Estimating the Temperature on the Reinforcing Bars of Composite Slabs Under Fire Conditions

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Abstract. A three-dimensional computational model based on finite elements was developed to evaluate the thermal behaviour of composite slabs with steel deck exposed to a standard fire. The resulting numerical temperatures are then used to obtain a new analytical method, which is an alternative to the simplified method provided by the standard, to accurately determine the temperatures at the reinforcing bars (rebar). The fitting of the analytical model to the numerical data was done by solving a linear least squares problem using the singular value decomposition. The resulting formula fits very well the numerical data, allowing to make predictions of the temperature in the rebar with an approximation error equal to zero and an estimating error at least 77% lower than that obtained with the proposal included in the standard.

Keywords: Transient heat transfer problem · Computational simulation · Concrete-steel slab · Fire rating · Least squares method · Singular value decomposition

1 Introduction

Steel-concrete composite slabs consist of a profiled steel deck which can be used as permanent formwork, and a reinforced concrete. Usually the concrete is reinforced with an anti-crack mesh on top and individual reinforcing bars (rebars) within the ribs (see Fig. 1). The use of composite slabs in buildings is very popular as these building elements offer some advantages for the structures, such as reducing the dead weight of the structures while speeding up the construction process.

Composite slabs may suffer considerable damage in the event of a fire, as the steel elements responsible for the slabs bending resistance capacity are significantly impaired in the event of a fire. To ensure that this building element is fire-resistant, in accordance with the regulations and standards, it is, therefore, necessary to carry out a thermal analysis prior to the static analysis. The fire resistance of this type of element is then determined by standard fire tests, taking into account load-bearing capacity (R), thermal

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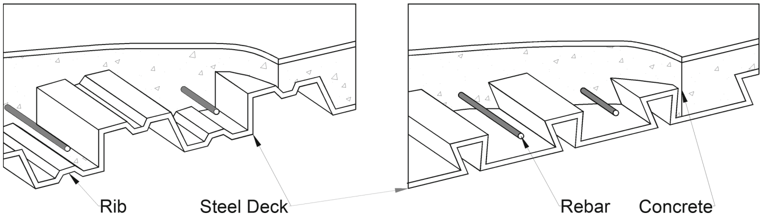


Fig. 1. Composite slab with trapezoidal (left) and re-entrant (right) steel deck.

insulation (I) and integrity (E). In order for a composite slab to demonstrate fire resistance according to the criteria of the European Standard EN13501-2 [7], it must be able to prevent large deformations or deformation velocities in case of fire, i.e. it must be load-bearing (R) and also provide thermal insulation that limits the temperature rise on the unexposed side (I). Finally, the composite slabs must prevent the passage of flames and hot gases through cracks or holes in order to contain the fire from below (E).

This paper deals with the determination of the thermal behaviour of composite slabs under a standard ISO -834 fire [11], focusing on the temperature evolution at the reinforcing bar (rebar). The rebar and the upper flange, web and lower flange of the steel deck are the structural components of the composite slab that are mainly affected by the temperature. An accurate and reliable estimation of the temperatures in these structural components is required, especially to determine the load-bearing criterion (R), as these temperatures have a direct influence on the reduction factors for the steel and concrete strength and thus on the bending resistance of the slabs.

Among the various ways to determine the fire rating of a composite slab, the development of standard experimental fire tests is the most expensive and time-consuming. Alternatively, Annex D of EN 1994-1-2 [6] provides the guidelines for estimating fire resistance based on the simplified calculation method, but this method is based on studies conducted long time ago and is currently outdated. The third method is to simulate computationally the experimental fire tests by means of numerical methods. Computer simulations are of great importance in this field because they allow a reliable and realistic description of the physical phenomena, including the effects of different fire scenarios, such as natural fires.

In [1,2], a series of computational simulations of the thermal effects on composite slabs where developed, with different steel deck geometries in a standard fire. The full-scale tests were simulated with 3D finite elements using Matlab Partial Differential Equations Toolbox (PDE Toolbox) [15]. The results were used to formulate a new proposal that enables to estimate of the temperatures in the slab components (lower and upper flange, web and rebar), which is an alternative to the EN1994-1-2 standard. Taking the numerical results as reference values, the proposed analytical method allows a more accurate estimation of the temperatures for different time values (45, 60, 90 and 120 min). However, since the coefficients of the new proposal are obtained by fitting the numerical data using the Generalised Reduced Gradient (GRG) optimisation method [13], there is no certainty that the solution is optimal or only quasi-optimal.

