An Efficient Formulation of the Critical Wave Groups Method for the Assessment of Ship Stability in Beam Seas

Panayiotis A. Anastopoulos and Kostas J. Spyrou

Abstract The paper presents a simplified setup of the “critical wave groups” method, suitable for swift probabilistic evaluations of ship capsize tendency due to beam-sea resonance. The simplifications proposed herein are twofold and aim at reducing the computational cost associated with the identification of the critical, for ship stability, wave episodes when these are represented by the “expected” wave groups for the ambient sea state. The first simplification concerns the initial conditions of the vessel at the moment of a wave group encounter which, according to the exact “critical wave groups” formulation, should be probabilistically treated. Instead, the simplified approach pursues reliable estimates by examining only the upright equilibrium state. Moreover, by focusing on sea states being highly probable to provoke resonance, fewer simulations need to be performed since, among all critical wave group candidates, the main probability contribution essentially comes from those having periods close to the natural period of the vessel in question. Considering these wave groups only constitutes the second simplification. Within this framework, regular wave trains are also tried to investigate the possibility of eliminating the computational burden due to the generation of the “expected” wave groups. The accuracy of both schemes in calculating the probability of extreme responses is assessed through comparisons with Monte Carlo simulations of roll motion.

Keywords Critical wave groups · Probability · Instability · Roll · Dynamics · Resonance · Extreme events

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1 Introduction

The study of large amplitude ship roll motions in stochastic beam seas is a non-trivial task expanding in both the fields of nonlinear dynamics and probability. As known, roll statistics deviate from Gaussianity with increasing level of nonlinearity, leading to probability distributions with heavy-tailed structure [6]. However, calculating the probability of extreme roll events by employing "brute force" methods suffers from a number of deficiencies. First, the accuracy of a "direct counting" definition of probability becomes questionable when dealing with rare events. At the same time, the fact that ship response is not essentially an ergodic random process for nonlinear systems calls for the use of ensemble averages in the direct counting procedure [4]. This, however, requires additional effort (comparing to an ergodic system) for generating a statistically meaningful amount of extremes since many short realizations need to be generated while their largest part, reflecting the "statistical" transients, will eventually be discarded (e.g. [3]). More so, even if temporal averages are to be used, one has to set-up the simulations carefully to sample throughout a relevant response sample space in the correct proportions without idly expending computational resources. Clearly, this type of sampling is not as straightforward as it would be for an ergodic dynamical system and thus, it may further deteriorate the efficacy of massive simulations in tracing the complex shape of the tails.

Various methods have been proposed to treat the so called "problem of rarity", described in the above. Extrapolation methods employ statistics based on a limited number of realizations to predict the probability of an event that is too rare to be observed. The concept derives from extreme value theory (EVT) which is built upon two main theorems providing asymptotic expressions for the distributions of the maximum (first theorem) and of the excesses over a threshold (second theorem) of a sample of independent and identically distributed random variables. Thus, the objective is the estimation of the parameters of an extreme value distribution through fitting to a set of experimental or simulation data. The effectiveness of the approach has been investigated in several studies and much effort has been put into addressing practical issues regarding its application for ship stability assessment (e.g. [5, 23]).

On the other hand, wave group methods offer an alternative solution to the problem by focusing on specific time intervals when dangerous wave events occur. One of them is the "critical wave groups" method which quantifies instability tendency through the probability of encountering any wave group that could have provoked the instability [20]. In the deterministic part of the method, regular wave trains are employed to identify critical, for ship stability, height thresholds. Then, in the probabilistic part, the probability of encountering any wave sequence higher than the specified thresholds is calculated using distributions of wave heights and periods derived either empirically, from simulations of the wave field, or theoretically, directly from the spectrum. A first attempt to validate the concept was presented by Shigunov et al. [15] who selected a modern 8000 TEU containership to calculate the probability of exceeding a 40° roll angle threshold. The results were tested against Monte Carlo simulations and fair coincidence was noted in the case of beam seas excitation.

Recently, Anastopoulos and Spyrou [3] demonstrated that the performance of the method in predicting extreme roll responses in beam seas is improved if critical thresholds are defined in terms of realistic wave group shapes being, in fact, the "most expected" representatives of the ambient sea state. In assessing the accuracy of the approach, it was concluded that treating the initial state of the vessel in a probabilistic context is essential when dealing with sea conditions causing very few extremes.

In this paper, a simplified setup of the most contemporary version of the "critical wave groups" method, presented in [3], is developed. The simplifications address the problem of exhaustively generating wave group environments in the process of determining the critical ones and expand in two directions. In the first, the idea is to focus our attention on identifying instability-causing wave groups for only one set of initial conditions of the ship at the beginning of the simulations. To determine the most relevant, for ship stability evaluation, initial state, well-established concepts of dynamical systems theory are invoked. In the second direction, the intention is to reduce the number of possible critical wave group shapes by exploiting the features of the "expected" wave groups derived for a given sea state. The method is applied to two ship models operating in qualitatively different, with respect to the frequency that extreme responses are realized, sea states in order to calculate the probability of exceedance for a number of roll angle thresholds. In this context, the conditions under which the simplified "critical wave groups" scheme produces comparable results with those obtained from Monte Carlo simulations of roll motion are investigated and the focus is set on the region of extreme responses where the accuracy of the latter is disputable.

2 A Simplified "Critical Wave Groups" Method

In the literature of ocean and coastal engineering, wave groups are traditionally defined as sequences of waves with heights exceeding a certain preset level and periods varying within a potentially narrow range [11, 13]. No doubt, such a definition can only rely on subjective criteria regarding the selection of an "appropriate", for the identification of wave grouping phenomena, height threshold. To overcome this issue in analyzing ship dynamics, wave groups are considered herein as sequences of waves which, given the variability of their periods, are sufficiently high to provoke instabilities.

