Research in Mathematics Education

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# Mathematical Creativity

## A Developmental Perspective

Springer

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# Foreword

As you are reading this book, millions of children and young people, who have their whole life laying ahead of them, are being taught in classrooms. Whenever I think of this, I wonder how these young people should be educated, what knowledge and skills do they really need to develop in order to be best equipped for their lives and the future world that we cannot foresee. Unequivocally, in the challenging and changing world that we live in, the role of education should not be restricted to enriching students' knowledge, but it should empower them to adapt to changes and approach problems creatively. Creative thinking constitutes the mechanism to manage change and challenge.

This book offers a widely useful compilation of theoretical frameworks, empirical findings, cases, and approaches to mathematical creativity across various ages. It is, I think, an important resource for those investigating mathematical creativity, for mathematics educators, policy makers, and teachers. First, it provides in a concise way these various aspects of mathematical creativity and an overall view of what is the state of the art on this topic. It highlights the similarities and differences of mathematical creativity across ages and presents some indicative research studies on creativity at different age groups, using different theoretical frameworks, research questions, and methodological tools. It is, therefore, with great pleasure that I accepted the invitation by Dr. Chamberlin, Dr. Liljedahl, and Dr. Savić, three well-known researchers in the field of mathematical creativity, to write the foreword for this book. I was excited to be given the opportunity to read all its chapters in advance.

In 1980, the National Council of Teachers of Mathematics (NCTM) identified gifted students of mathematics as the most neglected segment of research in mathematics education. Since then, a vast amount of quality research was developed for the identification of gifted students, and for creating appropriate materials for helping talented students to enrich their mathematical abilities. Nowadays, much focus is placed on the teaching of mathematics which provides for creativity not only for the gifted and talented students, but provides for all students the opportunity to appreciate the beauty of mathematics and to fully develop their talents and abilities. Creativity is a way of thinking in mathematics through different lenses. Mathematics understanding requires creative applications in the exploration of mathematics problems. Traditional teaching methods involving demonstrations and drill and practice using closed problems with predetermined answers insufficiently prepare students in mathematics. The essence of mathematics is thinking creatively, not simply arriving at the right answer.

In this line of thought, this book offers a detailed and elaborate picture of research on mathematical creativity, setting off from its origins, walking us through some of its major advances and bringing us to its current status, and finally openhandedly offering possible avenues for new research. The chapters blend nicely the theoretical background and literature of some of the most eminent theories in mathematics education and also present findings of some current empirical studies. The book is organized in four sections:

Section 1: History and Background of Mathematical Creativity
Section 2: Synthesis of Literature Finding for Researchers
Section 3: Recently Completed Empirical Studies in Mathematics Education
Section 4: Research Application and Editors' Summative Considerations

When discussing mathematical creativity, it is useful to start from its history. Thus, the first section refers to the history of research and definitions of creativity, providing at the same time the background of mathematical creativity. In the second chapter, one of the editors, Peter Liljedahl, provides an overview of the various strands of creativity research that have influenced mathematics education. He elaborates on the directions that research in mathematics creativity has taken and reveals the links among various theoretical frameworks in general, and specifically in mathematical creativity. This chapter is a useful tool for researchers to look at research on creativity in mathematics education, through different aspects that had been studied as well as through many underlying theoretical assumptions on creativity.

The history of research on creativity is closely related to the topic of the second chapter of the book, which reflects on mathematics creativity and society. The chapter by Chamberlin and Payne reveals various conceptions of creativity and their implicit and explicit value in society. This is mainly examined through the lens of national standard documents and international competitions. The researchers highlight the fact that one of the main reasons for which mathematical creativity is not advancing in the way we might have expected, is that rarely time and money are invested in the development of mathematical creativity in classrooms. Moreover, although the value of creativity is proclaimed in many curricula and policy documents, mathematics creativity is almost never or very rarely assessed in national or international level and thus teachers do not place adequate emphasis during their teaching. The authors aimed at showing the impetus connection of mathematics creativity to the society by underlying the chasm between the emphasis on creativity in curricula on one hand, and the resources invested by educational administrations on the other hand. It is not, of course, possible in a short article to deal with all aspects of the relation of mathematics creativity with the varsity of effects of society. Despite that, the article contributes to a fruitful discussion in answering questions such as how to best nurture mathematical creativity of students, and most importantly, why in most societies the administration undervalues the focus and significance of mathematics education creativity.

In the fourth chapter of this section, the three editors, Chamberlin, Liljedahl, and Savić, present the framework of the book and highlight its contribution. The editors wish to emphasize that mathematical creativity is for all students and not only restricted to a few talented students. To do so, they argue, that one needs to realize that mathematical creativity is not homogenous but its process and products differ in different ages. Therefore, they chose to discuss in this book, mathematical creativity and its development in ages 5–12 elementary school years, 13–18 secondary school years, and 19–23 tertiary education years.

In the second section, a synthesis of literature findings of three age groups are presented. The editors propose that research which focuses on mathematical creativity should take into consideration that persons, products, and processes are different at different ages. It appears that we need a more fine-grained analysis of what creativity may look like at different ages and how it might be developed. For instance, we need to provide a more detailed account of what mathematical creativity may look like at different ages, how it may develop and what the effects of various types of instruction on students' development of mathematical creativity are. The three chapters that follow offer a detailed account of what the empirical studies have shown until now, and what information is available about creativity in the three specific age groups. Understanding the development of creativity, learning about various attempts that were made to develop mathematical creativity and the impact that these attempts had, constitute important first steps for the development of better instruction for the development of students' mathematical creativity. The three chapters also present promising directions for future research which can be useful to people who want to pursue research in this field.

The second chapter of this section by Kozlowski and Chamberlin explores the way in which literature influenced research in mathematics creativity for individuals 5–12 years old. The literature explored in this article is organized in two main categories: academic oriented research and practice oriented research. In the third chapter of this section, Joklitschke offers a systematic overview of current empirical insights on mathematical creativity among secondary school students, while in the fourth chapter, Savić, Satyam, El Turkey, and Tang provide a broad view of research on mathematics creativity among students at the tertiary level. The authors indicate that far fewer research studies explored mathematical creativity among students of tertiary education in comparison to students of elementary and secondary education.

Actually, the second section of the book suggests that individuals are able to be creative in the sense that they are able to come up with novel ideas in the context of their age and abilities. Although there is a general agreement about which processes and abilities are important for the development of creativity, fully understanding the development of each process and its role in creativity is a more complex task. Research is not conclusive as to precisely indicating how creativity develops and what exactly is essential in fostering this development. Thus, the third section of the book, presents empirical studies which are related to a degree with developmental theories and processes thought to be important in the study of creativity in individuals. Specifically, in the third section of the book, five recent empirical research studies in mathematics education are presented. In a broad sense, section three presents research that highlights practices which contribute to the development of students' creativity. The authors refer to episodes in classrooms, to the differences of convergent and divergent thinking, to the progression of creativity, to the concept of group creativity, and finally to approaches which contribute to our understanding of the creative processes at play in educational environments. The chapter by Crespo and Dominguez presents the benefits of using different analytic lenses to understand children's creative mathematical thinking. The researchers invite readers to see through some episodes how children are working and what they are saying from different theoretical lenses. The realization that different theoretical frameworks could reveal or disguise the causes or the results of any learning experience is of fundamental importance. As teachers and researchers, we need to embrace this challenge and invest time and effort in making the right choices.

In the third chapter of this section, de Vink, Lazoner, Willemsen, Schoevers, and Kroesbergen investigate the contribution of convergent and divergent thinking in upper-elementary school children while working on problem posing and multiple solution tasks. This is a worthwhile topic, since we often see creative thinking being associated only with divergent thinking and even equated with divergent thinking. The researchers found that generally divergent and convergent thinking is evolving in a nonlinear process. Students often start from divergent thinking and then move to convergent thinking. The authors found that students with high achievements in mathematics tended to use more convergent thinking or a combination of convergent and divergent thinking. The realization of the important role that convergent thinking plays for the development of creative ideas may be an eye-opening experience and also reveals new directions for instruction which aim towards the development of mathematical creativity. It is possible that most often instruction that tried to facilitate mathematical creativity emphasized mostly divergent thinking without appreciating the combination and cyclic blending of convergent and divergent thinking.

Thinking of environments that will support the development of creativity and also of group creativity in school classrooms appears to be challenging and necessary. This is the topic that Liljedahl explores in the fourth chapter of this section. He uses the term burstiness, to describe the role of environment on group creativity. He outlines some of the key ingredients that are necessary for an environment to form fruitful ground for group creativity to occur such as the structure, diversity, psychological safety, welcome criticism, freedom to shift attention, focus, and opportunity for non-verbal communication. Liljedahl, presents an episode from secondary education and masterfully illustrates what these ingredients may look like in the mathematics classroom.

Although numerous studies explored what mathematics creativity may look like and how it may progress through various learning environments, we rarely find any studies that show how the perception of individuals' creativity changes during a learning course. A reason for this might be that most of the studies were conducted with young students who may not be mature enough to discuss their perceptions of mathematical creativity and also reflect on them.

In the fifth chapter of this section Karakok, Tang, Cilli-Turner, El Turkey, Satyam, and Savic explore the progression of four undergraduate students' perspectives on mathematical creativity. The original perspective of mathematical creativity of these four undergraduate students is that it involves unique, innovative, and original approaches. After the completion of the course, these students' perspective of creativity changed with the incorporation of different mathematical actions which appear to be more mathematically creative. Undergraduate mathematics students' perception of what mathematical creativity is and how it changes during a course is of outmost importance. Many of these undergraduate mathematics students will become mathematics teachers for the next generation. The way they perceive creativity will dictate the methods they will use to develop it. Thus, the successful development of mathematical creativity and the interruption of any vicious circles that inhibit its development, depend greatly on the perceptions that future mathematics teachers hold. Thus, we need to invest in such studies, and most importantly invest on future mathematics teachers who will take on the responsibility to educate future minds.

Numerous attempts have been made to develop mathematical creativity. Changes of available means and tools also have an impact on the methods used for the development of mathematical creativity. A book written in 2022 would not have been complete if it did not address a main concern and shift in the educational approaches that occurred worldwide as a result of the Covid-19 pandemic. Undoubtably, the Covid-19 pandemic brought to the forefront the need and possibilities of online learning. This raises the question whether online teaching will restrict development of mathematical creativity and if there are any ways in which one could develop mathematical creativity through online learning. In the sixth chapter of this section, Monahan and Munakata investigated through interviews the way in which seven instructors tried to incorporate creative teaching and learning in an online course which was prompted by the Covid-19 pandemic. The course was designed to support students to see the connections between mathematics and creativity. The researchers discuss the affordances and limitations of the online environment. It appears that online learning which so forcefully entered all levels of education in 2020, will not only constitute a teaching environment which was dictated by the restrictions imposed by Covid-19 but, looking at it more optimistically, it may offer new possibilities for the teaching of students of all ages worldwide. Of course, it is likely, that different methods and approaches will be needed for different age groups and mathematical processes and products may also be different among these populations.

The fifth and final section of the book offers an overview of the book and concluding thoughts on application, implications, and future directions. The authors discuss indicators, stages, assessment, processes, and products of creativity in the light of the development/maturation of creativity across the three age groups 5–12, 13–18, and 19–23. The authors discuss what they feel is still needed in research by highlighting application of research to scholars and practitioners.

This book is an important resource. It provides a useful compilation of ideas, theoretical backgrounds, empirical studies which address mathematical creativity of the general population across different ages. The literature review chapters, empirical studies presented, and reflective chapters offer the potential to researchers, mathematics educators, policy makers, teachers, and students to go beyond what they may learn from isolated research articles. The chapters of this book facilitate the reader to explore the field of mathematical creativity, make connections, and feed the development of new studies and theories in mathematical creativity. I hope that this book will become a useful tool for mathematics education researchers, teacher educators, professional developers, teachers, and students to learn and nurture mathematical creativity and creativity in general.

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# About the Author

**Demetra Pitta-Pantazi** is Full Professor of Mathematics Education in the Department of Education at the University of Cyprus. Her research interests focus on students' cognitive development of mathematical concepts, problem solving, mathematical creativity, integration of ICT in mathematics teaching, mathematics curriculum, and assessment. She is a member of the scientific committee for the development of the mathematics curriculum and textbooks, which are currently used in all primary schools in Cyprus. She is one of the associate editors of *Educational Studies in Mathematics* and a member of the editorial boards of the *International Journal of Science and Mathematics Education* and the *Journal of Mathematical Behavior*.

# Part I
# History and Background of Mathematical Creativity

# Chapter 1
# Creativity and Mathematics: A Beginning Look

**Alane Jordan Starko**

I am, perhaps, a strange person to be writing the first section of a book about creativity and mathematics. I am not a mathematician. I am a teacher and a teacher educator who is fascinated by creativity, particularly creativity in schools. Over more than 30 years, I've had the opportunity to speak to thousands of students and educators about the nature and support of creativity. In scores of presentations, conferences, and classes, I've begun by asking the group to name individuals or endeavors they believed to be creative. In all those efforts, I've never had anyone name a mathematician or a mathematical idea. Not ever. They have named individuals whose work was grounded in math to be sure; often the first person named is Albert Einstein, a theoretical physicist who spoke the language of mathematics fluently. But when the general person-on-the-street envisions creativity, they are much more likely to think about artists, musicians, and inventors than mathematicians.

There are many reasons for this. Most children develop their concept of mathematics in elementary school. There, for many years, school-math entailed rows of calculations to be completed with maximum speed and accuracy. Math problems always had a correct answer, easily located in the teacher's version of the text, and the students' job was to replicate it. Anything that deviated from that path was not considered creative; it was considered a mistake. The problem, of course, is that memorizing number facts has little to do with actual mathematics.

When I began studying creativity and tried to envision creative mathematics in schools, I came face-to-face with the notion of math-is-not-calculations. Early in the process, I interviewed a mathematician friend who talked about beauty and truth in equations in terms reminiscent of artists, composers, or philosophers. In the years since then, I've learned more about what mathematics is and is not. To readers who are mathematicians, this is painfully obvious. But for the rest of us, it is essential to

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understanding that creativity is fundamental to mathematics. In 1968, Halmos explained the nature of mathematics by first describing some of the things it is not.

> As a first step toward telling you what mathematicians do, let me tell you some of the things they do not do. To begin with, mathematicians have very little to do with numbers. You can no more expect a mathematician to be able to add a column of figures rapidly and correctly than you can expect a painter to draw a straight line or a surgeon to carve a turkey. Popular legend attributes such skills to these professions, but popular legend is wrong. (p. 376)
>
> Mathematics—this may surprise you or shock you some—is never deductive in its creation. The mathematician at work makes vague guesses, visualizes broad generalizations, and jumps to unwarranted conclusions. He arranges and rearranges his ideas, and he becomes convinced of their truth long before he can write down a logical proof. The conviction is not likely to come early—it usually comes after many attempts, many failures, many discouragements, and many false starts (p. 380–81).

Like creativity in any other discipline, creativity in mathematics entails new ideas, new applications, new discoveries of beauty. It supports our understanding of the universe and inspires awe in those who see its implications. Sadly, many of us learned mathematics without either a sense of wonder or belief in the value of guesses, failures, or false starts.

Fortunately, mathematics education has progressed dramatically since the days of math=number facts. Still, the journey from early number concepts to creative mathematics is a complex one. The authors of this book intend to guide readers on that journey, considering the development of mathematical creativity as a process of maturation and growing sophistication over time. It entails understanding both the nature of mathematics and the nature of creativity. Here, we'll start with creativity.

## 1.1 What Is Creativity?

There are many definitions of creativity (e.g., Kaufman et al., 2017; Kaufman & Sternberg, 2019). Since the mid-twentieth century, most definitions have included two major criteria for judging creativity: novelty and appropriateness. To be considered creative, an idea or product must be new and appropriate to some goal. Random novelty without function, such as might be produced by my cats walking across the keyboard, is not sufficient. As the century continued, it was recognized that novelty and appropriateness must be defined within some environment. Sometimes definitions take aim at the processes involved. An early effort in this direction was made by Guilford (1967, 1988), who defined components of creativity within his Structure of the Intellect model of intelligence. His identification of divergent thinking (fluency, flexibility, originality, and elaboration) as a key element of creativity continues to be important in much creativity research today. More recently, Kounios and Beeman (2015) defined creativity as "the ability to reinterpret something by breaking it down into its elements and recombining these elements in a surprising way to achieve some goal" (p. 9). Here, the elements of surprise and goal directedness echo the two traditional elements of creativity. Simonton (2016) proposed that surprise itself become a third criterion. In that view, being new in a repetitious or mundane way does not define creativity—it requires something that is novel in a surprising way. At its most basic, creativity involves the generation of a new—and possibly surprising—product (idea, artwork, invention, etc.) that is appropriate in some context. It can range from the everyday creativity I exercise when devising a meal from the random contents of my refrigerator or making mosaic switch plate covers, often dubbed "Little c creativity," to the "Big C Creativity" of those whose work changes the direction of their disciplines.

### *1.1.1 What Creativity Is Not*

Though the basics can seem straightforward, popular concepts of creativity are often confusing and prone to mythology (see, for example, Benedek et al., 2021). Some of the difficulties are rooted in the breadth of creative activities in the human experience, some in the varied aspects of creativity addressed in any given research study, some in the awe we feel when faced with the transformative power of "Big C" creativity. While we no longer believe creativity originates in the work of muses, the sense of mystery can remain. The list of creative myths is long, but a few are worth addressing specifically.

#### 1.1.1.1 Creativity Does Not Occur in the Right Brain

Creativity is a complex activity, requiring many kinds of cognitive and affective processes: considering likely areas for activity, producing diverse ideas, selecting from among ideas, viewing ideas from multiple perspectives, linking to prior knowledge and experience, critiquing possibilities, etc. Like any complex activity, it requires the whole brain. It is true that in some creative tasks, highly creative people use the right hemisphere of the brain more than less creative individuals. But everyone who has a whole intact brain uses all of it when attacking creative problems, as documented in activities from musical improvisation to story generation, designing book covers, and traditional creativity measures. In fact, explorations of the neurobiology of creativity, including multiple neural networks and coordination across networks, is some of the most vibrant creativity research today (Abraham, 2018; Vartanian, 2019).

#### 1.1.1.2 Creativity Is Not the Same as Intelligence or Expertise

There are several possible relationships between creativity and intelligence, varying with the measures and definitions used (Sternberg & O'Hara, 2000). Creativity has been hypothesized as part of intelligence; intelligence has been hypothesized as part of creativity. They have been viewed as overlapping in varied ways, or as differing uses of the same cognitive processes. One popular hypothesis postulates a threshold effect, holding that a minimum threshold of IQ of about 120 is necessary for major creative contributions. Above that level, the correlation between creativity and intelligence is seen as limited. This does not suggest that higher intelligence limits creativity (it doesn't), but rather that above 120, other personal and environmental factors may be more important than additional IQ points. The threshold hypothesis, while still popular, continues to be debated, as research is conflicting, particularly when examining real-world creativity rather than standardized assessments (see, for example, Jauk et al., 2013).

Similarly, creativity is not the same as expertise. One can be very knowledgeable about an area without producing original ideas. Unlike intelligence, large amounts of expertise and/or experience can impede creativity, if they lead individuals to routinized problem solving or to become so entrenched in current knowledge that they no longer seek fresh perspectives. Sternberg and Lubart (1995) postulated an upside-down U relationship between creativity and knowledge, in which too little knowledge impedes creativity and too much knowledge can also impede creativity, if it leads an individual to believe they have no need to seek more information or new problem-solving methods. In such cases, it may not be the expertise, per se, that is problematic, but complacency that can set in when individuals believe their knowledge to be sufficient. Expertise plus continued questioning may be a different matter entirely—perhaps as evidenced by Sternberg himself, who continues to develop new theories in an academic career well into its fourth decade.

#### 1.1.1.3 Creativity Is Not Just for a Lucky Few

As noted earlier, creativity takes many forms. While few individuals make the "Big C" contributions that change their disciplines in dramatic ways, there are many opportunities for creativity in smaller professional contributions and in the innovations that make daily life easier. The fact that activities may be "Little c" level in terms of the discipline doesn't limit their opportunity for creativity. In Maslow's words, "a first-rate soup is more creative than a second-rate painting" (1968, p. 136). The universality of creativity may be most easily envisioned in its early stages. Budding creativity evidenced in childhood play crosses time and cultures, while its more mature manifestations are impacted by personal values, characteristics, and experiences.

#### 1.1.1.4 Creativity Is Not Just a Phenomenon in the Arts

For many people, creativity is most immediately associated with the arts. We recognize great painters, poets, composers, and choreographers as creative in their fields. Certainly, creativity is essential in the arts. But additionally, every field needs creativity to move forward. Without creativity, there would be no progress in science, no new literature, no inventions or technology, no problem-solving for our myriad cultural dilemmas. And, fortunately for this book, creativity is important for original theorizing and problem solving in mathematics. The need for creativity across all areas of human endeavor means creativity is not a luxury (or worse yet, "fluff") to be seen as an unnecessary intrusion on education. It is essential to all human progress, and thus, essential in schools.

### *1.1.2 Mathematical Creativity*

One of the great debates of creativity research is whether creativity in particular disciplines, for example, mathematical or musical creativity, represents unique constructs or is simply "general" creativity applied to different content (Kaufman et al., 2017). One model that attempts to bridge the gap is the Amusement Park Theoretical model (APT, Baer & Kaufman, 2017). The APT model conceptualizes creativity as having initial requirements common to all creativity, such as intelligence and motivation (like the entrance tickets to amusement parks), and then increasingly specific general thematic areas, domains, and microdomains, in which the characteristics and requirements for creativity may vary (like varied height requirements for different rides). For example, when considering Katherine Johnson's creativity, one might consider her overall intelligence and motivation, but also how creativity might operate in the general area of mathematics, the domain of early computer science, and the micro domain enabling space exploration in the mid-twentieth century.

However the two are related, the creativity-basics of general creativity undergird concepts of mathematical creativity. Philosophies of mathematics differ as to whether mathematics is discovered, like the nature of sound waves, or invented, like the telephone. Regardless, creativity in mathematics may be seen as having two faces: discovering mathematical facts and creating proofs to support the discovered facts. Just as science requires questioning and data, so mathematics requires exploration, problems, proofs, and generalizations. It searches for new ideas, new processes, and original solutions, and is a far cry from the textbook-driven rows of problems some students have experienced.

In many ways, mathematical creativity resembles models and descriptions of other types of creativity. Hadamard's (1945) description of processes used in mathematical creativity mirrored Wallas' (1926) more general four-stage creativity model that included *preparation* for addressing a creative problem; a period of *incubation* representing time away from conscious consideration of the problem; *illumination* or the "aha!" experience of a new idea; and *verification*, in which the new idea is tested. Mathematical problem solving can involve essential elements of divergent thinking: fluency (many solutions), flexibility (many approaches to solution), and originality. Karwowski et al. (2017) described mathematical creative problem-solving as supported by creative abilities, openness, and independence, characteristics associated with general creativity since MacKinnon's research at the Institute of Personality Assessment and Research (IPAR) beginning in the 1950s (MacKinnon, 1978). Most contemporary models of creativity can be considered systems models, that is, they view creativity as the result of complex interactions of cognitive and affective variables considered in context. For example, Amabile's Componential model (Amabile, 1988; Amabile & Pratt, 2016) was a primary influence in recognizing the role of motivation in creativity. She described the necessity of individual domain skills and knowledge, creativity-relevant processes, and intrinsic motivation within situations conducive to creativity. Grégoire (2016) applied this thinking to mathematics, suggesting that mathematical creativity can be supported by addressing three dimensions of creativity: expertise, original thinking, and intrinsic motivation.

## 1.2 How Does Creativity Develop?

This book's authors are particular in their definition of "development" as regards creativity.

> In this book, development is not considered to be the development of creativity in the classroom, as influenced by overt pedagogical decisions or carefully selected curricula. Instead, it can be equated with a maturation process, which should not be left completely to chance (Chap. 3).

That is, the book is focused on the ways mathematical creativity matures across time. While the authors are careful to distinguish this idea from the notion of developing creativity in the classroom, the definition I'm more likely to utilize, it is clear they do not intend that creativity be ignored or left to develop on its own. This is wise. Virtually all current creativity research recognizes creativity existing within a social and emotional context. Those contexts influence how—and if—creativity will be possible or be recognized. Csiksentmilhalyi's (1988) fundamental question of "Where is creativity?" recognized "Big C's" naissance in the interactions of a person (or persons), a domain (discipline) and a field, the social structure of the domain. In the case of mathematical creativity, such interactions might include a mathematician's personal characteristics, motivation, and creativity; knowledge of the domain; and her interactions with the gatekeepers of the profession, such as journal editors. Hennessey (2015) has described these factors at an earlier educational level, examining the interactions among student characteristics, teachers' characteristics, the culture of the classroom, and the larger surrounding culture. Even as any living thing needs supportive conditions to mature, so does creativity. In considering the development and maturation of creativity, it is essential to consider the circumstances and influences that support it.

### *1.2.1 Creativity Across Time*

Relatively few researchers theorize about the development of creativity over time. Vygotsky situated his view of creativity in his sociocultural analysis of human thought, emphasizing the role of social and cultural interactions in the development of thought. He believed creativity developed in three major stages: the creativity and symbolic play of childhood, the increasingly abstract thinking of adolescence, and consciously purposeful creativity in adulthood. In all stages, it is influenced by surrounding social interactions. In childhood, these could be adults helping a child engage in symbolic play, while in adulthood, surrounding social and cultural needs can give direction to creative thought (Smolucha, 1992; Vygotsky, 1967).

In many ways, Vygotsky foreshadowed Bloom's (1985) studies of talent development, in which Bloom and colleagues studied the processes and influences through which individuals developed high levels of accomplishment in various fields. Those studied were all highly successful: concert pianists, sculptors, research neurologists, tennis champions, Olympic swimmers, and prize-winning research mathematicians. While the role and type of creativity varied across such diverse domains, the trajectories of the careers examined all entailed the development of creativity, and there was surprising consistency in stages of development, particularly considered the wide range of talents studied.

First, the authors recognized the long periods of training and support necessary for exceptional accomplishment, regardless of initial individual abilities. Mature creativity does not grow without care and attention. They also identified patterns of beginning, middle, and later stages of talent (creativity) development, requiring different types of instruction and support. Initial stages of talent development entailed finding and falling in love with a discipline. It was a time of joyful discovery. The timing varied by field. Whereas young people often became engaged in music or sports at a very young age, prospective scientists or mathematicians might not discover their specific area of study until high school or college. Teachers during the early years of talent development, whenever they occurred, helped students experience delight in discovery and envision what the field might be. Learning was often playful and supportive of exploration. The middle years of development entailed more rigorous study mastering the basics of a discipline, often requiring a more expert teacher. Emphasis was generally on precision and accuracy. Later years of talent development, particularly for those aspiring toward "Big C" creativity, often required yet a different type of teaching, supporting young people in finding their own voice, questions, or challenges rather than replicating those of the past. These stages of talent development have been used as an organizing framework for gifted education and for supporting creativity developmentally (Olszewski-Kubilius et al., 2018; Starko, 2018).

### *1.2.2 Talent Development in Mathematics*

The mathematicians studied by Bloom and colleagues were winners of the Sloan Research Fellowship, awarded to early career professionals in recognition of their "distinguished performance and a unique potential to make substantial contributions to their field," suggesting significant creative potential (Alfred P. Sloan Foundation, 2022). They came from homes that valued intellectual activity and encouraged curiosity. However, the "early years" teachers for these students were found in middle or high school, when students were first exposed to the patterns and processes of mathematics, and experienced math as problem solving with the opportunity for varied procedures. It is interesting to consider whether this might have been different had students experienced more actual mathematical discourse in their early years. For this group, middle years' teachers were generally found in college, particularly when undergraduate students had the opportunity to take more advanced graduate classes. In those years, the style of teaching seemed less essential than the knowledge base of the teacher, the commitment of the students to spend hours mastering essential content, and the teacher's commitment to help them succeed. Finally, the later years of high-level talent development required what Bloom (1985) described as a "master teacher" (p. 524). Only a handful of these individuals were seen to exist in any given field, so being accepted as a student in such a program required both skill and support. Mentorship with a master teacher, typically in a doctoral or post-doctoral environment, allowed young mathematicians to work alongside those who were doing the research that expands the field. In this type of environment, high-level mathematical creativity developed most successfully. Of course, this succession of progressively more expert teachers is not the only path to mathematical creativity. Srinivasa Ramanujan, for example, is known for his extraordinary contributions to mathematics, developed largely in isolation. Still, even he required correspondence with other mathematicians to integrate his ideas with standard procedures and bring his work to the field. As we consider the development of mathematical creativity, from early explorations in number sense to the abstractions of mature creativity, it seems best to consider the concept of "development" in both senses: the maturation and growing abstraction that are the focus of the book, and the supportive environments and actions that can allow it to flourish.

## 1.3 About This Section

The first section of the book contains three chapters. Chapter 1 (Liljedahl) presents an introduction to mathematical creativity. It overviews theoretical perspectives on creativity grounded in mathematical problem solving and reviews the ways mathematical creativity has been measured. It also describes some of the ways creativity, or creativity studies, can be divided, including the "Big C" "Little c" categories, and studies that emphasize creative persons, processes, products, or press.

Chapter 2 (Chamberlin & Payne) first reviews the development of general creativity research over time. The following section focuses specifically on mathematical creativity. Of particular interest to those focused on mathematics education is information on early interest in creativity by mathematicians. This is a stark contrast to the stereotypes of creativity existing only in the arts, or of mathematics as comprised only of increasing complex calculations. The next section emphasizes the value of mathematical creativity and its limited representation in the curriculum standards that shape today's education. Chamberlin and Payne emphasize the development of mathematical creativity as focused on maturation, dynamic and changing over time. They also examine factors that influence mathematical creativity, such as intelligence/content knowledge and affective variables. This section includes an introduction to the "Five Legs" theory (Chamberlin & Mann, 2021), which describes five affective factors influencing mathematical creativity. The final section of the chapter examines the application of Rhodes' (1961) creativity categories of person, place, and process to mathematical creativity.

Chapter 3 (Chamberlin, Liljedahl, & Savić) begins with an operational definition of development of mathematical creativity and its relationship to mathematical curriculum rigor. Development is associated with the Kaufman and Beghetto's (2009) 4 c's (levels) of creativity, illustrating how mathematical creativity may mature across time. The authors review the relationship of Rhodes' (1961) 4 P's (person, process, product, press) to creativity and its development. Finally, they discuss barriers to developing mathematical creativity, particularly in schools. These include the focus and/or breadth of standards, limitations in teachers' content or pedagogical content knowledge, developmentally inappropriate materials, and pressure to teach to standardized tests. A particularly striking (and very familiar) description is "When teachers are forced to hastily cover a rather extensive list of mathematical concepts, ample time for mathematical creativity to emerge… [is] compromised" (p. 48–49). The description of basic structural barriers is both realistic and daunting. Finally, the chapter addresses some of the affective variables that impact the development of creativity, including additional information on the "Five Legs Theory" that focuses specifically on mathematical creativity (Chamberlin & Mann, 2021). The theory includes affective dimensions that parallel those often described in general creativity research. For example, Iconoclasm can be seen as a particular aspect of the more general characteristics of risk-taking and courage. Impartiality entails flexible thinking and willingness to examine problems from multiple perspectives. Inquisitiveness mirrors curiosity and openness to experience. Like so much of what we know about mathematical creativity, these factors mirror general creativity characteristics, with a particular mathematical spin.

Considering the inclusion and support of creativity in mathematics education has the potential to transform the mathematics experiences of school children. With that transformation comes the opportunity to build a cohort of individuals with the vision and desire to develop mature mathematical creativity. If we are wise, that cohort will be both larger and more diverse than those who have gone before. The path may take us several steps closer to the "schools of curious delight" that have been my professional aspiration (Starko, 2022). With that hope, read on!

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# Chapter 2
# Creativity in Mathematics: An Overview of More Than 100 Years of Research

**Benjamin Rott, Maike Schindler, Lukas Baumanns, Julia Joklitschke, and Peter Liljedahl**

Research on creativity, invention, and innovation originates in different scientific disciplines and different decades. This diversity in theoretical backgrounds and definitions can still be seen in recent research, especially in mathematics education (see Joklitschke et al., 2021, for a review on notions of creativity in mathematics education research). The goal of this chapter is to provide an overview of different strands in creativity research that have been influential in mathematics education. We will furthermore elaborate on trends and explicate why creativity research is going in the direction that it is today. As the terminology is not always consistent, in this overview, we include research on related concepts like "creativity," "invention," "innovation," etc.

## 2.1 Research on Creativity Originating in (Mathematical) Problem-Solving

In this section, we present research on creativity that originates in the study of problem-solving processes along the lines of influential ideas—a summary in chronological order is given in Fig. 2.1.

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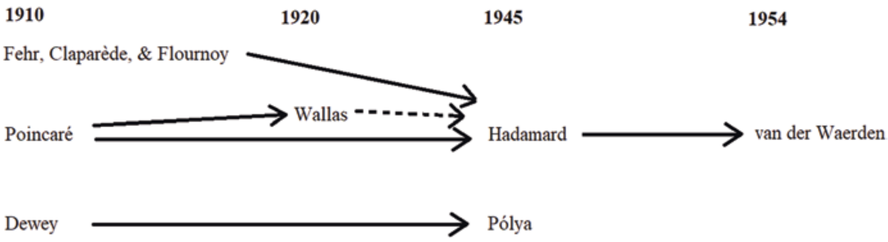


**Fig. 2.1** Chronological overview of the developments of different streams of creativity research from 1910 to 1954

The interest of mathematicians in the process of mathematical creation dates at least back to the beginning of the twentieth century. In 1899, in the first issue of the journal *L'Enseignement Mathématique*, Henri Poincaré wrote about intuition in mathematics and argued for more attention to this topic in mathematics instruction (Kilpatrick, 1992, p. 7). In 1908, Poincaré gave a presentation to the French Psychological Society in Paris with the title *L'Invention mathématique*, thereafter published in his book *Science et méthode* in 1908. In this presentation, he told the audience his famous story of a geological excursion on which he had a spontaneous idea that helped him solve a complex problem (Fuchsian functions) he was working on for some time:

> Just at this time, I left Caen, where I was living, to go on a geological excursion under the auspices of the School of Mines. The incidents of the travel made me forget my mathematical work. Having reached Coutances, we entered an omnibus to go some place or other. At the moment when I put my foot on the step, the idea came to me, without anything in my former thoughts seeming to have paved the way for it, that the transformations I had used to define the Fuchsian functions were identical with those of non-Euclidian geometry. I did not verify the idea; I should not have had time, as, upon taking my seat in the omnibus, I went on with a conversation already commenced, but I felt a perfect certainty. On my return to Caen, for conscience' sake, I verified the result at my leisure. (Poincaré, 1908, pp. 326 f.)

In his further writings about invention and creativity, Poincaré focused on the unconscious parts of processes of mathematical discovery. Such parts of creative processes are nowadays called "incubation" (i.e., the time in which the unconscious mind works on a problem), and "Illumination" or "aha!-moment" (i.e., a sudden idea or insight related to the solution of a problem). According to Poincaré, these two parts or phases of creative processes are preceded by a preparation phase (i.e., consciously trying to better understand and solve the problem) and should be followed by a verification phase (i.e., testing the idea from the illumination, turning it into a formal proof, etc.). As we will see, this Poincaré's story is now almost synonymous with the interplay of incubation and illumination and his conceptualization has often been picked up in research on creativity. It may have been possible that when he made such realizations, Poincaré did not deliberately make the connection between his ostensibly unconnected thoughts and their relationship to (mathematical) creativity.

Poincaré was not the only mathematician at the beginning of the twentieth century, who thought about mathematical creativity and invention. The mathematicians Henri Fehr and Charles-Ange Laisant, the editors of *L'Enseignement Mathématique*, assisted by the Swiss psychologists Théodore Flournoy and Édouard Claparède, published one of the first empirical studies regarding methods mathematicians use to solve high-level mathematical problems. They used a 30-item questionnaire to survey over 100 mathematicians on how they did mathematics, how they thought about the nature of their discipline, and how they achieved scientific progress therein. This questionnaire also included questions regarding the processes of inspiration ("Enquête sur", 1902; Fehr, 1905; Fehr et al., 1908—cited from Kilpatrick, 1992, p. 7). Poincaré saw the study of Fehr et al. as a confirmation of his conclusions (cf. Kilpatrick, 1992). However, Jacques Hadamard, another French mathematician, expressed some critique regarding the questionnaire study by Flournoy and Claparède, highlighting the lack of questions regarding failures and other topics as well as only "alleged mathematicians whose names are now completely unknown" (Hadamard, 1945, p. 10) as participants.

Hadamard did not only criticize others' ideas regarding mathematical creativity but did his own research. He had sent a questionnaire of his own to "first rate men" (ibid., p. 10)—including Henri Poincaré, George Pólya, Norbert Wiener, Hermann von Helmholtz, and Albert Einstein—to investigate creativity in the natural sciences. In 1943, Hadamard gave a series of lectures on mathematical invention at the École Libre des Hautes Études in New York City. These talks were subsequently published in his book *The Psychology of Invention in the Mathematical Field* (Hadamard, 1945).

Hadamard, probably influenced by theories from Gestaltist psychology, took Poincaré's ideas and turned them into a stage theory consisting of four separate stages stretched out over time. These stages are *initiation*, *incubation*, *illumination*, and *verification* (Hadamard, 1945). The first stage consists of deliberate and conscious work, trying to solve a scientific problem. When the solver is not able to come to a solution, s/he continues to work on the problem on an unconscious level, which is referred to as incubation. This second stage can last for any period of time—from minutes to weeks—and is inextricably linked to the conscious and intentional effort that precedes it (ibid.; van der Waerden, 1954). After the period of incubation, a rapid coming to mind of a solution, referred to as illumination, may occur. Thereafter, in the final stage, the correctness of the emergent idea needs to be evaluated. However, the verification may show that the solution revealed in the moment of illumination is, in fact, incorrect. For Hadamard (1945, p. 10; see also p. 64), such failures were as much a part of the creative process as the successes. The creative process should not be judged based on the correctness of the solution, which he pointed out when he criticized the questionnaire by Flournoy and Claparède:

> Moreover, the most essential question—I mean the one which concerns the genesis of discovery—suggests another one, which is not mentioned in the questionnaire [by Flournoy] though its interest is obvious. Mathematicians are asked how they have succeeded. Now, there are not only successes but also failures, and the reasons for failures would be at least as important to know.

> […] Who can be considered a mathematician, especially a mathematician whose creative processes are worthy of interest? Most of the answers which reached the inquirers [Maillet or Claparède and Flournoy] come from alleged mathematicians whose names are now completely unknown. This explains why they could not be asked for the reasons of their failures, which only first-rate men would dare to speak of. (Hadamard, 1945, p. 10)

Whereas Hadamard emphasized the unconscious aspects, especially incubation and illumination, Bartel Leendert van der Waerden (1954) adopted these ideas, focusing on the conscious parts of the mathematical process, especially the preparation phase in his book *Einfall und Überlegung—Beiträge zur Psychologie des mathematischen Denkens* [in English: Idea and Consideration—Contributions to the Psychology of Mathematical Thinking]. Van der Waerden distinguishes between *considerations*, which are products of conscious thoughts, and *ideas* as products of unconscious thoughts. Using the example of Archimedes' theorem "On the Sphere and the Cylinder," van der Waerden shows that large parts of this theorem can be found by considerations, with only two (unconscious) ideas being needed, which can be provoked by the conscious preparations (ibid., pp. 13 ff.).

Poincaré's ideas have not only been adopted by mathematicians, but also by psychologists. The most famous one in that area is the English psychologist Graham Wallas, who, in 1926, reflected upon problem-solving or creative processes in his book *The Art of Thought*. Building on Poincaré's work as well as the work of the German physicist Hermann von Helmholtz, Wallas describes four steps in creative processes, focusing on unconscious aspects of such processes. These four phases are preparation, incubation, illumination, and verification, which, of course, are the same phases that were later described by Hadamard (see above). The latter had known about Wallas' work, but based his work directly on Poincaré's ideas.

Even earlier, around the same time that Poincaré presented his ideas, the American psychologist John Dewey (1910) published the book *How We Think*. In this book, Dewey describes a general model of problem-solving processes, focusing—in contrast to Poincaré and Wallas—on conscious aspects. This work influenced George Pólya (cf. Neuhaus, 2001), who in 1945 published his first book, *How to Solve It*, in a series of several books on mathematical problem solving and discovery. Like Dewey, Pólya (1945) focused on conscious aspects of problem solving, specifically on heuristics. Pólya's four phases are very well known in the mathematics education community; for the sake of completeness, we name them here: (1) understanding the problem, (2) devising a plan, (3) carrying out the plan, and (4) looking back.

An overview of the work on creativity in chronological order is given in Fig. 2.1. It is important to note that—with the exception of Pólya's work—all studies referenced above followed the so-called "genius approach," meaning that creativity was seen as something that applies only to extraordinary people or at least renowned experts in their respective fields of science. However, already Hadamard (1945, p. 104) mentioned similarities in the processes regardless of the gravity of the findings: "between the work of a student who tries to solve a problem in geometry or algebra and a work of invention, one can say there is only a difference of degree."

To summarize, Poincaré's and Hadamard's insights into the subject of invention are an extensive exploration and extended argument for the existence of unconscious mental processes and the importance of not limiting research on creativity to (correct and elaborated) solutions.

The creative process, extended over time and being punctuated by the sudden appearance of a solution, has traditionally been researched through the a posteriori self-reports of this private and subjective experience (Hadamard, 1945; Liljedahl, 2013; Poincaré, 1952). More recently, however, Liljedahl (2013) has argued that illumination is largely an affective experience, which results in an observable emotive response.

Recent studies in the field of mathematical *problem solving*—having circumvented the genius approach (see below for details)—refer to creativity as an important factor for working on open-ended problems and problem fields (e.g., Haylock, 1997; Pehkonen, 1995; Levenson, 2011; Levenson & Molad, 2022; Molad & Levenson, 2020; Silver, 1995) or for teaching (especially in Japan: Neriage, cf. Becker & Shimada, 1997; Takahashi, 2021). However, most of those studies do not aim at further developing or better understanding the concept of creativity itself. For a recent study in which the problem-solving models of Dewey and Wallas are empirically tested, see Rott et al. (2021); amongst others, they come to the conclusion that in typical research settings—20–40 minute-processes—*incubation* and *illumination* are not suited to describe observed problem-solving processes. There are, however, studies in the field of *problem posing* that develop measures of mathematical creativity (for an overview, see Joklitschke et al., 2019). Such measures are addressed in the following section.

## 2.2 Quantitative Approaches to Measuring (Mathematical) Creativity (from Psychology)

Another approach to conceptualize creativity—not inspired by Poincaré's work—stems from psychology. In 1950, when he was the president of the American Psychological Association, Joy Paul Guilford (1950) complained that "the subject of creativity has been neglected by psychologists" (ibid., p. 444). Following this article, he did revive studying creativity in his scientific discipline. Guilford (1967) himself, with a background in research on intelligence, significantly contributed to this field by conceptualizing creativity as one component of intelligence and introducing the differentiation between convergent and divergent production: "Convergent production is in the area of logical deductions or at least the area of compelling inferences. Convergent production rather than divergent production is the prevailing function when the input information is sufficient to determine a unique answer. […] For example, if we ask, 'What is the opposite of HARD?'" (Guilford, 1967, p. 171). In comparison to convergent production, he describes divergent production as "a concept defined in accordance with a set of factors of intellectual ability that pertain primarily to information retrieval and with their tests, which call for a number of varied responses to each test item. […] [These] tests require examinees to produce their own answers, not to choose them from alternatives given to them" (Guilford, 1967, p. 138). Consequently, the act of recalling, as opposed to recognizing, can be considered a more rigorous demand cognitively and may serve as a metric for identifying creativity. In Guilford's theory, both divergent and convergent thinking contribute to creative thinking in a cyclic movement between both kinds of thinking (cf. Lubart, 2016).

Additionally, Guilford (1967, p. 169 ff.) developed the *Alternative Uses Task* as a measure of creative ability that was adopted by various researchers. This classic test asks what one can do with a simple object like a brick—think of as many uses as possible in a certain amount of time. For example, in addition to using it in building a wall, it could be used as a paperweight, a projectile, or a replacement for a hot water bottle when heated in an oven. Such answers as empirical data are then coded to identify four factors of divergent thinking, namely, *fluency* (the ability to produce a multitude of answers), *flexibility* (the capability to generate answers in various ways), *originality* (the ability to come up with unique answers), and *elaboration* (the level of details in answers). In Guilford's model, finding various and unusual ideas (i.e., fluency, flexibility, and originality) is attributed to divergent thinking, whereas giving details (i.e., elaboration) is linked to convergent thinking. As we will see, in mathematics education, divergent thinking is typically more often addressed than convergent thinking, up to a point where divergent thinking has become synonymous with creativity (Cropley & Reuter, 2018). Despite the almost obsession with divergent thinking, relative to convergent thinking, each are of importance in the emergence of mathematical creativity. In fact, Lee (2017, p. 996) refers to (mathematical) creativity as, "the confluence of divergent and convergent thinking" (see also Runco & Acar, 2012).

In 1974, Ellis Paul Torrance, building on ideas by Guilford, published the first version of the *Torrance Test of Creative Thinking* (TTCT, Torrance, 1974). Alongside very different ways of assessing creativity, the TTCT includes variations of the Alternative Uses Task (the "Unusual Uses Activities"). One of these different ways is the "Picture Completion Test" in which persons are asked to complete incomplete figures—for example, a given circle could be used to draw a round cake, a bicycle (using the circle as one wheel), an eye (with the circle as the pupil), a pig (with the circle as the nose), etc.

The idea of Guilford's Alternative Uses Task was adapted to mathematics education by Roza Leikin (presented) at PME 31 (Leikin & Lev, 2007). Instead of objects like a brick, they used mathematical problems that should be solved in as many ways as possible—so-called *Multiple Solution Tasks* (MSTs). For each solution, it is decided whether it is appropriate (a sort of *elaboration*). Then, the number *(n)* of appropriate solutions is counted to identify the score for *fluency*. The solutions are compared to each other to determine whether they rely on similar or different ideas, determining a score for *flexibility*. And, finally, solutions are compared within the solution space of a peer group and/or to solutions of experts to score their *originality*.

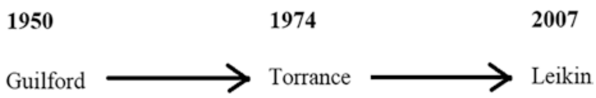


**Fig. 2.2** Overview of quantitative approaches to measuring (mathematical) creativity

Leikin's idea of using MSTs to measure mathematical creativity has been utilized by several researchers in the field. Various groups of researchers use MSTs to assess fluency, flexibility, and originality (and most often, elaboration is not addressed explicitly). However, scoring the dimensions or combining their scores to a general test score differs between such groups (e.g., Kattou et al., 2013; Pitta-Pantazi et al., 2013). Additionally, scoring systems are evolving over time (e.g., Leikin, 2016; Leikin & Lev, 2007).

In 2013 and 2016, this line of research came to prominence in the mathematics education community with a special issue in the journal *ZDM – Mathematics Education* (Leikin & Pitta-Pantazi, 2013) and a keynote by Roza Leikin at the 40th PME conference (Leikin, 2016).

This way of measuring creativity by analyzing products (i.e., written text and figures) with regard to categories inspired by Guilford is used not only with MSTs but also with open-ended tasks (Multiple Outcome Tasks, Leikin & Elgrably, 2022) and in problem-posing situations (e.g., Bonotto, 2013; Van Harpen & Sriraman, 2013; Van Harpen & Presmeg, 2013). A focus in problem-posing with respect to mathematical creativity pertains to the same constructs employed in problem solving (e.g., fluency, flexibility, and originality, but not often elaboration). In the context of problem-posing, fluency refers to the number of posed problems, flexibility refers to the diversity of posed problems (e.g., in terms of different mathematical ideas or strategies to be applied), and originality refers to the rareness of the posed problems with regard to all other problems that have been posed. See Fig. 2.2 for an overview of this line of research.

