A Definition of Sceptical Semantics in the Constellations Approach

Stefano Bistarelli and Francesco Santini(✉)

Dipartimento di Matematica e Informatica, Università degli Studi di Perugia, Perugia, Italy
{stefano.bistarelli,francesco.santini}@unipg.it

Abstract. We propose a different way to compute sceptical semantics in the constellations approach: we define the grounded, ideal, and eager extension of a *Probabilistic Argumentation Framework* by merging the subsets with the maximal probability of complete, preferred, semi-stable extensions respectively. Differently from the original work (i.e., [19]), the extension we propose is unique, as the principle of scepticism usually demands. This definition maintains some well-known properties, as set-inclusion among the three semantics. Moreover, we advance a quantitative relaxation of these semantics with the purpose to mitigate scepticism in case the result corresponds to empty-set, which is not very informative.

1 Introduction

Abstract Argumentation is a high-level language describing conflicting information, which can be simply represented by a set of arguments and a binary attack-relationship. Argumentation is "abstract" because the conflict between two arguments is not formally motivated, and the internal structure of an argument is not specified. Such an abstraction can be used to capture general properties of a debate, but it also fostered the enrichment of frameworks with additional information (e.g., probabilities).

In *uncertain reasoning* we can distinguish *qualitative* and *quantitative* approaches. The former ones focus on issues such as defeasibility, and default assumptions: computational models of argumentations are an example. The latter ones focus on the problem of quantifying the acceptance status of statements: an example is probabilistic reasoning. Probabilistic Argumentation frameworks (*PrAFs* for short) combine them by bringing together the qualitative view of argumentation and the probability values associated with arguments and attacks. The two main approaches on PrAFs in the literature consist in the *constellations* and *epistemic* approaches (see Sect. 3).

The authors are members of the INdAM Research group GNCS and of Consorzio CINI. This work has been partially supported by: project RACRA - funded by "Ricerca di Base 2018–2019" (Univeristy of Perugia), project DopUP - "REGIONE UMBRIA PSR" 2014–2020.

G. Gottlob et al. (Eds.): LPNMR 2022, LNAI 13416, pp. 62–74, 2022.
https://doi.org/10.1007/978-3-031-15707-3_6

In this paper we focus on the former: values determine the likelihood of arguments and attacks to be part of a framework, thus generating different frameworks with a different existence probability.

As advanced in several works in the related literature (e.g., [2,13]), the idea behind the grounded semantics in Abstract Argumentation is to accept only the arguments that must be accepted and to reject only the arguments that one cannot avoid to reject. This leads to the definition of the most *sceptical* (or *least committed*) semantics among those based on complete extensions. The ideal [14] and eager [10] semantics have been defined as less sceptical positions, since the grounded extension is a subset of the ideal extension, which is a subset of the grounded one [10]. In case of a sceptical approach, the existence of more than one possible argumentative position is often dealt with by taking the intersection of different extensions. For instance, the ideal semantics uses the intersection of all the preferred extensions in its definition. Even the grounded extension, though originally defined as the *least fixed-point* of a framework *characteristic function* [13], also corresponds to the intersection of complete extensions.

In this paper we propose a different way to compute sceptical extensions of a PrAF in the constellations approach [17,19]: hence we focus on the grounded, ideal, and eager semantics. The main goal is to propose a single extension for the whole set of frameworks induced from a given PrAF. On the contrary, in [19] all the most frequent subsets of arguments that belong to a grounded extension in induced frameworks are equally-good candidates.

A simple example is any PrAF where both $P(a) = P(b) = 1$, and a, b are not attacked: they are present in all the induced frameworks and they always belong to the grounded extension. Hence, either $\{a\}$ or $\{b\}$ have the same (highest possible) probability 1 to satisfy the grounded semantics.

To obtain unicity, we consider the probability of the intersection of the events *i)* a set of arguments is a subset of a complete/preferred/semi-stable extension in an induced framework, and *ii)* that framework is induced by the considered PrAF. Then, sceptical semantics can be seen as the union of argument-sets maximising the probability of these two events: by taking maximum-probability positions we realise scepticism in PrAFs.

With such an approach, we show that the sceptical extensions of a PrAF correspond to the intersection of their equivalents obtained on all the induced frameworks. As introduced before, this intersection operation is also often used in non-probabilistic frameworks to define classical sceptical semantics. By having characterised the intersection of possible frameworks in a quantitative (probabilistic) way, we end up with the opportunity of relaxing scepticism by taking less-than-maximal-probability sets. Therefore, the proposed approach offers a way to quantitatively relax scepticism, as classical ideal and eager semantics do in a qualitatively way with respect to the grounded extension instead. Being the grounded extension the intersection of all the complete extensions from all the induced frameworks, a (uninformative) result of empty-set is thus very likely.

The paper is organised as follows: Sect. 2 introduces the necessary background about semantics and PrAFs defined with the constellations approach.

Section 4 redefines the grounded semantics in PrAFs, with related formal results and examples, while in Sect. 5 we extend former results to the ideal and eager semantics. Section 3 summarises some of the related work about PrAFs, and finally Sect. 6 wraps up the paper with final conclusions and future work.

2 The Constellations Approach

An *Abstract Argumentation Framework* (AF, for short) [13] is a tuple $\mathcal{F} = (\mathsf{A}, \mathsf{R})$ where A is a set of arguments and R is the attack relation $\mathsf{R} \subseteq \mathsf{A} \times \mathsf{A}$.

A set $E \subseteq \mathsf{A}$ is *conflict-free* (in $\mathcal{F}$) if and only if there are no $a, b \in E$ with $a \rightarrow b$ (i.e., "a attacks b"). E is *admissible* (i.e., $E \in \mathbf{ad}(\mathcal{F})$) if and only if it is conflict-free and each $a \in E$ is defended by E, i.e., E attacks any attacker of a. Finally, the *range* of E in $\mathcal{F}$, i.e., $E^+_{\mathcal{F}}$, collects the same E and the set of arguments attacked by E: $E^+_{\mathcal{F}} = E \cup \{a \in \mathsf{A} \mid \exists b \in E : b \rightarrow a\}$. Argumentation semantics determine sets of jointly acceptable arguments, called *extensions*, by mapping each $\mathcal{F} = (\mathsf{A}, \mathsf{R})$ to a set $\sigma(\mathcal{F}) \subseteq 2^{\mathsf{A}}$, where 2^{A} is the power-set of A, and σ parametrically stands for any of the considered semantics. The extensions under complete, preferred, semi-stable, grounded, ideal and eager semantics are respectively defined as follows. Given $\mathcal{F} = (\mathsf{A}, \mathsf{R})$ and a set $E \subseteq \mathsf{A}$,