2.1 Mathematical Formulation

Let us assume that we are interested in estimating the probability that a vessel exceeds a limiting, from ship stability point of view, roll angle threshold φ_{cr}. The key idea of the "critical wave groups" method is to first identify the wave events that cause the exceedance and then, calculate the probability of encountering them. This is expressed by Eq. (1), presented below:

$$\Pr[\varphi > \varphi_{cr}] = \sum_k \underbrace{\Pr\left[\varphi > \varphi_{cr} | \left(\bigcup_q wg_{k,q}, ic_k\right)\right]}_{=1} \times \Pr\left[\bigcup_q wg_{k,q}, ic_k\right] \tag{1}$$

where $wg_{k,q}$ is a wave group event with characteristics q, determined for the k-th set of initial conditions $\{\varphi_0, \dot{\varphi}_0\}$ of the vessel at the moment of the encounter. Nonetheless, evaluating stability by considering a large number of initial states $\{\varphi_0, \dot{\varphi}_0\}$ can be time-consuming and eventually may become impractical in early design stages when decision-making requires swift calculations. In assessing the influence of initial conditions on transient ship dynamics, Thompson and co-workers discovered that capsize tendency is associated with a specific excitation level which is immensely independent of $\{\varphi_0, \dot{\varphi}_0\}$ for a given ship hull (e.g. [22]). This is due to the fact that, at this critical level, the erosion of the safe basin is sudden, rapid and, most importantly, starts "from within", i.e. in the vicinity of $\{\varphi_0, \dot{\varphi}_0\} = \{0, 0\}$. It can be argued, therefore, that examining only the upright equilibrium position of the vessel, corresponding to $k = 0$ in Eq. (1), can be somehow acceptable. Moreover, from a preliminary investigation on the sensitivity of the estimates of the "critical wave groups" method to the initial conditions, Themelis and Spyrou [21] concluded that retaining only $k = 0$ in Eq. (1) may be sufficient for sea states being highly probable to provoke resonance. Based on the above, the probability of exceeding φ_{cr} is approximated here through the following equation:

$$\begin{aligned}\Pr[\varphi > \varphi_{cr}] &= \sum_k \Pr\left[\bigcup_q wg_{k,q} \middle| ic_k\right] \times \Pr[ic_k] \\ &\approx \Pr\left[\bigcup_q wg_{0,q} \middle| ic_0\right] := \Pr\left[\bigcup_q wg_q^{(0)}\right]\end{aligned} \tag{2}$$

where the symbol ":=" is used to indicate the introduction of a new (more compact) notation (right-hand side) for the probability object appearing in the left-hand side. The superscript (0) implies that the probability calculations are performed over the set of instability-causing wave groups $\left(wg_q\right)$ determined for $\{\varphi_0, \dot{\varphi}_0\} = \{0, 0\}$ only. Equation (2) describes the essence of a swift "critical wave groups" approach being, in fact, a reduced order version of the method presented in [3] which duly accounts for the probabilistic nature of the initial state $\{\varphi_0, \dot{\varphi}_0\}$ of a vessel when hit by a wave group.

For large φ_{cr} one may assume that individual wave group occurrences are sufficiently rare to be treated as statistically independent. In the light of this, Eq. (2) is reformulated as [3]:

$$\Pr\left[\bigcup_q wg_q^{(0)}\right] = 1 - \Pr\left[\overline{\bigcup_q wg_q^{(0)}}\right]$$
$$= 1 - \Pr\left[\bigcap_q \overline{wg_q^{(0)}}\right] = 1 - \prod_q \left(1 - \Pr\left[wg_q^{(0)}\right]\right) \quad (3)$$

where the overbar denotes the complement of an event. A significant challenge in Eq. (3) is to ensure that wave groups causing $\varphi > \varphi_{cr}$ form a set of mutually exclusive and collectively exhaustive events. To avoid possible overlaps in the calculations, wave groups are classified with respect to their characteristics q being: (a) the run length j, i.e. the number of consecutive heights exceeding a critical threshold and (b) the range within which the periods of participating waves are considered to vary $T_{cr,m}$:

$$\Pr\left[wg_q^{(0)}\right] = \Pr\left[wg_{m,j}^{(0)}\right] = \Pr\left[\mathbf{H}_j > \mathbf{h}_{cr,j}, \mathbf{T}_j \in T_{cr,m}\right] \quad (4)$$

where $\mathbf{H}_j = \{H_1, \ldots, H_j\}$ and $\mathbf{T}_j = \{T_1, \ldots, T_j\}$ are vectors of random variables referring respectively to the heights H_n and periods T_n of an individual wave group event with run length j $(1 \le n \le j)$, $\mathbf{h}_{cr,j} = \{h_{cr,1}, \ldots, h_{cr,j}\}$ is a deterministic vector containing the heights of a critical wave group with run length j. It is remarked that, in (4), the vectorial inequality denotes comparisons between the corresponding components of the two vectors.

Eventually, the calculation of the right-hand side of Eq. (4) is performed in two parts: a deterministic one, focused on the identification of the so called "critical" wave groups, i.e. those wave successions leading to only slight exceedance of φ_{cr}; and a probabilistic part for calculating the probability of encountering any wave group higher than the determined critical. The implementation of the former is, in general, straightforward and requires a ship motion model and a method for systematically generating wave group excitations. Then, for a given set of wave group parameters $\{T_{cr,m}, j\}$, the associated $\mathbf{h}_{cr,j}$ vector can be determined through successive simulations, each of them testing a different, in terms of the heights of participating waves, group scenario, until the critical height sequence is detected [3]. As realized, the impact of the deterministic part on the effectiveness of the overall approach is explicitly related to the shape of the waveforms employed for representing critical wave groups. As for the probabilistic part of the approach, we follow the work of [10] who introduced the idea of modeling wave successions as Markov chains.[1] Nowadays, the concept enjoys wide acceptance by the scientific community since it has been successfully validated several times by both numerical simulations and real wave field measurements (e.g. [17]). Within the Markovian framework, the probability of encountering dangerous wave groups with certain specifications, as in Eq. (4), can be expressed as:

[1] In very simple terms, a Markov chain is a sequence of random events in which a future outcome depends solely on the event realized at the previous step.

$$\Pr\left[\mathbf{H}_j > \mathbf{h}_{cr,j}, \mathbf{T}_j \in T_{cr,m}\right] = p_0 \times \prod_{n=2}^{j} \int_{h_{cr,n}}^{+\infty} \int_{T_{cr,m}} f_{H_n,T_n|H_{n-1},T_{n-1}}(h_n, t_n|h_{n-1}, t_{n-1}) dt_n dh_n \quad (5)$$

where $m = 1, 2, ..., M$ denotes different cases of critical period segments and:

$$p_0 = \int_{h_{cr,1}}^{+\infty} \int_{T_{cr,m}} f_{H_1,T_1}(h_1, t_1) dt_1 dh_1 \quad (6)$$

In the above, $f_{H_n,T_n|H_{n-1},T_{n-1}}$ is the conditional probability density function (PDF) of wave height and period at time-step n given the values of these variables at the previous time step ($n - 1$), while f_{H_n,T_n} is the joint height-period PDF of a single wave. Hence, the product term in Eq. (5) gives the probability of encountering a critical (or worse) sequence of $j - 1$ waves given that an initial wave with height $h_1 > h_{cr,1}$ and period $t_1 \in T_{cr,m}$ is realized, while p_0 is the probability of actually experiencing an initial wave with these characteristics.

2.2 Wave Groups Construction Method

Based on the Markovian property of sea waves, [1] developed a method for predicting the shape of the "expected" wave groups for a given sea state. In its original version, the method requires as input the run length j and the characteristics (height h_c and period t_c) of the highest wave in order to generate the particular "expected" wave group for these specifications. In [3], a modified version of this method was discussed where the wave group construction algorithm could also allow for tuning the periods of individual waves to vary within a desired range $T_{cr,m}$. This, on the one hand, extends the capabilities of the "critical wave groups" method itself since the original construction process basically produces wave groups with periods varying only in the vicinity of the mean period of the assumed sea state. Therefore, when the natural period of the vessel in question is far from this regime, very few critical wave groups can be generated and eventually, the probability of stability failure is underestimated. On the other hand, having an additional design parameter ($T_{cr,m}$) entails a larger number of possible wave group formations which, in turn, have to be tested in the deterministic part of the method. Here, aiming at formulating a relatively simpler "critical wave groups" setup, requiring fewer simulations, we resort to the original construction algorithm of [1], yet knowing that the effectiveness of the current (more efficient) approach will presumably be challenged when the examined sea conditions are very unlikely to provoke instabilities due to resonance. The algorithm is implemented in two steps described, in brief, next.

Given the wave group specifications $\{j, h_c, t_c\}$, the first step is to predict the "expected" values (in time) of the heights and periods of the participating waves. To this end, the height h_c and period t_c of the highest wave, assumed to occupy the n-th position ($1 \leq n \leq j$) in the wave sequence, are used for initiating the following iterative scheme:

$$\overline{h}_n = \int_0^\infty h_n f_{H_n|H_{n-1},T_{n-1}}(h_n|h_{n-1}, t_{n-1})dh_n \tag{7}$$

$$\overline{t}_n = \int_0^\infty t_n f_{T_n|H_n,H_{n-1},T_{n-1}}(t_n|h_n, h_{n-1}, t_{n-1})dt_n \tag{8}$$

where the overbar is used to denote the expected value of the corresponding random variable. The integral kernels $f_{H_n|H_{n-1},T_{n-1}}$ and $f_{T_n|H_n,H_{n-1},T_{n-1}}$ are the transition PDFs of the Markov chain and can be obtained either from spectral methods [1] or by "direct counting" procedures based on Monte Carlo simulations of the wave field [2]. Provided the time reversibility property of this particular Markov chain, and since the characteristics of the highest wave are a priori known, at most $j - 1$ iterations of Eqs. (7) and (8) are required for predicting the "expected" characteristics of the surrounding waves. If, for example, the objective is to construct a wave group with $j = 5$ and highest wave encountered in the 4th position, then it is sufficient to predict the heights and periods $\{h_n, t_n\}$ of the 3 preceding waves ($n = 1, ..., 3$) and eventually set $\{h_5, t_5\} = \{h_3, t_3\}$ due to time reversibility. The concept applies to all wave group configurations, unless the highest wave occupies either the first or the last (j-th) position and thus, all $j - 1$ iterations need to be performed. Finally, to avoid any confusion due to the notation used herein, we emphasize that Eqs. (7) and (8) naturally differ from Eqs. (6) and (8) in [3] due to the absence of the $T_{cr,m}$ parameter from the current approach, as discussed in the above. Moreover, Eq. (8) improves Eq. (4) of [1] since it takes into account the correlation between the height and the period of a predicted wave.

The final step of the construction process deals with the generation of the wave group time-history at a fixed location x_0, given the height and period sequences obtained from the previous step. This can be formulated as an identification problem with respect to the b_i parameters appearing in the following expression for the water surface elevation η:

$$\eta(t) = \sum_{i=0}^{5j} b_i \sin(k_i x_0 - \omega_i t) \tag{9}$$

where j is the run length of the wave group under construction. The idea here is to consider Eq. (9) as an interpolating function passing through a number of key points (i.e. crests, troughs and zero-crossings) describing the wave group shape in the time domain. The coordinates of these points can be inferred from the "expected" height and period values derived from Eqs. (7) and (8) (more details on this part can be found elsewhere, e.g. in [2]). To ensure the uniqueness of the solution, the number of terms kept in the series expansion (i.e. $5j + 1$) is set equal to the number of imposed geometrical constraints and thus, it becomes a function of the run length j. The above constitute a well-defined (Hermite-type) interpolation problem for which closed-form expressions for the b_i parameters are available in the existing literature (e.g. [12]).

