Double Population Lattice Boltzmann Model for Magneto-Hydrodynamic Blood Flow in Stenotic Artery

Ikram Cherkaoui , Soufiene Bettaibi$^{(\boxtimes)}$, and Abdelwahed Barkaoui

Laboratoire des Energies Renouvelable et Matériaux Avancés,
Université Internationale de Rabat (UIR), Rocade Rabat-Salé, 11100 Rabat, Morocco
Bettaibisoufiene@gmail.com, soufiene.bettaibi@uir.ac.ma

Abstract. Atherosclerosis, which refers to a reduction in vessels diameter due to fatty deposits, is considered as the main cause of heart attacks, strokes, and peripheral vascular disease. The malfunctioning of cardiovascular system is mainly related to haemodynamics. However, the magnetic properties of blood are of great interest in haemodynamics. In this paper, a double population lattice Boltzmann model is suggested to investigate magnetohydrodynamic blood flow in stenotic artery. Blood is considered as a homogeneous fluid with magnetic properties. The rheological behavior of blood is presented by Carreau-Yasuda model. Blood flow is considered as incompressible and laminar. The vessel walls are assumed to be rigid. The proposed lattice Boltzmann model is found to be accurate, stable and effective. Findings are presented in terms of streamlines, velocity and wall shear stress profiles, based on a variety of parameters, including Reynolds and Hartmann number. The results show that the increase in magnetic intensity causes a considerable decrease in velocity and recirculation zones.

Keywords: Lattice Boltzmann approach · Atherosclerosis · Blood flow · Magneto-hydrodynamic

1 Introduction

The development of blood vessels pathologies such as stenosis, atherosclerosis and spasm disturb blood flow and lead to a malfunctioning of many organs. In order to detect the vessels diseases, a detailed knowledge of blood flow remains a necessity. The study of blood flow is the subject of different numerical methods. However, the traditional conventional computational fluid dynamics method (CFD) are limited and the implementation of boundary conditions still more complicated for complex geometries [1]. In addition, the resolution of mathematical equations used to present the system is complicated. Given the complexity of these equations, the analytical solutions of Navier-stokes equations are generally nonexistent and only an approximated numerical solution is existing. This justifies, the considerable development of the techniques and methods of

numerical computation in fluid mechanics (CFD) during these last decades. The continuous evolution of numerical methods is related to the computer resources development, what allows the numerical resolution of the equations governing fluid mechanics and heat transfer with great precision and for a wide range of complex geometries. Unlike numerical simulation methods based on the resolution of partial differential equations linking the macroscopic properties of fluids, the lattice gas automata (LGA) method makes it possible to find macroscopic variables such as velocity, pressure or pressure fields and temperature, by simulating the interactions between molecules. Lattice Boltzmann Method (LBM), a numerical method evolving from LGA, has gained popularity in the last few years. It has been used for simulating and modeling different systems including immiscible fluids [2], multiphase flows [3], heat transfer problems [4–7], isotropic turbulence [8] and porous media [9]. It has proven its effectiveness in the field of conventional fluid flows, particularly in complex geometries and porous media. It has attracted the attention of researchers for the simulation of flows in different applications. Higuera and Jimenez [10] proposed an important simplification in LBM by approximating the collision operator in Lattice Boltzmann Equation with a linearized one that assumes that the distribution is close to the equilibrium state. The success of lattice Boltzmann method is related, in large part, to the introduction of the Bhatnagar-Gross-Krook (BGK) collision operator characterized by its simplicity and ease of implementation. Bhatnagar-Gross-Krook (BGK) collision model is a simple linearized collision operator, introduced by Koelman [11] and Chen et al. [12]. The macroscopic Navier-Stokes equations are recovered by the Lattice BGK model through a Chapman-Enskog analysis [13]. The lattice Boltzmann method describes fluids in a mesoscopic scale and provides stable and efficient numerical calculations for the fluids macroscopic behavior [14–16]. The problem of taking into account the initial and boundary conditions was the subject of particular attention by the initiators of the LBM method. Stability and numerical precision are closely related to the nature of the boundary conditions. The lattice Boltzmann approach does not necessitate the resolution of a global system of equations, just information from surrounding nodes is required to describe variables evolution. Because of the nature of the explicit computation with locality, the lattice Boltzmann method is a cost-effective solution to communication between processors and hence excellent for parallel computation.