In the present work, new coefficients for the analytical method are proposed. To obtain the coefficients, the linear least squares method is used rather than an optimisation method. The resulting problem does not have full rank, so it must be solved using the singular value decomposition method. This method guarantees that the calculated solution is the one that minimises the sum of squared deviations.

This paper is structured as follows. In Sect. 2 the thermal problem to be solved is presented. Section 2.3 is devoted to the simplified method provided by the standard for the calculation of temperatures in the rebar. The new analytical proposal for estimating the temperatures at the rebar is proposed in Sect. 3. This section also includes a brief description of the calculation method used to perform the thermal analysis and of the different approaches used to obtain the new proposal. The article ends with the presentation of some final considerations in Sect. 4.

2 Transient Thermal Problem

This section is devoted to the description of the non-linear transient thermal problem that must be modelled and solved in the multi-domain body corresponding to the composite slab under standard fire conditions. The heat flux acting on the unexposed side depends on the ambient temperature and the heat flux acting on the fire exposed side depends on the standard fire defined by the ISO -834 fire curve [11].

2.1 Physical Multidomains

The 3D heat transfer problems are solved for four different composite slabs with different geometries shown in Fig. 2. Two composite slabs with trapezoidal geometry, Confraplus 60 and Polydeck 59s, and two slabs with re-entrant geometry, Multideck 50 and Bondek, were selected (see Fig. 2).

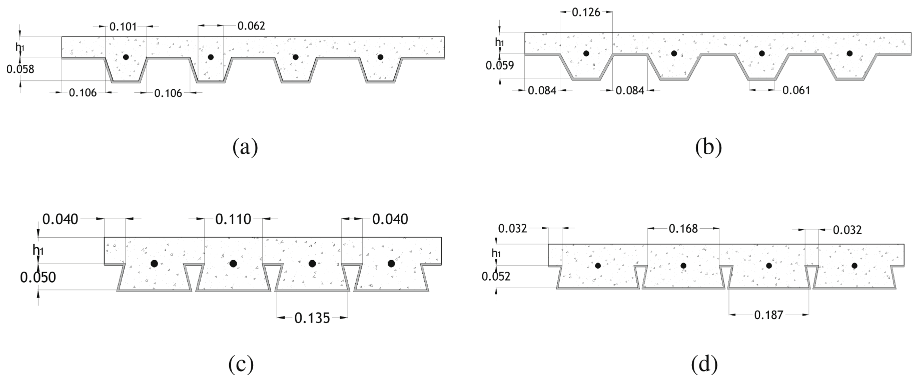


Fig. 2. Geometry and dimensions [mm] of the modelled slabs. (**a**) Confraplus 60. (**b**) Polydeck 59S. (**c**) Multideck 50. (**d**) Bondek.

The 3D computational models were developed in accordance with a realistic representation of the physical model of the composite slabs. The geometry of the models takes into account the exact shape of the surfaces from a representative volume of the slab. The selected cross-section has the side edges bounded by the centre of the upper flange and includes a rib and part of the anti-crack mesh. The length of the specimens is 200 mm in order to include the representative effect of all the components and the anti-crack mesh. The multidomain developed consists of four subdomains: the steel deck, the concrete, the reinforcing bars and the anti-crack. Thus, the materials that make up the physical sub-domains of the slabs are carbon steel (steel deck, rebar and anti-crack mesh) and concrete.

Confraplus 60 is a trapezoidal model profile manufactured by ArcelorMittal. The collaborating steel deck is made of S350 steel and the model uses a thickness of 1.25 mm. The geometric characteristics are shown in the Fig. 2a. The Polydeck 59S model is the second trapezoidal profile selected. The ArcelorMittal Polydeck 59S model, shown in Fig. 2b, consists of a steel deck with S450 steel and a thickness of 1 mm. The re-entrant model shown in Fig. 2c is the Multideck 50 manufactured by Kingspan Structural Products. This product has a steel deck with S450 steel grade and a thickness of 1 mm. The second type of re-entrant slab studied is Bondek, designed and manufactured by Lysaght. The slab consists of a steel profile with grade S350 and the model with a thickness of 1 mm was chosen (see Fig. 2d). These geometries were chosen based on geometric differences and current use.

