A Cellular Automaton Model of a Laser with Saturable Absorber Reproducing Laser Passive Q-switching

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Abstract. In this paper, we present a cellular automata model for a two-level laser which includes a saturable absorber. We show that the model reproduces laser passive Q-switching, a behavior in which intense short pulses of laser radiation are produced. Depending on the concentration of the absorbent, the automaton model qualitatively reproduces two operating states of the laser: a stable state and another oscillatory or pulsed state.

Keywords: Laser · Saturable absorber · Q-switching · Cellular automata

1 Introduction

Cellular automata (CA) have proven to be very successful in modeling complex systems in many areas of science and engineering [1,8,12]. One particularly interesting application is to model the dynamics of a laser, which is one of the most paradigmatic examples of a complex system. A CA model to describe laser dynamics was introduced in [3]. It describes the laser system as a collection of simple components: the atoms, electrons or molecules of the active medium of the laser cavity and the radiation laser photons that they produce. Local interactions among these components are described by the CA evolution rules based on the physical processes that occur in a laser system: stimulated emission, absorption, pumping, and noise. It was shown in [3] that different macroscopic laser properties are reproduced by the CA model as emerging properties induced by self-organization: the pumping threshold value, the emission of a laser beam

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above it, the temporal patterns (constant or oscillatory) of the radiation beam, and the dependence of the type of temporal pattern exhibited by a laser on its characteristic parameters.

Since then, it has been possible to model variants of a general laser system by modifying some of the ingredients of the CA model. And for instance in [4] a successful CA model of pulsed-pumped lasers was introduced. Also in [5] another CA model that reproduces antiphase dynamics in lasers was presented. This demonstrates the robustness and usefulness of the CA approach to model laser physics.

The basic idea of modeling laser physics using a microscopic or mesoscopic discrete model has been also developed further by Chusseau et al. to propose related Monte Carlo simulations of laser obtaining very good results for quantum-well and quantum-dot semiconductor lasers [2]. Also recently, Zhang et al. have proposed a CA model of nonlinear optical processes in a phase-change material inspired by this idea (in particular, for a polymorphic gallium film undergoing a light-induced structural phase transition) [13]. They have employed a CA model very similar to our laser model, a three-level system governed by only four transition rules and a sparse set of independent material and process parameters. They have found that their model can phenomenologically describe the complex, non-stationary, spatially inhomogeneous dynamics and resulting nonlinear optical properties of a medium undergoing a light-induced structural phase transition.

In this work, we go a step beyond the original model presented in [3] to introduce a new variant of that model that simulates a laser with a saturable absorber, capable of reproducing the behavior known as laser passive Q-switching. Laser Q-switching is a widely used technique by which a laser can be made to produce an output beam with intense light pulses by modulating the cavity losses, i.e. the Q factor (quality factor) of the cavity, which is the ratio of the stored energy to the energy dissipated per oscillation cycle [6,7,11]. Q factor determines the level of damping of the laser cavity: a laser with a low Q factor has higher losses and is thus more damped than a laser with a higher one. Q-switching is achieved by placing some type of variable attenuator in the laser optical cavity, which provides high attenuation (low Q-factor) for low intensities of laser light circulating through the cavity, and low attenuation (high Q-factor) for higher intensities. In this way, when the laser is switched on, the attenuation is very high, so that the intensity of laser radiation produced by stimulated emission increases only very gradually. Therefore, the pumped energy accumulates in a high population inversion. When the laser radiation intensity exceeds a certain threshold value, the variable attenuator quickly goes from low Q to high Q (the attenuation goes down). This, together with the high population inversion achieved, causes a rapid increase in laser radiation intensity by feedback from stimulated emission. This process consumes the population inversion until it is extinguished and returns to the starting point. The result is a short, intense pulse of laser light, called a giant pulse, which is repeated periodically. In lasers with passive Q-switching, one of the two variants of Q-switching, the variable attenuation is obtained by introducing a saturable absorber inside the laser cavity, a material whose transmission increases when light intensity exceeds some threshold. Some popular saturable absorbers are ion-doped crystals such as $Cr^{4+}:YAG, V^{3+}:YAG$, or $Co^{2+}:MgAl_2O_4$, where YAG stands for yttrium aluminum garnet ($Y_3Al_5O_{12}$).

