condition of partial *self-compensation* must be met: active powers of some nodes can be *pulled up* in the direction of their increase, and others—in the direction of decrease.

For further explanations we will use Fig. 5 that shows membership functions of fuzzy sets of the values loads' active power of the nodes *A*, *B*, and *C*.

Let after the distribution according to (6) the values of the loads' active power of indicated nodes *A*, *B*, *C* amounted to 3 MW, 3 MW, and 16 MW respectively (all of

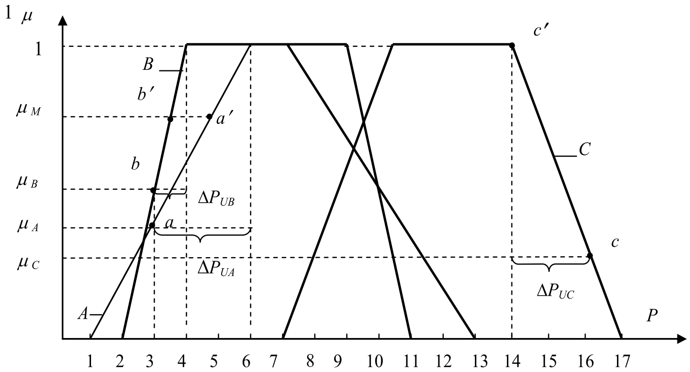

Fig. 5 Membership functions of fuzzy sets of loads' active power of the nodes *A*, *B*, and *C*

them are outside their intervals $\left[P^b, P^u\right]$ cores of membership functions and they correspond to the degrees of the membership μ_A, μ_B, μ_C.

To *pull up* the active powers of these loads to the boundaries of their intervals $\left[P^b, P^u\right]$ it is necessary to change their values to ΔP_{UA}, ΔP_{UB}, and ΔP_{UC} respectively, which will require 2 MW of *compensatory* unloading power (ΔP_C), because at *pull-up* 2 MW *self-compensate* (as a result of algebraic summation ΔP_{UA}, ΔP_{UB}, and ΔP_{UC}).

Let us consider the case when there is no possibility of *compensation*, that is the possibility of *self-compensation* must be determined (further it will become clear that the procedure considered below is also applicable to the case of partial *compensation*). Based on the condition for ensuring *compromise proximity* of the loads' active power values of unobserved nodes obtained using expression (6), and expert estimates for which the membership functions are presented by expressions of the form (5) and constructed, the loads' active power values of EN nodes and the degree belonging of these values to corresponding fuzzy sets should be determined.

It is obvious that if we limit ourselves to *pulling-up*-decreasing the load power of the node *C* only to the corresponding core's boundary of the trapezoidal membership function equal to 14 MW (see Fig. 5—due to such *pulling-up*-decreasing the new load's active power value in the node *C* will correspond to the point *c*' instead of the point *c* on the lateral side of the trapezoidal membership function) then load active powers in the nodes *A* and *B* can't be fully *pulled-up*-increased to the cores' boundaries of their trapezoidal membership functions due to the shortage of 2 MW of the *compensation* power.

To determine new values of load active powers of the nodes *A* and *B* and corresponding them membership degrees (equal to each other) to fuzzy sets of loads' active power values we will compose the balance equation of the remaining *unpulled-up* active power of the loads and the equality condition of the membership functions' values for the loads power of these nodes.

Note that the equation of the indicated balance is writing already taking into account *self-compensation.* Using (5) and introducing an additional lower subscript indicating belonging to node A or B we write for the example under consideration (see Fig. 5) the equation of the indicated balance and the equality condition of the membership functions' values

$$\left(P_A^b - P_A\right) + \left(P_B^b - P_B\right) = P_C; \tag{7}$$

$$\left(P_A - P_{A\,\min}\right) \big/ \left(P_A^b - P_{A\,\min}\right) = \left(P_B - P_{B\,\min}\right) \big/ \left(P_B^b - P_{B\,\min}\right) \tag{8}$$

Substituting the specific values presented in Fig. 5 we get

$$(6 - P_A) + (4 - P_B) = 2;$$

$$(P_A - 1)/(6 - 1) = (P_B - 2)/(4 - 2),$$

whence we find $P_A = 4.571\,\text{MW}$ and $P_B = 3.429\,\text{MW}$.

