An Analytical Model of Calculating the Flexural Strength of Encased SRC Composite T-beams with Full Interaction of Components

Tatiana Galinska, Dmytro Ovsii, Mukhlis Hajiyev, and Oleksandra Ovsii

Abstract The analytical model of calculation of flexural strength of composite steel-reinforced concrete (SRC) enclosed T-beams is proposed in the work. This model makes it possible to calculate the strength of the calculated sections of enclosed SRC T-beams taking into account their stress–strain state at the time of maximum bearing capacity. Comparison of experimental test data of enclosed steel-reinforced concrete T-beams and elements, which were performed by scientists of the world, with theoretical calculations of the proposed model confirmed the possibility of its use in the practice of their design. The following analytical dependencies can be used to solve two practical problems: checking the flexural strength and designing the optimal cross sections of span steel-reinforced concrete (SRC) elements in concrete casing in the form of a T-section.

Keywords Steel-reinforced concrete · Composite · T-beams · Flexural strength

1 Introduction

Composite elements of concrete and steel reinforcement can be divided into three types: a combination of reinforced concrete slabs and steel profiles; composite elements made of steel profile in concrete encasement, also known as reinforced concrete elements with rigid reinforcement; prefabricated monolithic composite elements made of external precast steel or reinforced concrete formwork filled with concrete. Composite steel reinforced concrete element allows to effectively taking advantage of the characteristics of structural steel and reinforced concrete and provides a design solution for elements that require high strength and ductility.

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Calculation of flexural strength of steel-reinforced concrete elements used in the design standards AISC 360–10 [1], Eurocode 4 [2], KBC 2014 [3], JGJ 138–2016 [4], JSCE 2009 [5] and DBN B.2.6 -160: 2010 [6], is based on four basic analytical models with different prerequisites that allow them to be designed by civil engineers.

According to Chen in [7], in the building codes of AISC [1] consistently from simple to complex set out for the convenience of engineers at designing, four analytical methods for determining the strength of steel-reinforced concrete elements in bending. Simple analytical methods are based on conservative calculations, using which in some cases such a design can lead to excessive reserves of strength and overconsumption of components of steel-reinforced concrete elements. Conversely, complex methods that require a lot of effort in calculations are expected to be more accurate than simple methods, which will lead to more cost-effective design. Therefore, information on the accuracy of each method is crucial for engineers to choose between conservatism and austerity.

As noted Chen in his work [7], AISC 360–10 [1] proposes the following analytical methods for determining the bending strength of reinforced concrete elements in a shell:

- the superposition of elastic stress;
- the plastic stress distribution on the steel section alone;
- the plastic stress distribution on the composite section (PSD on the composite section);
- strain compatibility method.

As Chen in [7], of the four analytical methods, the last two methods are allowed, according to the AISC specification [1], only if the structure provides anchor elements to prevent displacement, ensuring the joint interaction of the components in enclosed SRC T-beams at the operational and boundary stages.

Today, scientists continue to improve analytical models for calculating the bending strength of reinforced concrete and steel-reinforced concrete elements, which are based on the model of balanced failure (ideal failure), when the calculated cross section is simultaneously crushing compressed concrete and rupture of reinforsment bars, when stresses in cross section of the steel profile reaches stresses exceeding the yield strength.

The aim of the study is development a general algorithm and analytical dependences for calculating the flexural strength of encased steel-reinforced concrete (SRC) composite T-beams depending on the stress–strain state of their design cross section at the time of failure.

2 Analytical Model of Calculation of the Flexural Strength of Composite Enclosed SRC T-beams with Full Interaction of Components

The analytical model for the calculation of the flexural strength of enclosed steel-reinforced concrete (SRC) composite T-beams is the continuation of the authors' scientific research results aimed to improve their calculation procedure. The main theoretical methodological prerequisites for the calculation of the enclosed SRC composite T-beams have been previously developed by the authors in the following academic papers [8, 9]. In order to work out the analytical model for the calculation of the flexural strength of encased steel-reinforced concrete (SRC) composite.

T-beams with full interaction of components have defined the following prerequisites:

- strain distribution in cross-sections of an SRC T-beams at plastic (Composite-PSD) or elastic–plastic (Composite-SC) stages is carried out jointly by linear dependencies. Anchor elements are expected to be used to prevent shear and ensure the co-operation of components of steel-reinforced concrete elements in operational and boundary stages. The criterion for the limit state at the breaking moment of the design section of the SRC composite T-beams is the extremum criterion for achieving deformations of the compressive zone of the concrete with the limit value ε_{cu}, at which the flexural strength M_{Rb} of elements will be maximum:

  case a: $M_{plRb}(\varepsilon_{cu}, \varepsilon_a > \varepsilon_{au}) = max-$ is the plastic stage of destruction of encased SRC T-beams (Composite-PSD);

  case b: $M_{Rb}(\varepsilon_{cu}, \varepsilon_a = \varepsilon_{au}) = max-$ boundary stage of destruction of encased SRC T-beams—the border set between plastic stage (Composite-PSD) and elastic–plastic stage of destruction (Composite-SC);

  case c: $M_{Rb}(\varepsilon_{cu}, \varepsilon_a < \varepsilon_{au}) = max-$ elastic–plastic stage of destruction of encased SRC T-beams.