Modeling laser Q-switching using a CA instead of the standard approach based on macroscopic differential equations has different advantages: i) a CA model can be used in cases in which the differential equations are stiff and present convergence problems; ii) it is possible with a CA model to study specific spatial structures of the laser device, for example, structures of the absorbing medium (randomly or regularly distributed); iii) CA models can be implemented very efficiently on parallel computers, due to their intrinsic parallel nature; iV) once a basic CA model has been designed and validated, it is possible and relatively easy to study modifications of the model to deal with different variants of the phenomenon to be studied.

The structure of this paper is as follows. In Sect. 2 the classical description of laser passive Q-switching using rate equations is introduced and the main operation regimes obtained by integrating them are presented. The CA model is introduced in Sect. 3. Results are presented in Sect. 4. Finally, conclusions are drawn in Sect. 5.

2 Laser Rate Equations

The classical balance equations to formulate a two-level laser with a saturable absorber are [11]:

$$\frac{dn}{dt} = K_1\,N\,n - \frac{n}{\tau_n} - K_2 q n \tag{1}$$

$$\frac{dN}{dt} = R_1 - \frac{N}{\tau_N} - K_1\,N\,n \tag{2}$$

$$\frac{dq}{dt} = R_2 - \frac{q}{\tau_q} - K_2\,q\,n \tag{3}$$

where n is the number of photons, N is the population inversion and q is the saturable absorber. K_1 and K_2 are two coupling constants between the radiation and the lasing medium and between the radiation and the absorber. R_1 is the pumping of the laser medium and R_2 is a characteristic property of the absorber.

The laser rate equations allow us a simple interpretation of the different physical processes involved. The intensity of the laser, which is proportional to the number of photons n, increases with the stimulated emission (K_1Nn) and decreases due to the effect of the absorber ($-K_2qn$). The population inversion N, Eq. (2), which is the difference between the electrons that are in the higher and fundamental energy level of the laser active medium, increases due to an external pumping (R_1) and decreases due to stimulated emission. Regarding the absorber, its behavior is similar to that of the population inversion. But in this case, R_2 is a characteristic of the material, although its effect can be understood as if it were an external pump. The absorber q decreases its action as the number of photons increases (K_2qn), this being one of the main characteristics of this type of device: the laser is transparent for high intensity values. Each of the values of the three populations, photons, population inversion, and absorber has a lifetime that represents in each case the decay time of the photon in the resonant cavity $\tau_n = \gamma_n^{-1}$, the decay time of the electron in the upper level of the laser active medium $\tau_N = \gamma_N^{-1}$ and the decay time of the absorber in the active state $\tau_q = \gamma_q^{-1}$.

From the analysis of these laser rate equations, it has been established that for a single mode the main laser regimes are a constant wave (cw) and Q-switching state in which the power shows oscillations [10]. Other unstable laser operations can also be found but are out of the scope of this work [9].

Figure 1 shows the two main operation regimes of the laser which have been obtained by integrating the equations by the fourth order Runge-Kutta method, where we have used the following lifetimes of $\tau_n = 80, \tau_N = 10^3, \tau_q = 2$.

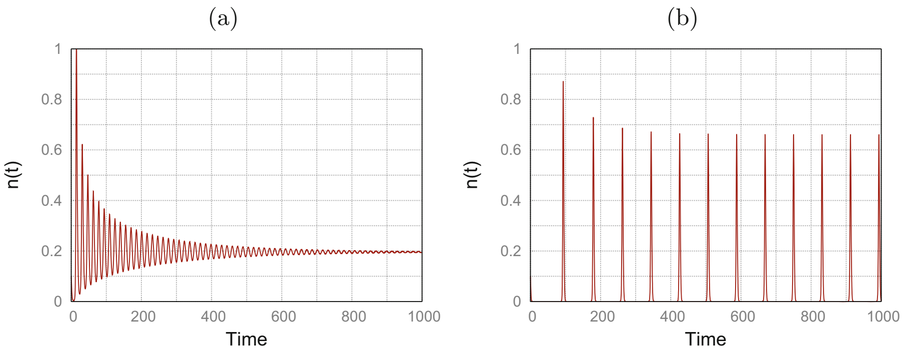

Fig. 1. Time series of the number of photons obtained from the laser rate equations. (a) After a damped transient the laser output is a constant wave. Parameters $R_1 = 0.1$, $R_2 = 0.5$. (b) Pulsed behavior for $R_1 = 0.05, R_2 = 0.9$.