In this paper, we propose an efficient and accurate lattice Boltzmann model for simulating magnetohydrodynamic blood flow in stenotic arteries. The unique feature of this modelization is that both velocity and magnetic fields are solved using the lattice Boltzmann technique, which allows to investigate the influence of strong magnetic field intensities on blood flow.

2   Mathematical Model

2.1   Problem Description

In this study, blood is considered as a homogeneous magnetic bio-fluid, incompressible and non-Newtonian with density $\rho = 1060\,\text{kg/m}^3$. The Vessel walls are assumed to be rigid and blood flow is considered laminar and steady. The diameter of the artery is $D = 6\,\text{mm}$. An idealized geometry of stenosis is considered (Fig. 1) in this study.
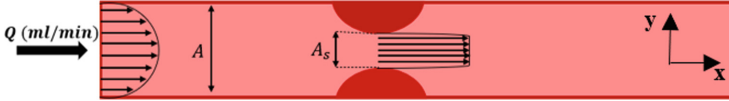

Fig. 1. Stenosis geometry

Stenosis refers to a reduction in the vessel section due to a deposition of fatty components on the walls. The geometry of the wall with the presence of stenosis is given by: $y(x) = D - h \sin\left[\frac{\pi(x-d)}{l}\right]$ where D is the diameter of health artery, h the width of the restricted zone, d the length of the inlet region and l the length of the restricted zone. The severity of the reduction zone (degree of stenosis DOS) can be calculated by the following equation: $DOS(\%) = (1 - \frac{A_s}{A}) \times 100$ where A_s is the restricted zone section and A is the section of healthy artery.

2.2   Equations

Taking into consideration the presented hypothesis, the 2-D incompressible, unsteady flow of blood as an electrically conductive fluid is described by Navier–Stokes equations, with an additional term presenting Lorentz force are written as:

$$\frac{\partial u}{\partial t} + (u.\nabla)u = -\frac{\nabla p}{\rho} + \nu \Delta u + \frac{\mathbf{j} \times \mathbf{B}}{\rho} + Q \tag{1}$$

$$\nabla.u = 0 \tag{2}$$

where ν is the fluid kinematic viscosity, ρ is the density, p is the pressure, $u = [u_x, u_y]$ the velocity, $\mathbf{B} = [B_x, B_y]$ the magnetic field, $\mathbf{j} = \nabla \times \mathbf{B}$ and $\mathbf{Q} = [Q_x, Q_y]$ the external body force vectors.

This research investigates the 2-dimensional, laminar and incompressible magnetohydrodynamic blood flow through a restricted vessel. The governing equations, including the impact of viscosity and energy dissipation due to the presence of magnetic field are given by the following equation:

$$\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} = 0 \tag{3}$$

$$\rho(u_x\frac{\partial u_x}{\partial x} + u_y\frac{\partial u_x}{\partial y}) = -\frac{\partial p}{\partial x} + \mu(\frac{\partial^2 u_x}{\partial x^2} + \frac{\partial^2 u_x}{\partial y^2}) - \sigma B_0^2 u_x \tag{4}$$

$$\rho(u_x\frac{\partial u_y}{\partial x} + u_y\frac{\partial u_y}{\partial y}) = -\frac{\partial p}{\partial x} + \mu(\frac{\partial^2 u_y}{\partial x^2} + \frac{\partial^2 u_y}{\partial y^2}) \tag{5}$$

The term $\sigma B_0^2 u_x$ in Eq. 4 depicts the magnetic body force $(\mathbf{j} \times \mathbf{B})$ per volume. Where $B = [B_x, B_0]$ and σ is the electrical conductivity of blood.