2.3 Equation of Roll Motion

In principle, "the critical wave groups" method is not biased towards any specific type of mathematical model (in fact, it can handle equally a simple ODE and a CFD model). However, since our intention is to evaluate the performance of the approach, massive Monte Carlo simulations will have to be carried out using the very same model of ship motion. Aiming at enhancing the reliability of Monte Carlo estimates (particularly in the tail region) through a large (and computationally inexpensive) simulation campaign, we adopt the following uncoupled equation, written in terms of the relative roll angle φ:

$$(I_{44} + A_{44})\ddot{\varphi} + D(\dot{\varphi}) + g\Delta GZ(\varphi) = M(t) \tag{10}$$

with I_{44} and A_{44} being the roll moment of inertia and the added moment of inertia, respectively, Δ is the ship displacement, g is the gravitational acceleration and D is the damping moment:

$$D(\dot{\varphi}) = B_1\dot{\varphi} + B_2\dot{\varphi}|\dot{\varphi}| \tag{11}$$

The restoring arm in still water is given as:

$$GZ(\varphi) = \sum_k C_k \varphi^k \tag{12}$$

When information about the roll moment amplitude operator $F_{roll}(\omega)$ is available, the wave group induced moment can be expressed, via Eq. (9), as:

$$M(t) = \sum_{i=0}^{5j} F_{roll}(\omega_i) b_i \sin(k_i x_0 - \omega_i t) \tag{13}$$

Alternatively, in the presence of long incident waves, the concept of instantaneous wave slope at the middle of the ship $\alpha(t) = \partial\eta(x, t)/\partial x|_{x=x_0}$ can be employed [24]:

$$M(t) = -I_{44}\ddot{\alpha}(t) \tag{14}$$

3 Results and Discussion

In this section, the simplified "critical wave groups" scheme is applied to two different ship models in order to compute the probability of exceedance $P_e = \Pr[\varphi > \varphi_{cr}]$ for several roll angle thresholds φ_{cr}. These are implied as possible limits of unacceptable behaviour given that Eq. (10) is a very simple roll model and it does not contain design information about deck submergence, downflooding angles etc. As a result, derived P_e values are considered to be referring to a parameterized limit for stability failure. Regarding the construction of the "expected" wave groups, the transition PDFs in Eqs. (7) and (8) are determined according to the copula-based methodology described in [1], yet the necessary correlation parameters are estimated from datasets produced from extensive simulations of the water surface displacement. In this way, the effectiveness of the Markov model in predicting the "expected" wave height and period sequences is enhanced. For calculating the probability in Eq. (5), the associated PDF $f_{H_n,T_n|H_{n-1},T_{n-1}}$ is also obtained by direct counting procedures based on the generated wave data. To investigate the possibility of eliminating the computational cost due to the generation of the "expected" wave groups, regular wave trains are also tried for representing critical group encounters. In both the regular and the "expected" wave group implementations of the approach the results are tested against Monte Carlo simulations of roll motion.

3.1 Ship Model 1

An ocean surveillance ship, referred in the study of [19], was selected as the first ship model. Main parameters of the vessel are given in Table 1 and the roll moment amplitude operator $F_{roll}(\omega)$ is presented in Fig. 1.

The ship is assumed to operate in conditions described by the Bretschneider spectrum with significant wave height $H_s = 4$m and peak period $T_p = 6$s (e.g. [13]):

$$S_{\eta\eta}(\omega) = \frac{1.25}{4}\frac{\omega_p^4}{\omega^5}H_s^2 \exp\left[-\frac{5}{4}\cdot\left(\frac{\omega_p}{\omega}\right)^4\right] \qquad (15)$$

Table 1 Main parameters of ship model 1

Parameter	Dimensional value	Dimensions
$I_{44} + A_{44}$	5.540×10^7	kg m^2
Δ	2.056×10^6	kg
B_1	5.263×10^6	kg m^2/s
B_2	2.875×10^6	kg m^2
C_1	3.167	m
C_3	−2.513	m

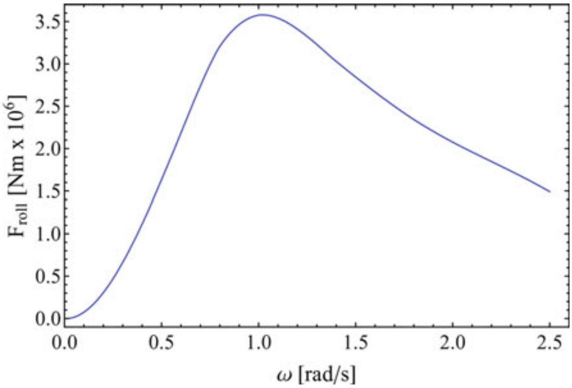

Fig. 1 Roll moment amplitude operator $F_{roll}(\omega)$ for ship model 1

were $\omega_p = 2\pi/T_p$ is the peak frequency. For the simulations of the wave field, the spectral representation method is adopted [18]:

$$\eta(t) = \sum_i \sqrt{2S_{\eta\eta}(\omega_i)\delta\omega_i}\cos(\omega_i t + \varepsilon_i) \tag{16}$$

were ε_i are random variables uniformly distributed over $[0, 2\pi)$, ω_i are the frequencies of the wave components and $\delta\omega_i$ is the frequency resolution. In total, 18,853 waves were analyzed from a set of 24 records of 1h produced within 7min on a modern laptop using the Fast Fourier Transform (FFT) approach [16].

