Safety Level of the IMO Second Generation Intact Stability Criteria

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Abstract The safety level of a criterion is defined as a probability that a failure will occur, if the criterion is met exactly, i.e. without any surplus. This chapter considers how the safety level can be evaluated, in principle, for the vulnerability assessment included in the Second Generation Intact Stability Criteria (SGISC). The chapter also provides a review of the background literature for the SGISC and considers the alignment of SGISC with Goal Based Standards and Formal Safety Assessment.

Keywords Intact stability · Safety level · Goal-based standards

1 Introduction

Goal-based standards (GBS) represent a significant paradigm-shift in regulation philosophy and practice. Instead of prescribing the means of achieving safety, GBS formulates the objective, leaving the freedom of achieving this objective to a designer (see, e.g. [28, 43]). GBS may be considered as the natural regulatory framework for deploying a risk-based or probabilistic approach. Indeed, for stability in particular, the probability of stability failure, as a universal indicator of danger, is a natural metric of the goal of safety and is naturally aligned with the GBS.

For example, the IMO Guidelines for the approval of alternatives and equivalents as provided in various IMO instruments [34] acknowledges that approval risk assessment and reliability analysis by Administrations is an increasingly acceptable practice, especially for novel designs. Also, risk analysis is an important part of a

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formal safety assessment (FSA), which is considered for use in the IMO rule-making process.

A comprehensive (and still up-to-date) review of a risk-based approach to intact stability can be found in [42]. The most difficult problem is the calculation of probability of stability failure in an absolute sense. In other words, what does the term "probability of stability failure" mean?

Stability failures in realistic sea conditions are rare and cannot be assessed by direct numerical simulation of reasonable fidelity. This problem of rarity (as defined in the Interim Guidelines on the second generation intact stability criteria, see [32]) inevitably generates need of using statistical extrapolation schemes. The ability to determine the probability of stability failure in an absolute sense means that an extrapolation method is capable of recovering the value of the probability of stability failure that would be observed from numerous lengthy data sets [72].

The derivation of probability of stability failure in an absolute sense allows consideration of intact-stability hazards together with other hazards, like fires, machinery failures etc., making intact stability fully assessable with risk analysis and FSA.

The next question is how the alignment with GBS propagates through the multi-tiered structure of the second generation intact stability criteria (SGISC). Since probabilistic criteria are expected to be used for direct stability assessment, such alignment appears quite trivial at the tier 3 of the SGISC. Indeed, probability of stability failure produced by the direct stability assessment directly "plugs-in" into FSA and risk analysis. This is more difficult however for vulnerability criteria, as there is less information available and the calculation methods are much simpler than the direct assessment. It is especially difficult for vulnerability criteria level 1. To address this challenge, a brief review of the physical background of the SGISC vulnerability criteria is carried out and an attempt is made to reveal their connection to a general probabilistic framework. Before this however, the background of the probabilistic formulation by means of which the stability failure modes can be assessed is explained.

The Interim Guidelines on the SGISC [32] define an intact stability failure as an event that includes the occurrence of very large roll (heel, list) angles or excessive rigid body accelerations, which may result in capsizing or impairs normal operation of the ship and could be dangerous to crew, passengers, cargo or ship equipment. The Interim Guidelines address five dynamic stability failure modes, including the dead ship condition, excessive acceleration, pure loss of stability, parametric rolling, and surf-riding/broaching.

2 Probabilistic Framework

Waves and wind are stochastic processes. Therefore, any stability failures caused by wind or waves are random events and they can be characterized by their probability of occurrence.

An objective of safe operation of a ship is the absence of stability failures during a ship's lifetime. The symbol X is used to denote a random event, the occurrence of at least one stability failure during a ship's lifetime. Then the complimentary event $\overline{X}$ is that no stability failures occur during a ship's lifetime. The bar above the symbol identifies it as a complimentary random event, i.e., that the event X does not occur. The likelihood of achieving this objective is characterized by the probability that no stability failure occurs during a ship's lifetime, $P\left(\overline{X}\right)$.

Stability failure is a random event and it might occur at any interval of time while the ship is in operation. The objective of safe operation is achieved when no stability failure occurs at any of the time intervals comprising the entire time of ship operation. Let us represent these intervals by a series of discrete time instances. Then the probability of failure is expressed as:

$$P\left(\overline{X}\right) = P_1\left(\overline{X}\right) \cdot P_2\left(\overline{X}\right) \cdot \cdots \cdot P_N\left(\overline{X}\right) = \prod_{i=1}^{N} P_i\left(\overline{X}\right). \tag{1}$$

For a particular ship, the probability, $P_i\left(\overline{X}\right)$ that no stability failure occurs in association to the ith time instant depends on the environmental (i.e., significant wave height, mean wind velocity, mean zero-crossing period of wave, etc.) and operational conditions (loading, speed, heading relative to the waves, etc.). Further justification of the probabilistic framework can be found in Chap. 1 of [9].

The short-term formulation is relevant for consideration of a particular sea state, i.e., when the significant wave height and the mean zero-crossing period can be associated with a particular cell of a scatter table (e.g. [30]).

For a ship at a given loading condition, heading and speed, the probability, $P_i\left(\overline{X}\right)$ remains constant for each time interval. Then the probability of no stability failure under the conditions of a realization of a sea state with significant wave height H_S and mean zero-crossing period T_Z is:

$$P\left(\overline{X}|H_S, T_Z\right) = \left(P_t\left(\overline{X}\right)\right)^n, \tag{2}$$

where $P_t\left(\overline{X}\right)$ is a probability that there will be no stability failure at a brief time period around some time instant; n is the number of such time instants.