Carreau-Yasuda Model

Human blood is a composed fluid, containing mainly plasma and blood cells. The plasma acts like a Newtonian fluid, its viscosity depends on the concentration of plasma proteins [17], whereas the whole blood has a non-Newtonian behavior. Many models have been developed in order to predicts the rheological behavior. Carreau-Yasuda model is one of the simplest and accurate models used in blood modeling. The viscosity depends on shear rate and modelled by the Carreau-Yasuda model [18] as following:

$$\mu(\dot{\gamma}) = \mu_\infty + (\mu_0 - \mu_\infty) + (1 + (\lambda\dot{\gamma})^\alpha)^{\frac{n-1}{\alpha}} \tag{6}$$

where μ is the viscosity, $\dot{\gamma}$ is the shear rate, $\mu_\infty = 0.0035\,Pa.s$ is the viscosity at infinite shear rate, $\mu_0 = 0.16\,Pa.s$ is the viscosity at the absence shear-rate, and $\lambda = 8.2$, $\alpha = 0.64$, and $n = 0.2128$ are material coefficients. The shear rate is given by:

$$\dot{\gamma} = 2\sqrt{D_{\mathrm{II}}} \tag{7}$$

where D_{II} is the second invariant of the strain rate tensor, given by:

$$D_{\mathrm{II}} = \sum_{\alpha,\beta=1}^{l} S_{\alpha\beta}S_{\alpha\beta} \tag{8}$$

where $l=2$ for a two-dimensional model.

For incompressible fluids, the stress tensor is written as:

$$\sigma_{\alpha\beta} = -p\delta_{\alpha\beta} + 2\mu S_{\alpha\beta} \tag{9}$$

where $\delta_{\alpha\beta}$ is the Kronecker delta and $S_{\alpha\beta}$ is the strain rate tensor, written as: $S_{\alpha\beta} = \frac{1}{2}(\nabla_\beta u_\alpha + \nabla_\alpha u_\beta)$

3 Numerical Model

3.1 Lattice Boltzmann Method with Single Relaxation Time (LBM-SRT)

Solving two linked lattice Boltzmann equations can be used to solve magnetohydrodynamic equations. The first equation covers fluid dynamics by forecasting

the development of the particle distribution function f_i, whereas the second equation incorporates a vector-valued function g_i that represents the evolution of the magnetic field. The two equations are discretized in a D2Q9 space (Fig. 2).
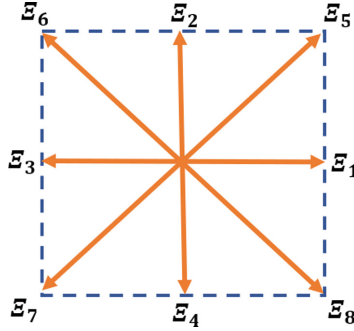

Fig. 2. D_2Q_9 model

The lattice Boltzmann approach (LBM) with single relaxation time, which is based on the Bhatnagar-Gross-Krook (BGK) approximation, is used to forecast the development of both fluid dynamics and magnetic fields. The fluid in the lattice Boltzmann approach is defined by a particle distribution function that develops in discrete space and time. As a result, the lattice Boltzmann equation is stated as:

$$f_i\left(\mathbf{x} + \mathbf{\Xi_i}\delta t, t + \delta t\right) - f_i\left(\mathbf{x}, t\right) = C_i\left(f\right) \tag{10}$$

where C_i is the collision operator, presenting the change in particles distribution after collision step. The lattice Bhatnagar-Gross-Krook (BGK) equation can be written as:

$$f_i\left(\mathbf{x} + \mathbf{\Xi_i}\delta t, t + \delta t\right) - f_i\left(\mathbf{x}, t\right) = -\frac{1}{\tau}\left[f_i\left(\mathbf{x}, t\right) - f_i^{eq}\left(\mathbf{x}, t\right)\right] \tag{11}$$

