A Database of Aristotelian Diagrams: Empirical Foundations for Logical Geometry

Lorenz Demey[1(✉)] and Hans Smessaert[2]

[1] Center for Logic and Philosophy of Science, KU Leuven
Kardinaal Mercierplein 2, 3000 Leuven, Belgium
lorenz.demey@kuleuven.be

[2] Department of Linguistics, KU Leuven
Blijde-Inkomststraat 21, 3000 Leuven, Belgium
hans.smessaert@kuleuven.be

Abstract. Aristotelian diagrams, such as the square of opposition, are among the oldest and most well-known types of logical diagrams. Within the burgeoning research program of logical geometry, we have been developing a comprehensive database of Aristotelian diagrams that occur in the extant literature: Leonardi.DB (the Leuven Ontology for Aristotelian Diagrams, and its corresponding Database). This paper presents an (intermediate) report on this development. We describe the philosophical background and main motivations for Leonardi.DB, focusing on how the database provides a solid empirical foundation for theoretical research within logical geometry. We also discuss some of the main methodological and technical aspects of the database development. As a proof-of-concept, we provide some examples of the new kinds of research that will be facilitated by Leonardi.DB, e.g. regarding broad trends in the usage and visual properties of Aristotelian diagrams.

Keywords: Aristotelian diagram · Square of opposition · Logical geometry · Leonardi.DB · Diagram database · Semantic Web

1 Introduction

Aristotelian diagrams are among the oldest and most well-known types of logical diagrams. The most famous example is the square of opposition (cf. Fig. 1), but there also exist many other, more complex examples. These diagrams have a rich history in philosophy and logic, and nowadays they are also used extensively in

The first author holds a Research Professorship (BOFZAP) from KU Leuven. This research was funded through the KU Leuven research projects 'Empirical Foundations for Logical Geometry: A Database of Aristotelian Diagrams' (3H180236, 2018–2020) and 'BITSHARE: Bitstring Semantics for Human and Artificial Reasoning' (3H190254, 2019–2023).

V. Giardino et al. (Eds.): Diagrams 2022, LNAI 13462, pp. 123–131, 2022.
https://doi.org/10.1007/978-3-031-15146-0_10

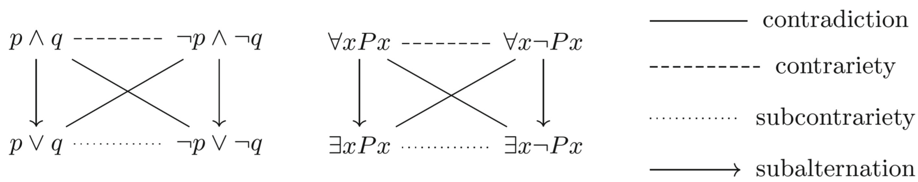

Fig. 1. Squares of opposition for propositional logic and first-order logic.

various other disciplines that deal with logical reasoning, such as linguistics, psychology and artificial intelligence [3,20]. Furthermore, in the past 15 years, it has become increasingly clear that Aristotelian diagrams are not only useful tools to explain or illustrate some logical notion, but can also be fruitfully studied as objects of independent mathematical and philosophical interest. This has given rise to the burgeoning research program of *logical geometry*.

One of the main aims of this research program has been to develop a comprehensive database of Aristotelian diagrams that occur in the extant literature. This has recently led to Leonardi.DB, i.e., the Leuven Ontology for Aristotelian Diagrams, and its corresponding Database, which is now fully available online.[1] The goal of this paper is to present a new (intermediate) report on this development.[2] Sect. 2 describes the philosophical background and main motivations for the development of this database. Section 3 describes some of its main methodological and technical (Semantic Web) aspects. Finally, Sect. 4 provides some examples of the new kinds of research that have become possible, and sketches some avenues for future research.