  The extremum criterion for the destruction of $M_{Rb}(\varepsilon_{cu}, \varepsilon_a = \varepsilon_{au}) = max$ was formulated similarly to the criterion $N(\varepsilon_{cu}, \varepsilon_s = \varepsilon_{su}) = max$, which was recommended by Mitrofanov in article [10] in order to calculate optimal compression RC elements.

- the effort N_c in the compression area of the encased SRC T-beam's cross-section is determined by mathematical relations, proposed by the scientists James K. Wight and James G. Macgregor and is now the basis for calculating the flexural strength of RC beams in Eurocode 2 and SRC beams in Eurocode 4 [2];
- the analysis of cross sections of encased SRC T-beams revealed that most part of cross-sections can be corrected to generalized characteristic design sections, which will be reduced to their vertical axis. The steel profile in the cross section of the encased SRC T-beams can be in the form of an I-beam or a U-shaped profile (Fig. 1);

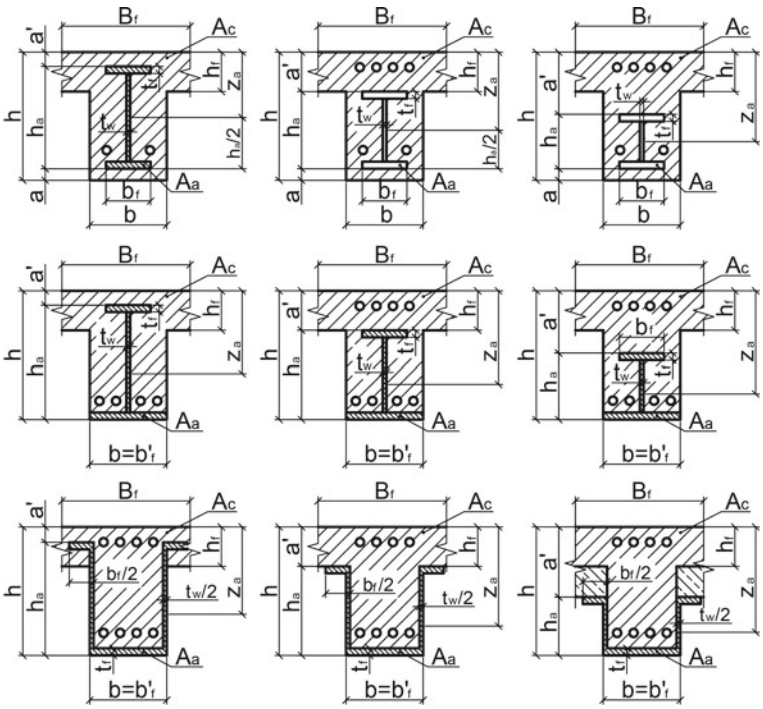

Fig. 1 Typological series of characteristic design sections of composite SRC T-beams in concrete casing in the form of a T-section

- to solve a multifactorial problem of determining the bending strength in sections of closed T-beams, it was necessary to bring one of them to the design transformed section, taking into account the type and magnitude of its reinforcement. To approximate the steel-reinforced concrete cross-section of the SRC T-beam to the steel–concrete section, it was proposed to use the value of the calculated compressive strength of reinforced concrete f_{zM} instead of the calculated compressive strength of concrete f_c, which the authors to determine in [11, 12, 17–21] depending on the type and coefficient of reinforcement ϖ:

$$f_{zM} = f_c \cdot k_z$$

$$f_c = f_{zM} / k_z$$

where k_z—parameter which depends on the mechanical reinforcement coefficient ϖ and options for reinforcing the section; presented in Tables 1, 2 and 3.

Table 1 Dependence of $\varpi - k_z$ by the flat bend for a rectangular section with single reinforcement [12]

ϖ	0	0,1	0,15	0,2	0,25	0,3	0,35	0,4	0,45	0,5	0,6	0,7	1	2	3
kz	0	0,568	0,828	1,071	1,299	1,511	1,706	1,885	2,028	2,07	2,14	2,95	2,31	2,476	2,542

Table 2 Dependence of $\varpi - k_z$ by the flat bend for a rectangular section with symmetrical reinforcement [12]

ϖ	0	0,1	0,15	0,2	0,25	0,3	0,35	0,4	0,45	0,5	0,6	0,7	1	2	3
kz	0	0,368	0,523	0,673	0,821	0,967	1,110	1,253	1,394	1,53	1,81	2,072	2,78	5,06	7,32

Table 3 Parameter values k_z for non-central compressed elements of rectangular cross-section with symmetrical reinforcement $\lambda \leq 4$ ($\lambda \leq 0$) [12]

ϖ	Relative initial eccentricity of longitudinal force application e_0/d									
	0,01	0,15	0,30	0,65	1,0	1,5	2,0	3,0	4,0	5,0
0,10	1,159	0,817	0,575	0,206	0,090	0,047	0,032	0,019	0,014	0,011
0,15	1,209	0,875	0,640	0,272	0,131	0,070	0,047	0,029	0,020	0,016
0,20	1,259	0,918	0,682	0,326	0,169	0,092	0,062	0,038	0,027	0,021
0,25	1,309	0,959	0,721	0,375	0,204	0,114	0,077	0,047	0,034	0,026
0,30	1,359	0,999	0,758	0,420	0,238	0,136	0,092	0,056	0,040	0,031
0,35	1,409	1,040	0,794	0,461	0,270	0,157	0,107	0,065	0,047	0,037
0,40	1,458	1,077	0,829	0,499	0,301	0,177	0,123	0,075	0,054	0,042
0,45	1,509	1,113	0,864	0,535	0,330	0,197	0,137	0,084	0,060	0,047
0,50	1,558	1,148	0,897	0,568	0,359	0,217	0,152	0,093	0,067	0,052
0,60	1,658	1,216	0,964	0,619	0,413	0,256	0,180	0,111	0,080	0,062
0,70	1,758	1,284	1,029	0,667	0,464	0,293	0,209	0,130	0,093	0,073
1,0	2,058	1,544	1,221	0,804	0,597	0,400	0,290	0,183	0,133	0,104
2,0	3,058	2,297	1,340	1,222	0,916	0,676	0,535	0,354	0,261	0,205
3,0	4,057	3,068	1,462	1,665	1,256	0,930	0,738	0,515	0,384	0,305

