Vibration Analysis of Laminated Composite Panels with Various Fiber Angles

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Abstract. Laminated composite structures have superior material properties such as high stiffness and high strength-to-weight ratio and have been increasingly used in advanced structures such as automobiles to replace conventional metal structures to reduce weight for enhanced energy efficiency. Composite structures in practical working condition are often subjected to external excitation force, which can result in severe vibration problems. This paper investigates the vibration characteristics of rectangular laminated composite panels with various fiber orientations subjected to harmonic loading. Both free and forced vibration analysis have been carried out to obtain natural frequencies and the steady-state dynamic responses. Both analytical and numerical FE methods based on the first-order shear deformation theory (FSDT) are used to carry out vibration analysis. The numerical FE analysis is employed to validate the accuracy of proposed analytical method. The vibration transmission behaviour of laminated composite panels is compared with that of steel panels. It is found that fiber orientations have significant influence on vibration characteristics of laminated composite panels. The results explicitly show that natural frequencies and dynamic responses could be altered by designing fiber orientation for vibration mitigation. The findings provide some improved understanding on the structural design of laminated composite structures for enhanced vibration transmission behaviour.

Keywords: Vibration analysis · Laminated composite panels · Fiber orientations · Natural frequencies · Mode shapes

1 Introduction

Laminated composite structures with superior properties such as high stiffness and high strength-to-weight ratio and light weight have been increasingly used in many advanced engineering structures [1]. In the automobile industry, the laminated composite structures

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have been increasingly used to replace the conventional metallic structures for saving weight and to increase the energy efficiency. The high-level vibrations can result in problems such as fatigue failure, destruction of mechanical system and high noise. There is a need for deeper understanding of vibration behaviour of laminated composite structures.

There have been a large number of studies on the free vibration behaviour of laminated composite panels based on several theories such as the classical laminate plate theory, first [2] and third-order shear deformation theory [3], and higher-order refined theory [4] to improve the accuracy of analytical methods. However, there have been much less studies reported on effects of the fiber orientations on vibration behaviour of the laminated composite panels subjected to complex dynamic loading. For instance, Dobyns [5] and Carvalho *et al.* [6] proposed a theoretical approach for vibration analysis of simply-supported rectangular laminated composite plate. Dynamic responses of laminated composite plates with respect to effects of various fiber orientations and stacking sequences were studied [7, 8]. The fibers could be tailored for vibration suppression designs [9, 10].

This paper presents an analytical approach and numerical FE method to examine the vibration behaviour of harmonically excited laminated composite panels with simply supported edges. Analytical method based on the first-order shear deformation theory (FSDT) can be used for special symmetric cross-ply laminated composite panel. For the laminated composite panels with various fiber orientations, the numerical ANSYS finite element (FE) method is used for investigation and validation of the analytical results. The dynamic responses are determined and compared with that of the steel panel. The effects of fiber orientations on the dynamic behaviour are examined.

2 Dynamic Analysis of Laminated Composite Panels

2.1 Model Description

Figure 1 shows the schematic illustration of a four-layered laminated composite rectangular panel with length a and width b and thickness h in the directions of OX, OY and OZ, respectively. Its four edges are simply supported and a harmonic external excitation force with amplitude $\tilde{f}_e$ and frequency ω is applied at the point $\mathrm{A}(x_e, y_e)$. Also a general point $\mathrm{B}(x_r, y_r)$ is labelled in the Fig. 1 for which the response is of interest. The angle θ_i that is measured from the positive direction of OX to the principal material axis is used to indicate the fiber orientation of the i-th layer.

