Effect of Pipe-End Material on Water Hammer and Cavitation in Viscoelastic Pipelines

Abdelaziz Ghodhbani[1(✉)], Lamjed Hadj-Taieb[1,2], and Sami Elaoud[1]

[1] Laboratory of Applied Fluid Mechanics, Process Engineering and Environment, National School of Engineers of Sfax, 3038 Sfax, Tunisia
abdelaziz.ghodhbani@stud.enis.tn

[2] College of Engineering, Department of Mechanical Engineering, Prince Sattam Bin Abdulaziz University, Al-Kharj, Saudi Arabia
L.hadjtaieb@psau.edu.sa

Abstract. Water hammer in viscoelastic pipelines can be accurately predicted by the classical model. However, discrepancies are observed in case of cavitation, even by use of the classical viscoelastic discrete gas cavity model (VE-DGCM). This paper deals with the improvement of the numerical solution of water hammer and cavitation in viscoelastic pipelines by involving the pipe-end rigidity in boundary conditions. The method of characteristics (MOC) is used to calculate fluid transient in both downstream and upstream-valve HDPE straight pipe, which are rigidly anchored. The classical viscoelastic modelling is used for both cavitating and non-cavitating flows. Pressure as well as circumferential strain are calculated at the valve and at the midstream of the pipe. Two types of pipe-end material are compared therein: the rigid end and the viscoelastic end. The former is considered as the rigid material of the pipe and the reservoir, while the latter is restricted to the pipe. The method consists in incorporating the end behavior in the boundary conditions of the problem. The classical viscoelastic discrete vapour cavity model (VE-DVCM) and the VE-DGCM are used to solve the problem. The calculation shows that the first assumption leads to more accurate results than the second in the two pipe locations.

Keywords: Water hammer · Column separation · Method of characteristics · Viscoelasticity

1 Introduction

Plastic pipes (such as polyethylene) are being increasingly used in hydraulic plants because of their high mechanical and chemical properties. Viscoelasticity can mainly attenuate the pressure fluctuations by increasing the dispersion of the travelling wave. Numerous researches focuses have been taken on water hammer modelling in viscoelastic pipelines. Fluid transient accidents in viscoelastic pipelines can be attenuated compared to metallic pipes. Chaker and Triki (2020) studied the branching strategy capacity to mitigate the cavitating flow induced into steel piping systems.

L. Walha et al. (Eds.): CMSM 2021, LNME, pp. 151–160, 2023.
https://doi.org/10.1007/978-3-031-14615-2_18

High-density polyethylene pipes (HDPE) is well used in hydraulic plants and fluid transient in HDPE can be predicted thanks to the classical viscoelastic model, in which the retarded circumferential strain is calculated. Mostly, unsteady friction (UF) is ignored because friction damping can be neglected against viscoelastic damping. Usually, the classical viscoelastic model (two-equation model) gives good accuracy of the result. However, in case of cavitating flow, the results present some errors. Several parameters can be investigated in order to improve the result. The creep-compliance functions are assumed to be the most important parameters. These functions are, in fact obtained from experimental tests and calibration.

In this paper, the emphasis is made on the effect of boundary conditions on the result. From dynamic point of view, both ends of the pipe are assumed to be fixed, so that junction coupling is ignored. The particularity of this purpose is to consider the type of the material and to discuss therefore its influence on the solution. Physically, the straight pipe is rigidly anchored at its two ends: the reservoir and the valve. Calculation at boundaries is in fact performed at the interfaces pipe-reservoir and pipe-valve. The mechanical behaviour at the interfaces can be considered either like the pipe (viscoelastic side) or like the rigid side (reservoir or valve). Hence, the simulation differs from one assumption to another.

2 Mathematical Formulation

2.1 Pipe Viscoelastic Behaviour

The viscoelastic behaviour is usually described by considering mechanical element to describe it; the spring for elastic response and the dashpot for viscous response. Usually, the generalized Kelvin-Voight model (Fig. 1) is used to describe the viscoelastic behaviour. This model consists in associating in parallel a springer and a damper. According to this model, the stress σ is related to the strain ε by the following (Keramat et al. 2012)

$$\sigma = E_0\varepsilon + \mu\dot{\varepsilon} \tag{1}$$

with E_0 is the Young's modulus of elasticity represented by the spring, μ is the viscosity represented by the damper.