Furthermore, Monte Carlo simulations of roll motion were set-up accordingly so as to estimate desired probabilities using ensemble averages, i.e. without assuming the ergodic property for the response [4]. The idea was to explore the roll probability space at a fixed time instant t_s beyond which the statistical properties of the response process remain practically constant. Specifically, a collection of approximately 15 $\times$ 10^5 short-duration realizations was simulated and the roll angle value sampled at $t_s = 150$s was kept from each realization for further analysis (details for duly selecting t_s are given in [3]). Then, exceedance probabilities for various roll angle thresholds φ_{cr} were computed through the number of observed exceedances over φ_{cr} divided by the sample size (15×10^5). To quantify the uncertainty of these direct counting estimates, the Wilson score confidence interval was preferred knowing that it is the most consistent with the nominal coverage probability among a number of binomial proportion-based intervals described in the literature [7]. At this point, it is important to distinguish Eq. (9) from Eq. (16) since the latter is a well-known model for representing stochastic processes, while the former is only a Fourier-based interpolation function, essentially not designed for Monte Carlo simulations. Therefore, for massively generating roll response time-histories through Eq. (10), the wave induced moment was obtained by multiplying each individual wave component in Eq. (16) by the corresponding $F_{roll}(\omega_i)$ amplitude.

Figure 2 illustrates the pattern of the predictions of Eqs. (7) and (8) for the examined sea conditions when various $\{h_c, t_c\}$ values (squares) are considered. The

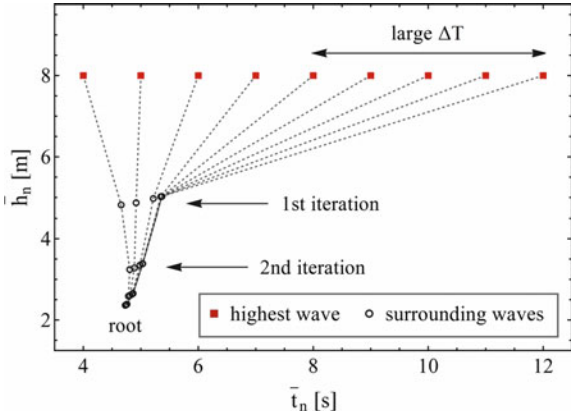

Fig. 2 Map of the "expected" height and period sequences generated for the Bretschneider spectrum

"expected" heights $\bar{h}_n$ and corresponding periods $\bar{t}_n$, as derived from successive iterations, are shown on the vertical and horizontal axes, respectively. The evolution of the procedure for a given set of $\{h_c, t_c\}$ parameters is denoted by circles along the dashed lines. The root of this tree-shaped diagram is the stationary state of the Markovian system and the structure of the "expected" wave groups is largely affected by the distance of $\{h_c, t_c\}$ from it. Since the width ΔT of a critical period range $T_{cr,m}$ is the difference of the shortest from the longest period in a generated sequence, it naturally coincides with this characteristic distance for wave groups with large j (i.e. when more iterations are applied). In this regard, the maximum period $t_{c,max}$ used in the calculations should exhibit an interesting relationship with the deduced probability values. Finally, as anticipated, there is a large concentration of points close to the mean period of the sea state, indicated by the abscissa of the root.

Figure 3 presents the results of the Monte Carlo simulations (solid line) and the associated 95% confidence intervals (dashed lines) in the tail region. The estimates of the "critical wave groups" method, obtained by employing the "expected" wave group forms for the ambient sea state with run lengths $j \leq 6$, have been superimposed on the same plot. For the latter method, two implementations (denoted by circles and diamonds) are presented corresponding to different values of the maximum period $t_{c,max}$ considered for initiating the iterations in Eqs. (7) and (8). As demonstrated in Fig. 2, for increasing t_c the highest wave progressively deviates from the periods of the surrounding waves leading to larger ΔT. Therefore, the tolerance for the detection of resonant phenomena is relaxed and the probability in Eq. (5) increases. However, including very distant, with respect to the root, t_c values may be irrelevant, and more importantly inaccurate, since the period of the highest wave distorts the grouping character of the rest period sequence. To avoid this issue, Fig. 2 could be utilized for identifying the region where the Markov chain predictions are insensitive to $t_{c,max}$. For the sea state in question, this happens beyond $t_c = 7$s and thus, for $t_{c,max} = 10$s the proposed method consistently overestimates the probability of exceedance P_e. On the contrary, more reliable estimates are provided for $t_{c,max} = 8$s. Interestingly, for practical instability limits $\varphi_{cr} \in [30\,\text{deg}, 40\,\text{deg}]$, the selection of

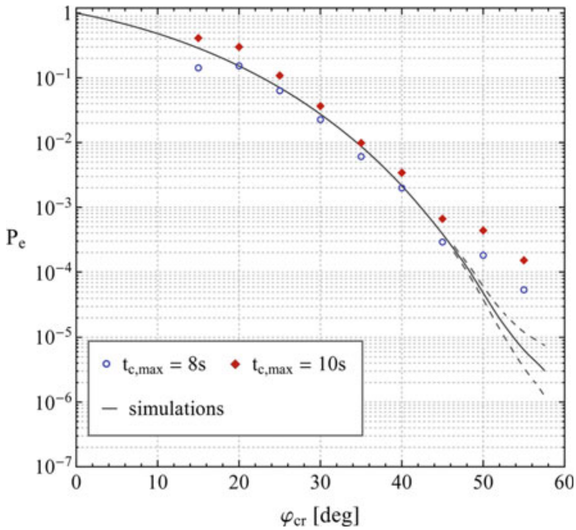

Fig. 3 Probability of exceedance for ship model 1 using irregular wave groups (circles and diamonds). Dashed lines indicate the 95% confidence intervals of the simulation-based estimates (solid line)

$t_{c,max}$ does not seem to be that important, given that in both the examined scenarios the method performs satisfactorily. In the tail region, though, both schemes predict more exceedances than the Monte Carlo approach; this, however, could be due to the very rarity of the extremes. The current method was not applied for $\varphi_{cr} < 15$ deg since in this regime ship response is, in principle, Gaussian [3]. In producing results for all 9 thresholds, the elapsed time per $t_{c,max}$ case was less than an hour on a modern desktop, including the most time-consuming part of the procedure being the construction of the "expected" wave groups.