where τ is the relaxation parameter, related to viscosity by the following: $\tau = \left(\frac{\nu}{c_s^2} + 0.5\right)$ with c_s is the lattice speed, given by $c_s = \frac{\delta x}{\sqrt{3}\delta t}$. δx and δt are the lattice width and time step respectively, chosen as $\delta x = \delta t = 1$ and $c = \frac{\delta x}{\delta t}$. f_i^{eq} is the equilibrium distribution function, which depends on the local fluid velocity and density. The equilibrium distribution function is given by:

$$f_i^{eq} = w_i\rho\left[1 + \frac{3\mathbf{\Xi_i}.\mathbf{u}}{c^2} + \frac{9\left(\mathbf{\Xi_i}.\mathbf{u}\right)^2}{2c^4} - \frac{3\mathbf{u}.\mathbf{u}}{2c^2}\right] \qquad i = 0 \rightarrow 8 \tag{12}$$

In the presence of external magnetic field, the equilibrium distribution function includes an additional term presenting the effect of magnetic field intensity on particles distribution. The equilibrium distribution function becomes:

$$f_i^{eq} = w_i \rho \left[1 + \frac{\mathbf{\Xi_i.u}}{c_s^2} + \frac{(\mathbf{\Xi_i.u})^2}{2c_s^4} - \frac{\mathbf{u.u}}{2c_s^2} \right] + \frac{\lambda_i}{2c_s^4} \left[\frac{1}{2} | \mathbf{\Xi_i} |^2 | \mathbf{b} |^2 - (\mathbf{\Xi_i.b})^2 \right] \quad (13)$$

where w_i and λ_i are the weighting factors defined in D_2Q_9 as following: $w_i = \lambda_i = \frac{4}{9}$ for $i = 0$, $w_i = \lambda_i = \frac{1}{9}$ for $i = 1, 2, 3, 4$ and $w_i = \lambda_i = \frac{1}{36}$ for $i = 5, 6, 7, 8$.

$$\begin{cases} \mathbf{\Xi_i} = (0,0) & i = 0 \\ \mathbf{\Xi_i} = \left(\cos\left[(i-1) \frac{\pi}{2} \right], \sin\left[(i-1) \frac{\pi}{2} \right] \right) c & i = 1, 2, 3, 4 \\ \mathbf{\Xi_i} = \left(\cos\left[(2i-9) \frac{\pi}{4} \right], \sin\left[(2i-9) \frac{\pi}{4} \right] \right) c\sqrt{2} & i = 5, 6, 7, 8 \end{cases} \quad (14)$$

The magnetic field evolution is described by the following lattice Boltzmann equation:

$$g_i (\mathbf{x} + \mathbf{\Xi_i}\delta t, t + \delta t) - g_i (\mathbf{x}, t) = -\frac{1}{\tau_m} [g_i (\mathbf{x}, t) - g_i^{eq} (\mathbf{x}, t)] \quad (15)$$

where τ_m is the magnetic relaxation parameter, related to magnetic resistivity η by: $\tau_m = \left(\frac{\eta}{c_s^2} + 0.5 \right)$. In 2-dimensional space, Eq. 15 is written as following:

$$g_{ix} (\mathbf{x} + \mathbf{\Xi_i}\delta t, t + \delta t) - g_{ix} (\mathbf{x}, t) = -\frac{1}{\tau_m} [g_{ix} (\mathbf{x}, t) - g_{ix}^{eq} (\mathbf{x}, t)] \quad (16)$$

$$g_{iy} (\mathbf{x} + \mathbf{\Xi_i}\delta t, t + \delta t) - g_{iy} (\mathbf{x}, t) = -\frac{1}{\tau_m} [g_{iy} (\mathbf{x}, t) - g_{iy}^{eq} (\mathbf{x}, t)] \quad (17)$$

The coupling between hydrodynamics and magnetic field takes place in the equilibrium functions:

$$g_{ix}^{eq} = \lambda_i \left[b_x + \frac{\Xi_{iy}}{c_s^2} (u_y b_x - u_x b_y) \right] \quad (18)$$

$$g_{iy}^{eq} = \lambda_i \left[b_y + \frac{\Xi_{ix}}{c_s^2} (u_x b_y - u_y b_x) \right] \quad (19)$$

In order to reproduce Navier-Stokes Equations, the following identities must hold:

$$\rho = \sum_{i=0}^{8} f_i \quad (20)$$

$$\rho\mathbf{u} = \sum_{i=0}^{8} f_i \mathbf{\Xi_i} \quad (21)$$

Unlike the other simulation methods that solve Poisson's equation to compute pressure, the pressure p can be directly computed from the equation of state $p = \rho c_s^2$.