3 A Cellular Automata Model for a Laser with a Saturable Absorber

The CA is defined in a two dimensional lattice of $N = L \times L$ cells with periodic boundary conditions. The state of each cell at a given time, $s_{ij}(t)$, is a vector of 3 values which includes the electronic state of the lasing medium $e \in \{0, 1\}$, the number of photons $f \in \{0, 1, 2, ..., max\}$ and the state of the absorber $q \in \{0, 1\}$.

$$s_{ij}(t) = \{e, f, q\} \tag{4}$$

Table 1. Set of parameters used in the simulations of the CA.

Parameter	Symbol	Value
Photons lifetime	τ_n	80
Population inversion lifetime	τ_N	10^3
Absorber lifetime	τ_q	2
Threshold for stimulated emission	K_1	2
Threshold for absorption	K_2	2
Transient time		50
Number of noise photons		100

The state of every cell changes in parallel according to the following transition rules:

Population Inversion

- Every electron in the ground state ($e = 0$) can be excited to the state $e = 1$ with a pumping probability R_1. Although in our model we speak of electrons, they are actually the two states of the laser medium. For this reason, we have not included any restrictions on the number of electrons in each level.
- An electron in the state $e = 1$ that is surrounded by a number of photons higher than a given threshold value K goes to $e = 0$. In this process, a new photon is created by stimulated emission. To evaluate this condition the photons in the Moore neighborhood of the cell are considered:

$$\Gamma_{ij} = \sum_{Neig} f_{i,j} \tag{5}$$

- An electron in the state $e = 1$ goes to $e = 0$ after a time τ_N. And this transition is considered to be not radiative.

Photons Evolution

When stimulated emission occurs one new photon is created:

$$f_{i,j}(t+1) = f_{i,j}(t) + 1 \tag{6}$$

Like the electrons, photons vanish after a given time τ_n.

The Absorber ($q \in \{0, 1\}$)

In our model, the absorber, in the same way as the inversion of the population, has only two possible states: an inactive state $q = 0$ in which it does not interact with radiation and the active state $q = 1$ in which said interaction does occur.

The evolution of the absorber is given by the following rules:

- If $q(t) = 0$ then with a probability R_2, which depends on the physical characteristics of the absorber, $q(t + 1) = 1$.