- $E \in \mathbf{co}(\mathcal{F})$ if and only if E is admissible in $\mathcal{F}$ and if $a \in \mathsf{A}$ is defended by E in $\mathcal{F}$ then $a \in E$,
- $E \in \mathbf{pr}(\mathcal{F})$ if and only if $E \in \mathbf{co}(\mathcal{F})$ and there is no $E' \in \mathbf{co}(\mathcal{F})$ s.t. $E' \supset E$,
- $E \in \mathbf{sst}(\mathcal{F})$ if and only if $E \in \mathbf{co}(\mathcal{F})$ and there is no $E' \in \mathbf{co}(\mathcal{F})$ s.t. $E'^+_{\mathcal{F}} \supset E^+_{\mathcal{F}}$,
- $E \in \mathbf{gr}(\mathcal{F})$ if and only if $E \in \mathbf{co}(\mathcal{F})$ and there is no $E' \in \mathbf{co}(\mathcal{F})$ s.t. $E' \subset E$,
- $E \in \mathbf{id}(\mathcal{F})$ if and only if E is admissible, $E \subseteq \bigcap \mathbf{pr}(\mathcal{F})$ and there is no admissible $E' \subseteq \bigcap \mathbf{pr}(\mathcal{F})$ s.t. $E' \supset E$,
- $E \in \mathbf{eg}(\mathcal{F})$ if and only if E is admissible, $E \subseteq \bigcap \mathbf{sst}(\mathcal{F})$ and there is no admissible $E' \subseteq \bigcap \mathbf{sst}(\mathcal{F})$ s.t. $E' \supset E$.

A *Probabilistic Argumentation Framework* (*PrAF*) [19] represents the set of all AFs that can potentially be induced from it. A PrAF is a Dung's framework where both arguments (A_p) and attacks (R_p) are associated with their likelihood of existence: i.e., $P_{\mathsf{A}_p} : \mathsf{A}_p \rightarrow (0, 1]$, and $P_{\mathsf{R}_p} : \mathsf{R}_p \rightarrow (0, 1]$: hence, $\mathcal{F}_p = (\mathsf{A}_p, \mathsf{R}_p, P_{\mathsf{A}_p}, P_{\mathsf{R}_p})$. An induced AF includes all the arguments and attacks with a likelihood of 1, as well as further components as specified by Definition 1.

Definition 1 (Inducing an AF [19]). *A Dung abstract framework* $\mathcal{F} = (\mathsf{A}, \mathsf{R})$ *is induced from a* $\mathcal{F}_p = (\mathsf{A}_p, \mathsf{R}_p, P_{\mathsf{A}_p}, P_{\mathsf{R}_p})$ *if and only if the remainder holds:* i) $\mathsf{A} \subseteq \mathsf{A}_p$, ii) $\mathsf{R} \subseteq (\mathsf{R}_p \cap (\mathsf{A} \times \mathsf{A}))$, iii) $\forall a \in \mathsf{A}_p$ *such that* $P_{\mathsf{A}_p}(a) = 1$*, then* $a \in \mathsf{A}$, iv) $\forall (a_i, a_j) \in \mathsf{R}_p$ *such that* $P_{\mathsf{R}_p}(a_i, a_j) = 1$ *and* $a_i, a_j \in \mathsf{A}$*, then* $(a_i, a_j) \in \mathsf{R}$*. We write* $\mathbb{I}(\mathcal{F}_p)$ *to represent the set of all AFs that can be induced from a* $\mathcal{F}_p$*.*

Therefore, arguments and attacks with a likelihood of 1 must be present in all the induced frameworks whenever possible (i.e., an attack also needs incident arguments to be present), while not-completely certain components can appear or not in an induced framework. The probability of an induced AF is computed as the joint probability of all the independent variables:

Definition 2 (Probability of induced $\mathcal{F}$ [19]). *With $\mathcal{F}_p = (\mathsf{A}_p, \mathsf{R}_p, P_{\mathsf{A}_p}, P_{\mathsf{R}_p})$, the probability of $\mathcal{F} = (\mathsf{A}, \mathsf{R}) \in \mathbb{I}(\mathcal{F}_p)$ is:*

$$P^{\mathbb{I}}_{\mathcal{F}_p}(\mathcal{F}) = \prod_{a_i \in \mathsf{A}} P_{\mathsf{A}_p}(a_i) \prod_{a_i \in (\mathsf{A}_p \setminus \mathsf{A})} (1 - P_{\mathsf{A}_p}(a_i))$$
$$\prod_{(a_i,a_j) \in \mathsf{R}} P_{\mathsf{R}_p}((a_i, a_j)) \prod_{(a_i,a_j) \in (\mathsf{R}_p \setminus \mathsf{R}) \; s.t. \; a_i, b_j \in \mathsf{A}} (1 - P_{\mathsf{R}_p}((a_i, a_j))$$

The set of possible worlds induced by a PrAF sums up to a probability of 1.

Proposition 1 [19]**.** *The sum of all the probability values of all the frameworks that can be induced from a $\mathcal{F}_p = (\mathsf{A}_p, \mathsf{R}_p, P_{\mathsf{A}_p}, P_{\mathsf{R}_p})$ is 1:*

$$\sum_{\mathcal{F}_i \in \mathbb{I}(\mathcal{F}_p)} P^{\mathbb{I}}_{\mathcal{F}_p}(\mathcal{F}_i) = 1$$

Definition 3 computes the likelihood of a set E of arguments being "consistent" with respect a given argumentation semantics σ.

Definition 3 (Extension probability [19]). *Given a $\mathcal{F}_p = (\mathsf{A}_p, \mathsf{R}_p, P_{\mathsf{A}_p}, P_{\mathsf{R}_p})$, the probability that a given set of arguments $B \subseteq P_{\mathsf{A}_p}$ satisfies a semantics σ is (function ξ is discussed in the following paragraph):*

$$P_\sigma(B, \mathcal{F}_p) = \sum_{\mathcal{F}_i \in \mathbb{I}(\mathcal{F}_p)} P^{\mathbb{I}}_{\mathcal{F}_p}(\mathcal{F}_i) \textit{ where } \xi^\sigma(\mathcal{F}_i, B) = \textit{true}$$

In [19] function $\xi^\sigma(\mathcal{F}_i, B)$ is said to return true if and only if the set of arguments B is deemed "consistent" using semantics σ when evaluated over a framework $\mathcal{F}_i$ induced from $\mathcal{F}_p$. For instance, we can consider ξ to return true if and only if $B \in \sigma(\mathcal{F}_i)$, that is if and only if B is an extension in $\mathcal{F}_i$ according to semantics σ, or if B is just a subset of a σ extension, as proposed in [19].