The constitutive equations for orthotropic lamina can be written as [1]:

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix}, \begin{bmatrix} \tau_{23} \\ \tau_{13} \end{bmatrix} = \begin{bmatrix} Q_{44} & 0 \\ 0 & Q_{55} \end{bmatrix} \begin{bmatrix} \gamma_{23} \\ \gamma_{13} \end{bmatrix}, \qquad \text{(1a-1b)}$$

where $Q_{ij}(i, j = 1, 2, 6)$ are the material constants in the material axes of the layer expressed by

$$Q_{11} = \frac{E_1}{1 - \nu_{12}\nu_{21}}, Q_{12} = \frac{\nu_{12}E_2}{1 - \nu_{12}\nu_{21}} = \frac{\nu_{12}E_1}{1 - \nu_{12}\nu_{21}}, Q_{22} = \frac{E_2}{1 - \nu_{12}\nu_{21}},$$

$$Q_{66} = G_{12}, Q_{44} = G_{23}, Q_{55} = G_{13},$$

where E_{11} and E_{22} are the in-plane moduli of elasticity in local coordinate system, G_{ij} are the shear moduli, while ν_{ij} are the Poisson's coefficients of the orthotropic materials. The stress-strain relations for the i-th orthotropic layer in the local coordinate system are

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & 0 \\ \bar{Q}_{12} & \bar{Q}_{22} & 0 \\ 0 & 0 & \bar{Q}_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}, \begin{bmatrix} \tau_{yz} \\ \tau_{xz} \end{bmatrix} = \begin{bmatrix} \bar{Q}_{44} & 0 \\ 0 & \bar{Q}_{55} \end{bmatrix} \begin{bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{bmatrix}. \quad (2)$$

The constitutive relations can be transformed from the local coordinate system to the global coordinate system by using the following transformation matrix:

$$[T_1] = \begin{bmatrix} \cos^2\theta & \sin^2\theta & 2\sin\theta\cos\theta \\ \sin^2\theta & \cos^2\theta & -2\sin\theta\cos\theta \\ -\sin\theta\cos\theta & \sin\theta\cos\theta & \cos^2\theta - \sin^2\theta \end{bmatrix}, [T_2] = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}. \quad (3)$$

The stiffness matrix for the i-th layer in global coordinate system can be obtained from

$$[\bar{Q}] = \left([T]^{(i)}\right)^{-1} [Q]^{(i)} [T]^{(i)}. \quad (4)$$

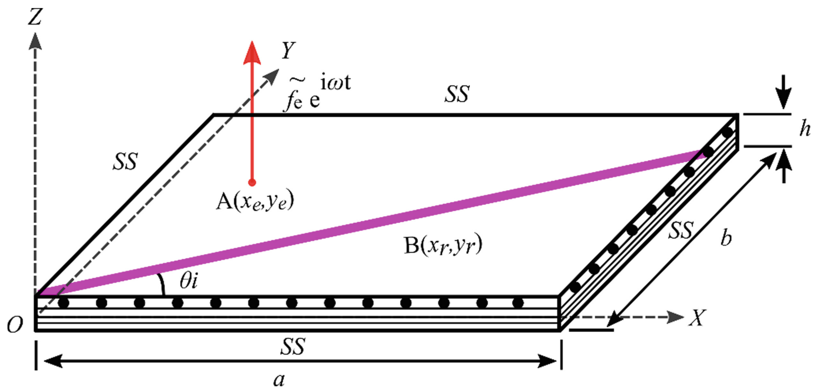

Fig. 1. Schematic illustrations of a laminated composite rectangular panel with simply supported (SS) edges.

2.2 Vibration Analysis of Laminated Composite Panels

For a symmetric cross-ply orthotropic plate, some of stress stiffness coefficients can be assumed as zero, $B_{ij} = 0, A_{16} = A_{26} = D_{16} = D_{26} = 0$. Hence, the equations of motion are

$$kA_{44}\left(w^0{}_{,yy} + \phi_{y,y}\right) + kA_{55}\left(w^0{}_{,xx} + \phi_{x,x}\right) + p_z = \rho h \ddot{w}^0, \quad (5a)$$

$$D_{11}\phi_{x,xx} + (D_{12} + D_{66})\phi_{y,xy} + D_{66}\phi_{x,yy} - kA_{55}\left(w^0{}_{,x} + \phi_x\right) = \frac{\rho h^3}{12}\ddot{\phi}_x, \tag{5b}$$

$$D_{66}\phi_{y,xx} + (D_{12} + D_{66})\phi_{x,xy} + D_{22}\phi_{y,yy} - kA_{44}\left(w^0{}_{,y} + \phi_y\right) = \frac{\rho h^3}{12}\ddot{\phi}_y, \tag{5c}$$

where h is the plate thickness, ρ is material density, w is the plate displacement in the OZ direction at the plate midplane, ϕ_x and ϕ_y are the shear rotations on the OY and OX directions. k is used to account for the effect of shear deformation, which is normally taken as $5/6$.