- a typological analysis of the cross-sections of an enclosed SRC T-beams showed that all their sections can be converted to one calculated transformable section in order to generalize their calculation method for bending. The converted design transformable cross-section of the enclosed SRC T-beams consists of a steel I-beam (steel profile) in concrete casing in the form of a T-section (Fig. 2);
- as a result of the generalization, we have selected six isolated cases of strain–stress state of the design section of the encased SRC T-beams at determining the flexural strength (Fig. 3). Differentiation of cases for encased SRC T-beams limit state depending on the position of the neutral axis in their section allows us to work out a stepwise algorithm of the analytical calculation model of their flexural strength and to obtain the basic calculated correspondences.

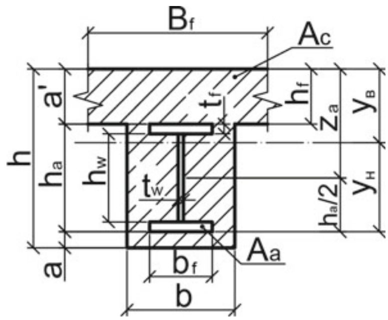

Fig. 2 The transformable calculated cross-section of the enclosed SRC T-beams

2.1 The Algorithm of Calculation of Flexural Strength Encased SRC T-beams

The purpose of the analytical method of design flexural strength of the encased SRC T-beam is determineting the limit value of the bending moment M_{Rb}, which perceives its design section, and to compare it with the external moment M from the load action: $M_{Rb} \geq M$ or $M_{plRb} \geq M$.

The succession of determining the encased SRC composite T-beam's flexural strength according to the proposed analytical method of design (mathematical method of calculation) is shown in the block diagram (Figs. 4 and 5).

At the first stage of calculation the flexural strength of the encased SRC T-beam at present parameters, dimensions of the design section and strength of the composites (ε_{cu}, ε_{au}, E_C, E_a, f_{cd}, f_y. A_C, A_a and h_w are calculated using Eqs. (1), (2) and (3)), we will find the following:

$$A_C = B_f \cdot T_f \tag{1}$$

$$A_a = 2 \cdot t_f \cdot b_f + h_w \cdot t_w \tag{2}$$

$$h_w = h_a - 2 \cdot t_f \tag{3}$$

– the result of the product $\alpha_a \cdot \mu$ Eq. (4) will be calculated as:

$$\alpha_a \cdot \mu = E_a \cdot A_a / (E_C \cdot A_C) \tag{4}$$

– let us check the constraint:

$$\alpha_a \cdot \mu \geq \alpha_a \cdot \mu_{onm},$$

– where $\alpha_a \cdot \mu_{onm}$ is the optimal value of the product, at which the maximum flexural strength will be equal to the mathematical relation: $M_{Rb}(\varepsilon_{cu}, \varepsilon_a = \varepsilon_{au}) = max$,