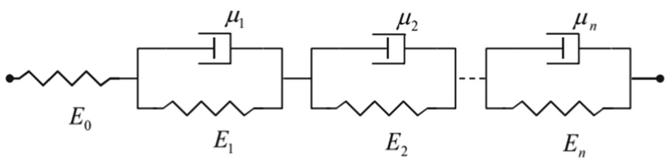

Fig. 1. The generalized Kelvin-Voight model

When subjected to an instantaneous stress σ, the linear viscoelastic material does not respond according to Hook's Law, but it has an instantaneous elastic strain and a retarded viscous strain (Covas et al. 2005)

$$\varepsilon(t) = \varepsilon^e + \varepsilon^r(t) \tag{2}$$

The Boltzmann's superposition principle establishes that for small strains, the combination of stresses acting independently results in linearly added strains. Hence, the total strain ε generated by a continuous application of stress σ is

$$\varepsilon = J_0\sigma + \sigma^* \frac{\partial J}{\partial t} \tag{3}$$

where J_0 is the instantaneous creep-compliance defined as $J_0 = 1/E_0$ for linearly elastic materials, J is the creep-compliance function of time t, and "*" denotes convolution. Equation (3) can be developed as (Covas et al. 2005)

$$\varepsilon(t) = J_0\sigma(t) + \int_0^t \sigma\left(t - t'\right)\frac{\partial J(t)}{\partial t'}dt' \tag{4}$$

The pipe material is assumed to be homogeneous, isotropic and it has linear viscoelastic behaviour for small strains. Poisson's ratio is constant so that the material behaviour depends only on the creep-functions. Hence, the total circumferential strain can be expressed by:

$$\varepsilon_\phi(t) = \frac{D_0}{2e_0}\left[p(t) - p_0\right]J_0 + \int_0^t \frac{D\left(t - t'\right)}{2e(t - t')}\left[p\left(t - t'\right) - p_0\right]\frac{\partial J\left(t'\right)}{\partial t'}dt' \tag{5}$$

where p is the pressure, D is the pipe inner diameter and e is the pipe wall thickness. The subscript 0 denotes the variable at time $t = 0$.

The first part of Eq. (5) corresponds to the elastic strain and the integral part to the retarded strain. For the generalized Kelvin-Voight model, the creep-compliance function $J(t)$ can be expressed as

$$J(t) = J_0 + \sum\nolimits_{k=1}^{n} J_k\left(1 - e^{-t/\tau_k}\right) \tag{6}$$

In which J_k is the creep-compliance of the spring of the Kelvin-Voight k-element defined with respect to modulus of elasticity E_k by $J_k = 1/E_k$, τ_k is the retardation time of the dashpot of the k-element given by $\tau_k = \mu_k/E_k$.

2.2 The Classical Viscoelastic Model (VEM)

First, we take into account the relationship between the pipe cross section A and the total circumferential strain ε_ϕ ($dA/d\varepsilon_\phi = 2A$), plus the two components (ε_ϕ^e, ε_ϕ^r) of strain, and the state equation of a barotropic fluid ($d\rho/\rho = dp/K$).

Then, using the continuity and momentum equations and neglecting the convective terms the classical water hammer model in viscoelastic pipes can be described by:

$$\frac{\partial H}{\partial t} + \frac{C_f^2}{gA}\frac{\partial q}{\partial z} + 2\frac{C_f^2}{g}\frac{\partial \varepsilon_\phi^r}{\partial t} = 0 \tag{7}$$

$$\frac{\partial H}{\partial z} + \frac{1}{gA}\frac{\partial q}{\partial t} + h_f = 0 \tag{8}$$

with H is the piezometric head, q is the discharge, C_f is the pressure wave-speed and h_f is the head loss which can be either steady or the sum of steady and unsteady terms.