2 Background and Motivation

Aristotelian diagrams are widely used across reasoning-related disciplines. After a relative decline in popularity in the 20th century,[3] they have witnessed a renewed surge of interest in the first two decades of the 21st century. To a considerable extent, this interest has crystallized around the SQUARE [1,2], and recently also the DIAGRAMS conference series. For example, recent research has focused on the role of Aristotelian diagrams in authors such as John Buridan [5] and Arthur Schopenhauer [14] and topics such as privative negation [12] and Hohfeld's legal concepts [16]. In logical geometry, Aristotelian diagrams are studied as objects of independent interest. From a logical perspective, we study the Boolean properties of these diagrams [18], the interface between opposition and implication relations [10,20], and their broader category-theoretic setting [26]. From a visual-geometric perspective, we study Aristotelian diagrams in terms of notions such as symmetry groups [7], central symmetry [19], vertex-first projections [21] and Euclidean distance [8]; from a visual-cognitive perspective,

[1] Cf. https://leonardi.logicalgeometry.org/.
[2] A first and very preliminary report can be found in [25].
[3] See [13] for the broader religious-cultural context of this temporary setback.

we focus on notions such as free rides [22] and derivative meaning [23]. Finally, there is ongoing research on the interface of Aristotelian diagrams with other types of logical diagrams, such as Hasse, duality and Euler diagrams [4,6,9,11].

Until now, systematic research on Aristotelian diagrams has largely remained an armchair enterprise. When a new theory about some logical, geometric, cognitive or other feature of Aristotelian diagrams is developed, it is checked against and/or illustrated by means of a small and well-delineated set of very well-known applications. Similarly, historical and philosophical reflection also starts from that same limited stock of well-known Aristotelian diagrams, coming from the historical canon of philosophy (e.g. Buridan, Schopenhauer).

To address this situation, we are currently developing a comprehensive database, which aims to collect all Aristotelian diagrams that have ever appeared in the extant literature, along with rich metadata annotations. This database will include the well-known Aristotelian diagrams mentioned above, but the vast majority of diagrams will come from lesser-known authors and applications. After all, it can reasonably be assumed that the distribution of Aristotelian diagrams throughout the literature obeys a version of *Zipf's law* [15]: the occurrence frequency of an Aristotelian diagram is inversely proportional to its frequency rank. We thus hypothesize that there is a small number of diagrams that are used very frequently (clearest example: the square of opposition), but that the overwhelming majority of diagrams is used less often. If the database is to be truly comprehensive in nature, it should not only include the small sample of frequently-used diagrams, but also the much larger number of rarely-used diagrams.

Once the database is sufficiently comprehensive, we envisage it will deliver three main benefits. First of all, it will provide a firm empirical basis for logical geometry, and thus help us to avoid idle armchair theorizing. Rather than developing, illustrating and testing our theories on the basis of a limited stock of well-known diagrams, we will be forced to take the lesser-known cases into account as well, which will lead to more empirically informed and nuanced theories. Secondly, we even expect to discover altogether new types of logical behavior in Aristotelian diagrams. After all, if a certain phenomenon only occurs in some lesser-known diagrams, then it will likely have gone unnoticed until now. However, by forcing us to take these lesser-known diagrams into account as well, the database will allow us to discover the new type of behavior after all. Finally, historical and philosophical research on Aristotelian diagrams often focuses on broader trends in the usage of Aristotelian diagrams across time periods or across scientific disciplines. For example, at the beginning of this section we already mentioned that in the 20th century, there was somewhat of a decline in the use of Aristotelian diagrams, and noted that [13] explains this in religious-cultural terms. To investigate this further, we first need to get a clearer quantitative picture of the situation: is there indeed a (statistically significant) decline in the use of Aristotelian diagrams in the 20th century? A comprehensive diagram database will enable us to carry out precisely such quantitative analyses.

3 Methodological and Technical Aspects

In this section we will describe some of the main methodological and technical aspects of the database that we are currently developing. The database is based on the Leuven Ontology for Aristotelian Diagrams (Leonardi), which was developed specifically for this purpose. The ontology consists of four main categories:

1. persons: e.g. authors, editors, translators, early Modern printers, etc.
2. sources: e.g. monographs, edited volumes, book chapters, journal articles, medieval manuscripts, incunabula, etc.
3. organization: e.g. publishing houses, libraries, national archives, etc.
4. diagrams: most importantly, the actual Aristotelian diagrams

Persons, sources and organizations are clearly auxiliary categories, and are thus annotated with only fairly basic metadata. For example, persons get annotated with their dates of birth and death, if these are known, and also with the most important renderings of their name. The latter is particularly relevant for medieval and early Modern people, e.g. *Jean Buridan* vs. *Johannes Buridanus*, or more extremely, *Juraj Dragišić* vs. *Georgius Benignus*. Whenever possible, we also provide links with other important datasets, e.g. the CERL Thesaurus concerning European book heritage.[4] The ontology is designed primarily to facilitate rich annotation of the actual diagrams. Every diagram in the database is annotated along the following dimensions:

1. administrative: e.g. dates of initial data entry and last modification, etc.
2. bibliographic: e.g. author, source, page/folio number, etc.
3. logical: e.g. Aristotelian family, Boolean complexity, formulas unique up to logical equivalence, presence of logical errors in the diagram, etc.
4. geometric: e.g. geometric shape, central symmetry, colinearity, etc.
5. vertices: e.g. words/symbols, logical system, linguistic/conceptual field, shape, presence of mnemonic support (e.g. the typical vowels A, E, I, O), etc.
6. edges: e.g. words/symbols, solid/dashed/dotted lines, arrowheads, etc.
7. style: e.g. presence of color, embellishments, etc.
8. additional info: e.g. research notes, connections with other diagrams, etc.

The Leonardi ontology has been implemented according to Semantic Web standards such as the Resource Description Framework (RDF), Linked Open Data (LOD) and (a computationally tractable subset of) the Web Ontology Language (OWL). More technical details and motivation are provided in our earlier paper [25]. Figure 2 displays a small but important part of the ontology, which can be used to describe a diagrams' vertices, edges, shape and general style features. The full ontology can be accessed online at https://logicalgeometry.org/assets/pdf/leonardi-schemata.pdf.

[4] See https://data.cerl.org/thesaurus/_search?lang=en. Note that these other datasets concern *people, books*, etc.; setting aside Leonardi.DB, we currently do not know of any comprehensive database which primarily consists of (logical) *diagrams*.

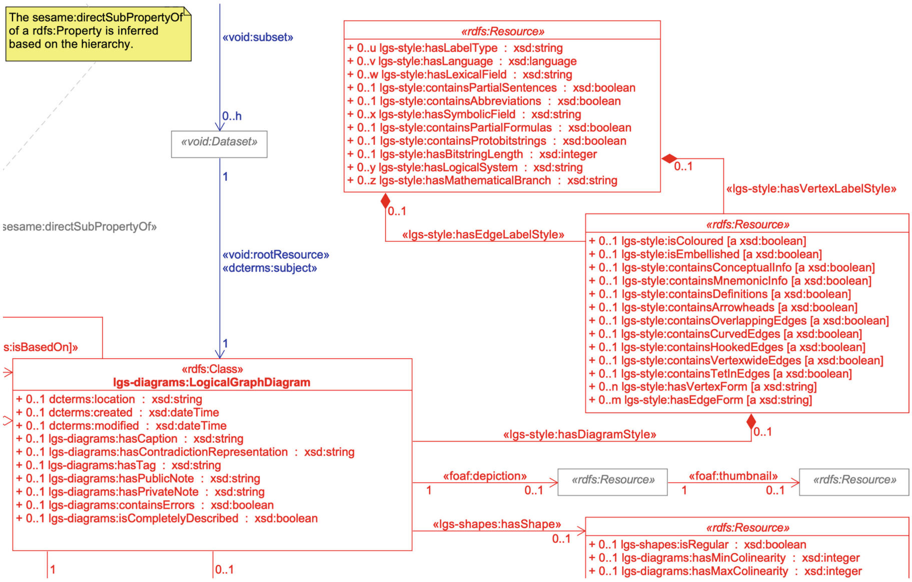

Fig. 2. A small part of the Leonardi ontology.

Data collection has thus far proceeded in a fairly straightforward fashion: we have focused on the numerous diagrams that are readily available, e.g. in research papers, textbooks, (digitized versions of) medieval manuscripts, incunabula, early Modern printed books, etc. In a later stage, data collection and processing will be done in a more comprehensive fashion, e.g. by systematically perusing bibliographic resources such as Risse's *Bibliographia Logica* [17] and online databases such as those of the Bibliothèque nationale de France (BnF) and the Bayerische Staatsbibliothek (BSB). We will return to this point in Sect. 4.

