Chapter 11
The Creative Mathematical Thinking Process

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11.1 Introduction

The value of creativity is increasingly recognized in mathematics education (Leikin & Sriraman, 2017). This increased interest fits well in the tradition of mathematicians like György Pólya and Jacques Hadamard, both of whom stressed more than 75 years ago that creativity is a driving force behind the discovery of new mathematical insights (Hadamard, 1954; Pólya, 1945). But, creativity is also important to those not involved in breaking new mathematical grounds, such as primary school children. Creativity helps them to integrate mathematical information and come up with different solutions or strategies to solve a problem (Hadamard, 1996; Mann, 2005), which is particularly important when children encounter a problem for which they have not yet learned a solution or solution strategy (Leikin, 2009). Indeed, research shows that children who score higher on measures of creativity also demonstrate higher mathematical performance (Jeon et al., 2011; Kattou et al., 2013; Schoevers et al., 2018). Prior research often studied mathematical creativity in a static way for instance by scoring children's performance on multiple-solution tasks in terms of the number of responses (fluency), variability of responses (flexibility), and uniqueness of responses (originality) (Assmus & Fritzlar, 2018). Such product-based measures of mathematical creativity, although informative, cannot unveil the creative thinking processes that led to a particular response or solution. If we want to support the development of creative thinking skills in mathematics education, more insight into the creative thinking process is required. This study therefore aspired to illuminate the use of creative thinking, in particular the use of divergent and convergent thought, in solving different types of mathematical problems.

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S. A. Chamberlin et al. (eds.), *Mathematical Creativity*, Research in Mathematics Education, https://doi.org/10.1007/978-3-031-14474-5_11

11.1.1 Divergent and Convergent Thinking

Mathematical creativity can be defined as “the cognitive act of combining known concepts in an adequate, but for the student new way, thereby increasing or extending the student’s (correct) understanding of mathematics” (Schoevers, 2019, p. 58). Guilford (1973) proposed that such new combinations of concepts (i.e., creative ideas) are conceived through divergent and convergent thinking. Divergent thinking refers to the process of generating ideas, like problem definitions, strategies, or solutions from a specific starting point, whereas convergent thinking concerns the process of selecting and evaluating ideas to arrive at the best possible solution (Brophy, 2001). Much creativity research has exclusively focused on divergent thinking (e.g., Jeon et al., 2011; Plucker et al., 2004), but researchers increasingly recognize the importance of convergent thinking too. If children rely on divergent thinking alone, they can generate many different creative ideas, including incorrect and unfeasible ones. Convergent thinking then helps to assess the value of these ideas for the task at hand (Brophy, 2001; Cropley, 2006).

Divergent and convergent thinking have been identified as separate constructs in former research (e.g., Barbot et al., 2016). However, as Cortes et al. (2019) proposed, task performance on either a divergent or a convergent thinking task could be a reflection of a mixture of both divergent and convergent thinking processes. Thus, previous results from research with divergent or convergent thinking tasks generally give little insight into children’s creative thinking process, as the exact process cannot be inferred from the creative product. To further illuminate the creative thinking process, it is necessary to make the shift from measuring creative products to measuring creative processes. Such an approach might shed more light on how creative ideas emerge in action (Corazza, 2016; Glăveanu, 2013). Conceiving creative ideas, for example a creative solution to a mathematics problem, is thought to consist of repeated cycles in which first divergent thinking and then convergent thinking is applied during different phases of creative problem-solving (Isaksen et al., 2011; Lubart, 2018). According to Wallas’s (1926) four-stage model of creativity, creative ideas are first prepared, followed by a process of incubation, an aha moment (illumination), and then evaluation and implementation of the idea (verification). Although these phases suggest a linear creative process, it is more likely that the phases can be implemented multiple times in different orders, with cycles of divergent and convergent thinking occurring in each phase (Lubart, 2018).

11.1.2 The Creative Mathematical Thinking Process

Various theories have been proposed as to how creative ideas arise in the mathematical domain. A well-known framework was introduced by Alan Schoenfeld (1982), who based his thoughts on earlier work by Polya. Schoenfeld proposed that (creative) problem-solving consists of a phase of reading the problem, analyzing task properties, exploring different possible solutions, planning how to reach a certain solution, implementing the solution properly, and lastly verifying (making sure the solution works). In general, phase models of creative problem-solving have been criticized for portraying creativity as a linear process that unfolds through a clearly defined sequence of steps (e.g., Lubart, 2018). However, it is more plausible that creative ideas also result from a messy process of going back and forth between steps, with cycles of divergent and convergent thinking embedded throughout (Lubart, 2018). Sheffield (2009) proposed such a non-linear process for mathematics. She suggested that creativity in mathematics is characterized by flexibility: students cycle through different activities such as creating, evaluating, and relating. The exact process can vary based on the problem and the amount of experience the student has.

One of the few studies that investigated the creative mathematical thinking process was a case study by Schindler and Lilienthal (2020) that depicted the creative problem-solving process of a high school student on a multiple-solution task. They indeed showed that such phase models might not be an accurate reflection of authentic creative problem-solving. Using a stimulated recall interview guided by recordings of the student's eye movements, Schindler and Lilienthal analyzed how new ideas emerge by coding the different parts of the student's creative problem-solving process and comparing it to existing models on creative problem-solving (e.g., Wallas's, 1926 model). They found that, compared to models like that of Wallas, phases could not be as clearly identified and that the sequence of phases did not seem to be as clear-cut. Instead of processing the different problem-solving phases step by step, the case study showed a cyclical process: the student constantly went back and forward between phases. For example, after generating an idea, the student was working on a solution. When he found out that this did not work, he discarded the approach and started looking for a new start and generating a new idea. Thus, Schindler and Lilienthal's case study provides initial evidence that for mathematics, the creative problem-solving process is not linear but rather cyclical. This notion provides support for previous claims made by Lubart (2018) and Sheffield (2009) about the general and the mathematical creative thinking process, respectively.