Example 1. In Fig. 1 we show an example of PrAF. Such a relatively small graph induces thirteen different frameworks: $\mathcal{F}_1 = (\{a, e\}, \{\})$, $\mathcal{F}_2 = (\{a, b, e\}, \{(a, b)\})$, $\mathcal{F}_3 = (\{a, c, e\}, \{\})$, $\mathcal{F}_4 = (\{a, b, c, e\}, \{(a, b)\})$, $\mathcal{F}_5 = (\{a, b, c, e\}, \{(a, b), (c, b)\})$, $\mathcal{F}_6 = (\{a, d, e\}, \{(d, e)\})$, $\mathcal{F}_7 = (\{a, b, d, e\}, \{(a, b), (d, e)\})$, $\mathcal{F}_8 = (\{a, c, d, e\}, \{(d, c), (d, e)\})$, $\mathcal{F}_9 = (\{a, c, d, e\}, \{(c, d), (d, c), (d, e)\})$, $\mathcal{F}_{10} = (\{a, b, c, d, e\}, \{(a, b), (d, c), (d, e)\})$, $\mathcal{F}_{11} = (\{a, b, c, d, e\}, \{(a, b), (c, b), (d, c), (d, e)\})$, $\mathcal{F}_{12} = (\{a, b, c, d, e\}, \{(a, b), (c, d), (d, c), (d, e)\})$, $\mathcal{F}_{13} = (\{a, b, c, d, e\}, \{(a, b), (c, b), (c, d), (d, c), (d, e)\})$, whose probabilities are $[0.09, 0.06, 0.21, 0.056, 0.084, 0.09, 0.06, 0.147, 0.063, 0.0392, 0.0588, 0.0168, 0.0252]$. These values clearly sum up to 1.

3 Related Work

In the literature there exist two main approaches to probabilistic argumentation: the constellations [19] and the epistemic approaches [22]. A third approach is proposed in [20]: in that case, the probability distribution over labellings [9] gives

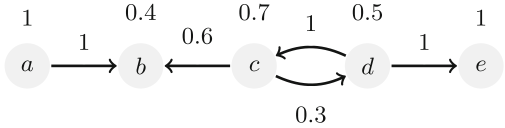

Fig. 1. An example of PrAF.

a form of probabilistic argumentation that overlaps with both the constellations and epistemic approaches.

In the constellations approach, the uncertainty resides in the topology of the considered AF: probability values label arguments and attacks. The authors of [15] provided the first proposal to extend abstract argumentation with a probability distribution over sets of arguments which they use with a version of assumption-based argumentation in which a subset of the rules are probabilistic rules. In [19] a probability distribution over the sub-graphs of the argument graph is introduced, and this can then be used to give a probability assignment for a set of arguments being an admissible set or extension of the argument graph. In [12] the authors characterise the different semantics from the approach of [19] in terms of probabilistic logic with the purpose of providing an uniform logical formalisation and also pave the way for future implementations. Complexity aspects related to computing the probability that a set of arguments is an extension according to a given semantics are instead presented in [16].

In the epistemic approach instead, the topology of a graph is fixed, but the more likely an agent is to believe in an argument, the less likely it is to believe in an argument attacking it. This reminds other related approached such as *ranking-based semantics* [1] and *weighted argumentation frameworks* [5,6,8]. For instance, in [3] the authors cast epistemic probabilities in the context of de Finetti's theory of subjective probability, and they analyse and revise the relevant rationality properties in relation with de Finetti's notion of coherence. However, most of the work in this directions is authored by M. Thimm [22] and A. Hunter [17]. In the first work, the author proposes a probabilistic approach assigning probabilities or degrees of belief to individual arguments. The presented semantics generalise the classical notions of semantics [13]. In the second work, the author starts from considering logic-based argumentation with uncertain arguments, but ends showing how this formalisation relates to uncertainty of abstract arguments. The two authors join their efforts in [18].

Some more related references concern the use of frameworks whose topology is not completely expressed, similarly to the constellations approach. For example, the work in [11] introduced the notion of *Partial Argumentation Framework* (*PAF*), which are defined by a set of arguments, an attack relation $\rightarrow \subseteq (\mathsf{A} \times \mathsf{A})$ specifying attacks *known to exist*, and an *ignorance relation ign* $\subseteq (\mathsf{A} \times \mathsf{A})$ specifying attacks whose existence is not known. This reflects the fact that some agents may ignore arguments pointed out by other agents, as well as how such arguments interact with her own ones: the goal of the authors is to merge different frameworks together, and thus not all the agents are assumed to share the same global set of arguments. *Incomplete Argumentation Frameworks* (*IAF*) further generalise PAFs, since they can represent uncertainty about the existence of individual arguments, uncertainty about the existence of individual attacks, or both simultaneously [4].

4 The Grounded Semantics of a PrAF

In [19] the authors suggests $\xi^{\mathbf{gr}}(\mathcal{F}_i, B)$ to return "true" when the set of arguments B is a subset of the grounded extension of $\mathcal{F}_i$ (see Definition 3). However, this choice for ξ leads to some issues related to the non-uniqueness of the grounded extension, which is indeed a desirable result being it also defined as a *unique*-status or *single-status* semantics in Dung's frameworks [2]. For example, the PrAF in Fig. 1 has two alternative choices for the grounded extension: $P_{\mathbf{gr}}(\emptyset, \mathcal{F}_p) = P_{\mathbf{gr}}(\{a\}, \mathcal{F}_p) = 1$.

This is the main motivation that moved us towards a different definition of the grounded semantics, with the purpose to have one single result (with the maximal probability), together with the need to connect its characteristic of scepticism to probabilistic frameworks: the grounded extension minimises the overall uncertainty and includes only the least questionable arguments present in complete extensions. In order to rephrase this characteristic into probabilistic frameworks, the grounded extension should include the arguments that are *most likely* included in the complete extensions of all the possible induced frameworks.

To accomplish this, we were inspired by the *law of total probability* to compute the average probability of an event U on the probability space defined by the events $\{V_n : n = 1, 2, \ldots, n\}$, which are a finite or countably infinite partition of such sample space: $P(U) = \sum_i P(U \cap V_i) = \sum_i P(U \mid V_i) \cdot P(V_i)$.

In our specific case, $P(V_i)$ describes the probability of $\mathcal{F}_i$ to be an induced framework of $\mathcal{F}_p$, that is $P^{\mathbb{I}}_{\mathcal{F}_p}(\mathcal{F}_i)$. Such a probability value is weighted by $P(U \mid V_i)$, which in our case is not really a probability but the frequency of a subset B to appear in the complete extensions of $\mathcal{F}_i$ instead.