As is obvious, the probability of no stability failure depends on time; the longer the time of exposure, the higher the probability of failure.

Equation (2) is interpreted as a particular case of the binomial distribution, which expresses the probability that a random event occurs k times out of n attempts—the probability of k failures occurring in n instants of time:

$$P(k) = C(n, k)p^k q^{n-k}, \tag{3}$$

where $C(n, k)$ is the number of k combinations out n without repetitions, p is the probability of stability failure at any given instant of time and q is the probability of the complimentary event, (i.e., that stability failure does not occur at any instant of time):

$$p = P_t(X); \quad q = 1 - P_t(X) = P_t\left(\overline{X}\right). \tag{4}$$

The Poisson distribution is the limit case of the binomial distribution for a large number of time instants, while the duration of each time instant is small:

$$P(k) = \frac{(\lambda T)^k}{k!} \cdot \exp(-\lambda T), \tag{5}$$

where T is a finite duration of time, while the condition (H_S, T_Z) exists and λ is the rate of random events (stability failures) per unit of time. The probability of no stability failures while the condition (H_S, T_Z) holds is given by the case $k = 0$:

$$P\left(\overline{X}|H_S, T_Z\right) = \exp(-\lambda T). \tag{6}$$

The probability of the complimentary event—at least one stability failure during time T is interpreted as the CDF of the time before the first event occurs. It is an exponential distribution with parameter λ:

$$P(X|H_S, T_Z) = CDF(T) = 1 - \exp(-\lambda T). \tag{7}$$

There are three assumptions, associated with the short-term formulation:

- Stability failures are independent random events;
- The probability of occurrence of a stability failure at a particular instant of time is infinitely small;
- Only one stability failure can occur at a particular instant of time.

The first assumption is inherited from (1), while the two others are the result of the limit transition from (3) to (5). A probabilistic model of random events using these three assumptions is known as a "Poisson flow of events".

The value of the stability failure rate λ depends on a ship's speed, heading and loading condition. The methods for the numerical evaluation of λ are failure-mode-specific. A key point is that λ is assumed constant for a particular speed, heading, loading and the environmental conditions (H_S, T_Z).

The lifetime of a ship is presented as a sequence of sea states described in a scatter table with N_S significant wave heights and N_T zero-crossing mean periods. Equation (1) for the probability of no stability failures over the lifetime of a ship, using (6), T_{LT} is rewritten as:

$$P\left(\overline{X}\right) = \prod_{i=1}^{N_S} \prod_{j=1}^{N_T} \exp\left(-\lambda_{i,j} T_{LT} f_{i,j}\right) = \exp(-\lambda_a T_{LT}), \tag{8}$$

where $f_{i,j}$ is the statistical frequency for the ith significant wave height and the jth mean zero-crossing period; λ_a is the rate of stability failures, averaged over the scatter table:

$$\lambda_a = \sum_{i=1}^{N_S} \sum_{j=1}^{N_T} f_{i,j} \lambda_{i,j}. \tag{9}$$

The probability of at least one stability failure is expressed through the complimentary probability to (8):

$$P(X) = 1 - \exp(-\lambda_a T_{LT}). \tag{10}$$

The criteria for different stability failure modes use different probabilistic formulations, but all of them are based on Eqs. (7), (9) and (10).

The safety level of the stability criterion is a measure of how remote the possibility of stability failure is if a ship meets the standard used with the criterion. Hence, the safety level of a vulnerability criterion is measured as a probability of stability failure of a ship that passes that standard. The idea to measure reliability of an intact stability criterion with a probability of stability failure while the criterion is satisfied exactly is not new, e.g. [69]. The English version is available in subsection 1.1 of [9], where the safety level is referred as a "guarantee".

While formulating the framework for the SGISC, two types of criteria were envisioned: deterministic and probabilistic [7]. A probabilistic criterion yields an estimate of probability of failure, the standard has a meaning as the acceptable probability of failure. Thus, determining the safety level of a probabilistic criterion is straightforward—it is equal to the standard.

To evaluate the safety level of a deterministic criterion, a random variable (or variables) needs to be found (or assumed) in a criterion's equation. A distribution for this random variation is to be determined or assumed. Then, the criterion's equation can be treated as a deterministic function of a random argument(s) and a distribution of the criterion value can be found. The safety level SL is determined as a probability of exceedance of a standard;

$$SL = P(C \geq St) = \int_{St}^{\infty} pdf(C)dC, \tag{11}$$

where C is a criteria and St is a standard.

Level 1 vulnerability criteria are deterministic, while all the level 2 vulnerability criteria are probabilistic. For three failure modes (excessive acceleration, pure loss of stability and surf-riding), wave steepness can be identified as a random variable that defines the safety level. Detailed consideration is given further in the text, while the distribution of wave steepness is described here.