Given this cyclical nature, divergent and convergent thinking might be intertwined throughout the creative mathematical process, as using both modes of thinking can help children to generate different possible solutions or strategies, as well as select the most fitting one and evaluate its quality (Assmus & Fritzlar, 2018; Mann, 2005; Tabach & Levenson, 2018). Previous research has related both divergent and convergent thinking to mathematical performance on different types of tasks (De Vink et al., 2021; Jeon et al., 2011; Kattou et al., 2013; Schoevers et al., 2018). It therefore stands to reason that both thinking modes contribute toward the emergence of creative ideas during mathematical problem-solving.

How often and how well children apply divergent and convergent thinking might differ depending on both the task and the child. In terms of the task, open tasks are proposed to be the most suitable for creative mathematical thinking as they usually allow for multiple responses and can take many different forms (e.g., posing mathematical problems or finding different solutions to a specific problem; Leikin, 2009). Indeed, creative thinking has been found to affect performance more on open mathematical tasks than on closed tasks (Leikin, 2009; Schoevers, 2019). In terms of child characteristics, mathematical achievement seems to be a key factor because children with higher mathematical achievement scores have shown higher creativity achievement scores than children with average or low mathematical achievement (Kroesbergen & Schoevers, 2017; Leikin, 2013). We therefore assumed that groups of children differing in mathematical achievement scores also show different creative thinking processes on a mathematical task.

11.1.3 The Current Study

This study is a qualitative investigation of the creative thinking processes of primary school children engaged in mathematical tasks. Two groups of children (characterized by high vs. low mathematical achievement, as determined by a general mathematics knowledge test) were asked about their creative problem-solving process. Children at the extreme ends of mathematical achievement were selected to gain insight into the role that mathematical knowledge plays in the mathematical creative thinking process. Comparing such extreme cases could help to determine whether the differences found in mathematical creativity task scores relate to their creative thinking processes. The fifth grade is an appropriate educational stage to study mathematical creativity because its mathematics curriculum contains complex problems (Noteboom et al., 2017) that require creative thinking skill. To get a more varied picture, two types of open mathematical tasks were used: a problem-posing task and a multiple-solution task. Furthermore, as open tasks allow for different types of responses, both easy and more difficult, these tasks were deemed appropriate for children with either high or low mathematical achievement.

11.2 Method

11.2.1 Participants

A group of 28 fifth-graders from eight Dutch primary schools participated in this study. These children were selected from a larger sample that participated in a research project on creativity in math and science education (De Vink et al., 2021; Willemsen et al., 2021). The children who participated in the current study were selected based on their most recent mathematics grade point average (GPA), as indicated by their scores on a standardized progress monitoring test (Janssen et al., 2007). This test consisted of multiple-choice questions on various topics, from basic arithmetic to geometry and fractions, and was found to have good internal consistency (KR-20 = .95, greatest lower bound = .97; Hop et al., 2016).

Table 11.1 Descriptive statistics of the low-achieving, high-achieving, and total sample

	Sex		Age	Math GPA
Group	Boys	Girls	*M (SD)*	*M (SD)*
Low achieving	6	6	10.65 (0.09)	208.50 (3.80)
High achieving	13	3	10.66 (0.12)	282.00 (2.58)
Total	19	9	10.66 (0.41)	250.50 (38.75)

Extreme case sampling was used to draw an illustrative sample of children for the current study who demonstrated either mathematical excellence or lower mathematical performance (Onwuegbuzie & Leech, 2007). Children whose mathematics GPA could be classified as the lowest or highest 15% of the sample were selected to participate. After removing eight children from the sample for various reasons (e.g., no permission for audio recording or illness during data collection), the final sample consisted of 28 children. Descriptive statistics are presented in Table 11.1. The children's parents were all of Dutch nationality, and about half of them (46.4%) earned an (applied) university degree. Ethical approval for this study was obtained from the local ethics committee (ECSW-2019-087). The children's parents gave informed consent for participation in the study, retrieval of mathematics scores from the school administration, and audio recording.

11.2.2 Mathematical Tasks

We used two tasks to assess how children applied divergent and convergent thinking during mathematical problem solving: a problem-posing task and a multiple-solution task. These tasks were selected from existing research instruments and combined in a test booklet.

The *problem-posing task* was taken from the geometrical creativity task (GCT, Schoevers et al., 2019). Children received a picture (a scenic view of two picnic tables and eight chairs in a forest) and were asked to generate different mathematical questions that their classmates could answer based on that picture. Children could, for example, pose the question "How many chairs should be added to the table if 10 people join for lunch?" This task was chosen because a picture is thought to call upon children's imagination, which is seen as an important element of mathematical creativity (Sriraman, 2005). The problem-posing task was administered first because it was the most open of the two tasks, and research has shown that creative performance is best elicited by starting with the task that has the most response possibilities (Moreau & Engeset, 2016).

The *multiple-solution task* originated from the mathematical creativity task (MCT, Kattou et al., 2013; Dutch translation by Schoevers et al., 2018). This task was chosen because it allows for both simple and more elegant solutions and therefore was suitable for both low- and high-achieving groups. The task asked children to formulate calculations on both sides of an equal sign that had the same answer.

To do this, children could use the digits 1, 2, 3, 4, 5, and 6 and the operators plus, minus, multiplication, division, and decimal point. Both operators and digits could be combined. A possible solution to this task would be to combine 2 + 2 and 5 – 1, as these calculations both equal 4. Children were instructed to formulate as many calculations as possible.

11.2.3 Procedure

Data was collected in December 2019 and January 2020. Children first participated in plenary creativity, and science and mathematical tasks as part of our larger creativity project in which the relation between creative thinking, mathematics, and science performance is assessed (De Vink et al., 2021; Willemsen et al., 2021). Next, the first author revisited the school after a couple of weeks to administer the current mathematical tasks. This ensured a relaxed setting for children as they were already familiar with the researcher and the different types of creativity and mathematical tasks. The tasks were administered to each child individually in a quiet area of the school. The administration of the two tasks took approximately half an hour. Audio recordings were made to capture the child's thoughts and conversations with the researcher.