The compatibility equations are

$$\frac{dH}{dt} \pm \frac{C_f}{gA}\frac{dq}{dt} + 2\frac{C_f^2}{g}\left(\frac{\partial \varepsilon_\phi^r}{\partial t}\right) \pm C_f h_f = 0 \tag{9}$$

The integration of the compatibility Eqs. (9) using an implicit finite difference scheme leads to (Ghodhbani 2021):

$$C_f^+ : H^P - H^A + \frac{C_f}{gA}\left(q^P - q^A\right) + 2\Delta t\frac{C_f^2}{g}\left(\frac{\partial \varepsilon_\phi^r}{\partial t}\right)^P + \Delta t C_f h_f^A = 0 \tag{10}$$

$$C_f^- : H^P - H^B + \frac{C_f}{gA}\left(q^P - q^B\right) + 2\Delta t\frac{C_f^2}{g}\left(\frac{\partial \varepsilon_\phi^r}{\partial t}\right)^P - \Delta t C_f h_f^B = 0 \tag{11}$$

where P (at time t) denotes the computation point and A and B (at time $t - \Delta t$ for rectangular grid) are information points already calculated in the previous iteration.

The retarded circumferential strain and its rate can be written with respect to the expression of k-element of the generalized Kelvin-Voight model as follows

$$\varepsilon_\phi^r(t) = \sum\nolimits_{k=1}^{n} \varepsilon_{\phi,k}^r(t) = \frac{D}{2e}\rho_f g\int_0^t \left[H\left(t - t'\right) - H_0\right] \sum\nolimits_{k=1}^{n} \frac{J_k}{\tau_k} e^{-t'/\tau_k} dt' \tag{12}$$

$$\frac{\partial \varepsilon_\phi^r(t)}{\partial t} = \sum\nolimits_{k=1}^{n} \frac{\partial \varepsilon_{\phi,k}^r(t)}{\partial t} \tag{13}$$

where

$$\begin{aligned} \varepsilon_{\phi,k}^r(t) \approx {} & e^{-\Delta t/\tau_k}\varepsilon_{\phi,k}^r(t-\Delta t) + \frac{J_k D\rho_f g}{2e}\left\{\left[1 - \frac{\tau_k}{\Delta t}\left(1 - e^{-\Delta t/\tau_k}\right)\right]H(t)\right. \\ & \left. + \left[\frac{\tau_k}{\Delta t}\left(1 - e^{-\Delta t/\tau_k}\right) - e^{-t/\tau_k}\right]H(t-\Delta t) - \left(1 - e^{-\Delta t/\tau_k}\right)H_0\right\} \end{aligned} \tag{14}$$

and

$$\begin{aligned} \frac{\partial \varepsilon_{\phi,k}^r(t)}{\partial t} \approx {} & \frac{e^{-\Delta t/\tau_k}}{\tau_k}\varepsilon_{\phi,k}^r(t-\Delta t) + \frac{J_k D\rho_f g}{2e\Delta t}\left(1 - e^{-\Delta t/\tau_k}\right)H(t) + \frac{D\rho_f g}{2e}\left\{\left[\frac{J_k}{\tau_k}e^{-\Delta t/\tau_k}\right.\right. \\ & \left.\left. - \frac{J_k}{\Delta t}\left(1 - e^{-\Delta t/\tau_k}\right)\right]H(t-\Delta t) - \frac{J_k}{\tau_k}e^{-\Delta t/\tau_k}H_0\right\} \end{aligned} \tag{15}$$

Noting that Eqs. (14) and (15) are also used to develop the viscoelastic discrete gas cavity model (VE-DGCM) in case of column separation in viscoelastic pipelines (Ghodhbani 2021). Since a staggered grid is used in such case, the time step used for calculation is $2\Delta t$ instead of the time step Δt.

2.3 The Viscoelastic Discrete Gas Cavity Model (VE-DGCM)

The Discrete Gas Cavity model (DGCM) has been used for column separation modelling in elastic pipelines. This model considers free gas volumes to simulate distributed free gas (Wylie and Streeter 1993). It was supposed that cavities were concentrated at the computational sections whereas pure liquid was assumed to remain in each computational reach. Gas volumes at each computational section expanded and contracted with respect to the pressure according to an isothermal perfect gas law. If a cavity forms, it is assumed that released gas stays in the cavity and does not immediately dissolve following a pressure rise. In contrast to vapour release, which takes only a few microseconds, the time for gas release is in the order of several seconds (Wylie and Streeter 1993).