Consider a short-term problem: a sea state is given where both the significant wave height H_s and the zero-crossing period T_z are known. A spectral density of wave elevations s_w is also defined. The availability of a joint probability density function (PDF) of the wave amplitude and the wave period is very useful for deriving the probability of stability failure since both of these variables affect ship stability. Such a PDF was proposed, for example, by [48] on the basis of normalized quantities, $H_w/\left(2\sqrt{2m_0}\right)$ for wave amplitude; and $\tau = T_w/T_{01}$ for wave period:

$$T_{01} = 2\pi \frac{m_0}{m_1};\, m_n = \int\limits_0^\infty s_w(\omega)\omega^n d\omega, \tag{12}$$

where n is an order of the spectral moment. The period, corresponding to the mean frequency is related to the mean zero-crossing frequency through the spectral moment, see e.g. [47] $T_z = 2\pi\sqrt{m_0/m_2}$. The joint distribution of a and τ is expressed as:

$$pdf(a,\tau) = \frac{2k_N}{\nu\sqrt{\pi}}\left(\frac{a}{\tau}\right)^2 \cdot \exp\left\{-a^2\left[1+\frac{1}{\nu^2}\left(1-\frac{1}{\tau}\right)^2\right]\right\} \tag{13}$$

where k_N is a normalizing factor taking into account the positivity of period and amplitude, while ν is a spectral width parameter:

$$k_N = \frac{1}{2}\cdot\frac{\sqrt{1+\nu^2}}{1+\sqrt{1+\nu^2}};\quad \nu^2 = \frac{m_0 m_1}{m_1^2} - 1. \tag{14}$$

Using the dispersion relation in deep water between a wave length λ_w and a wave period $T_w = \sqrt{2\pi\lambda_w/g}$ (where g is the gravity acceleration), the PDF (13) can be re-written for the wave length λ_w and the wave steepness $s = H_w/\lambda_w$, using well-known formulae for distribution of deterministic function of random arguments (see e.g. Sect. 6.7 of [65]—the derivation is not difficult as it is essentially a substitution of the variables:

$$pdf(\lambda_w, s) = k_{N1}\lambda_w^{3/2}s^2 \cdot \exp\left\{-2\left(\frac{\lambda_w \cdot s}{H_s}\right)^2\left[1+\frac{1}{\nu^2}\left(1-T_{01}\sqrt{\frac{g}{2\pi\lambda_w}}\right)^2\right]\right\}, \tag{15}$$

where the constant k_{N1} is defined through the normalizing factor k_N:

$$k_{N1} = \frac{8T_{01}\sqrt{g}}{\pi\nu H_s^3}k_N. \tag{16}$$

The distribution of the wave steepness is the marginal distribution of (15); it cannot be expressed in elementary functions:

$$pdf(s) = \int_0^\infty pdf(\lambda_w, s)d\lambda_w. \tag{17}$$

A probability that the wave steepness exceeds a critical value s_{cr} for a given sea state is defined with a significant wave height H_s and a zero-crossing period T_z:

$$P(s > s_{cr}|H_S, T_Z) = \int_{s_{cr}}^{s=1/7} pdf(s|H_S, T_Z)ds. \tag{18}$$

The critical wave steepness s_{cr} is defined for each failure mode, while $s = 1/7$ is the breaking wave limit.

All the level 2 vulnerability criteria are formulated as long-term probabilistic criteria, i.e. weight-averaged over all possible sea states using statistical weights from a wave scatter table (such as [30]). Thus, it makes sense to evaluate the safety level of the level 1 vulnerability criteria also as an average of the short-term value over a scatter table:

$$SL = P_a = \sum_{i=1}^{N_S}\sum_{j=1}^{N_T} f_{i,j} P\big(s > s_{cr}|H_{Si}, T_{Zj}\big), \tag{19}$$

where $f_{i,j}$ is the statistical frequency for the ith significant wave height H_{Si} and the jth mean zero-crossing period T_{Zj}.

3 Dead Ship Condition

The dead ship condition corresponds to the assumed situation considered by the severe wind and rolling criterion (also known as weather criterion), which is formulated in Sect. 2.3 of part A of the 2008 IS Code [31]. As it follows from its name, the main propulsion is assumed not to be available. As a result, a ship drifts under the action of wind and waves. A position of a ship relative to wind is defined by the distribution of the windage area. A conventional steam-era ship, with approximately symmetric windage area forward and aft, usually takes a near beam seas position. For modern ship types, this assumption is made in order to maximize the projected area and therefore the heeling moment. Gusty wind makes the ship to heel and roll motions have a non-zero mean. A hydrodynamic drag, generated by the drift creates an additional heeling moment and contributes to the roll mean value.

The objective of an assessment of stability in the dead ship condition is to ensure that a vessel can withstand the action of wind and waves; this is taken to mean that a roll angle does not exceed a prescribed limit. Three important elements are included in the dead ship condition assumed situation:

- A large roll angle is associated with the failure, while capsizing is not considered;

- A ship is subjected to the combined actions of wind and waves; and
- The dynamics of ship motions must be considered.

The weather criterion considers a specific instance when a ship experiences a peak roll angle to the windward side (roll back angle) and followed with the application of a wind gust. The dynamic roll angle is found with the energy balance method, which assumes that the work of the heeling moment is equal to the increase of the potential energy of heeling. The drift-generated drag is included in the lever of the wind heeling moment.

A diploma thesis explored the possibility of a probabilistic interpretation of the weather criterion [83]. It was realized, however, that this turns out to be quite ambiguous as the applied excitations follow a rather idealized structure.

The development and background for the vulnerability assessment of the SGISC are described in Bulian and Francescutto [12–14]. The level 1 vulnerability criteria is essentially the weather criterion with an extended table for the natural roll period [paragraph 2.2.2.4, 32]. The probabilistic interpretation of the level 1 criteria is a challenge and can be addressed only in a statistical sense, a detailed consideration of this problem is given by [62].