Before the start of the mathematical tasks, children were told that they would work on various types of mathematical tasks. They were explained that these tasks served to find out how different children approach mathematical tasks, that they would be asked to explain their responses and ideas, and that audio recordings would be made. Prior to each separate task, children were asked to read the instructions aloud. If children were not sure what to do after having read this information, they received help according to the standardized model for offering help during mathematical instruction from the Dutch guidelines on dyscalculia (Van Luit et al., 2014). To create an optimal atmosphere for creative thinking to occur, children were reminded throughout the tasks to share all of their ideas with the researchers (Sternberg, 2007; i.e., to think aloud). Research has shown that children are able to provide accurate think-aloud reports of mathematical problem-solving but benefit from using prompts while doing so (Reed et al., 2015; Robinson, 2001). Therefore, in addition to the ideas shared through think-aloud, the researcher used think-aloud prompts to ask children about their approach (e.g., "How did you think of this idea/solution?").

To minimize any possible bias toward achievement, children's mathematical achievement score was unknown to the researchers during the interview and coding process. A research assistant made a list of names and mathematics scores for children whose mathematics GPA could be classified as the lowest or highest 15% of the sample. A separate list with names, but no mathematics scores, was provided to the researchers during the interview and coding process so that no prior knowledge of children's achievement could affect their performance.

11.2.4 Data Analysis

After data collection, all audio recordings were transcribed verbatim. Next, ATLAS.ti (version 8) was used to perform directed content analysis (Hsieh & Shannon, 2005). This method was chosen because existing theories of divergent and convergent thinking formed the starting point of this study, and this study aspired to extend these theories to the domain of mathematics. The directed content analysis proceeded in three steps. First, operational definitions of mathematical creativity, divergent thinking, and convergent thinking were developed based on theory (see Table 11.2). Second, the researcher familiarized herself with the data by extensively reading each transcript and making notes with a first impression of each transcript. At this point, the transcripts were segmented into units that could be coded. A unit referred to a turn of the child, which can be defined as "one or more streams of speech bounded by speech of another, usually an interlocutor" (Crookes, 1990, p. 185). Third, the different turns received initial codes for mathematical creativity, divergent thinking, and convergent thinking using the operational definitions in Table 11.2.

During the initial coding phase, all turns with possible instances of mathematical creativity received the code "mathematical creativity." These turns were further

Table 11.2 Theoretical and operational definitions of mathematical creativity, divergent thinking, and convergent thinking for each task

	Mathematical creativity	Divergent thinking	Convergent thinking
Theoretical definition	"The cognitive act of combining known concepts in an adequate, but for the student new way, thereby increasing or extending the student's (correct) understanding of mathematics" (Schoevers, 2019, p. 58).	"Divergent thought from a single starting point generates varied ideas" (Brophy, 2001, p. 439).	"…whereas convergent thought starting from multiple points seeks one most true or useful conclusion" (Brophy, 2001, p. 439).
Operational definition Problem-posing task	The development of an idea that includes a combination of an element from the picture with a mathematical concept such as surface area in a way that is new to the child, resulting in an adequate question.	The process of generating a creative mathematical question based on the picture, as well as any corresponding elaboration or explanation.	The process of selecting or evaluating a creative mathematical question based on the picture.
Operational definition Multiple-solution task	The development of an idea that includes a combination of the given numbers with a type of calculation (e.g., multiplication) in a way that is new to the child, resulting in a correct calculation.	The process of generating a creative mathematical calculation, as well as any corresponding elaboration or explanation.	The process of generating a creative mathematical calculation, as well as any corresponding elaboration or explanation.

classified according to the mode of thinking (divergent or convergent), type of idea, and help given (see Table 11.7 in the appendix). Regarding thinking mode, we set out to label each creative turn as either divergent or convergent thinking. As we noticed during the coding process that many children used both divergent and convergent thinking in one turn, the code "divergent and convergent thinking" was added. It represented a combination of the operational definitions of divergent and convergent thinking for each task. Coding of the type of idea characterized the mathematical content that was central to the child's solution. This classification served to ensure that every mathematically creative idea actually represented a combination of concepts that was new for the child on this task (e.g., if the child thought of similar questions about surface area on the problem-posing task, this was not considered new for the child on this task). Codes for the type of idea were based on previous research that used the problem-posing and multiple-solution tasks presented here in larger samples (Schoevers et al., 2018; Schoevers et al., 2019). Since the problem-posing and multiple-solution tasks yielded different responses, separate codes for the type of idea were used for each (see Table 11.7). Lastly, every turn was binary coded to indicate whether children received any help to formulate their response or idea.

Since the tasks were used to measure the creative thinking process, and not the creative product (e.g., children also received help), no formal scores were calculated for fluency, flexibility, and originality. However, a descriptive comparison of the originality of ideas could be made between children based on previous research (Schoevers et al., 2018; Schoevers et al., 2019) that used the same tasks. These studies determined how original ideas were by comparing the frequency of a certain type of response to the frequency of other types of responses. For the problem-posing task, the Schoevers et al. studies showed a large variation in the questions that were generated, which means that quite a lot of responses could be seen as original. The questions that were generated the least often were (1) questions that made use of addition, subtraction, or division, (2) questions about ratio, (3) questions about volume, and (4) questions about circumference. An unoriginal response to the problem-posing task was any question that revolved around the concept of amount. For the multiple-solution task, original responses were (1) calculations with two numbers using division, (2) calculations with three or more numbers using subtraction or multiplication, (3) calculations using decimals, and (4) calculations using numbers consisting of three or more digits. Unoriginal responses for the multiple-solution task were calculations with two numbers using addition or mixed operations. To determine descriptive originality, the number of original responses was counted for every child.