In case of viscoelastic pipelines, the theory of the DGCM is applied and it leads to formulate the VE-DGCM. The piezometric head H at a given computational point P is obtained thanks to the following quadratic equation derived from the compatibility Eqs. (10) and (11) after considering discharges up and down the computational points

$$H^2 + \left(\widehat{Z} - Z - H_v\right)H - \widehat{Z}(Z + H_v) - \frac{p_0^* \alpha_0 \forall_m}{\rho_f g \widehat{A}} \tag{16}$$

which leads to

$$H = \frac{1}{2}\left(-\widehat{Z} + Z + H_v\right) - \left[\frac{1}{4}\left(\widehat{Z} - Z - H_v\right)^2 + \widehat{Z}(Z + H_v) + \frac{p_0^* \alpha_0 \forall_m}{\rho_f g \widehat{A}}\right] \tag{17}$$

in which Z denotes the elevation of the computational section, H_v is the vapour head of the liquid, α_0 is a void fraction at a given reference pressure p_0^*, ρ_f is the mass density of the liquid, g is the gravity acceleration, $\forall_m$ is the mixture volume. The quantities $\widehat{Z}$ and $\widehat{A}$ are defined as (Ghodhbani 2021).

$$\widehat{Z} = \frac{\widehat{\forall}_g}{\widehat{A}} \text{ and } \widehat{A} = \frac{4\Delta t \psi}{B\beta} \tag{18}$$

with Δt is the time step, ψ is a weighting factor used for calculation of the cavity volume (Wylie and Streeter 1993), $B = C_f/(gA)$ is the pipeline impedance, β is a constant defined with respect to the creep compliance functions as

$$\frac{1}{\beta} = 1 + \frac{D}{2e}\rho_f C_f^2 \sum_{k=1}^{N}\left(1 - e^{-2\Delta t/\tau_k}\right) \tag{19}$$

and the cavity volume $\widehat{\forall}_g$ at the computational point P is expressed with respect to the cavity volume at the earlier point Q in a staggered grid of the MOC

$$\widehat{\forall}_g = \widehat{\forall}_g^Q + 2\Delta t\left[\psi \hat{q} + (1 - \psi)\left(q_d^Q - q_u^Q\right)\right] \tag{20}$$

where $\hat{q}$ is a quantity obtained by

$$\hat{q} = q_d^P - q_u^P - \frac{2}{\beta \mathrm{B}} H^P \tag{21}$$

3 Results and Discussion

The analysis considers two cases: (i) the non-cavitating flow and (ii) the cavitating flow. The effect of the pipe-end material is tested through boundary conditions for the two cases. The pipe is discretized into N reaches, so that the calculation is performed at $N+1$ sections. To consider viscoelastic material at sections 1 and $N+1$, the expression (17) is used to express the retarded circumferential strain at these sections, however when the rigid-end condition is considered, the retarded circumferential strains are assigned to zero. Noting that unsteady friction is neglected from the classical VEM; only steady friction is considered because viscoelastic damping matters.

3.1 The Non-cavitating Flow Case

The experiment of Stoinov and Covas (Covas 2003) is used to validate numerical calculations obtained by the classical VEM. The calculations are performed at three locations at the High Density Polyethylene (HDPE) pipeline: transducer T1 at 270 m (the fixed downstream valve), transducer T5 at 116.42 m (the midstream of the pipe) and transducer T3 at 0 m (upstream end).

Two flow-cases are tested: laminar flow and turbulent flow. The analysis is the same for the two flows, but this work is restricted to the second. The simulation shows that the pressure magnitudes in case of rigid-end condition are closer to the experimental result than those of the viscoelastic-end case (Figs. 2 and 3). In fact, this discrepancy is realistic because the pipe ends are not PE made, and retarded circumferential strain at these locations should be omitted from boundary condition calculations (Ghodhbani 2021).