The energy equation governs the heat conduction inside the physical domain

$$\rho(T) C_p(T) \frac{\partial T}{\partial t} = \nabla \cdot (\lambda(T) \nabla T), \tag{1}$$

where T represents the temperature [°C], $\rho(T)$ is the specific mass [kg/m^3], $Cp(T)$ is the specific heat [J/kgK], $\lambda(T)$ is the thermal conductivity [W/mK], t is the time [s] and $\nabla = (\partial_x, \partial_y, \partial_z)$ is the gradient. Equation (1), is based on the heat flow balance, for the infinitesimal material volume, in each spatial direction.

The thermal properties ($\rho(T)$, $Cp(T)$ and $\lambda(T)$) of the materials that compose the slabs are determined by the Eurocodes [4–6] (steel and concrete), and are temperature dependent. Therefore, the specific mass $\rho(T)$, the specific heat $Cp(T)$ and the thermal conductivity $\lambda(T)$ vary with the temperature, introducing the non-linearity of the Eq. (1).

Once the heat flux on the surface exposed to fire changes with time, the Eq. (1) is time-dependent and holds a transient thermal state for the slab. Thus, to determine the temperature field along time, the solution of the Eq. (1) is required. Furthermore, to solve the problem correctly, it is necessary to apply the boundary conditions according to the ISO-834 fire curve in the physical domain [3].

2.2 Boundary Conditions Corresponding to a Standard Fire

To set the boundary conditions, we need to master the different types of heat transfer acting on the slabs, i.e. conduction, convection and radiation. The composite slabs are subjected to three main boundary conditions, namely the exposed surface, the non-exposed surface and the insulated surfaces. They all follow the guidelines of the Eurocode EN1991-1.2 [6].

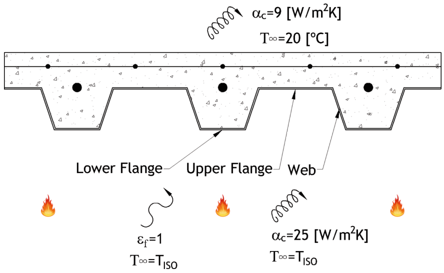


Fig. 3. Boundary conditions.

In the exposed side of the slab, the boundary conditions comprise the heat transfer by convection and radiation and are given by

$$\lambda(T)\nabla T.\overrightarrow{n} = \alpha_c(T_\infty - T) + \phi\varepsilon_m\varepsilon_f\sigma\left(T_\infty^4 - T^4\right) \tag{2}$$

where $\overrightarrow{n}$ is the unitary vector normal to the external face, ϕ is the view factor, α_c is the convection coefficient, ε_m is the emissivity of the material, ε_f is the emissivity of fire, σ is the Stefan-Boltzmann constant and T_∞ is the gas temperature of the fire compartment. Equation (2) represents the heat flux that arrives to the steel deck by radiation and convection based on the gas bulk temperature. The convection coefficient is $\alpha_c = 25$ W/m^{2}K, the emissivity of steel is $\varepsilon_m = 0.7$ and the fire emissivity is $\varepsilon_f = 1$. The boundary conditions parameters are represented in Fig. 3.

The view factor (ϕ) is a term that quantifies the geometric relation between the surface emitting radiation and the receiving surface. This parameter has no dimensions and depends on the rib surface's orientations and the distance between the radiative surfaces. The Crossed-Strings method, proposed by Hotell H. C. in 1950 [8], is used to determine the view factor.

Equation (2) includes the gas temperature, T_∞, of the fire compartment, which follows the standard fire curve ISO-834 ($T_\infty = T_{\text{ISO}}$) given by

$$T_{\text{ISO}} = 20 + 345\log_{10}(8t+1), \tag{3}$$

where T_{ISO} is given in [°C] and t in [min] [11].