## 2.3 Sorting the Field

As mentioned earlier, researchers in the field of creativity (e.g., Hadamard, Wallas, etc.) predominantly focused on self-reports of exceptional individuals as well as analyses of their works like literary works, musical compositions, technological inventions, or scientific discoveries (cf. Silver, 1997). Retrospectively, this was called the *genius view* of creativity—or "Big C" (Kaufman & Beghetto, 2009): It addresses the creativity of eminent individuals, which is often associated with exceptional creative contributions, that is, ideas or products that change the perception of the world (Sriraman, 2009; Sriraman et al., 2014). A typical example of the Big C-perspective on creativity in mathematics is Poincaré's (1948) work on Fuchsian functions (Sriraman, 2009).

However, researchers have turned away from the assumption that only geniuses can be creative and have focused their attention to ordinary or everyday creativity—or "little c," which means "everyday creativity" (Kaufman & Beghetto, 2009, p. 1; see also Feldhusen, 2006; Pehkonen, 1997; Sriraman et al., 2014). In the little c perspective on creativity, student solutions are considered creative if they "are unique and novel to the students in their particular environments" (Sheffield, 2018, p. 408). Sriraman et al. (2014) summarize the difference as follows:

> For a professional artist, some new, ground-breaking technique, product, or process that changes his or her field in some significant way would be creative, but for a mathematics student in lower secondary school, an unusual solution to a problem could be creative (Sriraman et al., 2014, p. 110).

In 2009, Kaufman and Beghetto (2009) summarized the research on what they call Big C and little c creativity and extended this differentiation by adding two further aspects, *mini c* and *pro c*. "Mini c is defined as the novel and personally meaningful interpretation of experiences, actions, and events" (p. 2), which emphasizes the creativity in the learning process more than little c, where students' creativity is scored based on their solutions to problems and outcomes. Mini c highlights the individual learning process taking place in its sociocultural context. Mini c refers to the personal level like drawing a picture that might even be a re-invention of previous work but is meaningful to the person and their learning process.

On the other hand, Pro c is a "category for individuals who are professional creators, but have not reached eminent status" (p. 4). Pro c takes into account the fact that for Big C, it is often required to reach an eminent status, and often this is only reached in a posthumous evaluation (ibid.). Therefore, in real-world practical applications, Pro c addresses professional creativity and outstanding contributions. Pro c refers to productions in professional domains or jobs such as the novels of a professional writer that may be endorsed in the professional discourse, for instance, by editors. On the other hand, Big C refers to creative contributions that receive large-scale recognition such as Einstein's contribution to physics.

In mathematics education, researchers often draw on the difference of Big C and little c to denote the difference of eminent individuals and those of students (e.g., Schindler et al., 2018: Schindler & Lilienthal, 2020; 2022). In mathematics education, when school or university students' creativity is addressed, we perceive a predominant trend that little c is addressed, for example, when MSTs or open-ended problems are addressed (e.g., Leikin & Lev, 2007; Levenson, 2011).

Within this aim to investigate students' creativity, a further distinction can be made, which relates to the question: What is creativity, or what is it that is considered to be creative? Rhodes (1961) identified four perspectives on creativity, the so-called "4P of creativity," which were laid out for mathematics education by Pitta-Pantazi et al. (2018), who furthermore categorized research on mathematical creativity around the classification provided by Rhodes. The 4P are understood as follows: Creativity can address four different aspects, a feature of either a person, a product, a process, or press (the latter of which addressed the environment). With creativity of a person, the person themselves and their characteristics are focused:

The predominant question is: Is this person creative? With creativity of a process, processes are in focus: The question arises if the process is a creative one. Often, prototypical phases of creative processes—such as the ones identified by Poincaré (1908) and Wallas (1926), i.e., preparation, incubation, illumination, and verification—are regarded and considered requisite for creative product to emerge. If the process involves incubation and illumination, mathematical processes are often regarded creative.

By creativity of a product, it is evaluated if the outcome, the result of a process is creative. Here, the question arises if the product is new or innovative, if it is original. And finally, with press, the emphasis is on the interplay of the individual and their environment, which draws on the assumption that persons in a vacuum cannot be creative, but rather respond to external factors. All these four aspects can be in focus in mathematics education research—also in a little c perspective: Regarding creativity, some researchers evaluate students' products, others investigate students' processes, while again others evaluate the interplay of individual and group creativity at school. However, what already Rhodes mentioned six decades ago still holds true in mathematics creativity research today: The interplay of these four aspects "causes fog in talk about creativity" (Rhodes, 1961, p. 307). Even if products are being evaluated, often conclusions are drawn on the persons' creativity, and sometimes inferences are being made about creative processes although the products were evaluated—not the processes themselves. This "fog" or confusion hints at the need for researchers in mathematics education to be clear about what it is that they consider being creative (or not), and to communicate clearly what aspects of creativity and what interplays they investigate. Pitta-Pantazi et al. (2018) use Rhodes' categorization and apply it successfully to mathematics education by showing how existing studies can be mapped based on Rhodes' categories, addressing processes, persons, press, and products.

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# Chapter 3
# Mathematical Creativity and Society

**Scott A. Chamberlin and Anna Payne**

## 3.1 A History of Mathematical Creativity

To understand the construct of mathematical creativity with a modicum of depth, it is informative to make sense of general creativity research conceptions first and then to realize its applications to mathematics. This book represents thoughts of researchers who view creativity through the lens of development and the tenets conceptualized in this chapter provide a basis for interpreting literature and ideas throughout the book.

## 3.2 Overview of Creativity Research

In this section, a concise overview of seminal research in the field of educational psychology is first presented. Subsequently, the research is applied to the domain of mathematics, in an attempt to elucidate the editors' conception of the construct known as mathematical creativity. The review of literature on general creativity is purposefully abridged because most readers of this book have substantial knowledge of the general construct.

Albert and Runco (1999) provide a comprehensive overview of the history of creativity, tracing it at least 2000 years to current. From their perspective, much of the formal emphasis on creativity research dates to 150–200 years ago. Albert and Runco mention that the construct of creativity was highly driven by a Western

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(European and American) influence. In specific, much of the early writing about creativity was Christian-oriented and pertained to evidence of creativity mostly in men. As the field progressed into the early twentieth century, visionaries such as Wallas (1926) provided models of why and how the creative process unfolds. In specific, Wallas suggested that the emergence of creative product(s) was a result of a four stage process which entailed: preparation, incubation, illumination, and verification. The stage that notoriously receives the largest amount of attention is that of incubation, because the stage suggests that creativity does not often comprise any one specific 'aha' moment or can be attributed to some sort of an epiphany. Instead, the emergence of creative product is more likely a result of considerable investment of cognitive or mental energy (Lykken, 2005) that may come from a struggle to make sense of an enigmatic situation.

Following several decades of discussion based largely on theory and mitigating the effect of empiricism about how the creative process unfolds, the next era of creativity research originated in the 1950s to 1960s. At this time, Guilford (1950) and Torrance (1966) were at the forefront of utilizing principles of psychometrics to quantify creative output. In specific, Guilford and Torrance each worked to develop respective instruments so that psychologists could ascertain the degree of creative thought. The next era of creative scholarly activity resulted in scholars such as Simonton (1988, 1999) and Sternberg and Davidson (1995) developing theories of creativity to provide insight with respect to what might be expected during the creative process. Of particular importance during this era was the contribution by Amabile (1983), in which she discussed the various components of creativity, called the *Componential Theory of Creativity*. Though this theory originated with a lens of organizational leadership, Amabile's theory ultimately was recognized in the more general domain of educational psychology as one with particular significance in describing the creative process. Amabile's work may have further served to provide insight about the indicators of creativity and/or how it might be manifested as a mathematical process. Imai (2000), traced initially to Hollands (1972) summarized the four indicators of creativity, fluency, flexibility, originality, and elaboration, in general literature and applied it to mathematical cognition.

## 3.3 An In-Depth Look at Mathematical Creativity

Imai (2000) and Hollands's (1972) work would have not occurred without substantial efforts by previous researchers in mathematics education. Just before educational psychologists' interest in creativity was born, dating at least to Chassell (1916), Poincaré (1908) is credited with the nascence of mathematical creativity discussion. Several decades later, Hadamard (1945) and Poincaré (1956) were instrumental in forwarding the initial discussion that mathematics, as a domain, could realize elements of creativity. Prior to their dialogue, some appeared to hold the belief that creativity had exclusive application to arts (creative and performing). Hadamard used terminology such as innovation in reference to how mathematics may apply to creativity. Poincaré suggested that intuition (Browder, 1983) was a requisite component for creativity to emerge. Thankfully, this notion that mathematicians were individuals engaged in rather mechanistic computations, as if they were a detached, rather emotionless computer on autopilot, was not accepted by many individuals in the world of creativity research.

Carlton's (1959) seminal dissertation on 14 outstanding mathematicians provided 21 characteristics, which are, to this day, enlightening. Krutetskii (1976) corroborated that mathematicians have an appreciation for aesthetics or beauty in solutions, thus suggesting that mathematicians do invest attention in creative process and product. By early to mid-1990, the topic of creativity had become a mainstay in professional organizations and academic journals. As an example, Silver (1997, 1994a, b) was the first to popularize the notion that problem-solving may be a vehicle to elicit creative thought, and a critical piece to problem-solving may not merely be solving problems, because the very act of posing problems might have proclivities to elicit creative mathematical processes and subsequently products. With the advent of a new millennium, the construct of mathematical creativity had realized considerable structure, through publications such as Liljedahl and Sriraman (2006), Sriraman's (2004), and subsequently, Nadjafikhah et al.'s (2012) commentary. In their work, they highlighted creativity and promoted the construct as a central component of mathematics education and mathematics psychology. The first publication, by Sriraman and Liljedahl, details a discussion that they had in which many (mis)conceptions about creativity were elucidated and clarified. As an example, one psychological principle discussed is the 'Aha' moment in relation to mathematical creativity. The 'Aha' moment in psychology is ostensibly an instantaneous moment when one (e.g., a problem solver) trying to make sense of a concept, has an epiphany or realization in which an illumination enables the mathematician to conclude work. In the final two publications, Sriraman, as well as Nadjafikhah, Yaftian, and Bakhshalizadeh, the authors provide desperately needed structure to the discussion of what mathematical creativity is. Each of these publications served to assist researchers in focusing their efforts in theoretical scholarly contributions and instrument design rather than to dictate specifically what mathematical creativity is. In fact, with the exception of Balka's (1974) seminal work in instrumentation to quantify mathematical creativity and Aiken Jr.'s (1973) comprehensive overview of mathematical creativity and the factors that may serve to indicate its presence, the work of the aforementioned scholars (Liljedahl, Sriraman, Nadjafikhah, Yaftian, and Bakhshalizadeh) has likely advanced the field considerably. More recently, efforts such as those by Singer (2018), Kattou et al. (2016), Goldin (2017), and Haavold (2018) have provided additional insight regarding conclusions reached based on empirical data and theory analysis to support conclusions about mathematical creativity.

## 3.4 Value of Mathematical Creativity

The value of mathematical creativity likely varies by nation. There are often two entities that may influence curricula and content taught in schools. First, leaders in commerce and industry may possess some indirect influence. Second, educational standards often dictate more specifically the curricula and what is taught in schools, as they are utilized as a guide for day-to-day instructional decisions. Prior to this discussion, it is incumbent upon the book editors to disclose that creativity has at least three notable purposes and another endless list of reasons for its importance.

First, creativity is the cornerstone of innovation. Without creative processes and products, the world would almost literally stand still in an era. Were this the case, the most advanced versions of automobiles, cellular phones, medical advances, and the creative arts would not have been realized. Speaking of mathematics, if mathematics advanced no farther than it is today, much knowledge would have already been generated. Nevertheless, future innovations that are predicated in the discipline of mathematics, such as engineering, physics, chemistry, and other STEM disciplines would not see advances. Second, and as a result of the stagnation of mathematical creativity, a discipline has outlived its usefulness when advances, through creativity, cease to be made. Third, creativity is valued simply for the enjoyment of doing mathematics. As Maslow (1943) illustrated when discussing motivation, once life's needs are met (e.g., air, water, food, shelter), an appreciation for aesthetics is realized. Words such as creativity, beauty, and aesthetics are nearly synonymous when discussing advances in mathematics. Third,

Following an exhaustive search, it was found that electronically accessing individual national standards documents for all countries became an impossible task. Hence, a central database of mathematics standards was utilized in which standards for each country was provided. This database was found at: http://timssandpirls.bc.edu/timss2015/encyclopedia/countries/. Two caveats are issued prior to sharing results about countries' emphasis on mathematical creativity. First, overviews were written by educational experts in respective countries, so the standards are open to interpretation in some countries more than others. Moreover, individuals constructing the summaries may have been instructed to concentrate specifically on content. Second, scant space was provided for the overview of national standards, so the synopses of them was greatly abbreviated relative to the size of the actual documents. Hence, the emphasis on mathematical creativity may not have been accurately reflected. A third explanation for the general paucity of emphasis on mathematical creativity exists. That is, international assessments are notorious for assessing mathematical performance and may rely on a regurgitation of standard, highly efficient mathematical procedures. Hence, in countries attempting to have high marks in international comparisons, creativity may intentionally be a casualty, so that acceleration may be achieved without distractions, though international data may not reveal this relationship empirically. In short, for countries that perform at the top of such international comparisons in Trends in International Mathematics and Science Study (TIMSS) and Programme for International Student Assessment

(PISA), creativity simply may not be a chief emphasis in grades K-12. Much of the emphasis on mathematical creativity, it may seem, originates at the university level. In fact, TIMSS appears to make no such claims that creativity is assessed. PISA, on the other hand (https://www.pisa.tum.de/en/domains/creative-thinking/), will be assessing mathematical creativity for the first time in 2022, but it will be done with a survey, rather than through having students create innovative solutions. Sadly, it may be the case that mathematical content on international assessments does not account for mathematical creativity and so mathematics instructors are not advised to facilitate it. Seeley (2004) made such a claim in reference to standardized assessments in the United States. An old adage in many of the westernized nations is, “If you assess it, they will teach it!” If there is any truth to this adage, then the repudiation of creativity assessment may be the reason that it is often neglected in national mathematics standards documents.

Listed in Table 3.1 are top performing countries in international comparisons, in order of their performance. The number of times that mathematical creativity was mentioned in the elementary and lower secondary documents is provided in parentheses.

Data from select regions and cities were removed as such geographical locales do not develop national standards

Despite what may be theorized as an almost patent neglect of mathematical creativity among top performing countries, literature in creativity education advocates for the infusion of it in day-to-day curricular decisions. This request for it may be predicated on the theorized by-products associated with it. For instance, it may be postulated that mathematics curricula heavy in creative demands may facilitate the development of students with flexible thinking, an appreciation for the aesthetics of mathematics (Chen, 2017; Lingefjärd & Hatami, 2020), and in the end may promote innovative thinking. Some industry experts feel that innovative mathematicians are more important than mathematicians capable of routinely completing voluminous

**Table 3.1** International mathematics assessment data

| | Grade 4 (TIMSS 2019) | Grade 8 (TIMSS 2019) | Grade 9 PISA (2018) |
|---|---|---|---|
| 1 | Singapore (0) | Singapore (0) | Singapore (0) |
| 2 | South Korea (1) | China-Taiwan (0) | China-Taiwan (0) |
| 3 | China-Taiwan (0) | South Korea (1) | South Korea (1) |
| 4 | Japan (1) | Japan (1) | The Netherlands (0) |
| 5 | Russia (0) | Russia (0) | Japan (1) |
| 6 | Northern Ireland (3) | Ireland (0) | Switzerland[a] |
| 7 | England (0) | Lithuania (0) | Poland (1) |
| 8 | Ireland (0) | Israel (0) | Belgium (0) |
| 9 | Latvia[a] | Australia (0) | Estonia (0) |
| 10 | Norway (0) | Hungary (0) | Canada (0) |

[a]Standards not listed
2019 TIMSS Results retrieved at: https://nces.ed.gov/timss/results19/#/math/intlcompare
2018 PISA Results retrieved at: https://nces.ed.gov/surveys/pisa/pisa2018/pdf/PISA2018_compiled.pdf

amounts of computations because computers can likely complete such work in a more precise and expeditious manner.

Given the almost universal neglect of mathematical creativity in elementary and secondary grade standards, it is not surprising that educational resources, in time and money, are scant in supporting its development (Plucker et al., 2004). When such a critical element of mathematics is wholly absent from standards documents and funding is not provided to facilitate it, professional development is often absent. It might be a safe assumption that mathematical creativity is often discussed all to infrequently in teacher preparation courses as well.

## 3.5 Organizational Framework of the Book

Of substantial importance to any academic book is its organizational framework. The editors of this book invested considerable time discussing the organizational framework because introducing the idea of mathematical creativity having a developmental component is rather a novel idea in the mathematics education literature. However, upon considering the influence of development in mathematical creativity, Drs. Chamberlin, Liljedahl, and Savic were surprised to learn that such a discussion had not ensued among scholars. When looking at the intersection of mathematical creativity and development, most scholars use the term development to refer to the development of mathematical creativity (Tubb et al., 2020), rather than mental development being a factor in the equation of mathematical creativity. Though not perfectly synonymous with the term development, another term that could be used is maturation. An operational definition of maturation for this book is, "a positive change in mental capabilities that leads to an enhanced penchant for advanced thinking." In substituting the term, maturation can have highly positive effects on the prospect of mathematical creativity; conversely, the lack of maturation or maturation at an abnormally slow rate, could have deleterious effects on mathematical creativity (process or product). In considering mathematical creativity in relation to development, several tenets are postulated.

Mathematical creativity is dynamic or ever-evolving. As mathematicians age, mature, and develop, the expectations regarding what constitutes mathematical creativity also advance. Hence, analysis of creativity becomes complex in upper levels of schooling. Mathematical creativity is, in part, influenced by other factors that are related to development. For instance, intelligence and cognition, as well as affect play a role in creative output in mathematics. Further, mathematical creativity can be viewed as composed of person, process, and product (Rhodes, 1961).

### *3.5.1 Mathematical Creativity Is Dynamic*

Unlike some domains, mathematics is one in which concepts build from year to year and they become increasingly complex as learners age (Gravemeijer et al., 2017). As an example, when learning to reason with rational numbers (Kainulainen et al., 2017), grade one students may learn initially about what constitutes a fraction and the relationship of part to whole. Subsequently, in grade two, students may learn how to combine parts (add) or remove a part from a whole (subtract) fractions. As they progress into later elementary and early secondary grade levels, students may learn to move from fractions to decimals to percentages. However, in other domains, such as humanities or social studies, the topics decided upon in educational standards may have little reason regarding why they are introduced at the respective ages.

Moreover, not only do content demands and mathematical concepts become increasingly complex as students age, but problem-solving demands grow in complexity precipitously (Gravemeijer et al., 2017). For instance, at very young ages, students are provided with problems to solve. However as they age and enter the tertiary system of education, problem solvers might start to generate their own problems and solve them with aids such as graphing calculators or computers. This responsibility of generating and answering one's own (novel) questions, may be consistent with future vocational demands, as they progress beyond formal education. Other demands placed on mathematicians may be, but are not limited to, critical thinking, creative thinking, problem-solving, data analysis, innovation, and mathematical modeling (Kozlowski et al., 2019; Vorhölter et al., 2014; Wagner, 2014).

The point about mathematics is that it grows in intricacy as students age. Understanding algebra, as an example, is predicated on one's ability to reason abstractly and such demands may not be requisite in younger grades. Geometry too, as does trigonometry, calculus, and a host of other university-related mathematics courses, has a component of abstraction involved. Hence, the target regarding what constitutes mathematical creativity seems to be moving as students age. A contribution that may be considered creative in grade three, for instance a particularly innovative method to divide fractions, may not hold such promise in algebra when it comes to making sense of factoring using the quadratic formula. This is likely due to that which constitutes creativity in the respective foci, as people age in relation to a model referred to as the Four-C model of creativity (Kaufman & Beghetto, 2009). In the Four-C Model of Creativity, there are four levels comprising (1) Mini-C, (2) Little-C, (3) Pro-C, and (4) Big-C. As individuals age, a developmental component is overlaid with the domain and contributions to fields become increasingly more sparse and less common. Hence, in looking at mathematical creativity, the expectation regarding what constitutes creativity is altered subtly from year to year to year.

The Four-C model is one initially created in the world of Educational Psychology. However, its applications are particularly salient in the domain of mathematics, as explicated previously. As the book develops, the understanding of mathematical creativity and its relationship to development and maturation will be comprehensively elucidated.

### 3.5.2 Mathematical Creativity Is Influenced by Affect, Intelligence, and Other Constructs

To minimize mathematical creativity to exclusively development is shortsighted. However, in looking at developmental, the relationship between development and other psychological constructs such as affect (students' feelings, emotions, and dispositions), intelligence which is not merely measured by intelligence quotient, and other constructs, is apparent. Affect, as an example (Chamberlin & Mann, 2021), appears to show some marked connections to mathematical creativity. Empirical evidence may suggest that affect that students bring to learning episodes can greatly influence the degree of mathematical creativity that is elicited (Akgul & Kahveci, 2016). Amabile et al. (2005) substantiated this claim in vocational settings. In fact, Chamberlin and Mann theorize that at least five factors, comprising Iconoclasm, Impartiality, Investment, Intuition, and Inquisitiveness, influence students' receptiveness and, subsequently, creative output.

To date, the Five Legs Theory of Creativity (Chamberlin & Mann, 2021) appears to be the only such theory that is specific and was designed with mathematics learning as the focus. Each of the Five Legs pertains to students' emotional states that can be altered by intentional teaching components. In specific, the two most salient teaching components are types of activities and classroom environment created. Iconoclasm pertains to a problem solver's penchant to challenge commonly accepted mathematical solutions, particularly when provided by an authority figure such as a teacher, mentor, or even an anonymous person on a website, while solving problems. Impartiality pertains to one's ability to be open-minded in developing mathematical solutions. In being Impartial, one does not have allegiance to any particular method or approach to solving a problem. Investment pertains to one's emotional dedication or commitment towards advancing content knowledge in a field, satisfying a curiosity, deepening one's own mathematical content knowledge in a field, or even resolving a discrepancy, such as with cognitive dissonance (Olson et al., 2006), that was previously unresolved. Intuition pertains to one's drive towards a particular response or solution, even if it is not one that is commonly endorsed. High levels of Intuition may result in increased levels of persistence. Similar to Intuition, Inquisitiveness (the Fifth Leg of Creativity) pertains to interest which may result in high levels of engagement. In looking at all Five Legs of Creativity, they should more appropriately be perceived as emotional states that may enhance the likelihood of creative process and subsequently creative output or product. When one or more of the Five Legs is at a low level, the likelihood of creativity emerging in mathematics is said to be compromised. Conversely, when multiple Legs, or better still all Five Legs are at high levels, the proclivity for mathematical creativity emerging is said to be high.

As well, intelligence plays a role in mathematical creativity. The debate lingers with respect to what degree of intelligence is prerequisite for mathematical creativity to emerge. Some hypothesize that specific intelligence is necessary for creativity to occur. The Threshold Theory of Creativity (Jung et al., 2009; Welter et al., 2016) maintains that an IQ of at least 120 is necessary for creative process and output to be realized. Conversely, others maintain that the Einstellung Effect (Ellis & Reingold, 2014) holds merit. In the Einstellung Effect, it is theorized that having too much experience and/or intelligence may impede one's penchant for creative responses because one's propensity to derive novel responses is hindered by preconceived notions of how to respond. It appears as though a combination of the two positions may be the most logical perspective through which to view creativity. It is possible that in mathematics, a specific level of intelligence, whatever that level is, must be apparent before creative output can occur. By the same token, exceedingly high levels of experience and/or intelligence may inhibit mathematical creativity, because individuals advanced in their field may rely on pre-established conventions to respond to problem-solving situations. Additional psychological constructs most certainly influence the likelihood of mathematical creativity emerging. For instance, content knowledge, advanced intellect in a specific domain, and expertise in solving problems appear to influence one's effectiveness in solving problems, though they have not been studied in sufficient detail to garner any conclusions. A final consideration in this chapter, regarding components that may influence mathematical creativity, with respect to development, pertains to the person, process, and product, from the Four P Model of Creativity (Rhodes, 1961).

### *3.5.3 Final Factors That Influence Mathematical Creativity*

The final factors mentioned in this book relevant to mathematical creativity are person, process, and product. Each of these components has multiple sub-constructs. As an example, a person may comprise, but is not limited to, individual characteristics and attributes such as affect, thinking styles (such as the ability to think in a divergent manner and/or to think flexibly), and personal behavior, as well as habits and traits. Critical to mathematical creativity is the person, as without the person, creative process and product would not occur. The person is the vehicle from which creativity originates. Moreover, the person is multifaceted as the person brings an ostensibly endless list of experiences and approaches to solving mathematical problems. As explicated earlier, it would appear as though having advanced experience, content knowledge in a domain, or intellect would significantly enhance the prospect of creative output. What does not appear to be discussed in considerable detail by Rhodes (1961), when considering the problem solver, is the prospect of a person interacting with one or more people, in seeking a solution. In this respect, a component of social mediation may serve to enhance or inhibit the prospect of creative mathematical outcomes. Without person, there is no process or product. Process is often discussed next, as it is an antecedent to the outcome, or product.

Process pertains to the acts in which a problem solver engages to generate a creative product. With respect to process, Rhodes (1961) was a believer in early work by Wallas (1926), and endorsed the four stages as comprising the following: preparation, incubation, illumination, and verification. In this process, preparation was a requisite and rather a self-explanatory construct-stage. In short, ample time is needed for the creative person to ruminate on a problem and to gather information in preparation to solve a problem. Though incubation, the second stage, may be the one most often attributed to Wallas, illumination may be the stage that generates particularly novel and useful products or solutions. Incubation, however, is not to be overlooked because it speaks of how creative solutions are not often miraculously derived instantaneously, especially when the mathematical problem is significantly challenging. Illumination pertains to the process of gaining some insight relative to a solution and can be considered the start of the culminating process. Verification too, has its role in the creative process, as it may be the catalyst that encourages problem solvers to refine their solution. The product is of primary importance as process and product are often the two components discussed on a regular basis in mathematics education. Products too, as opposed to the process used to develop the product(s), are often what is used to measure the creative output. In effect, the product becomes the artifact of the prospectively creative work because it is the process documented in physical form. Hence, products maintain a crucial role in the relationship of mathematical creativity and development. In fact, products may often be viewed by mathematics educators as the sole indicator of mathematical creativity, although it could be argued that mathematics educators have realized the importance of process (Brownell, 1947) long before many content peers did.

## 3.6 Conclusion

Mathematical creativity has been viewed through multiple lenses. A cursory review of mathematical creativity illustrates a progression of interest in research topics that initially began with theories and theoretical writings to explain (mathematical) creativity (e.g., Hadamard, 1945; Poincaré, 1908, 1956). Subsequently, instrument development was undertaken, with a flurry of publications in the 1960s and 1970s (Balka, 1974; Buckeye, 1970; Evans, 1965; Foster, 1970; Krutetskii, 1976; Manville, 1972; Meyer, 1970; Prouse, 1967; Spraker, 1960) to quantify and ascertain mathematical creativity scientifically.

Following the instrumentation development stage of the 1960s–1970s, the emphasis in mathematical creativity returned to a theoretical one. Silver's interest in problem posing as a vehicle to promote mathematical creativity garnered much attention and generated discussion among academics. Also, scholarly efforts were directed towards types of curricula that may elicit mathematical creativity. Chamberlin and Moon (2005), for instance, promoted the idea of using mathematical modeling activities to facilitate mathematical creativity. Similarly, Scherer et al. (2019) looked at the learning of computer programming as a means to develop mathematical creativity. As the book progresses, the importance of development will be explained in an attempt to learn about another piece of the mystery referred to as mathematical creativity.

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# Chapter 4
# Organizational Framework for Book and Conceptions of Mathematical Creativity

**Scott A. Chamberlin, Peter Liljedahl, and Miloš Savić**

## 4.1 Organizational Framework of Book

Of particular importance in this book on mathematical creativity and its relationship to student development is an operational definition with respect to what constitutes development. Specifically, mathematical creativity and development is operationally defined in this work as, "In relation to mathematical creativity, it is the authors' position that development can have rather dramatic effects on creative process and product, as can myriad other factors such as affect, classroom environment, and curricula. At the heart of development, in this book, mathematically creative processes are of central importance." More specifically, in this book, development is not considered to be the development of creativity in the classroom, as influenced by overt pedagogical decisions or carefully selected curricula. Instead, it can be equated with a maturation process, which should not be left completely to chance. Incidentally, this notion of creativity being something that teachers magically manipulate with some magical curricula is likely the most common conception of it when searches of academic databases transpire. As a caveat, the two conceptions of mathematical creativity, relative to development, are closely related. To clarify, development pertains to the maturation of learners and their corresponding ability to engage in mathematically creative processes, which typically results in creative products.

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For the sake of convenience, and because the aforementioned databases often partition pupils into one of several categories, three age categories have been adopted to guide the arrangement of the book:

- Elementary: ages 5–12
- Secondary: ages 13–18
- Tertiary: ages 19–23

Given the fact that conclusions reached in this book are predicated almost exclusively on empirical research in mathematics education, one admonition to readers is that some studies span more than one age category. As an example, a study may have been conducted in which students in grades 6, 7, and 8 are involved, thus including students age 12, 13, and 14. In such cases, authors have been asked to disclose the fact that the study may include more than one age category. In general, readers should use discretion in interpreting such passages and would be well-served by applying the findings to (a) both age ranges, or (b) the predominant age category of the participants. In the example above, since roughly two-thirds of students are in the Secondary age category, the findings might likely apply there foremost.

Another component to consider is that mathematics rigor may not advance in a linear fashion from grade to grade. Though experts in curriculum design and authors of standards may imply that it does, logic suggests otherwise. The amount of review from year to year may directly influence the extent of new material that can be introduced. It may also be postulated that the two most likely explanations for increasing rates of failure in mathematics is a consequence of reduced amount of review, which directly relates to the amount of new information that is introduced. Secondarily, the level of abstractness in mathematical domains likely increases as students age and this may influence student affect and cognition. For instance, algebra I is considerably more abstract than grade seven or eight mathematics is because much of that may be review. All of this (in)directly influences the degree to which student development influences the emergence of mathematical creativity. Taking into consideration that generally students mature at varying levels (Cromer et al., 2015; Hassler, 1991) and this evolution is overlaid with the construct of mathematical creativity, the complexity of this discussion is elevated considerably. Figure 4.1 illustrates the

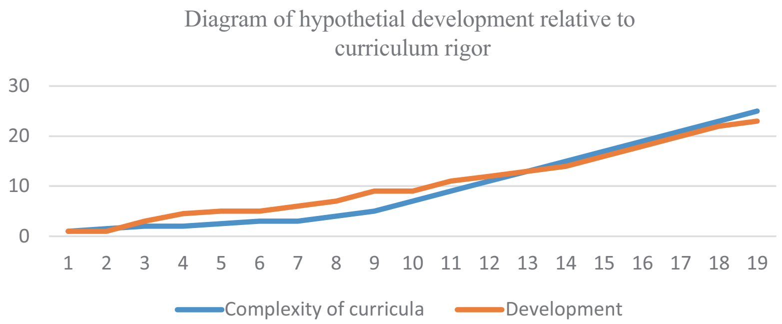


**Fig. 4.1** Development and curriculum rigor

varying level of development with the increased complexity of mathematical content in grades, but is a very rough approximation of the timeline.

This diagram of a hypothetical student is one in which readers can see the relationship between student development and (mathematical) curricular rigor. In it, neither the development of the student nor the increasing rigor of the curriculum advance in a consistently linear fashion. This is a result of a host of factors, notwithstanding the notion that not all students have precisely the same mathematics courses throughout the world, not all students mature in lockstep fashion (though some of our school systems are based on the fact that chronological advancement perfectly correlates with cognitive development), not all students begin school at the same time or in the same place, and many academic institutions cover material in more or less sophisticated manners than others.

Hence, while a student in school A may be doing quite well academically using the standardized assessment created by the district, another student merely miles away may have almost identical content knowledge and reasoning skills, but not fare as well on the district assessment because of its rigor. As an aside, the countless human factors that constitute the educational process are what makes generalizations in the soft-sciences so much less stable than they are in the hard-sciences.

The editors and authors of this book therefore would like readers to be aware that though the focal point of the book is predicated on data from high quality empirical work in various domains, all centered on mathematics and learning, some conclusions may apply to specific environments more than they do to others. Further, in extensive discussions about development and mathematical creativity, patterns were utilized to reach conclusions. As an example, whether one endorses Piaget's cognitive stages as valid (Inhelder & Piaget, 1958), it is generally believed that one's ability to reason abstractly is extended as learners age. Thus, a fourteen-year-old can likely reason abstractly in a much more efficient manner than an eight year old can, assuming normal circumstances and development.

Additional conversations are raised in this chapter. As an example, the discussion of development and mathematical creativity is considered in relation to some commonly accepted theories in the domain of educational psychology. Namely, Kaufman and Beghetto's (2009) Four C's and Rhodes' (1961) Four P's of Creativity are considered. Development and mathematical creativity is also discussed in relation to barriers that may impede it and in relation to determining factors in the learning equation: cognition, affect, and conation (Goldin, 2019).

## 4.2 Development and Mathematical Creativity in Relation to Creativity Models

Discussions of theoretical matters in a domain, in this case mathematics education, should not be undertaken devoid consideration of (a) empirical evidence that can be used to support or refute it, or (b) pre-existing theory. The former, empirical evidence, is considered throughout the book. The latter, pre-existing theory, is considered in this section and periodically revisited throughout the book. In this section, Kaufman and Beghetto's (2009) Four C's are discussed and then Rhodes' Four P's are discussed, with minimal discussion on press.

### *4.2.1 The Four C's*

In 2009, Kaufman and Beghetto proposed a general (not mathematical per se) model of creativity in which they outlined contributions that could be considered creative. This model was divided into four levels, each pertaining to creativity and hence the title The Four C's. The levels of creativity and a brief description of each level, is provided in Table 4.1.

According to Kaufman and Beghetto (2009), the Four C model was generated in response to the shortcomings in the previous Big-C (eminent creativity)/Little-C model (everyday creativity), as discussed by Merrotsy (2013), because Kaufman and Beghetto felt that this dichotomous model was one in which other, rather

**Table 4.1** Four C model of creativity with descriptions

| Levels of creativity | Description of levels |
|---|---|
| Mini-c | These are often contributions at a personal (Runco, 1996, 2004) or individual (Niu & Sternberg, 2002) level and examples of it may occur during general classroom learning. For instance, a teacher may witness a contribution that (s)he has never seen, but many other advanced mathematicians have made such a realization. Assessment may occur by oneself. |
| Little-c | These are often contributions considered to be *everyday* and may not require particularly advanced expertise in an area to emerge. They are, however, a bit more formal than those in mini-c. often, assessment may occur from a parent or teacher (not necessarily an expert in the domain). |
| Pro-c | These are often contributions that require some degree of work with a mentor, to induct one into an advanced state of a domain. These contributions are often in a professional field or endeavor. Fellow (expert) peers in a domain often assess pro-c contributions for value. |
| Big-C | These are often contributions that are incredibly rare and are recognized widely by experts in a domain as monumental or generational in nature. Such contributions are recognized across the domain as significant. As an example, in the world of health sciences, Jarvik's artificial heart is an example of a Big-C contribution (Kahn & Jehangir, 2014). |

*Note: The titles of the respective levels are recorded precisely as they were in the Kaufman and Beghetto (2009) article. It may be hypothesized that the mini and little levels start with a lower case letter to denote contributions that may be highly important to the individual, but not society per se. In the Pro and Big levels, each word starts with an upper case letter to denote (perhaps ostensibly) more significant contributions to society. Further, the final C, in Big-C, is upper case because it represents the highest level of contribution that an eminent individual can make. In fact, it is likely that accomplishing a Big-C contribution is what qualifies one as eminent in a field significant, contributions were omitted. Hence, their four-level model was in response to the faults in the two-level model and provided a great degree of specificity for increased precision in the discussion of creativity. The authors do mention several caveats with their model, perhaps the most significant of which is that they do not see the four levels as ones through which one must progress in sequential order. In fact, they mention that there may very well be situations in which one or more stages is skipped. Nevertheless, showing evidence of creativity in early stages may enhance the likelihood of advanced creative contributions surfacing eventually. It is also important to note that higher level creative contributions, such as Big-C and Pro-c, often require some degree of expertise. As an example, one is most unlikely to simply introduce oneself to a domain (e.g., astrophysics, veterinary medicine, or music) and realize a Pro-c or Big-C contribution within days. Often, such contributions require years of time, emotional, and cognitive investment.

How, one may inquire, does the Four C model have any application to the focus of this book, development and mathematical creativity? Simply stated, when learners are at a very young age, they cannot be expected to realize significant contributions because the requisite level of expertise is absent. In short, they may be at a neophyte level. Given the fact that Big-C contributions come from only some eminent individuals, perhaps every ten to twenty years, a seven year old grade one student will never generate such a contribution, at least in grade 1. Similarly, Pro-c contributions cannot happen at very young ages as the requisite amount of content expertise does not exist. However, in elementary and secondary grades, little-c becomes a possibility, as well as mini-c. This is because contributions from such individuals may be prospectively creative in mathematics and show promise for additional creative contributions. At the tertiary level, Pro-c becomes a possibility, but even then it is likely infrequent. As an example, in a senior level or early graduate level engineering course, a student may have a Pro-c contribution, but even that is unlikely in a school setting. Big-C contributions are often never realized by students, but a select group of professors or advanced professionals may ultimately have Big-C contributions. As an aside, Simonton (1997) suggested that what is known now as Big-C work often begins in one's twenties and may not likely come to fruition until one's forties (Table 4.2).

**Table 4.2** Explanation of age categories and likelihood of contributions

| Age category in book | Possible contributions level |
|---|---|
| Elementary (age 5–12) | Mini-c, little-c |
| Secondary (age 13–18) | Mini-c, little-c |
| Tertiary (age 19–23) | Mini-c, little-c, pro-c |

### *4.2.2 Person, Process, and Product: Portions of the Four P Model*

Of additional importance to discussions in this book is Rhodes' (1961) Four P model of Creativity. In this section, person, process, and product will be discussed in relation to development and mathematical creativity. The fourth P, press or environment, will receive less attention than the first four Ps, but will be discussed near the end of the chapter. Rhodes' conceptual definition of creativity included all four Ps. With the person as the centerpiece, he formulates a conceptual definition of the construct in this manner:

> My answer to the question, "What is creativity?" is this: The word creativity is a noun naming the phenomenon in which a person communicates a new concept (which is the product). Mental activity (or mental process) is implicit in the definition, and of course no one could conceive of a person living or operating in a vacuum, so the term press is also implicit. (p. 305).

In this respect, all four Ps are integral to the process of creative emergence. Editors of this book find this model equally applicable to the relationship of development and mathematical creativity. Person, according to Rhodes (1961), pertains to myriad factors, including, but not limited to: "personality, intellect, temperament, physique, traits, habits, attitudes, self-concept, value systems, defense mechanisms, and behavior" (p. 307). Note that most of these person attributes pertain to cognitive abilities or affective traits. Note additionally that the development, or state of maturation of the learner is not explicitly mentioned, though Rhodes does suggest from very early creativity research, that individuals that physically mature late, relative to their peers that may be early developing according to psychological and chronological norms, may hold some proclivity to be more flexible thinkers and thus more creative than their early maturing peers (Jones, 1957).

Just as the person is central to creative emergence, the (cognitive) process involved in generating creative products is instrumental. Many readers are interested in knowing which processes are specifically mentioned by Rhodes. Process, according to Rhodes (1961), pertains to, "motivation, perception, learning, thinking, and communicating" (p. 308). He posed several intriguing questions, all related to thinking and the motive that prospectively creative individuals have in opposition to individuals that may accept the status quo. Rhodes lists multiple reasoning processes relative to creativity, as he relates them to the Four Stages of Creative Thought (Wallas, 1926). As with person, process may have a developmental component to it for two reasons. First, it was believed by Rhodes that success in creative reasoning or process could be taught through formal approaches. Hence, the more advanced one is in chronological years, assuming formal approaches for thinking creatively were delivered, the more likely one may be to successfully engage in creative processes. Second, Rhodes closely tied creative process to success in preparation, incubation, inspiration, and verification, and he suggested that the more mature one is, the greater propensity one may have to engage in these periods successfully.

Success in creative processes, of which Rhodes mentioned many, ultimately leads to creative products. In fact, Rhodes (1961) referred to creative products as, "artifacts of thoughts" (p. 309). In essence, Rhodes suggested that products are the end-result or manifestation of processes. Without creative process, therefore, creative products would not materialize or come to a state of tangibility. The development of an initial product is of far greater significance than rather minor innovations or 'tweaks' to inventions after the initial creation. This, then links Rhodes' concept of products to Kaufman and Beghetto's Big C, in that generational inventions may not come about until later stages in life, once one has worked with a mentor for several years. This too suggests a developmental component in the emergence of creative products. Moreover, products of considerable importance are often the result of organizing and classifying thoughts. The processes of organizing and classifying information is not low level as no structure exists for systematically arranging information, until the original creator does so. Such a higher-order responsibility is typically incumbent upon the most advanced and eminent individuals in a field.

## 4.3 Barriers to Eliciting Creative Process and Product

Development and maturation likely play a large role in the emergence of mathematical creativity and they are one component in its facilitation. However, some may wonder if any barriers or negative attributes exist that may hinder creative process and product in mathematics. Likely, the most substantial factors pertain to a lack of qualifying characteristics among teachers, such as mathematical content knowledge, pedagogical content knowledge, and/or a lack of fundamental principles in mathematics psychology, such as a cognizance of creativity. In addition, several factors that may be beyond teachers' control likely impede creative processes and subsequently products. As an example, an overemphasis on standards and standardized assessments may obstruct time that could be devoted to mathematical creativity. This burden of an overcrowded curriculum may thwart time that could be invested in the Four Stages of creativity (Wallas, 1926).

There may be instances in which mathematics instructors or mentors, with insufficient understanding of mathematics content, mathematics education, and/or mathematical creativity fail to recognize windows, or opportunities, for creative process and product. Much of the failure of mathematical creativity to emerge may be a result of inappropriate development expectations, in coordination with the inability to infuse developmentally appropriate tasks and/or to create a developmentally appropriate environment. In some instances, it is likely that a basic modification to a problem's structure or mathematical information may make the task more developmentally appropriate than it was in its initial form. As an example, in some situations, quite engaging and highly open-ended problems may be negatively altered for use in textbooks with the insertion of organizational tools, designed to help students solve the problem. Such organizational tools may mathematize problems for students (Cobb et al., 1997; Lesh & Carmona, 2003), thereby serving to constrain thinking of individuals with creative potential. As an example, compare the two problems below:

**Problem A:**

Identify the relationship between column A and column B:

HINT: see if you can identify a pattern (e.g., an increase in value) that is consistent from column A to column B.

| A | B |
|---|---|
| 2 | 8 |
| 4 | 64 |
| 6 | 216 |
| 8 | 512 |

**Problem B:**

In a recent show on personal finances, the host mentioned that assuming normal financial circumstances, meaning not excessively aggressive or stagnant growth in a market, one could expect a set aside retirement amount to double in value every seven years. Given this claim, if a person wanted to retire with at least $1,000,000, what is the minimum amount of money that the person needs to have invested by age 30, assuming no additional contributions to retirement. Note: The person hopes to retire at age 65.

Though the two problems each pertain to the same mathematical concept, that is exponential growth, the second problem is not as leading as the first because the organizational tool is not provided. Further, no hint is offered to the problem solvers in Problem B. The second problem has an unknown, value that needs to be identified, and may result in more than one answer, since the problem is phrased as, "*Given this claim, if a person wanted to retire with at least $1,000,000, what is the minimum amount of money that the person need to have invested by age 30, assuming no additional contributions to retirement.*" Unfortunately, in some cases, highly engaging mathematical concepts may be compromised by curriculum developers that encourage the quickest path to the answer. In so doing, what Hiebert et al. (2000) refer to as 'sense-making' in mathematics is often thwarted.

In addition to problems that may arrest thinking, the overreliance and prospective overemphasis on standardized tests as a metric for gauging student mathematical proficiency may impede the emergence of mathematical creativity, as increased attention is invested in insuring that all standards, be they national, state, and/or district, are met. Though some standards are engaging, many may be reduced to rather mundane algorithmic procedures, ones in which the true aesthetics of mathematics (Breitenbach & Rizza, 2018) is not presented or appreciated.

To compound the problem, use of curricular materials that may constrain students' thinking, coupled with teachers that are expected to deliver an extensive quantity of mathematical concepts in a short period of time, and thus the likelihood that teachers hastily cover topics, is enhanced. When teachers are forced to hastily cover a rather extensive list of mathematical concepts, ample time for mathematical creativity to emerge, and in specific, ample time for preparation, incubation, illumination, and verification (Wallas, 1926) are also compromised. The advancement of creative process and product in mathematics is largely contingent upon mathematicians solving problems and having adequate time to pursue creative lines of thought. This notion of providing ample time to engage in cognition and thus prospectively engendering creativity theoretically helps students appreciate the battery of skills involved in being a mathematician. In so doing, comprehensive rather than partial, development of aspiring mathematicians may occur. Mathematicians, young and old alike, should realize that mathematics does not always comprise one demand. As an example, in young grades, mathematics students are often conditioned to realize that mathematics is a domain of precision (National Governor's Association & Council of Chief State School Officers, 2010; Otten et al., 2019). Hence, they may have a challenging time with the concept of estimation or approximation because they sense that precision is compromised or altogether neglected. In some instances, mathematics has demands of speed in processing (Clark et al., 2014; Lambert & Spinath, 2018), though some such demands are falsely created, such as timed tests (Sasanguie et al., 2013; Tsui & Mazzocco, 2007). For many students and teachers, therefore, slowing down to appreciate the beauty of mathematics (Koichu et al., 2017; Johnson & Steinerberger, 2019; Tjoe, 2016) is awkward. Similarly, providing ample time to consider various avenues to solve a problem may also be foreign, as students may not have engaged in such a process. Hence, though time is at a premium in many classes, and certainly mathematics is no exception, slowing down to appreciate the beauty and complexity of mathematics is incumbent upon mathematics educators with a desire to help young mathematicians truly mature. In fact, it was Papert (1980), in discussing Poincaré's (1908) conception of mathematical creativity, who intonated that appreciating the beauty of mathematics was an antecedent of mathematical creativity.

## 4.4 Additional Factors in the Relationship Between Mathematical Creativity and Development

Mathematical creativity is a multifaceted and complex construct in the domain of mathematical psychology and its occurrence may not be exclusively predicated on the state of cognitive factors. In fact, affect, as well as conation likely play a role in mathematical creativity. As a consequence, variables involved with precipitating it (e.g., person, process, and product) in mathematical learning episodes should not be left to happenstance. The environment (or press), as Rhodes (1961) suggested, likely plays a role in affective and conative states. Mathematics instructors must be purposeful in aligning problem solvers' developmental needs with mathematical creativity needs. When mathematics educators become intentional in fostering mathematical creativity with learners, it may enhance the probability of it emerging.

Affect, comprising beliefs, attitudes, and emotions (McLeod, 1989), as well as conation, or one's willingness to engage in a task (DeBellis & Goldin, 2006; Goldin, 2019), may play seminal roles in the emergence of mathematical creativity. In particular, it has been theorized (Chamberlin & Mann, 2021) and empirically shown that when affect and to a lesser extent conation are at optimal levels, the likelihood of mathematically creative process and subsequently product emerging is enhanced. In this final section of chapter three, the constructs of affect and conation are discussed in relation to mathematical creativity.

### 4.4.1 *Empirical Evidence of Affect/Conation Relationship to Mathematical Creativity*

Though empirical evidence to substantiate the connection between affect and creativity is sparse, some research does exist. For instance, Fernández-Abascal and Martín Díaz (2013) discuss three studies that they conducted in which positive and negative affect were investigated in relation to divergent thinking output. Generally, the studies showed that positive affect increased divergent thinking output, but negative affect had no particular influence on divergent thinking. Divergent thinking, by the way, is considered a metric for gauging creative output. Perhaps the most persuasive argument for the connection between affect and creativity comes from Davis, who, in 2009, showed such a connection based on a meta-analysis that he completed almost a decade previously. Baas et al. (2008) identified the same finding only one year earlier.