New values of the loads' active powers of the nodes A and B will be characterized by new values of the membership functions corresponding to points *a'* and *b'* on the trapezoids of the membership functions (see Fig. 5), which, taking into account (8), are equal to each other and equal to $\mu_M = 0.7143$.

Note that in the general case the equalities number of the form (8) is one less than the number of *pulled-up* load powers of the nodes.

Obviously, when implementing the *pull-up* procedure in the case of partial *compensation*, as was mentioned above, it is sufficient to take into account the corresponding value of the *compensation* power in the expression (7). The same *pull-up* procedure is applicable if not all values of active powers of the loads are subject to *pull-up*, but only which are characterized by the values of membership functions less than a certain value. In this case the expressions (7) and (8) are written for the conditions of *pulling up* only the indicated powers (rest powers are not subject to *pulling-up*). It should be noted that in the considered example it is possible to *pull-up* the powers of nodes A and B to the boundaries of their intervals (6 and 4 MW, respectively), but this purpose will require an additional unloading of 2 MW in the node C which already indicates a greater leaving from the values of loads' powers obtained according to the expression (6), towards expert estimates.

After initial values (approximations) determination of the loads' active power of unobserved nodes is completed, the initial values of the reactive power are determined. If there is relevant information that makes it possible to determine initial approximations of the reactive power of the loads of individual nodes, then it is used, otherwise indicated values are determined based on the equality assumption the tangents of the load power angles in each i-th node at the current time and at the corresponding hour of seasonal control measurements' day

$$Q_i = P_i \cdot Q_i^{sm} \big/ P_i^{sm}, \tag{9}$$

where P_i already takes into account the results of the implementation of the considered preliminary *pull-up* and *compensation* procedures.

At the next stage the minimization problem of the function $\Phi(\Delta V)$ (4) is carried out taking into account the restrictions (1)–(3) while fixing power balance components defined as the measurements of power flows through external electrical connections (set by the power constancy in corresponding boundary nodes).

A refinement of the load power values of unobserved nodes is directly related to the minimization of the function $\Phi(\Delta V)$ (4). Obviously, due to the action of various factors [7], in the process of $\Phi(\Delta V)$ minimizing only a certain (non-zero) minimum value can be achieved, depending on the restrictions on changing the load powers in the nodes: the absence of these restrictions with unreliable measurements can lead to unrealistic power estimates in individual nodes, and the severe restrictions presence of can hinder the $\Phi(\Delta V)$ minimization. If more accurate information is not available, then expert-defined minimum and maximum power values in particular nodes are accepted. If we use Fig. 3, then P^b and P^u or $P_{\min}$ and $P_{\max}$ can be taken as these restrictions.

Obviously, in the first case the restrictions may turn out to be too strict, while in the second case the opposite is true: a too wide range of possible power values is allowed. These restrictions are fuzzy in fact. The considered problem of the function $\Phi(\Delta V)$ (4) minimizing provided that some of the constraints are fuzzy is a problem of mathematical programming with fuzzy constraints. Therefore a slightly different approach to determining the restrictions on changing the loads power seems to be more reasonable.