With simply supported edges, we have

$$w = \phi_{x,x} = 0, \text{ at } x = 0, \text{ and } x = a.$$
$$w = \phi_{y,y} = 0, \text{ at } y = 0, \text{ and } y = b.$$

The solution of the dynamic response is based on expansions of the loads, displacement and rotations in double Fourier series. Each expression is the product of independent time coefficients by trigonometric functions, which satisfy the boundary condition:

$$\phi_x(x, y, t) = \sum_m \sum_n A_{mn}(t)\cos \alpha x \sin \beta y, \tag{6a}$$

$$\phi_y(x, y, t) = \sum_m \sum_n B_{mn}(t)\sin \alpha x \cos \beta y, \tag{6b}$$

$$w(x, y, t) = \sum_m \sum_n W_{mn}(t)\sin \alpha x \sin \beta y, \tag{6c}$$

where $\alpha = m\pi/a$, $\beta = n\pi/b$, m and n are integers, and $A_{mn}(t)$, $B_{mn}(t)$ and $W_{mn}(t)$ are the time-dependent coefficients. The load function is represented by

$$q(x, y, t) = \sum_m \sum_n Q_{mn}(t)\sin \alpha x \sin \beta y. \tag{7}$$

For a concentrated load located at the point A (x_e, y_e), we have

$$Q_{mn}(t) = \frac{4F(t)}{ab}\sin \alpha x_e \sin \beta y_e. \tag{8}$$

A substitution of Eq. (6) into Eq. (5) leads to

$$\begin{bmatrix} L_{11} & L_{12} & L_{13} \\ L_{12} & L_{22} & L_{23} \\ L_{13} & L_{23} & L_{33} \end{bmatrix} \begin{Bmatrix} A_{mn}(t) \\ B_{mn}(t) \\ W_{mn}(t) \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ Q_{mn}(t) - \mu \ddot{W}_{mn}(t) \end{Bmatrix}, \tag{9}$$

where $\mu = \rho h$ is mass per unit area and the elements of the symmetric matrix L_{ij} are

$$L_{11} = D_{11}(\alpha)^2 + D_{66}(\beta)^2 + \kappa A_{55}, L_{12} = (D_{12} + D_{66})\alpha\beta, L_{13} = \kappa A_{55}\alpha,$$

$$L_{22} = D_{66}(\alpha)^2 + D_{22}(\beta)^2 + \kappa A_{44}, L_{33} = \kappa A_{55}(\alpha)^2 + \kappa A_{44}(\beta)^2, L_{23} = \kappa A_{44}\beta.$$

The Eq. (9) can be reduced to a single differential equation by the following transformation:

$$A_{mn}(t) = \frac{L_{12}L_{23} - L_{13}L_{22}}{L_{11}L_{22} - L_{12}{}^2} W_{mn}(t), \tag{10a}$$

$$B_{mn}(t) = \frac{L_{12}L_{13} - L_{11}L_{23}}{L_{11}L_{22} - L_{12}{}^2} W_{mn}(t). \tag{10b}$$

The equation of motion for a rectangular panel can be expressed as

$$\ddot{W}_{mn}(t) + \omega_{mn}{}^2 W_{mn}(t) = \frac{Q_{mn}(t)}{\mu}, \tag{11}$$

where the natural frequency can be obtained from:

$$\omega_{mn} = \sqrt{\frac{L_{13}K_A + L_{23}K_B + L_{33}}{\mu}}. \tag{12}$$

To solve the equation of motion, the modal displacement can be written as

$$W_{mn}(t) = X\mathrm{e}^{\mathrm{i}(\omega t - \theta)} = \left(X\mathrm{e}^{-\mathrm{i}\theta}\right)\mathrm{e}^{\mathrm{i}\omega t} = \overline{W_{mn}}\mathrm{e}^{\mathrm{i}\omega t}. \tag{13}$$