11.3 Findings

For all children together, a total of 2197 turns was identified. Out of these turns, 585 (27%) received the code mathematical creativity. Subsequent coding of these turns showed that children predominantly used divergent thinking (76%), followed by a combination of divergent and convergent thinking (14%) and convergent thinking (10%).

11.3.1 Number of Creative Ideas

Children's turns were mostly coded as non-creative, as opposed to mathematically creative turns (see Table 11.3). This means that mathematical concepts were often not combined in a new way when children generated questions (problem-posing task) or calculations (multiple-solution task). Although many children were able to think of several questions or calculations, the underlying ideas often seemed to be quite similar. For example, many children started the problem-posing task by posing an "amount" question and re-used this concept in subsequent ideas, only changing their strategy when prompted by the researcher (e.g., "Can you also think of a different type of question/calculation?" or "Can you think of a question/calculation that incorporates addition?"). Such uniform strategy use also occurred on the multiple-solution task. Children would, for example, generate many calculations with two numbers using addition or mixed operations (e.g., addition and subtraction). The excerpt below illustrates how one child produced comparable responses to the problem-posing task. Only the first question received the code mathematical creativity because subsequent questions were a repetition of this first concept.

Child: I have a question. How many black chairs are there?

Researcher: How many black chairs are there. Yes, good one. How did you think of that one?

Child: Well, there are chairs, but here are also another two chairs, and then, you don't know whether you should count those.

Researcher: Yes.

Child: So, how many black chairs are there?

Researcher: Yes, smart. Then you can't be confused about which chairs the question is.

Child: This is quite hard.

Researcher: There is also a lot to see in the picture. … But, take your time; there is no rush.

Child: How many brown chairs are there?

Researcher: Yes, that's possible too. You can write that down.

……

Child: How many big trees are there?

Table 11.3 Number of creative ideas for the low-achieving and high-achieving group

	Creative ideas (%)	Non-creative ideas (%)
Low achieving	26	74
High achieving	27	73

Note: Percentage for this group of children

In addition, quite many turns involved clarification questions about the task in which no mathematically creative question or calculation was proposed. Even though children had worked on a multiple-solution task before, this type of task was still rather new to them. Some children started each task by extensively asking what was possible or what would be considered "good," despite instructions that there were no right or wrong answers as in a "regular" mathematical task. An example of such questions is presented below.

[Child reads the instruction aloud.]
Researcher: Yes, so on the next page, there is room to write down the questions your classmates can answer. Do you have an idea how you can approach this?
Child: And then … it has to be about mathematics?
Researcher: Yes. So, it should be about mathematics and about the picture, but anything that relates to those two things you can ask a question about.
Child: That's hard. And should it be easy questions? Or not?
Researcher: Any type of question.
Child: So, also a question on the level of first or second grade?

11.3.2 The Use of Divergent Thinking

To generate creative questions and solutions, children mostly seemed to make use of divergent thinking, with 76% of turns being coded as such. Many divergent thinking turns were statements of ideas, for example, "*And now, I have another mathematical question. What is the amount of chairs plus the amount of people?*" (problem-posing task) or "*Ehm … 6 times 1 and 3 times 2*" (multiple-solution task). Divergent thinking also concerned any elaboration or explanation of an idea, which differed considerably in terms of elaborateness. The initial idea statements of some children were short, for example, "*The next question is how many chairs are there?*" (problem-posing task). Other children immediately explained their ideas more elaborately, for example, "*I should probably pose a question about circumference, because with 1 square meter you are sitting in the middle, on top of the table. … and 2 people can sit at 1 square meter. … one on this side and one on the other side. … And 3 people can sit at the other square meter, because one can sit at the corner … so then, you should calculate the circumference as well*" (problem-posing task). This child had previously posed a question about the surface area of the table and how many people could be seated at it but realized that the circumference of the table would be a better way to calculate this.

Similar differences occurred when children were asked to explain how they conceived an idea. Some children explained that merely seeing an element of the picture or a number brought them to an idea, whereas others gave a more elaborate clarification. For example, one child mentioned thinking of a certain question on the problem-posing task because a similar question had been used in the mathematics class a few times. Children often related information from the task to a different setting. A few children mentioned that similar ideas or topics had been discussed in mathematics class, but also more remotely related settings or contexts were mentioned. For example, one child was imagining where the picture in the problem-posing task could have been taken and thought of Veluwe, a national park in the Netherlands. One child even seemed to activate such contextual knowledge, starting by asking himself "*Where are they and what are they doing?*" (problem-posing task). Something that most children had in common in terms of the divergent thinking statements was that they looked for elements that stood out. For example, some children decided to start their calculations in the multiple-solution task with the number 1, because it was the first number they noticed when reading the question. Some children also explicitly mentioned that they were trying to think of a question or calculation that was different from the ones that they had used before.

11.3.3 The Use of Convergent Thinking and Combinations of Divergent and Convergent Thinking

Children's spontaneous and prompted turns were less often coded as convergent thinking (10% of the turns) than divergent thinking (76% of the turns), both before and after researcher prompts. Convergent thinking was defined as the process of evaluating or selecting a mathematically creative question or calculation. Occasionally, children showed that they evaluated an idea or elaborated on why they selected a certain idea. For example, a child who proposed the question "*How many chairs are there*" on the problem-posing task later explained that she selected this question because easy questions were also allowed. Another child said, "*Yes, I had to think is this really a good and logical question?*" (problem-posing task). A recurring theme throughout the turns that received a convergent thinking code was that children either evaluated their own ideas as being simple or easy or mentioned specifically trying to think of or selecting ideas that were "easy to think of." For example, "*Yes, I just did a lot of easy calculations, except for this one!*" (multiple-solution task) or "*Using this method, I could go on easily*" (multiple-solution task). These statements might suggest that children did not want to challenge themselves (one child also said on the multiple-solution task "*I am not going to use divisions because I find that difficult*") or might be looking for a general rule or strategy they could use to generate many ideas. For example, one child said, "*I tried to make calculations that usually had 5 or 10 as outcome; it does not need to be very big*" (multiple-solution task). However, occasionally, children did mention looking for variability or different ideas, for example, "*I did not want to do the same thing every time, so then, I decided to do this*" (multiple-solution task).