Leonardi.DB is freely available online (cf. Footnote 1), as a service to the wider research community, but also in order to further increase its empirical coverage. In particular, database users are encouraged to submit new diagrams (along with the relevant metadata) that they have created or discovered in the extant literature. All data can be explored and queried via a user-friendly graphical user interface (GUI), and can be exported in various formats (BibTeX, HTML, RDF); cf. Fig. 3 for a simple example. The data can be queried and filtered in full detail using the RDF query language SPARQL. However, in order to optimize user-friendliness, the database GUI also enables quite advanced searches by simply clicking some buttons and ticking some boxes. We mention just one example. Suppose that a given diagram D in the database cannot be dated precisely; the most accurate dates that are available are the range 1000–1200. Now suppose that the user wants to query the database to return all diagrams from the period 1100–1500. Should D be among the results for this query? According to a loose interpretation, D should be included, since it is

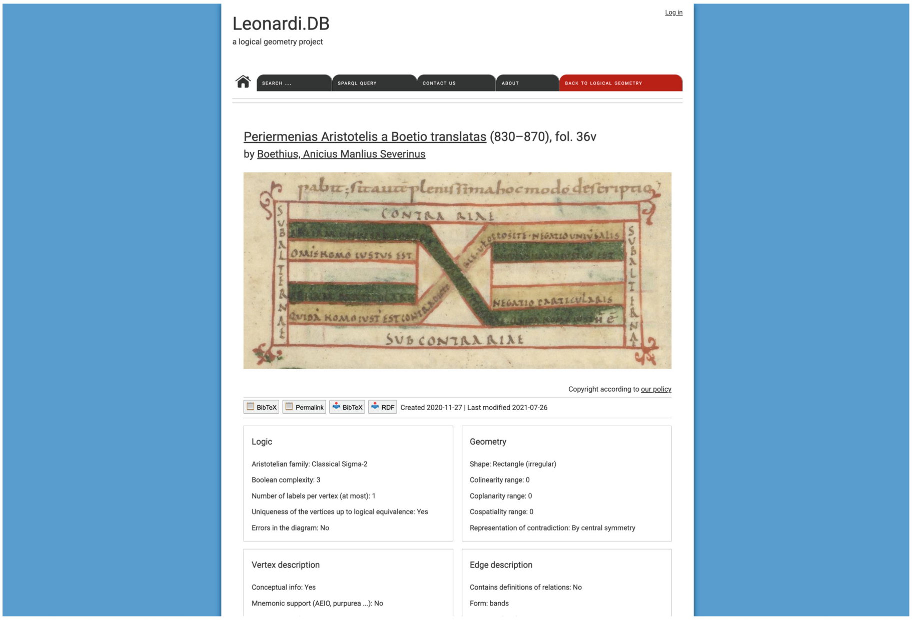

Fig. 3. A concrete diagram in Leonardi.DB, together with its annotation.

possible that D was created within the period specified in the query (mathematically: $[1000; 1200] \cap [1100; 1500] \neq \emptyset$); however, according to a strict interpretation, D should not be included, since it is not *certain* that D was created during the specified period (mathematically: $[1000; 1200] \not\subseteq [1100; 1500]$). Whenever the user wants to query the database based on chronological constraints like these, the GUI provides a simple checkbox that they can tick in order to indicate whether they want to adopt the 'loose' or rather the 'strict' interpretation.

4 New and Future Research Directions

At the time of writing (3 March 2022), Leonardi.DB contains annotations for 2461 Aristotelian diagrams (along with 1676 persons, 273 organizations, and 1616 sources). Although these numbers are not even close to the level of comprehensiveness that we are ultimately aiming for, the data volume and diversity are already sufficiently high to allow us to illustrate some of the new kinds of research that are facilitated by the diagram database. To make matters concrete, consider the following statement, taken from an earlier paper on logical geometry which was presented at DIAGRAMS 2016:

> we will only deal with Aristotelian diagrams in which negation is visually represented by means of central symmetry [...] both the logical condition (closed under negation) and the geometrical condition (central symmetry) are satisfied in nearly every Aristotelian diagram [7, p. 71]

Furthermore, when assessing this statement, it might make sense to set aside the last few decades, when more and more 'exotic' Aristotelian diagrams have begun to be studied (in the context of logical geometry and its immediate predecessors). Querying Leonardi.DB yields the following numerical results:

	Before 1950	After 1951	
Closed under negation, central symmetry	484	1584	2068
Closed under negation, no central symmetry	25	141	166
Not closed under negation	23	204	227
	532	1929	2461

We thus find that $(2068 + 166)/2461 = 90.8\%$ of all Aristotelian diagrams are closed under negation, and $2068/2461 = 84\%$ visualize negation by means of central symmetry. Furthermore, if we only consider the diagrams produced before 1950, these numbers further increase to 95.7% and 91%, respectively. Time period is clearly statistically significant; $\chi^2(2, N = 2461) = 25.78; p < 0.00001$. Furthermore, by further exploring the diagrams that are closed under negation but do not visualize this by means of central symmetry, we observed that many of them nevertheless do obey a kind of 'local' central symmetry. For example, in a cube diagram, we often found central symmetry *within* the front and back faces (so that the negation of the upper left front vertex occurs at the lower right *front* vertex, rather than at the lower right *back* vertex, as global central symmetry would require). This nicely illustrates how Leonardi.DB not only allows us to make more quantitatively precise statements about Aristotelian diagrams, but also triggers entirely new research questions and hypotheses.