In a damped system, the damping effect can be included by multiplying the stiffness term by a factor of $(1 + \mathrm{i}\eta)$, where η denotes the structural damping. The displacement can be obtained and expressed as:

$$\widetilde{w}(x, y, t) = \frac{4}{ab\mu} \sum_m \sum_n \sin\alpha x_e \sin\beta y_e \sin\alpha x \sin\beta y \times \frac{\tilde{f}_\mathrm{e}}{\omega_{mn}{}^2(1 + \mathrm{i}\eta) - \omega^2}\mathrm{e}^{\mathrm{i}\omega t}. \tag{14}$$

3 Results and Discussions

Here a rectangular panel with dimensions of 1 m by 0.5 m with total thickness of 0.005 m is considered. The material used for composite panel is T300/934 CFRP and the material properties are set as $E_{11} = 120$ GPa, $E_{22} = E_{33} = 7.9$ GPa, $G_{12} = G_{13} = 5.5$ GPa, $G_{23} = 1.58$ GPa, $\nu_{12} = \nu_{13} = 0.33$, $\nu_{23} = 0.022$, $\rho = 1580\,\mathrm{kg/m^3}$. The material properties used for the counterpart steel panel are listed as: $E = 205.8$ GPa, $\rho = 7800\,\mathrm{kg/m^3}$ and $\nu = 0.3$. The structural damping coefficient η is 0.01. Both analytical and numerical ANSYS FE methods are employed for the free and forced vibration analysis of different types of panels. The free vibration analysis of a steel panel is carried out analytically based on the classical plate theory (CPT) and that of laminated composite panels with specific fiber orientation such as $[0°]_4$ and $[90°]_4$ can be conducted by analytical method based on FSDT. The numerical FE method using element Shell 281 based on FSDT is used when the analytical method for the laminated composite panels with various fiber orientation is not available.

Table 1 shows first five natural frequencies (in Hz) of different steel and laminated composite panels. The numerical and analytical results show good agreement, verifying both methods. It is found that first five natural frequencies of the steel panel are larger than those of the $[0°]_4$ laminated composite panel. It shows that the fundamental frequency of the steel panel is larger than that of the $[0°]_4$, $[30°]_4$ and $[45°]_4$ laminated composite panels but lower than that of the $[60°]_4$ and $[90°]_4$ laminated composite panels. An increase in the fiber angle θ leads to the increase of fundamental frequencies. It is found that the second natural frequencies decrease firstly and then increase when angle θ changes from 0° to 90°. The third, fourth and fifth natural frequencies are increased with fibre orientation with a maximum value being reached and then they are decreased.

Table 1. First five natural frequencies (in Hz) of different types of panels

Fibre angles	Method	Mode number				
		1	2	3	4	5
Steel	CPT	61.0413	97.6661	158.7074	207.5405	244.1653
	FE	61.1679	97.8526	159.3501	209.7384	246.2012
$[0°]_4$	FSDT	34.0765	90.0339	91.7675	136.1537	187.8359
	FE	34.1228	90.2194	92.6991	136.8970	188.8816
$[30°]_4$	FE	46.2024	83.9190	134.5403	140.2765	195.2814
$[45°]_4$	FE	55.9063	83.4566	123.5800	172.6839	196.4226
$[60°]_4$	FE	67.6608	85.1224	114.8423	156.3408	208.4675
$[90°]_4$	FSDT	81.5943	90.0339	107.4791	136.1537	176.6830
	FE	81.8147	90.2194	107.7478	136.8970	178.6634

Figure 2 shows that dynamic responses of the steel panel and the $[0°]_4$ panel at the forcing position (0.5 m, 0.25 m) in an examined frequency range of 0–500Hz. The solid and dashed lines represent the analytical results for steel and laminated composite panels, respectively and the corresponding numerical results from FE are denoted by circles and squares. It shows that the analytical results have a great agreement with FE results especially in the low-frequency range. It is found that at a lower excitation frequency, the laminated composite panel exhibits relatively high vibration responses. It is found that the peak amplitudes of laminated composite panel are higher than those of the considered steel panel. The steel panel with heavy weight has relatively lower dynamic responses when subjected to the excitation force.