Another metric of creativity pertains to novelty. In 2013, Newton provided ample evidence of a connection between affect and novelty/original thought. In particular, the propensity for original thought is likely to be enhanced with positive affect, relative to negative affective states. Regarding conation, Schindler and Rott (2017) discussed the premise that task commitment plays a significant role in the emergence of creative output in mathematical settings. It would be a mischaracterization to suggest that the relationship between mathematical creativity and affect/conation is strong, when looking strictly at empirical evidence. In most cases, much of the literature is theoretical, as is that presented in the next section, and much of the empirical connection between affect and mathematical creativity is either out of the domain of mathematics, or tenuous, at best. Nevertheless, attention has been invested in this relationship in the research world of late.

### 4.4.2 *Five Legs Theory*

The Five Legs Theory (Chamberlin & Mann, 2021) is one in which five affective factors, comprising Iconoclasm, Impartiality, Investment, Intuition, and Inquisitiveness, are said to have a(n) (in)direct influence on mathematical process and product. This theory is deeply ensconced in empirical literature and the five subconstructs that comprise the theory have somewhat altered conceptions and operational definitions than the manner in which affect has been discussed previously in mathematics education literature. As an example, Iconoclasm pertains to, "the courage to challenge conventional mathematical ideas" (p. 21). Garnering such courage is not a trivial feat per se, as conventional mathematical ideas are ubiquitous in mathematical texts, online, and in teacher delivery of curricula in the classroom. "Impartiality is considered an openness to appreciate and see multiple perspectives and to consider utilizing unconventional ones" (p. 31). Much like Iconoclasm, the subconstruct of Impartiality may be rare in mathematical learning episodes and classrooms. In fact, the development of each of these five subconstructs may depend largely on the learner's characteristics and the environment (press) and atmosphere created by the teacher. Investment pertains to, "an emotional contribution to finding a solution to a task for one or more reasons" (p. 43). In the domain of mathematical psychology, investment shares several features with a financial investment, perhaps the most significant of which is that a sacrifice must be committed in an attempt to secure a reward. Moreover, inherent risk is involved in a financial, as well as an emotional commitment. In the Five Legs Theory, Intuition holds a slightly altered conception relative to what it does in layman's terms. Intuition, in this case, pertains to a, "drive toward a solution or response" (p. 53). Much like a highly skilled culinary artist may have an inclination to intermingle ingredients that have heretofore been unorthodox or foreign, a creative mathematician may have a drive to pursue a line of reasoning that has not been investigated, in an attempt to identify a new solution or proof to a problem. Inquisitiveness shares some attributes with interest in that it pertains to a curiosity which ultimately leads to high levels of engagement. The resultant product may be learners that are in Flow (Csikszentmihalyi & Csikszentmihalyi, 1993).

The question persists regarding specifically how affect and conation influence development and mathematical creativity. In general, two components are at work in this interaction. First, as learners age, their affective and conative states stabilize, according to Hart and Walker (1993). Second, as learners age, it is likely that those that engage in higher level mathematics have self-selected. This means that individuals matriculating higher level mathematics courses may be inclined to be more motivated than many of their younger counterparts. Hence, the propensity for creative output may be enhanced relative to its emergence in younger grades.

## 4.5 Conclusion

In this chapter, the framework for the book was outlined. In particular, the fundamental tenet of this book is that mathematical creativity has a developmental component to it. In this book, the term development is not used in the respect that it is in many literature reviews, that being to develop creativity in the mathematics classroom. Instead, development in this book can be thought of as maturation. Secondarily, development and mathematical creativity were discussed in relation to common creativity models as a concise investigation into their relationship. Next, barriers to mathematical creativity were explored and finally, additional factors in the discussion of development and mathematical creativity, such as affect and conation, were discussed.

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# Chapter 5
# Commentary on Section

**Deborah Moore-Russo**

The authors of Chaps. 6, 7, and 8 provided three informative literature reviews on mathematical creativity across the elementary, secondary, and tertiary levels. As the three sets of authors reported, there has been limited research, especially empirical research, in this area. I begin by summarizing the key aspects of each chapter. These summaries are followed with a commentary on the themes that appear across the three chapters.

## 5.1 Mathematical Creativity Research in the Elementary Grades

Kozlowski and Chamberlin (Chap. 6) provided a review of the literature on mathematical creativity in the elementary grades. They differentiated between academic-oriented research and practice-oriented research, and this dual categorization organized how they reported the literature cited in the chapter.

In looking at academic-oriented studies, the authors provided historical context citing work from mathematics, psychology, cognitive science, mathematics education, and mathematical psychology. The authors used maturation, the mental development of a child, as a lens for the chapter. This focus on development was also noted in much of the research cited as well as in Kozlowski and Chamberlin's sentiments that understanding the relationships between creativity, age, and development merits further study.

In their review of the practice-oriented research, the authors focused on different instructional tasks and activities, including some that integrated digital resources. In

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the practice-oriented category, they also considered the didactic contract (Brousseau, 1997) and affective development, including both traits and states, as two environmental aspects that relate to mathematical creativity.

After categorized reporting of the literature, the authors provided a synthesis of the current research with ideas for future research initiatives. The closing discussion continued with the two-category system considering promising directions for both academic-oriented and practice-oriented research on mathematical creativity for elementary students. One direction that was suggested for future academic-oriented research involves the relationship between a child's age, maturation, and mathematical creativity. Interestingly, the authors reported that it is the rapid rate at which elementary students develop that makes the study of creativity so challenging in children. The authors addressed four possible ways that maturational development might be taken into account. For example, the use of repeated measures to account for development was suggested.

Two possible future directions noted by the authors for practice-oriented research involve studying how certain digital resources and how a holistic approach in the classroom might nurture mathematical creativity in children. Kozlowski and Chamberlin reported that disparate classroom practices (e.g., use of open-ended tasks) are studied currently in terms of their individual impacts on creativity. The authors suggested that combining different classroom factors and studying how together they might produce a more holistic creative environment could be a promising direction for mathematical creativity research related to elementary-aged children.

## 5.2 Empirical Findings on Creative in Mathematics Among Secondary School Students

Joklitschke (Chap. 7) provided an overview of current research on mathematical creativity, focusing specifically on empirical research, at the secondary school level. The author decided to use this focus since previous work has already considered different approaches to creativity in mathematics and the theoretical assumptions that currently undergird mathematical creativity research.

The outline and style of this chapter contrasted with that of the previous chapter. Joklitschke used theoretical background, research methods, data analysis, results, and discussion sections to organize the chapter, similar to the organization that one would encounter in the empirical studies on which she reported. After introducing commonly used theoretical frameworks, the chapter provided information on the search criteria and how these criteria led to the inclusion of the 22 articles in the review, most of which (14 of 22) were quantitative studies. The author reported on common perspectives that were noted across many of the 22 articles, such as the phases of preparation, incubation, illumination, and verification (Wallas, 1926). Many of these same themes also appeared in Chaps. 6 and 8, and I address them later in my commentary. A particularly interesting part of Chap. 7 related how some of the articles reviewed by the authors attempted to better understand creativity by creating models that in turn were validated in the studies.

Joklitschke stated that the current research landscape for mathematical creativity is diverse, with few published studies devoted solely to the topic of creativity. She offered possible explanations for this, but also reported that current research frequently considers how creativity correlates to different psychometric constructs (e.g., achievement, giftedness). In addition, the author noted the heterogeneity of the methodological approaches in the mathematical creativity studies she considered.

The end of the chapter started with a general outlook for the field. Here, the author pondered whether interaction, collaboration, or classroom-based research might become more prevalent in the study of mathematical creativity. Joklitschke then looked at some of the research results in light of creativity as fluency, flexibility, and originality (Torrance, 1966). Before considering limitations of the chapter, she dedicated a paragraph to an interesting finding on the lack of historical references in the works she reviewed.

## 5.3 Mathematical Creativity at the Tertiary Level: A Systematic Review of the Literature

Savić, Satyam, El Turkey, and Tang (Chap. 8) provided a broad picture of the research on mathematical creativity at the tertiary level. The chapter focused on the developmental nature of creativity, looking at how mathematical creativity is fostered. This differed from the developmental perspective in Chap. 6, where "developmental" referenced the maturation and mental development of children. The authors limited the research to college and university mathematics courses and chose not to include research done in tertiary mathematics courses exclusively designed for preservice teachers, although they mentioned that there is value in such work since it could influence how future teachers view mathematics and mathematical creativity.

As is the case for Chap. 7, Savić and colleagues included methods, results, discussion, and future directions sections. The authors focused on 29 articles for the review and described the search criteria that were used to identify these studies. There were three interesting findings reported. The first was that 27 of the 29 articles have been published since 2012, and 17 of the 29 were published after 2018. They concluded that this shows how new this area of investigation is. The second finding was the authors called 11 of the 29 studies "descriptive." These 11 articles did not use any coding, nor did they involve quantitative or qualitative methodologies. The remaining 18 studies were rather evenly split between quantitative and qualitative research methods. The third finding was that almost 38% of the articles in the literature review came from only two journals. Six of the 29 articles came from a special edition of the *Journal of Humanistic Mathematics*, while another five were from different issues of *Problems, Resources, and Issues in Mathematics Undergraduate Studies* (a.k.a. *PRIMUS*).

This chapter also pointed out different rubrics that are being used to study mathematical creativity at the tertiary level, of which two are of note. One was the creativity-in-progress rubric on proving (El Turkey et al., 2018; Savić et al., 2017). Another was Lithner's (2008) imitative/creative reasoning framework.

Savić and colleagues reported that while five of the articles reviewed had a focus on calculus, there was an array of mathematical topics covered. This lack of focus on a single mathematical topic or content area inspired the authors to write "that mathematical creativity can be fostered in any aspect of tertiary mathematics education" (p. 111). The authors concluded the chapter with reasons why they believe that mathematical creativity is important for mathematics education at the tertiary level.

## 5.4 Themes

All three chapters mentioned the increase in the research in mathematics education that focused on, or at least took into consideration, mathematical creativity. This research topic seems to be rapidly growing, and the authors in this section all reported that there is limited existing research and that more investigation in on mathematical creativity would be of great value to those in mathematics education.

## 5.5 Mathematical Creativity: A Complex Topic

In Chap. 7, Joklitschke referred to the "complex literature landscape" of mathematical creativity and discussed how few recent publications have been solely devoted to the study of creativity. Creativity is often studied in context with other constructs, be they intellectual, personality, or affective. Some constructs mentioned in different places of this section as being studied with mathematical creativity included giftedness, intelligence, achievement, persistence, motivation, openness, self-confidence, self-efficacy, autonomy, and "an ability to deal with messiness" (Kozlowski & Chamberlin, p. 75). Joklitschke stated that creativity may be seen and studied as more of a "side effect" of other constructs than a construct in its own right.

Because of its overlap or connection with other constructs, one of the methodological challenges mentioned by Kozlowski and Chamberlin is how to measure mathematical creativity. Joklitschke suggested the research community lacks methods capable of shedding light on creativity from other perspectives; she proposed the possible use of EEGs and eye tracking. Savić and colleagues in Chap. 8 related that, in one study included in their literature review, the research team concluded that "evaluating creativity is a difficult task" (Blyman et al., 2020, p. 169).

Moreover, mathematical creativity is complex to understand because it is not a steady state measure. Joklitschke cited works that considered how increases in knowledge may cause creativity to increase, with a possible dip around eighth grade during an increased algebraic approach in the mathematics curriculum. Kozlowski and Chamberlin also discussed how researchers have reported spikes, slumps, and leveling in an individual's creative outputs.

## 5.6 Mathematical Creativity: Where It Lives and How It Is Understood

Mathematical creativity is not rooted in a single academic area. Kozlowski and Chamberlin stated that mathematical creativity is "unique because many of its characteristics can be attributed to various fields" (p. 66). They began Chap. 6 outlining how this construct cuts across many academic areas. In the closing of Chap. 7, Joklitschke brought forth four reasons as to the finding she called "striking" that few historical references are included in the empirical parts of the research she reviewed. A fifth reason that might be added to her list would be that a deep study of creativity requires reading across many academic areas.

Since creativity cuts across many areas of academia, it is not surprising that there are numerous ways that creativity is framed. This was noted in all three chapters in this section. All three sets of authors cited literature that uses the framing of creativity as being related to processes, persons, press (i.e., environment), and products (Rhodes, 1961). Joklitschke, in particular, raised the point that the products were often used as data for studying creativity, especially in quantitative studies, while qualitative studies were more likely to study processes. The authors of Chaps. 6 and 7 mentioned studies that frame creativity as fluency, flexibility, and originality (Torrance, 1966). Mathematical creativity as divergent thinking was also mentioned in by the authors of Chaps. 6 and 7 in the articles they reviewed.

## 5.7 Mathematical Creativity in the Classroom

All three sets of authors in this section relayed that both teaching actions (often focusing on particular types of instructional tasks) and the educational environment fostered in classrooms have been reported, or assumed, to impact mathematical creativity. In Chap. 6, Kozlowski and Chamberlin used practitioner-oriented research as one of their categories, which they considered to focus on "advancing implementable concepts [that] may have a direct influence on what teachers do to facilitate creativity in mathematics classrooms" (p. 66). In addition, Savić and colleagues stated that "there is a need for enhancing students' creativity in mathematics classrooms at the tertiary level" (p. 106).

Considering instructional tasks and their relation to mathematical creativity, Kozlowski and Chamberlin discussed and differentiated between open-ended tasks and multiple solution tasks. In Chap. 7, Joklitschke mentioned that tasks can either capture or promote mathematical creativity, and she mentioned multiple solutions tasks as being part of several studies she reviewed. In fact, multiple solution tasks were the most mentioned instructional practice that received research attention across the three chapters in the section.

The learning environment and how mathematical creativity is fostered in classrooms was also a common topic in the literature cited in this section. Savić and colleagues, in Chap. 8, reported that over a third of the studies they reviewed involved the researchers describing how they nurtured creativity in their own classrooms and that almost all the studies (25 of 29) assumed that creativity "could be fostered or developed in the classroom" (p. 110). Joklitschke reported that the classroom "setting or way of instruction plays a prominent role" (p. 91) in much of the mathematical creativity research at the secondary level. For example, she reported on one study that investigated a classroom described as having an inquiry-based environment and another that analyzed the interactions of pairs working with problems posed in a dynamic geometry environment. Kozlowski and Chamberlin also reported on environment and mathematical creativity. For example, one study they used in their review involved an instructional environment that encouraged students to explore alternative strategies to solve problems.

## 5.8 Concluding Thoughts

Mathematical creativity is a growing research topic in mathematics education. The three literature reviews provided a solid foundation to understand the existing body of literature on creativity that spans from elementary to tertiary mathematics. As all the chapters related, there are still numerous areas that merit exploration. For example, one interesting suggestion was that the study of creativity should not be limited to the individual but should expand to consider the collective.

The literature reviews in this section set the groundwork for others to carry on with the study of mathematical creativity in elementary, secondary, and tertiary mathematics. What's more, as someone who has dabbled ever so slightly in mathematical creativity research as related to pre-and in-service teachers (Moore-Russo & Demler, 2018; Moore-Russo et al., 2020), I would love to see a research team build on the methodologies employed in the literature reviews in this section to investigate future and current teachers' notions of mathematical creativity and how it should be nurtured.

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# Part II
# Synthesis of Literature on Mathematical Creativity

# Chapter 6
# Mathematical Creativity Research in the Elementary Grades

**Joseph S. Kozlowski and Scott A. Chamberlin**

## 6.1 Mathematical Creativity Research in the Elementary Grades

Mathematical creativity (MC) in the elementary classroom has become a focal point for some mathematics education research (e.g., Sriraman & Haavóld, 2017) as well as practitioner frameworks (e.g., twenty-first-century learning framework; National Education Association, 2010). Furthermore, a chapter titled "Creativity and Giftedness in Mathematics Education: A Pragmatic View" (Sriraman & Haavóld, 2017) was included in the most recent *Compendium for Research in Mathematics Education* (Cai, 2017) in the section about promising topics for future research in mathematics education. Therefore, a first step in MC research is to understand what the empirical work suggests on the topic at the elementary level. In this systematic literature review, information is presented in the following structure. The first section contains results of the literature review on mathematical creativity (MC) research in grades K-6 which is categorized into two main groups; *academic-oriented research* and *practice-oriented research*. The second portion of the paper contains a synthesis of current research, with carefully designed questions to help direct future initiatives in the MC research community.

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## 6.2 Mathematical Creativity Research: Academic-Oriented and Practice-Oriented

The two following sections contain MC research organized into two main strands called *academic-oriented research* and *practice-oriented research.* These two strands share a major similarity in that they focus on empirical research on MC in grades K-6. However, their goals are primarily distinct. Academic-oriented research is defined as empirical research on MC that is directed at advancing theory, methods, and nuanced psychological details about MC. Ultimately, academic-oriented research comprises two foci. First, it is focused on advancing future research and, second, it is conducted to answer research questions built from the corpus of literature among scholars, academics, and theoreticians. Practice-oriented research is defined as empirical research on MC that is directed at advancing practices, instruction, skills, curricular materials, or any other aspect of the field of MC. Ultimately, practice-oriented research is focused on advancing implementable concepts and may have a direct influence on what teachers do to facilitate creativity in mathematics classrooms.

## 6.3 Academic-Oriented Research on Mathematical Creativity: Impacting Future Research

Some research on MC is directly aimed at advancing research efforts. In this paper, such research is referred to as academic-oriented research. The construct of MC is unique because many of its characteristics can be attributed to various fields. For example, the process of MC has deep roots in psychology and cognitive science, the mathematical output informs mathematics research, and the process of learning and teaching of MC informs mathematics educational (and mathematical psychology) research. Sriraman (2017) critiques the field of mathematics education for not understanding the depth of knowledge about MC that has existed for decades across various fields (i.e., psychology, cognitive science, mathematics). Regardless, current research efforts in (a) psychology and cognitive science, and (b) mathematics education and mathematics psychology are aggressively working to advance understanding of MC and drive future research. It is important to note that psychology and cognitive science are minimally disparate domains, but, for the sake of this literature review, they have been grouped into one category.

### *6.3.1 Psychology and Cognitive Science Research*

Several themes guide the review of the literature. First, the preponderance of (modern) research that pertains to MC originated in psychology and cognitive science as well as mathematics education and mathematical psychology. Though the domain of mathematics may not contribute significant work relevant to the construct of MC today, it was mathematicians that were historically responsible for highlighting the importance of MC (Emch, 1900; Hadamard, 1945; Poincaré, 1913). In the first half of the twentieth century, research and theoretical writings about MC pertained to two emphases. The first emphasis was the recognition of the beauty or (a)esthetics of mathematics. This notion was foreign to many outside of the domain of mathematics because beautiful mathematical solutions seemed at odds with the stereotypical mathematics procedures. Examples of that which was aesthetically pleasing or highly sophisticated had almost exclusively resided in the domain of creative arts, both performing and visual, and applying the construct of aesthetics in mathematical solutions was considered peculiar. The second focus in creativity studies situated in the domain of mathematics pertained to thought processes in which mathematicians engaged. For instance, synonyms employed early for the process of creativity were invention and imagination (Hadamard, 1945). Earlier, Poincaré suggested that science and mathematics may be perceived as a wholly unemotional discipline, but that for innovations to transpire, creative individuals must drive such efforts. One commonality in each of the two aforementioned foci pertained to the rather unidimensional view of MC as novel, at the expense of fluency, flexibility, and elaboration.

As MC interest burgeoned, much of the empirical work was shouldered by researchers in the domain of psychology and cognitive science. In this section, recent work garners much of the attention, though some seminal work is discussed to situate and help readers make sense of recent work on MC. An important caveat in the literature is clarified here, which pertains to the term *development*. This review contains a discussion about development of MC and considers it as maturation, and that this development affects the emergence of MC. This is distinct from myriad references relevant to MC which use the term development to reference the process of developing creativity in students over a finite (e.g., several weeks, months) period of time. Hence, many hallmark studies are not discussed in this review, as they were not germane to MC and development as used in the context of this book, which focuses on mental development (maturation) and its effects on MC. Also, literature in this section has been performed by scholars in psychology and cognitive sciences.

Perhaps the first researcher of consequence to investigate the relationship between development (maturation) and MC was Torrance (1968). In his study on creativity in multiple domains, he ascertained that fluency and flexibility realized considerable decreases of at least one half of a standard deviation among fourth graders, but that elaboration did not incur a statistically significant drop. He did this by analyzing data from 100 randomly selected participants, out of 350 students in a 6-year period as they progressed through third, fourth, and fifth grades. In fact, over time, elaboration was the only factor that experienced statistically significant gains. This slump occurs in about 50% of the students and it is typically recovered in later years (e.g., sixth or seventh grades). Raina (1980), incidentally, found similar results to Torrance when investigating fourth graders in India, and Sak and Maker (2006) found a much less prominent, though statistically it did exist, slump in MC in fourth grade. Sak and Maker's findings show at least some evidence of a fourth-grade slump, having reviewed literature that spanned a 50 year period. To this day, no credible evidence exists to disprove this finding.

However, this study was replicated, to some extent, by Charles and Runco (2001) 33 years later. In their study, they determined that when third, fourth, and fifth grade students' performance was analyzed, there was not a slump in student creativity in fluency, when looking at divergent thinking. It is important to note several admonitions with respect to the outcome of the Charles and Runco study. First, instrumentation was likely far more sophisticated in the Charles and Runco investigation than it was previously. Second, one-third of a century after Torrance's work, student demographics likely changed. Third, though fluency did enjoy a slight increase, there was a fourth-grade decrease in a category referred to as highly appropriate ideas.

One of the most comprehensive discussions, at least during that era, relevant to the relationship between students' advancing age and creative output was forwarded by Simonton (1984). In covering this topic, he stated that the theory was initially posited by Lehman (1953). In his theory, Lehman suggested that in sciences, of which mathematics is considered a sub-domain, creativity generally has a substantial surge early in one's career, reduced in later levels, and even later in one's career is almost fully eliminated, after the vast majority of creative ideas are extinguished. Simonton stated that Lehman's work, much like Piaget's theories of child development, endured strong criticism, but later (Simonton, 1977, 1980, 1984) such criticism of Lehman's work was shown to be without merit through Simonton's empirical efforts. Hence, there is some value to the claim that initially, highly creative individuals may encounter a spike in output, but such a spike often levels, as creative individuals age. Perhaps the most notable critique of applying Simonton's findings to the discussion of MC among elementary age students is that (a) the findings were not exclusive to mathematics, though mathematicians were one of the major sub-domains, and (b) the age of creative individuals is not specified in the research. Nevertheless, his findings do warrant consideration in light of development and MC.

Much of this work culminated with Sak and Maker's (2006) investigation on children's development in relation to MC. They reiterated, as several previous scholars had (Anderson, 1992; Runco, 1991; Maker Runco, 2003), that growth in creativity, relative to age, was curvilinear. One finding of instrumental importance in their work was that with advanced age, it is assumed that a corresponding advance in domain knowledge transpires, thus enhancing the likelihood for increasing divergent thinking and fluency. Hence, it could be argued, as per Sak and Maker's work, that an advance in domain knowledge (or a developmental level), as well as age, is what enables increasing amounts of creative process and product among young, elementary age, mathematicians. Though much of the attention on creativity and development in the domain of mathematics has centered on the fourth-grade dip, additional studies have provided insight. Studies by scholars such as Kattou et al. (2016) and Hetrozoni et al. (2019) are perhaps of most applicability to this chapter.

The Kattou et al. (2016) investigation was informative to the intersection of MC and development in two respects. First, in the literature review, the authors illustrate that no consensus appears to exist regarding age and MC. In particular, Wu et al. (2005) were unable to establish a definitive relationship between age and MC. Two admonitions exist, however, with their claim. First, not all research presented came directly from the domain of mathematics. Second, in their review, age is considered synonymous with development and though it likely correlates quite highly, it cannot be considered a perfect synonym with learner development. The second important finding from Kattou and colleagues arose because they analyzed the relationship of MC and several factors. Age was found to be one of several factors that influenced the amount of creative output and along with personality characteristics, it was found to be less important than cognitive characteristics in the emergence of MC. Again, though age presumably correlates quite highly with development, this finding may not be completely accurate because it is predicated on the notion that development follows a perfect trajectory with age.

In 2019, Hetrozoni and colleagues investigated whether age 9–11 high functioning children with autistic spectrum disorder (HFASD) held similar capabilities for MC as their age 9–11 typically developing (TD) counterparts. In the world of creative arts, it appears as though under certain circumstances, HFASD students outperformed their TD peers (Liu et al., 2011; Ten Eycke & Müller, 2015). In mathematical problem solving, TD students outperformed HFASD students (Bae et al., 2015). Results with this degree of inconclusiveness may precipitate researchers to question the applicability to their sample and focus. Hence, Hetrozoni et al. investigated the aforementioned comparison between HFASD and TD students in MC. In their study, they found that each group performed similarly, with the TD group outperforming the HFASD group in fluency and originality and the HFASD group outperforming the TD group in overall creativity in the Creating Equal Number task. The overall finding is that HFASD students may appear to have similar capabilities in MC output relative to their TD peers. The caveat with this study is that the two comparison groups were somewhat small ($n = 20$ in each group), hence the findings may be questioned.

To conclude that creativity and development research among elementary mathematics students is confined only to psychology and cognitive sciences would be naïve. In fact, the domains of mathematics education and mathematical psychology, as well as mathematics have studies of note. In the next sections, they are discussed.

### *6.3.2 Mathematics Education and Psychology Research*

One impetus for this book is that scant empirical research exists regarding MC and its relationship to maturation and development. In this section, research from experts in the field of mathematics education and psychology is shared. An important study at the intersection of MC and student development came from Hong and Aqui (2004), in which they identified prospective differences in mathematics students identified as academically gifted, but not creatively gifted, in relation to mathematics students identified as creatively gifted, but not academically gifted. In their study, a significant difference existed in the two groups in cognitive resourcefulness (i.e., ability to utilize various cognitive strategies), with the group identified as creatively gifted the more advanced. Though this characteristic may seem trivial, cognitive resourcefulness may be thought of as the single distinction in mathematically gifted and creatively gifted mathematicians because the added ability to think quickly in multiple avenues likely provides greater adaptability and perhaps pliability in considering multiple solutions simultaneously. Hence, creatively gifted students may be more advanced, with respect to development, than their academically gifted peers. One caveat with the Hong and Aqui research is that it was conducted with secondary students, so that finding may not be directly generalizable to elementary age students. Nevertheless, the finding does hold promise in distinguishing between the two groups, especially when considering development and creativity in mathematics.

In a previous study, Haylock (1987a, b) found two characteristics, Overcoming Fixations (i.e., breaking away from mental sets and stereotyped solutions; OF) and Divergent Production (i.e., production of atypical responses; DP) of mathematically creative students that resulted in high levels of mathematics achievement. In short, according to Haylock, 11- and 12-year-old individuals with more well-developed capabilities to think divergently have greater success in mathematical achievement and specifically may be inclined to show greater levels of MC than their less well-developed counterparts. In addition, individuals with the ability to overcome fixations, or not be attached to one single problem-solving approach, will fare better than their less well-developed peers in mathematical achievement and creativity.

As well, there appears to be some evidence that upper elementary students that can endure through ego depletion may be (more) well-developed and mature in their ability to work hard, apply effort, and persist while solving problems than their peers (Price & Yates, 2015). Such characteristics, according to Price and Yates (2015), often translate into additional positive attributes that result in highly creative responses in mathematics, such as a determination to succeed, resilience, and an openness to accept challenge and difficult choices.

## 6.4 Practice-Oriented Research on Mathematical Creativity: Impacting Future Practice

Some empirical research on MC is directly aimed at advancing practice, which is referred to as practice-oriented research in this chapter. Much practice-oriented research on MC is categorized into one of the two following groups, which structures this section: (a) instructional tasks that relate to MC and (b) environmental aspects that relate to MC.

### *6.4.1 Instructional Tasks*

One specific area of practice-oriented research focuses on instructional tasks that support, elicit, or relate to mathematics creativity (Bicer, 2021; Haylock, 1997; Kwon et al., 2006; Leikin, 2009; Levenson, 2011, 2013; Levenson et al., 2018; Silver, 1997; Sinclair et al., 2013). Research on MC instructional tasks is classified by the authors as practice-oriented researcher because it is geared at explicating specific activities that teachers can implement in the classroom to support MC. Bicer (2021) conducted a systematic literature review on MC fostering instructional practices and found that "problem-solving, problem-posing, open-ended questions, multiple solution tasks, tasks with multiple outcomes, modeling and model eliciting activities, technology integration (manipulatives, computers, and graphic calculators), extendable tasks, and emphasizing abstractness of mathematic" (p. 261) were important. The section of this chapter will focus on open-ended tasks, multiple solution tasks, and technological integration as current instructional practices that are receiving research attention due to their promising nature to support MC in elementary students. It is important to note that robust research has been conducted on mathematical modeling and MC, but will not be reviewed in this short chapter. For research on mathematical modeling and elementary MC, see literature such as (Amit & Gilat, 2012; Chamberlin & Moon, 2005; Lesh & Caylor, 2007).

#### 6.4.1.1 Open-Ended and Multiple Solution Tasks

Open-ended tasks and multiple solution tasks (MST) have been shown by researchers to benefit MC in elementary classrooms (Haylock, 1997; Kwon et al., 2006; Leikin, 2009; Levenson, 2011, 2013). Although these tasks are similar, there is a distinction between their manifestation and use in the classroom. Levenson et al. (2018) describe the distinction by focusing on the ending, or the goal. Open-ended tasks typically do not have one right or wrong ending answer; there may be a variety of final answers that satisfy the task requirements. However, MSTs typically have one correct answer and individuals are asked to arrive at that answer in various ways. A related topic pertains to what are called moderately open tasks (Bokhove & Jones, 2018). The idea of 'moderately open' tasks is that a certain degree of constraints in a task (neither no constraints at all nor too much constraints) can induce creative thinking within mathematics, since constraints are part of the concept of creativity itself.

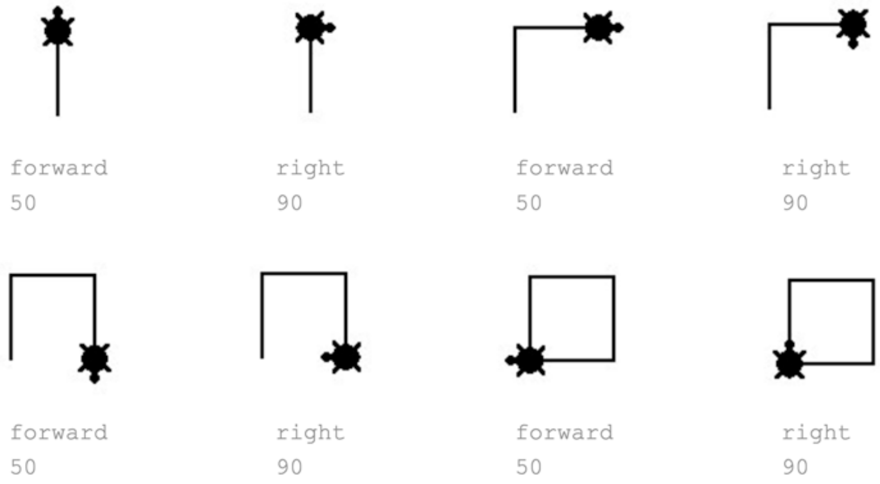
Levenson (2013) conducted a study on 43 graduate students regarding their perception of elementary classroom activities that would occasion MC. Qualitative analysis revealed that participants found problems that required a variety of solution strategies – a correct answer that could be garnered in various ways – were beneficial to MC. One participant stated "In my opinion the task promotes MC because ...in the wording of the question 'suggest different ways' and 'give examples' there is an opening for various possible solutions" (p. 285). In essence, future teachers described how MSTs were beneficial to elementary-aged students in MC because they required the students to generate various solution strategies to garner a correct response. This teacher perception of creativity-fostering mathematical activity demonstrated how MSTs are important pedagogical tools that allow students to engage in divergent production, which was discussed earlier as an important characteristic of creativity-fostering tasks.

#### 6.4.1.2 Technological Integrations to Support MC

The second theme of instructional tasks that has been shown to support elementary MC is incorporation of specific technologies and aspects of virtuality (Papert, 1980; Sinclair et al., 2013). A pioneering effort in technological advances to support elementary-aged children's mathematics was Papert's (1980) Logo programming and Turtle Geometry. This primitive system had a robotic triangle that sat on the floor and was programmed to do basic movements programmed from a computer (e.g., rotate 90 degrees, move forward 100 'turtle steps'). The floor-based turtle soon moved onto the computer screen where a line was traced behind the turtle as it was programmed to move around the screen. Logo and Turtle Geometry evolved through Resnick et al.' (2009) work at MIT into what is now Scratch. Importantly, this type of technology was thought by Papert to be key in supporting MC. Papert discusses that through this technology, students are able to incorporate their bodies (i.e., body syntonicity) and through this bodily engagement experience, construct creative mathematics. Papert describes that extralogical elements of mathematics (i.e., aesthetics, beauty) may surface from the unconscious not primarily based on logical accurateness, but on combinatorial or creative thought, and this technology allowed students to create their own mathematical objects and investigate the relationships between them.

Sinclair et al. (2013) supported the hypotheses of Papert and found a unique distinction between realizing the possible (purely logical) and actualizing the virtual (creating something ontologically new) when using technology. Sinclair et al. conducted a study with 6–9-year-old students to determine if specific technologies promoted creative mathematics. Students in the study created new mathematical objects and relationships as they used gestures and created diagrams of virtual mathematical topics. For example, students in Sinclair et al.'s study constructed mental maps of areas outside the scope of the computer screen to decide on geometrical intersection of lines, constructed creative hypothesis and mathematical proofs about intersecting lines, and used gestures to create impossible mathematical line objects.

Taken together, tasks that foster MC, and that are related to MC, are an important research topic for the future and already some are known about benefits of certain tasks and activities such as open-ended questions, MSTs, and integrating specific technologies. One of the methodological challenges in understanding the relationship between tasks and MC is the difficult nature of measuring the psychological construct of MC. Although progress is being made on MC assessments, considerable work is still needed to rigorously provide an MC measurement instrument that could be used to determine the statistical effects of instructional tasks on MC change over time.




## *6.4.2 Environmental Aspects That Relate to MC*

Another theme of practice-oriented research is environmental aspects that relate to MC. Two specific environmental aspects that garner research interest and are promising for future research are the didactic contract of mathematics teaching and affective development.

### 6.4.2.1 The Didactic Contract of Mathematics Teaching

The didactic contract describes the learning and teaching expectations that are inherent between a teacher and the pupils (Brousseau, 1997). For example, a teacher in a mathematics classroom may expect the 3rd grade pupils who memorize the multiplication table to be able to transfer these same multiplication calculations to other contexts; the pupil may or may not be aware of this expectation. Conversely, a teacher in a mathematics classroom may expect the grade students to actively form mathematical arguments to justify solutions; the pupil may or may not be aware of this expectation. In just these two situations, a variety of didactic contracts are 'signed' between the teacher and the pupil, depending on the teacher expectations and whether or not the pupil perceives such expectations. Ultimately, certain didactic contracts have been supported through research to support creative mathematical thought.

Sarrazy and Novotná (2013) studied the mathematical didactic contract in relation to MC with 155, 9–10-year-old students. Two groups of students completed the same lessons on a single mathematic topic. One group used a *devolving* type of didactic contract (e.g., variability in instruction and expectation, active group work, complex initial problems, community analysis and critique of solutions). The other group used an *institutionalizing* type of didactic contract (e.g., show-remember-apply, question-answer, teacher evaluation of solutions). Overall, the results indicated that the institutionalizing contract resulted in higher creativity scores than the devolving. However, when analyzed more closely, results showed that advanced-performing students scored higher in the devolving contract but medium- and lower-performing students scored higher in the institutionalizing contract. These results are interesting because they suggest an environmental aspect that may be promising for a certain group of students when considering the effects on MC. For example, when it comes to supporting MC, certain didactic approaches such as allowing for ambiguity and variability in instruction may be promising for advanced-performing students, whereas other didactic approaches such as question-answer and show-remember-apply may be more promising for medium- and lower-performing students.

#### 6.4.2.2 Classroom Affective Development

Though the research on student affective traits and states and its influence on MC, through a developmental lens, is not particularly advanced, some studies exist that inform the field about the interrelationship. Studies included in this section pertain to student emotions, such as anxiety, metacognition, personality characteristics, and emotional quotient.

As an overview of this section, Gregoire's (2016) discussion of MC which is bolstered with a discussion of models relevant to MC, is particularly insightful. A salient personality characteristic mentioned by Grégoire is persistence. In his discussion of three historical examples of MC, the author discusses a discovery in topology by Russian mathematician Perelman. When mathematicians pressed him about the discovery, he suggested that he had been working on it for several years and that the discovery could not have come about without work of several individuals. His discovery though in an advanced state of academia, has implications for elementary students. Earlier in the article, thoughts by Sriraman (2005), Polya (1954), and Hadamard (1945) suggest that the chief difference in creativity among high level academics and elementary students may merely be one of degree (of complexity). In addition, Grégoire illustrates that an advanced state of maturity or development in additional affective characteristics might enhance the likelihood of MC emerging. As an example, a high degree of motivation, as well as openness, flexibility, self-confidence, and autonomy appear to be inherent in individuals that have recurring creative output.

In a cursorily related study on affect, divergent thinking, and mathematics achievement among elementary age girls, Wallace and Russ (2015) found that having a wide array of positive affective states resulted in increased divergent thinking and mathematics achievement. In turn, such qualities resulted in high levels of original thinking in mathematics. The caveat with this longitudinal study was that it was conducted with a very small *n* of participants. Initially, 46 girls participated in the study, but 4 years later, only 31 of the original participants were able to contribute data.

In following one facet of Grégoire's study, Greensfield and Deutsch (2016) studied positive mathematical emotions among 12 of the top female participants in the Israel International Math Competition for Girls. Using extensive interviews, one finding was that foundational experiences laid in elementary (childhood) years were a prerequisite to positive emotional states towards mathematics. Female mathematicians interviewed in the study described how seminal childhood experiences (e.g., working on puzzles with parents, doing quizzes, engaging in thinking games) provided them with opportunities to realize the importance of mathematics in their family. Other characteristics that were mentioned by participants were determination and persistence, an enjoyment of doing mathematics, and a fascination with tasks that may elicit creative process and product. It was hypothesized that being able to direct positive emotions in mathematical settings comes at the expense of inordinately negative emotions, which may ultimately result in overall positive experiences in mathematics. This, in part, explained these eminent young females' success in the high-level mathematics competition.

Studies such as those discussed may have encouraged Syaiful et al. (2020) to investigate the relationship between one's emotional quotient – ability to manage and use one's own emotions – and their creative output in mathematics. In using two instruments, one to assess creative output and one to assess one's emotional quotient, it was determined, with an *n* of 82 junior high students, that the emotional quotient scores explained 71.6% of the variance in creative performance, while the remaining 28.4% was explained by other variables. In short, such data suggest that one's emotional quotient plays a considerable role in one's creative output. Though the study was conducted with students barely senior to upper elementary, it is believed that this study has implications for elementary students' affective states, their development, and their creative output.

In another study, Bonnett et al. (2017) investigated the effect of multiple factors, including the manner in which encouraging problem solvers to explore alternative strategies in creativity-based problems may refine mastery-oriented goals. Utilizing an *n* of 24, 12 boys and 12 girls, with an arithmetic mean age 8 years and 9 months, the researchers found that their approach not only helped students see learning as an ongoing and iterative process (i.e., refining mastery-oriented goals) but also had many other positive affective attributes that emerged as by-products. As an example, increased levels of metacognition, persistence, and the ability to deal with messiness or unexpected results, thus yielding problem solver comfort, were resultant effects of the experience. Likely, the greatest criticism of the investigation was the very low *n* of participants. However, the results are encouraging and the investigation was conducted with grade 3 students, so generalizability to elementary students may not be in question.

## 6.5 Next Steps: Answering Some of the Field's Most Immediate Questions

This review on elementary MC was organized into two main categories: academic-oriented research and practice-oriented research. Both warrant a closing discussion to bring light to the next steps in terms of moving the field forward and answering questions that still exist.

### *6.5.1 Promising Directions for Academic-Oriented Research on MC for Elementary Students*

Considering the current state of extant literature, a few promising areas of academic-oriented researcher exist that warrant future investigation. One pressing direction of future research surrounds the aforementioned topic regarding the relationship between age, development, and MC at the elementary age. Part of the remaining challenge moving forward with this topic relates to the rapid development of individuals at this young age. Young children's minds are developing at a rate which is very hard to keep up with from a researcher's perspective. Trying to psychometrically evaluate a students' MC is challenging when every week, month, and year of schooling at the K-6 grade years, makes such a difference in development. A few ways to account for this could be to increase research methods that utilize repeated measures to capture the ongoing change, to more clearly specify age when conducting such research (e.g., 8 years, 2 months, 2 weeks), to apply mixed-effect modeling to account for groups of ages, and to use more longitudinal methods. Another pertinent topic to this question is the state of early childhood research on MC. Although substantive research exists on early childhood creativity in general (see Saracho, 2012 for a comprehensive perspective on creativity and early childhood), there is little empirical research on domain-specific MC for this young age group, with few exceptions (Ariba & Luneta, 2018; Krummheuer et al., 2013; Shen & Edwards, 2017). Increased understanding of how the youngest of minds creatively think about and produce mathematics would support MC researchers trying to understand the K-6 population.

### *6.5.2 Promising Directions for Practice-Oriented Research on MC for Elementary Students*

Like academic-oriented research, several promising areas exist within practice-oriented research that warrant future investigation. One promising direction for future practice-oriented research is understanding the influence that new technologies could have on MC of elementary children. Technology continues to develop and change at a pace that is astonishing. Sixty years ago, teachers began to use overhead projectors to portray instructional material to children. Fast forward, and many children have personal devices with instant access to millions of apps, videos, activities, and flexible learning opportunities; there are even educational robotics and augmented/virtual reality devices. It is utterly unknown how many of these new technological opportunities relate to the MC of young children. Specifically, due to the ephemeral nature of specific technological apps or devices, it is important to start to understand broadly the components or nature of certain technologies that support MC. This would then be able to be more generalizable to future technology products as commercial and academic enterprises continue to revolutionize available products.

Finally, a promising area of future practice-oriented research is presented in Kozlowski and Chamberlin (2020), in which they discuss a possible combination of classroom factors (e.g., tasks, environmental aspects, discourse) that may produce a holistically creative environment. Current research is being conducted to investigate disparate classroom practices and their relationship to MC (e.g., a specific task); however, what is missing is the evaluation of multiple classroom factors and how their concomitant use impacts MC. For example, is offering an open-ended task enough to really foster MC, or do certain didactic contracts also need to be in place, coupled with specific affective characteristics, to truly support MC? Another thought is that open-ended tasks may enhance the likelihood of mathematical creativity emerging, but alone, they do not guarantee it. These questions are not yet answered and yet are pivotal in truly understanding how to support students' creativity in mathematics.

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# Chapter 7
# Literature Review on Empirical Findings on Creativity in Mathematics Among Secondary School Students

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For several years, research in mathematics education has increasingly focused on the area of creativity. There are several reasons for this development; for example, the importance of the free and creative individual engagement of students with mathematical problem solving is gaining recognition (National Council of Teachers of Mathematics, 2005; Sekretariat der Ständigen Konferenz der Kultusminister der Länder in der Bundesrepublik Deutschland, 2015). Additionally, social transformation requires a high degree of creative abilities and skills in the context of twenty-first-century skills when considering increasing technological needs.

Theoretical discussions as well as empirical research are increasingly devoted to the topic of creativity in teaching and learning mathematics. And here quite different foci are set. Often, the respective research is concerned with the relation to giftedness or special achievements. But also, the (further) development of instruments to measure creativity is often considered. In other areas, creativity is seen as an important part of reasoning processes. The challenge in this chapter is to maintain an overview in this increasingly complex literature landscape.

Previous research in systematizing this thematic area has tended to focus on different approaches to creativity in mathematics (Sriraman, 2009), illustrate what perspectives can be taken (e.g., focusing on products or on processes) (Pitta-Pantazi et al., 2018), or to explore more the basic theoretical assumptions that underlie current research (Joklitschke et al., 2021).

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However, we see a lack of research regarding an overview of current, empirical research on mathematical creativity in mathematics education and its respective findings. We believe that it can be a great added value for both researchers and teachers to have access to a systematic overview. Since both the thematic domains and the mathematical knowledge vary according to the age level, it makes sense to focus likewise in this systematic view of creativity research. This could provide a first insight, for example, for the further orientation of one's own research, or one could get first ideas about which tasks and problems are used to capture or also promote mathematical creativity in students. Therefore, our research aim is to provide a systematic overview of the current, empirical insights on the topic of creativity in mathematics education among secondary school students.

## 7.1 Theoretical Background

The study of creativity is attracting growing interest (Craft, 2003; Hersh & John-Steiner, 2017)—not only because it is increasingly a matter of fostering students individually but also because society is striving for solutions to problems, which is becoming ever more complex. New demands require extraordinary solutions. Here, for sure, creativity is sought. When we think of cyber security, big data, or artificial intelligence, for example, mathematics plays a very central role (Leikin & Pitta-Pantazi, 2013). These two goals, the fostering of the individual and the development of society, should of course also be addressed at school—twenty-first-century skills can serve as a guideline for these two goals and help to identify key competencies (Binkley et al., 2012). Although there is no clear definition of the twenty-first-century skills, a comparison of various classifications shows that creativity, among other things, is always listed, and therefore, a central element (Maass et al., 2019).

Along with this increase in importance, research on creativity in mathematics education is also growing (Leikin & Pitta-Pantazi, 2013). And with this, the theoretical foundations and findings are multifaceted (Leikin & Pitta-Pantazi, 2013; Sriraman, 2009). As in other disciplines, this diversity of research can lead to a feeling of disorientation. Here, systematizations and literature reviews can help to better guide the research.

In this regard, one way of systematizing this heterogeneous research landscape of creativity was elaborated by Rhodes (1961) and adapted in mathematics education by Pitta-Pantazi et al. (2018). Rhodes, in his work on creativity and in his search for a definition of creativity, elaborated four strands, which he calls the four P's of Creativity. Those four P's stand for Person, Process, Product, and Press. Creativity can thus be seen as a personality trait of a *Person*. The *Process* focuses on the ways of thinking that can be considered creative. The *Product* view considers the outcomes and *Press* is understood as the interaction with the environment. With this classification, research on creativity can be studied more focused and also communicated more transparently. Moreover, Rhodes added that "only in unity do the four strands operate functionally. It is this very fact of synthesis that causes fog in talk about creativity" (Rhodes, 1961, p. 307). Pitta-Pantazi et al. (2018) adapted this systematization to mathematics education research and cited a number of research papers under the respective strands. However, this collection does not represent a comprehensive overview.

Another overview is given by Sriraman (2009). Sriraman (2009) presents selected studies that are sorted according to their approach. Six approaches are distinguished: the mystical approach, the pragmatic approach, the psychodynamic approach, the psychometric approach, the cognitive approach, and the social-personality approach. Following the systematization of creativity by Sternberg and Lubart (1999), Sriraman's work classifies studies in mathematics education research along these approaches. However, so far there is no such overview that sorts the current research accordingly or identifies future directions of research.

Another review is provided by Joklitschke et al. (2021). By means of a systematic protocol, the authors analyzed journal articles from 2007 to 2019 and elaborated the theoretical foundations on which current mathematics education research in the field of creativity is based. With this approach, five essential notions were identified:

1. Creativity as Flexibility, Fluency, and/or Other Characteristics: This notion traces back to the psychologist and intelligence researcher Guilford (1967) as well as to the psychologist Torrance (1974) and describes conceptualizations in which creativity is understood, for example, through fluency, flexibility, and originality (as, for example in mathematics education in Leikin, 2009).
2. Creativity as Divergent Thinking also derives from Guilford (1967) and describes that creative thinking is primarily characterized by thinking in different directions. In some other understandings of this notion, it is emphasized that both aspects are indispensable for creative thinking: divergent and convergent thinking.
3. Creativity as a Sequence of Stages focuses mainly on the creative process, in different stages can be identified: According to the mathematician Hadamard (1945), these stages are Preparation, Incubation, Illumination, and Verification. Mathematics education researchers such as Liljedahl (2013) use this view, emphasizing the affective component of the "aha!"-experience in students.
4. Creativity in the Sense of Creative Mathematical Reasoning (CMR) stems directly from mathematics education and focuses on reasoning processes of learners (Lithner, 2008). Here, reasoning sequences are newly formed or re-created. This contrasts with imitative reasoning, in which remembered or algorithmic reasoning sequences are recalled and
5. Person-, Product-, Process-, and/or Behavior-Based Notion of Creativity includes conceptualizations based on the 4 P's of Creativity explained above (Rhodes, 1961).