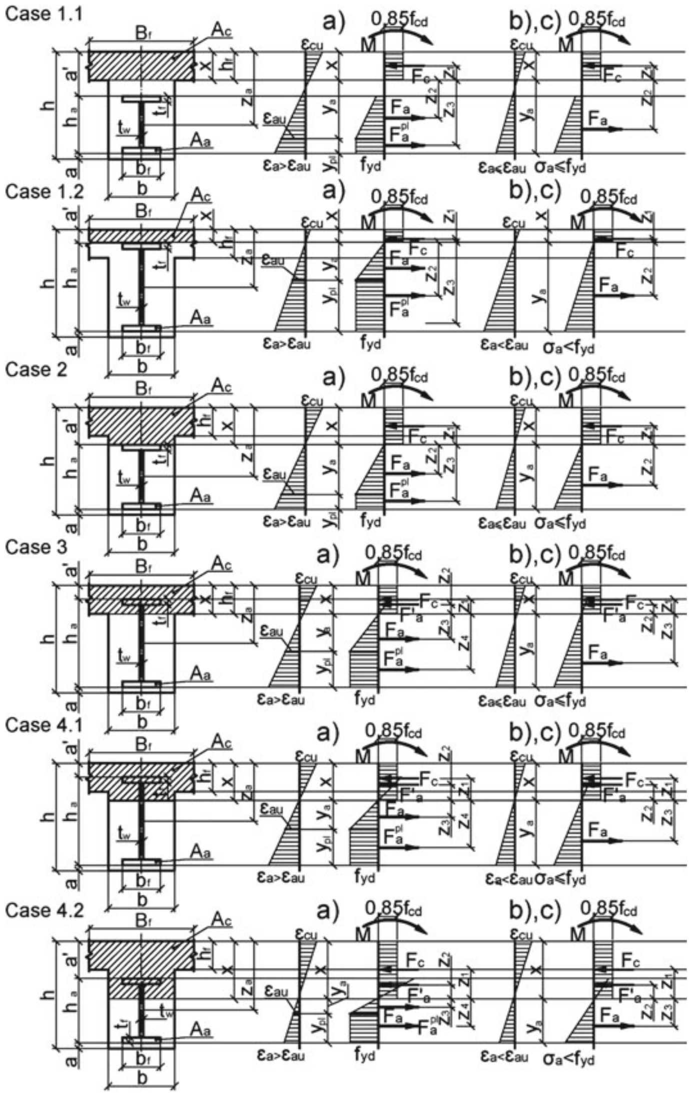

Fig. 3 Cases of the limit stress–strain state for calculated transformed cross-section of the enclosed SRC T-beams design at determining their flexural strength

when the deformations in the extreme fiber of the compressive zone of the concrete reach the value $\varepsilon_C = \varepsilon_{cu}$, and in the steel segment they reach the value $\varepsilon_a = \varepsilon_{au}$. The values of the product quantities $a_a \cdot \mu_{onm}$ can be determined from the data in the tables given in the authors' research work [9], or using the mathematical relations given in Eqs. (5), (6), (7), (8) and (9):

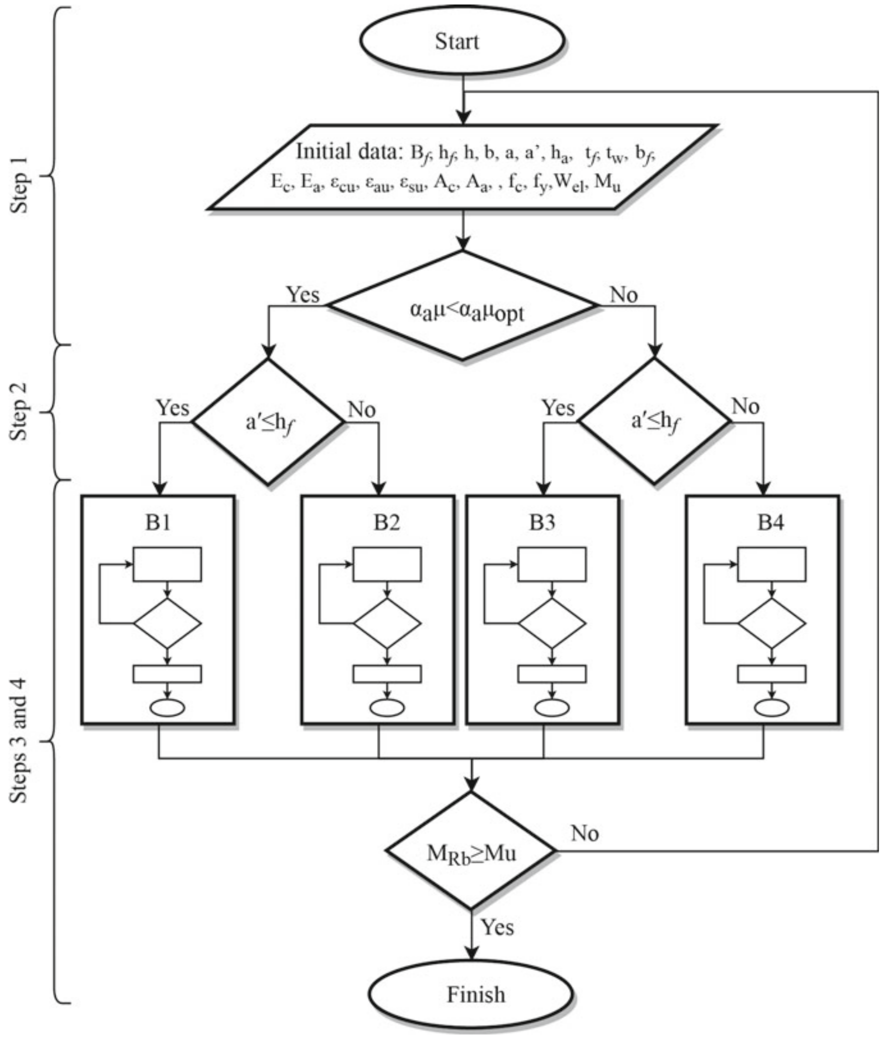

Fig. 4 The block diagram of the succession of determining flexural strength the encased SRC T-beam

$$\alpha_a = E_a/E_C \tag{5}$$

$$\mu_{onm} = (1 - \Delta_\varepsilon)/\{\alpha_a \cdot [2 - (\Delta_c + \Delta_h) \cdot (1 + \Delta_\varepsilon)]\} \tag{6}$$