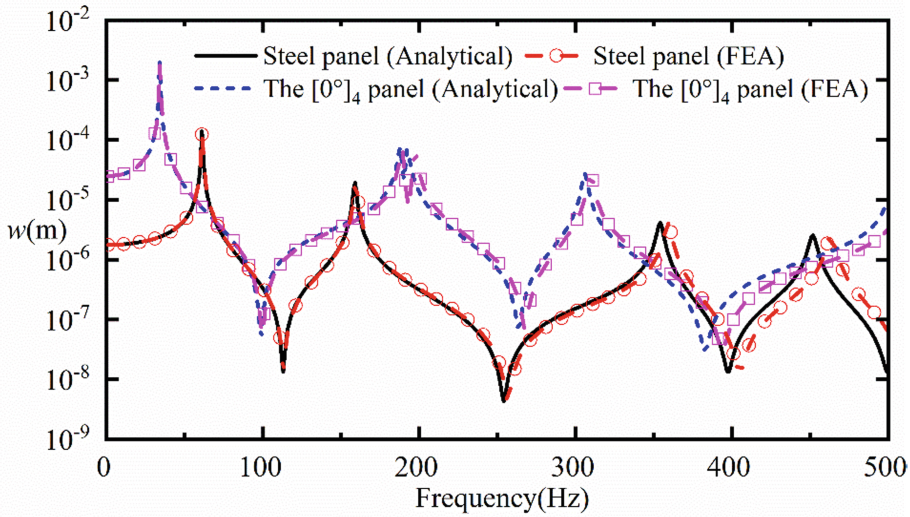

Fig. 2. Dynamic responses at the centre point (0.5 m, 0.25 m) on the steel panel and the laminated composite $[0°]_4$ panel with the excitation at the same point.

Figure 3 shows the variations of the dynamic responses at the forcing position with the excitation frequency. In this case, the effects of fiber orientations such as 0°, 30°, 45° and 60° on the dynamic responses are examined. It is found that with the increase of fibre angles from 0° to 60°, the first resonance peak is shifted to the higher frequency and the peak values are effectively reduced. The first resonance frequency of the $[60°]_4$ panel is larger than that of steel panel. The frequency range between the first and the second peaks of the laminated composite panels are narrowed with the increase of the fiber angle. The frequency range between the first and second peaks of the $[0°]_4$ panel is the widest. In the frequency band, the panel has relatively low dynamic response. In the frequency range approximately from 200 to 350 Hz, the vibration level of the steel panel is lower than that of laminated composite panels, whereas those of the laminated composite panels in the frequency range of 150 to 200 Hz are lowered. It demonstrates that reinforced composite laminates have an ability of reducing vibration levels in a described excitation frequency range and the peak values of dynamic response can be reduced by tailoring the fiber orientations.

In Fig. 4, the vibration transmission is investigated by examining the influences of fiber angles on the ratio R_t between the dynamic response amplitude of point B (0.75 m, 0.25 m) to that of the input point A (0.25 m, 0.25 m). In a lower frequency range of 0–55 Hz, it is found that the response amplitude ratio of the $[0°]_4$ panel is larger than in other cases. With the increase of fiber angle, the first peak of R_t is moved to larger frequencies. The first peak value of the $[60°]_4$ panel is the lowest, which indicates a low vibration transmission level. The first and second peak values of the $[0°]_4$ panel and the steel panel are close with the same vibration transmission behaviour. The frequency range between the first and second peak can be narrowed by increasing the fiber angle from 0° to 60°.