The top part of the composite slab (unexposed side) is also an important side to determine the temperature evolution. After all, it will determine the heat transfer from the slab to the above compartment. Following the standard EN1991-1-2 recommendations, the heat effect on the unexposed side may be defined by the heat flux by convection, using $\alpha_c = 9$ W/m^{2}K, to include the radiation effect [5]. The boundary condition in the upper surface of the slab is given by Eq. (4),

$$\lambda(T)\nabla T.\overrightarrow{n} = \alpha_c(T - T_\infty) \tag{4}$$

where T_∞ is the room temperature. The adiabatic boundary conditions, given by Eq. (5), are applied to the other four surfaces of the slab (front, back, left and right):

$$\lambda(T)\nabla T.\vec{n} = 0. \quad (5)$$

2.3 Analytical Method Provided by the Standard Eurocode

The simplified calculation method for the load-bearing criterion (R) presented in Eurocode EN1994 1.2 [6] can be applied to simply supported composite slabs when subjected to a ISO -834 standard fire [11]. In order to calculate the bending moment resistance of the composite slab (sagging moment), the standard provides the following analytical method for estimating the temperatures at the rebar (θ_r):

$$\theta_r = c_0 + c_1\frac{u_3}{h_2} + c_2 z + c_3\frac{A}{L_r} + c_4\alpha + c_5\frac{1}{l_3} \quad (6)$$

where the temperature θ_r are given in [$^\circ C$]. The parameter l_3 is the distance within the ribs, u_3 represents the distance from the middle of the ribar to the lower flange in [mm], h_2 is the height of the rib in [mm], the z-factor represents the position of the rebar concerning the slab rib given by

$$\frac{1}{z} = \frac{1}{\sqrt{\frac{1}{u_1}}} + \frac{1}{\sqrt{\frac{1}{u_2}}} + \frac{1}{\sqrt{\frac{1}{u_3}}} \quad (7)$$

in [mm$^{-0.5}$], α corresponds to the angle formed between the web component of the steel deck and the horizontal direction in degrees [°], A/L_r is the ratio between the concrete volume and exposed area per meter of rib length of the steel deck, it is given in [mm], and its calculation is performed through

$$\frac{A}{L_r} = \frac{h_2\left(\frac{l_1+l_2}{2}\right)}{l_2 + 2\sqrt{h_2^2 + \left(\frac{l_1-l_2}{2}\right)^2}}. \quad (8)$$

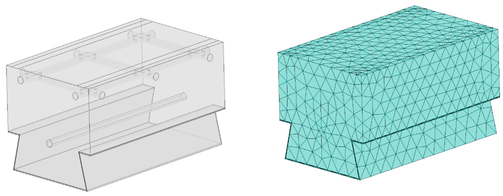
The coefficients c_i in Eq. (6) are given by EN1994 1.2 [6], which depends on the time of fire resistance (fire rating) that must be achieved (60 min, 90 min or 120 min).

3 Improving the Analytical Method with Numerical Results

To improve the simplified method proposed by the standard, the thermal problem was solved computationally for different values of the concrete thickness (h_1). The values of the numerical temperatures obtained were used to define an alternative analytical method to the simplified method.

3.1 Computational Solution by Finite Elements Method

The Eq. (1) is discretised by finite elements within the physical subdomains corresponding to the different materials. Figure 4a shows the geometry of a representative volume of a composite slab Multideck, and Fig. 4b shows the corresponding mesh, both generated by Matlab PDE Toolbox.



(a) (b)

Fig. 4. Representative volumes of the Multideck 50 modelled in Matlab. (**a**) Geometry. (**b**) Finite-elements mesh.

The discretization of Eq. (1) by finite elements method (FEM) leads to the energy matrix formulae

$$C(T)\dot{T} + K(T)T = F \tag{9}$$

where $\dot{T}$ is the vector of time derivatives of the nodal temperatures, C is the capacitance matrix, K is the conductivity matrix and F is the vector of the thermal loads that includes the boundary conditions (for details see, for example, [12]). The solution of the first order non-linear system of ordinary differential Eqs. (9), considering $T(t_0) = T^0$ and the respective boundary conditions, enables to determine the temperature at each node of the mesh, illustrated in Fig. 4b, over the time interval $\left[t_0,\ t_f\right]$.

Matlab (R2021a) PDE toolbox was used to develop and solve the nonlinear transient thermal analysis. The finite element model of the composite slab uses only the tetrahedron finite element type. This finite element is defined by four nodes and uses linear interpolation functions. The resulting mesh includes the four sub-domains concrete, steel deck, rebar and anti-crack mesh, each of which has its own thermal properties. The solution of Eq. (9) is computed by the built-in function `ode15s` [18].

The finite element computational model described here has been validated in previous work (see [1]) with the experimental results published by Lim and Wade [14] and Piloto et al. [17].