Joklitschke et al. (2021) present a comprehensive literature review that provides an up-to-date overview of the currently prevailing basic theoretical assumptions in mathematics education research.

However, what is missing so far is an overview that is similarly systematic and complete, but relates to the empirical implementations of current research. We feel there is a need to systematize the current research topics, the current research findings, as well as the specific tasks that characterized the research domain.

Therefore, we would like to systematically aggregate and present current findings in mathematics education research. For this purpose, we have conducted a systematic literature review. This has the substantial advantage that the different steps starting with a guiding research interest, through a clear data acquisition and to data analysis and its interpretation to condensed results are presented in a methodologically clear way and are thus easily traceable. Since there is evidence that mathematical creativity can also depend on prior knowledge (e.g., Tabach & Friedlander, 2013), it makes sense to narrow such a review with respect to age level. In this chapter, therefore, the focus will be on secondary school students. Especially with regard to twenty-first-century skills, this age group is particularly interesting. At the end of their school career, they decide on their future professional career and this decision is influenced by their personal dispositions as well as by the current demands of society. Here, their skill of creativity can also play an important role.

With this approach, we would like to answer the following questions:

1. How and in what contexts (e.g., other constructs) is creativity in mathematics education research assessed in secondary school students?
2. What are the main results of current research in mathematical creativity?
3. What particular tasks and problems are typically used to capture mathematical creativity?

## 7.2 Methods

The aim of this paper is to provide a systematic overview of the current, empirical insights on the topic of creativity in mathematics education among secondary school students. Other publications have already approached the topic of mathematical creativity and presented current theoretical references in research (Joklitschke et al., 2021). We build on this dataset in the present work. In the review at hand, we will now attend to the empirical parts of the respective publications. We used mathematics education-related databases as well as psychologically oriented databases to find all relevant articles published in journals in the time from 2007 to 2019. For this purpose, we used the following search terms:

- creative*
- aha*
- divergent think*
- illuminat*
- invent*
- innovate*

- overcom* fixation
- bisociat*

After performing the search and excluding doubled entries, our search resulted in 473 articles (see Fig. 7.1, left column). These articles were then further filtered using title, abstract, and keyword. Articles that did not mention creativity as an essential component were discarded. For example, when it is clear that the issue is an everyday understanding of creativity, or when it is about creative teaching, or when it is clear that creativity is used as a motivation (for example, in the context of twenty-first-century skills) (as in Duijzer et al., 2019). In a next step, the articles were read and excluded if they did not contain creativity as a central topic. In the last step, the school level was considered, so that articles were sorted out that did pertain to secondary school level—samples composed of different school levels were also included. If this information was not explicitly given, for example, because only the age of the students was apparent, then a brief research on the school system in the respective country was conducted and the school level was estimated. Finally, after several steps of sorting those articles and—for this chapter—to further narrow the literature with regard to the grade level (secondary school level), we finally included 22 articles to this literature review. The search procedure is illustrated in the flowchart in Fig. 7.1.

At this point, a big difference between the initially found articles and the finally included articles can be noticed. On the one hand, this was due to the fact that the search with the specified word stems also found articles that, for example, report on an *invent*ory or use phrases such as "Fortunately, theory and research illuminate learning trajectories that help all children meet these standards" (Clements et al., 2019, p. 11). On the other hand, the term creativity is also a kind of umbrella term that is often used in everyday language.

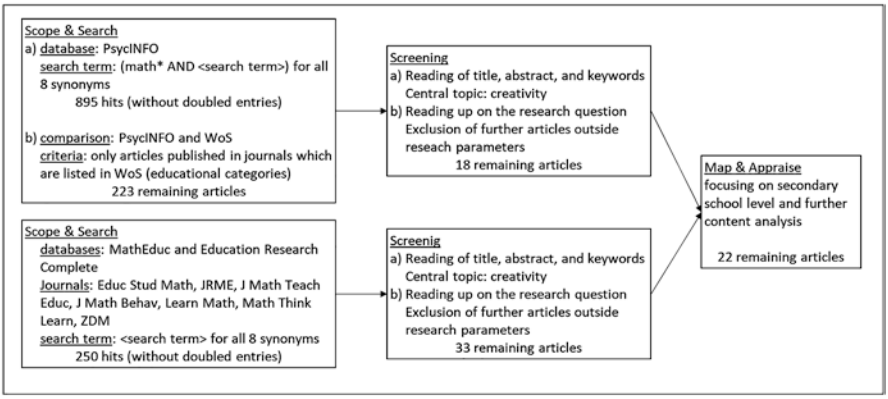



**Fig. 7.1** Flowchart of search procedure. In total, 22 articles were included for this review. The first and second columns stem from Joklitschke et al. (2021)

## 7.3 Data Analysis

Since we intend to provide a summary synthesis rather than to present the existing literature in its breadth and heterogeneity, as was the case with Joklitschke et al. (2021), we follow the guidelines for aggregative literature review according to Newman and Gough (2019) with our analysis: “aggregative synthesis logic focusses on the minimization of bias and thus selection pays particular attention to homogeneity between studies” (Newman & Gough, 2019, p. 5). We intensively read all articles, coded, and summarized them with regard to different categories. Those categories provided partially very general information (e.g., short title, or the country where the study was conducted), and partially arose directly from our research interest (e.g., which tasks were used to assess creativity?). With this focus, all articles were summarized using the following categories:

- Short title
- Country where study was conducted
- Country of first author
- Research question/aim of research
- Research design
- Sample (age, class, number of students)
- Qualitative/quantitative/mixed methods/other
- Methodology
- 4 P’s (which P is addressed? Person, Product, Process, or Press)
- Tasks for assessing mathematical creativity
- Data analysis with focus on parts regarding creativity
- Main results

To follow our research interest, we qualitatively analyzed the data (Mayring, 2015). For this, we looked for commonalities and also special characteristics in each category. These findings were then discussed in a panel of experts. In doing so, we mainly followed the criteria of a qualitative content analysis and subsequently, inductively formed different clusters, which are presented as “perspectives” in the following results.

## 7.4 Results

In this section, we will discuss the current research landscape on the topic of creativity in mathematics education. For this purpose, different focal points (we call them *perspectives*) are set, which we will discuss in depth in the following sections.

It must be noted that the perspectives listed are not intended to constitute disjoint sets. Rather, they are clusters with respect to various criteria that we consider relevant. Therefore, some papers are listed in more than one section. We would also like to stress that this review is a synthesis of existing literature. This automatically means that the individual empirical findings published in the respective articles have

**Table 7.1** Overview of the distribution of articles depending on research approach. The third column focuses of Rhodes' 4 P's of creativity

| Research approach | Number | 4 P's |
|---|---|---|
| Quantitative | 14 | 10 × product<br>3 × product → person<br>1 × product → process |
| Qualitative | 6 | 4 × process<br>1 × process → person<br>1 × person and press |
| Mixed methods | 2 | 1 × product<br>1 × (person; product → person) |

been greatly condensed and thus cannot be adequately reproduced in their entirety in the review at hand. For detailed questions, we therefore refer the reader to the original studies themselves.

Twenty-two papers were included in our final analysis. The majority, namely, 14 of these, were quantitatively oriented (see Table 7.1). In addition, there were six articles that had a qualitative research approach. The remaining two articles (Kim & Kim, 2010; Levav-Waynberg & Leikin, 2012) took both quantitative and qualitative methodological approaches (mixed methods) in both articles; however, the quantitative part predominated. The articles were also analyzed in terms of the 4 P's of creativity (right column) and therefore specify the focus that the empirical parts of the articles pursued (arrows indicate that there is an inference from one P to another P). Products were used exclusively to conduct quantitative analyses and, in two cases, these were used to infer other constructs (once to the person and once to the process). In qualitative research, on the other hand, process analysis is predominantly used, and mixed methods studies address different domains of the 4 P's.

In addition to this first, quite general, overview, various perspectives on current research are taken. First, we analyze articles that focus on a deeper understanding of creativity and consider models of creativity (Perspective I). Subsequently, we provide an insight into the relationship with other constructs and consider particular treatments (Perspectives II and III). In Perspective IV, we pertain to further noteworthy studies and present their results. In the concluding Perspective V, we take a step back from the methodological approaches and the empirical findings and present typical problems and tasks that were used to assess mathematical creativity.

### 7.4.1 *Perspective I: Understanding Creativity and Validation of Creativity Models*

Of the 22 included articles, 4 reflect on creativity, either by investigating creativity in isolation (i.e., without examining other constructs) or by establishing models of creativity and validating them. In this perspective, both qualitative and quantitative works can be found.

An example of a qualitative article that belongs to this perspective is Meyer (2010). The article uses abduction to provide a theoretical framework and to analyze the creative processes. With this work, Meyer leans on Peirce's theory of induction, deduction, and abduction (cf. Peirce, 1998), whereby "the abduction is the decisive inference for the discovery of mathematical coherences, which are not implied in the (construction of the) premises" (Meyer, 2010, p. 202). With focus on creativity, Meyer states that reconstruction of an abduction gives information of the degree of students' discovery and therefore also of the degree of creativity. For example, if the rule of an abduction is not known to the student, and the student "invents" this rule in the abductive process, as well as the case as it is characteristic for the abduction, one can speak of "creative abduction."

Likewise, Palatnik and Koichu (2019) focus on the examination of creativity in a qualitative way. Using a single case study, the authors show what conditions are needed to obtain what they call "flashes of insights." They argue that such a flash of insight can occur when both intellectual triggers (a cognitive challenge) and emotional triggers (such as the pressure to fail a course) come together. The discussion is strongly reminiscent of Hadamard's process model, who calls the moment of idea generation illumination (Hadamard, 1945; see Chap. 2 for more information).

The quantitative articles assigned to this perspective are about model and test validation to capture creativity. However, it should be noted that in both cases other constructs are included.

For instance, in Peng et al. (2013), classroom goal structures and self-determination and how they could affect creativity are investigated (see Fig. 7.2; also for sample items). Creativity is evaluated by Peng and colleagues using two components: overcoming fixations and divergent production (by means of fluency, flexibility, and originality) as described by Haylock (1987). To give a small insight into how classroom goal structures and self-determination are assessed, an illustrative example for each component is given below.

- Mastery approach goal structure: "Maths teachers care about whether we master or understand the learning materials, instead of our test scores."
- Mastery avoidance goal structure: "Maths teachers often ask us to avoid making mistakes in mathematical assignments."
- Performance approach goal structure: "Maths teachers are most concerned about how to increase our mathematical scores."
- Performance avoidance goal structure: "Maths teachers tell us that the purpose of learning maths is to avoid being regarded as incapable."
- Autonomous motivation: "I enjoy doing my mathematical homework a lot."
- Controlled motivation: "I do mathematical homework because I want avoid being punished by my teacher." (p. 57).

It was found that classroom goal structures have a significant impact on students' self-determination and this again has a significant impact on both overcoming fixation and divergent production. More precisely, "mastery-approach is the classroom goal structure that exerts the greatest effect on creativity via autonomous

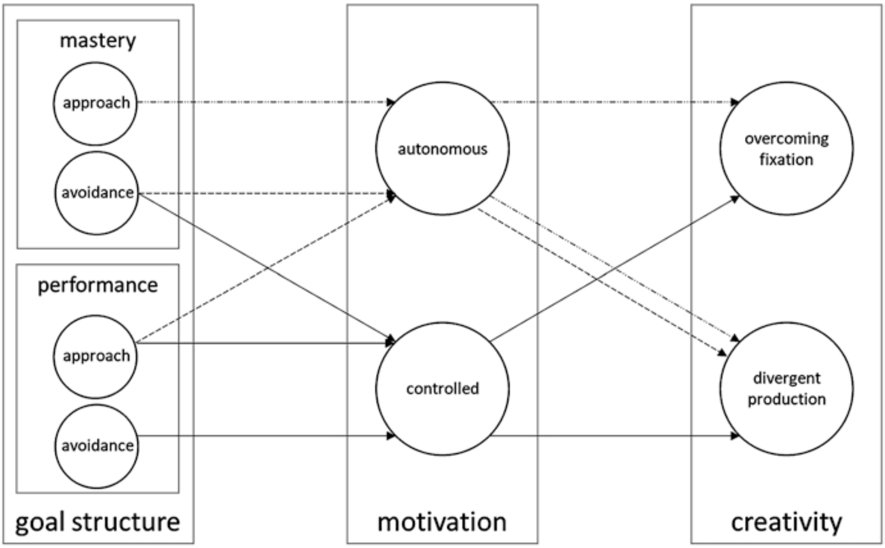



**Fig. 7.2** Simplified model of classroom goal structures (left column), self-determination motivations (middle row), and mathematical creativity (right column) (Peng et al., 2013, p. 56). Notes: The dashed lines as well as the lines that are dashed and dotted show a significant correlation (explanation in the paragraph above). For the purpose of clarity, the relations between the different classroom goal structures have been omitted

motivation" (see dashed and dotted line). Mastery-avoidance and performance-approach have a significant effect on divergent production then, only if they significantly influence autonomous motivation (purely dashed line). Interestingly, the performance-avoidance goal structure, when it affects controlled motivation, has no effect on creativity.

The model validation of Ayas and Sak (2014) is quite different from the other articles. Even though this is also a domain-specific view of creativity, the focus is not on mathematic creativity, but is defined more broadly as scientific creativity in five areas, one of which is called change graph, which is labelled as interdisciplinary. In all these areas, creativity is assessed via open-ended problems and products are quantified by the components fluency, flexibility, and originality (the total score is calculated using logarithm functions, among others). The model was tested with $N = 693$ sixth graders participating in an educational program for gifted students. Using a confirmatory factor analysis, this test (called C-SAT; Creative Scientific Ability Test) was to be validated and "can be used as an objective measure of scientific creativity both in research and in the identification of scientifically creative [sixth grade] students" (Ayas & Sak, 2014, p. 195). As a second focus, the relationship with mathematical achievement is reviewed in this article and it was shown that there is a positive relationship between the creativity score and both the math grade and the performance on a test of mathematical talent.

### 7.4.2 Perspective II: Relation and Correlation to Other Constructs

The great majority of articles put creativity in context with at least one other construct being researched. Since creativity has been systematically conceptualized, strong relationships to constructs such as intelligence or giftedness have repeatedly emerged. These relationships are also still evident in current mathematics education research.

In a classical correlation study (with one-way ANCOVA analysis), Chen et al. (2016) found that mathematically and scientifically talented students ($n = 84$; from senior high school and university) perform better in mathematical divergent thinking ability tests (which are also classically used to assess creativity) than non-gifted students, regardless of their intelligence. However, other studies examining the relationship with giftedness do not assume independence of intelligence (Leikin et al., 2017). In this study, a group comparison was conducted between students which were either (1) non-gifted and excelling in school mathematics; (2) generally gifted and excelling in school mathematics; or (3) were super-mathematically gifted. The comparison was conducted along three dimensions, namely, (a) domain-general cognitive traits; (b) domain-specific (mathematical) creativity; and (c) neuro-cognitive functioning expressed in event-related potentials (ERPs). Methodologically, non-parametric Kruskal-Wallis test and Mann-Whitney tests were used. Even though the most relevant results apply to the characterization of different types of super mathematically gifted, it should be emphasized that the results also showed that they strongly depend on the different tasks. Thus, an important issue for interpreting creativity scores in relation to intelligence might be the nature of the creativity task presented to the students. In contrast to Chen et al. (2016), creativity tasks used to obtain those results from Leikin et al. (2017) were particularly challenging, insight-based tasks. Furthermore, those insight-based MSTs might also be "beneficial for both the identification of mathematical giftedness and the ability grouping process" (Levav-Waynberg & Leikin, 2012, p. 85). The task dependency and the influence of intelligence were also confirmed in a study by Leikin and Lev (2013). In addition, correlational studies have been used to investigate the extent to which different levels of knowledge have an influence on creativity. In this regard, a quasi-longitudinal study with $n = 76$ students from elementary school (fourth grade) to junior high (ninth grade) provides insight, showing that an increase in mathematical knowledge (over learning time in several school years; from fourth to ninth grade) also causes creativity to increase (Tabach & Friedlander, 2013)—with one exception: in eighth grade, creativity seems to temporarily decrease. This phenomenon can be explained by the increased processing of creativity tasks with algebraic approaches. In the long run, creativity performance seems to benefit from the strongly algebraic learning material in eighth grade (Tabach & Friedlander, 2013).

### 7.4.3 *Perspective III: Reflecting on Instructions and Interventions*

As seen above, there are various studies using group comparisons to examine correlations and relationships between creativity and other different constructs—such as creativity and mathematical knowledge. When reviewing the included articles, it turns out that the setting or the way of instruction also plays a prominent role. This is a focus in this section.

Levav-Waynberg and Leikin (2012) inquired into the influence of extra geometry instruction (in terms of geometrical Multiple Solution Tasks) and its consequences on (among other things) students' creativity. In a pretest-posttest design with control group ($n_{\text{experimental}} = 229$, $n_{\text{control}} = 74$; tenth grade), classes of the experimental group worked continuously with MSTs over a period of one year. The work on these special creativity problems not only promoted fluency and flexibility but also their so-called *connectedness* (the relative number of theorems used). Furthermore, the study showed that the third typical component of creativity, originality, was independent of the intervention, indicating that this aspect seems to be rather stable.

As outlined in the systematic literature review on theoretical assumptions and related notions, creative mathematical reasoning also seems to play an increasingly prominent role (Joklitschke et al., 2021). For the theoretical basis of this notion, see Lithner (2008). In an intervention study ($n_{\text{CMR}} = 25$, $n_{\text{AR}} = 23$; all students attended upper secondary school or universities) with matched pair design, Norqvist et al. (2019) investigated to what extent and why learning through creative mathematical reasoning can be more effective than imitative reasoning. In contrast, the pretest-posttest design consisted of only three sessions. The intervention unit consisted of a computer-based practice. The participants' activity was recorded with an eye tracker and for the analysis, these data were used to examine dwell time in relation to different areas of interest. It was found that while the AR (algorithmic reasoning) task group was more successful during the intervention, the CMR (creative mathematical reasoning) group outperformed during the post-test. The eye-tracking analysis offers a possible explanation for this phenomenon, since students in the CMR group focused primarily on illusions. This is considered to be more conceptual and sustainable. Students of the AR group focused more on formula, which has less of a long-term effect (Norqvist et al., 2019).

The concept of creative mathematical reasoning has also been used to investigate the relationship between CMR, students' collaboration, and their use of dynamic geometry software (Granberg & Olsson, 2015). Thirty-six students (aged 16 and 17) worked in pairs on a problem using the dynamic geometry software GeoGebra. Here, special attention was paid to their joint problem space and on the reconstruction of the reasoning sequences. Granberg and Olsson (2015, p. 61) showed that "students used GeoGebra to collaborate and to engage in creative reasoning."

Another study in which the interaction between learners plays a special role investigated "creative reasoning within the shifts of knowledge in an inquiry-based classroom" (Hershkowitz et al., 2017, p. 25). For this purpose, class discussions in a 10-h lesson unit were analyzed in small steps with regard to so-called *knowledge agents*, who provide a certain chain of reasoning, and the *followers*, who take over those of reasoning processes. The classroom-based research showed impressively that all students who used CMR became knowledge agents, whereas students who were not assessed as creative did not necessarily function as knowledge agents.

### *7.4.4 Perspective IV: Articles That Do Not Fit Perspectives I–III*

In addition to the studies presented above, other articles should be mentioned. In three of the articles, the field of autonomy in combination with creativity is examined. One of these studies (Peng et al., 2013) has already been presented in Perspective I. In this study, a model was validated that measures the impact of teaching culture on self-determination motivation and this in turn measures the impact on creativity. In this context, self-determination motivation was assessed by questionnaire and includes two scales, namely, autonomous motivation scale and controlled motivation scale. According to the results, only the autonomous motivation scale, measured by intrinsic motivation and the identified regulation, has a significant influence on creativity, whereas the controlled motivation scale does not show a significant correlation. Furthermore, this study could also be considered classroom-based research, even though the data collection was done through questionnaires and pen-and-paper tests, because, it starts from the effect of the teaching culture.

The second article focuses on the support of gifted students. Here, the relevance of mathematical modeling on (a) creative production ability and (b) self-directed learning was investigated in a 6-month support program. In order to investigate creativity, the mathematical modeling process of the students was examined, focusing specifically on student behavior. It is not precisely clear to the authors to what extent Kim and Kim (2010) focus on processes, behaviors, or products. When conceptualizing creativity, the authors draw on a previously developed model for the concept of creativity, which is not prominent in the literature (Joklitschke et al., 2021)—they conceptualize the creative product production model in mathematics (similar to Renzulli, 2002) composed of rings, namely, mathematical thinking, mathematical knowledge, and mathematical inquiry skill. The self-directed learning attitude was surveyed by means of a questionnaire. Unfortunately, both aspects are hardly connected with each other, but are almost isolated in the context of mathematical modeling. For creativity, a focused group analysis showed that modeling tasks were “problem situation calling for a maximum of creativity” (Kim & Kim, 2010, p. 116). For self-directed learning, a t-test with a comparison group showed that students in the modeling course exhibited a higher level of self-directed learning.

A third article (Kordaki, 2015) also deals with self-directed learning. In this article, the problems that are usually characteristic for creativity research play the crucial role: As variously studied and discussed previously, MSTs are used to investigate, measure, and promote creativity (Kattou et al., 2013; Leikin & Lev, 2007; Pitta-Pantazi, 2017). The researcher investigated the role of MSTs in students' development of multiple representation on a specific computer-based learning environment (Kordaki, 2015). In a comparative study with 20 14-year-old students, it was shown that MSTs, which explicitly demand multiple solutions, "can efficiently support students in expressing themselves to their fullest extent" in the context of this specific digital computer environment (Kordaki, 2015, p. 509).

It is also noticeable that in almost all articles, creativity is assessed through special tasks or tests. In only one paper, creativity is measured by a self-report. Liu et al. (2015) examine differences in creativity between below- and above-average achievers and between classroom and extracurricular instruction. To capture creativity, students completed a questionnaire that was used to represent creativity—one of two items was "I have been challenged to come up with new ideas" (p. 145). Since the database is also quite small despite a large sample ($N = 381$) due to only two items used for creativity, the results are not intended to be further relevant to this review.

### *7.4.5 Perspective V: Problems and Tasks for Assessment*

Following the insights that we have gained in the previous sections about the contexts, settings, and other constructs with which creativity is studied, the focus will now be put to the specific tasks and problems with which creativity is assessed. Of course, we cannot present all tasks that were presented in the included articles. Therefore, we will present characteristic examples that are typical or even used in more than one publication. Tasks that were used to capture other constructs are not presented—the focus is clearly on creativity.

A historical classic among tasks that capture creativity is the 9-dot-grid (see Fig. 7.3). The task is to draw an area with an area of 2 $cm^2$ in a field of nine arranged dots in many different ways (Haylock, 1987).

In this review, for example, Chen et al. (2016) used this task in a further developed form. For the analysis of the students' products, the components fluency, flexibility, and originality were scored to measure creativity, whereby "the scoring system is based on the New Creative Thinking Test developed by Wu et al. (1999)" (Chen et al., 2016, p. 249). It should be stressed that no total score for creativity was determined here, but the scores of the individual components were used to relate them to other scores, such as intelligence. Similarly, this task was also used by Peng et al. (2013). Here, the task was "Draw a figure to form 2 $cm^2$ for its measure of area within a nine-dot square. Please draw as many figures as you can in given 10 minutes" (p. 57). Even though the 9-dot-grid in the two articles does not carry the label of an MST, this task could still be assigned to this class of tasks. Interestingly, no reference is made to this particular type of task anywhere.

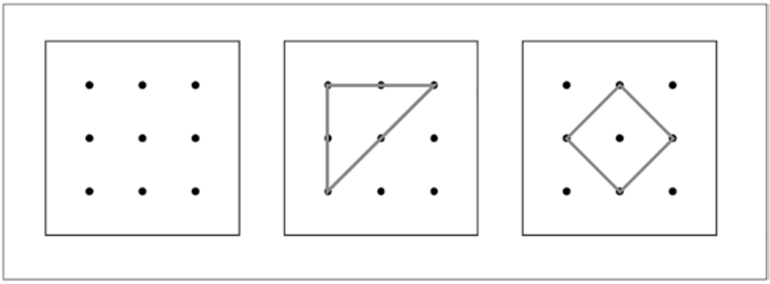


**Fig. 7.3** Example of the 9-dot-grid with two possible, but very common, solutions to the task of finding a 2 $cm^2$-area

Dor and Tom walk from the train station to the hotel. They start out at the same time. Dor walks half of the time at speed $v_1$ and half of the time at speed $v_2$.
Tom walks half way at speed $v_1$ and half way at speed $v_2$.
Who gets to the hotel first: Dor or Tom?

**Fig. 7.4** Example of a text based MST (Leikin et al., 2017, p. 112)

Of course, MSTs were represented in the present review. In fact, in Leikin and Lev (2013) all the problems that were used in this study to measure mathematical creativity are illustrated. The Movement Problem (See Fig. 7.4), which was used in Leikin et al. (2017) as well, was highlighted as a problem that particularly provokes insight-based solutions—as also discussed in Perspective II.

The instruction is the same for all MSTs: "The students were explicitly asked to solve each problem in as many ways as they can" (Leikin et al., 2017, p. 114; Leikin & Lev, 2013, p. 189; and nearly the same for Levav-Waynberg & Leikin, 2012, p. 77). The products of the students are then evaluated with respect to the categories Fluency, Flexibility, and Originality. Fluency refers to the number of solutions the student included. Flexibility expresses the diversity of the solutions. Originality describes the rarity of solutions or the level of insight. Finally, an overall creativity score is obtained, which multiplicatively offsets the flexibility and originality components: $\mathrm{Cr} = \sum_{i=1}^{n} \mathrm{Flx}_i \cdot \mathrm{Or}_i$, for $n$ approaches. Detailed explanations of the calculation scheme can be found in Leikin (2009).

Furthermore, for the assessment of creativity, especially with the help of MSTs, geometric tasks are also frequently used. An example of this can be seen in Fig. 7.5 (Levav-Waynberg & Leikin, 2012, p. 74). Here, the question is to find a proof for the perpendicularity of a triangle in a variety of ways.

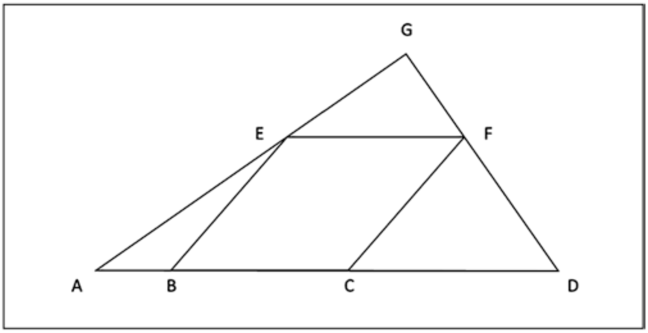



**Fig. 7.5** Example of a geometrical MST (Levav-Waynberg & Leikin, 2012, p. 74). Instruction: "In triangle *AGD*, points *E* and *F* are on *AG* and *DG*, respectively, and points *B* and *C* are on *AD* (see drawing). Given that $EF = FC = CB = BE$, prove that triangle *AGD* is a right triangle." (Levav-Waynberg & Leikin, 2012, p. 74)

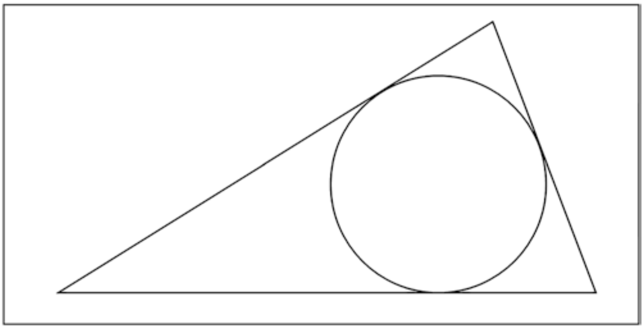
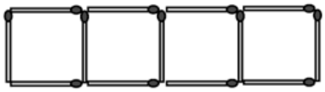


**Fig. 7.6** Examples of a semi-structured problem posing situation (cf. van Harpen & Presmeg, 2013)

A similar approach to MSTs in problem-solving research, as shown above, can also be found in problem-posing research. Posing mathematical problems is often referred to as an act of creative invention (Silver, 1997). In our review, there are two articles that investigate mathematical creativity through problem-posing abilities. van Harpen and Presmeg (2013), for example, asked US and Chinese high school students to pose as many problems as they can that are related to a given picture of a triangle and its inscribed circle (among other problem-posing situations) (see Fig. 7.6). The responses, i.e., the posed problems, were analyzed with respect to the categories Fluency, Flexibility, and Originality, similar to the approach of analyzing responses to MSTs presented above. In the context of problem posing, fluency refers to the number of posed problems, flexibility refers to the diversity of posed problems (e.g., in terms of different mathematical ideas or strategies to be applied), and originality refers to the rareness of the posed problems compared within the solution space of the peer group.

A

When squares are put in a row it looks like the figure to the right. 13 matches are needed for four squares.



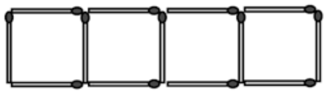
If x is the number of squares then the number of matches y can be calculated by the function
y=3x+1

Example: If 4 squares are put in a row then
y=3x+1=3·4+1=13 matches are needed.

**How many matches are needed to get 6 squares in a row?**

B

When squares are put in a row it looks like the figure to the right. 13 matches are needed for four squares.



**How many matches are needed to get 6 squares in a row?**

**Fig. 7.7** Examples of a (**a**) AR task and a (**b**) CMR task

As shown in Perspective III, some articles also deal with the processes of mathematical creativity, with the help of a certain intervention. In this context, creative mathematical reasoning has already been discussed. A typical task that has been published more frequently (sometimes with slight modifications) in this context involves laying a row of squares of matchsticks. In Jonsson et al. (2014), for example, performances of students who were given either algorithmic reasoning or creative mathematical reasoning tasks were compared. In Fig. 7.7, two analogous tasks can be seen for this purpose, illustrating the difference between these tasks that induce either AR or CMR (Jonsson et al., 2014, p. 24).

## 7.5 Discussion and Outlook

Current research on mathematical creativity in secondary students looks widely diverse. In order to bring some organization to this complex array of the research landscape, a systematic, aggregative literature review (cf. Newman & Gough, 2019) was conducted that highlights commonalities in the analyzed articles.

It is interesting to note that few publications in the recent period (2007–2019) have been devoted to the topic of creativity by itself. One possible explanation would be that researchers believe the construct has already been sufficiently explored. However, this is implausible as creativity has been identified as a very complex phenomenon (Chamberlin, 2020; Joklitschke et al., 2021; Kozlowski et al., 2019; Kupers et al., 2019). Another explanation could be that there is currently a lack of methods to further explore creativity compared to established approaches. New methods could include EEGs (electroencephalogram; a recording of brain activity), CT scans, or eye tracking, all of which could help to learn more about creative processes. There are already studies using eye tracking to investigate such processes (Dietrich & Kanso, 2010; Leikin et al., 2017; Norqvist et al., 2019; Schindler & Lilienthal, 2020, 2022).

Summarizing the current state of research reveals a dominance of quantitative research. Out of twenty-two articles, fourteen are quantitative, six are qualitative, and two use a mixed-methods approach, both with a quantitative focus. Referring to the *4 P's of creativity* (Rhodes, 1961), there is an obvious relation: All analyzed quantitative studies use products as a data basis. In a few cases, this is used to draw conclusions about the processes or the person. The qualitative studies focus in five cases on the process, whereby, in one of them, this process is used to propose statements about the creativity of the person. In a sixth qualitative article, the focus is on the person and press. This distribution seems to be relatively stable for the period under consideration, so that no trends can be derived that would indicate a shift to qualitative research in the future. What is also noticeable is that the focus in the respective publications on one of the *4 P's* does not always seem to be consistently implemented. For example, by analyzing products, statements are made about the creativity of products and inferences are drawn about the creativity in the process. However, such a conclusion cannot be derived in every case (Liljedahl & Rott, 2017).

From a content perspective, the focus of current research on mathematical creativity on secondary school level is regularly dedicated to the investigation of correlations to other psychometric constructs—most often intelligence, giftedness, or mathematical achievement. Such studies investigating the connection to other constructs were mostly quantitative in nature and exclusively focused on products or inferred from products to processes or to persons. Investigating specifically these constructs is plausible since creativity has often been conceptualized as part of intelligence. Furthermore, it should be acknowledged that there are easily accessible measures for the most frequently considered secondary constructs, such as IQ, the grade in class, or participation in courses that require a high level of mathematical knowledge. Nevertheless, results of such studies are inconsistent—as, for example, in the case of the relationship between intelligence and mathematical creativity. A possible explanation for this can be found in the methodological implementation, since the tasks for assessing creativity differed greatly in their requirements (compare problem settings in Chen et al., 2016, and in Leikin & Lev, 2013).

Regarding methods for measuring mathematical creativity, the majority of researchers appears to agree that this construct cannot validly be measured indirectly, that is via the use of questionnaires, but instead product evaluations or process analyses are needed.

Taking a step back from classical correlational studies and looking at publications that are focused on a specific setting, the methodological approaches are definitely more heterogeneous—as both qualitative and quantitative approaches are used in this regard. It is also understandable that results in this area that can hardly be summarized in a review. This is due to the fact that the research interests in this area are very wide-ranging and therefore the analyses are very heterogeneous. Overall, it is striking that there seems to be low research interest on topics, in which a particular setting/treatment is in the foreground. Classroom-based research or the interaction between students or students and teachers are rarely studied, at least.

Nevertheless, there are first indications that, for example, working with tasks that intend CMR could be beneficial for students' learning, discussing, and thinking (Hershkowitz et al., 2017). While there is no clear trend towards additional research focusing on interactions and collaborations, it is clear that ample research opportunities in this area exist. With caution, one could therefore hypothesize that interaction studies, collaboration studies, or classroom-based research could become frequent subjects of future studies, since there are already first steps in this direction and there are also recent publications that were not part of the review, but dedicated to these connections between creativity and, for example, collaboration (Khaliq & Rasool, 2019; Lee et al., 2021; Levenson & Molad, 2022; Schindler & Lilienthal, 2022).

There are two further, probably important, results when considering the widely used conceptualization of mathematical creativity as fluency, flexibility, and originality. First, originality seems to be a relatively personality-stable component, whereas fluency and flexibility are trainable over time (Levav-Waynberg & Leikin, 2012). Second, in the synopsis of the tasks belonging to the notion of creativity composed of fluency, flexibility, and originality, it is noticeable that they focus exclusively on fluency. We are not aware of any study that systematically examines the instructions for MSTs, which might warrant a close look. Differences that may occur could be investigated if the students were given instructions such as "Find different ways to solve the problem," which would rather focus on flexibility, or "Find several different ways to solve the problem" or even "Find rare ways to solve the problem."

When we relate our findings to the historical traces laid out in Chap. 2 of this book, it is striking that we have seen little to no such historical references in the empirical parts of the reviewed articles. This phenomenon may entail four explanatory options. First, the field of creativity research seems to have evolved rapidly, so that tracing back to the beginnings of the research is no longer considered relevant. Second, it could also be that current research is seen as more or less detached—this hides the danger that important basic assumptions are reinterpreted and may no longer live up to their original meaning, so that concerns about validity could arise. Third, an explanation could also lie in the relationship with other variables studied. As shown, many publications examine the connection with other constructs. Thus, it is also conceivable that creativity is seen and investigated more as a side effect of these other constructs. And fourth, because of the dynamic relevance of creativity (e.g., twenty-first-century skills; Beswick & Fraser, 2019), it could also be that researchers see creativity as a new, emerging construct and may neglect the fact that it has a rich history of work. Referring back to the theoretical parts, we carefully hypothesize that it may be due to the multiple constructs that also need to be given some attention (third reason).

Shortly before the finalization of this chapter, a paper by Leikin and Sriraman (2022) was published that summarizes the current state of the art of empirical research on creativity in mathematics education. Leikin and Sriraman identify three major themes of this research field. These major themes largely confirm the perspectives we have identified in our analysis. In addition, we identified further perspectives, such as Perspective I, the understanding of creativity and validation of creativity models.

Limitations of the present review include the fact that due to the rigorous sampling procedure, only a selection of articles and journals was reviewed. Certainly, the picture of the research landscape would look slightly different if, for example, conference papers, articles from conference proceedings, or even other mathematics education journals had been included. Moreover, the findings obtained are only addressed to the secondary level. A next step would be to obtain results for other school levels and to compare both results. Another important point is to keep the developments hypothesized and discussed research gaps in view and to investigate them further. Some gaps of research and foci that could be considered in the future are listed in the following:

- To better systematize and compare research, it would be helpful if researchers always stated clearly and consistently which of the *4 P's of creativity* their research addresses.
- Future research could focus on creativity in collaborative work (also targeting at *Press*).
- Future research could focus on classroom-based environments (also targeting at *Press*).
- Future research could focus on creative processes similar to those described by Hadamard, for example, by using long-term problems.
- Future research could focus on finding different types of creativity, for example, different process types or different types of persons.

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# Chapter 8
# Mathematical Creativity at the Tertiary Level: A Systematic Review of the Literature

**Miloš Savić, V. Rani Satyam, Houssein El Turkey, and Gail Tang**

## 8.1 Introduction

Van Nuys (2019) shared results from a study in which it was stated that creativity is the most needed skill for employees in companies in 2019 and 2020. Specifically in STEM, careers will be uncertain and require flexibility and, most importantly, creativity (Wilson et al., 2017). Creativity is an important piece of mathematical thinking according to many prominent mathematicians (Borwein et al., 2018; Karakok et al., 2015), and thus is important to foster in future mathematicians. As well, the Mathematical Association of America's Committee on the Undergraduate Program in Mathematics (Zorn et al., 2015) has emphasized the importance of mathematical creativity in its latest guidelines: "[A] successful major offers a program of courses to gradually and intentionally leads [sic] students from basic to advanced levels of critical and analytical thinking, while encouraging creativity and excitement about mathematics" (p. 9). Under Cognitive Goals and Recommendations, the guidelines also state that "[T]hese major programs should include activities designed to promote students' progress in learning to approach mathematical problems with curiosity and creativity and persist in the face of difficulties" (p. 10). Whether the focus is on industry, academia, or the classroom, creativity is ubiquitously important.

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Much of the above motivations for tertiary mathematical creativity fostering lie in the developmental perspective of creativity, that a person's creativity can be developed or fostered. Kozbelt et al. (2010) stated that developmental theories of creativity "help us to understand the roots of creativity, as suggested by the background of unambiguously creative persons, but they also often suggest how to design environments so that the creative potentials of children will be fulfilled" (p. 26). Developmental creativity pertains to the development of creative processes, persons, and press (environment), according to Rhodes (1961), whereas creative products are the end-results: "Although products are not the primary focus of developmental theories, they still play an important, but often tacit, role" (p. 26). Kozbelt et al. (2010) described that in the developmental theory of creativity, studies that have analysis of the creative process, including moments that influenced development of one's creativity, are important, as well as longitudinal studies to examine such development over time.

An abundance of scholarly work pertains to how to develop creativity in the primary and secondary classrooms (e.g., Beghetto & Kaufman, 2010; Starko, 2013), but the tertiary perspective is still growing (Kozlowski & Si, 2019). This literature constitutes an aggregate of the tertiary mathematics education literature on mathematical creativity, allowing researchers in the field to survey and add to the previous studies. Furthermore, because there is a need for enhancing students' creativity in mathematics classroom at the tertiary level, we explored the following research questions through systematic literature review: (i) What is the current state of research on tertiary mathematical creativity? (ii) To what extent is the developmental perspective of creativity present in current research?

## 8.2 Method

In this systematic literature review, we followed the guidelines set out by Newman and Gough (2020). We set our research questions and then searched the literature. Because this is a first review of the tertiary mathematical creativity literature, our review may be considered a "scoping review" (Newman & Gough, 2020, p. 15), as we are not taking a full conceptual framework. Scoping reviews "summarize literature in a topic area" and are an "effective means of highlighting the relevant literature to the researcher" (O'Flaherty & Phillips, 2015). In the second research question, we are using the developmental lens described above, which will be used as an analysis tool and rather than as a selection criterion.

We first used Google Scholar to search for publications in which tertiary mathematical creativity was studied. The first search was conducted with the terms "___ math creativity," where the blank was tertiary, undergraduate, and post-secondary. The second search was conducted substituting the blank with content-specific topics within tertiary mathematics: calculus, graph theory, real analysis, abstract algebra, differential equations, discrete math, precalculus, college algebra. Finally, we substituted the blanks with two terms separately, proof and proving, as they are important mathematical activities in upper-division courses. We restricted the selection of articles to content that is taught in tertiary mathematics and mathematics-only tracks for a focused systematic literature review. Therefore, in the search, we did not include articles about pre-service teachers or any article that used tertiary mathematics as a subset of a general, all-grades mathematics education article, although we acknowledge that there could be an intersection of both pre-service courses and topics such as number theory. This is not to discredit pre-service math courses at all, as they are important in the preparation of future teachers. We also narrowed our results to journal articles, book chapters, and dissertation publications. Accounting for all the criteria above, we found 29 artifacts total.

Each article was then put into a spreadsheet with author(s), title, journal, year, content topic (if specified), methods, results, and any other important information. We then analyzed each column, making observations about common themes.[1] We now present those themes.

## 8.3 Results

The two journals that had the largest number of articles were the Journal of Humanistic Mathematics (JHM, six articles) and Problems, Resources, and Issues in Mathematics Undergraduate Studies (PRIMUS, five articles). The JHM articles were all from a special issue that was guest-edited by our research group, which explains the frequency of articles from that journal. We believe that the number of PRIMUS articles is due to the position of the journal as a practitioner journal in tertiary mathematics, so mathematics instructors interested in mathematical creativity in their classroom may publish here. Each of these 11 articles had a description of how the authors fostered students' creativity in their own courses. For example, Kasman (2014) described a project system, including how they assessed creativity, in a course for students that required a minimum of one math course for graduation (i.e., a general education course). They used a rubric to value several aspects of graph theory or voting problems, one of which was creativity (worth 3 points out of 20). Kasman reported that the creativity in both mathematics and their aesthetics made them "delighted during the grading of these projects" (p. 489). Mayes-Tang (2020) also wrote about a first-year general education course where students created new geometrical concepts and built upon those concepts throughout the course. The author described fostering creativity by prompting the students to find properties or theorems with their created concept, and to present an end-of-semester, semester-long project on their new geometrical concepts, including the semester-long prompt to "find as many properties as you can for your newly-defined creation and formulate relevant theorems about it" (p. 267). They concluded with eight recommendations to implementing creativity-focused courses, including to "look

[1] Data analysis was concluded at the end of August 2020.

for creative moments in each class" and "set a grading structure for creativity-focused assessments that rewards effort and reflection over sophistication of results" (p. 271). Munakata et al. (2021) studied a general education mathematics course that focused on creativity. They studied both student effects, including seeing math differently, seeing math creativity, frustration, collaboration; and teacher effects, including not knowing what to expect and feeling out of their comfort zone. All three practitioner papers had both creativity and terminal math courses. This small number may be due to the experience being the last math requirement for students, coupled with the minimal requirements for content (Kasman, 2014). The other two PRIMUS articles (El Turkey et al., 2018; Omar et al., 2019) are from our research group and are situated within proof-based courses. Both offer a rubric, the Creativity-in-Progress Rubric on Proving (presented in full in Savić et al., 2017)), as a basis for actions in the classroom. The rest of the 18 articles were published in separate journals or books. Of the 29 articles, 27 were written in the last 10 years (2012–2021), of which 17 were written in the last 3 years (2019–2021). This indicates that the field of mathematical creativity in tertiary education is recently growing. Figure 8.1 shows an infographic of articles by year.

The most popular topic out of the 29 articles was calculus, with five articles. Three of the five articles were quantitative, including creating and validating a "learning model based on open-ended questions… to improve students' creativity in calculus learning in a valid and practical way" (Arsyad et al., 2017, p. 144). Mac an Bhaird et al. (2017) coded tasks from business, science, and pure calculus courses using Lithner's (2008) imitative/creative reasoning framework. They found that, in the business and science calculus classes, tasks were mostly imitative, and tasks on tests were almost 100% algorithmic (which is a subset of imitative). The authors

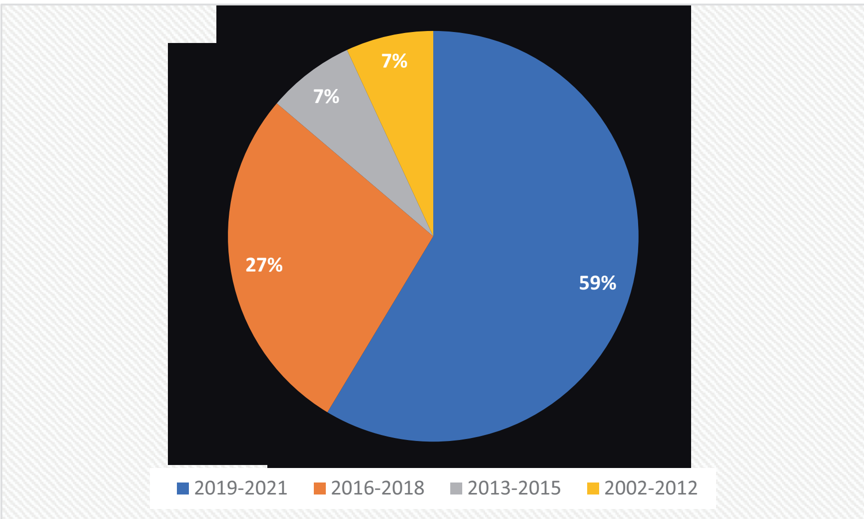


**Fig. 8.1** Tertiary math creativity research by year

end with a reflection on business and science courses, claiming that they "need not have a lower proportion of CR [creative reasoning] tasks" (p. 160). Blyman et al. (2020) discussed a rubric that they used in calculus to assess math creativity with pre-post-semester tasks. They had mixed results and concluded that "evaluating creativity is a difficult task" (p. 169). The other two calculus articles included derivative TACTivities for moving and manipulating derivative calculations (Hodge-Zickerman et al., 2020), and an investigation of Hawaii Algebra Learning Project (HALP) and its strong positive impact on mathematical achievement and creativity (Roble, 2017). In the case of Roble, mathematical creativity was defined by Leikin's (2009) use of Torrance's (1966) fluency, flexibility, and originality categories.

The second-most discussed topics in our literature review were graph theory and combinatorics (four articles). There were two articles on the same course in combinatorics (Karakok, 2021; Omar et al., 2019), and there were two other articles that mentioned problems in combinatorics and graph theory in order to foster creativity (Hoshino, 2018; Zazkis & Holton, 2009). The latter two graph theory articles were (1) a systematic literature review of graph theory with a consideration for how the problems can foster creativity (Suriyah et al., 2020); and (2) an article about how an online application with graph theory fosters creativity (Wahyuningsih et al., 2020).

In our investigation, we found that the most articles (11) were descriptive, meaning that the authors described what happened in their classrooms or courses and how they fostered (or attempted to foster) mathematical creativity (e.g., Marciniak, 2020; Monahan et al., 2020). These 11 articles also did not use qualitative, quantitative, or any other coding techniques. These are separate from the three theoretical pieces that did not use coding (Grégoire, 2016; Hafizi & Kamarudin, 2020; Savic, 2016). For example, Grégoire (2016) claimed there was interplay between the intellectual abilities, personality of the student, and the educational environment. Hafizi and Kamarudin's (2020) main claim was that there was a growth of creativity research in Malaysia specifically in higher education and detailed mathematical creativity research happening in the country. Finally, Savić (2016) combined the theories of problem solving and creativity while discussing proof research.