On the other hand, the effectiveness of the "critical wave groups" method deteriorates when regular wave trains with $j \leq 6$ are employed, as shown in Fig. 4. In the same spirit, two different cases of critical period range widths ΔT were studied and both were found to consistently underestimate the probability of exceedance below 40°. For larger angles, though, the accuracy of the method is improved, particularly for $\Delta T = 2$s. This is in accordance with the work of [22] who concluded that in analyzing capsize tendency,[2] it is sufficient to consider only the upright equilibrium initial state of the vessel when hit by a regular wave train. In the context of the "critical wave groups" method, this was verified by Themelis and Spyrou [21] who observed that the effect of the initial state becomes weaker (in terms of probability) for sea conditions associated with resonant phenomena. Another important aspect of the regular-wave implementation of the current approach is that, in contrast to the "expected" wave groups-based scheme, there is no guidance (at the moment at least) for selecting ΔT accordingly. Only in retrospect it can be deduced that setting $\Delta T = 1$s is too strict. Finally, although here calculations were performed for 11 thresholds, the associated computational cost per ΔT scenario was only few minutes

[2] Defined as the escape from the potential energy well; thus, implying the exceedance of a large roll threshold.

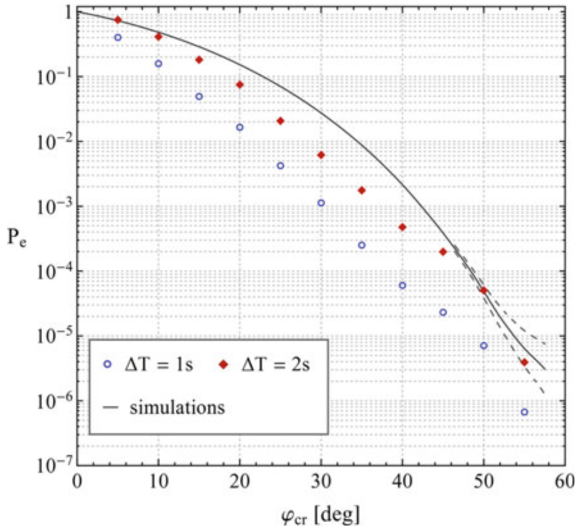

Fig. 4 Probability of exceedance for ship model 1 using regular wave groups (circles and diamonds). Dashed lines indicate the 95% confidence intervals of the simulation-based estimates (solid line)

on a modern desktop. The dramatic speed-up comes from the time spent in generating wave group environments given that for regular waves it is practically negligible.

In the deterministic part of the method, critical wave group parameters, identified for $\varphi_{crit} = 45$deg, are summarized in Fig. 5 in the form "transient capsize diagrams" [14]. These are plots of the wave steepness H/λ of a critical wave group against its period T, here normalized with the natural period of the vessel $T_o = 5.9$s. In the case of regular wave trains, the boundary between the "stable" and "unstable" regions is shown by solid lines, while for the "expected" wave groups, short and long dashed lines are utilized for indicating the boundary location when defined in terms of the mean and maximum steepness, respectively, of the participating waves. As one obtains two boundary lines for this case, shading has been applied to enhance the contrast against the regular-waves curve. For $j = 2$, height thresholds produced by regular and the "expected" wave trains are, in the mean sense, relatively close. For $j = 3$, however, the dangerous zone is enlarged when considering irregular wave groups. The shift of instability region towards the area of long waves has already been reported in [2].

3.2 Ship Model 2

A modern 4800 TEU Panamax containership with main parameters listed in Table 2 and natural period $T_o = 15.2$s was considered for a second case study. The restoring

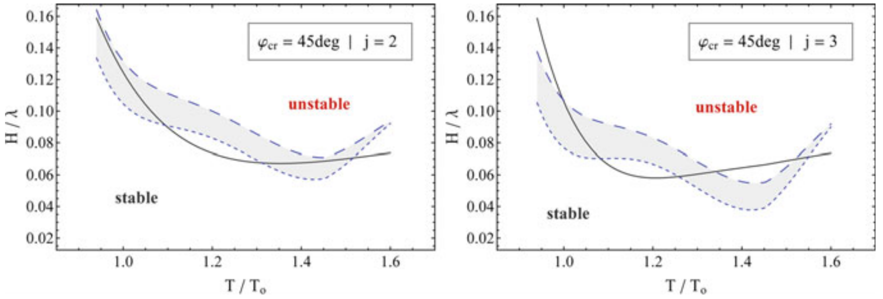

Fig. 5 Transient capsize diagrams for ship model 1 corresponding to run lengths $j = 2$ (left panel) and $j = 3$ (right panel); in both cases instability is defined as the exceedance of $\varphi_{crit} = 45$deg

arm coefficients in Eq. (12) were provided directly from the loading manual of the vessel, while roll damping estimates were obtained by applying Ikeda's method [9]. Since no information was available about the F_{roll} function, wave forcing was approximated by Eq. (14). In this application, the JONSWAP spectrum with $H_s =$ 10m, $T_p = 14$s and $\gamma = 1.932$ was selected to describe the sea state of operation (e.g. [8]):

$$S_{\eta\eta}(\omega) = (1 - 0.287 \ln \gamma) S_B(\omega) \gamma^{\exp\left[-\frac{1}{2}\left(\frac{\omega-\omega_p}{0.08\omega_p}\right)^2\right]} \quad (17)$$

where $S_B(\omega)$ is the Bretschneider spectrum. Again, wave group statistics were extracted by simulating 24 records of 1h length using Eq. (16). These produced a total population of 7,875 waves within only few minutes on a modern desktop. Roll motion time-histories were generated using the same setup as for ship model 1, yet sampled at $t_s = 200$s.