$$\Delta_C = a/h \tag{7}$$

$$\Delta_\varepsilon = \varepsilon_{cu}/\varepsilon_{au} \tag{8}$$

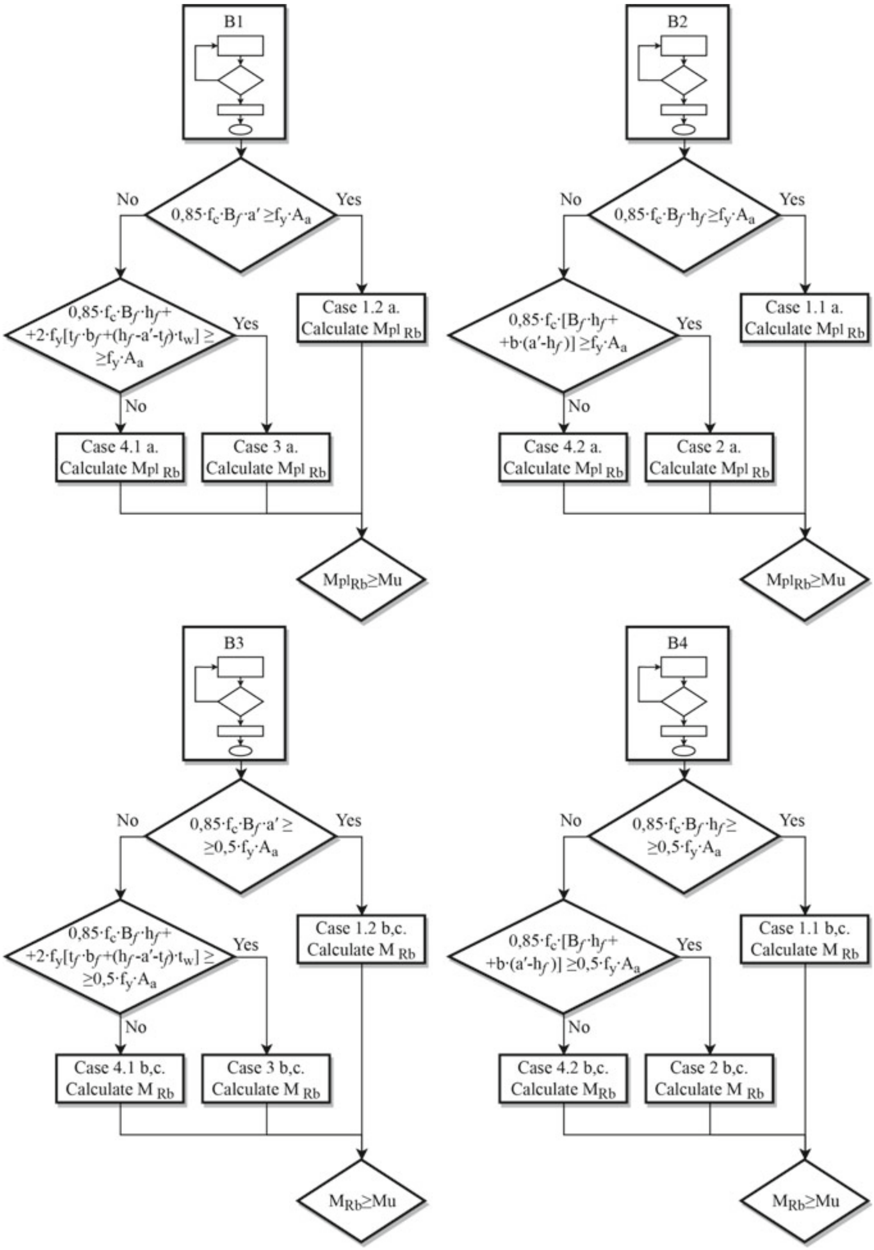

Fig. 5 Elements B1, B2, B3, B4 of the block diagram of succession for determining flexural strength of encased SRC T-beam

$$\Delta_h = h_a / h \tag{9}$$

At the second stage of calculation is determined the position of the steel I-beam profile in height relative to the shelf and the wall of the T-section of the beam, provided:

$$a' \le h_a$$

At the third stage of calculation, depending on the variant of the stress–strain state of the T-beam section, is determined the possibility of the neutral horizontal axis of the section within the height (a') of the T-section shelves under the action of external moment (M) under the conditions:

$$0,85 \cdot f_c \cdot B_f \cdot h_f \ge f_y \cdot A_a;$$

$$0,85 \cdot f_c \cdot B_f \cdot a' \ge 0,5 \cdot f_y \cdot A_a.$$

If the conditions $0,85 \cdot f_c \cdot B_f \cdot h_f \ge f_y \cdot A_a$ and $0,85 \cdot f_c \cdot B_f \cdot a' \ge 0,5 \cdot f_y \cdot A_a$ are met, the neutral horizontal axis of the section of the element passes within the height of the shelf (a') and outside the steel I-beam profile. Next, determine the height of the compressed zone of concrete (x_C) and the value of the bending moment (M_{Rd}) for the calculated cross section of encased SRC T-beam in the stress–strain state in cases 1.1a 1.1b, c, or 1.2a, 1.2b, c (Fig. 5) according to the dependencies below in Table 4.

The value of flexural strength (M_{Rd}) in the calculated cross section of the encased SRC T-beam is compared with the value of the moment from external forces (M). The cross-sectional strength of the encased SRC T-beam will be provided provided that:

$$M_{Rd} \ge M.$$

If the bending strength condition of the encased SRC composite T-beam is not satisfied, it is necessary to increase the size of their calculated cross-section in the direction of increase and accept the materials of their components with higher values of strength characteristics, and then repeat the calculation.