Definition 4 (Probability B is a subset of a complete extension in $\mathcal{F}_p$). *Given $\mathcal{F}_p = (\mathsf{A}_p, \mathsf{R}_p, P_{\mathsf{A}_p}, P_{\mathsf{R}_p})$, the probability of $B \in \mathsf{A}_p$ to be a subset of the complete extensions in $\mathcal{F}_p$ is computed as:*

$$P(B)^{\mathbf{co}}_{\mathcal{F}_p} = \sum_{\mathcal{F}_i \in \mathbb{I}(\mathcal{F}_p)} (|E \in \mathbf{co}(\mathcal{F}_i)|\, s.t.\, B \subseteq E) \,/\, |\mathbf{co}(\mathcal{F}_i)| \cdot P^{\mathbb{I}}_{\mathcal{F}_p}(\mathcal{F}_i)$$

For example, if B is a subset of half of the complete extensions in an induced $\mathcal{F}_i$ and $P^{\mathbb{I}}_{\mathcal{F}_p}(\mathcal{F}_i) = 0.25$, then the contribution of $\mathcal{F}_i$ to $P(B)^{\mathbf{co}}_{\mathcal{F}_p}$ is $0.5 \cdot 0.25 = 0.125$. To compute the total contribution one has to consider all the $\mathcal{F}_i$.

It is now possible to define the grounded semantics of a PrAF as the union of the subsets of maximal-probability, with probability as defined in Definition 4.

Definition 5 (Grounded semantics). *Given $\mathcal{F}_p = (\mathsf{A}_p, \mathsf{R}_p, P_{\mathsf{A}_p}, P_{\mathsf{R}_p})$, a family of sets S, the grounded extension is defined as the union $\bigcup$ of subsets of maximal $P(B)^{\mathbf{co}}_{\mathcal{F}_p}$:*

$$\mathbf{gr}(\mathcal{F}_p) = \bigcup\{B \mid P(B)^{\mathbf{co}}_{\mathcal{F}_p} \text{ is maximal}\} \tag{1}$$

Example 2. Given the PrAF in Fig. 1, the thirteen induced frameworks has the following sets of complete extensions: $\mathbf{co}(\mathcal{F}_1) = \{\{a, e\}\}$, $\mathbf{co}(\mathcal{F}_2) = \{\{a, e\}\}$, $\mathbf{co}(\mathcal{F}_3) = \{\{a, c, e\}\}$, $\mathbf{co}(\mathcal{F}_4) = \{\{a, c, e\}\}$, $\mathbf{co}(\mathcal{F}_5) = \{\{a, c, e\}\}$, $\mathbf{co}(\mathcal{F}_6) = \{\{a, d\}\}$, $\mathbf{co}(\mathcal{F}_7) = \{\{a, d\}\}$, $\mathbf{co}(\mathcal{F}_8) = \{\{a, d\}\}$, $\mathbf{co}(\mathcal{F}_9) = \{\{a\}, \{a, d\}, \{a, c, e\}\}$, $\mathbf{co}(\mathcal{F}_{10}) = \{\{a, d\}\}$, $\mathbf{co}(\mathcal{F}_{11}) = \{\{a, d\}\}$, and $\mathbf{co}(\mathcal{F}_{12}) = \{\{a\}, \{a, d\}, \{a, c, e\}\}$, and $\mathbf{co}(\mathcal{F}_{13}) = \{\{a\}, \{a, d\}, \{a, c, e\}\}$. There exist ten possible subsets of complete extensions in $\mathcal{F}_p$, that is $\emptyset, \{a\}, \{c\}, \{d\}, \{e\}, \{a, c\}, \{a, d\}, \{a, e\}, \{c, e\}, \{a, c, e\}$, whose probability as defined in Definition 4 are respectively defined by the array $[1, 1, 0.385, 0.43, 0.535, 0.385, 0.43, 0.535, 0.385, 0.385]$. Hence, as stated by Definition 5, $\mathbf{gr}(\mathcal{F}_p) = \emptyset \cup \{a\} = \{a\}$, since both $\emptyset$ and $\{a\}$ have a probability of 1.

The next remark explains why it is possible to replace "maximal probability" with $P(B)^{\mathbf{co}}_{\mathcal{F}_p} = 1$ in Definition 5 without changing the result.

Remark 1. Note that the probability of empty-set is always maximal, since it is trivially a subset of any $E \in \mathbf{co}(\mathcal{F}_i)$: as a consequence, empty-set will always be considered with a probability of 1 in the union of sets in Eq. 1. For the same reason, any subset B with a probability strictly less than 1 will never be considered to be part of the grounded extension. Therefore, maximal and equal to 1 probabilities will be interchangeably used in the rest of the paper.

The grounded semantics always results in a single extension.

Proposition 2 (Unicity). *The grounded extension in $\mathcal{F}_p = (\mathsf{A}_p, \mathsf{R}_p, P_{\mathsf{A}_p}, P_{\mathsf{R}_p})$ is unique.*

Proof. It straightforwardly follows from the fact that the grounded extension is defined as the union of some sets of arguments.

In addition, when from $\mathcal{F}_p$ it is possible to induce a single framework, i.e., $|\mathbb{I}(\mathcal{F}_p)| = 1$, then the grounded semantics corresponds to its classical definition given by P. M. Dung in [13]. This allows to reconnect to classical abstract argumentation in case of no uncertainty in the framework topology.

Theorem 1 (Correspondence with Dung). *Given $\mathcal{F}_p = (\mathsf{A}_p, \mathsf{R}_p, P_{\mathsf{A}_p}, P_{\mathsf{R}_p})$ such that $\mathbb{I}(\mathcal{F}_p) = \{\mathcal{F}\}$, then $\mathbf{gr}(\mathcal{F}_p) = \mathbf{gr}(\mathcal{F})$.*

Proof. Since we only have one induced framework $\mathcal{F}$ by hypothesis, whose probability is 1 according to Proposition 1, then $P(B)^{\mathbf{co}}_{\mathcal{F}_p} = \sum_{\mathcal{F}_i \in \mathbb{I}(\mathcal{F}_p)} (|E \in \mathbf{co}(\mathcal{F}_i)| \; s.t. \; B \subseteq E) \,/\, |\mathbf{co}(\mathcal{F}_i)| \cdot P^{\mathbb{I}}_{\mathcal{F}_p}(\mathcal{F}_i) = (|E \in \mathbf{co}(\mathcal{F})| \; s.t. \; B \subseteq E) \,/\, |\mathbf{co}(\mathcal{F})|$. Since $\mathbf{gr}(\mathcal{F})$ is defined as the intersection of all the complete extensions in $\mathcal{F}$, then we have that $P(\mathbf{gr}(\mathcal{F}))^{\mathbf{co}}_{\mathcal{F}_p}$ is 1, while for any $B \neq \emptyset$ and $B \neq \mathbf{gr}(\mathcal{F})$, $P(B)^{\mathbf{co}}_{\mathcal{F}_p} < 1$. From Definition 5 we obtain $\mathbf{gr}(\mathcal{F}_p) = \mathbf{gr}(\mathcal{F})$.