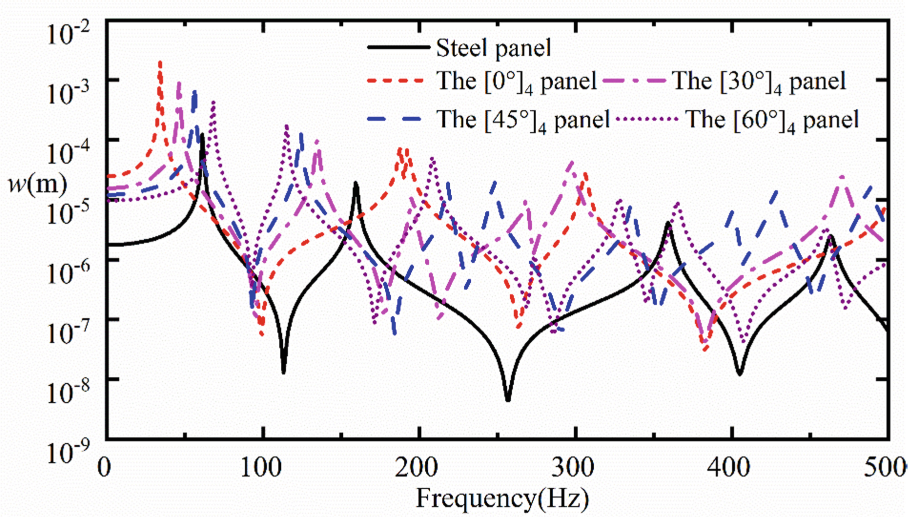

Fig. 3. Dynamic responses at the centre point (0.5 m, 0.25 m) on the steel panel and laminated composite panels with different fiber angles. —: the steel panel; ------: the $[0°]_4$ panel; -·-·-·-: the $[30°]_4$ panel; - - -: the $[45°]_4$ panel; ······: the $[60°]_4$ panel.

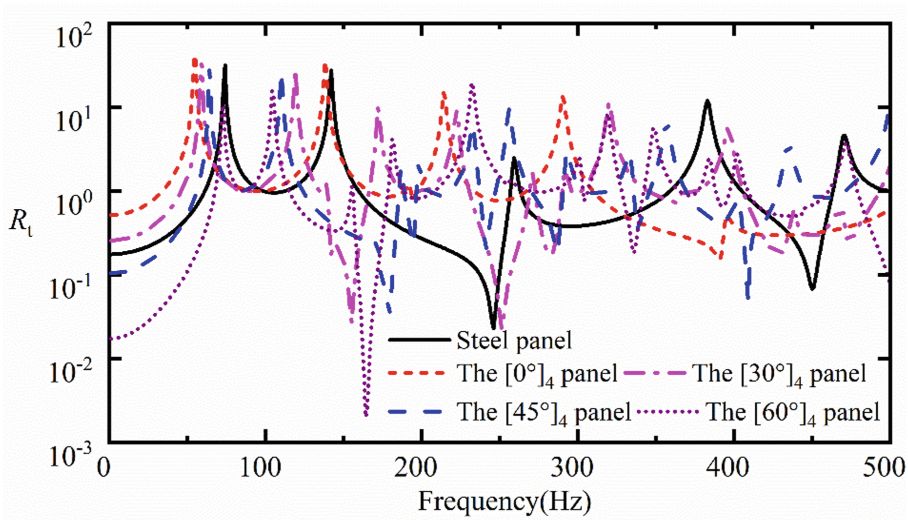

Fig. 4. Ratio between response amplitudes at Points B (0.75m, 0.25m) and A (0.25 m, 0.25 m) of the steel panel and laminated composite panels with different fiber angles. —: the steel panel; ------: the $[0°]_4$ panel; -·-·-·-: the $[30°]_4$ panel; - - -: the $[45°]_4$ panel; ······: the $[60°]_4$ panel.

4 Conclusions

This paper presented the free and forced vibration characteristics of rectangular laminated composite panels with various fiber orientations such as $[0°]_4$, $[30°]_4$, $[45°]_4$ and $[60°]_4$, with comparisons to vibration performance of a steel panel. The analytical method based on the FSDT has been shown to be accurate by verification with numerical ANSYS finite element results. This proposed method can be applied in investigation of forced vibration response at any designated positions. For panels with complex fiber orientations, the numerical FE method has been employed. The results showed that resonance peak values of dynamic responses of the steel panel are generally lower than that of laminated composite panels. The vibration responses and vibration transmission at a prescribed frequency can be reduced and designed by tailoring fiber orientation. The findings show the potential advantages of laminated composite panels with large design space to reduce the vibration level with reduced weight.

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