In future research, we plan to scale up data collection and annotation through machine learning algorithms. The Aristotelian diagrams that have already been manually annotated are sufficiently representative to constitute a good training set. We hope to draw inspiration from the work of Sørensen and Johansen [24]: they have developed a regional convoluted neural network (r-CNN) within the Python-based deep learning platform Keras, which can quite accurately detect diagrams in a corpus of mathematical texts.

References

1. Béziau, J.Y., Jacquette, D. (eds.): Around and Beyond the Square of Opposition. Springer, Basel (2012). https://doi.org/10.1007/978-3-0348-0379-3
2. Béziau, J.Y., Vandoulakis, I. (eds.): The Exoteric Square of Opposition. Springer, Basel (2022)
3. Ciucci, D., Dubois, D., Prade, H.: Structures of opposition induced by relations. The Boolean and the gradual cases. Ann. Math. Artif. Intell. **76**, 351–373 (2016)
4. Demey, L.: Algebraic aspects of duality diagrams. In: Cox, P., Plimmer, B., Rodgers, P. (eds.) Diagrams 2012. LNCS (LNAI), vol. 7352, pp. 300–302. Springer, Heidelberg (2012). https://doi.org/10.1007/978-3-642-31223-6_32

5. Demey, L.: Boolean considerations on John Buridan's octagons of opposition. Hist. Philos. Logic **40**(2), 116–134 (2019)
6. Demey, L., Smessaert, H.: The relationship between Aristotelian and Hasse diagrams. In: Dwyer, T., Purchase, H., Delaney, A. (eds.) Diagrams 2014. LNCS (LNAI), vol. 8578, pp. 213–227. Springer, Heidelberg (2014). https://doi.org/10.1007/978-3-662-44043-8_23
7. Demey, L., Smessaert, H.: The interaction between logic and geometry in Aristotelian diagrams. In: Jamnik, M., Uesaka, Y., Elzer Schwartz, S. (eds.) Diagrams 2016. LNCS (LNAI), vol. 9781, pp. 67–82. Springer, Cham (2016). https://doi.org/10.1007/978-3-319-42333-3_6
8. Demey, L., Smessaert, H.: Logical and geometrical distance in polyhedral Aristotelian diagrams in knowledge representation. Symmetry **9**(10), 204 (2017)
9. Demey, L., Smessaert, H.: Aristotelian and duality relations beyond the square of opposition. In: Chapman, P., Stapleton, G., Moktefi, A., Perez-Kriz, S., Bellucci, F. (eds.) Diagrams 2018. LNCS (LNAI), vol. 10871, pp. 640–656. Springer, Cham (2018). https://doi.org/10.1007/978-3-319-91376-6_57
10. Demey, L., Smessaert, H.: Using multigraphs to study the interaction between opposition, implication and duality relations in logical squares. In: Pietarinen, A.-V., Chapman, P., Bosveld-de Smet, L., Giardino, V., Corter, J., Linker, S. (eds.) Diagrams 2020. LNCS (LNAI), vol. 12169, pp. 385–393. Springer, Cham (2020). https://doi.org/10.1007/978-3-030-54249-8_30
11. Demey, L., Smessaert, H.: From Euler diagrams to Aristotelian diagrams. In: V. Giardino et al. (eds.) Diagrams 2022. LNCS (LNAI), vol. 13462, pp. 279–295. Springer, Cham (2022). https://doi.org/10.1007/978-3-031-15146-0_24
12. García Cruz, J.D.: What kind of opposition-forming operator is privation? In: Basu, A., Stapleton, G., Linker, S., Legg, C., Manalo, E., Viana, P. (eds.) Diagrams 2021. LNCS (LNAI), vol. 12909, pp. 118–131. Springer, Cham (2021). https://doi.org/10.1007/978-3-030-86062-2_11
13. Jaspers, D., Seuren, P.: The square of opposition in Catholic hands: a chapter in the history of 20th-century logic. Logique et Anal. (N.S.) **59**(233), 1–35 (2016)
14. Lemanski, J., Demey, L.: Schopenhauer's partition diagrams and logical geometry. In: Basu, A., Stapleton, G., Linker, S., Legg, C., Manalo, E., Viana, P. (eds.) Diagrams 2021. LNCS (LNAI), vol. 12909, pp. 149–165. Springer, Cham (2021). https://doi.org/10.1007/978-3-030-86062-2_13
15. Li, W.: Zipf's law everywhere. Glottometrics **5**, 14–21 (2002)
16. Pascucci, M., Sileno, G.: The search for symmetry in Hohfeldian modalities. In: Basu, A., Stapleton, G., Linker, S., Legg, C., Manalo, E., Viana, P. (eds.) Diagrams 2021. LNCS (LNAI), vol. 12909, pp. 87–102. Springer, Cham (2021). https://doi.org/10.1007/978-3-030-86062-2_9
17. Risse, W.: Bibliographia Logica. 4 vols., esp. I: 1472–1800 and II: 1801–1969. Georg Olms, Hildesheim (1965–1979)
18. Smessaert, H.: Boolean differences between two hexagonal extensions of the logical square of oppositions. In: Cox, P., Plimmer, B., Rodgers, P. (eds.) Diagrams 2012. LNCS (LNAI), vol. 7352, pp. 193–199. Springer, Heidelberg (2012). https://doi.org/10.1007/978-3-642-31223-6_21
19. Smessaert, H., Demey, L.: Logical and geometrical complementarities between Aristotelian diagrams. In: Dwyer, T., Purchase, H., Delaney, A. (eds.) Diagrams 2014. LNCS (LNAI), vol. 8578, pp. 246–260. Springer, Heidelberg (2014). https://doi.org/10.1007/978-3-662-44043-8_26
20. Smessaert, H., Demey, L.: Logical geometries and information in the square of opposition. J. Logic Lang. Inform. **23**, 527–565 (2014)