3.2 Improving the New Proposal with an Optimization Method

In [1] a parametric study was developed to determine the influence of concrete thickness h_1 on the temperatures used to determine the fire resistance of composite slabs according to the load-bearing criterion (R). A total of 20 numerical simulations were performed. The simulations considered h_1 values of 60, 70, 90, 110 and 125 mm for the two trapezoidal geometries and of 50, 70, 90, 110 and 125 mm for the two re-entrant geometries. These values are the most frequently used dimensions in building practise.

Based on the results of the parametric study, new coefficients c_i where proposed for the simplified method given by the Eq. (6). In addition, the inclusion of a new term responsible for the inclusion of the effects of the variation of the thickness h_1 of the concrete slab was proposed. The thickness h_1 was explicitly included in the mathematical model multiplied by coefficient c_6:

$$\theta_{\text{new}} = c_0 + c_1 \frac{u_3}{h_2} + c_2\, z + c_3 \frac{A}{L_r} + c_4\, \alpha + c_5 \frac{1}{l_3} + c_6\, h_1. \tag{10}$$

The coefficients for these new proposal methods were determined by fitting the mathematical model represented by the Eqs. (10) to the numerical results of the parametric study with h_1 considering the four different composite slab geometries. The coefficients were determined by minimising the sum of the squared deviations between the numerical and analytical temperatures. This sum was minimised using the Generalised Reduced Gradient (GRG) optimisation method [13].

Table 1. Coefficients of the new proposal obtained by optimization method (New OP).

Fire rating	c_0	c_1	c_2	c_3	c_4	c_5	c_6
45 min	99.82	100.20	106.00	–11.83	2.07	–3983.08	–0.06
60 min	–880.00	923.77	389.18	–30.70	2.96	–5263.73	–0.12
90 min	117.69	961.63	–526.70	28.09	0.74	–5803.21	–0.35
120 min	–151.22	834.65	31.95	–8.06	2.21	–7000.32	–0.60

The resulting coefficients for the estimation of temperatures on the rebar, through Eqs. (10), are included in Table 1. It is worth mentioning that, in addition to the standard fire resistance ratings of 60, 90, and 120 min, the new proposal also comprises the coefficients for the fire rating of 45 min. Analyzing the new coefficients presented in the Table 1, it turns out that c_6 is the smallest of the coefficients, showing that h_1 has a reduced effect on the rebar temperature, compared to the other parameters. However, it is a non-negligible effect that increases with time of fire resistance ratings. Although, as the coefficients in Table 1 were obtained by a numerical optimization method, there is no certainty that the solution is optimal. Consequently, a new approach is proposed to obtain these coefficients based on the linear least-squares method.

3.3 Improving the New Proposal by the Linear Least Squares Method

For each fire rating, new coefficients c_j, $j = 0, 1, 2, \ldots, 6$, must be provided so that the values calculated by Eq. (10) are approximately equal to the corresponding numerical temperatures θ_i, with $i = 1, 2, \ldots, 20$, for the four different types of steel deck geometries and for the five different values of h_1. This results in the need for the approximate solution of the following overdetermined linear system of equations

$$\begin{cases} c_0 + c_1a_{1.2} + c_2a_{1.3} + c_3a_{1.4} + c_4a_{1.5} + c_5a_{1.6} + c_6a_{1.7} \approx \theta_0 \\ c_0 + c_1a_{2.2} + c_2a_{2.3} + c_3a_{2.4} + c_4a_{2.5} + c_5a_{2.6} + c_6a_{2.7} \approx \theta_1 \\ \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \\ c_0 + c_1a_{20.2} + c_2a_{20.3} + c_3a_{20.4} + c_4a_{20.5} + c_5a_{20.6} + c_6a_{20.7} \approx \theta_{19} \end{cases} \tag{11}$$

where $a_{i.2} = u_3/h_2$, $a_{i.3} = z$, $a_{i.4} = A/L_r$, $a_{i.5} = \alpha$, and $a_{i.6} = 1/l_3$ represents the geometrical parameters of one of the four composite slabs (Confraplus 60, Polydeck 59S, Multideck 50, or Bondek), and $a_{i.7} = h_1$ is the corresponding concrete thickness (50, 70, 90, 110 or 125 mm).