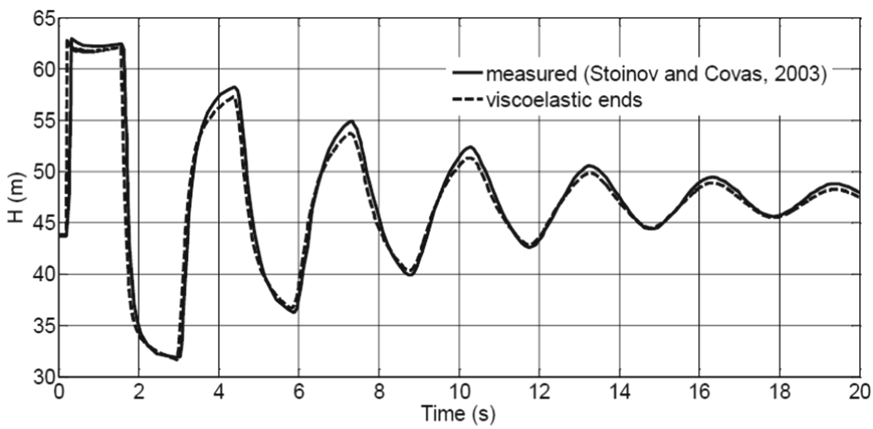

Fig. 2. Piezometric head at the valve using the viscoelastic-end condition.

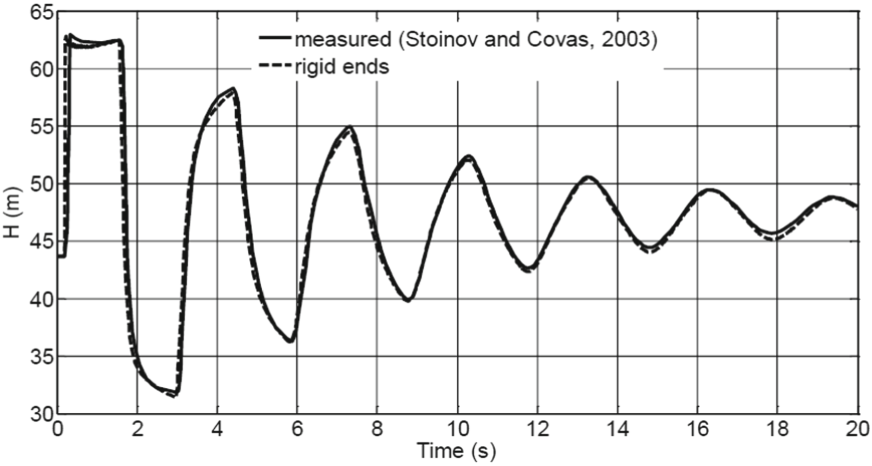

Fig. 3. Piezometric head at the valve using the rigid-end condition.

As shown for pressure, Figs. 4 and 5 show the circumferential strain histories at the valve. The same description given below for piezometric head histories is still valid for the circumferential strains, namely the effect of the end mechanical behaviour. Obviously, the rigid end assumption leads to more accuracy regarding strain magnitudes. As mentioned above, this latter assumption seems more realistic than the viscoelastic end assumption.

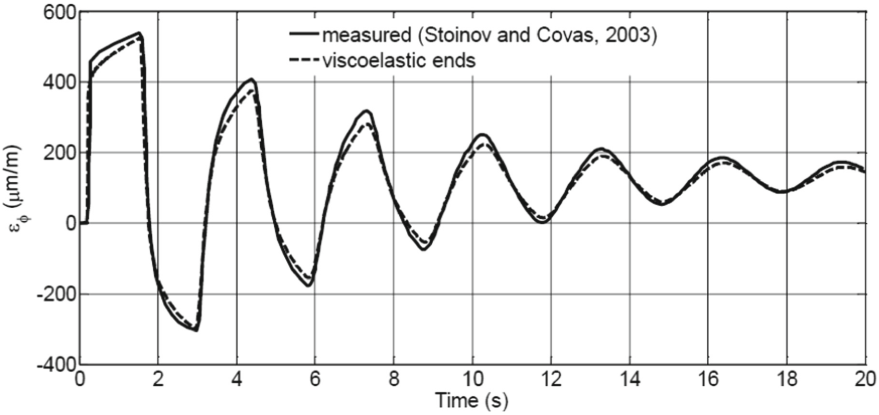

Fig. 4. Circ. strain at the valve using the viscoelastic-end condition.