Example 3. If we consider a PrAF $\mathcal{F}_p$ s.t. $\mathsf{A}_p = \{a, b, c, d\}$, $\mathsf{R}_p = \{(a, b), (b, c), (c, d)\}$, $P_{\mathsf{A}_p} = \{1, 1, 1, 1\}$, $P_{\mathsf{R}_p} = \{1, 1, 1\}$, we have a single induced framework whose complete extension is $\{a, c\}$. Hence, its subsets are $\{\emptyset, \{c\}, \{a\}, \{a, c\}\}$ with probabilities $[1.0, 1.0, 1.0, 1.0]$. The union of all these maximal-probability subsets is equivalent to $\mathbf{gr}(\mathcal{F}_p) = \{a, c\}$.

Theorem 2 states that the definition of grounded extension given in Definition 5 adheres to the principle often used to enforce scepticism in Abstract Argumentation: as introduced in Sect. 1, the intersection of different extensions leads to only those arguments that are taken in all of them, thus eliminating uncertainty. Also in the case of PrAFs, the grounded extension of each induced $\mathcal{F}_i$ is the intersection of complete extensions, while the grounded extension of the entire $\mathcal{F}_p$ is the intersection of all the different grounded extensions for each $\mathcal{F}_i$.

Theorem 2 (Intersection of grounded ext.s). *Being $\mathcal{F}_p = (\mathsf{A}_p, \mathsf{R}_p, P_{\mathsf{A}_p}, P_{\mathsf{R}_p})$ any PrAF, then the grounded extension defined in Definition 5 corresponds to:*

$$\mathbf{gr}(\mathcal{F}_p) = \bigcap_{\mathcal{F}_i \in \mathbb{I}(\mathcal{F}_p)} \mathbf{gr}(\mathcal{F}_i)$$

Proof. Given any $a \in \mathsf{A}_p$, if $\forall \mathcal{F}_i \in \mathbb{I}(\mathcal{F}_p)$ (except the empty framework, if it exists) $a \in \mathbf{gr}(\mathcal{F}_i)$ then $P(\{a\})^{\mathbf{co}}_{\mathcal{F}_p}$, since a is included in any complete extension of each $\mathcal{F}_i$. According to Definition 3.2 we have that $\bigcup_a \{a\} = \mathbf{gr}(\mathcal{F}_p)$, because all these sets have maximal probability. On the contrary, if $\exists \mathcal{F}_i \in \mathbb{I}(\mathcal{F}_p). a \notin \mathbf{gr}(\mathcal{F}_i)$ then $P(\{a\})^{\mathbf{co}}_{\mathcal{F}_p}$ is not maximal (i.e., it has a probability strictly lower than 1).

Note that from Theorem 2 we directly derive that $\mathbf{gr}(\mathcal{F}_p)$ is conflict-free in $\mathcal{F}_p$, even if it make little sense to check it in a PrAF, since an attack may or may not exist depending on the induced framework.

Corollary 1 underlines the scepticism behind the definition of grounded semantics given in Definition 5. If an induced framework such that its grounded extension is empty-set exists, or equivalently empty-set is a complete extension of that framework, then the grounded extension of the whole PrAF is empty-set as well.

Corollary 1 (Empty-set dominance). *If $\exists \mathcal{F}_i \in \mathbb{I}(\mathcal{F}_p)$ s.t. $\emptyset \in \mathbf{co}(\mathcal{F}_i)$, then $\mathbf{gr}(\mathcal{F}_p) = \emptyset$.*

Proof. We have that $P(\emptyset)^{\mathbf{co}}_{\mathcal{F}_p} = 1$ for any $\mathcal{F}_i \in \mathbb{I}(\mathcal{F}_p)$. If $\emptyset \in \mathbf{co}(\mathcal{F}_i)$, then $\forall B \neq \emptyset$ we have that $B \nsubseteq (E = \emptyset)$. For this reason, $P(B)^{\mathbf{co}}_{\mathcal{F}_p} < 1$ for any $B \neq \emptyset$. Since Definition 3.2 aggregates maximal-probability subsets only, then $\mathbf{gr}(\mathcal{F}_p)$ corresponds to the union of empty-set only.

In the definition of the grounded semantics for probabilistic frameworks we tried to stick to the principle of scepticism, according to which empty-set is clearly the most sceptical position to be taken. In the next example we recap the previous formal results.

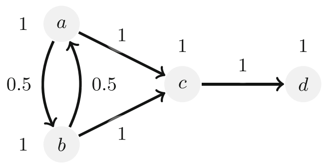

Fig. 2. An example of PrAF.

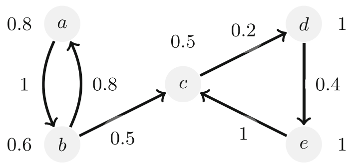

Fig. 3. A second example of PrAF.

Example 4. In Fig. 2 we show an example of a PrAF that induces the four frameworks represented in Fig. 4. These frameworks respectively have $\mathbf{co}(\mathcal{F}_1) = \{\{a, b, d\}\}$, $\mathbf{co}(\mathcal{F}_2) = \{\{a, d\}\}$, $\mathbf{co}(\mathcal{F}_3) = \{\{b, d\}\}$, $\mathbf{co}(\mathcal{F}_3) = \{\emptyset, \{a, d\}, \{b, d\}\}$. Thus, subsets of complete extensions are $\emptyset$, $\{a\}$, $\{b\}$, $\{d\}$, $\{a, b\}$, $\{a, d\}$, $\{b, d\}$, $\{a, b, d\}$ and their probability is $[1, 0.583, 0.583, 0.916, 0.25, 0.583, 0.583, 0.25]$. The four grounded extensions are $\mathbf{gr}(\mathcal{F}_1) = \{a, b, d\}$, $\mathbf{gr}(\mathcal{F}_2) = \{a, d\}$, $\mathbf{gr}(\mathcal{F}_3) = \{b, d\}$, and $\mathbf{gr}(\mathcal{F}_4) = \emptyset$. The intersection of all the grounded extensions from the four frameworks is $\emptyset$.