The macroscopic magnetic properties are given by:

$$b_x = \sum_{i=0}^{8} g_{ix} \tag{22}$$

$$b_y = \sum_{i=0}^{8} g_{iy} \tag{23}$$

3.2   Boundary Conditions

The problem of taking into account the initial and boundary conditions was the subject of particular attention by the initiators of the lattice Boltzmann method. Stability and numerical precision are closely related to the nature of the boundary conditions. In order to simulate blood flow in stenotic artery in the presence of magnetic field, we implement the Zou-He boundary condition in the inlet and the Bounce back boundary condition in the walls for both velocity and magnetic field (Fig. 3).
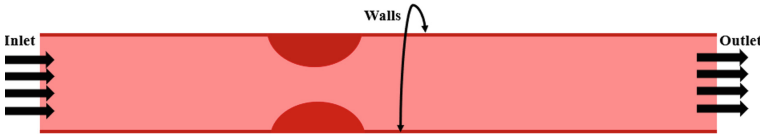

Fig. 3. Boundary conditions

Zou-He Boundary Condition. The Zou-He boundary condition is used to apply certain flux condition in the inlet. The velocity at the inlet is given by the profile of poiseuille:

$$\begin{cases} u_x(y) = 4u_{max}\left(\frac{y}{D} - \frac{y^2}{D^2}\right) \\ \qquad u_y = 0 \end{cases} \tag{24}$$

After streaming, the unknown density and distribution functions f_1, f_5, f_8 at the inlet are given by:

$$\begin{aligned} \rho &= \frac{1}{1-u_x}\left[f_0 + f_2 + f_4 + 2(f_3 + f_6 + f_7)\right], \\ f_1 &= f_3 + \frac{2}{3}\rho u_x, \\ f_5 &= f_7 - \frac{1}{2}(f_2 - f_4) + \frac{1}{2}\rho u_y + \frac{1}{6}\rho u_x, \\ f_8 &= f_6 + \frac{1}{2}(f_2 - f_4) - \frac{1}{2}\rho u_y + \frac{1}{6}\rho u_x. \end{aligned} \tag{25}$$

3.3   Bounce Back

In the walls, the mid way bounce back boundary condition is applied. This boundary condition is equivalent to no-slip boundary condition, which means that the velocity is zero in the walls. The wall is placed halfway between a wall grid point and a fluid grid point. The bounce back boundary condition assumes that particles hitting the wall disperse back to the fluid following their entering path (Fig. 4).
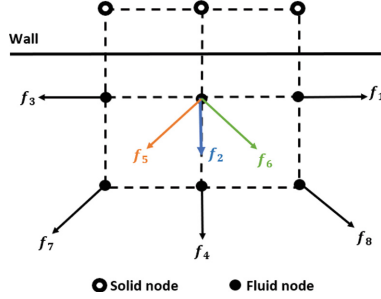

Fig. 4. Mid-way bounce back boundary condition

The unknown distribution functions at the wall are given by:

$$\begin{aligned} f_5(x, y, t) &= f_7(x, y, t) \\ f_2(x, y, t) &= f_4(x, y, t) \\ f_6(x, y, t) &= f_8(x, y, t) \end{aligned} \tag{26}$$