Table 4 Analytical dependences of the calculation of flexural strength of the encased SRC T-beams depending on the case of strain–stress state of their design section

№ ca-se	Analytical dependences of the calculation of flexural strength of the encased SRC T-beams in the determinate case of the boundary strain–stress state of its design section at the breaking moment
Case 1.1a	When the conditions $\alpha_a\mu < \alpha\mu;\ \ a' \le h_f;\ \ 0,85 \cdot f_c \cdot B_f \cdot a' \ge f_y \cdot A_a$ are met The height of the compressive zone of the concrete will be: $x = (A_a \cdot f_y)/(0,85 \cdot f_c \cdot B_f)$ Flexural strength will be: $M_{plRb} = A_a \cdot f_y \cdot (z_a - x/2);$ $z_a = a' + h_a/2$
Case 1.2a	When the conditions $\alpha_a\mu < \alpha\mu;\ \ a' \ge h_f;\ \ 0,85 \cdot f_c \cdot B_f \cdot h_f \ge f_y \cdot A_a$ are met The height of the compressive zone of the concrete will be: $x = (A_a \cdot f_y)/(0,85 \cdot f_c \cdot B_f)$ Flexural strength will be: $M_{plRb} = A_a \cdot f_y \cdot (z_a - x/2);$ $z_a = a' + h_a/2$
Case 2a	When the conditions $\alpha_a\mu < \alpha\mu;\ \ a' \ge h_f;\ \ 0,85 \cdot f_c \cdot \left[B_f \cdot h_f + b \cdot \left(a' - h_f\right)\right] \ge f_y \cdot A_a$ are met The height of the compressive zone of the concrete will be: $x = (A_a \cdot f_y - 0,85 \cdot f_c \cdot B_f \cdot h_f)/(0,85 \cdot f_c \cdot b) + h_f$ Flexural strength will be: $M_{plRb} = A_a \cdot f_y \cdot (z_a - y_c)$ $z_a = a' + h_a/2$ $y_c = \frac{(B_f - b)\cdot(h_f^2/2) + b \cdot x^2/2}{(B_f - b)\cdot h_f + b \cdot x}$
Case 3a	When the conditions $\alpha_a\mu < \alpha\mu;\ \ a' \le h_f;$ $0,85 \cdot f_c \cdot B_f \cdot h_f + 2 \cdot f_y[t_f \cdot b_f + (h_f - a' - t_f) \cdot t_w] \ge f_y \cdot A_a$ are met The height of the compressive zone of the concrete will be: $x = (A_a \cdot f_y - 2 \cdot f_y[t_f \cdot b_f + (h_f - a' - t_f) \cdot t_w])/(0,85 \cdot f_c \cdot B_f)$ Flexural strength will be: $M_{plRb} = 0,85 \cdot f_c \cdot B_f \cdot \frac{x^2}{2} + f_y \cdot \left[W_{pl} + (z_a - x)^2\right];$ $z_a = a' + h_a/2$
Case 4.1a	When the conditions $\alpha_a\mu < \alpha\mu;\ \ a' \le h_f;$ $0,85 \cdot f_c \cdot B_f \cdot h_f + 2 \cdot f_y[t_f \cdot b_f + (h_f - a' - t_f) \cdot t_w] < f_y \cdot A_a$ are met The height of the compressive zone of the concrete will be: $x = (A_a \cdot f_y + 2 \cdot f_y[t_w \cdot a' + t_f \cdot (t_w - b_f) \cdot t_w])/(0,85 \cdot f_c \cdot b + 2 \cdot f_y \cdot t_w)$ Flexural strength will be: $M_{plRb} = \left[\left(B_f - b\right) \cdot h_f \cdot \left(x - \frac{h_f}{2}\right) + b \cdot \frac{x^2}{2}\right] \cdot 0,85 \cdot f_c + f_y \times [W_{pl} + (z_a - x)^2];$ $z_a = a' + h_a/2$

(continued)

Table 4 (continued)