The next most-used method was quantitative, which had eight publications that studied ways of gauging whether a student was creative, with two articles citing Torrance's Tests for Creative Thinking (Asahid & Lomibao, 2020; Singh & Kushwaha, 2019). There were seven qualitative studies, two of which were not part of our research group. The first one (Roble, 2017) discussed Multiple Solution Tasks (MSTs, Leikin, 2009) and pre-post testing of non-routine problems for achievement, along with student interviews about struggles in mathematics. Adiredja and Zandieh (2020) introduced anti-deficit perspectives in mathematical creativity, noting that students have creative examples of basis in linear algebra, and can generate mathematical creativity collectively as well as individually. Finally, there was a dissertation that had both qualitative and quantitative methods (Regier, 2020). The quantitative part was focused on fostering creativity in the classroom and its impact on self-efficacy. Surveys were created with influence from Cilli-Turner et al. (2019) for the students to gauge how their teachers provided

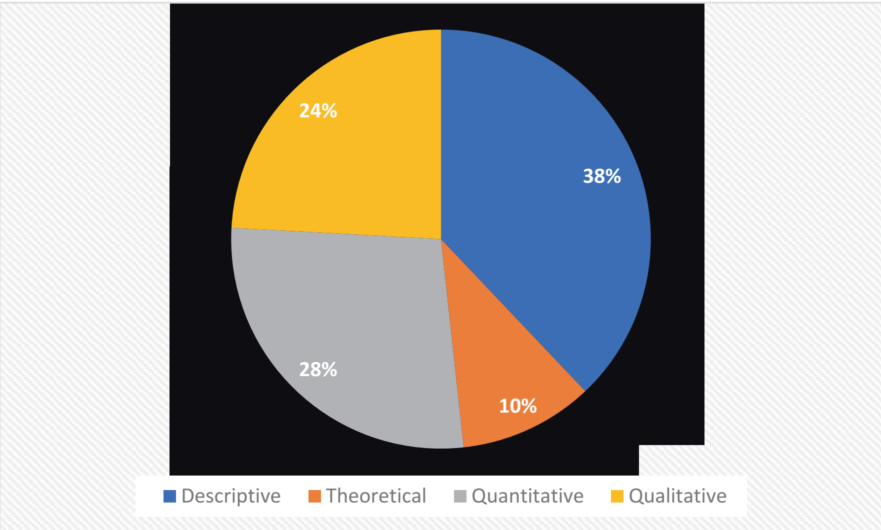


**Fig. 8.2** Tertiary math creativity research by method

opportunities for them to be creative. The qualitative portions studied connections between problem-posing and motivation as well as linked fostering creativity with self-efficacy, which was also presented in Regier and Savic (2020). This indicates a balance of methods for research, which is demonstrated in Fig. 8.2.

Using the Kozbelt et al.'s (2010) definition of developmental theories of creativity mentioned in the introduction, most of the articles found (25 of 29, or 86.2%) took a developmental perspective, meaning that they assumed that creativity could be fostered or developed in classrooms. The four articles that we perceived as not developmental were all quantitative studies (Arsyad et al., 2017; Mac an Bhaird et al., 2017; Singh & Kushwaha, 2019; Tularam & Hulsman, 2015). However, the Arsyad et al. (2017) article was about creating a tool that could ultimately have the impact of increasing students' creativity, which is a secondary outcome of the developmental perspective. This secondary outcome is true for Mac an Bhaird et al. (2017), who wanted to look at reasoning tasks in summative assessments for the development of "mathematical reasoning skills" since it is an "important objective of teaching of mathematics at all levels, in particular at university" (p. 160).

## 8.4 Discussion and Future Research Directions

There is certainly much more that can be done in tertiary mathematics education with creativity. Compared to its place in primary and secondary education, mathematical creativity is new to the realm of tertiary mathematics education. We hope this chapter encourages the field to consider mathematical creativity at this level, including being creative in research methods, tools, and approaches to fostering creativity.

Our systematic literature review showed that very high or high-quality journals[2] have published only one article on creativity (i.e., Regier & Savic, 2020), in our search using the words "undergraduate/tertiary/post-secondary mathematical creativity." This speaks to how new the research subfield is, how much more work needs to be done for mathematical creativity at the tertiary level to be more valued, or how journals need to consider publishing more research on mathematical creativity at the tertiary level. Regardless of the reason, there is a gap between the value of creativity in the three areas of industry, academia, and the classroom with the publication rates of tertiary mathematical creativity. Most of the articles described what happened in a classroom; future syntheses should cross-examine each descriptive article and see the common themes or ideas. Mathematical content was not focused on one topic or area of mathematics. This makes us believe that mathematical creativity can be fostered in any aspect of tertiary mathematics education. In fact, according to Ervynck (2002), mathematical creativity *should* be fostered in *every* aspect of tertiary mathematics education.

There were limitations to this scoping review, including the limit on the keywords in searching. For example, in pre-service mathematics education, which was not considered in this chapter, there have been a number of articles on mathematical creativity, including those that conceptualize what teachers believe as mathematically creative (Bolden et al., 2010; Moore-Russo & Demler, 2018). This research at the pre-service level can have a huge influence on what future primary and secondary students see as mathematics (Aiken Jr, 1973; as cited by Fetterly, 2010). Future systematic reviews will hopefully take this review as a first step towards cataloging and broadening mathematical creativity.

## 8.5 Conclusion

Why is mathematical creativity so important in tertiary mathematics education? For some students, this might be their last experience of mathematics, so there is one last chance to change their beliefs about mathematics as more exploratory (Kasman, 2014; Mayes-Tang, 2020; Monahan et al., 2020; Munakata et al., 2021). For others, they will continue on to graduate school in mathematics, and creativity is a chance for them to feel like mathematicians (Omar et al., 2019). Mathematical creativity can also be a catalyst towards a more equitable classroom (Luria et al., 2017), although much more empirical work is needed to validate that theoretical claim

[2] We are using the rankings by Williams & Leatham (2017) to define a very high or high-quality journal. These journals include Journal of Research in Mathematics Education (JRME), Educational Studies in Mathematics (ESM), Journal of Mathematical Behavior (JMB), ZDM – Mathematics Education, For the Learning of Mathematics (FLM), Mathematical Thinking and Learning (MTL), and Journal of Mathematics Teacher Education (JMTE).

(Kozlowski & Si, 2019). All these reasons for fostering mathematical creativity have at their core a developmental perspective that centers students. Also, all these reasons need more research to understand how mathematical creativity impacts students, including teaching actions that can foster creativity (Satyam et al., accepted) and the impacts on students' affect (Tang et al., accepted). We also need to expand our knowledge from the individual to the collective, thinking of fostering creativity in groups or teams (Heath, 2021) as not many of the articles include this perspective.

Based on the results of this review, we implore instructors of tertiary mathematics, many of whom are mathematicians, to consider a developmental perspective on creativity. Hirst (1971), when discussing creativity in mathematics education, stated:

There must be a recognition that worth-while investigations can take place at a lower level than the full-blown research problem, and the purpose of these must be seen as contributing to the student's mathematical development, and not the furtherance of the boundaries of the subject. (p. 28)

In the 50 years since that quote, we have seen momentum only recently towards this perspective. We hope that by examining this systematic literature review, researchers and instructors can add to the developmental perspective of tertiary mathematical creativity.

## Appendix A: Table of all 29 Articles/Book Chapters Listed by Alphabetical Last Name

| Author | Year | Title | Journal | Content | Methods | Develop? |
|---|---|---|---|---|---|---|
| Adams, Margaret | 2020 | Three Creativity-Fostering Projects Implemented in a Statistics Class | Journal of Humanistic Mathematics | Statistics | Rhodes 4P | Yes |
| Adiredja, Aditya P; Zandieh, Michelle | 2020 | Everyday examples in linear algebra: Individual and collective creativity | Journal of Humanistic Mathematics | Linear Algebra | Qualitative: Interviews, coding for originality of basis, vector space | Yes |
| Arney, Chris | 2002 | Building Creativity Through Mathematics, Interdisciplinary Projects, and Teaching with Technology | Changing Core Mathematics | All | Description of course | Yes |

(continued)

| Author | Year | Title | Journal | Content | Methods | Develop? |
|---|---|---|---|---|---|---|
| Arsyad, Nurdin; Rahman, Abdul; AHMAR, Ansari Saleh | 2017 | Developing a self-learning model based on open-ended questions to increase the students' creativity in calculus | Global Journal of Engineering Education | Calculus | Quantitative | No |
| Asahid, Remelyn L; Lomibao, Laila S | 2020 | Embedding Proof-Writing in Phenomenon-based Learning to Promote Students' Mathematical Creativity | American Journal of Educational Research | Mixed, but students in Diff Eq | Quantitative | Yes |
| Blyman, Kayla K; Arney, Kristin M; Adams, Bryan; Hudson, Tara A | 2020 | Does Your Course Effectively Promote Creativity? Introducing the Mathematical Problem Solving Creativity Rubric | Journal of Humanistic Mathematics | Calculus | Quantitative pre-post problem solving | Yes |
| El Turkey, Houssein; Tang, Gail; Savic, Milos; Karakok, Gulden; Cilli-Turner, Emily; Plaxco, David | 2018 | The creativity-in-progress rubric on proving: Two teaching implementations and students' reported usage | PRIMUS | Transition-to-proof, number theory | Reflections, student work | Yes |
| Grégoire, Jacques | 2016 | Understanding creativity in mathematics for improving mathematical education | Journal of Cognitive Education and Psychology | NA | Theoretical | Yes |
| Hafizi, Mardiah Hafizah Muhammad; Kamarudin, Nurzatulshima | 2020 | Creativity in mathematics: Malaysian perspective | Universal Journal of Educational Research | NA | Theoretical | Yes |

(continued)

| Author | Year | Title | Journal | Content | Methods | Develop? |
|---|---|---|---|---|---|---|
| Hodge-Zickerman, Angie; Stade, Eric; York, Cindy S; Rech, Janice | 2020 | TACTivities: Fostering Creativity Through Tactile Learning Activities | Journal of Humanistic Mathematics | Calculus | Descriptions of projects | Yes |
| Hoshino, Richard | 2018 | Supporting Mathematical Creativity Through Problem Solving | Teaching and Learning Secondary School Mathematics | Graph Theory, Combinatorics | Descriptions of problems | Yes |
| Karakok, Gulden | 2021 | Exploration of Students' Mathematical Creativity with Actor-Oriented Transfer to Develop Actor-Oriented Creativity | Transfer of Learning: Progressive Perspectives for Mathematics Education and Related Fields | Combinatorics | Qualitative: Case-study analysis | Yes |
| Kasman, Reva | 2014 | Balancing structure and creativity in culminating projects for liberal arts mathematics | PRIMUS | Math for Liberal Arts (voting theory, graph theory) | Descriptions of projects | Yes |
| Mac an Bhaird, Ciarán; Nolan, Brien C; O'Shea, Ann; Pfeiffer, Kirsten | 2017 | A study of creative reasoning opportunities in assessments in undergraduate calculus courses | Research in Mathematics Education | Business, Science, and Pure calculus | Quantitative: Coding tasks with Lithner's IR CR | No |
| Marciniak, Malgorzata A | 2020 | Creative Assignments in Upper Level Undergraduate Courses Inspired by Mentoring Undergraduate Research Projects | Journal of Humanistic Mathematics | Differential Equations | Descriptions of projects | Yes |
| Mayes-Tang, Sarah | 2020 | Designing Opportunities for Mathematical Creativity: Three Ways to Modify an Existing Course | PRIMUS | First year seminar | Reflections and end of class discussion | Yes |

(continued)

| Author | Year | Title | Journal | Content | Methods | Develop? |
|---|---|---|---|---|---|---|
| Monahan, Ceire; Munakata, Mika; Vaidya, Ashwin; Gandini, Sean | 2020 | Inspiring Mathematical Creativity Through Juggling | Journal of Humanistic Mathematics | Gen ed terminal course | Description of class, journals, notes of class and focus groups. | Yes |
| Munakata, Mika; Vaidya, Ashwin; Monahan, Ceire; Krupa, Erin | 2021 | Promoting Creativity in General Education Mathematics Courses | PRIMUS | Gen ed terminal course | Description of class, journals, notes of class and focus groups. | Yes |
| Omar, Mohamed; Karakok, Gulden; Savic, Milos; Turkey, Houssein El; Tang, Gail | 2019 | I felt like a mathematician: Problems and assessment to promote creative effort | Primus | Combinatorics | Qualitative study: interviews, classroom artifacts – Best for teaching | Yes |
| Regier, Paul | 2020 | The impact of creativity-fostering mathematics instruction on student self-efficacy and motivation | Dissertation | Multiple | Qualitative, Quantitative | Yes |
| Regier, Paul; Savic, Milos | 2020 | How teaching to foster mathematical creativity may impact student self-efficacy for proving | The Journal of Mathematical Behavior | Introduction to proofs course | Qualitative: Teaching observations, interviews, coding for self-efficacy and sources | Yes |
| Roble, Dennis B | 2017 | Communicating and valuing students' productive struggle and creativity in calculus | Turkish Online Journal of Design Art and Communication | Calculus | Qualitative surveys, MST (Leikin, 2009) after HALP | Yes |
| Savic, Milos | 2016 | Mathematical problem-solving via Wallas' four stages of creativity: Implications for the undergraduate classroom | The Mathematics Enthusiast | NA | Theoretical | Yes |

(continued)

| Author | Year | Title | Journal | Content | Methods | Develop? |
|---|---|---|---|---|---|---|
| Savic, Milos; Karakok, Gulden; Tang, Gail; El Turkey, Houssein; Naccarato, Emilie | 2017 | Formative assessment of creativity in undergraduate mathematics: Using a creativity-in-progress rubric (CPR) on proving | Creativity and giftedness | Introduction to proofs course | Qualitative: Student work | Yes |
| Singh, Ram Dhani; Kushwaha, Sarita | 2019 | Components of Creativity and Mathematical Achievement in Undergraduate Students | Parisheelan | NA | Quantitative | No |
| Suriyah, Puput; Waluya, Stevanus Budi; Rochmad, Rochmad; Wardono, Wardono | 2020 | Graph Theory as A Tool for Growing Mathematical Creativity | Jurnal Pendidikan Edutama | Graph Theory | Systematic literature review | Yes |
| Tularam, Gurudeo Anand; Hulsman, Kees | 2015 | A Study of Students' Conceptual, Procedural Knowledge, Logical Thinking and Creativity During the First Year of Tertiary Mathematics. | International Journal for Mathematics Teaching and Learning | Precalculus | Quantitative: Likert 1–5, based on connection making | No |
| Wahyuningsih, Sapti; Satyananda, Darmawan; Qohar, Abd; Atan, Noor | 2020 | An Integration of “" Online Interactive Apps” for Learning Application of Graph Theory to Enhance Creative Problem Solving of Mathematics Students | International Journal of Interactive Mobile Technologies | Graph Theory | Quantitative: Creative PS scale | Yes |
| Zazkis, Rina; Holton, Derek | 2009 | Snapshots of Creativity in Undergraduate Mathematics Education | Creativity in mathematics and the education of gifted students | Various | Descriptions of problems, classrooms, and previous work | Yes |

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# Chapter 9
# Mathematical Creativity from an Educational Perspective: Reflecting on Recent Empirical Studies

**Esther S. Levenson**

## 9.1 To Comment Is to Reflect

Writing a commentary on others' research is an opportunity to reflect on one's own. Inevitably, you end up comparing the theoretical frameworks, methods, and messages of others to those used in your own work. The five chapters in this section have certainly afforded me many such opportunities. In terms of theoretical frameworks, Crespo and Lominguez challenged me to see beyond what I usually see. In their chapter, they compare a human/language-centric lens to a materialist posthuman lens on children's creative thinking. At the heart of their study is the question of where creativity is located. Is it only in the linguistic productions of humans, or does it emerge from the "agentive encounters between children and all the materials they work with"? These questions continue to follow the reader, as they read the other chapters in this section. Liljedahl puts forth a different theory, specifically directed at group creativity. He posits that group creative processes are synonymous with *burstiness*, a concept he borrows from group psychology. Whereas in my own studies of collective creativity (e.g., Levenson & Molad, 2022) I borrowed from theories of group learning (e.g., Martin et al., 2006), analyzing dialogical moves, co-actions, and interactions within a group, burstiness focuses on turn taking, where a bursty conversation is characterized by "multiple brief, focused sequences of turns at talk with reduced openings, and closings" (Woodruff & Aoki, 2004, p. 434). The more bursty a conversation, the quicker there is an exchange of ideas.

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In terms of methods, like de Vink et al., I have conducted quantitative studies related to students' mathematical creativity (Molad et al., 2020). Like Crespo and Dominguez and Liljedahl, I have conducted and qualitatively analyzed classroom observations, seeking evidence of mathematical creativity (Levenson, 2011). However, Karakok et al. utilized a phenomenological case study design, a method that seems to be less employed in mathematics education research and even less so when studying classroom mathematical creativity. Karakok et al. were interested in studying undergraduate students' experiences in an introduction-to-proof course and how these experiences shaped their developing perspectives on mathematical creativity. As such, they collected multiple reflections from the students regarding what they had learned, as well as their reflections on creativity and mathematical creativity. Bleiler (2015) stated that when participants' reflections serve as the data, the methodological framework is more accurately called "interpretative phenomenological analysis" as the research focuses on "lived experiences of individuals as those experiences are reflected on and interpreted by the individuals themselves" (p. 234). Reading Karakok et al.'s chapter, one appreciates how the ordinary experience of writing proofs can become a meaningful and significant experience for a student.

Finally, there is the message of each study. Of course, each study, with its own unique questions, ends with a unique message. However, I did find a common message running through all five chapters, the message that within educational contexts, creative processes, more than creative products, are the essence of mathematical creativity. In my own work (e.g., Levenson, 2011, 2014; Molad et al., 2020), I have not always been able to separate the study of creative processes from creative products. In one of my first studies related to collective mathematical creativity, I noted the following:

> Regarding collective flexibility … it becomes more difficult to separate the product from the process. On the one hand, when discussing flexibility as a product, we may look at the solutions produced by the group which employed different strategies … Regarding the process, we may say that flexibility was marked by a certain adaptation of previous solutions…. (Levenson, 2011, p. 229)

In a more recent study (Molad et al., 2020), my colleagues and I were challenged by one reviewer who claimed that we were mixing up the two notions. In response, we took the following stance: "Although research has not confirmed that creative products are always the result of creative processes, we regard the products of these tasks as expressions of creativity" (Molad et al., 2020, p. 202). In the introduction to his chapter in this book, Liljedahl succinctly clarifies that there are two distinct approaches to measuring creativity. The first approach is similar to the one my colleagues and I adopted and assumes that creative products act as a proxy for determining the creativity of a process. The second approach does not make this assumption and instead values the creative process itself and not because it may be a means to a novel solution. The question then remains: what processes may be called creative? Which then leads to the question of how to promote these creative processes.

## 9.2 Creative Processes: What Are They?

Liljedahl and de Vink et al. offer brief histories of the discourse on creative processes, which I shall not repeat here. Interestingly, it seems that each paper adopts for their own research different views of creative processes, not always very explicitly. Crespo and Dominguez use the term “creative mathematical thinking” and link it with inventing novel ways of expressing mathematical ideas and flexibly handling real and imaginary worlds. de Vink et al. also related to imagination, although in a more implicit manner. They chose to implement a specific problem-posing task because they believed it would provoke students’ imagination. In Karakok et al.’s study, one of the students described her perspective of mathematical creativity as using your imagination to create something original. Previously, Whitcombe (1988) linked imagination to creativity, stating the following:

> For although it is probably true that mathematics originated, thousands of years ago, as a consequence of real-world activity and observation, it soon developed ideas and concepts that are beyond actual human imagination. (p. 15)

Liljedahl seems to view imagination as a way to perhaps kick-start the creative process. In his chapter, creative processes include shifting attention from one idea to another and exchanging ideas without pauses (in line with the theory of burstiness). Monahan and Munakata specifically relate to flexible thinking when viewing teachers’ creativity as they shifted to an online environment. Of the five chapters in this section, de Vink et al. were most clear regarding thinking processes that make up creativity – divergent thinking and convergent thinking.

As de Vink et al. noted, divergent thinking is often thought of as the hallmark of creative thinking and is often measured in terms of the fluency, flexibility, and originality of ideas produced (Haylock, 1997; Silver, 1997). Yet, divergent thinking might also produce random useless ideas. According to Runco (1996), creativity is “manifested in the intentions and motivation to transform the objective world into original interpretations, coupled with the ability to decide when this is useful and when it is not” (p. 4). In other words, divergent thinking along with convergent thinking may be necessary for creativity. Convergent thinking happens when the solver logically strives to find a solution to a problem, seeking to understand the logical connections among knowledge elements in the problem. In fact, de Vink et al. posit that children use both types of thinking when solving mathematics problems and that these processes are often intertwined, not necessarily following a certain order.

Interestingly, what de Vink et al. found among fifth-grade students was also noticed among a group of mathematics education researchers (Tabach & Levenson, 2019). Yet, at least two important differences may be noticed between those studies (besides the age of the participants). First, in de Vink et al.’s study, they analyzed students’ divergent and convergent thinking processes only for what they deemed as mathematically creative turns, when mathematical concepts were combined in a new way. In Tabach and Levenson’s study, thinking processes for all responses were analyzed. Second and more importantly, in de Vink et al.’s study, students’ responses all consisted of individual solutions (or individual questions for the problem-posing task), whereas in Tabach and Levenson's study, two types of responses were given. The first type, like in de Vink et al.'s study, consisted of individual solutions. However, the second type of response consisted of sets of solutions, where each set description explicitly referred to the existence of infinitely many solutions according to the set characteristic. In de Vink et al.'s study, most ideas were created by divergent thinking. Similarly, in Tabach and Levenson's study, divergent thinking led to relative fluency in producing the isolated solutions. Yet, in line with Runco, de Vink et al. noted that ideas should not only be generated, but also evaluated. In their study, original ideas were generated by divergent thinking followed by convergent thinking. However, in Tabach and Levenson's study, it was convergent thinking that led to different sets in which each set contained an infinite number of solutions. Then, after reaching a solution set with an infinite number of solutions, participants continued to search for an additional solution set. Thus, it may be said that convergent thinking processes also led to divergent thinking processes. Both de Vink et al. and Tabach and Levenson call for additional research regarding these types of thinking.

## 9.3 Creative Processes: How Can We Foster Them?

How to foster creative processes is discussed in all five chapters in this section. Of course, all chapters mention the activities that stimulated creative processes. de Vink et al. utilized intentionally designed mathematical tasks that have been shown in previous studies (e.g., Kattou et al., 2013) to elicit mathematical creativity. Three studies (Crespo & Dominguez, Karakok et al., Liljedahl) were classroom-based, where the mathematics activities were part of the regular curriculum. Monahan and Munakata focused on the teachers' activities. What struck me as interesting was that most of the studies made notice of the tasks, but did not dwell on them as the prime factor impacting on creative processes. In my own studies (e.g., Levenson & Molad, 2022), I have always spent a fair amount of space describing the tasks used, why they were chosen, how they were used in previous studies, and how they could be used to measure mathematical creativity. I even investigated teachers' perspectives on mathematical creativity by having them choose tasks they believed could occasion mathematical creativity (Levenson, 2015). Upon reflection, many of my previous studies focused on the products or related to both processes and products without quite separating the two. Perhaps because the chapters in this section focus on creative processes, they place more emphasis on the environment that may foster these processes, rather than on the tasks themselves, not that focusing on environments is a new idea. Rhodes, in 1961, identified four strands of creativity research he called the 4Ps: product, person, process, and press, where press refers to the environment, both the physical and social environment.

Regarding the social environment, several of the chapters in this section discuss the interactions between students as well as the interactions between the students and the teacher. In line with previous research (Levenson, 2011), they point out the need for creating a safe environment, where students can share ideas, make mistakes, and ask questions, without fear of being rebuffed (Karakok et al., Liljedahl). The role of the teacher is to foster autonomy, to not show students the "right way" before they have a chance to explore on their own (Crespo & Dominguez). Finally, a teacher who demonstrates enthusiasm for creativity can pass on this enthusiasm to students (Monahan & Munakata).

The social environment is especially significant when groups of students work together. For example, Crespo and Dominguez describe a class environment fostered by the teacher, where children show care and responsibility when working in groups. When a teacher is not present, a student in the group may take on the role of a social leader, encouraging collaboration and listening to others, collecting ideas, and putting them forth to the group (Levenson & Molad, 2022). Interestingly, in Karakok et al.'s study, it is the students themselves who take note of the social environment in group work, especially that questioning one's own thinking, as well as others' thinking, causes creativity to be "contagious." Adding to these remarks, Liljedahl stated that the makeup of a group is an important factor in supporting creativity. Specifically, he suggests random groups, where neither the students nor the teacher forms the groups. Such randomness allows for diversity while simultaneously breaking down social barriers. Although Crespo and Dominguez do not explicitly focus on diversity, they do mention that the students in their study were multilingual and multicultural.

The environment also relates to the physical surroundings of those working on a problem (e.g., a divergent production task). In the workplace, for example, factors such as adequate light, ventilation, noise levels, and even the square footage of the work area were found to affect workers' perceived ability to be creative (De Alencar & De Bruno-Faria, 1997; Stokols et al., 2002). Studies of learning environments found that an open space containing moveable furniture, where multiple spaces for group work are available and there is access to a variety of materials and resources, can support creativity (e.g., Richardson & Mishra, 2018). Crespo and Dominguez also emphasized the availability of materials stating that the classroom was "flooded" with materials.

In my own studies, I have been less aware of the physical environment than perhaps I ought to have been. In the classrooms I observed, students were either sitting at their own desks or sitting around a table working in groups. The teacher was either at the front of the classroom or walking between groups of students during group work. I thought back to those classrooms when reading Liljedahl's chapter. Liljedahl claims that groups should "stand and work on vertical non-permanent (erasable) surfaces … making work visible to the teacher and other groups." He further describes how the furniture in a classroom ought to be arranged. His suggestions are in line with the theory of burstiness he proposes, as they afford fluid communication. While Liljedahl stressed the affordances of an environment in support of creativity, Monahan and Munakata focused on how the constraints of an environment (specifically caused by the Covid-19 pandemic) can also benefit creativity. In their study, they noted that the change in instructional environments, from face-to-face to online, offered teachers the opportunity to question their assumptions about teaching, which served as a catalyst for the teachers' and student' creativity. It also led to resourcefulness, which they claim is a trait of creativity. For the students, instructors noted that breakout rooms encouraged collaboration, allowing students to freely express their ideas. Unlike the abovementioned studies, for Crespo and Dominguez, the physical environment is more than just a catalyst for creativity; it is where creativity lies. In line with their materialist posthuman lens, creativity emerges "from the agentive encounters between children and all the materials they work with."

## 9.4 Some Pre-reading Suggestions

The five chapters in this section all present current research focused on mathematical creativity. Participants in those studies were of various ages, from young elementary school students, middle school students, high school students, undergraduate students, and teachers. One may be tempted to read only chapters that pertain to a certain age-group, perhaps the age of participants you, the reader, are most interested in. This would be a mistake. Instead, I challenge the reader to ask how the research described in each chapter may impact on one's own research. How can you use the theory put forth in Crespo and Dominguez's chapter to see creativity where it might have previously been hidden? Could Liljedahl's design for a creativity-promoting environment be implemented in schools in your country? How might you foster burstiness in online groups? What might be the perspective of your students regarding mathematical creativity? Before reading the following chapters, my suggestion is to reflect on your perspectives of how mathematical creativity can be developed and then be open to others' perspectives.

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# Part III
# New Empirical Research on Mathematical Creativity

# Chapter 10
# Now You See It, Now You Don't: Why The Choice of Theoretical Lens Matters When Exploring Children's Creative Mathematical Thinking

**Sandra Crespo and Higinio Dominguez**

## 10.1 Introduction

As educators and researchers in the field of mathematics education, we have invested considerable amounts of time in the company of children, observing and learning with and from them inside and outside of the classroom. The young children in our families have also challenged us to stop projecting narrow adult perspectives on them but instead allowing children to take the lead on how they see, experience, and make sense of the world. It is with this understanding that we write this chapter and continue to explore the ideas we shared in Dominguez et al. (2020) about children's creative mathematical thinking. Here, we focus more specifically on what we stand to learn when we promote dialogue between contrasting analytical lenses – a dialogue that can help us enhance our understanding of children's creative mathematical thinking.

Our point is that traditional research on children's mathematical creativity has documented a small portion – perhaps too small – of the abundant, complex, and interconnected creativity that is possible to see when one considers other analytical lenses. Consider the following exchange in a culturally and linguistically diverse kindergarten classroom, the authors were invited to observe and pay attention to what you, the reader, focus on and why.

*Teacher* (displaying a large red rectangle for the class): What shape is this?
*Students* (simultaneously): Red. Rectangle.
*Teacher*: What shape is this?

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*Students* (in unison): Redtangle.
*Teacher*: You mean Rectangle. Good. How come it isn't a square? Do you remember?
*Student A* (softly): Because it has two short sides and two long sides.
*Teacher*: Good job. What were you going to say, Student B?
*Student B*: A square has four ends, and a rectangle has four ends.
*Teacher*: They do both have four ends. But what is special about the rectangle? Student A said it. What did he say?
*Students C and D*: There's two big sides and two small sides.
*Teacher*: Good. (Displays a square) And what shape is this, then?
*Students*: Square.
*Teacher*: Square. How do we know this is a square?
*Student B*: It has four ends.
*Student E*: And it is littlerer.
*Teacher*: It doesn't matter that it's smaller. It could be the same size. Because of why? Student B?
*Student A* (speaking out of turn): They're all the same length.
*Teacher*: They're all the same length, Student A says.
*Student B*: Because they're all the same length.
*Teacher*: They're all the same length.

When we have shared this dialogue with other mathematics educators, their analytical lenses primarily focus on the mathematical correctness of the ideas shared. They seem bothered by the imprecise language in this exchange, and they often draw upon a binary correct/incorrect framework to classify the children's thinking. Dismissed in this kind of analysis is the linguistic creativity the students in this example have used to express their mathematical thinking. They seem to miss and perhaps dismiss the possibility that students have invented a novel way to refer simultaneously to the color and type of shape (redtangle) and that they found a new use for the word "ends" and apply it in the mathematics classroom. Yet to us, this classroom interaction exemplifies the creation of a space marked by creativity, criticality, and even transgression, as suggested by García and Wei (2014) and Wei (2011) in their discussion of translanguaging as a pedagogical and learning practice often observed in multilingual classrooms. Similar to art, translanguaging has a strong liberatory orientation to what is possible. It encourages creative forms of expression that have no bounds to the past nor allegiance to canonical perspectives. It encourages new meanings and new forms of sensemaking.

Mathematics educators and researchers alike are familiar with the pedagogy used in the above transcript and its linear organization of turn taking that promotes a focus on who says what, to whom, and with what frequency. The focus on language in this neatly organized transcript hides the "messiness" of human thinking, including sensorial, bodily, and kinetic aspects (Sheets-Johnstone, 2011), all of which play an intrinsic role in how students make sense of mathematical ideas. Our point here is that common notions of creative mathematical thinking reduce every instance of mathematical creativity to *a kind of language* that is individual, cognitive (exists in the speaker's mind), and normative. There are compelling critiques of this narrow perspective on language because it has been used as an oppressive tool to categorize and judge students' creative sensemaking (DeLanda, 2008; Deleuze, 1994). We argue for a liberating view of children's creative mathematical thinking that recognizes that creativity needs space, freedom, and movement, all aspects that transcription traditions fail to capture.

As we have noted elsewhere, studies of children's creative mathematical thinking have tended to focus on individual children's responses typically collected in interview settings and not necessarily within the fluid context of whole class instructional interactions. These studies have been important in recognizing the potential and existence of children's mathematical creativity, including for very young preschool children. However, these studies do not consider the dynamic and relational nature of mathematical creativity, which is the type of children's mathematics that we are interested in documenting.

## 10.2 On Seeing and Not Seeing Mathematical Creativity

Because mathematics education research on children's mathematical thinking is mostly grounded on clinical interviews conducted by university researchers, it is often the case that the children's interactions with the social and material worlds are rendered invisible and unrelated to their creative thinking. This body of research has reinforced notions of creative mathematics as individualistic cognitive work, as it overemphasizes the value of children's language productions over the process of their creative work with different materials, the contextual and cultural aspects of that work, and their linguistic creativity. It is these aspects of students' creative mathematical thinking that we argue are critical and important to reconsider in this body of research.

In our experience observing children interact with one another in the mathematics classroom and in the playground, we find that these settings provide many opportunities to study their creative mathematical thinking. This was the case in the following episode from a lesson we observed in another kindergarten classroom in a school also serving a culturally and linguistically diverse student population. The school is a field site for prospective teachers in our teacher education program. In this episode, children were invited to explore measurement as a human invention that requires joint labor, intense conversations, and collaborative meaning-making.

The lesson contextualized the practice of measuring as part of a broader conversation about environmentally responsible and sustainable fishing practices – if a fish is too small, it has to be released, and if the fish measures up to the teacher's proposed standard of 10 unit cubes long, then it can be kept (Fig. 10.1). The children were organized into *fishing groups*, and they could hardly contain their excitement. The students were eager to role play being part of a fishing crew. The teacher showed them how to "catch" a paper fish with a homemade fishing rod and invited them to guess the size of the fish. Their teacher measured the length of the paper fish twice

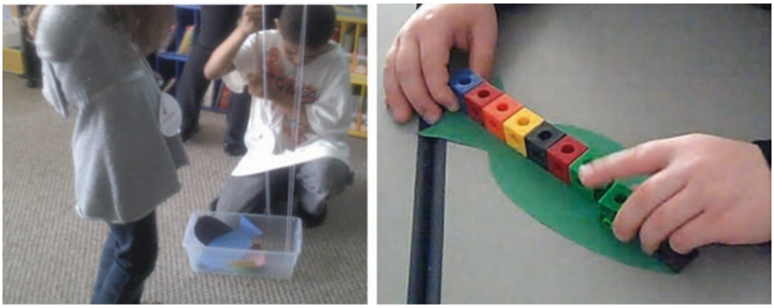

**Fig. 10.1** Measuring fish lesson

to encourage the children to recheck their measures. She measured the fish lengthwise from the tip of the nose to the end of the tail, the first time pushing the cubes together and the second time by leaving uneven gaps in between them. The children gasped when the two measures turned out to be different (11 unit cubes and eight unit cubes). One of the children pointed out that although the unifix cubes were the same size, the gaps between them were different. With this realization, the children began measuring their own fish while the teacher encouraged them to help each other recheck their measures.

To check their accuracy, one of the groups decided to take turns to measure the fish they had caught. While they all agreed with their measurement of 10 cubes, the group grappled with the fact that 10 was right at the cutoff value of keeping or releasing the fish. When their teacher told the group that it was up to them to decide what to do, the children went back to remeasuring and discussing what they could do. Other groups were similarly debating their measurements. One of the groups found two different measurements for their fish, some claiming the fish was nine cubes and others only five. When the teacher asked the students whether this was a kind of problem with two answers, two students said no, one said maybe, and the fourth one convincingly said "yes!" The teacher then asked the students to show how they were getting these two different measurements. A student showed the teacher that they were getting nine cubes by placing the cubes starting at the nose, leaving no gaps, lined up end-to-end as straight as possible until reaching the tail of the fish. Then another student showed how they counted five cubes by doing the same but from the top fin on the back of the fish to the fin on its belly. After a couple of more interactions and some clarifications, the group decided they wanted to measure the fish's "longness" (from nose to tail) *and* also the "wideness" (from back to belly), and if any of these measures was 10 or more, then they would keep the fish, otherwise they would return it to the bucket.

Researchers in mathematics education will read the above classroom episode differently depending on the theoretical lenses they use. We have found that the dominant approach to research on children's mathematical creativity draws on classical notions of creative thinking (e.g., Haylock, 1997), which focuses on children's divergent thinking and associates creativity with giftedness, thus suggesting an innate nature of creativity. Qualities in the children's responses to given tasks are associated with *fluency*, *flexibility*, *originality*, and *elaboration*, which are then considered as indicators of creative thinking (Haylock, 1987). Using this lens on the "measuring fish" episode highlights and makes visible the students' eagerness for role playing and flexibility in handling real and imaginary worlds even inside the rigid structures of the mathematics classroom. We can also connect the students' attention to the spacing or the gaps between the unifix cubes as an original and unusual realization for this particular age group as noticing the "negative" space between objects and images tends to be associated with divergent thinking. Another aspect of mathematical creativity that can be noticed with this more classical lens on children's creative thinking is their willingness to entertain multiple truths and consider multiple possibilities (when deciding that there can be multiple acceptable ways of measuring the fish), another marker of divergent thinking.

There are, however, important aspects of children's thinking that are rendered invisible when only using a human-/language-centric lens on children's creativity in mathematics. By human-centric, we mean a lens that privileges humans and their actions, whereas the materials with which this human works and what both human and materials achieve together are not central to the research analyses. By language-centric, we mean a lens that reduces all data to linguistic productions, thus negating other forms of expressivity that are not language-based. A lens that moves creativity from its assumed innateness must address the question of where then creativity is located.

A materialist posthuman lens, in contrast, considers creativity as emerging from the agentive encounters between children and all the materials they work with – in the example provided, the paper fish, plastic bucket, homemade fishing rods, unit cubes, classroom floor, light in the room, students' bodies and, importantly, the task itself. Thus, the creative moments identified with this lens are no longer seen as individual, innate, or child-centered; the gaps between the unit cubes were produced by the material nature of the cubes. A different material, such as a strip of paper folded in an accordion fashion, would not have permitted children to demonstrate this same type of creative thinking. The children's eagerness to engage with the task would have been different if the task employed a different storyline. Similarly, the children's willingness to actively engage in decision-making related to their measuring activity was triggered by imagining material (and perhaps even moral) consequences of their collective actions on the world around them. The point here is that a material posthumanist approach to analyzing this classroom episode allows us to see both human and nonhuman bodies as agentive partners. This is a significant point in the dialogue between these contrastive analyses because human-/language-centric approaches have shaped most of the research literature in mathematics education, including research with a focus on equity, which has struggled to find nonhierarchical lenses on children's mathematical thinking.

## 10.3 Children's Mathematical Thinking in a Fractions Lesson

To further illustrate the dialogue between contrasting lenses on children's creative mathematical thinking, we have selected the following example from our international collaboration that took us first to Chile and then back to Texas as part of a teaching-research exchange (see Dominguez et al., 2020). This final example occurred in a classroom that is unusual for at least three reasons. First, the teacher had established a culture of trust and reciprocity as a result of her five-year participation in a research project led by the chapter's second author. Her classroom truly was "a public space of debates in which the students are encouraged to show openness toward others, responsibility, solidarity, care, and critical awareness" (Radford, 2016, p. 5). Second, the teacher was consistently interested in exploring mathematical concepts in depth instead of following the superficial treatment found in most mathematics curricula. Again, this was a classroom the authors were invited into by the teacher, and their role was to be participant observers during a week-long visit to this teacher's classroom.

In the unit explored, the focus was on the meaning of fractions as measurement that is often underemphasized in the US elementary mathematics curricula where the part-whole meaning is favored. The emphasis on this meaning has been associated with children's difficulty understanding fractions greater than one (Thompson & Saldanha, 2003). Other countries, such as Japan, emphasize this meaning when the concept of fractions is first introduced to students (Watanabe, 2006). Third, the instructional approach was consistently transdisciplinary. In this example, the teacher fused the importance of teaching the measurement meaning of fractions with the importance of examining access to healthy foods in the students' community by considering how access to food quality is associated with racial/economic segregation.

The multilingual and multicultural students in this third-grade classroom were investigating the consequences of their food choices on their own health and the intersection of access to healthy foods and racial segregation in their community by physically measuring the amount of sugar they were consuming in a typical day. The classroom was flooded with materials (e.g., buckets filled with sugar, Ziploc bags, plastic spoons, worksheets, notebooks, pencils, document camera, and laptops) and information (e.g., screen projecting the students' favorite food labels, copies of food labels on students' desks, and questions and answers traveling across the small groups).

To help maintain the uninterrupted fluidity of interactions (and intra-actions) between her students and the abundance of materials and information, the teacher gave them an approximation of one spoonful as more or less equal to 5 g of sugar. Mathematically, though, the teacher chose this approximation to have students work with the concept of fifths. The encounter between children and this fraction occurred when they had to measure an amount of sugar in a favorite beverage that contained 56 g. This number prompted students to count by fives (by scooping spoonfuls of sugar) but only until 55. Then, the question that emerged in this class was as follows: How do we measure that one extra gram of sugar in order to have 56 g?

In one group, led by students Giselle and José Luis, students first approached the task by doing something likely familiar to them: drawing a circle and attempting to split it into five parts. They erased these parts multiple times because the parts did not come out equal. As Higinio acknowledged their effort and difficulty of drawing fifths, they accepted their drawing as an important referent for their next activity, and José Luis exclaimed, "It's like the tire of a car!" Next, they moved to working with the sugar. In this part of the video, we observed the level of care for others, responsibility, and critical awareness that characterizes a well-organized collective engaged in joint labor (Radford, 2016). While some members of the group volunteer to measure the sugar, others assume the flexible roles of record keepers, material seekers, accuracy observers, and even camera persons helping the research team capture the episode. The record keepers invented a system of keeping track of the spoons counted by drawing 11 circles and putting a checkmark for each spoonful counted (Fig. 10.2).

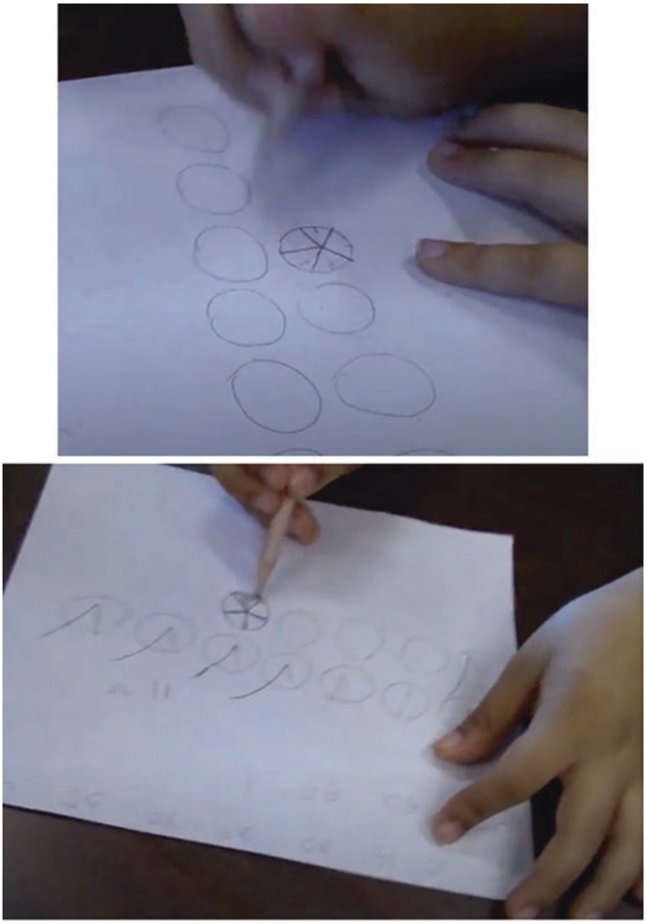

**Fig. 10.2** Keeping track of how many spoons of sugar to make 56 g


**Fig. 10.3** Splitting a rectangle of sugar into fifths

When confronted with the task of splitting one spoonful into fifths, students referred back to the circle split into five parts. Using the flat side of the ruler, they leveled one more spoon with sugar and emptied it onto the table with the goal of splitting it into fifths. With the same ruler but now using the straightedge of it, Giselle began pushing the mound of sugar into an elongated shape that made all of us around the table evoke the shape of a rectangle. Hearing this observation, Giselle continued using the straightedge to reshape the sugar into the shape of a rectangle (Fig. 10.3). When Sandra arrived to visit this group, student José Luis provided an update on their developing strategy.

José Luis: We made it into a rectangle, and then, we put a line to get it.
Sandra: Deja ver, ¿cómo lo van a hacer? [*Let me see. How are you going to do it?]*
(Using the spoon handle, Giselle marks three lines on the rectangle of sugar.)
Higinio: Those are fourths.
Sandra: ¿Cómo vamos a hacer los quintos? [*How are we going to make fifths?]*
José Luis: We need to make it longer! We need to make it longer! (Giselle elongates the rectangle of sugar and this time marks four lines, thus creating fifths.)
Higinio: Oh, I like that!
Everyone: Seeing Giselle improvise the handle of the spoon as a tool for marking lines along the rectangle, all of us began counting 1, 2, 3, 4, and 5 for each part she was making (Fig. 10.3).

In the next two sections, we analyze this episode more closely using two different lenses – one that centers human activity and another that decenters it. We have structured our analysis by focusing on the same three dimensions: the roles that language, materials, and agentivity play in children's creative mathematical thinking. We focus on the same three dimensions in order to establish a dialogue between the two analytic approaches and so that we can explore what each lens helps to make visible and invisible.

## 10.4 A Human-/Language-Centric Lens on Children's Creative Thinking

It is important to establish right from the start that no matter which lens we use to analyze the above classroom episode, we can find moments of children's creative mathematical thinking. Even with a classical lens on creative thinking, we can see the creative problem-solving of the group when the children realized that it is not that simple to make identical spoons of sugar in practice as it is in one's mind or in the abstract. Something that seems as natural and simple in our everyday life as to get a spoonful of sugar is now understood as a much more complex concept when considering how to make the spoonfuls hold exactly the same amount of sugar, as suggested by the groups' careful leveling of sugar with the ruler. This insight is a hallmark of mathematical thinking, and when the children use different approaches to solve this problem, we are then moving into the territory of children's creative mathematical thinking.

Realizing that there was a lot of variation in the size of the spoonfuls of sugar (some were too high, too low, had clumps, were not entirely full, and so on) the children needed to create a method for standardizing their unit of measure and make their spoonfuls of sugar as equal as possible. In addition to standardizing the unit of measure, they also realized that it was very easy to lose track of the number of spoons of sugar and that they could not tell by only looking at the Ziploc how many spoons they had emptied into it. Yet another challenge was how to split a spoon of sugar into fifths. All three of these challenges were happening in tandem, and they were all embedded within the same task. In other words, this group of children were able to work on three interrelated tasks in tandem while also moving across oral and written language in two different languages, with multiple tools, and while negotiating social interactions with their peers and with the adults in the group. Let us now consider the role of *agentivity*, *language*, and *materials* on children's creative mathematical thinking from a human-/language-centric analytic lens.

### *10.4.1 Agentivity*

A human-/language-centric lens on children's creative thinking attends to how children either are provided with or take on agentive roles in group participation. It pays particular attention to qualities of students' participation, such as active or passive or productive or unproductive participation (e.g., Webb et al., 2019). This lens associates the quality of children's participation with student learning and with creative thinking. Traditionally, this lens has been used to sort and classify children in terms of their active or passive learning behaviors and in terms of productive or unproductive contributions to the intellectual work of the group. With this lens, the children's participation in the above episode is high. Children did not wait for the adults to tell them what to do; they grabbed the materials and began to tackle the challenge of making 56 g of sugar. We can then say that all of the students were eagerly and actively participating, none of them took over the task, no one was excluded from the task, and none of them seemed to disengage. All hands on deck is one way to characterize the children's agentive participation as each of them took on a role within the group. Importantly, they did so flexibly as they each took turns playing different roles – filling up the spoon with sugar, trying to make it "even," and trying to keep track of their counting.

### *10.4.2 Language*

In a human-/language-centric perspective, language is viewed as a primary resource for making mathematical sense. With this primacy comes an emphasis on assessing the value of children's linguistic contributions. In the episode, student José Luis appears to be more verbal than his peers as he expresses his thinking publicly without being prompted. The other two students, Giselle in particular, seem much quieter. From the human-/language-centric lens, this difference in the amount of talk between these students can be interpreted as indicating more or less advanced mathematical thinking. However, prioritizing more spoken language used as an indication of more advanced mathematical thinking takes away from the advanced mathematical thinking Giselle displayed in pouring the sugar onto paper and reshaping it so it can be broken up into five parts. Additionally, when the focus is on how children use language in mathematics, we tend to invoke normative concerns with the mathematics register (Pimm, 1987; Setati, 2005). For example, in the above episode while students were talking about rectangles and lines and making a rectangle longer, adults were using the more formal mathematical terms to refer to fractions as fourths and fifths. A focus on this contrastive use of language easily invisibilizes children's creative work with fractions.