4   Model Validation

The results given by the suggested lattice Boltzmann model for hemodynamic are compared with in vivo measurements conducted by H. Park et al. [19] in the case of a stenosed aorta with a stenosis degree of 34%. The in vivo measurements were performed by surgically attaching a stenotic clip to a live rat model. The hemodynamic information are obtained by using X-ray PIV method. Figure 5 shows a comparison of velocity field in the stenotic aorta. It is shown that velocity increases considerably in the stenotic section reaching its maximum value of 8 mm/s in both numerical and experimental results. The results found by lattice Boltzmann model are in good agreement with in vivo measurements. It can be concluded that the proposed lattice Boltzmann model is accurate and effective in the treatment of blood flow in stenosed vessels.
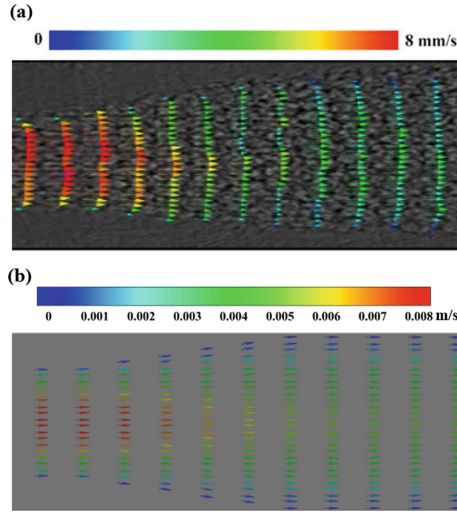
Fig. 5. Comparison of velocity field of in vivo measurement (a) and lattice Boltzmann model (b)

5   Results and Discussion

The aim of this study is to investigate the impact of imposed external magnetic field on blood flow characteristics, mainly velocity and wall shear stress (WSS), in a stenotic artery. The numerical simulations have been carried out for a Reynolds number $Re = 360$ and various Hartmann numbers $Ha = 0$, $Ha = 5$, $Ha = 10$, $Ha = 15$ and $Ha = 20$. The Obtained results of blood flow simulations, indicate that for a fixed stenosis degree (50% in this case), the velocity decreases due to the increasing magnetic field intensity.

Figure 6 and Fig. 7 show the effect of various magnetic field intensities on velocity profiles and recirculation zones in the downstream region of an artery with a 50% stenosis degree. It is apparent that increasing the magnetic field lowers the recirculation zone, resulting in a reduction in hydrodynamic stresses in this area. For $Ha = 20$, velocity at the stenotic section reduces by approximately 80%, and recirculation zones vanish. This decrease in velocity is produced by RBC aggregation, which increases when blood is exposed to a magnetic field. Our findings are consistent with those reported in the literature by Ilyani et al. [20] who studied the magnetohydrodynamic (MHD) effects on blood flow and discovered that a magnetic field decreases blood flow rate. In their method, a term containing Lorentz force is introduced to Navier-Stokes equations, resulting in a magnetic field with just one conceivable direction. In contrast, in our model, each particle is connected with a vector with nine possible directions, describing the evolution of magnetic field.
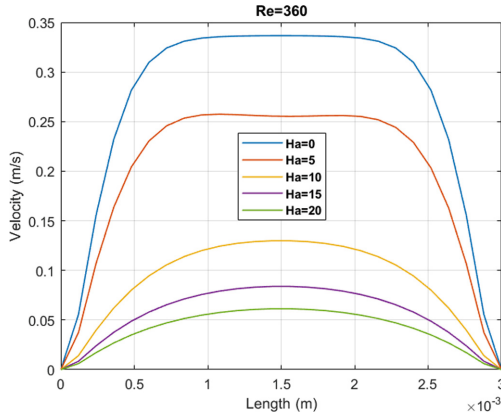
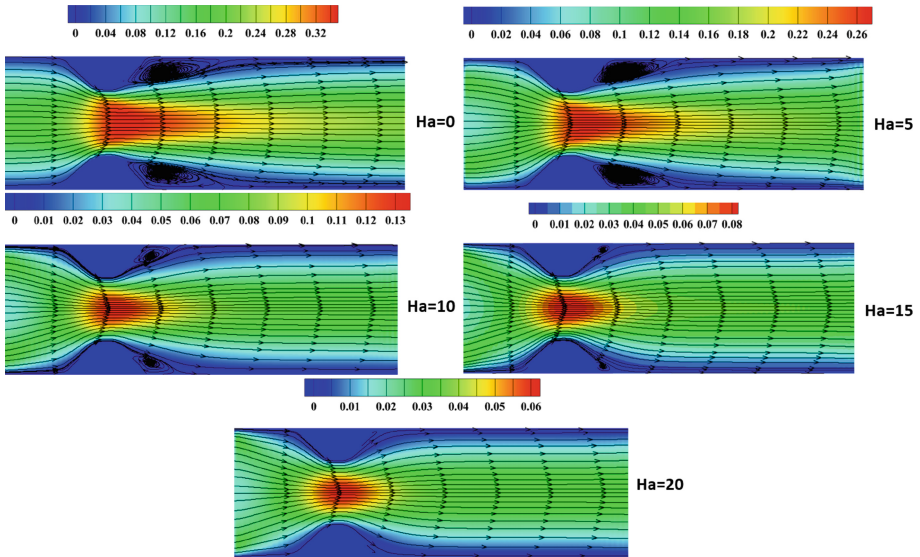**Fig. 6.** Velocity profiles at the restricted zone for Re = 360 and Ha = 0, 5, 10, 15 and 20