The results obtained from the implementation of the "critical wave groups" method when ship model 2 is excited by the "expected" wave groups for the given sea state and by regular wave trains are presented in Figs. 6 and 7, respectively. Regarding the accuracy of the approach for roll angle thresholds up to 40°, again it is enhanced when irregular waveforms are employed. Beyond 43°, the Monte Carlo simulations did not predict any extremes due to the problem of rarity; while in the same regime both schemes of the current method seem to reliably extrapolate the

Table 2 Main parameters of ship model 2

Parameter	Dimensional value	Dimensions	Parameter	Dimensional value	Dimensions
I_{44}	1.020×10^{10}	kg m^2	C_1	2.851	m
A_{44}	1.021×10^9	kg m^2	C_3	5.407	m
Δ	6.820×10^7	kg	C_5	−18.169	m
B_1	4.829×10^8	kg m^2/s	C_7	14.278	m
B_2	6.316×10^8	kg m^2	C_9	−3.677	m

trend of the direct counting estimates. As for the selection of $t_{c,max}$, it was based on the same methodology described for ship model 1. Specifically, $t_{c,max} = 15$s was anticipated to perform better since beyond this value the "expected" height and period sequences were found practically independent of the assumed $\{h_c, t_c\}$. In the regular-wave version of the approach, it can be argued that setting $\Delta T = 2$s is more suitable for calculating exceedance probabilities P_e associated with intermediate roll angle thresholds, while $\Delta T = 1$s appears more suitable for extrapolation, as documented also in [3]. The specific sea state was consciously selected for demonstrating the extrapolation character of the proposed method since although it is very likely to provoke resonance, H_s is not high enough for inducing many extremes. More so, [21] have observed that, for such sea states, considering various initial conditions within the "critical wave groups" framework is rather unnecessary.

Finally, Fig. 8 compares regular and irregular critical wave trains with $j = 2$ and $j = 3$ in terms of their contribution to the total probability of exceedance P_e. The calculations were made for the critical period parameters that provided the best agreement with the simulation results in Figs. 6 and 7. Thus, $\Delta T = 2$s and $t_{c,max} = 15$s were selected for the regular and the irregular case, respectively. The contribution of run lengths with $j > 6$ to the total probability of exceedance was found negligible.

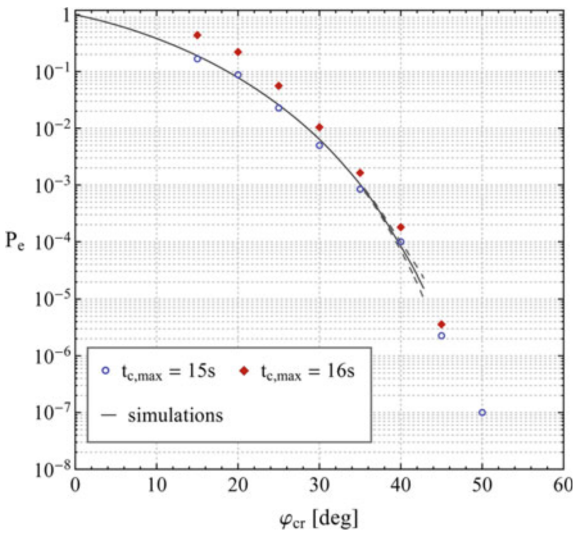

Fig. 6 Probability of exceedance for ship model 2 using irregular wave groups (circles and diamonds). Dashed lines indicate the 95% confidence intervals of the simulation-based estimates (solid line)

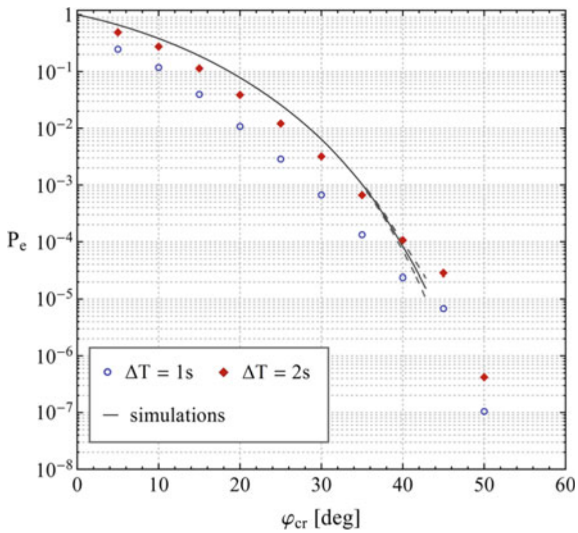

Fig. 7 Probability of exceedance for ship model 2 using regular wave groups (circles and diamonds). Dashed lines indicate the 95% confidence intervals of the simulation-based estimates (solid line)

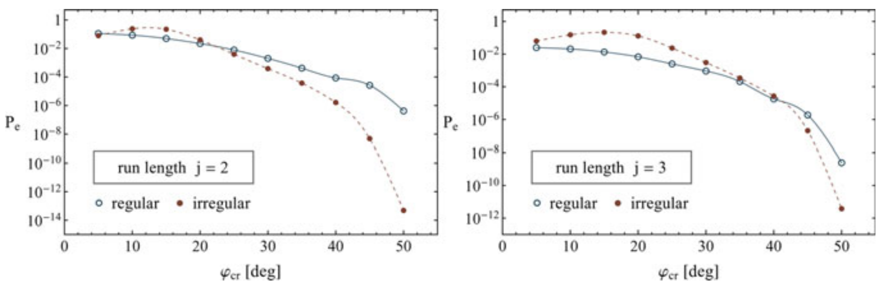

Fig. 8 Contribution of individual run lengths j to the probability of exceedance for ship model 2

4 Concluding Remarks

In this study, a simple and computationally efficient "critical wave groups" method was developed for calculating the probability of large-amplitude ship motions in beam seas. The method is focused on providing swift estimates by examining only one scenario of initial conditions of the vessel when approached by the "expected" wave groups for the ambient state. To investigate the possibility of further simplification, since the generation of realistic wave environments is time-consuming, the method was applied also using regular wave trains. The effectiveness of both the regular and the "expected" wave group-based schemes was assessed through comparisons with the predictions of Monte Carlo simulations of roll motion. The results indicate that the proposed method performs better when the “expected” wave groups are utilized for representing critical, for ship stability, wave episodes, particularly because period successions are modeled in a realistic manner. Since the degree of variability allowed in the wave period groupings is crucial for the accuracy of the method, guidelines were formulated for duly selecting it. This contributed not only in obtaining reliable estimates for practical limiting angles (e.g. 40°), but also in extrapolating in the tail region where the efficiency of Monte Carlo simulations is generally low.