Convergent thinking often co-occurred with divergent thinking. Such divergent-and-convergent-thinking turns often incorporated the initial creation of an idea and a selection or evaluation that further refined the idea. Examples are "*Ehm, I saw two* *tables, and I thought I can't ask a question about the chairs. Yes, maybe it would be possible. But then, I thought no; I should ask how long can the table be, because that also is related to mathematics*" (problem-posing task) or "*Yes, you could try and see which calculations have 1 as outcome. And when you have had those, you can do it with 2 as outcome. And then, for example, with lower numbers, because when it is hard, it is best to start small*" (multiple-solution task).

In terms of sequence, children often generated multiple ideas first (i.e., divergent thinking) before switching to convergent thinking or a combination of divergent and convergent thinking. That is, for most children, the first couple of turns were coded as divergent thinking. These turns either conveyed different ideas that were then translated into a specific question/problem or calculation or a multitude of more finished ideas (i.e., different actual questions/problems or calculations). After children had started thinking of different ideas, they also started applying convergent thinking or combinations of divergent and convergent thinking. Since both tasks required children to think of multiple ideas, this sequence was repeated several times. Most often, a few divergent thinking turns were identified before a turn with a combination of divergent and convergent thinking or pure convergent thinking occurred. An example of a sequence of divergent and convergent thinking for the problem-posing task is presented below.

Child (divergent): Maybe, a question is how many chairs are at the table?

Researcher: Yes, seems like a good one. You can write that down. And how did you think of that question?

Child (divergent): Well, when I read it had to be a question related to mathematics, I immediately thought of amount.

Researcher: Yes.

Child (convergent): Those kinds of questions are usually the normal basic questions you can ask.

Researcher: Yes, very good. Did you think of amount before you decided to do something with the chairs?

Child (divergent and convergent): Yes. But, you should be able to answer these questions right? So, you can't ask like how long is the table?

Researcher: Yes, it is actually possible to ask that. As long as your question relates to the picture.

Child: Okay.

Researcher: So, it would be possible to ask how long the table is.

[Child writes this question down]

Researcher: Yes good one too. And how did you think of that?

Child (divergent and convergent): Ehm, I saw two tables, and I thought I can't ask a question about the chairs. Yes, maybe it would be possible. But then, I thought no; I should ask how long can the table be, because that also is related to mathematics.

11.3.4 Differences Between Children and Tasks

The observations presented thus far focused on the general characterization of the creative mathematical problem-solving process. However, we also observed differences between children and across tasks. In terms of the sequence of divergent and convergent thinking, most children used some convergent thinking or a combination of divergent and convergent thinking after a couple of turns of divergent thinking. However, three children used only divergent thinking and no convergent thinking or a combination of divergent and convergent thinking on the two tasks. A trend for these children seemed to be that they did not generate many different questions or mathematical calculations that included a new combination of concepts and as such had relatively few turns that received the code mathematical creativity, to begin with. One of these children also received a relatively large amount of help.

Another notable difference between children concerned the creativity of the ideas that were generated using divergent and convergent thinking or the combination of both. Specifically, we compared the use of divergent and convergent thinking between children of whom more than two ideas on the two tasks were coded as original (n = 4) and children of whom no ideas were coded as original on the two tasks (n = 7) (see Table 11.4). Although originality is a judgment of the creative product, a comparison of divergent and convergent thinking between children who differed in terms of the originality of their ideas can still yield valuable insights into their creative thinking process. First of all, this comparison showed that children who did not generate original ideas required a little more help (in 60% of the mathematically creative turns) than children who generated original ideas (in 40% of the turns). The group of children who generated multiple original ideas made more use of convergent thinking on the problem-posing task. On the multiple-solution task however, children who generated original ideas differed from children who did not generate original ideas in the use of divergent thinking and the combination of divergent and convergent thinking. Children who generated original ideas used less divergent thinking, while using more combined approaches with divergent and convergent thinking. An example of such an approach is given below. This child thought of a mathematical calculation including a three-digit number on the multiple-solution task.

Table 11.4 Number of original ideas for the low-achieving and high-achieving group

	0 original ideas	1 original idea	2 original ideas	3 original ideas
Low achieving	5	5	1	1
High achieving	2	9	2	3

Child (divergent and convergent): Yes. I think I am going to choose … ehm … for example, 124 is something that is possible to make with those numbers.

Researchers: Yes, you can.

Child (divergent): Ehm, and then, I am using plus 1, but you can also do 123, that is one less, plus 2.

[Child writes down 124 + 1 = 123 + 2].

Researcher: Yes, exactly, that is how you can do it. You thought of this one quickly. How did you do that?

Child (divergent and convergent): Well, I just saw 123, but I thought that is an uneven number, so it is not so convenient, so then, I decided not to use that. I then went to the 4, and that seemed like it was convenient, so I thought let's make it simple and just add 1. But then, I thought, hey that is 125. So, to the 123, that's already here, you can add 2 and make 125, and it works out.

We also contrasted children with high ($n = 16$) and low ($n = 12$) prior mathematical achievement. Findings showed that, although divergent thinking prevailed in both subgroups, children with high mathematical achievement scores had a slightly more balanced ratio (see Table 11.5) of divergent thinking to convergent thinking and combinations of divergent and convergent thinking than children with low mathematics achievement scores. On the problem-posing task, children with high mathematical achievement used slightly less divergent thinking and more convergent thinking than children with low mathematical achievement. On the multiple-solution task, the high mathematical achievement group used less divergent thinking and more combinations of divergent and convergent thinking than the low mathematical achievement group. Therefore, the differences between high and low achievers resembled those between children who generated original versus non-original ideas. This was also reflected in the fact that most original ideas were generated by children with high mathematical achievement, whereas children with low mathematical achievement generated more unoriginal ideas. The amount of help received did not differ substantially between the two groups.