№ ca-se	Analytical dependences of the calculation of flexural strength of the encased SRC T-beams in the determinate case of the boundary strain–stress state of its design section at the breaking moment
Case 4.2a	When the conditions $\alpha_a\mu < \alpha\mu;\ \ a' > h_f;$ $0,85 \cdot f_c \cdot \left[B_f \cdot h_f + b \cdot \left(a' - h_f\right)\right] < f_y \cdot A_a$ are met The height of the compressive zone of the concrete will be: $x = (A_a \cdot f_y + 2 \cdot f_y[t_w \cdot a' + t_f \cdot (t_w - b_f) \cdot t_w])/(0,85 \cdot f_c \cdot b + 2 \cdot f_y \cdot t_w)$ Flexural strength will be: $M_{plRb} = \left[\left(B_f - b\right) \cdot h_f \cdot \left(x - \frac{h_f}{2}\right) + b \cdot \frac{x^2}{2}\right] \cdot 0,85 \cdot f_c + f_y \times [W_{pl} + (z_a - x)^2];$ $z_a = a' + h_a/2$
Case 1.1b, c	When the conditions $\alpha_a\mu > \alpha\mu;\ \ a' \le h_f;\ \ 0,85 \cdot f_c \cdot B_f \cdot a' \ge 0,5 \cdot f_y \cdot A_a$ are met The height of the compressive zone of the concrete will be: $x = (0,5 \cdot A_a \cdot f_y)/(0,85 \cdot f_c \cdot B_f)$ Flexural strength will be: $M_{Rb} = 0,5 \cdot A_a \cdot \sigma_a \cdot (z_a - x/2);$ $z_a = a' + h_a/2$ $\varepsilon_a = (\varepsilon_{cu} \cdot (h_a + a' - x))/x;\qquad \sigma_a = \varepsilon_a \cdot E_a$
Case 1.2b, c	When the conditions $\alpha_a\mu > \alpha\mu;\ \ a' \ge h_f;\ \ 0,85 \cdot f_c \cdot B_f \cdot h_f \ge 0,5 \cdot f_y \cdot A_a$ are met The height of the compressive zone of the concrete will be: $x = (0,5 \cdot A_a \cdot f_y)/(0,85 \cdot f_c \cdot B_f)$ Flexural strength will be: $M_{Rb} = 0,5 \cdot A_a \cdot \sigma_a \cdot (z_a - x/2);$ $z_a = a' + h_a/2$ $\varepsilon_a = (\varepsilon_{cu} \cdot (h_a + a' - x))/x;\qquad \sigma_a = \varepsilon_a \cdot E_a$
Case 2 b, c	When the conditions $\alpha_a\mu > \alpha\mu;\ \ a' \ge h_f;$ $0,85 \cdot f_c \cdot \left[B_f \cdot h_f + b \cdot \left(a' - h_f\right)\right] \ge 0,5 \cdot f_y \cdot A_a$ are met The height of the compressive zone of the concrete will be: $x = (0,5 \cdot A_a \cdot f_y - 0,85 \cdot f_c \cdot B_f \cdot h_f)/(0,85 \cdot f_c \cdot b) + h_f$ Flexural strength will be: $M_{Rb} = 0,5 \cdot A_a \cdot \sigma_a \cdot (z_a - y_c);\ z_a = a' + h_a/2$ $y_c = \frac{(B_f - b) \cdot (h_f^2/2) + b \cdot x^2/2}{(B_f - b) \cdot h_f + b \cdot x};$ $z_a = a' + h_a/2 \qquad \varepsilon_a = (\varepsilon_{cu} \cdot (h_a + a' - x))/x;\qquad \sigma_a = \varepsilon_a \cdot E_a$

(continued)

Table 4 (continued)

№ ca-se	Analytical dependences of the calculation of flexural strength of the encased SRC T-beams in the determinate case of the boundary strain–stress state of its design section at the breaking moment
Case 3 b, c	When the conditions $\alpha_a\mu > \alpha\mu;\quad a' \le h_f;$ $0,85 \cdot f_c \cdot B_f \cdot h_f + 2 \cdot f_y[t_f \cdot b_f + (h_f - a' - t_f) \cdot t_w] \ge 0,5 \cdot f_y \cdot A_a$ are met The height of the compressive zone of the concrete will be: $x = (0,5 \cdot A_a \cdot f_y - 2 \cdot f_y[t_f \cdot b_f + (h_f - a' - t_f) \cdot t_w])/(0,85 \cdot f_c \cdot B_f)$ Flexural strength will be: $M_{Rb} = 0,85 \cdot f_c \cdot B_f \cdot \frac{x^2}{2} + 0,5 \cdot \sigma_a \cdot \left[W_{pl} + (z_a - x)^2\right];$ $z_a = a' + h_a/2 \qquad \varepsilon_a = (\varepsilon_{cu} \cdot (h_a + a' - x))/x; \qquad \sigma_a = \varepsilon_a \cdot E_a$
Case 4.1 b, c	When the conditions $\alpha_a\mu > \alpha\mu;\quad a' \le h_f;$ $0,85 \cdot f_c \cdot B_f \cdot h_f + 2 \cdot f_y[t_f \cdot b_f + (h_f - a' - t_f) \cdot t_w] < 0,5 \cdot f_y \cdot A_a$ are met The height of the compressive zone of the concrete will be: $x = (0,5 \cdot A_a \cdot f_y + 2 \cdot f_y[t_w \cdot a' + t_f \cdot (t_w - b_f) \cdot t_w])/(0,85 \cdot f_c \cdot b + 2 \cdot f_y \cdot t_w)$ Flexural strength will be: $M_{Rb} =$ $\left[\left(B_f - b\right) \cdot h_f \cdot \left(x - \frac{h_f}{2}\right) + b \cdot \frac{x^2}{2}\right] \cdot 0,85 \cdot f_c + 0,5 \cdot \sigma_a \times [W_{pl} + (z_a - x)^2];$ $z_a = a' + h_a/2 \qquad \varepsilon_a = (\varepsilon_{cu} \cdot (h_a + a' - x))/x; \qquad \sigma_a = \varepsilon_a \cdot E_a$
Case 4.2 b, c	When the conditions $\alpha_a\mu > \alpha\mu;\quad a' > h_f;$ $0,85 \cdot f_c \cdot \left[B_f \cdot h_f + b \cdot \left(a' - h_f\right)\right] < 0,5 \cdot f_y \cdot A_a$ are met The height of the compressive zone of the concrete will be: $x = (0,5 \cdot A_a \cdot f_y + 2 \cdot f_y[t_w \cdot a' + t_f \cdot (t_w - b_f) \cdot t_w])/(0,85 \cdot f_c \cdot b + 2 \cdot f_y \cdot t_w)$ Flexural strength will be: $M_{Rb} =$ $\left[\left(B_f - b\right) \cdot h_f \cdot \left(x - \frac{h_f}{2}\right) + b \cdot \frac{x^2}{2}\right] \cdot 0,85 \cdot f_c + 0,5 \cdot \sigma_a \times [W_{pl} + (z_a - x)^2];$ $z_a = a' + h_a/2 \qquad \varepsilon_a = (\varepsilon_{cu} \cdot (h_a + a' - x))/x; \qquad \sigma_a = \varepsilon_a \cdot E_a$

2.2 *Analytical Dependences of Encased SRC T-beams Flexural Strength Depending on the Case of Strain–Stress State of the Design Section at the Breaking Moment*

Analytical dependences of calculation flexural strength of encased steel-reinforced concrete (SRC) T-beams, depending on the case of strain–stress state of their design section at the breaking are shown in Table 4.