The level 2 vulnerability criteria has a probabilistic formulation through the random exceedance of a prescribed limit from either side of the ship. The exceedance or upcrossing of a threshold is defined as a random event when the current value of a stochastic process equals the leeward threshold φ_{fail+} and the first derivative is positive $\left(\varphi(t) = \varphi_{fail+} \cap \dot{\varphi}(t) > 0\right)$. The general formula for the rate of exceedance or upcrossing of φ_{fail+} is:

$$\lambda_{fail+} = \int_{0}^{\infty} \dot{\varphi} \cdot pdf\left(\varphi = \varphi_{fail+}, \dot{\varphi}\right) d\dot{\varphi}, \tag{20}$$

where $pdf\left(\varphi = \varphi_{fail+}, \dot{\varphi}\right)$ is a joint PDF of the roll angle and the roll rate computed at the level of stability failure in the leeward direction.

The failure or exceedance through the windward side is expressed through the downcrossing of the level φ_{fail-}. The rate of downcrossing is expressed in a similar way to (11), but under the condition of a negative roll rate:

$$\lambda_{fail-} = \int_{-\infty}^{0} \dot{\varphi} \cdot pdf\left(\varphi = \varphi_{fail-}, \dot{\varphi}\right) d\dot{\varphi}. \tag{21}$$

The failure in the dead ship condition is the exceedance of either φ_{fail+} or φ_{fail-}, which is an assumption that both exceedances are rare (i.e., that this does not happen within the time interval of the auto-correlation of ship motion being significant). This failure is expressed through a simple sum of (11) and (13);

$$\lambda_{DS}(H_S, T_Z) = \int_{0}^{\infty} \dot{\varphi} \cdot pdf\left(\varphi = \varphi_{fail+}, \dot{\varphi}\right) d\dot{\varphi}$$

$$+ \int_{-\infty}^{0} \dot{\varphi} \cdot pdf\left(\varphi = \varphi_{fail-}, \dot{\varphi}\right) d\dot{\varphi}. \qquad (22)$$

The short term solution (13) requires knowledge of the joint distribution of roll angles and rates that needs to include large roll angles.

The evaluation of the joint distribution of roll angles and rates including large roll angles is a non-trivial task. The distribution is non-Gaussian and its shape depends on the hull geometry (mostly on the freeboard that effects the shape of the GZ curve (e.g. subsection 8.6.2 of [9]). For a non-Gaussian distribution, the roll angles and roll rates may be dependent while uncorrelated because only stochastic processes with a normal joint distribution are independent when they are uncorrelated. However, there are some indications that the assumption of independence of the roll angles and roll rates is applicable for beam seas [8]. A method for modeling the non-Gaussian distribution of large roll angles through Fokker-Plank-Kolmogorov equation was proposed by Maki [49, 50]. The influence of the hull geometry on the PDF is preserved despite the existence of white noise excitation. The actual PDF is obtained by scaling with the results of a numerical simulation.

An approximate approach was proposed by Bulian and Francescutto [12–14] and it was used for the level 2 vulnerability criterion of the dead ship condition stability failure mode. The idea is to use a linear roll process and to adjust a level of failure to account for nonlinearity. The values of this equivalent level of failure are computed from the first integral equations, which express the energy balance over one quarter of a period of roll oscillation.

4 Excessive Accelerations

The second generation intact stability criteria extends the definition of an intact stability failure to the lateral acceleration that exceeds a prescribed limit [paragraph 1.1.2.2.3 of 32]. The lateral accelerations were a main factor in two fatal accidents with the container carriers *Chicago Express* in September of 2008 [6] and *Guayas* in September of 2009 [5]. Fatalities and injuries were sustained by crew members who fell and were thrown across the navigation bridge in the course of these accidents. Both accidents occurred in stormy conditions in which large roll angles were observed. Both cases were characterized by a very high GM value (7.7 m and 5.6 m, respectively). Synchronous roll resonance is believed to be the main reason for both accidents. The situation was exacerbated by a decrease of roll damping (caused by slow speed) and by the high location of the navigation bridge. Development of the vulnerability assessment is described in [70], while the validation of a direct stability assessment for this failure mode is addressed in [45].

The criteria consider the highest point on a ship where passengers or crew may be present. The evaluation of the acceleration at the point is based on the kinematics of a point of a rigid body involved in an arbitrary motion.

For mathematical models with reduced degrees of freedom, further simplifications are appropriate [see Appendix 3, 36]. Roll acceleration combined with the lateral component of the gravity acceleration is considered to be the main factor causing the failure. Separation of the contribution of the roll acceleration from the contribution of other motions leads to the following equation for the lateral acceleration:

$$a_{ALat} = (a_{z0} - g)\sin\varphi + z_A\ddot{\varphi} + a_{y0}\cos\varphi. \tag{23}$$

The contribution from other degrees of freedom is accounted as horizontal a_{y0} and vertical a_{z0} components of the acceleration caused by motions other than roll, expressed in an earth-fixed coordinate system; where z_A is the coordinate of the point of interest expressed in ship-fixed coordinate system. Note that a difference in (23) and formula (1) in Ref. [Appendix 3, 36] is caused by the difference in coordinate systems applied.

A further simplification of (23) involves a linear assumption for roll motion. For both known accidents involving excessive accelerations, the maxima of the GZ curve were located above 50°. Therefore, the observed roll angles (20°–30°) were in the nearly linear range. The assumption of linearity could be justified by the fact that the excessive accelerations are expected in case of high metacentric heights, when the GZ curve is dominated by the “initial stability” in the expected range of roll angles.