Finally, the two different tasks were compared. The problem-posing task and multiple-solution task were quite comparable with regard to the occurrence of mathematically creative ideas and the use of divergent and convergent thinking. Table 11.6 shows the absolute and relative frequency of divergent thinking, convergent thinking, and a combination of the two for each task. On the problem-posing

Table 11.5 Use of divergent thinking, convergent thinking, and divergent and convergent thinking for the low-achieving and high-achieving group

	Divergent thinking (%)	Convergent thinking (%)	Divergent and convergent thinking (%)
Low achieving	80	10	10
High achieving	72	11	17

Note: Percentage for this group of children

Table 11.6 Use of divergent thinking, convergent thinking, and divergent and convergent thinking for the different tasks

	Divergent thinking (%)	Convergent thinking (%)	Divergent and convergent thinking (%)
Problem-posing task	80	6	14
Multiple-solution task	73	13	14

Note: Percentage for this task

task, around 28% of turns received the code mathematical creativity. On the multiple-solution task, this percentage was slightly lower (22%). Divergent thinking was predominant in both tasks, with a slightly higher occurrence rate on the problem-posing task. On the other hand, the use of convergent thinking was slightly higher on the multiple-solution task than on the problem-posing task. The use of a combination of divergent and convergent thinking was comparable between tasks. A notable difference between the two tasks concerned how elaborate ideas were, and therefore, also, extensive ideas were explained. Probably, the slightly larger percentage of divergent thinking used in the problem-posing task was related to the fact that children thought of complete questions here that included several elements (e.g., mathematical operations combined with several concepts from the picture). On the other hand, the multiple-solution task could be completed using relatively short calculations, which might lead to less elaborate divergent thinking processes and explanations thereof.

11.4 Discussion

This study investigated children's use of divergent and convergent thinking on two mathematical tasks: a problem-posing task and a multiple-solution task. Sixteen children with high mathematical achievement scores and twelve children with low mathematical achievement scores were asked how they thought of different creative ideas using think-aloud prompts. Their ideas were coded using qualitative content analysis. Specifically, the use of divergent and convergent thinking or a combination of the two was identified for every mathematically creative question (problem-posing task) or calculation (multiple-solution task).

11.4.1 The Use of Divergent and Convergent Thinking

Relatively few ideas were coded as creative ideas (27%) compared to uncreative ideas (73%). Although children could generate multiple ideas for both tasks, it was more difficult for them to think of diverse ideas and especially original ones. This result is in line with previous research that showed that most high school students are able to produce ideas fluently but that flexibility and originality of solutions are more difficult to achieve (Leikin, 2013). The novelty of the tasks may also have impeded children's conception of creative ideas. Both tasks in this study differed from the closed tasks in Dutch mathematical textbooks (Van Zanten & Van den Heuvel-Panhuizen, 2018). Although new tasks could evoke creative ideas because they require children to search for different ways to solve a mathematical problem (Levenson, 2013), the newness of the task might also have caused some children to feel insecure. This was apparent from the many questions children asked at the beginning of each task about when a response would be considered "correct." Furthermore, some children needed additional explanations to understand what each task involved. The researcher emphasized in her explanation that there were many possible solutions, a response assumed to contribute to mathematical creativity (Kozlowski et al., 2019). Still, some children might have been preoccupied with finding the "right" answers because their mathematical instruction often focuses on one specific solution or solution strategy.

Despite the fact that most ideas were coded as uncreative, children also produced several creative ones, mostly through divergent thinking, and three children even relied exclusively on this mode of thinking. This finding is in line with Tabach and Levenson's (2018) suggestion that tasks with (infinitely) many solutions can lead to "excessive" divergent thinking. That is, such tasks might enable children to produce many, but sometimes infeasible or ineffective, ideas. For all children, at least some of these ideas represented a new combination of mathematical concepts and therefore received the code mathematical creativity. However, children who used divergent thinking and convergent thinking, either concurrently or in separate turns, generated the most original ideas. This finding is in line with previous research in which children with high divergent and convergent thinking skills also scored high on a multiple-solution task (de Vink et al., 2021). This finding also corroborates creative thinking theories that advocate the role of both divergent and convergent thinking (e.g., Brophy, 2001; Cropley, 2006; Guilford, 1973). It seems that, for mathematical creativity, ideas should not only be generated but also selected and evaluated to produce ideas of high quality.

Some authors have portrayed the creative thinking process as a linear series of steps (e.g., Wallas, 1926) whereas others characterize it as a "messy" process in which children alternately employ divergent and convergent thinking throughout the task (Isaksen et al., 2011; Lubart, 2018; Schindler & Lilienthal, 2020; Sheffield, 2009). Our study supports the latter view and indicates that switching between divergent and convergent thinking occurred somewhat irregularly: children often had multiple repetitions of divergent thinking, after which one or two turns of convergent thinking followed. This sequence seems to be a reflection of divergent thinking as an inherently exploratory process, that is, an "idea search in multiple directions […], which is inherently an exploration of a thought space" (Lubart, 2018, p. 7). Likely, children first explore several possibilities, which are then evaluated and combined into one solution. Previous research has not only identified divergent and convergent thinking as separate constructs but also showed that both thought processes might be intertwined (Barbot et al., 2016; Cortes et al., 2019). In line with this connectedness between divergent and convergent thinking, we also found instances of a combination of divergent and convergent thinking. Such combinations show that although divergent and convergent thinking are seen as separate constructs, their processes might indeed be intertwined (Barbot et al., 2016; Cortes et al., 2019). This connectedness is demonstrated at the task level with turns of divergent and convergent thinking. Turns that were coded with a combination of divergent and convergent thinking show that this connectedness extends to the turn level as well. These combination turns could reflect a micro-cycle in which the child alternated between divergent and convergent thinking. Analysis with other more fine-grained methods (e.g., preceding the turn level) could be used to unveil what such micro-cycles look like. This finding illustrates the need for more process-based research on divergent and convergent thinking, as static measures might not fully capture the complexity of the creative thinking process.