3 Comparisons Between Experimental and Analytical Results of Bending Strength Calculation Encased SRC Composite T-beams

In order to compare the proposed analytical model of calculation of flexural strength of encased SRC composite T-beams, we have used the results of experimental studies of scientists, such as: A.P. Vasiliev (specimens of beams 4, 5a, 5b, 6a, 6b, 7, 9a, 9b, 10a, 10b, 11a, 11b, 13a, 13b) [13]; B.A. Kalaturov (specimens of beams BG-1… BG-4, BG-6…BG-9) [14]; Myong-Keun Kwak (specimens of beams SB200, SB250-B, SB300-A, SB300-C, SB300-E) [15]; Cl. Goralski (specimens of beams S1… S4) [16].

When comparing the results, we have determined arithmetic mean ($\overline{X}$), root-mean-square deviation (σ_{n-1}) and coefficient of variation (ν).

The comparisons of experimental results (M^{test}) and analytical findings of the calculation of flexural strength of encased SRC T-beams (M^{calc}) are given in Table 5.

Comparison of experimental and theoretical strength values of 31 specimens of encased SRC composite T-beams beams, which components are bond, leads to the following statistical indicators:

– for partial factors for concrete $\gamma_C = 1,0$ and for material property, also accounting for model uncertainties and dimensional variations $\gamma_M = 1,0$ – $\overline{X} = 1,195;\ \sigma_{n-1} = 0,025;\ \nu = 2,1\%$;
– for partial factors for concrete $\gamma_C > 1,0$ and for material property, also accounting for model uncertainties and dimensional variations $\gamma_M > 1,0$ – $\overline{X} = 1,242;\ \sigma_{n-1} = 0,019;\ \nu = 1,5\%$.

4 Conclusions

The algorithm and analytical dependences of the method for calculating the flexural strength of encased steel-reinforced concrete (SRC) composite T-beams are presented in the academic paper. Comparative analysis of the experimental findings with theoretical calculations of flexural strength of encased SRC composite T-beams has showed their adequate convergence, which allows to apply the proposed analytical dependencies in the design practice.

The model of balanced failure (ideal failure) allows for optimal (rational) design of reinforced concrete elements working on the bend, with minimal costs, taking into account design constraints, such as: design of structures taking into account the type of loads acting on them, and formation (typing) case of their ultimate stress–strain state depending on the defined ultimate criteria for the destruction of their components.

Table 5 Comparison of experimental bending moments with theoretical

№	Author	Specimen	M^{test} kNm	$M^{cala}_{\gamma=1,0}$kNm	$\frac{M^{test}}{M^{cala}_{\gamma=1,0}}$	$M^{cala}_{\gamma=1,0}$ kNm	$\frac{M^{test}}{M^{cala}_{\gamma=1,0}}$
1	Vasiliev [13]	4	97,5	96,5	1,01	84,9	1,15
2		5 a	105,0	89,0	1,18	75,9	1,38
3		5 b	95,0	89,0	1,07	75,9	1,25
4		6 a	97,5	94,2	1,04	81,1	1,20
5		6 b	97,5	94,2	1,04	81,1	1,20
6		7	85,0	77,7	1,09	70,1	1,21
7		9 a	132,5	99,7	1,33	98,9	1,34
8		9 b	132,5	99,7	1,33	98,9	1,34
9		10 a	150,0	106,8	1,40	105,9	1,42
10		10 b	155,5	106,8	1,46	105,9	1,47
11		11 a	183,0	129,4	1,41	117,3	1,56
12		11 b	172,5	120,8	1,43	109,5	1,58
13		13 a	167,0	146,0	1,14	132,2	1,26
14		13 b	187,0	146,5	1,28	132,7	1,41
15	Kalaturov [14]	BG-1	1071,0	808,2	1,33	740,4	1,45
16		BG -2	1092,0	794,6	1,37	716,6	1,52
17		BG -3	1060,0	738,3	1,44	665,9	1,59
18		BG -4	1013,0	707,6	1,43	638,7	1,59
19		BG -6	918,0	794,2	1,16	725,4	1,27
20		BG -7	906,0	745,4	1,22	683,0	1,33
21		BG -8	904,0	738,0	1,22	672,8	1,34
22		BG -9	988,0	754,0	1,31	683,0	1,45
23	Myong-Keun Kwak [15]	SB200	411,0	393,2	1,05	377,7	1,09
24		SB250-B	819,3	764,2	1,07	706,4	1,16
25		SB300-A	923,5	598,5	1,54	559,1	1,65
26		SB300-C	1046,0	657,2	1,59	607,1	1,72
27		SB300-E	1157,4	816,6	1,42	743,5	1,56
28	Goralski [16]	S1	3001,0	2506,8	1,20	2292,8	1,31
29		S2	2981,0	2490,1	1,20	2270,1	1,31
30		S3	1728,0	1619,2	1,07	1461,5	1,18
31		S4	1703,0	1617,3	1,05	1459,0	1,17

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