The assumption of linearity of the GZ curve allows the use of frequency-domain models for irregular roll motions. The response amplitude operator (RAO) for lateral acceleration is expressed through the magnitude of the lateral acceleration, which is derived from (23). Further simplifications include expressing the influence of other degrees of freedom through a location coefficient k_L, which takes into account the simultaneous action of roll, yaw and pitch motions:

$$a_{ALat}(\omega) = \varphi_a(\omega)k_L\left(g + \omega^2 z_A\right). \tag{24}$$

where ω is a wave frequency; a formula for k_L is given in paragraph 2.3.2.1 of [32]. Formula (24) is used for the vulnerability assessment on both level 1 and level 2; further simplifications for level 1 are aimed at the elements of roll motions.

Level 1 vulnerability criterion is a deterministic criterion and uses a characteristic roll angle that depends on wave steepness. The wave steepness is interpreted as a random variable and is used for the safety level evaluation, see Eq. (19). Table 2.3.2.1 in [32] provides values of wave steepness as a function of the natural roll period. These values of wave steepness can be considered as critical for use in Eq. (19).

The well-justified linear assumption for roll motions leads to a normal distribution of lateral accelerations for the considered failure mode scenario (i.e., when the GM value is large). The rate of the upcrossing, λ_{Lat}, of the acceptable level of lateral acceleration, $R_2 = 9.81$ m/s^2, is expressed as

$$\lambda_{Lat} = \frac{1}{2\pi}\frac{\sigma_{\dot{a}_{ALat}}}{\sigma_{a_{ALat}}}\exp\left(-\frac{R_2^2}{2\sigma_{a_{ALat}}^2}\right), \tag{25}$$

where $\sigma_{a_{ALat}}$ is the standard deviation of lateral accelerations, while $\sigma_{\dot{a}_{ALat}}$ is the standard deviation of a temporal derivative of the lateral acceleration.

There are certain difficulties in the frequency-domain evaluation of the standard deviation of a temporal derivative of the lateral acceleration. This is related to the accepted formulation of the spectral density of wave elevations that may have a convergence problem when used with high-order frequency derivatives. To avoid this problem, the level 2 vulnerability criterion kept only the exponential part of Eq. (25) and the standard $R_{\mathrm{EA2}} = 0.00039$ was calibrated for this truncated formula. This calibration requires additional consideration for evaluation of the safety level for the excessive acceleration vulnerability criteria level, which remains as future work.

5 Pure Loss of Stability

At a given draft, the ship waterplane may be narrow at the bow and stern, while near the midship section it is relatively full. At the same time, the waterplane is full at the full depth level. These basic geometry features may lead to decreased stability while a crest of a longitudinal wave is located near the midship section. Sometimes, this loss of stability may be so significant (even completely negative) that a ship capsizes or heels over to a large angle. This type of stability failure is referred as "pure loss of stability on a wave crest" or just "pure loss of stability". A universally accepted theory describing a failure caused by pure loss of stability is not available at this writing.

The fact that stability decreases when a ship is located in the wave crest has been known to naval architecture for well over a century [64]. It was observed on a segmented model in 1949 [46]. However, practical calculation methods were not available until the 1960s [57]. A decade later, it was recognized as a separate mode of stability failure [60] while observing capsizing due to this phenomenon in free-running model experiments in San Francisco Bay. Kan et al. [20] de Kat and Thomas [39] and others also demonstrated capsizing due to pure loss of stability by free-running model experiments in seakeeping and maneuvering basins.

There are a number of single large roll accidents in which pure loss of stability may have been a trigger: the rail ferry *Aratere* in March 2006 [55], the ro-ro ship *M/V Finnbirch* in November 2006 [82], the fast ferry *M/V Ariake* in November 2009 [38], and the container ship *M/V Svendborg Maersk* in February 2014 [18].

The main feature of pure loss of stability is the significant change of the stiffness of the dynamical system. The stiffness may even become completely negative, which has the effect of turning a dynamical system into a "repeller" (see e.g., [4] or [81]).

Key elements of the GZ curve can be assessed as stochastic processes in irregular waves. The first attempt to describe the behavior of the instantaneous GM was made by [21] using a Gaussian distribution. Later, [11] applied it to describe pure loss of stability. However, it was found that the behavior of stability elements in irregular waves is too complex to be described by a normal distribution [10]. This complex

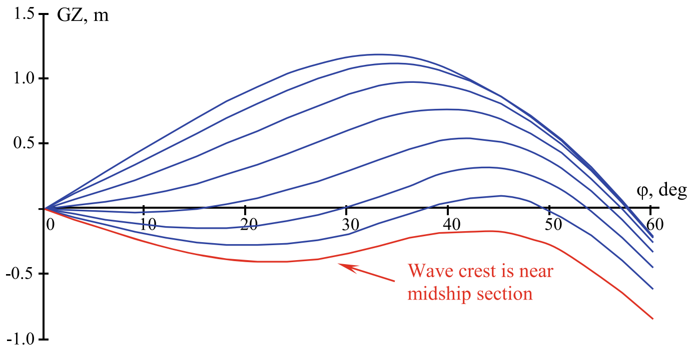

Fig. 1 GZ curve variation in wave

probabilistic behavior of the stability elements in irregular waves makes the [23] effective wave the preferable practical solution. The accuracy of Grim's effective wave was studied by [88, 86] and found sufficient for practical use. To account for surging [86] "modulated" Grim effective wave, it allows modeling the effect of increased timing of decreased stability.