In this case it is convenient to use the concept of a level set α of a fuzzy set [9]. Formally, the level set α of the fuzzy set P_L in P will be the set $(P_{L\alpha})$ composed of elements $p \in P$ whose membership degrees in the fuzzy set P_L is not less than the number α, i.e.

$$P_{L\alpha} = \left\{ p | p \in P,\ \mu_{P_L}(p) \geq \alpha \right\}.$$

Decomposition of the P_L fuzzy set by its sets of level α makes it possible to move although to subjective (expert) but more differentiated and clear restrictions if we introduce the *degree of subjective tolerance* to do it possible to change the load power (here and below for brevity we will use the *degree of subjective tolerance*) defined as $(1 - \alpha)$. The greater is the *degree of subjective tolerance*, the greater is the interval of possible values of the load power of the unobserved node (zero value of the *degree of subjective tolerance* corresponds to $\alpha = 1$).

For example, the level α_i of the fuzzy set (see Fig. 4) corresponds to the *degree of subjective tolerance* equal to $(1 - \alpha_i)$ while the range of possible values of the active power of the node is limited by the values $P^b_{\alpha i}$ and $P^u_{\alpha i}$ (with zero *degree of subjective tolerance* the specified interval is limited by the values of P^b and P^u). It should be emphasized that the *degree of subjective tolerance* does not have a probabilistic interpretation and reflects only the expert (subjective) knowledge about the load power changing possibility.

This approach to the restrictions determining imposed on the possible values of nodes' load powers allows to minimize the function $\Phi(\Delta V)$ (4) with different *degrees of subjective tolerance.*

If in this minimization process it turns out that established power limitations in a number of the nodes prevent to decrease in individual terms of the function $\Phi(\Delta V)$ then this is due either to measurements unreliability of the corresponding EN operational condition parameters which are used in (4) or with expert's knowledge discrepancy (representations) about real possibilities of the load power changes in above nodes.

If the second case occurs then by increasing the *degree of subjective tolerance* for indicated nodes (by increasing respectively and the interval of the possible change in the load power) and continuing the process of $\Phi(\Delta V)$ minimizing it is possible to achieve a decrease in the values of the corresponding terms $\varphi_i(\Delta v_i)$ of the function $\Phi(\Delta V)$.

Load power values of indicated unobserved nodes obtained as a result of the EN state estimation do it possible also to correct subjective expert ideas on possible values of these nodes load power.

3 Illustrative Example

As an illustration of the above approach use to replenishment in the missing information and obtaining *expert-calculation* estimates of the EN operational condition parameters the EN fragment with the rated voltage of 110 kV was used (see Fig. 6), and EN lines' parameters are given in Table 1. Let us first give a brief description

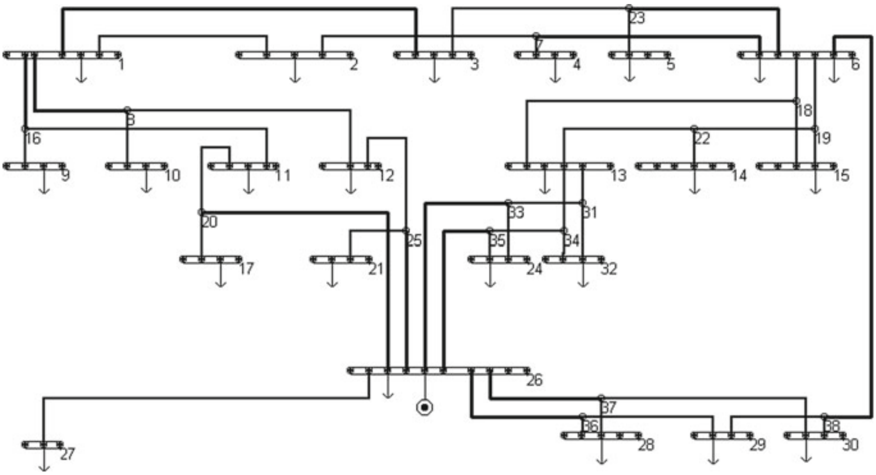

Fig. 6 EN fragment (nodes connection diagram)