Because of the aforementioned motivations, having defined the grounded extension of a PrAF as the intersection of the grounded extensions of each induced framework, the probability for a framework to be empty-set is clearly high. This result is clearly not very informative, and it is a possible drawback of being too much sceptical. However, by having defined a quantitative approach to the definition of the grounded semantics, we can also think of relaxing scepticism by imposing a lower threshold on probability when merging the subsets as suggested in Definition 4, instead of taking the most probable subsets.

Definition 6 (*t*-relaxed grounded semantics). *Given* $\mathcal{F}_p = (\mathsf{A}_p, \mathsf{R}_p, P_{\mathsf{A}_p}, P_{\mathsf{R}_p})$*, a family of sets* S*,* $t \in [0, 1]$*, the* t*-relaxed grounded extension is defined as the union of subsets whose* $P(B \subseteq (E \in \mathbf{co}(\mathcal{F}_p)))$ *is greater-equal than* t*:*

$$\overset{t}{\mathbf{gr}}(\mathcal{F}_p) = \bigcup\{B \mid \ P(B)^{\mathbf{co}}_{\mathcal{F}_p} \geq t\} \tag{2}$$

Example 5. If we set $t = 0.9$, the t-grounded extension of the PrAF in Fig. 2 is $\{d\}$, since the probability of this subset is 0.916, and $\emptyset \cup \{d\} = \{d\}$.

Clearly, $\overset{1}{\mathbf{gr}}(\mathcal{F}_p)$ corresponds to $\mathbf{gr}(\mathcal{F}_p)$ in Definition 5.

5 Further Sceptical Semantics

There are two other sceptical semantics in the literature, which share the unique-statusness with the grounded: the ideal and eager semantics. Both **eg** and **id** have been designed to relax scepticism of the former one: $\mathbf{gr}(\mathcal{F}) \subseteq \mathbf{id}(\mathcal{F}) \subseteq \mathbf{eg}(\mathcal{F})$ [10].

Because of their importance and closeness to the grounded semantics, in the following of this section we provide a probabilistic definition of these two semantics, in the style of what proposed in Sect. 4.

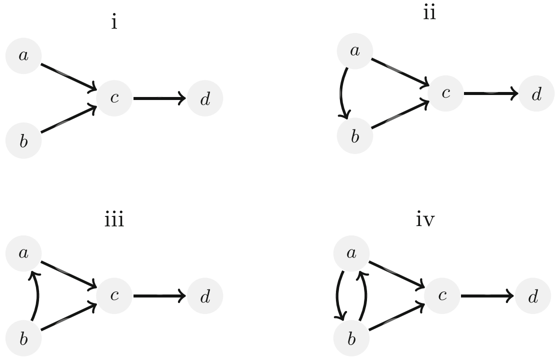

Fig. 4. The four frameworks induced by the PrAF in Fig 2.

Definition 7 (Probability of id and eg). *Given* $\mathcal{F}_p = (\mathsf{A}_p, \mathsf{R}_p, P_{\mathsf{A}_p}, P_{\mathsf{R}_p})$*, the probability of* $B \in \mathsf{A}_p$ *to be a subset of the preferred/semi-stable extensions in* $\mathcal{F}_p$*, are respectively computed as:*

$$P(B)^{\mathbf{pr}}_{\mathcal{F}_p} = \sum_{\mathcal{F}_i \in \mathbb{I}(\mathcal{F}_p)} (|E \in \mathbf{pr}(\mathcal{F}_i)|\, s.t.\, B \subseteq E) \,/\, |\mathbf{pr}(\mathcal{F}_i)| \cdot P^{\mathbb{I}}_{\mathcal{F}_p}(\mathcal{F}_i) \tag{3}$$

$$P(B)^{\mathbf{sst}}_{\mathcal{F}_p} = \sum_{\mathcal{F}_i \in \mathbb{I}(\mathcal{F}_p)} (|E \in \mathbf{sst}(\mathcal{F}_i)|\, s.t.\, B \subseteq E) \,/\, |\mathbf{sst}(\mathcal{F}_i)| \cdot P^{\mathbb{I}}_{\mathcal{F}_p}(\mathcal{F}_i) \tag{4}$$

In Definition 8 we use Eq. 3 and Eq. 4 to propose a definition of respectively ideal and eager semantics in PrAFs.

Definition 8 (Ideal/eager semantics). *Given* $\mathcal{F}_p = (\mathsf{A}_p, \mathsf{R}_p, P_{\mathsf{A}_p}, P_{\mathsf{R}_p})$*, the ideal/eager extensions are defined as the union of maximal-probability subsets, as defined in Eq. 3 and Eq. 4 respectively:*

$$\mathbf{id}(\mathcal{F}_p) = \bigcup\{B \mid\; P(B)^{\mathbf{pr}}_{\mathcal{F}_p} \textit{ is maximal}\}, \;\; \mathbf{eg}(\mathcal{F}_p) = \bigcup\{B \mid\; P(B)^{\mathbf{sst}}_{\mathcal{F}_p} \textit{ is maximal}\}$$

Example 6. The grounded, ideal, and eager extensions in Fig. 2 are $\emptyset$. The grounded extension in Fig. 2b is $\emptyset$, while the ideal and the eager ones are $\{a\}$: in this case, the induced frameworks are $\mathcal{F}_1 = (\{a,b\}, \{(a,b),(b,a),(b,b)\})$ and $\mathcal{F}_2 = (\{a,b\}, \{(a,b),(b,b)\})$. Thus, subsets of complete extensions are $\emptyset$ and $\{a\}$, with probability of 1 and 0.5.

Remark 2. As in Remark 1, empty-set is always contained in every preferred and semi-stable extension, and thus $P(\emptyset)^{\mathbf{pr}}_{\mathcal{F}_p} = P(\emptyset)^{\mathbf{sst}}_{\mathcal{F}_p} = 1$ for any possible $\mathcal{F}_p$. For this reason, requiring maximal probability or a probability value equal to 1 is equivalent in Definition 8. Both the ideal and eager extensions will always be made of only subsets B with a probability of 1.

In case of a single induced framework from $\mathcal{F}_p$, the ideal and eager extensions correspond to their classical definition in [14] and [10] respectively.

Proposition 3 (Unicity of ideal and eager). *The ideal and eager extensions in* $\mathcal{F}_p = (\mathsf{A}_p, \mathsf{R}_p, P_{\mathsf{A}_p}, P_{\mathsf{R}_p})$ *are unique.*

Proof. It straightforwardly follows from the fact that the ideal and eager extensions are defined as the union of argument sets.