### *10.4.3 Materials*

In a human-/language-centric perspective, materials are considered as resources that mediate sensemaking (e.g., Hall & Nemirovsky, 2012). Using this perspective, we see children drawing a circle and attempting to split it into equal parts multiple times and then moving to creating a rectangle of sugar with a ruler. All this activity is mediated by multiple materials, and the assumption is that this activity is goal-oriented, in that students want to create five unit fractions of fifths. Put differently, the materials lend support to the human-driven mathematical activity. We can see persistence in the children's multiple attempts at drawing and splitting a circle into five equal parts or elongating the sugar rectangle to fit more equal parts. This suggests that the children are using the materials to make real what they have imagined in their minds and are wielding and shaping the materials in unexpected ways. This perspective assumes that the students' persistence is driven by the human desire to problem solve, and the materials are only in the service of this human desire.

## 10.5 A Materialist Posthuman Lens on Children's Creative Mathematical Thinking

We now revisit our fractions episode with the same focus on agentivity, language, and materials but from a materialist posthuman analytical perspective. Our purpose is to make visible what this lens allows us to see/not see in children's creative mathematical thinking.

### *10.5.1 Agentivity*

Central in a materialist posthuman lens is the recognition that every body – human or otherwise – is agentive (Barad, 2003, 2007). Humans establish relationships with their material worlds precisely because the materials in these worlds are vibrant (Bennett, 2010), alive (Cajete, 2000; Barnhardt & Kawagley, 2005; Martínez Parédez, 1964), and in continuous correspondence – as in exchanging letters with the humans (Ingold, 2011; Roth, 2016). Yet, the source of this agentivity is not in the individual: no child, no bucket of sugar, and no drawing possesses agentivity in and of itself. Put differently, none of these material bodies bring to their coming together a preexisting agentivity. Rather, agentivity emerges from the moment – unique, uncertain, changing – that these bodies come into these assemblages (Deleuze, 1994). The agentivity observed earlier of José Luis taking the lead in explaining to Sandra the creative nature of his group's strategy emerges, according to this materialist posthuman lens, not from the individual child, not as a premeditated act, but rather as part of the group's welcoming of a new member, Sandra, as she came to visit this group. All members of the group are agentive at every moment in the interaction. Without Giselle manipulating the materials, the recorders inventing a visual record of their fraction measuring activity, or the peers as observers of the accuracy in measurement, José Luis' explanation of their invented strategy would not have been as forceful and convincing as it was. This is because José Luis is not seen as an individual bringing his own agentivity to this interaction.

### *10.5.2 Language*

Posthumanism offers a strong critique on the role of language in human affairs. Far from being the common interpretation of language as a resource for making sense in mathematics, these critiques reveal the imperialist pretensions of language and its tendencies to reduce every form of expressivity to language (DeLanda, 2008). Rather than focusing on this human-/language-centric approach to language, a posthumanist approach highlights its role as one form among many other forms of material expressivity (Deleuze, 1994; de Freitas, 2016; Sinclair et al., 2013). In the episode provided, language – or bilingualism, more precisely – emerged among many other material forms, all of which flew into and across a space filled with material expressivities. The voice of José Luis exclaiming “It’s like the tire of a car!” shared the expressivity (a continuity) of his group members’ drawing of a circle split into five parts. Fresh in this drawing were faint lines that had been erased and redrawn multiple times by multiple hands. Clinging to those faint lines were eraser residue and graphite dust still vibrating on the paper. Similarly, the expressivity of José Luis’ urgency as he repeats “We need to make it longer, we need to make it longer!” finds a continuity in the amorphous amount of sugar taking the elongated shape of a rectangle. Participating in this orchestration of expressivities are José Luis’ voice, Giselle’s hands, the ruler, the moving sugar, and the eyes of all of us following this assemblage of language-material expressivities. Language is therefore not a resource *for* making mathematical sense but rather one of many expressivities that emerge from materials coming into assemblages.

### *10.5.3 Materials*

A materialist posthumanist lens breaks away from the tradition of hierarchies that position humans as in control of the material world and, instead, proposes an ontological equivalence among all bodies, human and more than human (Barad, 2003, 2007). This tenet has inspired analyses that are variously named as flattened out (DeLanda, 2008; Hultman & Lenz Taguchi, 2010) or nonhierarchical (Dominguez, 2021). This is an important aspect of decentering the human in the sense that what matters in flattened out analyses is the multiple continuities between and across materials. Thus, the initial circle split into five parts, the check marks inside the 11 circles as a record system, the shaping of the sugar into a rectangle, the students’ bodies and all their senses, and their bilingualism are all equal partners in this creative process of making fifths. Similarly, the pencil, eraser, and hands drawing the circle multiple times; the notebook and eyes keeping an accurate record of spoonfuls measured; and the ruler that acts as a leveling tool for the sugar in a spoon only to later become a plow that pushes the sugar into a rectangle are all equal partners with the students. One needs the other, and no one claims a hierarchical position or ownership of the creative act. When children in these rare but possible classrooms learn to see themselves as equal partners with materials, issues of competition for correct answers and status dissolve, giving way to spaces filled with reciprocity (Dominguez, 2021) and joint labor (Radford, 2016).

## 10.6 Further Thoughts: Dialogue Between Analytic Lenses

In this chapter, we argue for the benefit of using multiple analytic lenses in order to better understand children's creative mathematical thinking. We have shown how theoretical lenses that place a primary focus on human interaction can limit what counts as creativity in children's mathematical thinking because humans constitute only one of many forces engaged in acts of creativity (Watts, 2013). In contrast, theories that decenter and reposition the human as an equal partner of the material world promote a view of creativity as emerging from the dynamic and fluid encounters between humans and materials (de Freitas, 2016; Sinclair et al., 2013). We see the dialogue between these contrastive perspectives as necessary and urgent because it can help mathematics educators begin to shift and question the idea that children's creativity in mathematics is a rare occurrence. Bringing a human and posthuman analysis on children's thinking into dialogue can help mathematics educators begin to see mathematical creativity as a common everyday aspect of children's sense-making. We argue that this reframing is necessary and foundational to support the elusive agenda of providing more rigorous, equitable, and justice-oriented mathematics education to all children.

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# Chapter 11
# The Creative Mathematical Thinking Process

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## 11.1 Introduction

The value of creativity is increasingly recognized in mathematics education (Leikin & Sriraman, 2017). This increased interest fits well in the tradition of mathematicians like György Pólya and Jacques Hadamard, both of whom stressed more than 75 years ago that creativity is a driving force behind the discovery of new mathematical insights (Hadamard, 1954; Pólya, 1945). But, creativity is also important to those not involved in breaking new mathematical grounds, such as primary school children. Creativity helps them to integrate mathematical information and come up with different solutions or strategies to solve a problem (Hadamard, 1996; Mann, 2005), which is particularly important when children encounter a problem for which they have not yet learned a solution or solution strategy (Leikin, 2009). Indeed, research shows that children who score higher on measures of creativity also demonstrate higher mathematical performance (Jeon et al., 2011; Kattou et al., 2013; Schoevers et al., 2018). Prior research often studied mathematical creativity in a static way for instance by scoring children's performance on multiple-solution tasks in terms of the number of responses (fluency), variability of responses (flexibility), and uniqueness of responses (originality) (Assmus & Fritzlar, 2018). Such product-based measures of mathematical creativity, although informative, cannot unveil the creative thinking processes that led to a particular response or solution. If we want to support the development of creative thinking skills in mathematics education, more insight into the creative thinking process is required. This study therefore aspired to illuminate the use of creative thinking, in particular the use of divergent and convergent thought, in solving different types of mathematical problems.

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### 11.1.1 Divergent and Convergent Thinking

Mathematical creativity can be defined as “the cognitive act of combining known concepts in an adequate, but for the student new way, thereby increasing or extending the student’s (correct) understanding of mathematics” (Schoevers, 2019, p. 58). Guilford (1973) proposed that such new combinations of concepts (i.e., creative ideas) are conceived through divergent and convergent thinking. Divergent thinking refers to the process of generating ideas, like problem definitions, strategies, or solutions from a specific starting point, whereas convergent thinking concerns the process of selecting and evaluating ideas to arrive at the best possible solution (Brophy, 2001). Much creativity research has exclusively focused on divergent thinking (e.g., Jeon et al., 2011; Plucker et al., 2004), but researchers increasingly recognize the importance of convergent thinking too. If children rely on divergent thinking alone, they can generate many different creative ideas, including incorrect and unfeasible ones. Convergent thinking then helps to assess the value of these ideas for the task at hand (Brophy, 2001; Cropley, 2006).

Divergent and convergent thinking have been identified as separate constructs in former research (e.g., Barbot et al., 2016). However, as Cortes et al. (2019) proposed, task performance on either a divergent or a convergent thinking task could be a reflection of a mixture of both divergent and convergent thinking processes. Thus, previous results from research with divergent or convergent thinking tasks generally give little insight into children’s creative thinking process, as the exact process cannot be inferred from the creative product. To further illuminate the creative thinking process, it is necessary to make the shift from measuring creative products to measuring creative processes. Such an approach might shed more light on how creative ideas emerge in action (Corazza, 2016; Glăveanu, 2013). Conceiving creative ideas, for example a creative solution to a mathematics problem, is thought to consist of repeated cycles in which first divergent thinking and then convergent thinking is applied during different phases of creative problem-solving (Isaksen et al., 2011; Lubart, 2018). According to Wallas’s (1926) four-stage model of creativity, creative ideas are first prepared, followed by a process of incubation, an aha moment (illumination), and then evaluation and implementation of the idea (verification). Although these phases suggest a linear creative process, it is more likely that the phases can be implemented multiple times in different orders, with cycles of divergent and convergent thinking occurring in each phase (Lubart, 2018).

### 11.1.2 The Creative Mathematical Thinking Process

Various theories have been proposed as to how creative ideas arise in the mathematical domain. A well-known framework was introduced by Alan Schoenfeld (1982), who based his thoughts on earlier work by Polya. Schoenfeld proposed that (creative) problem-solving consists of a phase of reading the problem, analyzing task properties, exploring different possible solutions, planning how to reach a certain solution, implementing the solution properly, and lastly verifying (making sure the solution works). In general, phase models of creative problem-solving have been criticized for portraying creativity as a linear process that unfolds through a clearly defined sequence of steps (e.g., Lubart, 2018). However, it is more plausible that creative ideas also result from a messy process of going back and forth between steps, with cycles of divergent and convergent thinking embedded throughout (Lubart, 2018). Sheffield (2009) proposed such a non-linear process for mathematics. She suggested that creativity in mathematics is characterized by flexibility: students cycle through different activities such as creating, evaluating, and relating. The exact process can vary based on the problem and the amount of experience the student has.

One of the few studies that investigated the creative mathematical thinking process was a case study by Schindler and Lilienthal (2020) that depicted the creative problem-solving process of a high school student on a multiple-solution task. They indeed showed that such phase models might not be an accurate reflection of authentic creative problem-solving. Using a stimulated recall interview guided by recordings of the student's eye movements, Schindler and Lilienthal analyzed how new ideas emerge by coding the different parts of the student's creative problem-solving process and comparing it to existing models on creative problem-solving (e.g., Wallas's, 1926 model). They found that, compared to models like that of Wallas, phases could not be as clearly identified and that the sequence of phases did not seem to be as clear-cut. Instead of processing the different problem-solving phases step by step, the case study showed a cyclical process: the student constantly went back and forward between phases. For example, after generating an idea, the student was working on a solution. When he found out that this did not work, he discarded the approach and started looking for a new start and generating a new idea. Thus, Schindler and Lilienthal's case study provides initial evidence that for mathematics, the creative problem-solving process is not linear but rather cyclical. This notion provides support for previous claims made by Lubart (2018) and Sheffield (2009) about the general and the mathematical creative thinking process, respectively.

Given this cyclical nature, divergent and convergent thinking might be intertwined throughout the creative mathematical process, as using both modes of thinking can help children to generate different possible solutions or strategies, as well as select the most fitting one and evaluate its quality (Assmus & Fritzlar, 2018; Mann, 2005; Tabach & Levenson, 2018). Previous research has related both divergent and convergent thinking to mathematical performance on different types of tasks (De Vink et al., 2021; Jeon et al., 2011; Kattou et al., 2013; Schoevers et al., 2018). It therefore stands to reason that both thinking modes contribute toward the emergence of creative ideas during mathematical problem-solving.

How often and how well children apply divergent and convergent thinking might differ depending on both the task and the child. In terms of the task, open tasks are proposed to be the most suitable for creative mathematical thinking as they usually allow for multiple responses and can take many different forms (e.g., posing mathematical problems or finding different solutions to a specific problem; Leikin, 2009). Indeed, creative thinking has been found to affect performance more on open mathematical tasks than on closed tasks (Leikin, 2009; Schoevers, 2019). In terms of child characteristics, mathematical achievement seems to be a key factor because children with higher mathematical achievement scores have shown higher creativity achievement scores than children with average or low mathematical achievement (Kroesbergen & Schoevers, 2017; Leikin, 2013). We therefore assumed that groups of children differing in mathematical achievement scores also show different creative thinking processes on a mathematical task.

### *11.1.3 The Current Study*

This study is a qualitative investigation of the creative thinking processes of primary school children engaged in mathematical tasks. Two groups of children (characterized by high vs. low mathematical achievement, as determined by a general mathematics knowledge test) were asked about their creative problem-solving process. Children at the extreme ends of mathematical achievement were selected to gain insight into the role that mathematical knowledge plays in the mathematical creative thinking process. Comparing such extreme cases could help to determine whether the differences found in mathematical creativity task scores relate to their creative thinking processes. The fifth grade is an appropriate educational stage to study mathematical creativity because its mathematics curriculum contains complex problems (Noteboom et al., 2017) that require creative thinking skill. To get a more varied picture, two types of open mathematical tasks were used: a problem-posing task and a multiple-solution task. Furthermore, as open tasks allow for different types of responses, both easy and more difficult, these tasks were deemed appropriate for children with either high or low mathematical achievement.

## 11.2 Method

### *11.2.1 Participants*

A group of 28 fifth-graders from eight Dutch primary schools participated in this study. These children were selected from a larger sample that participated in a research project on creativity in math and science education (De Vink et al., 2021; Willemsen et al., 2021). The children who participated in the current study were selected based on their most recent mathematics grade point average (GPA), as indicated by their scores on a standardized progress monitoring test (Janssen et al., 2007). This test consisted of multiple-choice questions on various topics, from basic arithmetic to geometry and fractions, and was found to have good internal consistency (KR-20 = .95, greatest lower bound = .97; Hop et al., 2016).

**Table 11.1** Descriptive statistics of the low-achieving, high-achieving, and total sample

| | Sex | | Age | Math GPA |
|---|---|---|---|---|
| Group | Boys | Girls | *M (SD)* | *M (SD)* |
| Low achieving | 6 | 6 | 10.65 (0.09) | 208.50 (3.80) |
| High achieving | 13 | 3 | 10.66 (0.12) | 282.00 (2.58) |
| Total | 19 | 9 | 10.66 (0.41) | 250.50 (38.75) |

Extreme case sampling was used to draw an illustrative sample of children for the current study who demonstrated either mathematical excellence or lower mathematical performance (Onwuegbuzie & Leech, 2007). Children whose mathematics GPA could be classified as the lowest or highest 15% of the sample were selected to participate. After removing eight children from the sample for various reasons (e.g., no permission for audio recording or illness during data collection), the final sample consisted of 28 children. Descriptive statistics are presented in Table 11.1. The children's parents were all of Dutch nationality, and about half of them (46.4%) earned an (applied) university degree. Ethical approval for this study was obtained from the local ethics committee (ECSW-2019-087). The children's parents gave informed consent for participation in the study, retrieval of mathematics scores from the school administration, and audio recording.

### *11.2.2 Mathematical Tasks*

We used two tasks to assess how children applied divergent and convergent thinking during mathematical problem solving: a problem-posing task and a multiple-solution task. These tasks were selected from existing research instruments and combined in a test booklet.

The *problem-posing task* was taken from the geometrical creativity task (GCT, Schoevers et al., 2019). Children received a picture (a scenic view of two picnic tables and eight chairs in a forest) and were asked to generate different mathematical questions that their classmates could answer based on that picture. Children could, for example, pose the question "How many chairs should be added to the table if 10 people join for lunch?" This task was chosen because a picture is thought to call upon children's imagination, which is seen as an important element of mathematical creativity (Sriraman, 2005). The problem-posing task was administered first because it was the most open of the two tasks, and research has shown that creative performance is best elicited by starting with the task that has the most response possibilities (Moreau & Engeset, 2016).

The *multiple-solution task* originated from the mathematical creativity task (MCT, Kattou et al., 2013; Dutch translation by Schoevers et al., 2018). This task was chosen because it allows for both simple and more elegant solutions and therefore was suitable for both low- and high-achieving groups. The task asked children to formulate calculations on both sides of an equal sign that had the same answer.

To do this, children could use the digits 1, 2, 3, 4, 5, and 6 and the operators plus, minus, multiplication, division, and decimal point. Both operators and digits could be combined. A possible solution to this task would be to combine 2 + 2 and 5 – 1, as these calculations both equal 4. Children were instructed to formulate as many calculations as possible.

### *11.2.3 Procedure*

Data was collected in December 2019 and January 2020. Children first participated in plenary creativity, and science and mathematical tasks as part of our larger creativity project in which the relation between creative thinking, mathematics, and science performance is assessed (De Vink et al., 2021; Willemsen et al., 2021). Next, the first author revisited the school after a couple of weeks to administer the current mathematical tasks. This ensured a relaxed setting for children as they were already familiar with the researcher and the different types of creativity and mathematical tasks. The tasks were administered to each child individually in a quiet area of the school. The administration of the two tasks took approximately half an hour. Audio recordings were made to capture the child's thoughts and conversations with the researcher.

Before the start of the mathematical tasks, children were told that they would work on various types of mathematical tasks. They were explained that these tasks served to find out how different children approach mathematical tasks, that they would be asked to explain their responses and ideas, and that audio recordings would be made. Prior to each separate task, children were asked to read the instructions aloud. If children were not sure what to do after having read this information, they received help according to the standardized model for offering help during mathematical instruction from the Dutch guidelines on dyscalculia (Van Luit et al., 2014). To create an optimal atmosphere for creative thinking to occur, children were reminded throughout the tasks to share all of their ideas with the researchers (Sternberg, 2007; i.e., to think aloud). Research has shown that children are able to provide accurate think-aloud reports of mathematical problem-solving but benefit from using prompts while doing so (Reed et al., 2015; Robinson, 2001). Therefore, in addition to the ideas shared through think-aloud, the researcher used think-aloud prompts to ask children about their approach (e.g., "How did you think of this idea/solution?").

To minimize any possible bias toward achievement, children's mathematical achievement score was unknown to the researchers during the interview and coding process. A research assistant made a list of names and mathematics scores for children whose mathematics GPA could be classified as the lowest or highest 15% of the sample. A separate list with names, but no mathematics scores, was provided to the researchers during the interview and coding process so that no prior knowledge of children's achievement could affect their performance.

### *11.2.4 Data Analysis*

After data collection, all audio recordings were transcribed verbatim. Next, ATLAS.ti (version 8) was used to perform directed content analysis (Hsieh & Shannon, 2005). This method was chosen because existing theories of divergent and convergent thinking formed the starting point of this study, and this study aspired to extend these theories to the domain of mathematics. The directed content analysis proceeded in three steps. First, operational definitions of mathematical creativity, divergent thinking, and convergent thinking were developed based on theory (see Table 11.2). Second, the researcher familiarized herself with the data by extensively reading each transcript and making notes with a first impression of each transcript. At this point, the transcripts were segmented into units that could be coded. A unit referred to a turn of the child, which can be defined as "one or more streams of speech bounded by speech of another, usually an interlocutor" (Crookes, 1990, p. 185). Third, the different turns received initial codes for mathematical creativity, divergent thinking, and convergent thinking using the operational definitions in Table 11.2.

During the initial coding phase, all turns with possible instances of mathematical creativity received the code "mathematical creativity." These turns were further

**Table 11.2** Theoretical and operational definitions of mathematical creativity, divergent thinking, and convergent thinking for each task

| | Mathematical creativity | Divergent thinking | Convergent thinking |
|---|---|---|---|
| Theoretical definition | "The cognitive act of combining known concepts in an adequate, but for the student new way, thereby increasing or extending the student's (correct) understanding of mathematics" (Schoevers, 2019, p. 58). | "Divergent thought from a single starting point generates varied ideas" (Brophy, 2001, p. 439). | "…whereas convergent thought starting from multiple points seeks one most true or useful conclusion" (Brophy, 2001, p. 439). |
| Operational definition Problem-posing task | The development of an idea that includes a combination of an element from the picture with a mathematical concept such as surface area in a way that is new to the child, resulting in an adequate question. | The process of generating a creative mathematical question based on the picture, as well as any corresponding elaboration or explanation. | The process of selecting or evaluating a creative mathematical question based on the picture. |
| Operational definition Multiple-solution task | The development of an idea that includes a combination of the given numbers with a type of calculation (e.g., multiplication) in a way that is new to the child, resulting in a correct calculation. | The process of generating a creative mathematical calculation, as well as any corresponding elaboration or explanation. | The process of generating a creative mathematical calculation, as well as any corresponding elaboration or explanation. |

classified according to the mode of thinking (divergent or convergent), type of idea, and help given (see Table 11.7 in the appendix). Regarding thinking mode, we set out to label each creative turn as either divergent or convergent thinking. As we noticed during the coding process that many children used both divergent and convergent thinking in one turn, the code “divergent and convergent thinking” was added. It represented a combination of the operational definitions of divergent and convergent thinking for each task. Coding of the type of idea characterized the mathematical content that was central to the child’s solution. This classification served to ensure that every mathematically creative idea actually represented a combination of concepts that was new for the child on this task (e.g., if the child thought of similar questions about surface area on the problem-posing task, this was not considered new for the child on this task). Codes for the type of idea were based on previous research that used the problem-posing and multiple-solution tasks presented here in larger samples (Schoevers et al., 2018; Schoevers et al., 2019). Since the problem-posing and multiple-solution tasks yielded different responses, separate codes for the type of idea were used for each (see Table 11.7). Lastly, every turn was binary coded to indicate whether children received any help to formulate their response or idea.

Since the tasks were used to measure the creative thinking process, and not the creative product (e.g., children also received help), no formal scores were calculated for fluency, flexibility, and originality. However, a descriptive comparison of the originality of ideas could be made between children based on previous research (Schoevers et al., 2018; Schoevers et al., 2019) that used the same tasks. These studies determined how original ideas were by comparing the frequency of a certain type of response to the frequency of other types of responses. For the problem-posing task, the Schoevers et al. studies showed a large variation in the questions that were generated, which means that quite a lot of responses could be seen as original. The questions that were generated the least often were (1) questions that made use of addition, subtraction, or division, (2) questions about ratio, (3) questions about volume, and (4) questions about circumference. An unoriginal response to the problem-posing task was any question that revolved around the concept of amount. For the multiple-solution task, original responses were (1) calculations with two numbers using division, (2) calculations with three or more numbers using subtraction or multiplication, (3) calculations using decimals, and (4) calculations using numbers consisting of three or more digits. Unoriginal responses for the multiple-solution task were calculations with two numbers using addition or mixed operations. To determine descriptive originality, the number of original responses was counted for every child.

## 11.3 Findings

For all children together, a total of 2197 turns was identified. Out of these turns, 585 (27%) received the code mathematical creativity. Subsequent coding of these turns showed that children predominantly used divergent thinking (76%), followed by a combination of divergent and convergent thinking (14%) and convergent thinking (10%).

### *11.3.1 Number of Creative Ideas*

Children's turns were mostly coded as non-creative, as opposed to mathematically creative turns (see Table 11.3). This means that mathematical concepts were often not combined in a new way when children generated questions (problem-posing task) or calculations (multiple-solution task). Although many children were able to think of several questions or calculations, the underlying ideas often seemed to be quite similar. For example, many children started the problem-posing task by posing an "amount" question and re-used this concept in subsequent ideas, only changing their strategy when prompted by the researcher (e.g., "Can you also think of a different type of question/calculation?" or "Can you think of a question/calculation that incorporates addition?"). Such uniform strategy use also occurred on the multiple-solution task. Children would, for example, generate many calculations with two numbers using addition or mixed operations (e.g., addition and subtraction). The excerpt below illustrates how one child produced comparable responses to the problem-posing task. Only the first question received the code mathematical creativity because subsequent questions were a repetition of this first concept.

*Child: I have a question. How many black chairs are there?*

Researcher: How many black chairs are there. Yes, good one. How did you think of that one?

*Child: Well, there are chairs, but here are also another two chairs, and then, you don't know whether you should count those.*

Researcher: Yes.

*Child: So, how many black chairs are there?*

Researcher: Yes, smart. Then you can't be confused about which chairs the question is.

*Child: This is quite hard.*

Researcher: There is also a lot to see in the picture. … But, take your time; there is no rush.

*Child: How many brown chairs are there?*

Researcher: Yes, that's possible too. You can write that down.

……

*Child: How many big trees are there?*

**Table 11.3** Number of creative ideas for the low-achieving and high-achieving group

| | Creative ideas (%) | Non-creative ideas (%) |
|---|---|---|
| Low achieving | 26 | 74 |
| High achieving | 27 | 73 |

*Note:* Percentage for this group of children

In addition, quite many turns involved clarification questions about the task in which no mathematically creative question or calculation was proposed. Even though children had worked on a multiple-solution task before, this type of task was still rather new to them. Some children started each task by extensively asking what was possible or what would be considered "good," despite instructions that there were no right or wrong answers as in a "regular" mathematical task. An example of such questions is presented below.

*[Child reads the instruction aloud.]*
Researcher: Yes, so on the next page, there is room to write down the questions your classmates can answer. Do you have an idea how you can approach this?
*Child: And then … it has to be about mathematics?*
Researcher: Yes. So, it should be about mathematics and about the picture, but anything that relates to those two things you can ask a question about.
*Child: That's hard. And should it be easy questions? Or not?*
Researcher: Any type of question.
*Child: So, also a question on the level of first or second grade?*

### *11.3.2 The Use of Divergent Thinking*

To generate creative questions and solutions, children mostly seemed to make use of divergent thinking, with 76% of turns being coded as such. Many divergent thinking turns were statements of ideas, for example, "*And now, I have another mathematical question. What is the amount of chairs plus the amount of people?*" (problem-posing task) or "*Ehm … 6 times 1 and 3 times 2*" (multiple-solution task). Divergent thinking also concerned any elaboration or explanation of an idea, which differed considerably in terms of elaborateness. The initial idea statements of some children were short, for example, "*The next question is how many chairs are there?*" (problem-posing task). Other children immediately explained their ideas more elaborately, for example, "*I should probably pose a question about circumference, because with 1 square meter you are sitting in the middle, on top of the table. … and 2 people can sit at 1 square meter. … one on this side and one on the other side. … And 3 people can sit at the other square meter, because one can sit at the corner … so then, you should calculate the circumference as well*" (problem-posing task). This child had previously posed a question about the surface area of the table and how many people could be seated at it but realized that the circumference of the table would be a better way to calculate this.

Similar differences occurred when children were asked to explain how they conceived an idea. Some children explained that merely seeing an element of the picture or a number brought them to an idea, whereas others gave a more elaborate clarification. For example, one child mentioned thinking of a certain question on the problem-posing task because a similar question had been used in the mathematics class a few times. Children often related information from the task to a different setting. A few children mentioned that similar ideas or topics had been discussed in mathematics class, but also more remotely related settings or contexts were mentioned. For example, one child was imagining where the picture in the problem-posing task could have been taken and thought of Veluwe, a national park in the Netherlands. One child even seemed to activate such contextual knowledge, starting by asking himself "*Where are they and what are they doing?*" (problem-posing task). Something that most children had in common in terms of the divergent thinking statements was that they looked for elements that stood out. For example, some children decided to start their calculations in the multiple-solution task with the number 1, because it was the first number they noticed when reading the question. Some children also explicitly mentioned that they were trying to think of a question or calculation that was different from the ones that they had used before.

### *11.3.3 The Use of Convergent Thinking and Combinations of Divergent and Convergent Thinking*

Children's spontaneous and prompted turns were less often coded as convergent thinking (10% of the turns) than divergent thinking (76% of the turns), both before and after researcher prompts. Convergent thinking was defined as the process of evaluating or selecting a mathematically creative question or calculation. Occasionally, children showed that they evaluated an idea or elaborated on why they selected a certain idea. For example, a child who proposed the question "*How many chairs are there*" on the problem-posing task later explained that she selected this question because easy questions were also allowed. Another child said, "*Yes, I had to think is this really a good and logical question?*" (problem-posing task). A recurring theme throughout the turns that received a convergent thinking code was that children either evaluated their own ideas as being simple or easy or mentioned specifically trying to think of or selecting ideas that were "easy to think of." For example, "*Yes, I just did a lot of easy calculations, except for this one!*" (multiple-solution task) or "*Using this method, I could go on easily*" (multiple-solution task). These statements might suggest that children did not want to challenge themselves (one child also said on the multiple-solution task "*I am not going to use divisions because I find that difficult*") or might be looking for a general rule or strategy they could use to generate many ideas. For example, one child said, "*I tried to make calculations that usually had 5 or 10 as outcome; it does not need to be very big*" (multiple-solution task). However, occasionally, children did mention looking for variability or different ideas, for example, "*I did not want to do the same thing every time, so then, I decided to do this*" (multiple-solution task).

Convergent thinking often co-occurred with divergent thinking. Such divergent-and-convergent-thinking turns often incorporated the initial creation of an idea and a selection or evaluation that further refined the idea. Examples are "*Ehm, I saw two* *tables, and I thought I can't ask a question about the chairs. Yes, maybe it would be possible. But then, I thought no; I should ask how long can the table be, because that also is related to mathematics*" (problem-posing task) or "*Yes, you could try and see which calculations have 1 as outcome. And when you have had those, you can do it with 2 as outcome. And then, for example, with lower numbers, because when it is hard, it is best to start small*" (multiple-solution task).

In terms of sequence, children often generated multiple ideas first (i.e., divergent thinking) before switching to convergent thinking or a combination of divergent and convergent thinking. That is, for most children, the first couple of turns were coded as divergent thinking. These turns either conveyed different ideas that were then translated into a specific question/problem or calculation or a multitude of more finished ideas (i.e., different actual questions/problems or calculations). After children had started thinking of different ideas, they also started applying convergent thinking or combinations of divergent and convergent thinking. Since both tasks required children to think of multiple ideas, this sequence was repeated several times. Most often, a few divergent thinking turns were identified before a turn with a combination of divergent and convergent thinking or pure convergent thinking occurred. An example of a sequence of divergent and convergent thinking for the problem-posing task is presented below.

*Child (divergent): Maybe, a question is how many chairs are at the table?*

Researcher: Yes, seems like a good one. You can write that down. And how did you think of that question?

*Child (divergent): Well, when I read it had to be a question related to mathematics, I immediately thought of amount.*

Researcher: Yes.

*Child (convergent): Those kinds of questions are usually the normal basic questions you can ask.*

Researcher: Yes, very good. Did you think of amount before you decided to do something with the chairs?

*Child (divergent and convergent): Yes. But, you should be able to answer these questions right? So, you can't ask like how long is the table?*

Researcher: Yes, it is actually possible to ask that. As long as your question relates to the picture.

*Child: Okay.*

Researcher: So, it would be possible to ask how long the table is.

*[Child writes this question down]*

Researcher: Yes good one too. And how did you think of that?

*Child (divergent and convergent): Ehm, I saw two tables, and I thought I can't ask a question about the chairs. Yes, maybe it would be possible. But then, I thought no; I should ask how long can the table be, because that also is related to mathematics.*

### *11.3.4 Differences Between Children and Tasks*

The observations presented thus far focused on the general characterization of the creative mathematical problem-solving process. However, we also observed differences between children and across tasks. In terms of the sequence of divergent and convergent thinking, most children used some convergent thinking or a combination of divergent and convergent thinking after a couple of turns of divergent thinking. However, three children used only divergent thinking and no convergent thinking or a combination of divergent and convergent thinking on the two tasks. A trend for these children seemed to be that they did not generate many different questions or mathematical calculations that included a new combination of concepts and as such had relatively few turns that received the code mathematical creativity, to begin with. One of these children also received a relatively large amount of help.

Another notable difference between children concerned the creativity of the ideas that were generated using divergent and convergent thinking or the combination of both. Specifically, we compared the use of divergent and convergent thinking between children of whom more than two ideas on the two tasks were coded as original ($n = 4$) and children of whom no ideas were coded as original on the two tasks ($n = 7$) (see Table 11.4). Although originality is a judgment of the creative product, a comparison of divergent and convergent thinking between children who differed in terms of the originality of their ideas can still yield valuable insights into their creative thinking process. First of all, this comparison showed that children who did not generate original ideas required a little more help (in 60% of the mathematically creative turns) than children who generated original ideas (in 40% of the turns). The group of children who generated multiple original ideas made more use of convergent thinking on the problem-posing task. On the multiple-solution task however, children who generated original ideas differed from children who did not generate original ideas in the use of divergent thinking and the combination of divergent and convergent thinking. Children who generated original ideas used less divergent thinking, while using more combined approaches with divergent and convergent thinking. An example of such an approach is given below. This child thought of a mathematical calculation including a three-digit number on the multiple-solution task.

**Table 11.4** Number of original ideas for the low-achieving and high-achieving group

| | 0 original ideas | 1 original idea | 2 original ideas | 3 original ideas |
|---|---|---|---|---|
| Low achieving | 5 | 5 | 1 | 1 |
| High achieving | 2 | 9 | 2 | 3 |

*Child (divergent and convergent): Yes. I think I am going to choose … ehm … for example, 124 is something that is possible to make with those numbers.*

Researchers: Yes, you can.

*Child (divergent): Ehm, and then, I am using plus 1, but you can also do 123, that is one less, plus 2.*

*[Child writes down 124 + 1 = 123 + 2].*

Researcher: Yes, exactly, that is how you can do it. You thought of this one quickly. How did you do that?

*Child (divergent and convergent): Well, I just saw 123, but I thought that is an uneven number, so it is not so convenient, so then, I decided not to use that. I then went to the 4, and that seemed like it was convenient, so I thought let's make it simple and just add 1. But then, I thought, hey that is 125. So, to the 123, that's already here, you can add 2 and make 125, and it works out.*

We also contrasted children with high ($n = 16$) and low ($n = 12$) prior mathematical achievement. Findings showed that, although divergent thinking prevailed in both subgroups, children with high mathematical achievement scores had a slightly more balanced ratio (see Table 11.5) of divergent thinking to convergent thinking and combinations of divergent and convergent thinking than children with low mathematics achievement scores. On the problem-posing task, children with high mathematical achievement used slightly less divergent thinking and more convergent thinking than children with low mathematical achievement. On the multiple-solution task, the high mathematical achievement group used less divergent thinking and more combinations of divergent and convergent thinking than the low mathematical achievement group. Therefore, the differences between high and low achievers resembled those between children who generated original versus non-original ideas. This was also reflected in the fact that most original ideas were generated by children with high mathematical achievement, whereas children with low mathematical achievement generated more unoriginal ideas. The amount of help received did not differ substantially between the two groups.

Finally, the two different tasks were compared. The problem-posing task and multiple-solution task were quite comparable with regard to the occurrence of mathematically creative ideas and the use of divergent and convergent thinking. Table 11.6 shows the absolute and relative frequency of divergent thinking, convergent thinking, and a combination of the two for each task. On the problem-posing

**Table 11.5** Use of divergent thinking, convergent thinking, and divergent and convergent thinking for the low-achieving and high-achieving group

| | Divergent thinking (%) | Convergent thinking (%) | Divergent and convergent thinking (%) |
|---|---|---|---|
| Low achieving | 80 | 10 | 10 |
| High achieving | 72 | 11 | 17 |

*Note:* Percentage for this group of children

**Table 11.6** Use of divergent thinking, convergent thinking, and divergent and convergent thinking for the different tasks

| | Divergent thinking (%) | Convergent thinking (%) | Divergent and convergent thinking (%) |
|---|---|---|---|
| Problem-posing task | 80 | 6 | 14 |
| Multiple-solution task | 73 | 13 | 14 |

*Note:* Percentage for this task

task, around 28% of turns received the code mathematical creativity. On the multiple-solution task, this percentage was slightly lower (22%). Divergent thinking was predominant in both tasks, with a slightly higher occurrence rate on the problem-posing task. On the other hand, the use of convergent thinking was slightly higher on the multiple-solution task than on the problem-posing task. The use of a combination of divergent and convergent thinking was comparable between tasks. A notable difference between the two tasks concerned how elaborate ideas were, and therefore, also, extensive ideas were explained. Probably, the slightly larger percentage of divergent thinking used in the problem-posing task was related to the fact that children thought of complete questions here that included several elements (e.g., mathematical operations combined with several concepts from the picture). On the other hand, the multiple-solution task could be completed using relatively short calculations, which might lead to less elaborate divergent thinking processes and explanations thereof.

## 11.4 Discussion

This study investigated children's use of divergent and convergent thinking on two mathematical tasks: a problem-posing task and a multiple-solution task. Sixteen children with high mathematical achievement scores and twelve children with low mathematical achievement scores were asked how they thought of different creative ideas using think-aloud prompts. Their ideas were coded using qualitative content analysis. Specifically, the use of divergent and convergent thinking or a combination of the two was identified for every mathematically creative question (problem-posing task) or calculation (multiple-solution task).

### *11.4.1 The Use of Divergent and Convergent Thinking*

Relatively few ideas were coded as creative ideas (27%) compared to uncreative ideas (73%). Although children could generate multiple ideas for both tasks, it was more difficult for them to think of diverse ideas and especially original ones. This result is in line with previous research that showed that most high school students are able to produce ideas fluently but that flexibility and originality of solutions are more difficult to achieve (Leikin, 2013). The novelty of the tasks may also have impeded children's conception of creative ideas. Both tasks in this study differed from the closed tasks in Dutch mathematical textbooks (Van Zanten & Van den Heuvel-Panhuizen, 2018). Although new tasks could evoke creative ideas because they require children to search for different ways to solve a mathematical problem (Levenson, 2013), the newness of the task might also have caused some children to feel insecure. This was apparent from the many questions children asked at the beginning of each task about when a response would be considered "correct." Furthermore, some children needed additional explanations to understand what each task involved. The researcher emphasized in her explanation that there were many possible solutions, a response assumed to contribute to mathematical creativity (Kozlowski et al., 2019). Still, some children might have been preoccupied with finding the "right" answers because their mathematical instruction often focuses on one specific solution or solution strategy.

Despite the fact that most ideas were coded as uncreative, children also produced several creative ones, mostly through divergent thinking, and three children even relied exclusively on this mode of thinking. This finding is in line with Tabach and Levenson's (2018) suggestion that tasks with (infinitely) many solutions can lead to "excessive" divergent thinking. That is, such tasks might enable children to produce many, but sometimes infeasible or ineffective, ideas. For all children, at least some of these ideas represented a new combination of mathematical concepts and therefore received the code mathematical creativity. However, children who used divergent thinking and convergent thinking, either concurrently or in separate turns, generated the most original ideas. This finding is in line with previous research in which children with high divergent and convergent thinking skills also scored high on a multiple-solution task (de Vink et al., 2021). This finding also corroborates creative thinking theories that advocate the role of both divergent and convergent thinking (e.g., Brophy, 2001; Cropley, 2006; Guilford, 1973). It seems that, for mathematical creativity, ideas should not only be generated but also selected and evaluated to produce ideas of high quality.

Some authors have portrayed the creative thinking process as a linear series of steps (e.g., Wallas, 1926) whereas others characterize it as a "messy" process in which children alternately employ divergent and convergent thinking throughout the task (Isaksen et al., 2011; Lubart, 2018; Schindler & Lilienthal, 2020; Sheffield, 2009). Our study supports the latter view and indicates that switching between divergent and convergent thinking occurred somewhat irregularly: children often had multiple repetitions of divergent thinking, after which one or two turns of convergent thinking followed. This sequence seems to be a reflection of divergent thinking as an inherently exploratory process, that is, an "idea search in multiple directions […], which is inherently an exploration of a thought space" (Lubart, 2018, p. 7). Likely, children first explore several possibilities, which are then evaluated and combined into one solution. Previous research has not only identified divergent and convergent thinking as separate constructs but also showed that both thought processes might be intertwined (Barbot et al., 2016; Cortes et al., 2019). In line with this connectedness between divergent and convergent thinking, we also found instances of a combination of divergent and convergent thinking. Such combinations show that although divergent and convergent thinking are seen as separate constructs, their processes might indeed be intertwined (Barbot et al., 2016; Cortes et al., 2019). This connectedness is demonstrated at the task level with turns of divergent and convergent thinking. Turns that were coded with a combination of divergent and convergent thinking show that this connectedness extends to the turn level as well. These combination turns could reflect a micro-cycle in which the child alternated between divergent and convergent thinking. Analysis with other more fine-grained methods (e.g., preceding the turn level) could be used to unveil what such micro-cycles look like. This finding illustrates the need for more process-based research on divergent and convergent thinking, as static measures might not fully capture the complexity of the creative thinking process.

### *11.4.2 The Role of Mathematical Achievement and Task Type*

We also observed qualitative differences between children with high and low mathematics achievement scores. Previous research has shown higher mathematical achievement to be associated with higher mathematical creativity (Jeon et al., 2011; Kattou et al., 2013; Kroesbergen & Schoevers, 2017; Leikin, 2013; Schoevers et al., 2018). In this study, ideas of children with high mathematical achievement scores were coded more often as original than the ideas of children with low mathematical achievement scores. It is important to note that this finding, just like the other findings, is of qualitative nature and not statistically significant. Thus, we cannot conclude that children with high and low mathematical achievement scores differ in their ability to generate original ideas. Rather, this might be related to differences in the use of creative thinking skill. In this study, children with high mathematical achievement scores more often used convergent thinking, or combinations of divergent and convergent thinking, than children with low mathematics achievement scores. Mathematics education emphasizes convergent thinking (i.e., looking for one correct answer instead of generating different ideas or strategies; Levenson, 2013). Thus, it seems plausible that children with high mathematical achievement scores do well on tests of achievement by applying convergent thinking skill. These experiences with convergent thinking might have helped this group to select the most original ideas, while children with low mathematics achievement found this more difficult.

We found minor differences between children's creative performance on the two types of tasks. On the problem-posing task, we found a slightly higher percentage of creative ideas than on the multiple-solution task. Children used relatively more divergent thinking and less convergent thinking on the problem-posing task than they did on the multiple-solution task. To our knowledge, no research has yet compared children's creative thinking processes on these types of tasks. Therefore, it is difficult to determine whether this observation is specific to our study or whether other researchers might find the same results. It seems plausible that the problem-posing task elicits more creative ideas because it contains fewer constraints (Medeiros et al., 2014). The only constraints to generate a question on this task are the topic of mathematics and the elements within the picture. The multiple-solution task, by contrast, required children to use the given numbers and operators and to match the outcome of the two calculations. These restrictions may have caused a slightly smaller number of creative ideas. Another explanation, however, is that the problem-posing task was presented first and that children suffered more from fatigue or inattention on the multiple-solution task. The fact that this task is also slightly more similar to regular textbook mathematical tasks might explain why children made more use of convergent thinking on this task than on the problem-posing task.

### *11.4.3 Future Studies and Limitations*

This study provided a first look into how children with low or high mathematical achievement scores generate, select, and evaluate mathematically creative ideas on two types of mathematical tasks. Although we tried to make the research setting as relaxed and natural as possible for the children, it remains unclear whether our findings are typical of creative thinking in regular mathematics classrooms. Another possible limitation concerns the use of children's verbal expressions as a proxy for creative thinking. Although research has shown that children are capable of explaining their thinking on a mathematical task (Reed et al., 2015; Robinson, 2001), we do not know whether the ability to verbalize their thoughts differed between children, for example, as a result of differences in language ability or emotional factors like shyness. Therefore, the validity of the insight that we obtained into the creative thinking process might be higher for some children than for other children. Lastly, an important limitation of the current study is the binary coding of mathematical achievement as either "high" or "low." We used extreme case sampling as a way of creating an illustrative sample and labeled the groups accordingly. It is important to stress that this label is based on a single test score and, hence, not necessarily reflective of children's general mathematical skills or abilities. It does, however, provide insight into creative differences that can be observed between children who might score lower or higher on a more traditional mathematics test.

We recommend future research to contrast various types of measures of creative thinking processes in one sample to improve measurement reliability. For example, Schindler and Lilienthal (2020) combined eye-tracking with stimulated recall interviews to capture children's creative thinking. Furthermore, future research could contrast different types of tasks, as well as groups of children characterized by different individual features. For example, Bokhove and Jones (2018) have argued that mathematical creativity is not limited to open tasks but can also be displayed on tasks that are "moderately closed" (i.e., tasks that have some constraints but also allow for multiple solutions/strategies). It would be interesting to assess whether children's creative thinking process on such a task differs from creative thinking on more open tasks. Furthermore, differences between children, especially regarding cognitive characteristics such as executive functions, might play a large role in the creative mathematical thinking process and should therefore be a topic of further research. For example, it would be interesting to assess what role inhibition plays in the creative mathematical thinking process, because for children with high mathematical achievement, reduced inhibition aids mathematical creativity, whereas for children with low mathematical achievement, strong inhibition seems important (Stolte et al., 2019). Given the developmental nature of such cognitive characteristics, another interesting avenue for future research would be to examine what the creative mathematical thinking process looks like in different age groups (e.g., upper vs. lower primary school students).

## 11.5 Conclusion and Implications

This study showed that many children find it difficult to come up with new ideas and stick to ideas similar to the ones they had generated before. Whereas this inclination might not harm when solving mathematical problems that rely on automated knowledge, it becomes problematic when problems become more difficult and children can no longer rely on learned procedures. Our findings further indicate that convergent thinking is important in conceiving mathematically creative ideas (cf. Brophy, 2001; Cropley, 2006; de Vink et al., 2021; Tabach & Levenson, 2018). Primary math teachers are recommended to model and explain the use of divergent and convergent thinking in their classes, as the interplay between divergent and convergent thinking seems imperative. The use of different types of problems, both (moderately) closed and open, is recommended for children to gain experience with different ways of creative problem-solving. Finally, we recommend combining the learning of new mathematical facts or procedures with creative thinking, both divergent and convergent. It is important that children are not only taught that creative thinking is important, but also taught *how* to do this. The lower frequency of ideas coded as creativity in the group of children with low mathematical achievement scores shows that this group might need different support to come up with creative ideas. The dominant focus on convergent thinking in mathematics education might disfavor a certain group of children, both in terms of creative thinking and mathematical achievement.