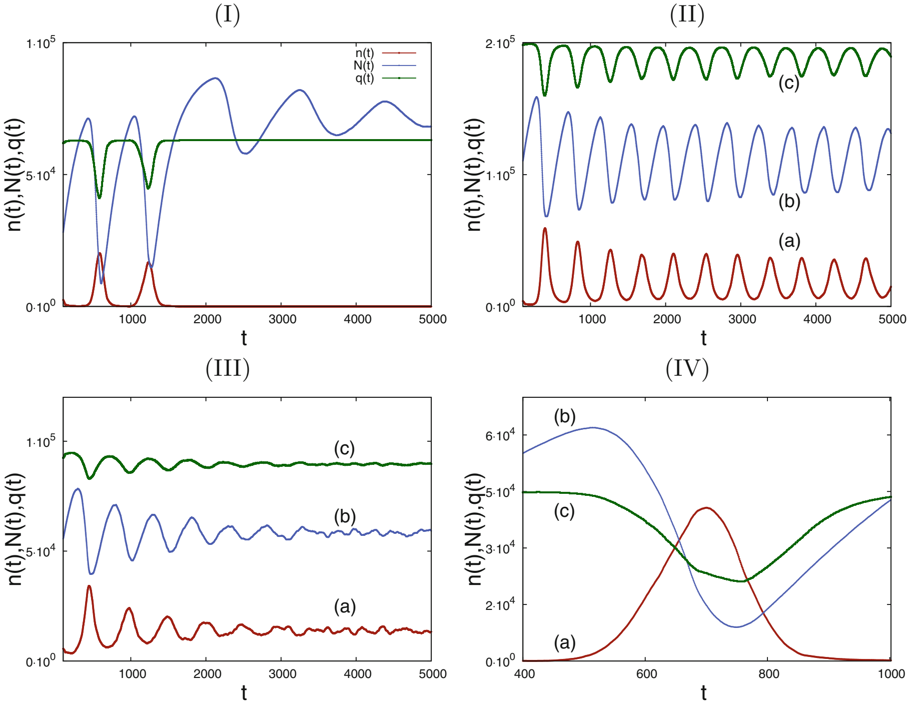

Fig. 2. Time series of **(a)** the number of photons $n(t)$, **(b)** the population inversion $N(t)$ and **(c)** the absorber $q(t)$. For clarity, the last two data sets (b and c) have been shifted slightly along the y-axis in I, II and III. The lattice size is 300×300 cells, the parameters $d_a = 0.5, R_2 = 0.5$. The other parameters used in these simulation are shown in Table 1. The different laser outputs depending on the pumping probability R_1 are: (I) No laser output $R_1 = 0.0035$. (II) Oscilatory behaviour $R_1 = 0.004$. (III) Constant wave $R_1 = 0.007$. (IV) Detail of a pulse corresponding to case (II).

– When the absorber is in the excited state, $q(t) = 1$, it eliminates photons if $\Gamma_{ij} \geq K_2$ and decays to the state $q(t+1) = 0$. This is a deterministic process. In our simulations, we have considered the case in which the absorber eliminates all the photons in the corresponding cell position.
– Also the absorber in the excited state decays to the inactive state after a certain number of time steps τ_q if $\Gamma_{ij} < K_2$.

Another important aspect to take into consideration is that in this lattice model, unlike in the balance equations, it is possible to take into account the spatial distribution of the absorber inside the system. In each and every point of the network we have considered that it can host population inversion and photons. But not so for the case of the absorber where it will be only present in a certain number of cells so that we can introduce a given density of points with absorber d_a.

4 Results

The simulations have been carried out in networks of 300×300 cells with periodic boundary conditions. The values of the different parameters used in most of the simulations are shown in Table 1. Initially the population inversion is null and the random distribution of absorbers is also in the ground state. During a temporary transient we introduce a small amount of noise photons into the system to initiate the action of the laser.

The number of parameters (seven, see Table 1) that define the system are too many to address in this preliminary work an exhaustive study of all the behaviors that can be shown by the CA model. In this way, the simulations that we present below have been carried out by setting the values of the lifetimes and of the constants K_1 and K_2 as indicated in the Table 1. The values of τ_n and τ_N were the typical ones used in previous studies [3]. And as for τ_q we take a value small enough and less than τ_n.

4.1 Dependence with the Pumping Probability R_1

First, we analyze the possible behaviors of the model as we modify the pumping probability of the lasing medium R_1 having fixed $R_2 = 0.5$ and the density of absorber cells $d_a = 0.5$. Figure 2(I) shows the time series of the number of photons $n(t)$, the population inversion $N(t)$ and the absorber in the excited state $q(t)$. For small values of R_1 after a small transient no laser signal is produced. The absorber reaches a fixed value while the population inversion shows damped oscillations until it reaches a fixed value as well.

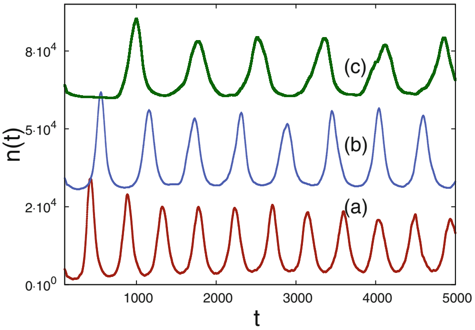

Fig. 3. Dependence of the laser output on the parameter R_2: a) 0.1, b) 0.4, c) 0.8. The pumping is $R_1 = 0.004$ and the density of absorber cells is $d_a = 0.5$.

By increasing the value of R_1 above a certain threshold value ($\approx$0.003), the laser shows an oscillatory state that is stable over time as can be seen in Fig. 2(II). A further increase in R_1, Fig. 2(III) results in the disappearance of the oscillatory state after a transient period during which the output intensity dampens and now the laser shows a constant wave output.