Theorem 3 (Ideal/eager correspondence). *Given* $\mathcal{F}_p = (\mathsf{A}_p, \mathsf{R}_p, P_{\mathsf{A}_p}, P_{\mathsf{R}_p})$ *s.t.* $\mathbb{I}(\mathcal{F}_p) = \{\mathcal{F}\}$*, then* $\mathbf{id}(\mathcal{F}_p) = \mathbf{id}(\mathcal{F})$ *and* $\mathbf{eg}(\mathcal{F}_p) = \mathbf{eg}(\mathcal{F})$*.*

Proof. Since we only have one induced framework $\mathcal{F}$ by hypothesis, whose probability is 1 according to Prop. 2.1, then $P(B)^{\mathbf{pr}}_{\mathcal{F}_p} = \sum_{\mathcal{F}_i \in \mathbb{I}(\mathcal{F}_p)} (|E \in \mathbf{pr}(\mathcal{F}_i)| \; s.t. \; B \subseteq E) \,/\, |\mathbf{pr}(\mathcal{F}_i)| \cdot P^{\mathbb{I}}_{\mathcal{F}_p}(\mathcal{F}_i) = (|E \in \mathbf{pr}(\mathcal{F})| \; s.t. \; B \subseteq E) \,/\, |\mathbf{pr}(\mathcal{F})|$. Since $\mathbf{id}(\mathcal{F})$ is defined as the intersection of all the preferred extensions in $\mathcal{F}$, then we have that $P(\mathbf{id}(\mathcal{F}))^{\mathbf{pr}}_{\mathcal{F}_p}$ is 1, while for any $B \neq \emptyset$ and $B \neq \mathbf{id}(\mathcal{F})$, $P(B)^{\mathbf{pr}}_{\mathcal{F}_p} < 1$. From Definition 3.2 we obtain $\mathbf{id}(\mathcal{F}_p) = \mathbf{id}(\mathcal{F})$. Similar considerations hold for the eager semantics, with respect to semi-stable extensions.

Even for these two sceptical semantics we can prove that they can be both obtained by intersecting all the respectively ideal/eager extensions on all the induced frameworks, as Theorem 2 shows for the grounded semantics.

Theorem 4. (Intersection of extensions). *Being* $\mathcal{F}_p = (\mathsf{A}_p, \mathsf{R}_p, P_{\mathsf{A}_p}, P_{\mathsf{R}_p})$ *any PrAF, then the ideal and eager extensions respectively correspond to:*

$$\mathbf{id}(\mathcal{F}_p) = \bigcap_{\mathcal{F}_i \in \mathbb{I}(\mathcal{F}_p)} \mathbf{id}(F_i) \qquad \mathbf{eg}(\mathcal{F}_p) = \bigcap_{\mathcal{F}_i \in \mathbb{I}(\mathcal{F}_p)} \mathbf{eg}(F_i)$$

Proof. The proof follows the same approach adopted in Theorem 2.

From Theorem 3, in case of a single induced framework $\mathcal{F}$ we straightforwardly inherit the result that $\mathbf{gr}(\mathcal{F}_p) \subseteq \mathbf{id}(\mathcal{F}_p) \subseteq \mathbf{eg}(\mathcal{F}_p)$ from previous works [10,14]. The following theorem extends this result to more than one induced framework, that is, to all possible PrAFs.

Theorem 5 (Sceptical semantics inclusion). *The subset inclusion* $\mathbf{gr}(\mathcal{F}_p) \subseteq \mathbf{id}(\mathcal{F}_p) \subseteq \mathbf{eg}(\mathcal{F}_p)$ *holds for any PrAF* $\mathcal{F}_p$*.*

Proof. Since for each $\mathcal{F}_i \in \mathbb{I}(\mathcal{F}_p)$ it holds that $\mathbf{gr}(\mathcal{F}_i) \subseteq \mathbf{id}(\mathcal{F}_i) \subseteq \mathbf{eg}(\mathcal{F}_i)$ from Theorem 1 and Theorem 3, then from Theorem 4 the intersection of all $\mathbf{gr}(\mathcal{F}_i)/\mathbf{id}(\mathcal{F}_i)$ is included in the intersection of respectively $\mathbf{id}(\mathcal{F}_i)/\mathbf{eg}(\mathcal{F}_i)$.

It is then possible to relax the ideal and eager extensions as shown for the grounded extension in Definition 6 (similar motivations in mitigating scepticism).

Definition 9 (*t*-relaxed ideal and eager). *Given* $\mathcal{F}_p = (\mathsf{A}_p, \mathsf{R}_p, P_{\mathsf{A}_p}, P_{\mathsf{R}_p})$ *and* $t \in [0, 1]$*, the* t*-relaxed ideal/eager extension is defined as the union of subsets whose* $P(B)^{\mathbf{pr}}_{\mathcal{F}_p}/P(B)^{\mathbf{sst}}_{\mathcal{F}_p}$ *is greater-equal than* t*:*

$$\overset{t}{\mathbf{id}}(\mathcal{F}_p) = \bigcup \{B \mid \; P(B)^{\mathbf{pr}}_{\mathcal{F}_p} \geq t\} \qquad \overset{t}{\mathbf{eg}}(\mathcal{F}_p) = \bigcup \{B \mid \; P(B)^{\mathbf{pr}}_{\mathcal{F}_p} \geq t\}$$

Example 7. The PrAF in Fig. 3 induces 42 different frameworks (which we do not report here for the sake of conciseness); $\mathbf{eg}(\mathcal{F}_p) = \emptyset$, while $\overset{0.601}{\mathbf{eg}}(\mathcal{F}_p)$ is $\{a, d\}$, which happens to be the eager extension if we consider the same framework in the classical Dung's setting. Moreover, $\overset{0.6}{\mathbf{eg}}(\mathcal{F}_p) = \{a, d, e\}$, since $P(\{d\})^{\mathbf{sst}}_{\mathcal{F}_p} = 0.97008$, $P(\{a\})^{\mathbf{sst}}_{\mathcal{F}_p} = 0.60416$, and $P(\{e\})^{\mathbf{sst}}_{\mathcal{F}_p} = 0.6$.

Clearly, by increasing the threshold it is possible to progressively include more arguments.