11.4.2 The Role of Mathematical Achievement and Task Type

We also observed qualitative differences between children with high and low mathematics achievement scores. Previous research has shown higher mathematical achievement to be associated with higher mathematical creativity (Jeon et al., 2011; Kattou et al., 2013; Kroesbergen & Schoevers, 2017; Leikin, 2013; Schoevers et al., 2018). In this study, ideas of children with high mathematical achievement scores were coded more often as original than the ideas of children with low mathematical achievement scores. It is important to note that this finding, just like the other findings, is of qualitative nature and not statistically significant. Thus, we cannot conclude that children with high and low mathematical achievement scores differ in their ability to generate original ideas. Rather, this might be related to differences in the use of creative thinking skill. In this study, children with high mathematical achievement scores more often used convergent thinking, or combinations of divergent and convergent thinking, than children with low mathematics achievement scores. Mathematics education emphasizes convergent thinking (i.e., looking for one correct answer instead of generating different ideas or strategies; Levenson, 2013). Thus, it seems plausible that children with high mathematical achievement scores do well on tests of achievement by applying convergent thinking skill. These experiences with convergent thinking might have helped this group to select the most original ideas, while children with low mathematics achievement found this more difficult.

We found minor differences between children's creative performance on the two types of tasks. On the problem-posing task, we found a slightly higher percentage of creative ideas than on the multiple-solution task. Children used relatively more divergent thinking and less convergent thinking on the problem-posing task than they did on the multiple-solution task. To our knowledge, no research has yet compared children's creative thinking processes on these types of tasks. Therefore, it is difficult to determine whether this observation is specific to our study or whether other researchers might find the same results. It seems plausible that the problem-posing task elicits more creative ideas because it contains fewer constraints (Medeiros et al., 2014). The only constraints to generate a question on this task are the topic of mathematics and the elements within the picture. The multiple-solution task, by contrast, required children to use the given numbers and operators and to match the outcome of the two calculations. These restrictions may have caused a slightly smaller number of creative ideas. Another explanation, however, is that the problem-posing task was presented first and that children suffered more from fatigue or inattention on the multiple-solution task. The fact that this task is also slightly more similar to regular textbook mathematical tasks might explain why children made more use of convergent thinking on this task than on the problem-posing task.

11.4.3 Future Studies and Limitations

This study provided a first look into how children with low or high mathematical achievement scores generate, select, and evaluate mathematically creative ideas on two types of mathematical tasks. Although we tried to make the research setting as relaxed and natural as possible for the children, it remains unclear whether our findings are typical of creative thinking in regular mathematics classrooms. Another possible limitation concerns the use of children's verbal expressions as a proxy for creative thinking. Although research has shown that children are capable of explaining their thinking on a mathematical task (Reed et al., 2015; Robinson, 2001), we do not know whether the ability to verbalize their thoughts differed between children, for example, as a result of differences in language ability or emotional factors like shyness. Therefore, the validity of the insight that we obtained into the creative thinking process might be higher for some children than for other children. Lastly, an important limitation of the current study is the binary coding of mathematical achievement as either "high" or "low." We used extreme case sampling as a way of creating an illustrative sample and labeled the groups accordingly. It is important to stress that this label is based on a single test score and, hence, not necessarily reflective of children's general mathematical skills or abilities. It does, however, provide insight into creative differences that can be observed between children who might score lower or higher on a more traditional mathematics test.

We recommend future research to contrast various types of measures of creative thinking processes in one sample to improve measurement reliability. For example, Schindler and Lilienthal (2020) combined eye-tracking with stimulated recall interviews to capture children's creative thinking. Furthermore, future research could contrast different types of tasks, as well as groups of children characterized by different individual features. For example, Bokhove and Jones (2018) have argued that mathematical creativity is not limited to open tasks but can also be displayed on tasks that are “moderately closed” (i.e., tasks that have some constraints but also allow for multiple solutions/strategies). It would be interesting to assess whether children’s creative thinking process on such a task differs from creative thinking on more open tasks. Furthermore, differences between children, especially regarding cognitive characteristics such as executive functions, might play a large role in the creative mathematical thinking process and should therefore be a topic of further research. For example, it would be interesting to assess what role inhibition plays in the creative mathematical thinking process, because for children with high mathematical achievement, reduced inhibition aids mathematical creativity, whereas for children with low mathematical achievement, strong inhibition seems important (Stolte et al., 2019). Given the developmental nature of such cognitive characteristics, another interesting avenue for future research would be to examine what the creative mathematical thinking process looks like in different age groups (e.g., upper vs. lower primary school students).

11.5 Conclusion and Implications

This study showed that many children find it difficult to come up with new ideas and stick to ideas similar to the ones they had generated before. Whereas this inclination might not harm when solving mathematical problems that rely on automated knowledge, it becomes problematic when problems become more difficult and children can no longer rely on learned procedures. Our findings further indicate that convergent thinking is important in conceiving mathematically creative ideas (cf. Brophy, 2001; Cropley, 2006; de Vink et al., 2021; Tabach & Levenson, 2018). Primary math teachers are recommended to model and explain the use of divergent and convergent thinking in their classes, as the interplay between divergent and convergent thinking seems imperative. The use of different types of problems, both (moderately) closed and open, is recommended for children to gain experience with different ways of creative problem-solving. Finally, we recommend combining the learning of new mathematical facts or procedures with creative thinking, both divergent and convergent. It is important that children are not only taught that creative thinking is important, but also taught *how* to do this. The lower frequency of ideas coded as creativity in the group of children with low mathematical achievement scores shows that this group might need different support to come up with creative ideas. The dominant focus on convergent thinking in mathematics education might disfavor a certain group of children, both in terms of creative thinking and mathematical achievement.