Table 1 EN lines parameters' values

The node numbers to which line's poles are attached		Line's resistance, Ohm	Line's reactance, Ohm	Line's capacitive conductivity, Siemens × 10^{-6}
Pole 1	Pole 2			
1	2	1.93	6.52	48.1
1	3	1.93	6.52	89.2
1	8	2.28	7.89	62.2
1	16	2.28	7.89	68.5
2	7	1.62	2.78	37.1
3	23	2.02	3.46	81.3
5	23	0.42	0.73	39.3
6	19	0.065	0.17	14.2
6	18	0.3	0.67	19.2
7	4	0.75	1.28	62.4
7	6	1.65	3.05	42.7
8	10	0.65	0.99	35.2
8	12	0.14	0.49	70.6
11	20	0.69	1.23	79.4
12	25	0.74	1.14	81.7
13	31	0.16	0.28	6.20
13	34	0.16	0.28	5.35
16	9	0.65	0.99	62.1
16	11	0.14	0.49	18.1
18	13	2.07	3.5	4.69
18	15	0.47	0.81	5.49
19	15	0.22	0.38	79.2
19	22	0.26	0.58	43.4
20	17	0.75	1.28	68.3
20	26	1.81	4.63	60.4
22	13	1.39	2.39	59.9
22	14	0.43	2.42	7.21
23	6	0.75	1.28	0.749
25	21	0.75	1.28	86.6
25	26	1.81	4.63	60.7
26	27	1.48	2	8.35
26	36	1.02	2.6	42.3
26	37	1.02	2.6	22.9
29	38	0.05	0.09	88.7

(continued)

Table 1 (continued)

The node numbers to which line's poles are attached		Line's resistance, Ohm	Line's reactance, Ohm	Line's capacitive conductivity, Siemens $\times 10^{-6}$
Pole 1	Pole 2			
31	32	0.023	0.043	96.3
31	33	0.023	0.043	27.4
33	24	0.72	1.24	22.7
33	26	0.56	1.9	30.6
34	32	0.023	0.043	94.4
34	35	0.69	1.19	84.5
35	24	0.72	1.24	46.3
35	26	0.56	1.9	94.7
36	28	0.32	0.56	95.3
36	29	0.82	1.41	75.7
37	28	0.32	0.56	31.0
37	30	0.82	1.41	69.3
38	6	4.08	8.65	74.2
38	30	0.1	0.1	42.6

of this EN fragment in terms of available information on measured EN operational condition parameters which are used in EN state estimation.

The node (busbar) number 26 is the node of the input to the EN of the generated power. On this busbar a constant voltage value is ensured (according to telemetering information this value is 115 kV). The power flows telemetering from this node is characterized as a higher reliability compared to other telemetering information.

It is known that at the busbar with number 1 the load is absent and the loads at the busbars with numbers 2 and 13 are the power flows through observed external electrical connections which are taken into account in this way as the loads (these power flows along with power flows from tires 26 are used to determine the balance of power supplied to the EN).

When the problem is solving the values of power flows through external EN connections are taking fixed and they are represented by constant power load model taking account corresponding signs (P = const, Q = const) with telemetered data use ($P_2 = 4.02$; $Q_2 = 1.96$; $P_{13} = -9.0$; $Q_{13} = 0.0$).

When initial approximations of unobserved nodes (busbars) loads' power were determining in the nodes with 5, 24, and 32 numbers the procedures of *pull-up* and *compensation* considered above were performed. Before minimizing the objective function $\Phi(\Delta V)$ (4) its value was equal to 401.1663 and after minimizing became equal to 34.8496.

State estimation results and measured values of active and reactive power flows (respectively in MW and MVAr) through EN lines are given in Table 2 where in the columns marked P_{T1}, Q_{T1}, P_{T2}, and Q_{T2} telemetry data measured at the side of the first (index T1) and second (index T2) ends of the lines (such ends we'll call poles) are presented, and in the columns marked P_1, Q_1, P_2, and Q_2 the values of active and reactive power flows obtained at the sides of respective poles as the EN state estimation results are given.