Fig. 7. Velocity streamlines for Re = 360 and Ha = 0, 5, 10, 15 and 20

The WSS is one of the most critical hemodynamic variables in cardiovascular diseases, it has a major impact on stenosis pregression. Figure 8 presents the effect of an external magnetic field on WSS in a stenosed artery for various values of Hartmann number. It is shown that the wall shear stress reaches its maximum at the restricted zone, this is caused by the reduction in diameter in that region. In the stenotic section, the non-Newtonian behavior of blood is more
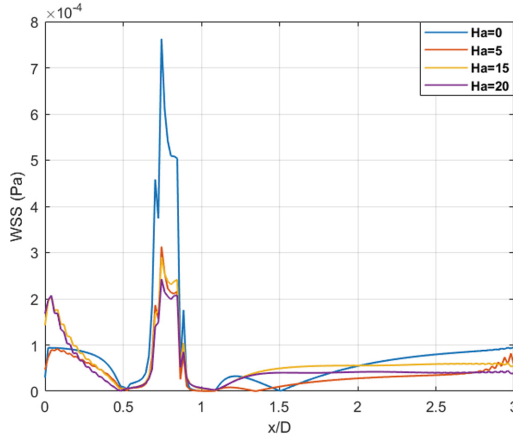
Fig. 8. Wall shear stress (WSS) for Re = 360 and Ha = 0, 5, 15 and 20

noticeable due to red blood cells aggregation in that zone. Figure 8 shows that the WSS in the stenotic region is reduced considerably by applying an external magnetic field. It is found that the WSS decreases by increasing the magnetic field intensity.

6   Conclusion

A simulation of 2-D steady and laminar magnetohydrodynamic blood flow is conducted using a double population lattice Boltzmann model. The blood vessels are assumed to be rigid and blood is considered as non-Newtonian and its rheological behavior is modelled by Carreau-Yasuda model. The effect of magnetic field intensity on blood flow is investigated. The findings show the effectiveness of the proposed Lattice Boltzmann model to study magnetohydrodynamic blood flow problem. In the other hand, it is found that the velocity profiles, recirculation zones and WSS decrease by increasing the magnetic field strength. Which can have interesting application in modulating blood flow rate during medical surgeries and in the treatment of hypertension and other cardiovascular diseases.