## Appendix

**Table 11.7** Codebook

| Code | Subcodes | Problem-posing task | Multiple-solution task |
|---|---|---|---|
| 1_ Mathematical creativity | | The development of an idea that includes a combination of an element from the picture with a mathematical concept such as surface area in a, for the child, new way, resulting in an adequate question (e.g., *the development of the idea* "*How many square meters is the table?*"). | The development of an idea that includes a combination of the given numbers with a type of calculation (e.g., including decimals) in a, for the child, new way, resulting in a correct calculation (e.g., *the development of the idea* "*4,5 + 3 & 6 + 1,5*"). |
| 2_Divergent thinking | | The process of generating a mathematically creative question[a] based on the picture, as well as any corresponding elaboration or explanation (e.g., "*Oh, I have got one! How many square meter is the table? …. Because 2 people can sit at 1 square meter*"). | The process of generating a mathematically creative calculation[a], as well as any corresponding elaboration or explanation (e.g., "*Hmm, what can I do with this? … Ahh, okay … yes, 4,5 plus 3 and 6 plus 1,5*"). |
| 3_Convergent thinking | | The process of selecting or evaluating a mathematically creative question based on the picture (e.g., "*Yes, I had to think for a while whether it is actually a good and logical question*"). | The process of selecting or evaluating a mathematically creative calculation (e.g., "*I wanted to have an easy calculation, and I could think of this fast*"). |
| 4_Divergent and convergent thinking | | The process of generating and selecting or evaluating a mathematically creative question based on the picture, as well as any corresponding elaboration or explanation (e.g., "*I thought the surface are of the whole landscape is a bit too much … So, I was looking for something small that you could know the surface area of*"). | The process of generating and selecting or evaluating a mathematically creative calculating, as well as any corresponding elaboration or explanation (e.g., "*Well, I started thinking what I could with a comma, because I have not used it yet, so then, I thought of this question*"). |

(continued)

**Table 11.7** (continued)

| Code | Subcodes | Problem-posing task | Multiple-solution task |
|---|---|---|---|
| 5_Type of idea | _Adding | The generation of a mathematically creative question in which something is added (e.g., "*What is the amount of chairs plus the amount of people?*"). | N. A. |
| | _Amount | The generation of a mathematically creative question about an amount (e.g., "*How many chairs are there?*"). | N. A. |
| | _Circumference | The generation of a mathematically creative question about circumference (e.g., "*What is the circumference of the table?*"). | N. A. |
| | _Estimate | The generation of a mathematically creative question in which something is estimated (e.g., "*How many pebbles are there on the ground?*"). | N. A. |
| | _Multiplying | The generation of a mathematically creative question in which something is multiplied (e.g., "*If you multiply the amount of chairs by 2, how many chairs are there?*"). | N. A. |
| | _Ratio | The generation of a mathematically creative question about ratio (e.g., "*How many people can sit at the table?*"). | N. A. |
| | _Subtracting | The generation of a mathematically creative question in which something is subtracted (e.g., "*There are six chairs. I take two away; how many are left?*"). | N. A. |

(continued)

**Table 11.7** (continued)

| Code | Subcodes | Problem-posing task | Multiple-solution task |
|---|---|---|---|
| | _Surface area and size | The generation of a mathematically creative question about surface area and size (e.g., "*How many square meter is the garden?*"). | N. A. |
| | _Combination | The generation of a mathematically creative question in which any of the above concepts are combined (e.g., "*How many chairs are at a table on average?*"). | N. A. |
| | _Other | The generation of a mathematically creative question that does not fit with any of the other codes for type of idea (e.g., "*What kind of shape is the table?*"). | N. A. |
| | _2 number plus | N. A. | The generation of a mathematically creative calculation that uses two numbers and the operator plus (e.g., 2 + 2 = 3 + 1). |
| | _2 number minus | N. A. | The generation of a mathematically creative calculation that uses two numbers and the operator minus (e.g., 4–2 = 6 – 4). |
| | _2 number multiply | N. A. | The generation of a mathematically creative calculation that uses two numbers and the operator multiply (e.g., 2 × 3 = 1 × 6). |
| | _2 number mixed | N. A. | The generation of a mathematically creative calculation that uses two numbers and mixed operators (e.g., 2 + 4 = 2 × 3). |
| | _3 number plus | N. A. | The generation of a mathematically creative calculation that uses three numbers and the operator plus (e.g., 2 + 2 + 2 = 3 + 3). |
| | _3 number minus | N. A. | The generation of a mathematically creative calculation that uses three numbers and the operator minus (e.g., 6 – 2 – 2 = 5 – 3). |
| | _3 number multiply | N. A. | The generation of a mathematically creative calculation that uses three numbers and the operator multiply (e.g., 3 × 3 × 2 = 6 × 3). |

(continued)

**Table 11.7** (continued)

| Code | Subcodes | Problem-posing task | Multiple-solution task |
|---|---|---|---|
| | _3 number mixed | N. A. | The generation of a mathematically creative calculation that uses three numbers and mixed operators (e.g., $4 \times 2 + 1 = 6 + 2 + 1$). |
| | _2-digit number | N. A. | The generation of mathematically creative calculation that uses a number composed of two digits (e.g., $15 + 13 = 14 + 14$). |
| | _3-digit number | N. A. | The generation of mathematically creative calculation that uses a number composed of three digits (e.g., $124 + 146 = 142 + 126 + 2$). |
| | _decimal number | N. A. | The generation of a mathematically creative calculation that uses a decimal number (e.g., $1.4 + 4.6 = 3 + 3$). |
| Help | | The child received help during the process of generating, selecting or evaluating a mathematically creative question based on the picture. | The child received help during the process of generating, selecting, or evaluating a mathematically creative calculation. |

*Note:* [a]"Mathematically creative question" or "mathematically creative calculation" in this table refers to a question or calculation as defined under the code "mathematical creativity"

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# Chapter 12
# Group Creativity

**Peter Liljedahl**

## 12.1 Introduction

*Creative* is a modifier that can be used to refer to a product (*what a creative solution*), a process (*be creative*), a person (*she is so creative*), or an environment (*this is such a creative space*) (Liljedahl, 2008; Liljedahl & Allan, 2013, 2017; Mooney, 1963; Pitta-Pantazi et al., 2018; Rhodes, 1961). Each of these usages has formed a discourse in the creativity literature.

The work on creative products has its roots in the work of the psychologist J. P. Guilford (1950, 1967) who felt that creativity had not been adequately treated as a psychological phenomenon. In an effort to quantify creativity, he created the *Alternate Use Task*, which asked participants to find as many different uses for an everyday object. Responses to this task were then analyzed for *fluency* (how many uses were found), *flexibility* (were there fundamentally different uses found), *originality* (how unique were the responses), and *elaboration* (how detailed the responses were). E. P. Torrance (1966) extended Guilford's ideas to create the *Torrance Test of Creative Thinking*, which not only looked at alternate uses of objects but also asked participants to use everyday objects to solve problems and scored their performance on the same four categories as Guilford: fluency, flexibility, originality, and elaboration.

These approaches to quantifying creativity all have their roots in the assumption that for creativity to have happened, then something must have been created—it cannot be a creative process if nothing is created (Bailin, 1994). And what is created acts as a proxy for determining the *creativity* of the process that spawned it (Getzels & Jackson, 1962; Torrance, 1966). The second discourse on creativity—creative process—does not make this assumption.

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Unlike the discourse on creative products, which began in psychology and migrated to mathematics, the discourse on the creative process has its roots in mathematics with the work of Henri Poincaré (1854–1912) who, in 1908, gave a presentation to the French Psychological Society in Paris entitled *L'Invention mathématique* (Poincaré, 1952). In this presentation, he shares a story of a creative process.

Just at this time, I left Caen, where I was living, to go on a geological excursion under the auspices of the School of Mines. The incident of the travel made me forget my mathematical work. Having reached Coutances, we entered an omnibus to go some place or other. At the moment when I put my foot on the step, the idea came to me, without anything in my former thoughts seeming to have paved the way for it, that the transformations I had used to define the Fuchsian functions were identical with those of non-Euclidean geometry. I did not verify the idea; I should not have had the time, as, upon taking my seat in the omnibus, I went on with the conversation already commenced, but I felt a perfect certainty. On my return to Caen, for conscience' sake, I verified the results at my leisure (Poincaré, 1952, p. 53).

Poincaré argued that the creative process lived at the junction between the conscious and the unconscious (or subconscious) mind and that what felt like illumination was actually the transference of an idea from the unconscious to the conscious. Jacques Hadamard (1865–1963), a contemporary and a friend of Poincaré, extended these ideas. Using an instrument originally created by two French psychologists (Édouard Claparède and Théodore Flournoy), he surveyed his friends—mathematicians such as Henri Poincaré and Albert Einstein. In 1943, Hadamard gave a series of lectures on mathematical invention at the École Libre des Hautes Études in New York City. These talks were subsequently published as *The Psychology of Mathematical Invention in the Mathematical Field* (1945).

Hadamard's classic work positions the subject of invention at the crossroads of mathematics and psychology. It not only provides an entertaining look at the eccentric nature of mathematicians and their rituals but also outlines the beliefs of mid-twentieth-century mathematicians about the means by which they arrive at new mathematics. It is an extensive exploration and extended argument for the existence of unconscious mental processes. In essence, Hadamard took the ideas that Poincaré had posed and, borrowing a conceptual framework for the characterization of the creative process from the Gestaltists of the time (Wallas, 1926), turned them into a stage theory—*initiation*, *incubation*, *illumination*, and *verification*. This theory still stands as the most viable and reasonable description of the process of mathematical creativity.

After Hadamard (1945), the work on the creative process splits into two unique discourses—*descriptive* and *prescriptive*. Although both of these discourses have their roots in the four stage process that Wallas (1926) proposed, they make use of these stages in very different ways. The descriptive discourse sees all four stages as important and inevitable part of the creative process (Kneller, 1965; Koestler, 1964). For example, Csíkszentmihályi (1996), in his work on "flow," attends to each of the stages, with much attention paid to the fluid area between conscious and unconscious work, or initiation and incubation. On the other hand, the *prescriptive* discourse primarily focuses on the first stage, *initiation*, and is best summarized as a cause-and-effect discussion of creativity wherein the thinking processes during the initiation stage are the cause and the creative outcome are the effects (Ghiselin, 1952). Some of the literature claims that the seeds of creativity lie in being able to think about a problem or situation analogically (Johnson-Laird, 1989). Other literature claims that utilizing specific thinking tools such as imagination, empathy, and embodiment (Root-Bernstein & Root-Bernstein, 1999) helps kick start the process. In all of these cases, the underlying theory is that the eventual presentation of a creative idea will be precipitated by the conscious and deliberate efforts during the initiation stage. Taken together, whereas the prescriptive discourse sees the creative process as a means to an end, the literature following the descriptive discourse sees creative process as an end unto itself.

The third discourse on creativity pertains to the person. This discourse is dominated by two distinct characteristics, *habit* (Bailin, 1994) and *genius* (Silver, 1997). Habit has to do with the personal habits as well as the habits of mind of people that have been deemed to be creative (Pehkonen, 1997). However, creative people are most easily identified through their reputation for genius (Silver, 1997). Consequently, this discourse is often dominated by the analyses of the habits of geniuses as is seen in the work of Ghiselin (1952), Koestler (1964), and Kneller (1965) who draw on historical personalities such as Albert Einstein, Henri Poincaré, Vincent van Gogh, D. H. Lawrence, Samuel Taylor Coleridge, Igor Stravinsky, and Wolfgang Amadeus Mozart, to name a few. The result of this sort of treatment is that creative acts are viewed as rare mental feats, which are produced by extraordinary individuals who use extraordinary thought processes (Weisberg, 1999).

The first three discourses on creativity—product, process, and person—although not explicitly so, assumes that creativity is a solitary activity. Guilford and Torrance were interested in the fluency, flexibility, originality, and elaboration of individual research participants. Claparède and Flournoy, Poincaré (1952), Hadamard (1945), and Wallas (1926) were all interested in the creative process of a person working alone. And the discourse on creative people, by definition, looks at individuals.

The final discourse on creativity—environment—does not make this assumption. Yes, environment has a significant role in the contribution to (Goldin, 2002), and sustaining (Csíkszentmihályi, 1996) of, the creativity of an individual—whether that be a product, process, or person (Mooney, 1963; Rhodes, 1961).

The creation of a creative product and the interaction of a creative person and a creative process does not occur in a vacuum […] Hence, it is impossible to separate creativity from the context in which it takes place (Pitta-Pantazi et al., 2018, p. 41).

But the discourse on environment does not limit us to thinking about creativity of the individual. We can explore the role that environment has on fostering and maintaining the creativity of groups. This is exactly what I am interested in looking at in this chapter—the role of environment on group creativity. More specifically, I am interested in looking at the role of a collective problem-solving environment called a *thinking classroom* in fostering the group creativity and what it is about this environment that achieves this.

## 12.2 Building Thinking Classrooms

*Building Thinking Classrooms* (Liljedahl, 2020) is a teaching framework that I developed in response to the realization that much of what happens during a mathematics lesson is not thinking—at least not the type of thinking that we know students need to be doing to have ongoing success in mathematics. In particular, the baseline data that emerged from this research showed that in a typical lesson only about 20% of students do any thinking at all and, even then, only for approximately 20% of the lesson. That is, in a typical 60-min lesson of 30 students, 5–7 students will spend 8–16 min thinking, while the rest of students spend no time thinking (Liljedahl, 2020). The research has further shown that the normative practices present in many classrooms are not only allowing student to not think but are also actually promoting, in both explicit and implicit ways, nonthinking behaviors such as mimicking (Liljedahl, 2020; Liljedahl & Allan, 2013). These normative structures permeate classrooms around the world and are so entrenched that they transcend the idea of classroom norms (Yackel & Cobb, 1996) and can only be described as institutional norms (Liu & Liljedahl, 2012)—norms that have extended beyond the classroom and have become ensconced in the very institution of school and the fabric of what it means to teach.

Much of how classrooms look and much of what happens in them today is guided by these institutional norms—norms that have not changed since the inception of an industrial-age model of public education. Yes, desks look different now, and we have gone from blackboards to greenboards to whiteboards to smartboards, but students are still sitting, and teachers are still standing. Although there have been many innovations in assessment, technology, and pedagogy, much of the foundational structure of school remain the same. I realized that if we want to promote and sustain thinking in the classroom, these norms were going to have to change (Liljedahl, 2020). So, I embarked on a massive long-term study into what kinds of changes were necessary for thinking to flourish in the classroom.

Over the course of 15 years, and through the conducting of thousands of micro-experiments with over 400 K-12 practicing teachers, a series of 14 practices eventually emerged that not only broke away from the aforementioned institutional normative ways of teaching but have also been empirically proven to get more students thinking and thinking for longer (Liljedahl, 2020). Each of these practices is a response to a question—a question which, in turn, served as a variable in the research.[1] Each of these questions/variables and the emperically emergent optimal thinking practices is briefly described below.

1. *What are the types of tasks we should use?* If we want students to think, then we have to give them something to think about—and that comes in the form of a task. Good thinking tasks are tasks that are novel to the students—they have not seen them before. They need to have a low floor (accessible to all students) and

[1] More details about the methodologies involved and the results can be found in Liljedahl (2020, 2016).

a high ceiling (have evolving complexity that allows all students to eventually feel challenged). At the beginning, highly engaging, non-curricular tasks are used, but after a period of time, they can be gradually replaced with curricular thinking tasks.

2. *How should collaborative groups be formed?* At the beginning of each lesson, students should be placed into visibly random groups of three to work on the thinking tasks. If the lesson is significantly longer than 1 h, these groups should be re-randomized approximately every hour.
3. *Where should students work?* Groups should stand and work on vertical non-permanent (erasable) surfaces (VNPS) such as whiteboards, blackboards, or windows, making work visible to the teacher and other groups.
4. *How should we arrange the furniture in the classroom?* The classroom should be de-fronted with desks placed in a random configuration around the room (but away from the walls) and the teacher addresses the class from a variety of locations within the room. Further, the teacher's desk should not be on the same wall as the projector and screen.
5. *How should we answer questions?* Students only ask three types of questions: (1) proximity questions, which are asked when the teacher is close; (2) stop thinking questions, e.g., "is this right" or "are we doing this right"; and (3) keep thinking questions, which are clarifying or extending questions they ask so they can get back to work. Teachers should answer only the third type of question.
6. *When, where, and how should we give tasks?* Tasks should be given verbally and visually (non-textually), in the first 5 min of the lesson, and from a noncentral location in the room with students standing in loose formation around the teacher. If there are data, diagrams, or long expressions in the task, then these are written or projected on a wall, but the instructions pertaining to the activity of the task should be given verbally.
7. *What should homework look like?* Rather than *assigning* homework or practice questions, students should be given the *opportunity* to do 4–6 questions for them to check their understanding. Students should have the freedom to work on these in self-selected groups or on their own, and on the vertical nonpermanent surfaces or in their desks. They should be for self-evaluation and not marked or checked.
8. *How should we foster student autonomy?* Students should interact with other groups extensively, both for the purposes of extending their work and getting help. As much as possible, the teacher should encourage this interaction by directing students toward other groups.
9. *How should we use hints and extensions to further student understanding?* The teacher should maintain student engagement through a judicious and timely use of hints and extensions to maintain a balance between the challenge of the current task and the abilities of the students working on it.
10. *How should we consolidate a lesson?* When every group has passed a minimum threshold, the teacher should pull the students together to debrief what they have been doing. This debriefing should begin at a level that every student in the room can participate in and use group VNPS work to illustrate and exemplify the mathematics emerging out of the group activity.

11. *How should students take notes?* Students should make notes to their future selves. Students should have autonomy of what goes in these notes and how they are formatted and should be based on work that has already taken place.
12. *What should we choose to evaluate?* Evaluation should honor the activities of a thinking classroom—evaluate what you value. If you value perseverance, evaluate perseverance; if you value collaboration, evaluate collaboration; and so on.
13. *How should we do formative assessment?* Formative assessment should be focused primarily on informing students about where they are and where they are going in their learning. This requires, by necessity, a number of different activities from observation to check your understanding questions to unmarked quizzes where the teacher helps students to decode their demonstrated understandings.
14. *How should we grade?* Reporting out of students' performance should be based on the analysis of the data, rather than the counting of points, that is collected for each student within a reporting cycle. These data need to be analyzed on a differentiated basis and be focused on discerning the learning that a student has demonstrated.

Although each of these 14 practices, on their own and in concert, have been empirically shown to contribute to an increase in student thinking in the classroom (Liljedahl, 2020), the visually defining quality of a thinking classroom is that students work together to solve thinking tasks in random groups of three while standing at vertical whiteboards (see Fig. 12.1).

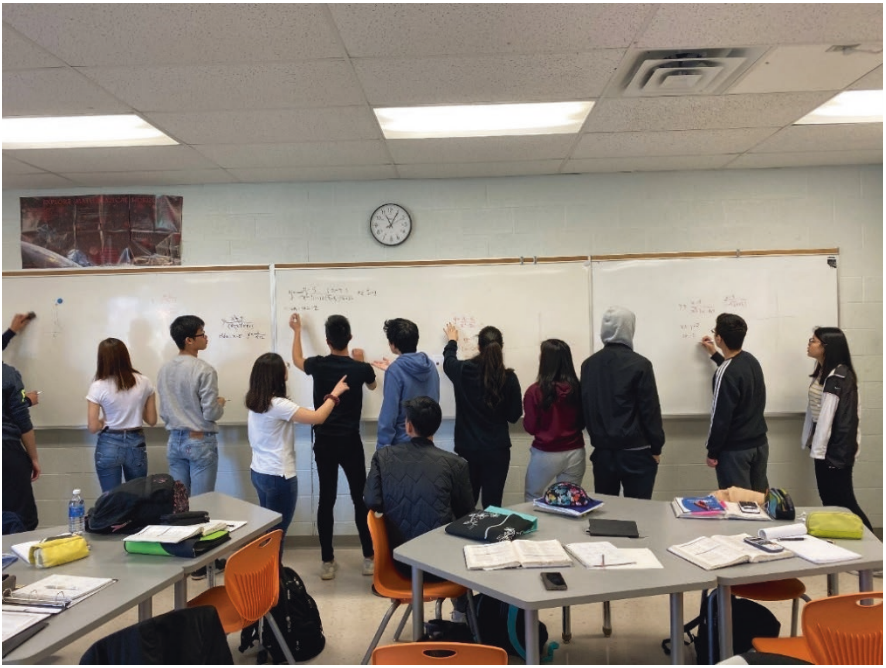

**Fig. 12.1** A thinking classroom

When put together, these 14 practices build a classroom ethos, routine, and culture of students thinking individually and collectively to do and learn mathematics. And it radically improves on the baseline data stated above. When these thinking classroom practices are enacted, rather than 20% of students thinking, we are now seeing upward of 90% of students thinking. And rather than thinking for 8–16 min, students are now thinking for 50–85 min.

But these practices do more than create an environment conducive to thinking. It also creates an environment conducive to group creativity. By the mere fact that students are working on unfamiliar tasks—curricular or non-curricular—they are going to get stuck. And, in some cases, they are going to need creative insights to get unstuck. But the creativity that is exhibited in these settings does not always follow the four-step creative process that we see in individuals (Hadamard, 1945; Wallas, 1926). For students working collectively, we need to look for different markers of creativity. I draw on the phenomenon of *burstiness* for these markers as well as environments that allow for these markers to flourish.

## 12.3 Burstiness

When we think of a group being creative, one of the images that comes to mind is a brainstorming session. Ironically, this is both a naïve and flawed understanding of what group creativity looks like in a group. Brainstorming is just the throwing out of ideas into a common space. Most of these ideas are routine derivations or extensions of what is already known. Every once in a while, however, an idea may be offered that is the product of an individual creative process. This offering, in and of itself, is not enough to say that a group is being creative. Even if multiple creative ideas are offered by individuals of the group, the group is not understood to be creative. Brainstorming is not group creativity. It may lead to group creativity. But it is not yet group creativity. A group is said to be creative when *burstiness* occurs.

*Burstiness* is a term that describes how rapidly members of a group are taking turns in conversation. The more rapid the exchange of ideas, the more bursty the communication is until there is a point where members of the group are walking on top of each other, excitedly interrupting each other, adding to each others' ideas, and building off each other to spin out new ideas. "Burstiness is when everybody is speaking and responding to each other in a short amount of time" (Woolley, quoted by Vallance, 2020) and "is like the best moments in improv jazz. Someone plays a note, someone else jumps in with a harmony, and pretty soon, you have a collective sound that no one planned" (Grant, 2018).

Burstiness is not brainstorming. It is what may emerge out of brainstorming. "Burstiness is a sign that you're not stuck in one of those dysfunctional brainstorming sessions. It's when a group reaches its creative peak because everyone is participating freely and contributing ideas" (Woolley, quoted by Vallance, 2020).

Burstiness is a concept that emerged out of group psychology and the realization that, against all odds, teams of collaborators working asynchronously online can and do sometimes become highly productive. Traditional organizational literature, which looked at the ingredients necessary for successful collaboration within brick-and-mortar organization, could not explain how any level of collective intelligence could emerge from such settings (Boudreau et al., 2014; Woolley et al., 2010). But it did. Digging into these successful online asynchronous collaborations revealed that in instances of productivity, there was a marked decrease in the time between communication until the point where the work shifted from being asynchronous to synchronous. And thus, was born the idea of burstiness.

Emerging first as a marker of productivity, burstiness has since been linked to innovation (Grant, 2018) and creativity (Vallance, 2020), and it has shifted from describing these outcomes in online asynchronous collaborative groups to face-to-face synchronous collaborative groups. And like often happens, the research has shifted from finding descriptive markers of burstiness to finding prescriptive ingredients necessary for burstiness to occur. This prescriptive work has found that there are key ingredients necessary for an environment to be ripe for burstiness—and group creativity—to occur (Grant, 2018; Marghetis et al., 2019; Riedl & Wooley, 2020; Vallance, 2020). In what follows, I present seven of these key ingredients.

1. *Some Structure*: Lack of focus can occur if the work environment is too open. The work environment needs to allow for the unencumbered flow of ideas (Vallance, 2020) while at the same time providing enough structure to ensure that everyone can get their ideas out (Grant, 2018). In essence, there needs to be a structure around which the group can organize their collective work.
2. *Diversity*: As with many things, when it comes to burstiness, diversity is a strength. As such, the group needs to be made of people with different backgrounds, different knowledge, and different ways of thinking. When everyone is the same, "they do worse at creative problem-solving, but they think they do better, because they're more comfortable. Diverse groups are more creative" (Grant, 2018).
3. *Psychological Safety*: Group members need to feel safe to contributing ideas and know that they will not be punished or humiliated for their ideas, questions, concerns, or mistakes (Grant, 2018).
4. *Welcome Criticism*: This does not mean that ideas are above criticism. Burstiness does not happen unless good ideas come to the surface and that can not happen without a process in place to weed out less good ideas. As such, group members need to feel safe in offering criticism of ideas as well as feel safe in having their ideas criticized. The key here is that the critique is about the idea, not about the person (Grant, 2018).
5. *Freedom to Shift Attention*: One of the key markers of burstiness is the ability of a group member, or the group as a whole, to shift their attention between ideas. As such, the structure needs to provide a workspace that will not only allow for the presentation and representation of lots of different ideas but also afford the freedom to shift their attention between these many ideas (Marghetis et al., 2019).
6. *Focus*: As mentioned, diversity is a good thing, but too much diversity can lead to a lack of focus—especially as it pertains to ideas. This is why brainstorming

is not the same as burstiness. Brainstorming has too much diversity—too big a range of idea. Burstiness cannot happen until the group settles on a smaller subset of ideas and begins to focus their energy on those (Riedl & Wooley, 2020).

7. *Opportunity for Nonverbal Communication*: Burstiness is signaled by group members talking over each other as ideas are pilled on top of ideas. From a purely verbal perspective, this can appear as quite a rude behavior as group members interrupt each other. From a nonverbal perspective, however, it isn't rude. The use of body language to both read and signal emotions and intentions allows burstiness to not be experienced as rude and to remain productive (Riedl & Wooley, 2020). As such, the work environment needs to allow for an ease of nonverbal communication.

## 12.4 Method

As mentioned, my intention in this chapter is to look at the role of environment on group creativity. More specifically, I am interested in looking at the role of a *thinking classroom* in fostering the group creativity and what it is about *thinking classrooms* that achieves this. To this end, the data for the work presented here comes from a grade 11 Foundations of Mathematics (FoM 11) course wherein the teacher is enacting the *Building Thinking Classrooms* framework of teaching.

### *12.4.1 Course and Participants*

At the time of data collection, students could choose from one of three grade 11 mathematics courses: Pre-calculus 11 (PC 11), Foundations of Mathematics 11 (FoM 11), and Apprenticeship and Workplace 11 (A&W 11). Any one of the three will satisfy the high school graduation requirement, but only PC 11 and FoM 11 are eligible for admission to university. From a content perspective, the FoM and PC sequence of courses are equally rigorous. The difference is that, whereas PC 11 (and 12) is made up of exclusively continuous mathematics topics, FoM 11 (and 12) covers content from both continuous and discrete mathematics. Having said that, PC 11 and 12 are perceived to be more rigorous because they will allow students to apply to university programs that require calculus. The distribution of students across these three courses varies from school to school, depending on what percentage of students have ambitions to attend university, college, vocational school, or trade schools.

The school in which the data was gathered was in a middle-class neighborhood, graduation rates were high, and the majority of students anticipated going on to some sort of postsecondary education. As such, 50% of students in a grade 11 mathematics course[2] were in PC 11, 30% in FoM 11, and 20% in A&W 11. The particular FoM 11 course in which the data was collected consisted of 28 students (ages 16–19). Of these, 20 were in grade 11, six were in grade 12, and two were older than grade 12.

The data was gathered in late November. That is to say, the teacher had had plenty of time to establish the thinking classroom practices as norms in the classroom and to build a thinking classroom culture among her students.

### *12.4.2 The Lesson*

The lesson during which the data was collected began by the teacher gathering the students around her at one of the vertical whiteboards in the room. She then proceeded to play a game of tic-tac-toe with the students. She played badly—by design—and the student won. She acknowledged that the student had beaten her on one of the diagonals and asked the class, "how many different lines are there where the student could have won?" There was some murmuring among students, and then, the class began to call out, "8." She asked them to clarify and to show her where the eight wins were. She then asked, "what defines a win?" The class immediately responded with "three in a row," to which she stated, "rows are horizontal, and columns are vertical." The students modified their answer to "three on a line," to which she nodded with agreement.

The teacher then held up a picture of a 3D tic-tac-toe board (see Fig. 12.2) and asked, "if there are eight ways to win on 2D tic-tac-toe board, how many ways are there to win on a 3D tic-tac-toe board?" She then put the students into random groups of three using a deck of cards and sent them off to work at vertical nonpermanent surfaces spread around the room.

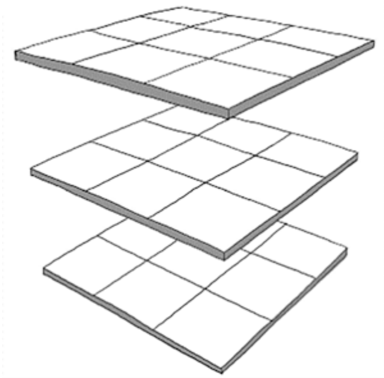

**Fig. 12.2** A 3D tic-tac-toe board

[2] Because of failure and acceleration, not all grade 11 students are in a grade 11 mathematics course. Likewise, not all students in a grade 11 mathematics course are grade 11 students.

### 12.4.3 The Data

The data comes from one of the groups working on this problem. The group was comprised of Adam, Betty, and Christine—all pseudonyms. The data consists of an audio recording of their work plus the field notes that I took during my observation of their work. In what follows, I provide synopsis of their problem-solving episode. This episode took 42 min from the time the group got to their whiteboard to the time the teacher drew the activity to a close. For purposes of brevity, I present their work through a combination of narrative, portions of their transcribed discussions, and excerpts from my field notes.

### 12.4.4 The Episode

Immediately upon getting to their whiteboard, Adam suggested that the answer might be $8^3$. Betty asked him where that came from, and Mark hypothesized that because "there are 8 ways to win in regular tic-tac-toe, there might be 8 × 8 × 8 ways to win in 3D tic-tac-toe." Christine pushed them off this idea by saying, "Hmm. I'm not sure. That seems too easy. Let's try counting and see where we get to."

With this, Betty drew a 3D tic-tac-toe board. Christine started off by stating, "we know there are 8 ways to win on each board," to which Adam immediately added, "and 9 straight up and down." There was some discussion (4 min) about this before Betty wrote 3 × 8 + 9 on the board. After this point, the group spent the next 8 min discussing a variety of different diagonals that could be considered wins. Eventually, they all agreed that there are four diagonals that start in a corner of the topmost gameboard, goes through the middle square of the middle gameboard, and finishes in a corner on the bottom game board (see Fig. 12.3).

During this discussion, Betty misunderstood what Christine was saying and drew in the diagonal that starts in a corner of the top game board, goes through a middle

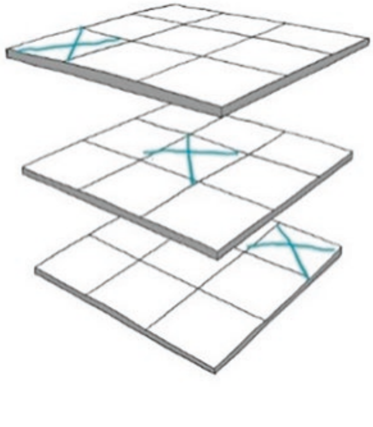

**Fig. 12.3** Diagonal through the middle

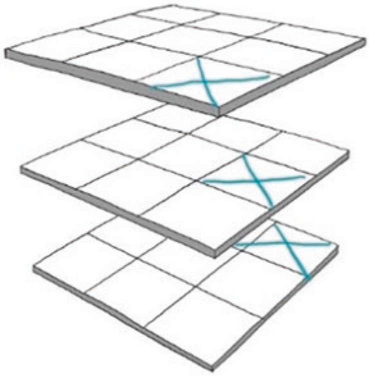

**Fig. 12.4** Diagonal along the edge

square along one of the edges of the middle board, and finishes in a corner on the bottom board (see Fig. 12.4). Both Adam and Christine dismissed this and pointed at the middle square of the middle board, "it goes through here."

After they had all agreed on the four diagonals going through the middle of the middle board (see Fig. 12.3), Betty brought their attention back to the diagonal she had drawn (see Fig. 12.4), "so, what about this one?" They all agreed that that was a win and that there were also four such wins. But then something interesting happened.

1. Christine Wait. It can go the other way as well.
2. Adam [8s. pause] Hmm. You mean up?
3. Betty [5s. pause] Up and down are the same win.
4. Christine [no pause] No. It can go this way [drawing a new line (see Fig. 12.5)].
5. Adam [5s. pause] Ok.
6. Betty [no pause] Yes. That's a win.
7. Adam [6s. pause] So, there are four more wins?
8. Betty [12s. pause] Wait. Didn't we already count those?
9. Christine [25s. pause] Ok. Let's try again. If we look at just this corner …[pointing at one of the corners on the top board].
10. Betty [no pause] … There are three wins …
11. Adam [no pause] … Three diagonals.
12. Christine [no pause] Right. One that way …
13. Betty [at the same time as Christine] … That way and that way.
14. Adam [3s. pause] Ok. [Adam writes a three in the corner they are pointing at (see Fig. 12.6).]
15. Christine [1s. pause] And these are also three [writes 3's in the other corners (see Fig. 12.6)].
16. Betty [no pause] [Betty writes 1's in the four edge squares (see Fig. 12.6).]
17. Adam [no pause] [Adam writes 0's in the middle square (see Fig. 12.6).]

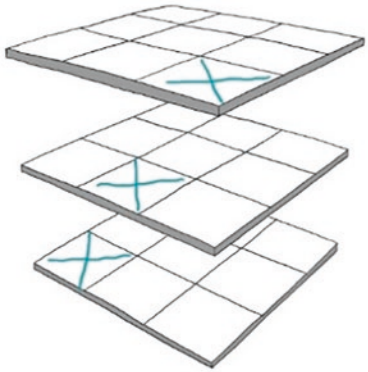

**Fig. 12.5** Diagonal along a different edge

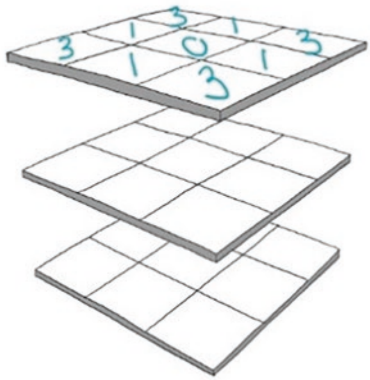

**Fig. 12.6** Diagonal wins from a cell

18. Betty [15s. pause] So, there are 24 plus nine plus …
19. Christine [at the same time as Betty] … Plus three plus three plus …
20. Adam [at the same time as Christine] … Plus one plus one plus one plus one.
21. Christine [5s. pause] So, 49?
22. Adam [5s. pause] Yeah.
23. Betty [15s. pause] Hmm. [10s. pause] What if we made this four [drawing a new 3 × 3 grid and putting a four in one of the corners (see Fig. 12.7)]?
24. Christine [no pause] Like, four ways to win from that corner going down.
25. Adam [no pause] Instead of the vertical [pointing at the 9 on the board].
26. Christine [2s. pause] Then, these are 2's and this is a 1 [pointing at the squares along the edges and the middle].
27. Betty [no pause] [writes in the numbers on the grid (see Fig. 12.7)].
28. Adam [5s. pause] So, now its 24 plus four time four plus four times two plus one …

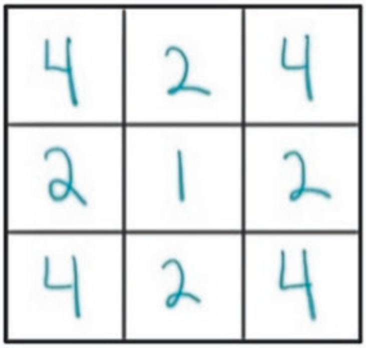


**Fig. 12.7** Diagonal wins from a cell plus vertical wins

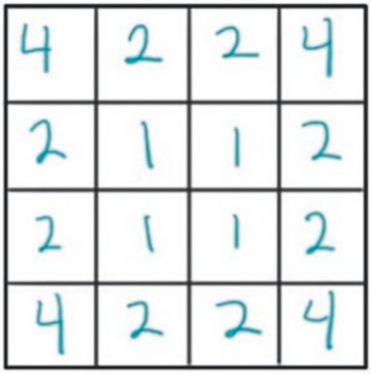


**Fig. 12.8** Wins from a cell on 4 × 4 × 4 game

29. Christine [5s. pause] 25?
30. Adam [5s. pause] Yeah, 25.

At this point, the group pauses for almost 2 min. Christine looks at her phone, and Adam looks around the room. Betty, on the other hand, is looking at the whiteboard.

31. Betty Let's do a 4 × 4 × 4. [Betty draws a 4 × 4 grid (see Fig. 12.8).]
32. Adam [no pause] [takes the marker from Betty and writes in the 4's and 2's and 1's (see Fig. 12.8)].
33. Christine [20s. pause] These are like constants [waving her hand over the 4 × 4 grid].
34. Betty [no pause] There are just more of them.
35. Adam [no pause] Except the 4's ….
36. Christine [no pause] … There are always four 4's ….
37. Betty [no pause] … Because there are always four corners [pointing at the four corners (see Fig. 12.8)].
38. Christine [15s. pause] I think we can generalize this.

At this point, the group begins a process of generalizing the problem to an $n \times n \times n$ game. They work in a more determined and purposeful way, making conjectures, pausing to discuss the conjectures, and checking their thinking. Eventually, they arrive at a generalization for the problem, simplifying this generalization, and checking their answer with another groups. That group had arrived at a different answer, so there was a fair bit of discussion before the two groups agreed on the final solution. During this discussion, the other group adopted Adam, Betty, and Christine's (Adam's group) way of tracking wins. So too did other groups who saw what Adam's group was doing. Adam, Betty, and Christine spent some time interacting with other groups about their way of counting wins that cut through the different game boards.

## 12.5 Analysis I: Burstiness

Within the lesson that I observed, coming up with the way of annotating each cell of the top game board (see Figs. 12.7 and 12.8) was unique to Adam's group. Others adopted it when they saw what Adam's group was doing, but only Adam's group came up with it on their own. In the 50 or so settings in which I have used this task, as well as in the 10 or so settings where I have seen it used, very few groups come up with the type of annotation that Adam's group used (less than 10%). It is a relatively novel solution. We might even want to call it a creative solution. But was the process that generated it creative? By looking at the time intervals between interactions, we can see where there were moments where the ideas were piling on each other as the group members exchanged thoughts without pauses, even talking overtop of each other. At these moments, the group was bursting—and by extension, exhibiting group creativity (Vallance, 2020).

Although the entire period between lines 1 and 37 can be seen as one single extended burst, we can also look at this time span as a series of five individual bursts. In what follows, I comment on each of these.

### *12.5.1 Burst 1: Lines 9–17*

The first burst was triggered by Christine when she suggested that they only look at one corner on the top board. This burst consisted of all three very rapidly talking about the different diagonals that can stem from one corner, eventually prompting Adam to write a 3 in that corner (see Fig. 12.6). This was followed immediately by Betty and Christine filling in the rest of the 3's and adding 1's to the non-corner cells along the edges (see Fig. 12.6). This burst is characterized by a shift from thinking about diagonals as lines in three-dimensional space and trying to track where those diagonals are to thinking about the cells that diagonals can start from and how many start from that same point. The minute this shift was initiated by Christine, both Betty and Adam's thinking switched over, and they started piling ideas on top of each other.

### *12.5.2 Burst 2: Lines 18–20*

This burst is triggered when Betty shifts the attention away from tracking and annotating diagonals to adding them all up. Adam and Christina quickly jump on this way of thinking as they all begin to talk overtop of each other. Although there are no ideas emerging out of this burst, it is a burst, nonetheless.

### *12.5.3 Burst 3: Lines 23–27*

The third burst was triggered by Betty when she suggested that they incorporate the vertical win into their notation scheme. Like with the first burst, this rethink about what they were doing was immediately picked up by Christine and Adam as they started piling on their ideas. This burst is marked by a shift in thinking of vertical wins as different from diagonal wins to a way of thinking about all the wins that cut through the different game boards—both diagonal and vertical—as belonging to the same category.

### *12.5.4 Burst 4: Lines 31–32*

This short burst is initiated by Betty when she suggests that they consider a $4 \times 4 \times 4$ game. This suggestion immediately brought Adam back to the group, and he jumped on Betty's suggestion without hesitation, adding in the relevant numbers. Although the transcript does not reveal this, Christine is fully focused on what Adam is doing, and I suspect, had there been more than one marker, Christine would have been writing on the grid at the same time as Adam. This burst is indicative of a shift of attention away from distractions and is triggered by Betty suggesting an extension to what they had already been doing.

### *12.5.5 Burst 5: Lines 33–37*

The final burst is triggered by Christine's suggestion that the numbers on the grid are constants. This prompts both Betty and Adam to join with Christine to think about the fact that the grid will always be populated by 4's and 2's and 1's, and what changes is how many of each that there are. This burst is marked by a shift of thinking about the $4 \times 4 \times 4$ game as a specific task to the $4 \times 4 \times 4$ game as an example of a more general game—the group started looking at the general in the particular (Mason & Pimm, 1984) in the $4 \times 4 \times 4$ game.

Vallance (2020) claims that burstiness is synonymous with group creativity, and hence, that moments of burstiness can be used to identify if a group is engaged in a collaborative creative experience. For the most part, the data presented here supports this claim. From the data, this seems to be mostly true in that four of the bursts (1, 3, 4, and 5) were marked by a creative shift in thinking about the task at hand—the most significant of which occurred in the first burst where thinking about diagonals moves from lines to starting points. But the data also shows that burst is occurring without evidence of group creativity—the group was just adding up the numbers in the grid. Regardless of whether a burst correlated with creative thinking or not, however, is that all five bursts were triggered by a shift of attention. So, burstiness, at least within this data set, is indicative of a shift of attention—and some shifts of attention can trigger group creativity.

The question that remains to be answered, now, is—*in what way did the environment occasion these incidences of burstiness*?

## 12.6 Analysis II: Environment

In what follows, I unpack the thinking classroom through the lens of the seven ingredients necessary to occasion burstiness to see in what ways the environment that Adam, Betty, and Christine were working in contributed to the burstiness we saw in the aforementioned episode.

### *12.6.1 Some Structure*

In order for burstiness to occur, there needs to be enough freedom for ideas to flow unencumbered (Vallance, 2020) while at the same time providing enough structure to ensure that ideas can come out (Grant, 2018). The thinking classroom, although providing students with lots of autonomy and freedom, has some well-defined structures—chief among is that students work at vertical nonpermanent surfaces (VNPS). Although a constant structure within a thinking classroom, these workspaces provide a huge degree of freedom as to how to use the space. It is a workspace that is both easy and familiar to output ideas onto. And it is large enough to hold multiple ideas from multiple group members on at the same time. For example, the whiteboard that Adam, Betty, and Christine worked on had all of the ideas that were shared in the episode above on the whiteboard at the same time.

### 12.6.2 Diversity

Another structure that is ubiquitous in a thinking classroom is that students work in random groups. Early on in the research, we discovered that the optimal group size was three. Groups of two struggled more than groups of three, and groups of four almost always devolved into a group of three plus one. Groups of three were optimal. We didn't know why—we just knew that groups of three were optimal. The explanation for why came to us from complexity theory, which tells us that in order for a group to be generative, it needs to have both redundancy and diversity (Davis & Simmt, 2003). Redundancy are the things that a group has in common—common language, common knowledge, and common notation. Without these commonalities, students cannot even begin to collaborate. But if all they have is redundancy, the group will not produce anything more than an individual member of the group could. To be generative, they also need to have diversity or the things that individual members of the group bring that are not shared by the others—different ideas, different viewpoints, different representations, etc. Random groups of three seems to have the perfect balance of redundancy and diversity.

This is not to say that randomness guarantees diversity. It doesn't. And it is always the case that when randomly assigning groups of three there will be some groups that are more diverse than others. It even happens sometimes that three very strong and like-minded students will be randomly put into the same group. When this happens, it is often the case that the teachers will declare, a priori, that this group is going to "kick but" on the task at hand. Ironically, this turns out to almost never be true. It is my experience that homogeneous groups of strong students are almost always outperformed by groups with more diverse abilities. Diverse groups offer up more ideas and interrogate these ideas and each other more—all of which are necessary for burstiness to occur.

Although not a guarantee of diversity, randomness does increase the likelihood that diversity can occur. For example, the *Building Thinking Classrooms* research showed that when students group themselves, they tend to select their partners from within their friend groups (Liljedahl, 2020). Friends tend to be more like-minded, have related histories and experiences, and have similar viewpoints and dispositions. Friend groups are less diverse. Likewise, when teachers make groups, they often form homogeneous groups (Liljedahl, 2020) as homogenous groups have long been promoted as a way to help teachers manage the challenges of differentiated instruction. Homogenous groups are not diverse. And even when teachers decide to form heterogeneous groups, their criteria for doing so often include concerns for behavior, social issues, and maintaining peace and quiet (Liljedahl, 2014, 2020)—all of which can diminish the diversity that can be achieved. Random groups cuts through all that.

### 12.6.3 *Psychological Safety*

Although not guaranteeing diversity, what creating random groups does is create an environment where students have psychological safety. In working with different students each day, students get to know everyone in the class which, in turn, builds community (Liljedahl, 2020). And with this comes an elimination of social barriers (Liljedahl, 2014) and the unlocking and mobilization of empathy among students (Liljedahl, 2020). The students begin to care about and care for each other more when random groupings are used. And they start to see themselves as more capable. In a study of how thinking classrooms shifted self-efficacy beliefs in students, I found that random groups were one of the most significant ways for teachers to persuade (Bandura, 1986, 1994, 1997) students, in nonverbal ways, that they were all equally capable (Liljedahl, in press).

With this increase in self-efficacy, mobilization of empathy, and greater sense of community, students feel less at risk and, as a result, are more willing to take risks. This is further supported by working on VNPS. My research has shown that working on erasable surfaces reduces the risk that students feel when outputting ideas (Liljedahl, 2016, 2019, 2020) where errors can easily be erased. Taken together, the structures of a thinking classroom increase the psychological safety for students and, as a result, increases the output of ideas.

### 12.6.4 *Welcome Criticism*

This is not to say that all these ideas are equally good. Burstiness often begins with brainstorming. We saw that in the above episode when Adam threw out the idea that the answer could be $8^3$. He didn't have any real expectation that this was the right answer—it was just an idea. Brainstorming is a good way to get lots of ideas out, but for burstiness to occur, groups need to be able to cut through these ideas and push forward those ideas that are better. Individual members of a group need to be able to not just offer up ideas; they need to interrogate these ideas and be ok with their ideas being interrogated. The psychological safety that thinking classrooms affords helps with this. The sense of community, feelings of empathy, and increased self-efficacy allow students to welcome the criticism that is necessary for good ideas to emerge and burstiness to occur.

### 12.6.5 *Freedom to Shift Attention*

But this is not to say that burstiness is only about focus on one idea. Burstiness occurs when there is freedom for group members to easily shift their attention between multiple good ideas (Marghetis et al., 2019). In the episode with Adam, Betty, and

Christine, we saw several instances of the group shifting their attention from one idea to the next. Being able to represent multiple ideas on vertical nonpermanent surfaces facilitates this shift in attention. So too does the ability to see other groups' work. Although Adam, Betty, and Christine did not do this very much, it is interesting to note that their way of representing how many wins originate from a particular cell (see Fig. 12.6) did spread to five other groups in the room. Although some groups adopted this method through direct interaction with the Adam, Betty, and Christine, other just noticed the idea as they were looking around the room. Good ideas can come both from inside the group and from outside the group. It doesn't matter. A good idea is a good idea and, regardless of where it comes from, can trigger a burst in another group. The freedom with which to shift attention between ideas in a thinking classroom, your own group's or others', is key for this to happen.

### 12.6.6 Focus

Access to too many ideas, however, can lead to a lack of focus, which, in turn, prevents burstiness from occurring. This is why brainstorming is not the same as burstiness. Brainstorming has too much diversity—too big a range of idea. Burstiness cannot happen until the group settles on a smaller subset of ideas and begins to focus their energy on those (Riedl & Wooley, 2020). Access to the multitude of ideas that are being shared by 10 different groups working on vertical whiteboards could create too much diversity and, as such, lead to a lack of focus, except it doesn't. And the reason it doesn't is that, although there is a lot of diversity of ideas among 30 students working in 10 groups, they are all working on the same task or sequence of tasks. This constrains the diversity and creates the focus necessary for burstiness to potentially occur.

### 12.6.7 Opportunity for Nonverbal Communication

In a thinking classroom, students are communicating with each other across five channels of communication. The first channel is the *verbal channel*—they are talking to each other. The second channel is the *representational channel*. As students are talking to each other, one member of the group is also writing what is being said through symbols and diagrams on the VNPS. Although only one member of the group has the ability to represent their or other's thinking in this way, all members of the group have the ability to point at the symbol and diagrams on the VNPS as they explain their thinking. This is the *pointing channel*, and it is a very well used channel in a thinking classroom. We saw this in several points in the episode. The fourth channel in a thinking classroom is the *tool channel*. Although not used in the particular episode above, in a thinking classroom, students often work with manipulatives and digital tools, and they use these tools to communicate their thinking with each other. The final channel of communication is the *gesture channel*. They use this channel to communicate ideas, to affirm or correct ideas, and to communicate emotions and intentions. Although only one of these five channels is verbal, the other four channels work in conjunction to the verbal channel to support the verbal communication of ideas. The gesture channel, in particular, allows group members to signal intention in such a way that the piling on of ideas during moments of burstiness is welcomed and not seen as rude instances of interrupting.