The solution of the system (11), in the least squares sense, corresponds to the solution $\mathbf{x} = [c_0\, c_1, \ldots, c_6]^T \in \mathbb{R}^7$, which minimises the sum of the squared difference between the two sides of the 20 equations, viz,

$$\min_{\mathbf{x}} \sum_{i=1}^{20} \left(\theta_i - \sum_{j=0}^{6} c_i a_{ij} \right)^2 . \tag{12}$$

The system (11) can be written in the matrix form as

$$\begin{bmatrix} 1 & a_{1.2} & a_{1.3} & a_{1.4} & a_{1.5} & a_{1.6} & a_{1.7} \\ 1 & a_{2.2} & a_{2.3} & a_{2.4} & a_{2.5} & a_{2.6} & a_{2.7} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & a_{20.2} & a_{20.3} & a_{20.4} & a_{20.5} & a_{20.6} & a_{20.7} \end{bmatrix} \cdot \begin{bmatrix} c_0 \\ c_1 \\ \vdots \\ c_6 \end{bmatrix} \approx \begin{bmatrix} \theta_0 \\ \theta_1 \\ \vdots \\ \theta_{20} \end{bmatrix} \Leftrightarrow \mathbf{Ax} \approx \mathbf{b} \tag{13}$$

where $\mathbf{A} \in \mathbb{R}^{20\times 7}$ and $\mathbf{b} \in \mathbb{R}^{20\times 1}$. The condition given by Eq. (12) is equivalent to the minimization of the squared norm of the residual, $\|\mathbf{r}\| = \|\mathbf{b} - \mathbf{Ax}\|$, of the system $\mathbf{Ax} \approx \mathbf{b}$, i.e.,

$$\min_{\mathbf{x}} \|\mathbf{b} - \mathbf{Ax}\|^2 . \tag{14}$$

This least squares problem could be solved in different ways if **A** had full column rank, but **A** had only rank 5. In all four composite slab models, $u_3 = h_2$ is verified so that the second column of **A** is equal to the first ($a_{i.2} = 1$ for $i = 1, \ldots, 20$). As a result, the least squares problem has multiple solutions. However, of all the solutions to the Eq. (14), there is only one that also minimises the norm of **x** (see for example [10]). This solution can be obtained by decomposing it into singular values. The singular value decomposition of **A** is

$$\mathbf{A} = \mathbf{U}\,\mathbf{D}\,\mathbf{V}^T . \tag{15}$$

where $\mathbf{U} \in \mathbb{R}^{20\times 20}$, $\mathbf{D} \in \mathbb{R}^{7\times 7}$ and $\mathbf{V} \in \mathbb{R}^{7\times 7}$. The matrix **D** is a diagonal matrix with the singular values of the matrix **A**. Two of them are zeros because of the rank of the matrix **A**. These matrices can be decomposed to separate the singular values that are not zero:

$$\mathbf{U} = [\mathbf{U}_1\, \mathbf{U}_2] , \quad \mathbf{D} = \begin{bmatrix} \mathbf{D}_1 & 0 \\ 0 & 0 \end{bmatrix} , \quad \text{and} \quad \mathbf{V} = \begin{bmatrix} \mathbf{V}_1^T \\ \mathbf{V}_2^T \end{bmatrix} \tag{16}$$

where $\mathbf{U}_1 \in \mathbb{R}^{20\times5}$, $\mathbf{D}_1 \in \mathbb{R}^{5\times5}$ and $\mathbf{V}_1 \in \mathbb{R}^{5\times5}$. The matrix $\mathbf{D}_1$ includes the non-singular values of $\mathbf{A}$. The solution of the least squares problem (14) is then given by

$$\mathbf{x} = \mathbf{V}_1\mathbf{D}_1^{-1}\mathbf{U}_1^T\mathbf{b}. \tag{17}$$

The new coefficients obtained by this method are presented in Table 2.

Table 2. Coefficients for the new proposal obtained by least squares method (New LS).