Most importantly, given that only few minutes were needed for completing the calculations with respect to a single roll angle threshold, the current approach appears very suitable for preliminary ship stability evaluations. At the same time, however, it is designed specifically for sea states being highly probable to provoke resonance since otherwise most critical wave encounters will presumably remain unidentified due to the very nature of the “expected” wave groups. More so, in non-resonant sea states the effect of initial conditions becomes quite important [21] and thus, the more detailed version of the “critical wave groups” method, discussed in [3], should be invoked. As a final remark, methods for quantifying the uncertainty of the estimates obtained via the “critical wave groups” approach are currently investigated and results will hopefully be presented in future studies.

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References

1. Anastopoulos PA, Spyrou KJ, Bassler CC, Belenky VL (2016) Towards an improved critical wave groups method for the probabilistic assessment of large ship motions in irregular seas. Probab Eng Mech 44:18–27
2. Anastopoulos PA, Spyrou KJ (2016) Ship dynamic stability assessment based on realistic wave group excitations. Ocean Eng 120:256–263
3. Anastopoulos PA, Spyrou KJ (2019) Evaluation of the critical wave groups method in calculating the probability of ship capsize in beam seas. Ocean Eng 187:106213
4. Belenky VL, Degtyarev AB, Boukhanovsky AV (1998) Probabilistic qualities of nonlinear stochastic rolling. Ocean Eng 25(1):1–25
5. Belenky VL, Weems K, Pipiras V, Glotzer D, Sapsis TP (2018) Tail structure of roll and metric of capsizing in irregular waves. In: Proceedings of the 32nd symposium on naval hydrodynamics, Hamburg, Germany
6. Belenky VL, Glozter D, Pipiras V, Sapsis TP (2019) Distribution tail structure and extreme value analysis of constrained piecewise linear oscillators. Probab Eng Mech 57:1–13
7. Brown LD, Cai TT, DasGupta A (2001) Interval estimation for a binomial proportion. Stat Sci 16(2):101–117
8. Det Norske Veritas (DNV) (2011) Modelling and analysis of marine operations. Report No. H103
9. Ikeda Y, Himeno Y, Tanaka N (1978) A prediction method for ship roll damping. Report No. 00405 of the Department of Naval Architecture, University of Osaka Prefecture
10. Kimura A (1980) Statistical properties of random wave groups. In: Proceedings of the 17th international coastal engineering conference, Sydney, Australia, pp 2955–2973

11. Masson D, Chandler P (1993) Wave groups: a closer look at spectral methods. Coast Eng 20:249–275
12. Nathan A (1975) Trigonometric interpolation of function and derivative data. Inf Control 28(3):192–203
13. Ochi M (1998) Ocean waves: the stochastic approach. Cambridge University Press, Cambridge. ISBN: 978-0-521-01767-1
14. Rainey RCT, Thompson JMT (1991) The transient capsize diagram—a new method of quantifying stability in waves. J Ship Res 35(1):58–62
15. Shigunov V, Themelis N, Spyrou KJ (2019) Critical wave groups vs. direct Monte-Carlo simulations for typical stability failure modes of a container ship. In: Belenky V, Spyrou K, van Walree F, Almeida Santos Neves M, Umeda N (eds) Contemporary ideas on ship stability. Fluid mechanics and its applications vol 119, Springer, pp 407–421. ISBN: 978-3-030-00514-6
16. Shinozuka M, Deodatis G (1991) Simulation of stochastic processes by spectral representation. Appl Mech Rev 44(4):191–204
17. Stansell P, Wolfram J, Linfoot B (2002) Statistics of wave groups measured in the northern North Sea: comparisons between time series and spectral predictions. Appl Ocean Res 24:91–106
18. St Denis M, Pierson WJ (1953) On the motions of ships in confused seas. Transactions of SNAME 61:280–332
19. Su Z (2012) Nonlinear response and stability analysis of vessel rolling motion in random waves using stochastic dynamical systems. Ph.D. Thesis, Texas A&M University, United States
20. Themelis N, Spyrou KJ (2007) Probabilistic assessment of ship stability. Transactions of SNAME 115:181–206
21. Themelis N, Spyrou KJ (2008) Probabilistic assessment of ship stability based on the concept of critical wave groups. In: Proceedings of the 10th international ship stability workshop, Daejeon, Korea, pp 115–125
22. Thompson JMT, Rainey RCT, Soliman MS (1990) Ship stability criteria based on chaotic transients from incursive fractals. Phil Trans R Soc A 332(1624):149–167
23. Campbell B, Weems K, Belenky VL, Pipiras V, Sapsis TP (2023) Envelope peaks over threshold (EPOT) application and verification. Chapter 16 in: Spyrou K, Belenky VL, Katayama T, Bačkalov I, Francescutto A (eds) Contemporary ideas on ship stability—from dynamics to criteria. Fluid mechanics and its applications vol 134, Springer, pp 265–289. ISBN: 978-3-031-16328-9
24. Wright JHG, Marshfield WB (1980) Ship roll response and capsize behavior in beam seas. RINA Transactions 122:129–148