The Fig. 2(IV) is an expanded figure of one pulse corresponding to Q-switching. It is interesting to observe that the behavior captured by the CA model reproduces qualitatively the physics of the laser with a saturable absorber. The absorber q reduces its value near the peak in the number of photons due to the bleaching effect.

4.2 Dependence with R_2

We have limited the dependency with the parameter R_2 to the case of the oscillatory state, previously described, being $R_1 = 0.004$ and $d_a = 0.5$. Figure 3 shows the time series of the number of photons for three values of R_2. We have found that as long as R_1 is greater than the threshold value, the oscillatory behavior is maintained as R_2 is modified. But the frequency of pulses decreases as R_2 increases.

4.3 The Effect of the Density of the Absorber

With the discrete model presented here, we can investigate the result of varying the concentration d_a of possible absorbing cells in the lattice. That is an important issue in the preparation of materials with adequate characteristics. Figure 4 is a heat map obtained from the analysis of the time series of $n(t)$ for a fixed value of R_1. Higher values (yellow) are assigned to regular oscillations, and lower values (violet) appear when the signal is constant, down to the null value (black) when there is no output.

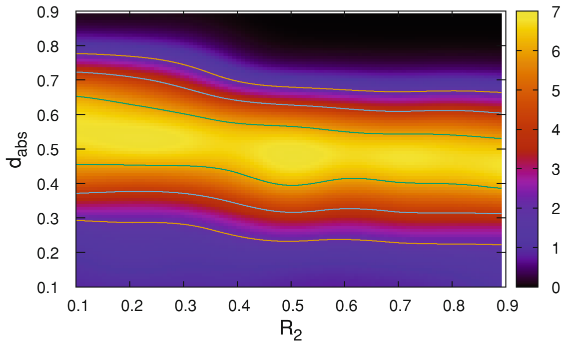

Fig. 4. The different kinds of laser outputs as a function of the density of the absorber and the R_2 parameter. The pumping probability is fixed at $R_1 = 0.004$. a) Black color: there is no laser output for high density of the absorber ($d_a > 0.75$). b) Violet color: constant output of the laser intensity. c) Red color: damped oscillations. d) Yellow color: oscillatory behaviors with an almost constant value of the maximum intensity are observed in the range ($0.45 < d_a < 0.75$) whereas damped oscillations are observed when $d_a < 0.45$. (Color figure online)

Figure 5 shows the time series of the intensity for three values of the density d_a and a fixed value of R_2. We find that in the absence of the absorber (Fig. 5-(a)) the signal has a constant value. As the density increases, the behavior goes from a constant value to an oscillatory behavior; first of all, there are damped oscillations and later they are maintained over time (Fig. 5-(b)). A further increase in the density causes the laser action to stop (Fig. 5-(c)).

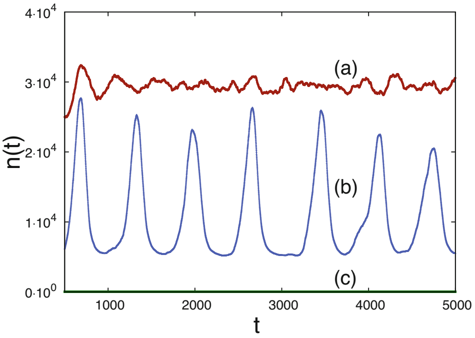

Fig. 5. Time series of the laser intensity for three values of the density d_a of absorber cells in the lattice: a) 0, b) 0.4 and c) 0.8. Other parameter values: $R_1 = 0.004$ and $R_2 = 0.4$. Without the saturable absorber the laser output is a steady state with some noise. The presence of the saturable absorber makes the laser to pulse.

5 Conclusions

In this work we present an extension to a previous CA model used to simulate the laser physics in which a passive saturable absorber is included. Despite its simplicity, the model qualitatively reproduces the main phenomenology of such systems: the inclusion of the absorber can cause the laser to pulse.

Depending on the different parameters that define the system, we have carried out a study modifying the pumping probability R_1 and the parameter R_2 which is a property of the absorber. In this way we obtain that the laser signal can be constant, a damped oscillation, a maintained oscillation (Q-switching) and the absence of laser output.

Finally, this discrete model allows us to analyze the different behaviors that can take place in lasers by modifying the density and location of the points that act as absorbers, something that is not possible with the laser rate equations.

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