Appendix

Table 11.7 Codebook

Code	Subcodes	Problem-posing task	Multiple-solution task
1_ Mathematical creativity		The development of an idea that includes a combination of an element from the picture with a mathematical concept such as surface area in a, for the child, new way, resulting in an adequate question (e.g., *the development of the idea* "*How many square meters is the table?*").	The development of an idea that includes a combination of the given numbers with a type of calculation (e.g., including decimals) in a, for the child, new way, resulting in a correct calculation (e.g., *the development of the idea* "*4,5 + 3 & 6 + 1,5*").
2_Divergent thinking		The process of generating a mathematically creative question[a] based on the picture, as well as any corresponding elaboration or explanation (e.g., "*Oh, I have got one! How many square meter is the table? …. Because 2 people can sit at 1 square meter*").	The process of generating a mathematically creative calculation[a], as well as any corresponding elaboration or explanation (e.g., "*Hmm, what can I do with this? … Ahh, okay … yes, 4,5 plus 3 and 6 plus 1,5*").
3_Convergent thinking		The process of selecting or evaluating a mathematically creative question based on the picture (e.g., "*Yes, I had to think for a while whether it is actually a good and logical question*").	The process of selecting or evaluating a mathematically creative calculation (e.g., "*I wanted to have an easy calculation, and I could think of this fast*").
4_Divergent and convergent thinking		The process of generating and selecting or evaluating a mathematically creative question based on the picture, as well as any corresponding elaboration or explanation (e.g., "*I thought the surface are of the whole landscape is a bit too much … So, I was looking for something small that you could know the surface area of*").	The process of generating and selecting or evaluating a mathematically creative calculating, as well as any corresponding elaboration or explanation (e.g., "*Well, I started thinking what I could with a comma, because I have not used it yet, so then, I thought of this question*").

(continued)

Table 11.7 (continued)

Code	Subcodes	Problem-posing task	Multiple-solution task
5_Type of idea	_Adding	The generation of a mathematically creative question in which something is added (e.g., "*What is the amount of chairs plus the amount of people?*").	N. A.
	_Amount	The generation of a mathematically creative question about an amount (e.g., "*How many chairs are there?*").	N. A.
	_Circumference	The generation of a mathematically creative question about circumference (e.g., "*What is the circumference of the table?*").	N. A.
	_Estimate	The generation of a mathematically creative question in which something is estimated (e.g., "*How many pebbles are there on the ground?*").	N. A.
	_Multiplying	The generation of a mathematically creative question in which something is multiplied (e.g., "*If you multiply the amount of chairs by 2, how many chairs are there?*").	N. A.
	_Ratio	The generation of a mathematically creative question about ratio (e.g., "*How many people can sit at the table?*").	N. A.
	_Subtracting	The generation of a mathematically creative question in which something is subtracted (e.g., "*There are six chairs. I take two away; how many are left?*").	N. A.

(continued)

Table 11.7 (continued)

Code	Subcodes	Problem-posing task	Multiple-solution task
	_Surface area and size	The generation of a mathematically creative question about surface area and size (e.g., "*How many square meter is the garden?*").	N. A.
	_Combination	The generation of a mathematically creative question in which any of the above concepts are combined (e.g., "*How many chairs are at a table on average?*").	N. A.
	_Other	The generation of a mathematically creative question that does not fit with any of the other codes for type of idea (e.g., "*What kind of shape is the table?*").	N. A.
	_2 number plus	N. A.	The generation of a mathematically creative calculation that uses two numbers and the operator plus (e.g., $2 + 2 = 3 + 1$).
	_2 number minus	N. A.	The generation of a mathematically creative calculation that uses two numbers and the operator minus (e.g., $4–2 = 6 – 4$).
	_2 number multiply	N. A.	The generation of a mathematically creative calculation that uses two numbers and the operator multiply (e.g., $2 \times 3 = 1 \times 6$).
	_2 number mixed	N. A.	The generation of a mathematically creative calculation that uses two numbers and mixed operators (e.g., $2 + 4 = 2 \times 3$).
	_3 number plus	N. A.	The generation of a mathematically creative calculation that uses three numbers and the operator plus (e.g., $2 + 2 + 2 = 3 + 3$).
	_3 number minus	N. A.	The generation of a mathematically creative calculation that uses three numbers and the operator minus (e.g., $6 – 2 – 2 = 5 – 3$).
	_3 number multiply	N. A.	The generation of a mathematically creative calculation that uses three numbers and the operator multiply (e.g., $3 \times 3 \times 2 = 6 \times 3$).

(continued)

Table 11.7 (continued)

Code	Subcodes	Problem-posing task	Multiple-solution task
	_3 number mixed	N. A.	The generation of a mathematically creative calculation that uses three numbers and mixed operators (e.g., $4 \times 2 + 1 = 6 + 2 + 1$).
	_2-digit number	N. A.	The generation of mathematically creative calculation that uses a number composed of two digits (e.g., $15 + 13 = 14 + 14$).
	_3-digit number	N. A.	The generation of mathematically creative calculation that uses a number composed of three digits (e.g., $124 + 146 = 142 + 126 + 2$).
	_decimal number	N. A.	The generation of a mathematically creative calculation that uses a decimal number (e.g., $1.4 + 4.6 = 3 + 3$).
Help		The child received help during the process of generating, selecting or evaluating a mathematically creative question based on the picture.	The child received help during the process of generating, selecting, or evaluating a mathematically creative calculation.

Note: [a]"Mathematically creative question" or "mathematically creative calculation" in this table refers to a question or calculation as defined under the code "mathematical creativity"

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