21. Smessaert, H., Demey, L.: Visualising the Boolean Algebra $\mathbb{B}_4$ in 3D. In: Jamnik, M., Uesaka, Y., Elzer Schwartz, S. (eds.) Diagrams 2016. LNCS (LNAI), vol. 9781, pp. 289–292. Springer, Cham (2016). https://doi.org/10.1007/978-3-319-42333-3_26
22. Smessaert, H., Shimojima, A., Demey, L.: Free rides in logical space diagrams versus Aristotelian diagrams. In: Pietarinen, A.-V., Chapman, P., Bosveld-de Smet, L., Giardino, V., Corter, J., Linker, S. (eds.) Diagrams 2020. LNCS (LNAI), vol. 12169, pp. 419–435. Springer, Cham (2020). https://doi.org/10.1007/978-3-030-54249-8_33
23. Smessaert, H., Shimojima, A., Demey, L.: On the cognitive potential of derivative meaning in Aristotelian diagrams. In: Basu, A., Stapleton, G., Linker, S., Legg, C., Manalo, E., Viana, P. (eds.) Diagrams 2021. LNCS (LNAI), vol. 12909, pp. 495–511. Springer, Cham (2021). https://doi.org/10.1007/978-3-030-86062-2_51
24. Sørensen, H.K., Johansen, M.W.: Counting mathematical diagrams with machine learning. In: Pietarinen, A.-V., Chapman, P., Bosveld-de Smet, L., Giardino, V., Corter, J., Linker, S. (eds.) Diagrams 2020. LNCS (LNAI), vol. 12169, pp. 26–33. Springer, Cham (2020). https://doi.org/10.1007/978-3-030-54249-8_3
25. Termont, W., Demey, L., Smessaert, H.: First steps toward a Digital Access to Textual Cultural Heritage. In: Poster presentation at the third international conference on digital access to textual cultural heritage (DATeCH 2019) (2019)
26. Vignero, L.: Combining and relating Aristotelian diagrams. In: Basu, A., Stapleton, G., Linker, S., Legg, C., Manalo, E., Viana, P. (eds.) Diagrams 2021. LNCS (LNAI), vol. 12909, pp. 221–228. Springer, Cham (2021). https://doi.org/10.1007/978-3-030-86062-2_20