The background of the vulnerability assessment for pure loss of stability is described in [44] and [90]. A low-freeboard extended weather deck has a significant influence on the physics of pure loss of stability failure because this feature can allow a large volume of green water on the ship that changes the dynamics of the ship and the stability variation may not be sole factor causing a stability failure. The results of experimental and numerical studies can be found in [91] and [29].

The essential feature of pure loss of stability is a negative stability experienced by a ship during a certain interval of time. Figure 1 shows an example of the variation of the GZ curve computed for a 260 m long containership with a draft of 8.4 m encountering a following wave of a length equal to the ship length. The height of the vertical center of gravity is assumed as the maximum to satisfy the 2008 IS Code [31], part A, which corresponds to a GM of 0.39 m.

The roll stiffness, shown by the GZ curve, changes from positive to negative and then from negative to positive during the passing of a longitudinal wave (i.e. a "wave pass"). The solution of the roll equation experiences drastic changes with these stiffness variations. To understand this better, consider an equation of roll motion that includes the variation of the restoring arm $GZ(\varphi, t)$, depicted in Fig. 1.

$$(I_{xx} + A_{44})\ddot{\varphi} + B_{44}\dot{\varphi} + \Delta g GZ(\varphi, t) = 0, \tag{26}$$

where A_{44} is the added mass in roll, B_{44} is the roll damping.

The drastic changes can be illustrated with the solutions of the linear equations for positive and negative stiffness. For the positive stiffness, the linear roll equation and its solution are well known:

$$(I_x + A_{44})\ddot{\varphi} + B_{44}\dot{\varphi} + \Delta g GM\varphi = 0, \tag{27}$$

$$\varphi = \varphi_A \sin(\omega_d t + \varepsilon), \tag{28}$$

where φ_A and ε are arbitrary constants, depending on initial conditions and $\omega_d = \sqrt{\omega_0^2 - \delta^2}$ is the frequency of small damped roll oscillations, ω_0 is the natural roll frequency and $\delta = B_{44}/2(I_x + A_{44})$ is the roll damping coefficient.

The solution of the linear roll equation with the negative stiffness is quite different because it is exponential rather than oscillatory where the eigenvalues $\lambda_{1,2} = -\delta \pm \sqrt{\omega_1^2 + \delta^2}$ are real where ω_1^2 is a stiffness coefficient: $\omega_1^2 = \Delta g|GM_1|/(I_x + A_{44})$ (frequency is no longer relevant as $GM_1 < 0$)

$$(I_x + A_{44})\ddot{\varphi} + B_{44}\dot{\varphi} - \Delta g|GM_1|\varphi = 0, \tag{29}$$

$$\varphi = C_1 \exp(\lambda_1 t) + C_2 \exp(\lambda_2 t), \tag{30}$$

where $C_{1,2}$ are arbitrary constants which depend on the initial conditions. Dynamical system described by the (29) is known as "repeller".

One of the eigenvalues of the solution (30) is negative, but the other one is positive. The solution (30) is unlimited. The time t_f necessary to reach a large roll angle φ_f is approximated by neglecting the exponential function with the negative eigenvalue:

$$t_f = \frac{1}{\lambda_1} \ln\left(\frac{\varphi_f}{C_1}\right). \tag{31}$$

The linear solution (30) is only appropriate for a qualitative description because the actual phenomenon includes large amplitudes of roll and time-dependent stiffness. An equivalent linear stiffness may be derived by balancing the potential energy that corresponds to the initial time-dependent stiffness with the potential energy of a linear system. This approach further may lead to an analytical formulation for dynamics of pure loss of stability in regular waves, see Spyrou [77]. However, the level 2 vulnerability assessment of SGISC in [32] does not include any mathematical model of dynamics and instead focuses on the stability variation in waves. Following the ideas presented in Spyrou [77], alternative vulnerability criteria were proposed that include dynamical considerations, see [62].

The level 1 vulnerability criteria for pure loss of stability is described in Sect. 2.4.2 of [32]. This is essentially a simplified calculation of GM with the ship assumed to be situated with the crest of a longitudinal wave at amidships. The critical value of the wave steepness is set to 0.0334 and can be used in Eq. (19) to evaluate the safety level.

6 Parametric Roll

Parametric roll resonance is an amplification of roll motion caused by periodic variation of stability in longitudinal waves.

The large-scale container loss that occurred on the container carrier M/V APL China in October 1998 was attributed to parametric roll beyond reasonable doubt [22]. Parametric roll can be also suspected as cause of accident of M/V Pacific Sun [54]. Beyond the IMO, the problem of parametric roll has being addressed by classification societies [1, 16] and the International Towing Tank Conference [37].

The theoretical possibility of parametric roll was studied in [58] and the observation of this phenomenon in a model test is described in [59].

The background for vulnerability assessment for parametric roll is described in Spyrou [75], Bulian and Francescutto [15, 14] and Sakai et al. [67]. The probabilistic treatment for the level 2 vulnerability criteria is based on the application of the Grim's effective wave [23] where the calculation of the encounter wave period for the Grim effective wave is considered in [66].