References

1. Luo, L.-S., Krafczyk, M., Shyy, W.: Lattice Boltzmann method for computational fluid dynamics. Encycl. Aerosp. Eng. **56**, 651–660 (2010)
2. Gunstensen, A.K., Rothman, D.H.: Lattice Boltzmann model of immiscible fluids. Phys. Rev. A **43**(8), 4320 (1991)
3. Grunau, D., Chen, S., Eggert, K.: A lattice Boltzmann model for multiphase fluid flows. Phys. Fluids A **5**(10), 2557–2562 (1993)
4. Bettaibi, S., Kuznik, F., Sediki, E.: Hybrid lattice Boltzmann finite difference simulation of mixed convection flows in a lid driven square cavity. Phys. Lett. A **378**, 2429–2435 (2014)

5. Bettaibi, S., Sediki, E., Kuznik, F., Succi, S.: Lattice Boltzmann simulation of mixed convection heat transfer in a driven cavity with non-uniform heating of the bottom wall. Commun. Theor. Phys. **63**(1), 91 (2015)
6. Bettaibi, S., Kuznik, F., Sediki, E.: Hybrid LBM-MRT model coupled with finite difference method for double diffusive mixed convection in rectangular enclosure with insulated moving lid. Phys. A: Stat. Mech. Appl. **444**, 311–326 (2016)
7. Bettaibi, S., Kuznik, F., Sediki, E., Succi, S.: Numerical study of thermal diffusion and diffusion thermo effects in a differentially heated and salted driven cavity using MRT-lattice Boltzmann finite difference model. Int. J. Appl. Mech. **13**(04), 2150049 (2021)
8. Chen, S., et al.: Lattice Boltzmann computational fluid dynamics in three dimensions. J. Stat. Phys. **68**(3), 379–400 (1992)
9. Lallemand, P., Luo, L.-S.: Theory of the lattice Boltzmann method: dispersion, dissipation, isotropy, Galilean invariance, and stability. Phys. Rev. E **61**(6), 6546 (2000)
10. Higuera, F.J., Jiménez, J.: Boltzmann approach to lattice gas simulations. EPL (Europhys. Lett.) **9**(7), 663 (1989)
11. Koelman, J.M.V.A.: A simple lattice Boltzmann scheme for Navier-Stokes fluid flow. EPL (Europhys. Lett.) **15**(6), 603 (1991)
12. Chen, S., et al.: Lattice Boltzmann model for simulation of magnetohydrodynamics. Phys. Rev. Lett. **67**(27), 3776 (1991)
13. Shu, C., Peng, Y., Chew, Y.T.: Simulation of natural convection in a square cavity by Taylor series expansion-and least squares-based lattice Boltzmann method. Int. J. Mod. Phys. C **13**(10), 1399–1414 (2002)
14. Bettaibi, S., Jellouli, O.: Double diffusive mixed convection with thermodiffusion effect in a driven cavity by lattice Boltzmann method. LNTCS **12599**, 209–221 (2021)
15. Cherkaoui, I., Bettaibi, S., Barkaoui, A., Kuznik, F.: Magnetohydrodynamic blood flow study in stenotic coronary artery using lattice Boltzmann method. Comput. Methods Programs Biomed. **221**, 106850 (2022)
16. Mhamdi, B., Bettaibi, S., Jellouli, O., Chafra, M.: MRT-lattice Boltzmann hybrid model for the double diffusive mixed convection with thermodiffusion effect. Nat. Comput. 1–14 (2022). https://doi.org/10.1007/s11047-022-09884-4
17. Alexander, D.E.: Biological materials blur boundaries. Nat. Mach. 99–120 (2017). https://doi.org/10.1016/B978-0-12-804404-9.00004-9
18. Abraham, F., Behr, M., Heinkenschloss, M.: Shape optimisation in steady blood flow: a numerical study of non-Newtonian effects. Comput. Methods Biomech. Biomed. Eng. **8**(2), 127–137 (2005)
19. Park, H., Park, J.H., Lee, S.J.: In vivo measurement of hemodynamic information in stenosed rat blood vessels using X-ray PIV. Sci. Rep. **6**(1), 1–8 (2016)
20. Ilyani, A., Norsarahaida, A., Tasawar, H.: Magnetohydro-dynamic effects on blood flow through an irregular stenosis. Int. J. Numer. Methods Fluids **67**(11), 1624–1636 (2011)