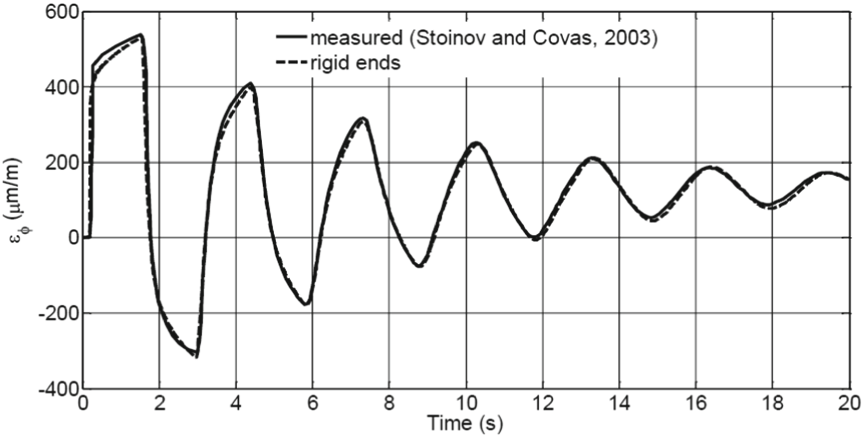

Fig. 5. Circ. strain at the valve using the rigid-end condition.

3.2 The Cavitating Flow Case

The cavitating flow case is studied by considering the experiment of Carriço (2008), which was carried out for a rigidly restrained HDPE pipe with upstream valve (Soares et al. 2009).

In this study, numerical results obtained by the viscoelastic model with cavitation (VE-DGCM) are compared against experimental results. Figures 6 displays comparison between the two end conditions and shows that the rigid-end condition is more realistic and more accurate regarding magnitudes. Regarding timing, it is observed that the two solutions are almost similar and they give good agreement compared to the experiment.

Although the timing concordance exhibited by the VE-DGCM, the numerical solution steel presents high discrepancies compared to the experimental result for the two

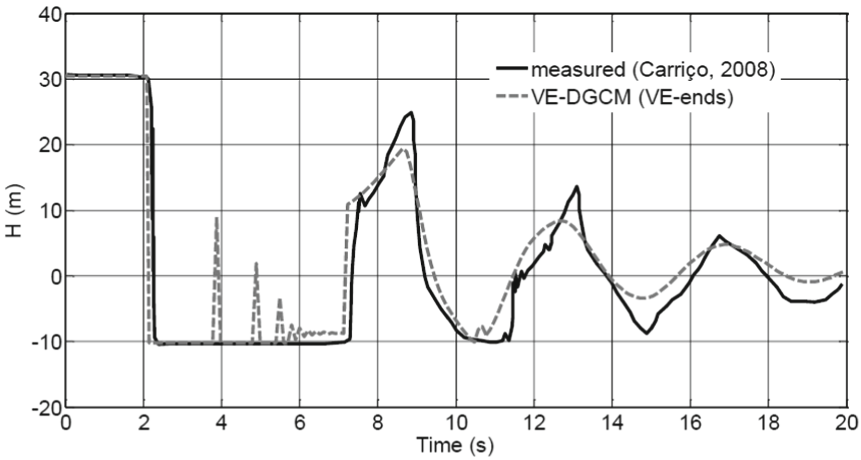

Fig. 6. Piezometric head at the valve using the viscoelastic-end condition.

cases of end-material. These discrepancies is likely due to the effect of ignoring fluid-structure interaction (FSI) in calculation. In addition, the experimental measurement shows particular shape like the short duration pressure pulse observed in case of active column separation flow regime in elastic pipes. The effect of viscoelasticity on pressure history is less important in case of column separation because of the short duration pressure pulse. Other discrepancies are due to the DGCM hypothesis. Some unrealistic oscillations are observed in case of the VE-end condition (Fig. 6). However, these discrepancies disappear in Fig. 7, where the rigid-end condition is considered.