Shaded rows in Table 2 contain telemetering values of EN operational condition parameters which should be fixed (remain unchanged) during the EN state estimation procedure execution (the results of the EN state estimation should practically coincide with them). This requirement is satisfied with satisfactory accuracy when setting values of corresponding weight coefficients ρ_i in the expression (4) will be an order of magnitude larger in comparison with weight coefficients relating to others telemetered EN operational condition parameters.

Table 3 contains for EN nodes estimated EN operational condition parameters. Shaded rows indicate the values which were fixed (remained unchanged) during state estimation procedure's execution. According to obtained estimates the total active

Table 2 State estimation results and TI values of active and reactive power flows

The node numbers to which line's poles are attached		P_1	Q_1	P_{T1}	Q_{T1}	P_2	Q_2	P_{T2}	Q_{T2}
Pole 1	Pole 2								
1	2	−1.84	0.23	–	–	1.84	−0.24	–	–
1	3	−4.37	−1.48	–	–	4.36	1.47	–	–
1	8	3.67	1.20	4.07	1.59	−3.67	−1.21	–	–
1	16	2.55	0.05	2.79	1.37	−2.55	−0.05	–	–
2	7	2.176	2.20	–	–	−2.18	−2.20	–	–
3	23	6.78	4.21	6.75	4.01	−6.79	−4.22	–	–
5	23	1.87	0.92	–	–	−1.87	−0.92	–	–
6	18	10.25	2.75	8.99	3.61	−10.25	−2.76	–	–
6	19	8.94	8.24	8.03	8.01	−8.94	−8.25	–	–
7	4	−1.64	−0.89	–	–	1.64	0.89	1.59	0.78
7	6	3.82	3.09	–	–	−3.82	−3.09	–	–
8	10	−4.37	−2.83	–	–	4.37	2.83	4.5	–
8	12	8.03	4.04	–	–	−8.04	−4.04	−7.91	−4.04
11	20	12.28	5.23	12.8	5.37	−12.29	−5.24	–	–
12	25	14.24	6.22	14.57	–	−14.26	−6.24	–	–
13	34	14.24	2.22	–	–	−14.24	−2.22	–	–
13	31	16.05	2.61	–	–	−16.05	−2.61	–	–

(continued)

Table 2 (continued)

The node numbers to which line's poles are attached		P_1	Q_1	P_{T1}	Q_{T1}	P_2	Q_2	P_{T2}	Q_{T2}
Pole 1	Pole 2								
16	9	−5.51	−4.59	–	–	5.51	4.59	5.5	–
16	11	8.06	4.64	–	–	−8.06	−4.65	−8.08	−4.7
18	13	15.77	2.45	–	–	−15.81	−2.51	–	–
18	15	−5.52	0.31	–	–	5.51	−0.32	4.55	0
19	15	1.56	0.68	–	–	−1.56	−0.69	−2.56	−0.68
19	22	7.38	7.56	–	–	−7.38	−7.56	–	–
20	17	−5.16	−3.94	–	–	5.15	3.93	5.62	–
20	26	17.45	9.18	–	–	−17.50	−9.32	−17.5	−9.32
22	13	23.42	2.21	–	–	−23.48	−2.31	–	–
22	14	−16.04	5.35	–	–	16.03	−5.41	15.1	–
23	6	8.66	5.14	–	–	−8.67	−5.15	−8.5	−5.03
25	21	−0.97	−0.78	–	–	0.97	0.78	1.28	0.79
25	26	15.23	7.02	–	–	−15.27	−7.12	−15.27	−7.12
26	27	−11.70	−10.50	−11.7	−10.5	11.67	10.46	–	–
26	36	−20.51	−8.45	−20.51	−9.07	20.47	8.35	–	–
26	37	−20.53	−8.42	−20.53	−7.8	20.49	8.33	–	–
29	38	−3.43	1.50	−4.4	−1.9	3.43	−1.50	–	–
31	33	23.51	4.66	–	–	−23.51	−4.66	–	–
31	32	−7.46	−2.05	–	–	7.46	2.05	–	–
33	24	−0.24	−1.61	–	–	0.24	1.61	–	–
33	26	23.75	6.27	–	–	−23.78	−6.36	−23.7	−6.67
34	32	4.46	0.85	–	–	−4.46	−0.85	–	–
34	35	9.79	1.38	–	–	−9.79	−1.39	–	–
35	24	−8.41	−2.72	–	–	8.41	2.71	–	–
35	26	18.20	4.11	–	–	−18.22	−4.16	−18.3	−3.74
36	28	−5.16	−2.12	–	–	5.156	2.12	4.35	–
36	29	−15.31	−6.23	–	–	15.29	6.20	14.5	6.25
37	28	−5.07	−2.24	–	–	5.07	2.24	5.0	2.1
37	30	−15.43	−6.09	–	–	15.41	6.06	–	–
38	6	−4.40	0.73	–	–	4.39	−0.74	4.05	–
38	30	0.96	0.77	–	–	−0.96	−0.77	−0.5	0.92