Fire rating	c_0	c_1	c_2	c_3	c_4	c_5	c_6
45 min	1074.7	1074.7	–2005.1	123.02	–2.3751	–1833.9	–0.056628
60 min	1364.8	1364.8	–2510.3	154.60	–3.2784	–2307.9	–0.12413
90 min	1794.8	1794.8	–3272.5	203.24	–4.6096	–3018.4	–0.35139
120 min	2005.7	2005.7	–3580.4	222.61	–5.2642	–3328.4	–0.60330

3.4 Comparison of the Results

To compare the accuracy of the new coefficients, Table 3 shows the values of error measurements obtained with the new proposal, given by Eq. (10), and with the simplified method proposed by standard EN1994-1.2 [6] and given by Eq. (6). In the case of the new proposal, the results were obtained with the new coefficients derived in the previous section by the least squares method (New LS) and with the coefficients previously obtained by optimisation (New OP) and presented in the Table 1.

In Table 3, the values obtained by the analytical methods y_i for $i = 1,2,\ldots,20$ given by the simplified method or by the new proposal are compared with the numerical results θ_i for $i = 1,2,\ldots,20$ (right-hand side of the system (13)). According to the recommendations of Chai and Draxler [9], the Root Mean-Squared Error (RMSE) is used as an error measure to compare the results:

$$\text{RMSE} = \sqrt{\frac{1}{20}\sum_{i=1}^{20}(\theta_i - y_i)^2}, \tag{18}$$

and the Bias, given by

$$\text{Bias} = \frac{1}{20}\sum_{i=1}^{20}(\theta_i - y_i). \tag{19}$$

To complement the RMSE and Bias metrics, the Standard Deviation of the Error (SDE) is also considered. The SDE simply corresponds to:

$$\text{SDE} = \sqrt{\text{RMSE}^2 - \text{Bias}^2}. \tag{20}$$

From a statical point of view, the Bias is a basic indicator of the *systematic error* in a prediction and the SDE is the equivalent indicator of the *random error*. In the artificial

Table 3. Errors measures of the different proposals

Fire rating:	45 min			60 min			90 min			120 min		
Proposal	Bias	RMSE	SDE	Bias	RMSE	SDE	Bias	RMSE	SDE	Bias	RMSE	SDE
New LS	0	1.163	1.163	0	2.192	2.192	0	6.539	6.539	0	6.538	6.539
New OP	–10.62	21.38	18.56	–13.90	28.36	24.72	–15.64	31.69	27.56	–15.64	31.689	27.56
EN1994-1.2	–	–	–	46.21	54.18	28.28	202.2	204.1	28.29	202.16	204.13	28.29

intelligence (AI) context, the bias represents the *approximation error* and the SDE the *estimation error* [16].

The results obtained in Table 3 show that the temperatures obtained with the new proposal, with the coefficients obtained by the least squares method (New LS), fit the numerical results very well. It allows the smallest errors for all fire ratings, including a zero value for the bias (estimation error). In the context of machine learning (ML), the reduction to zero of the approximation error shows that the new proposal overfits the available numerical data [16]. The other errors are very small compared to the others proposals. Compared to the formulae provided in the standards, this new proposal makes it possible to reduce the SDE (approximation error) by 77%, in the case of fire ratings of 90 and 120 min, and by 92%, in the case of a fire rating of 45 min.

It can also be observed that the temperatures estimated by the simplified method of Eurocode EN1994-1.2 are usually lower than the numerical temperatures, as the Bias is always positive. Compared to the standard proposal, the new proposal, with New OP coefficients, helps to improve the temperature estimation for each fire rating time. However, they are not as efficient in reducing errors as the coefficients obtained by least squares (New LS).

4 Conclusion

In this work, a realistic computational model was used to simulate the thermal behaviour of composite slabs subjected to standard fire conditions. The results of the numerical simulations make it possible to determine the temperatures for different slab geometries and compare them with those resulting from the simplified calculation method of the standard EN1994-1.2.

The numerical values were compared with the simplified analytical method provided in the standard for estimating temperatures in the reinforcing bar (rebar). The results show that the simplified method gives temperatures that are very far from the numerical values, indicating that its formulation needs to be revised.

To improve the analytical calculation proposal, new coefficients are presented, obtained by fitting the numerical data using a linear least squares method. This method leads to the resolution of an overdetermined linear system where the coefficient matrix has an incomplete column rank, resulting in the problem having multiple solutions. Resolving this system by decomposition into singular values guarantees obtaining a solution that minimises the squared norm of the residue of the linear system. The results show that the new coefficients enable to improve the estimation of the temperatures in the rebar, allowing to zero the estimation error and to get an approximation error at least 77% lower than that obtained with the formulae included in the standard.

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