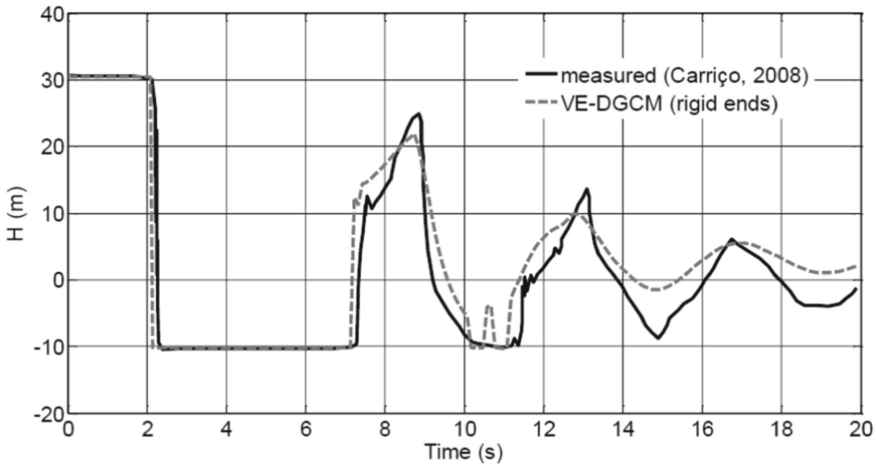

Fig. 7. Piezometric head at the valve using the rigid-end condition.

4 Conclusion

Transient in viscoelastic pipelines has been studied in the present work, where two hydraulic cases have been discussed: the non-cavitating flow and the cavitating flow. The case studies used to validate the result has not considered the axial movement of the pipe, so that junction coupling has not been considered. The simulation considers two boundary conditions, which affect the result: the rigid-end condition and the viscoelastic-end condition. In case of water hammer without cavitation (case 1), the classical VEM is sufficient to predict column separation in HDPE pipeline. This is because junction coupling has been avoided and Poisson coupling modelling has no significant effect in case of restrained pipeline. The simulation shows that the result obtained by the classical VEM is in good agreement with the experimental results. In case of cavitating flow, the VE-DGCM has been successfully used to predict column separation in HDPE pipes. The simulation shows that the VE-DGCM simulates better column separation especially when the rigid-end condition is considered.

References

Bergant, A., et al.: Discrete vapour cavity model with efficient and accurate unsteady friction term. Department of Mathematics and Computer Science (2006)

Chaker, M.A., Triki, A.: Investigating the removal of hydraulic cavitation from pressurized steel piping systems. In: Aifaoui, N., et al. (eds.) CMSM 2019. LNME, pp. 92–101. Springer, Cham (2020). https://doi.org/10.1007/978-3-030-27146-6_11

Covas, D.I.C.: Inverse transient analysis for leak detection and calibration of water pipe systems modelling special dynamic effects. Ph.D. thesis, Department of Civil and Environmental Engineering, Imperial College of Science, Technology and Medicine, London, UK (2003)

Covas, D.I.C., et al: The dynamic effect of pipe wall viscoelasticity in hydraulic transients. Part I-model development, calibration and verification. Department of Civil and Environmental Engineering, Imperial College of Science, Technology and Medicine, London, UK (2005)

Ghodhbani, A.: Modelling and simulation of transient cavitation in pipelines. Doctoral thesis, University of Sfax (2021)

Keramat, A., et al.: Fluid-structure interaction with pipe-line viscoelasticity during water hammer. J. Fluids Struct. **28**, 434–455 (2012)

Soares, A.K., et al.: Unsteady flow with cavitation in viscoelastic pipes. Int. J. Fluid Mach. Syst. **4**, 269–277 (2009). Department of Sanitary and Environmental Engineering, Federal University of Mato Grosso Ave, Cuiabá, Brazil

Wylie, E.B., Streeter, V.L.: Fluid Transients. McGraw-Hill, New York (1993)