and reactive power input into the EN is 127.5034 MW and 54.3208 MVAr respectively, and the consumed active and reactive power is 127.0343 MW and 53.2609 MVAr respectively.

From preliminary analysis of obtained results it can be concluded that overall "picture" is relatively satisfactory but the question regarding the differences between individual calculated values and corresponding telemetering values of reactive power flows needs some explanation. For example (see Table 2), such reactive power values are shown from first poles side for the lines (*i-j*): 1-16, 6-18, 29-38 and from the second pole side for the line 38-30. One of the reasons for these differences may be telemetering errors because in the EN operation practice the dispatchers, as a rule, pay less attention to telemetering values of reactive power flows then others EN operational condition parameters.

Table 3 State estimation results of EN operational condition parameters (shaded rows indicate the values which remained unchanged during state estimation procedure's execution)

Node's number	P_L, MW	Q_L, MVAr	U, kV	φ_u, degrees	I, A
1	0.0	0.0	114.14	−0.432	0.0
2	4.0200	1.9600	114.12	−0.487	22.6
3	11.1475	5.6778	113.98	−0.545	63.4
4	1.6390	0.8911	114.18	−0.483	9.4
5	1.8700	0.9200	114.21	−0.484	10.5
6	11.0918	2.0060	114.34	−0.448	56.9
9	5.5110	4.5878	114.12	−0.355	36.3
10	4.3657	2.8289	114.24	−0.328	26.3
11	4.2187	0.5818	114.22	−0.330	21.5
12	6.2088	2.1807	114.32	−0.303	33.2
13	−9.0	0.0	114.74	−0.202	45.3
14	16.0319	−5.4065	114.47	−0.613	85.3
15	3.9536	−1.0019	114.36	−0.441	20.6
17	5.1540	3.9328	114.27	−0.296	32.8
21	0.9727	0.7755	114.46	−0.255	6.3
24	8.6500	4.3200	114.76	−0.177	48.6
27	11.6723	10.463	114.67	−0.034	78.9
28	10.2226	4.3583	114.60	−0.204	56.0
29	11.8595	7.6947	114.44	−0.266	71.3
30	14.4451	5.2913	114.44	−0.268	77.6
32	3.0000	1.2000	114.77	−0.186	16.3

4 Conclusions

Distribution ENs traditionally are unobservable (partially observable only). That is why in order to ensure EN expert-computational observability and to obtain corresponding EN state estimation results this approach is focused on the use, additionally to telemetered EN operational condition parameters, also the results of seasonal control measurements processing and experts' estimates of active power loads of unobservable EN's nodes. Performed calculations' results including the given illustrative example of the EN's fragment with the rated voltage of 110 kV confirm operability and suitability of the proposed approach for practical application in the dispatching system of distribution ENs' load dispatch centers.

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