Proposition 4. *For any* $\mathcal{F}_p = (\mathsf{A}_p, \mathsf{R}_p, P_{\mathsf{A}_p}, P_{\mathsf{R}_p})$ *and thresholds* $t_1, t_2 \in [0, 1]$, *if* $t_2 < t_1$ *then* $\overset{t_1}{\mathbf{gr}}(\mathcal{F}_p) \subseteq \overset{t_2}{\mathbf{gr}}(\mathcal{F}_p)$, $\overset{t_1}{\mathbf{id}}(\mathcal{F}_p) \subseteq \overset{t_2}{\mathbf{id}}(\mathcal{F}_p)$, *and* $\overset{t_1}{\mathbf{eg}}(\mathcal{F}_p) \subseteq \overset{t_2}{\mathbf{eg}}(\mathcal{F}_p)$.

6 Conclusions and Future Work

In this paper we have provided a probabilistic view of sceptical semantics in the constellations approach, since we have focused on the grounded, ideal, and eager extensions. The purpose was to compute clear and single solutions for these semantics, as it happens in Dung's frameworks. To achieve this, we have computed how frequently subsets of arguments appear in complete, preferred, and semi-stable extensions by considering all the induced frameworks. Then, by merging maximal-probability subsets among them we enforce the idea of scepticism in PrAFs. Such a quantitative approach reconnects to the qualitative one often used in argumentation, that is the intersection of different alternatives. However, by using probability values we now also have a quantitative means to relax scepticism, besides using the ideal and eager semantics proposed in Sect. 5. The presented framework has been implemented with a Python script that calls the Docker container of ConArg [7] to enumerate complete, preferred and semi-stable extensions on all the induced frameworks.

In the future we plan to enrich the paper with a definition of more credulous semantics, for example the preferred and stable ones, while still satisfying classical implications among semantics. We will also investigate similarities and differences w.r.t. *standard epistemic extensions* [18], which are directly related to Dung's semantics. Finally, the presented framework can be equipped with a more fine-grained probabilistic logic that explicitly takes epistemic uncertainty and belief (and disbelief as well) into account: i.e., *subjective logic* [21].

References

1. Amgoud, L., Ben-Naim, J., Doder, D., Vesic, S.: Acceptability semantics for weighted argumentation frameworks. In: Proceedings of the Twenty-Sixth International Joint Conference on Artificial Intelligence, pp. 56–62. ijcai.org (2017)
2. Baroni, P., Caminada, M., Giacomin, M.: An introduction to argumentation semantics. Knowl. Eng. Rev. **26**(4), 365–410 (2011)
3. Baroni, P., Giacomin, M., Vicig, P.: On rationality conditions for epistemic probabilities in abstract argumentation. In: Computational Models of Argument - Proceedings of COMMA, FAIA, vol. 266, pp. 121–132. IOS Press (2014)

4. Baumeister, D., Neugebauer, D., Rothe, J.: Collective acceptability in abstract argumentation. FLAP **8**(6), 1503–1542 (2021)
5. Bistarelli, S., Rossi, F., Santini, F.: ConArg: a tool for classical and weighted argumentation. In: Computational Models of Argument - Proceedings of COMMA, FAIA, vol. 287, pp. 463–464. IOS Press (2016)
6. Bistarelli, S., Rossi, F., Santini, F.: A ConArg-based library for abstract argumentation. In: 29th IEEE International Conference on Tools with Artificial Intelligence, ICTAI, pp. 374–381. IEEE Computer Society (2017)
7. Bistarelli, S., Rossi, F., Santini, F.: ConArgLib: an argumentation library with support to search strategies and parallel search. J. Exp. Theor. Artif. Intell. **33**(6), 891–918 (2021)
8. Bistarelli, S., Santini, F.: Weighted argumentation. FLAP **8**(6), 1589–1622 (2021)
9. Caminada, M.: On the issue of reinstatement in argumentation. In: Fisher, M., van der Hoek, W., Konev, B., Lisitsa, A. (eds.) JELIA 2006. LNCS (LNAI), vol. 4160, pp. 111–123. Springer, Heidelberg (2006). https://doi.org/10.1007/11853886_11
10. Caminada, M.: Comparing two unique extension semantics for formal argumentation: ideal and eager. In: Belgian-Dutch Conference on Artificial Intelligence (BNAIC), pp. 81–87 (2007)
11. Coste-Marquis, S., Devred, C., Konieczny, S., Lagasquie-Schiex, M., Marquis, P.: On the merging of Dung's argumentation systems. Artif. Intell. **171**(10–15), 730–753 (2007)
12. Doder, D., Woltran, S.: Probabilistic argumentation frameworks – a logical approach. In: Straccia, U., Calì, A. (eds.) SUM 2014. LNCS (LNAI), vol. 8720, pp. 134–147. Springer, Cham (2014). https://doi.org/10.1007/978-3-319-11508-5_12
13. Dung, P.M.: On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming and n-person games. Artif. Intell. **77**(2), 321–357 (1995)
14. Dung, P.M., Mancarella, P., Toni, F.: Computing ideal sceptical argumentation. Artif. Intell. **171**(10–15), 642–674 (2007)
15. Dung, P.M., Thang, P.M.: Towards (probabilistic) argumentation for jury-based dispute resolution. In: Computational Models of Argument: Proceedings of COMMA, FAIA, vol. 216, pp. 171–182. IOS Press (2010)
16. Fazzinga, B., Flesca, S., Parisi, F.: On the complexity of probabilistic abstract argumentation frameworks. ACM Trans. Comput. Log. **16**(3), 22:1–22:39 (2015)
17. Hunter, A.: A probabilistic approach to modelling uncertain logical arguments. Int. J. Approx. Reason. **54**(1), 47–81 (2013)
18. Hunter, A., Thimm, M.: Probabilistic reasoning with abstract argumentation frameworks. J. Artif. Intell. Res. **59**, 565–611 (2017)
19. Li, H., Oren, N., Norman, T.J.: Probabilistic argumentation frameworks. In: Modgil, S., Oren, N., Toni, F. (eds.) TAFA 2011. LNCS (LNAI), vol. 7132, pp. 1–16. Springer, Heidelberg (2012). https://doi.org/10.1007/978-3-642-29184-5_1
20. Riveret, R., Governatori, G.: On learning attacks in probabilistic abstract argumentation. In: Proceedings of the 2016 International Conference on Autonomous Agents & Multiagent Systems, pp. 653–661. ACM (2016)
21. Santini, F., Jøsang, A., Pini, M.S.: Are my arguments trustworthy? Abstract argumentation with subjective logic. In: 21st International Conference on Information Fusion, FUSION, pp. 1982–1989. IEEE (2018)
22. Thimm, M.: A probabilistic semantics for abstract argumentation. In: ECAI - 20th European Conference on Artificial Intelligence, FAIA, vol. 242, pp. 750–755. IOS Press (2012)