The Mathieu equation is the simplest mathematical model of parametric roll and it has been extensively used to analyze this phenomenon (e.g. [61]).

$$\frac{d^2x}{d\tau^2} + (p + q \cdot \cos(\tau)) \cdot x = 0, \tag{32}$$

where the variable x is related to roll motion through an exponential formula (to eliminate damping), τ is non-dimensional time, p and q are numerical parameters related to calm water GM and the magnitude of the GM variation in waves, respectively.

The Mathieu equation is a linear differential equation with variable coefficients, but its solution cannot be expressed with elementary functions. The solution is considered to be a specialized function, known as the Mathieu function. It is tabulated and is included in advanced mathematical software packages. The Mathieu functions may exhibit two types of behavior: bounded and unbounded, each depending on values of the parameters p and q. A graphical representation of this dependence is known as Ince-Strutt diagram. Formulae for the approximation of the boundaries between the bounded and unbounded types of Mathieu functions can be found in [27].

Due to its linear nature, the Mathieu equation cannot yield an amplitude of steady-state parametric roll. To evaluate roll amplitude caused by parametric roll, the nonlinearity of a GZ curve must be included, see e.g. [61]. To avoid the complexity of a nonlinear differential equation, Spyrou [75] proposed to use a transient solution of the Mathieu equation to formulate a criterion and a standard for development of dangerous parametric roll:

$$\frac{\delta GM_1}{GM} \leq 2\frac{\ln f + \ln 2}{\pi n} + \frac{4\delta}{\omega_0}, \tag{33}$$

where δGM_1 is an amplitude of variation of the metacentric height, δ is roll damping coefficient, ω_0 is natural roll frequency, and f is an amplification factor of parametric resonance achieved during n roll oscillations. Two factors are included in the standard, which are shown on the right-hand side of Eq. (33): damping δ and amplification f.

The level 1 vulnerability criterion for parametric roll and its appropriate standard is described in section 2.5.2 of [32]. With the exception of ships with sharp bilges, the standard R_{PR} in paragraph 2.5.2.1 of [32] consists of two components:

$$R_{PR} = 0.17 + C_{bk}, \tag{34}$$

where the value C_{bk} accounts for the contribution of the bilge keels and is computed as a function of the area of the bilge keels, length, beam and midship section coefficient.

The value 0.17 may be attributed to the transient and roll damping of the bare hull and appendages other than bilge keels. To estimate a safety level for the parametric roll criterion, a conservative assumption can be taken that the entire value of 0.17 is attributed to the transient. For the initiation of parametric roll, the amplification factor has to be more than 1.0, but this is another conservative assumption. According to Eq. (33), the value 0.17 will be achieved during approximately 2.6 roll oscillations, which is equal to approximately 5 wave encounters.

Further consideration requires a probabilistic characterization of encountering several waves of certain parameters. The wave group approach seems to be the most natural one. First, the application of the wave group approach to ship dynamics was proposed by Spyrou and Themelis [80]. This was followed by an application of this approach to a long-term probabilistic assessment of stability during the voyage [84] as well as the application to broaching [89] and parametric roll [52]. For the current state-of-the-art on wave groups, see [2, 26] as well as [71].

Since the wave group is defined as N_W waves that exceed a certain threshold a_G, the event of the encounter of a wave group can be considered as the upcrossing of the threshold followed by $N_W - 1$ waves with peaks exceeding the threshold. If the threshold is set high enough, the event of the encounter can be considered to follow Poisson flow with the rate:

$$\xi_G(N_W) = \xi_G(a_G) P\left(\bigcup_{i=2}^{N_W} (a_i > a_G)\right), \tag{35}$$

where the $\xi(a_G)$ is a rate of upcrossing of the threshold a_G by the water surface. Assuming a normal distribution for wave elevations, the rate is expressed as

$$\xi_G(a_G) = \frac{1}{2\pi}\sqrt{\frac{V_D}{V_W}} \exp\left(-\frac{a_G^2}{2V_W}\right), \tag{36}$$

where V_W is the variance of the wave elevation and V_D is the variance of the derivative of wave elevations. Further modeling of the wave group follows [84].

The properties of wave amplitudes are described using envelope theory. The assumption that only amplitudes of two consecutive waves are dependent is made because the autocorrelation function of the wave envelope usually goes to zero within two mean periods. With this, the set of amplitudes of consecutive waves can be represented by a Markov chain and the rate of encounter of a wave group with N_W waves can be written as

$$\xi_G(N_W) = \xi_G(a_G)(P(a_2 > a_G|a_1\rangle a_G))^{N_W-1}. \tag{37}$$

The conditional probability that the second wave exceeds the threshold a_G as the first wave exceeds it as well is calculated from the joint distribution of two consecutive amplitudes available from envelope theory:

$$f(a_1, a_2) = \frac{a_1 a_2}{V_W^2 p^2}\exp\left(-\frac{a_1^2 + a_2^2}{2V_W p^2}\right)\mathbf{I}_0\left(\frac{a_1 a_2\sqrt{1-p^2}}{2V_W p^2}\right), \tag{38}$$

where $\mathbf{I}_0$ is the modified Bessel function of the first kind and zero order (the standard function is included in most mathematical handbooks and software packages) and p is the parameter derived from the spectrum:

$$p = \sqrt{1 - k^2 - r^2}, \tag{39}$$

where $k()$ and $r()$ are the autocorrelation function and the result of the sine transformation of the wave spectrum density $s_w(\omega)$, which is computed for the time lag τ that is equal to a period corresponding to the mean frequency $\tau = T_{01}$:

$$k(\tau) = \int_0^\infty s_w(\omega)\cos(\omega\tau)d\omega; \quad r(\tau) = \int_0^\infty s_w(\omega)\sin(\omega\tau)d\omega. \tag{40}$$

The formulation of the level 1 vulnerability criterion in paragraph 2.5.2.2 uses the wave steepness value $s_{PR} = 0.0167$. For the wave length equal to ship length, $a_G = 0.5s_{PR}L$. If the wave length that causes significant stability variation is between λ_{w1} and λ_{w2} (say, between 1 and 2 ship length), then a rate encounter of a wave capable of causing parametric roll can be approximated as:

$$\xi_G(N_W) = (P(a_2 > a_G|a_1\rangle a_G))^{N_W-1}\int_{\lambda_{w1}}^{\lambda_{w2}} \xi_G(0.5s_{PR}\lambda_w)\mathrm{pdf}(\lambda_w)d\lambda_w, \tag{41}$$

where $\mathrm{pdf}(\lambda_w)$ is the marginal pdf for wave lengths.

Equation (41) describes the short-term rate of encounter of wave groups capable of causing parametric roll according to the level 1 vulnerability criteria. This equation preserves the dependence on time. If the safety level is needed in a time-independent form (like Eq. 11), then the rate of encounter (11) may be substituted by a probability of exceedance of the wave amplitude using the Rayleigh distribution:

$$p_G(N_W) = (P(a_2 > a_G|a_1\rangle a_G))^{N_W-1} \int_{\lambda_{w1}}^{\lambda_{w2}} p_G(0.5 s_{PR}\lambda_w)\mathrm{pdf}(\lambda_w)d\lambda_w;$$

$$p_G(a_G) = \frac{a_G}{2\pi V_W}\exp\left(-\frac{a_G^2}{2V_W}\right). \tag{42}$$

The long-term safety level is computed by averaging Eq. (41) or (42) over all the sea states represented in the wave scatter table.

The level 2 vulnerability criteria is described in Sect. 2.5.3 of [32]. Both C1 and C2 are probabilistic criteria; the safety level is equal to the corresponding standard identified in paragraph 2.5.3.1 of [32].

7 Surf-Riding and Broaching

Broaching (a shortening of "broaching-to") is a violent uncontrollable turn (or large yaw rate) that occurs despite maximum steering efforts to maintain course that are often accompanied with a dangerously large heel angle that leads to a partial or total stability failure. Surf-riding is a transition from a periodic surging motion to a situation where the ship takes on the speed of the wave. Surf-riding is the most common pre-requisite for broaching, but it is not the only one [74].

Broaching-to is believed to be a primary reason behind the capsizing of the Papua New Guinean passenger ship *M/V Rabaul Queen* on the route from Kimbe to Lae on February 2nd, 2012, which caused the death of at least 142 and possible as many as 161 persons [3].

As a phenomenon, broaching was known as a major threat from the age of sail [79]. The scientific description of broaching and surf-riding date to the middle of the twentieth century [19, 17, 92, 93]. The early phase plane analysis [53] pointed towards the true nature of the phenomena. The development of nonlinear dynamics [25], together with the analysis of surf-riding experiments [40] prompted the modern understanding of the physics of surf-riding and broaching [73, 74, 78, 85].

A recommendation on the avoidance of surf-riding is included in [33], which is the same criterion that was used for the level 1 vulnerability assessment based on [87]. The level 2 vulnerability assessment is based on the Melnikov analysis [56]. The initial application of linearized resistance is described in Kan [41], while versions that include nonlinear resistance is available in [51, 76] and [68]. These versions, however, used an assumption of small damping in the surge equation. Wu et al. [93] theoretically demonstrated the validity of the small damping assumption.

The physics of surf-riding and the justification of the criteria are described in another chapter of this book [63]. The level-1 vulnerability criterion is described in Sect. 2.6.1 of [32]. The criterion was developed by assuming the wave steepness to be 0.1, which means that the safety level can be computed using Eq. (19). The level 2 vulnerability criterion is described in 2.6.2 of [32]. Because this is a probabilistic criterion, its safety level is equal to its standard, which is defined in paragraph 2.6.3.2 of [32].

8 Final Remarks

This chapter has summarized the scientific background of the IMO second generation intact stability criteria (SGISC). In particular, the implementation of the concept of "safety level" in the SGISC, that is the probability of failure if a criterion is satisfied, has been analyzed.

There are two types of the criteria in SGISC: probabilistic and deterministic. An assessment of the safety level of a probabilistic criterion is straightforward since it equals the associated standard. Evaluation of the safety level for a deterministic criterion is more challenged because the random elements need to be identified in the criterion's formulation. While the criterion may be treated as a deterministic function of random variables, the details, however, may not be so straight forward.

The SGISC covers five intact stability failure modes: dead ship condition, excessive accelerations, pure loss of stability parametric roll, surf-riding and broaching. For the level 2 vulnerability criteria, the derivation of the safety level is simple, since for all failure modes these criteria are probabilistic. However, as all the level 1 criteria are deterministic, the safety level for these criteria requires probabilistic interpretation as follows:

- For the dead ship condition failure mode, there is no robust probabilistic interpretation;
- For excessive accelerations, pure loss of stability and surf-riding failure modes, the probabilistic consideration is that of wave steepness; and
- For the parametric roll failure mode, a combination of a simplified wave group method and a probabilistic consideration of wave steepness is needed.

Considerations of the safety level of these criteria are still quite abstract, but a detailed numerical analysis remains for the future work.

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