## 12.7 Conclusions

Creativity does not happen in a vacuum. "It is impossible to separate creativity from the context in which it takes place" (Pitta-Pantazi et al., 2018, p. 41). This context consists of not only the people that someone works with but also the environment within which this work is being done. In this chapter, I was interested in looking closer at the construct of group creativity—something that has been largely overlooked in the creativity research in mathematics education—and the role that environment plays in fostering group creativity. Using the construct of *burstiness* (Grant, 2018; Marghetis et al., 2019; Riedl & Wooley, 2020; Vallance, 2020) to identify instances of group creativity, I was able to not only illuminate what creativity looks like in a collaborative setting but also illuminate the important role that environment plays in occasioning burstiness. Structure, diversity, psychological safety, welcome criticism, freedom to shift attention, focus, and opportunity for nonverbal communication are all contributed to the burstiness seen in the data—and all, in turn, were contributed to through the construct of the thinking classroom framework (Liljedahl, 2020).

The thinking classroom, through its combination of structure and freedoms to move within these structures, creates an ideal environment for burstiness, and group creativity, to flourish. By having students work on thinking tasks at vertical whiteboards (or their proxies) in random groups of three, groups are provided with an environment that allows for the piling on of ideas necessary for burstiness to begin. At the same time, the thinking classroom creates an environment within which students feel safe to take risks, offer ideas, and welcome the criticism necessary for the very best of these ideas to move forward. The structures of a thinking classroom, in turn, makes these ideas not only visible to every member of the groups but also every member of other groups in the room, which, in turn, can seed burstiness among other groups.

However, the structures of a thinking classroom are not mere descriptions of an environment wherein burstiness, and group creativity, can occur. A thinking classroom is prescriptive—it can be built. The *Building Thinking Classrooms* framework emerged out of 15 years of research as a framework for how to construct an environment that not only fosters thinking among students—individually and collectively—but also necessitates it. And as it turns out, it also build an environment that fosters group creativity.

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# Chapter 13
# "Creativity Is Contagious" and "Collective": Progressions of Undergraduate Students' Perspectives on Mathematical Creativity

**Gulden Karakok, Gail Tang, Emily Cilli-Turner, Houssein El Turkey, V. Rani Satyam, and Miloš Savić**

## 13.1 Introduction

*It is shocking to me that the creativity involved in math is often overlooked … A professor at my school is dedicated to integrating creativity in her coursework to ensure students are engaged and excited throughout the entire course. It was in her course that my interest in the topic of number theory grew.* Peyton, a first-generation, female student.

Peyton was one of the participants of the presented study, and this short anecdotal quote was from personal communication a year after the study. As Peyton points out, mathematical creativity is often underemphasized in mathematics courses that tertiary level students take, even though there are numerous policy and curriculum standard documents, both in the United States and internationally, emphasizing creativity as an important and needed skill when learning mathematics

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(e.g., Askew, 2013; Schumacher & Siegel, 2015). Peyton's quote also highlights the potential impacts of explicitly valuing and providing ways for students to enhance their mathematical creativity in mathematics courses. We, the Creativity Research Group, recognize the need to emphasize mathematical creativity, and in our research studies (e.g., Cilli-Turner et al., 2019, 2020; El Turkey et al., 2018; Omar et al., 2019; Savić et al., 2017; Tang et al., 2015), we aim to explore the ways in which students' mathematical creativity can be explicitly valued and enhanced at the tertiary level mathematics courses. We agree with Nadjafikhah et al.'s (2012) claim that fostering mathematical creativity should be one of the goals of any education system.

The lack of a universally agreed-upon definition of mathematical creativity (Mann, 2006) should not prevent us from nurturing all our students' existing mathematical creativity in our courses. Since mathematics is so prevalent and acts as a gatekeeper in science, technology, engineering, and mathematics (STEM) fields, "[t]eaching engineers (and other STEM disciplines) to think creatively is absolutely essential to a society's ability to generate wealth, and as a result provide a stable, safe, healthy and productive environment for its citizens" (Cropley, 2015, p. 140). The number of studies examining students' mathematical creativity and the ways to enhance it at the tertiary level is slowly growing; however, compared to the number of studies at the primary and secondary school mathematics level, it is still sparse. Furthermore, most of the studies focus on the quantitative outcomes of mathematical creativity; there is a need to understand the phenomenon of mathematical creativity through students' lived experiences in order to build classroom experiences to support it.

To complement the existing knowledge and expand our understanding of perspectives that students bring to mathematics, we share results from one of our studies conducted in an introduction-to-proofs course to explore the progression of undergraduate students' perspectives of mathematical creativity. This progression was examined through several data sources collected chronologically, including pre-course survey, students' reflection assignments (RAs), in-class conversations, post-course survey, and one end-of-semester interview. Students' perceived development of their own mathematical creativity guided our understanding of these progressions. Using a phenomenological study design, we explore students' lived experiences of mathematical creativity in a proof course.

## 13.2 Background Literature

Guilford (1950), in his presidential address to the American Psychological Association, urged researchers and educators to find ways to enhance the creative promise of learners. However, from a research perspective, this has been a hard task as there are more than 100 definitions of mathematical creativity (Mann, 2006). In fact, Borwein et al. (2014) demonstrated that many mathematicians had different ideas about mathematical creativity. Some conceptualizations of creativity focus on emphasizing whether the end product is original and useful (Runco & Jaeger, 2012), while others describe mathematical creativity as a process that involves different modes of thinking, some of an unusual nature (Balka, 1974).

Most frequently, we observe researchers focusing on quantitative measures with an end product orientation using the Torrance (1966) categories of *fluency*, *flexibility*, *originality*, and *elaboration* as a framework for data analysis. *Fluency* in general refers to the number of meaningful and relevant ideas as a response to a problem or a stimulus. *Flexibility* is defined as the number of groups or categories of responses, whereas *originality* (or novelty) is a unique production or unusual thinking. *Elaboration* is defined as the ability to produce a detailed plan and relates to the generalization of ideas.

Leikin (2013), for example, used a point system to evaluate three of these categories (fluency, flexibility, and originality) in students' work. While Leikin acknowledged that solutions must be "appropriate" – "The notion of appropriateness has replaced the notion of correctness" (p. 391) – an expert (e.g., an instructor or a researcher) was the one who judged what is or should be appropriate or original. Furthermore, even though quantitative approaches can measure how creative a product is from the perspective of an expert, they obscure completely the perspective of the student writing the solution and how others (e.g., students or instructors of the courses) may perceive it.

In one of our earlier studies, we explored university students' and mathematicians' perspectives of mathematical creativity using three process categories: taking risks, making connections, and creating ideas (Tang et al., 2015). We found that students rarely (9% of students' responses) associated making connections (e.g., synthesizing different mathematical content) with creativity when compared to mathematicians (38% of mathematicians' responses). This study provided motivation to think about explicitly valuing and discussing the processes that are deemed to be important in the existing literature to develop mathematical creativity (El Turkey et al., 2018). Furthermore, we recognize that most of the existing definitions of mathematical creativity were derived from experts' (e.g., research mathematicians, other experts in education and psychology fields) experiences or perspectives; however, it is (as) important to consider students' voices and experiences with mathematical creativity. Researchers have been calling for studies that bring students' voices forward (e.g., Roos, 2019, on the topic of inclusion), and it is time for us to hear undergraduate students' perspectives on mathematical creativity.

With this overarching goal to gain an understanding of students' perspectives on mathematical creativity, we conducted the present study in an introduction-to-proofs course and examined the data collected at various stages of the course. Earlier results from this study used hypothesis coding (Saldaña, 2013) on students' end-of-semester interviews only and indicated that students' perspectives were related to Torrance's (1966) originality category with codes such as uniqueness, original, and innovative ways (Cilli-Turner et al., 2019). We also observed "being flexible" and "trying different ways" in students' perspectives, which related to Torrance's (1966) flexibility and fluency categories (Cilli-Turner et al., 2019). We further explored the sources of students' perspectives (Cilli-Turner et al., 2020) and noticed that students during these interviews mentioned their experience in the introduction-to-proofs course.

These results initiated our work here examining the progression of these students' perspectives of mathematical creativity from the start of the introduction-to-proofs course until its end. More precisely, we aim to address the research question: what are the progressions of students' perspectives on mathematical creativity through their experiences in a semester-long introduction-to-proofs course?

## 13.3 Theoretical Perspective and Methodology

In our mathematical creativity research projects (see http://www.creativityresearchgroup.com for a complete list of references), we use a developmental perspective of creativity (Kozbelt et al., 2010) that contends that creativity develops over time and emphasizes the role of the environment in the development of creativity. Such an environment should provide students with authentic mathematical tasks and opportunities to interact with others (Sriraman, 2005).

We operationalize mathematical creativity as "a process of offering new solutions or insights that are unexpected for the student, with respect to their mathematical background or the problems [they've] seen before" (Savić et al., 2017, p. 1419). This definition focuses on the process (Pelczer & Rodríguez, 2011) of creation, rather than the product that is created at the end of a process (Runco & Jaeger, 2012). This orientation allows for a dynamic view rather than a static one to capture nuances in the individual's thinking and experiences. Furthermore, our definition takes a relativistic perspective – creativity relative to the student – in contrast to absolute creativity in the field of mathematics (Leikin, 2009). For example, Levenson (2013), using a similar viewpoint, focused on the discussion of ideas by individual students and how these ideas helped in developing a product of collective mathematical creativity in fifth- and sixth-grade mathematics classrooms. Levenson also emphasized the teachers' roles in facilitating these discussions. The developmental perspective of creativity interlaced with our operational definition of mathematical creativity is our theoretical perspective.

We utilized a phenomenological case study design (Patton, 2002) to explore the progression of students' perspectives on mathematical creativity through their experience in the introduction-to-proofs course. This methodology was suitable for our investigation as it is "particularly effective at bringing to the fore the experiences and perceptions of individuals from their own perspectives, and, therefore, at challenging structural or normative assumptions" (Lester, 1999, p. 1). This allowed us to focus on students' perspectives of mathematical creativity rather than experts' views (e.g., mathematicians' or researchers' views) or normative assumptions.

## 13.4 Method

### *13.4.1 Setting*

The research setting for this study was an introduction-to-proofs course at a small liberal arts college in southwestern United States. Students met twice a week for 15 weeks for approximately an hour and a half. The course topics typically included sets, logic, and various proof techniques (e.g., direct proof, induction, contradiction, contraposition). The instructor of the course implemented inquiry-based learning (IBL) pedagogy (see https://www.inquirybasedlearning.org/) and adapted Ernst's (2017) textbook. Students often worked in small groups and presented their proofs to the class, which was followed by class discussions. There were three exams during the semester and a final exam in week 16. Students had daily assignments with problem sets for which they had unlimited opportunities to re-work their proofs during the semester using the feedback provided by the class community (peers and instructor).

Almost every week, students also submitted a reflection (short writing) assignment (RA). The content of these reflections varied from week to week. For example, for the first three assignments, students were asked to read articles on topics such as the importance of discussions in mathematics courses, the importance of reflection, and the impact of IBL. Then, they wrote a minimum of a half-page reflection on what they learned and found meaningful. There was also one RA for each of the three exams. For the RA in week 5 (RA#5), students were asked to reflect on their views of creativity and mathematical creativity and how these views were similar to and different from each other.

Students were introduced to the Creativity-in-Progress Reflection (CPR) on Proving (e.g., Karakok et al., 2016; Savić et al., 2017, named Creativity-in-Progress Rubric in our earlier publications) in class during week 7 (see Appendix 1 for the version used in this course). The CPR was developed as a formative assessment tool with two categories (making connections and taking risks) that incorporated aspects of fluency, flexibility, originality, and elaboration from existing research. The *making connections* category is defined as the process of connecting the proving task with definitions, theorems, multiple representations, and examples from both the current course and possible experiences from previous courses. The *taking risks* category is defined as the process of actively attempting a proof to demonstrate flexibility in using multiple approaches or techniques, posing questions about reasoning within the attempts, and evaluating those attempts. The CPR also provides three general development levels: *beginning*, *developing*, and *advancing*, each of which marks a student's progress on a given task along a continuum, as our way to communicate the possible states of growth.

The CPR was given to the students without the word "creativity" in the title to not steer students' attention to that word but rather used the title "Progress Rubric on Proving" to focus on "progress" (see Appendix 1). Students were asked to reflect, using the CPR, on their proofs (including all scratch work) on various assignments and exams. In addition, the instructor asked students to use the CPR in class on their peer's presented work to discuss ways in which it demonstrated various aspects of the CPR and to engage students to think about how to move their own thinking forward to advancing levels on the rubric categories.

### *13.4.2 Participants*

Fifteen students out of 17 enrolled students in the course agreed for their submitted course work and their utterances from class recordings to be used for the study. We conducted one 60- to 90-min audio-video recorded interview with seven participants who volunteered for these interviews. In this chapter, we present results from four participants to demonstrate the uniqueness of each participant's perspectives on mathematics and mathematical creativity, while cross-case analysis resulted in common themes on what contributed to the development of their mathematical creativity, from their perspectives, in this course. All four students were female and first-generation students, and Table 13.1 summarizes the four participants' information.

These four participants were selected among seven participants using both convenience and maximum variation sampling methods (Patton, 2002). With convenience sampling, we considered the most complete data set from participants to gain better insights into their lived experiences in the course. In addition, we examined all seven students' pre- and post-course survey entries, perspectives of mathematical creativity and progressions, background information (e.g., majors, courses taken), and self-identified mathematical ability for maximum variation. With maximum variation, we do not mean that the other three participants did not have any variation in their progression of perspectives; rather, with these selected four students, we were able to demonstrate varieties in progressions for all seven students. In other words, a maximum variation (heterogeneity) sampling (Patton, 2002) was utilized to provide "high-quality, detailed descriptions of each case, which are useful for documenting uniqueness" and document "shared patterns that cut across cases" (Patton, 2002, p. 235).

**Table 13.1** Four participants' information

| Participants (pseudonyms) | Ethnicity | Major | Math courses concurrently enrolled |
|---|---|---|---|
| Alice | Latine[a] | Mathematics | Number theory |
| Stephanie | White | Mathematics | Number theory, Calculus 3 |
| Peyton | White | Economics (math minor) | None |
| Olivia | Latine[a] | Biology (math minor) | None |

[a]See https://www.vox.com/the-highlight/2019/10/15/20914347/latin-latina-latino-latinx-

### 13.4.3 Data Collection and Analysis

There were several data sources collected for the study: surveys, audio recordings of class sessions, reflection assignments, and one 60- to 90-min, audio-video recorded interview with seven students. Our first round of analysis of these data started by examining the interview transcripts from all seven students (Cilli-Turner et al., 2019, 2020), which addressed the research question: what are tertiary students’ perspectives of mathematical creativity?

We follow these results by using a case study with the selected students to further explore their experiences throughout the course. To address our research question for this study, the following data sources were organized chronologically: open questions on pre- and post-course surveys (collected during weeks 1 and 15), reflection assignments (RA#5 on creativity and mathematical creativity; RA#8 and RA#12 on which students discussed their creative moments on exams 2 and 3, respectively; and any other available RAs), class discussions on the CPR (weeks 7 and 9), and the interview at the end of the semester (conducted in weeks 15 and 16).

Students were given pre- and post-course surveys with the same questions. There were three open-ended questions where students typed their answers. For example, one of the questions was as follows: to be good in math, you need to … because …. The other survey questions asked students to rate their perceived abilities in doing mathematics, attitudes about mathematics, and agreement with statements related to doing mathematics (e.g., doing mathematics involves creativity, taking risks is important in doing mathematics). Data from the surveys were included in the data analysis to gain insights into participants’ views of mathematics in general.

Each student participated in one 60- to 90-min audio-video recorded interview at the end of the semester. Interviews were transcribed in their entirety. At the beginning of the interview, all participants were asked what mathematical creativity meant to them with follow-up clarifying questions. We then asked students to expand on their RA#5, if they felt creative in the course, and to give a specific moment from the course as an example of their mathematical creativity. Similarly, they were asked if they thought other students were creative and asked to give examples. Additional questions were about what students thought contributed to their and others’ creativity and if and how they utilized the CPR in their work in this course.

Data analysis methods were chosen to fit the phenomenological study design: themes were created to describe each student’s experience throughout the course to understand the phenomenon of mathematical creativity and the progression of their perspectives of it. Analyses sought “descriptions of what [students] experience and how it is that they experience what they experience” (Patton, 2002, p. 107) related to mathematical creativity. This follows the theoretical perspective of developmental creativity; each student’s experiences in the environment (the course) will be different and progress based on what they perceive in the environment over time.

The chronologically organized data for each participant was read several times prior to the start of the systematic process of coding. In these initial readings, the goals were to get a sense of what each participant uttered (as captured in transcripts) or wrote about their beliefs, skills, and perspectives on mathematics, creativity, and mathematical creativity by identifying relevant texts (Auerbach & Silverstein, 2003), which are texts (e.g., a portion of a transcript or written work) that include words, sentences, or phrases related to research questions from all data sources. The first author turned these relevant texts into narratives using bracketing (Patton, 2002) to briefly describe the participant's experience with the phenomenon of mathematical creativity at that point in time. After the narratives were created for each participant, the relevant texts were coded further into ideas that were repeated by the participants at least three or more times, and participants' own words or phrases were used to describe these codes (Auerbach & Silverstein, 2003). For example, some repeated ideas were metaphors students used, such as "thinking outside the box," or words, such as "different ways" and "making connection." These repeated ideas were checked with the results of the earlier studies, such as Cilli-Turner et al. (2019, 2020), for triangulation purposes (Patton, 2002). Then, we looked for occurrences of the repeated ideas for each participant to gain insight into the progression of their perspectives on mathematical creativity.

The first author continued by examining the narratives to add more nuances related to the participants' experiences with the repeating ideas. These revised narratives were re-examined and re-written (process described in van Manen, 1990) to capture overarching themes for each participant's experience. One important aspect of our data analysis method was to examine each participant's perceived development of their own mathematical creativity. The mathematical actions (e.g., taking risks, posing questions) that participants used when explaining their mathematical creativity (e.g., "I made connection to … ") in RA#8, RA#12, and during the interview provided us additional insights into participants' operationalization of and, thus, their conception of mathematical creativity. In other words, such self-reflections of ability brought "to the fore [the students'] experiences and perceptions" of mathematical creativity "from their own perspectives" (Lester, 1999, p. 1).

Analysis of pre- and post-course survey responses provided corroborating evidence of the progression of participants' perspectives. Even though open-ended survey questions were not asking about mathematical creativity, participants' responses captured their initial and end-of-the-course views related to mathematics, because "it has long been accepted that we understand new phenomena [of mathematical creativity] in terms of the understanding we already possess" (Spangler & Williams, 2019, p. 4).

## 13.5 Results

We first present each participant's perspectives of mathematical creativity and their progression of them through our developed narratives. Then, the results of the cross-case analysis are presented to share the uniqueness and similarities of perspectives and progressions of the four participants' perspectives. We start with Alice's narrative and then present condensed versions (due to space limitations) of Stephanie, Olivia, and Peyton's narratives. Alice's narrative was provided in a longer version to exemplify the chronological data analysis process only. In the condensed versions, we provide participants' initial perspectives captured in RA#5 and address our research question on the progression of perspectives, by making references to the repeated ideas.

### *13.5.1 Progression of Alice's Perspective*

Alice initially described her view of being creative in mathematics by referring to "finding different and innovative ways to come to a solution or a number of solutions" (RA#5). At this point, she associated creativity with the idea of "memorization" in mathematics: "It may also help to use creativity to help remember theorem[s] or formulas such as creating a song to remember the quadratic formula." This memorization idea aligns with her rating of the statement "the best way to do well in math is to memorize all the formulas" on the pre-course survey. Alice was the only student (among the seven) who slightly agreed (rated 4 out of 6 on the agreement scale) with this statement, whereas other participants disagreed with this sentence in varying degrees (i.e., strongly to slightly disagree).

In this initial perspective of mathematical creativity, Alice also mentioned that being creative in mathematics helps to understand concepts "easier," which, for her, seemed to be about creating ways to memorize formulas or methods of solution. For example, in week 7, during a small group discussion in which students were asked to comment on other students' RA #5 entries (which were presented without any student name), Alice shared her view of being mathematically creative as "not necessarily like making it colorful or pretty, just being able to find a different way to approach it or maybe a shorter way or a simpler way" (classroom transcript, week 7). In this discussion, she said, "That's what tripped me up when I started to get to college cause it's like 'Oh yeah! Here are all the ways you can do it!' and I'm like, 'there are that many ways? Really!?'" Her remarks made her group members laugh, and one of them said, "Let's just stick with one!" Alice continued, "One! I can memorize one. I don't know if I can memorize 5." It seems that the idea of having multiple ways to solve problems was a new and challenging experience for her in college mathematics courses, and she related this experience to mathematical creativity at this point in the semester.

After the CPR was introduced to the students, Alice seemed to relate her own proof construction process to various subcategories of *making connections* category in the CPR (repeated idea). On both exam 2 and exam 3 reflection assignments (RA#8 and RA#12, respectively), for example, she mentioned trying to connect a theorem to the proof statement. "I was being creative when I tried to connect a previous theorem on the test to help prove another theorem on the test" (RA#8). As the repeated idea of making connections was observed in data after the CPR was introduced, we claim that it became part of Alice's perspective of mathematical creativity. It also seems that Alice started to incorporate the CPR language to examine her own work and found this practice (making connections) useful for her to better understand the course.

The end-of-the-semester interview provides additional evidence of the progression of Alice's perspective from "different and innovative ways" to making connections. We observed other aspects of her perspective that were not repeated or uttered by Alice prior to this interview. Alice described creativity in mathematics as:

> … coming up with like new and different techniques to be able to solve um problems… to be able to prove theorems, specifically for [this course]. Um, it kind of means just, kind of using like a trick um something that's not really common, or maybe like a different representation to show the same thing that no one has really used (interview, week 16).

She, yet again, repeated the finding different ways ("different techniques") idea but included using tricks or different representations in her description. For Alice, "using a trick" mathematical action (process) was related to making connections between theorems and the proof statement, which helped her to understand a proof more easily.

> I guess finding kind of like a trick or … being able to find the connections between theorems or being able to use one theorem to solve another or using like a lemma to solve part of a theorem, just to make it a lot more … easier so the theorem's not … a page and a half long … [I]n any case, just having, being able to find … a technique that works that doesn't necessarily make everything longer. It kind of just makes it more … easier to understand too (interview, week 16).

As Alice's perspective progressed to incorporate making connections, her view of the function of different ways or approaches moved away from memorization of formulas for understanding. For instance, Alice strongly disagreed with the post-course survey statement, "the best way to do well in math is to memorize all the formulas" with which she was in slight agreement at the beginning of the course.

In addition, Alice "tried to feel" creative, and she did feel creative through her attempts to make connections, even though she identified herself as "struggling a little bit trying to make connections between um theorems, um and being able to like create lemmas to be able to uh fit into proofs." However, when she indeed made connections, she perceived this part of her understanding, "when I make a connection I'm like 'Yes! Like, I understand.' Like, it makes me really happy when I'm able to make a connection" (interview, week 16). It seems that making connections not only helped her feel creative but also impacted her emotion ("makes me really happy").

Alice's perspective of mathematical creativity is centered around the idea of finding different solutions. Throughout the course, Alice incorporated strategies (e.g., using different examples, representations, etc.) to find these different solutions. She believed that her mathematical creativity ability developed when utilizing these strategies. She not only mentioned *making connection* repeatedly but also internalized these mathematical actions for her own mathematical creativity. For this reason, we claim Alice's perspective of mathematical creativity progressed to include making connections. It seems that at the end of the semester, Alice recognized the ways in which making connections provided her the tools to improve her own mathematical creativity and to understand the concepts "better" and "easier" that were not (just) memorization of formulas.

### 13.5.2 Progression of Stephanie's Perspective

Stephanie's perspective of mathematical creativity centers around the metaphor of "taking the road less traveled" (repeated idea) that makes the most sense to the individual. This perspective progressed throughout the course to recognize the interplay between an individual's mathematical creativity and collective mathematical creativity. Stephanie initially explained her perspective of mathematical creativity in terms of using "imagination to innovate something original" (RA#5) in an arts setting. However, in this reflection, she acknowledged that creativity can be found in other contexts and real-life situations. She claimed that mathematics professors "tend to teach the road most traveled to get to the solution, but more times than not, there are other ways to get to the correct solution," (RA#5) and, for her, finding these other ways to get answers was being mathematically creative.

Stephanie referred to a person's creativity relating to their own process of finding different solutions (repeated idea) and taking paths that were less traveled (repeated idea) again after week 5. However, when these ideas were repeated, she included the purpose of taking such different paths as personal sense-making and understanding of mathematics. Furthermore, for her, determining the less traveled path required her to see other's paths. For example, when students were asked to reflect on their creative moments on exam 3 (week 12), Stephanie wrote:

> It is hard to determine if there were any "creative moments," as for me, creativity is the path less traveled. I do not know how my classmates proved any of them, and so, I don't know if any of my proofs were creative.

During the interview, Stephanie first described what it means to be mathematically creative as "the same as being creative in anything else. It's taking the road less traveled [repeated idea]. It's not just doing what the herd is doing but finding your own way to get to where you need to be." She again emphasized the importance of sense-making and understanding during the process of finding solutions, "It's finding the solution but doing it in a way that makes the most sense to you."

As she identified her and her classmates' creativity to be developed throughout the course, she mentioned that seeing others' works in class was an important aspect of the course that contributed to these developments. When she elaborated on this idea, she stated that for her, "creativity is both individual and collective … [another student's] creative moment, I could then use to expand on and do something a little different to have my own creative moment." She believed that all of these creative moments of students were "not the road most traveled," and she viewed these moments to be an integral part of the "road we are traveling together, and yet each time we're changing it to be what we need it to be, expanding on it and having our own creative moments, based on a creative moment somebody else had before us."

For her individual creativity, she focused on the subcategory of tricks and tools of the CPR "to create something new … to have that one thing that's like 'Wow, that's awesome!'" She also wanted to do this for the collective creativity of the class; she wanted to "bring forth a new tool that we could all use as a class." She reflected in the interview that she "started to look at creativity a little bit different through the course" as she noticed many different "paths" the other students were taking both in their choice of proof technique and using conceptually different ideas.

Even though Stephanie repeatedly referred to the metaphor of “taking the road less traveled” to describe her perspective of mathematical creativity, her desire to compare her “road” to others’ was not only to assess her own creativity but also for her to develop her creativity through other’s creative moments. She believed that finding tools and tricks and sharing these with others, persistence, and being flexible to try different things were important for all of them to develop their individual and collective creativity.

### *13.5.3 Progression of Peyton’s Perspective*

Peyton’s perspective on mathematical creativity progressed throughout the semester from believing that “there’s no need for creativity in mathematics” to recognizing that mathematical creativity is within the process of producing proofs. Peyton described creativity as to “be able to come up with original and innovative ideas” (repeated idea) initially and identified herself as not creative but wished to be (RA#5). She mentioned in RA#5 that prior to the introduction to this proofs course, she was on the “spectrum that generally believes that there’s no need for creativity in mathematics.” She enjoyed mathematics initially because she could get the correct answer if she understood the material. This course had proven her initial ideas to be untrue and “every time I see someone else’s answer to a proof, I am amazed at how he or she came to that answer … Overall, I realized math has a lot of room for creativity” (RA#5).

Peyton’s perspectives on creativity started to evolve at the very beginning of the course (as evidenced in RA#5), and she described her view of creativity in the interview (week 16) through a process that involves reflection in thinking, “[b]ecause every step requires more thinking, and every step requires you to figure out your next step, and so that’s where the creativity comes in.” We notice an additional progression of Peyton’s perspective on mathematical creativity prior to the end-of-the-semester interview. For example, when she reflected on a quote given for RA#8, she shared her noticing as follows:

> … that creativity does not necessarily need to be a “spontaneous” and brand new discovery, because creativity can be presented in many ways. This is especially significant for me because I generally assume that creativity does in fact require spontaneous and new discoveries…But reading [the] quote emphasizes the fault in that logic. True creativity lies in a person’s ability to use resources in ways to improve his or her own thought process.

Even though Peyton did not perceive herself as mathematically creative throughout the semester, she was able to recognize other students’ mathematical creativity and hence thought she probably had mathematically creative moments as well. She believed that her mathematical creativity developed in this course, in particular “it helped me to realize or recognize that there does not always have to be one set process in math” (interview, week 16). Overall, Peyton’s perspective of mathematical creativity progressed from the ability “to come up with original and innovative ideas” (RA#5) spontaneously to recognizing that it is a process where “true creativity lies in a person’s ability to use resources in ways to improve on his or her own thought process” (interview, week 16).

### *13.5.4 Progression of Olivia’s Perspective*

Olivia’s perspective of mathematical creativity centers around the metaphor of “thinking outside the box,” which she initially associated with “trying something new that is often different from others” (repeated idea of uniqueness) (RA#5). Throughout the semester, her perspective evolved to include other aspects of mathematical creativity related to the mathematical actions described in the categories of the CPR. She believed that creativity was contagious, and the course discussions of their (students’) proofs, the thinking processes behind such proofs, and the CPR contributed to her perception of creativity and the development of her own mathematical creativity. At the end of the semester, she described mathematical creativity as “really thinking outside of the box [repeated idea] and being able to be comfortable or at least willing to take risks and not just follow a standard format … but being willing to be flexible and try different approaches.”

She initially thought of creativity in art-related contexts and believed she did not have any of those qualities and hence never thought of herself as creative. However, in this course, she noticed that “there are mathematical ways of being creative, so I was able to get a better understanding as the semester went on of what that [creativity] meant in a different context” such as mathematics and proving. She noticed that students in the course developed their mathematical creativity, including her. She mentioned that early in the semester, they (students) were “kind of not really feeling confident in our abilities to be creative”; however, later in the semester, “it was really interesting to see students that were quiet, reserved early on, like show their work later in the semester and they had done something like totally cool and amazing.”

Olivia mentioned many aspects of the course contributing to her and her classmates’ development of mathematical creativity. For example, the IBL structure of the course was an important aspect because “you really try to make connections, and it forces you to get creative because you have, um, very little like understanding of the right way to do it, so it kind of throws that out of a student’s mind.” She viewed not having pre-exposure to the “right way” allowed them to be “free,” and working with other students helped to develop different methods to approach problems. She said that these course activities aligned with her idea that “creativity can be contagious.” Active class engagement in which students share their work and discuss their mathematical thinking was a way that Olivia thought the creativity spread out among students and developed their mathematical creativity.

Olivia’s beliefs about mathematics in general also showed some changes from the pre- to the post-course survey. On the pre-course survey, she stated that to be good in mathematics, one needs to “think outside the box [repeated idea] and be comfortable with abstract thinking because math is not one dimensional.” To her, thinking mathematically meant “to think critically and use what I know and apply it to any given situation.” On the post-course survey, Olivia explained being good in mathematics through flexibility, persistence, creativity, and the process of evaluation. These shifts from the pre- to the post-course survey served as corroborating evidence for the progression of Olivia’s perspective of mathematical creativity.

Overall, Olivia internalized several mathematical actions from the CPR to help her “to think outside the box” and be mathematically creative: willing to take risks, being flexible, making connections, and posing questions to further her thinking.

## 13.6 Uniqueness and Similarities in Progressions Across Participants

All four participants’ progressions of their perspectives of mathematical creativity had some unique aspects from their experiences in the course. Alice’s perspective of mathematical creativity was centered around the idea of finding new, different techniques and solutions. Throughout the course, Alice incorporated strategies (e.g., using different examples, representations, etc.) to find these different solutions. She believed that her mathematical creativity developed utilizing these strategies, which was unique in her experience. For Alice, creativity is “being able to find the connections” to make mathematics easier to understand.

Stephanie’s perspective of mathematical creativity was centered around the metaphor of “taking the road less traveled” and progressed to include the importance of others’ contributions to have collective creativity. Stephanie identified the subcategory of tricks and tools of the CPR as an important strategy to develop individual and collective creativity, which was unique to her experience. For Stephanie, creativity is “taking the road less traveled” individually and contributes to the “the road we are traveling together” to create collective creativity.

Peyton’s perspective of mathematical creativity progressed from believing that “there is no need for creativity in mathematics” to viewing it as the ability “to come up with original and innovative ideas” spontaneously, to recognizing it as a process where “true creativity lies in a person’s ability to use resources in ways to improve on his or her own thought process.” Peyton mentioned many aspects of the course contributing to her recognition of mathematical creativity and the development of mathematical creativity of others. Peyton’s experience was unique in the way that she started to recognize many aspects of the course and continuously reflect on these experiences to integrate them into her perspective. For Peyton, creativity is finding your own ways and about the process of “getting from the beginning to the end” within the idea production.

Olivia’s perspective of mathematical creativity centered around the metaphor of “thinking outside the box” and identified many strategies such as taking risks, being flexible, and posing questions to develop mathematical creativity. As she believed that creativity was contagious, posing questions to understand her and others’ thinking was important to her. She wrote her own questions on her scratch paper, which was a new practice for her. In addition, she repeatedly talked about flexibility in terms of working on and finding many different solutions, which was another unique aspect. For Olivia, creativity is “contagious” and requires one to be “comfortable or at least willing to take risks.”

There were similar repeated ideas in all four students’ experiences with the phenomenon of mathematical creativity. Some of these repeated ideas were about aspects that contributed to the development of their mathematical creativity: course structure (e.g., IBL format) and the use of the CPR, and the instructor’s actions.

The IBL format of the course was new to all four participants, and in this course, students actively engaged in constructing their own proofs and sharing their mathematical thinking, processes, and proofs with each other. All students mentioned in this course structure were not pre-exposed to the “right way” or “one way” of proving, and they had to do what they thought “made sense to them” or “was going to work.” They had to “swim or sink,” and they were all in this together, so “you start to work together, and you start to build relationships with [classmates], and you work off of each other’s creativity.” Thus, this format helped students develop not only their proving skills but also their mathematical creativity.

Examining each other’s proofs was helpful to all four participants for different reasons. Alice valued this practice because it helped her to reflect on her own process to make sure she understands the mathematical concepts. She appreciated the use of the CPR in this process as it helped to notice the use of strategies mentioned in the CPR. For Stephanie, examining others’ proofs was important to see if she “took the road less traveled” and to learn what other tools and tricks that she could use in her future proof work. However, she felt uncomfortable to use the CPR as she felt like she was being “judgy” (classroom transcript, week 9). Olivia, and other students (including Alice), pointed out that the purpose of using the CPR was to understand each other’s thinking processes. Examining proofs “opened” Peyton’s “eyes to realize that there really are so many different ways to go about doing things, especially in math,” and for Olivia, this was the essence of mathematical creativity and helped her to develop her own.

All four students noticed certain instructor actions that provided them opportunity to develop their mathematical creativity. Peyton considered the instructor’s guidance and determination not to interfere with their learning process to be important. Olivia noticed that the instructor created a “safe,” “free-spirited” environment that allowed them to take risks. She noticed even small actions by the instructor, such as sitting down at students’ table to join the conversation, which helped create this environment and facilitated the message “you know you guys kind of run the show type of deal” (interview week 16). Stephanie thought the instructor’s persistence in not giving out answers or confirming correctness was an important action for them to develop their mathematical creativity collectively. She acknowledged the instructor’s intentions for a “swim or sink” approach as pushing them to focus on processes by guiding them through questions.

## 13.7 Conclusion

The purpose of this chapter was to present students’ perspectives of mathematical creativity and how such perspectives develop in a course environment. The developmental orientation of creativity in this phenomenological study provided us the opportunity to hear student voices and notice the ways in which they experienced the phenomenon of mathematical creativity in the classroom environment that was carefully designed by the instructor who made explicit choices. The IBL implementation coupled with the use of CPR provided students opportunities to experience multiple ways of thinking, examining proof processes, and developing individual strategies. They internalized these experiences as part of mathematical creativity and utilized them to enhance their own mathematical creativity.

The results of this study contribute to our existing knowledge of creativity in several ways. The instructional designs (e.g., the course structure, teaching actions) can nurture students' perceived self-abilities of mathematical creativity and shape (progress) their perspectives of mathematical creativity. The CPR provides strategies for students to develop their own mathematical creativity in unique ways and provides additional tools to understand mathematics. The instructors' actions not only motivate students to form a classroom learning community but also develop collective creativity. All these carefully engineered instructional efforts made creativity contagious.

The research design allowed us to break free from normative assumptions of mathematical creativity. We (the researcher and the instructor) purposefully did not evaluate students' mathematical creativity ability but rather attended to participants' voices to understand the formation of their perspectives. As we believe both the ability and perspectives of mathematical creativity are dynamic and shape continuously, our results only present the progressions of participants' perspectives at the time of the study. In the opening anecdotal quote, we have a glimpse of Peyton's perspective a year after the study. In this research study design, we also incorporated students' views of mathematics (through examining their pre- and post-course survey entries) as these perspectives intertwine with views of mathematical creativity.

In our current studies, we are exploring some of these experiences further. For example, we have been expanding on explicit instructors' actions to enhance mathematical creativity in Calculus courses (Tang et al., 2020). As discussed in this chapter, we noticed that students utilized the CPR not only to develop their mathematical creativity but also to better understand mathematics. We are exploring this connection with Calculus students' lived experiences (Cilli-Turner et al., forthcoming).

We also observed that many participants of this study connected their experiences with emotions that they felt in the course with the phenomenon of mathematical creativity (e.g., Alice's "makes me really happy", Stephanie's excitement ("Wow!")). This line of observations led us to focus on affective domains in mathematical creativity in our current work as well (Cilli-Turner et al., forthcoming).

In closing, we invite both researchers and instructors to design environments for students to notice their mathematical creativity in their own way and spread it to others. The developmental perspective of creativity provides us a roadmap for the "road we are traveling together" to understand the research construct of mathematical creativity "collectively" (Stephanie, week 16).

# Appendix 1

**Progress Rubric on Proving**

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| MAKING CONNECTIONS: | Beginning | Developing | Advancing |
|---|---|---|---|
| Between Definitions/Theorems | Recognizes some relevant definitions/theorems from the course or textbook with no attempts to connect them in their proving | Recognizes some relevant definitions/theorems from the course and attempts to connect them in their proving | Implements relevant definitions/theorems from the course and/or other resources outside the course in their proving |
| Between Representations[1] | Provides a representation with no attempts to connect it to another representation | Provides multiple representations and recognizes connections between representations | Provides multiple representations and uses connections between different representations |
| Between Examples | Generates one or two specific examples with no attempt to connect them | Generates one or two specific examples and recognizes a connection between them | Generates several specific examples and uses the key idea synthesized from their generation |

[1] We define a *mathematical representation* similar to NCTM's (2000) definition. It includes written work in the form of diagrams, graphical displays, and symbolic expressions. We also include linguistic expressions as a form of lexical or oral representation. For example, a student can use the lexical or oral representation, "the intersection of sets $A$ and $B$"; a Venn Diagram to depict his/her mathematical thinking; a symbolic representation $A \cap B$; or set notation $\{x | x \in A \text{ and } x \in B\}$ (which is also a symbolic representation). Note the last two representations are in the same category, e.g. symbolic, but they are still considered two different representations.

**Progress Rubric on Proving**

| TAKING RISKS: | Beginning | Developing | Advancing |
|---|---|---|---|
| Tools and Tricks[2] | Uses a tool or trick that is algorithmic or conventional for the course or the student | Uses a tool or trick that is model-based or partly unconventional[3] for the course or the student | Creates a tool or trick that is unconventional for the course or the student |
| Flexibility[4] | Begins a proof attempt (or more than one proof attempt), but uses only one approach | Acknowledges and/or uses more than one proving approach, but only draws on one proof technique | Uses more than one proof technique |
| Posing Questions | Recognizes there should be a question asked, but does not pose a question[5] | Poses questions clarifying a statement of a definition or theorem | Poses questions about reasoning within a proof |
| Evaluation of Proof Attempt | Examines surface-level[6] features of a proof attempt | Examines an entire proof attempt for logical or structural flow | Examines and *revises* an entire proof attempt for logical or structural flow |

[2] Based on the Originality category from Leikin (2009).
[3] Learned in a different context.
[4] A proof attempt is a continuous, sustained line of reasoning focused on a single theorem or conjecture. A proof approach is a proof attempt in which a new or different (to the prover) idea is introduced. Finally, a proof technique is a proof approach that addresses the overall logical structure of the proof. Common proof techniques include induction, proof by cases, direct proof, contradiction, and contrapositive.
[5] For example, a student writes a "?" next to something.
[6] Surface-level features include technical, computational, and line-to-line logical details.

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# Chapter 14
# The Role of Creativity in Teaching Mathematics Online

**Ceire Monahan and Mika Munakata**

## 14.1 Introduction

In this chapter, we report on a study that explored creativity in the context of the teaching and learning of mathematics in an online setting. Seven instructors of a general education mathematics course were followed during the fall 2020 semester to determine how the transition to an online platform served as a "useful" constraint as they implemented lessons on creative thinking in mathematics. We outline our motivation for the study, provide the particular context in which it took place, report on our findings, and provide implications for future research and practice.

There were two main catalysts for our study: our existing projects on creativity in mathematics and the pandemic. We were part of a research team on a National Science Foundation-funded project, "Engaged Learning through Creativity in Mathematics and Science" (CMS), that ran from September 2016 to August 2021. The main objective of the CMS project was to develop, implement, and disseminate modules that encouraged students to think creatively as they explored mathematics. The CMS project was motivated by a survey that showed that undergraduate science and mathematics students thought of disciplines related to the arts as creative but their own disciplines as lacking in creativity (Munakata & Vaidya, 2013). This result was in keeping with other studies that have shown that creativity and the sciences (including mathematics) are rarely recognized together (Kaufman & Baer, 2004). This, along with other works (Boaler, 2016; Neumann, 2007), inspired us to consider how we can convince students of the intertwined nature of mathematics and creativity. Over its five-year duration, the CMS project involved a professional development program for teachers (Monahan, 2020), collaborations with the campus performing arts center (Monahan et al., 2020) and a community college, and a permanent change to our undergraduate course offerings.

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As part of the CMS project, we redesigned a course formerly called Contemporary Applied Mathematics for Everyone (MATH 106), resulting in the name being officially changed to Creative Thinking through Mathematics. The course is a regular offering for non-science and non-mathematics majors at our 4-year university. Whereas the course previously focused on mathematical applications in the social sciences (e.g., voting theory, apportionment) and daily matters (e.g., interest rates, scheduling, bin packing), we revised it to reflect a way of thinking rather than aligning mathematics with certain contexts. Over the course of three years, the two authors and a colleague co-designed, taught, revised, and retaught a set of modules on creative thinking in mathematics (Munakata et al., 2021). In redesigning the course, we relied on research on creativity (e.g., Guilford, 1968; Rhodes, 1961; Sternberg, 2006) and mathematical creativity (Chamberlin & Moon, 2005; DeHaan, 2009; Shriki, 2010). The redesigned course is the context for this study.The shift to online teaching due to the pandemic served as the other context for this study. Although the shift to online teaching in Spring 2020 was sudden and unexpected, by Fall 2020, the instructors had had time to reflect and plan for more meaningful online interactions with their students. We were interested in determining the ways in which the added constraint of online teaching encouraged instructors to think more deeply about what it means to teach creatively, given that MATH 106 was already focused on teaching creatively and encouraging students to be creative and also because it was not initially designed for an online setting. Furthermore, we sought to determine the ways in which our modules on creative thinking were amenable to an online setting. Specifically, our research questions were as follows:

1. In what ways does the transition from in-person to online teaching and learning encourage creativity?
2. What features of a course focused on creativity in mathematics are amenable to this shift?

## 14.2 Related Literature

Although creativity is a topic of interest in many disciplines, there is no agreed-upon definition for the term (Cropley, 2000). While some researchers have identified personal characteristics and thinking styles of someone considered creative (Neumann, 2007; Sternberg, 2006), others have explored the object of creativity (Rhodes, 1961; Runco, 2004). Specifically, Rhodes (1961) identified four categories of creativity, referred to as the "4Ps": product, person, process, and press. Although the original 4Ps include press as the final category, we have renamed that construct place, for the purposes of this study. Under these categories, product is defined as what is created, person focuses on specific attributes of a creative person, process is the thinking behind a creative endeavor, and place is the environment in which creativity occurs. Researchers have explored creativity in mathematics since the late 1940s and argued that mathematics is creative when people demonstrate its novel and useful nature by combining familiar ideas in new ways (Poincaré, 1908/1952). Other researchers have identified creativity in mathematics as one's ability to demonstrate non-algorithmic decision-making (Ervynck, 1991) and generate novel solutions to problems (Chamberlin & Moon, 2005; DeHaan, 2009; Shriki, 2010). Similarly, Boaler (2016) identified mathematical creativity as a flexible mental construct and highlighted the importance of including creativity in the teaching and learning of mathematics. It is important to note that researchers have distinguished *teaching creatively* from *teaching for creativity* (Bolden et al., 2010; Department for Educational and Employment, 1999). *Teaching creatively* includes instances where teachers demonstrate creativity to make learning interesting and meaningful for students, while *teaching for creativity* identifies ways in which teachers can encourage students to be creative learners. The two perspectives are both useful in mathematics education and are both relevant to our study. New situations, in our case a shift to online teaching and learning, can provide constraints that encourage teacher and student creativity. A constraint is a limitation or restriction, which some researchers argue encourages and promotes creativity (Stokes, 2001; Stokes & Fisher, 2005). Several studies have described how constraints (e.g., lack of resources, competitors, social demands) promote creativity in real-world settings (Peterson et al., 2013), among artists (Dahl & Moreau, 2007; Stokes, 2001), and in linguistics (Costello & Keane, 2000). One's ability to work within these constraints influences how one engages in the problem-solving process (Peterson et al., 2013). However, little research has focused on how constraints might promote creativity in teaching and learning mathematics, particularly in an online setting. Given that constraints are inevitable, Peterson et al. (2013) identified the need to support people to work within constraints. This literature framed our research questions and data analysis to better understand how a shift from in-person to online mathematics instruction promoted teacher and student creativity.

## 14.3 Methods

This study involved seven of nine instructors of MATH 106 in Fall 2020; in the first semester, the course ran under its new name and focus. Whereas the transition to online in Spring 2020 occurred without warning, all instructors were aware that courses would be predominantly online when they signed up to teach the course. Of the eight sections reported in this study, two were held in hybrid mode where students had the option of attending class in person.

While there is some flexibility on the part of the instructor to choose the particular content focus, the official course catalog includes the following description:

> Explorations of mathematics that foster creative thinking and interdisciplinary approaches. Topics include fractals, symmetry, recreational mathematics, projective geometry, probability, statistics, and mathematics of arts and design. Students are encouraged to broaden their understanding of the meaning and utility of mathematical and creative thinking.

MATH 106 is a coordinated course. It is typically taught by adjunct instructors, with many having taught it for multiple semesters. Instructor workshops and information sessions are held prior to and during each semester. During these workshops, instructors review course objectives, discuss creativity in mathematics, and share ideas for lessons. Instructors also are a part of an online community page, where they are encouraged to share lessons and ideas.

## 14.4 Data Collection and Analysis

### *14.4.1 Interviews*

The seven instructors were interviewed at the end of the semester by one of the authors. These interviews took place throughout the month of December. We used a semi-structured interview protocol (Appendix 14.A) to guide the interviews. The interviews were conducted via Zoom and ranged from 38 to 70 minutes. Each session was transcribed verbatim, and the entirety of these transcripts was entered into an Excel spreadsheet, organized by utterances. Utterances, which became our units of analysis, were deemed as passages in the transcripts that were separated by change of topic or idea. After partitioning the transcript in this way, we identified passages that were related to creativity in teaching in an online setting, creativity of students or instructors, and any products that seemed creative. Once the units of analysis were identified, we independently read each highlighted utterance and generated possible codes. Because we were using the 4Ps framework (product, person, process, place) to identify the object of creativity, those became our level 1 codes (see Table 14.1). Level 2 and 3 codes (see Table 14.2) were generated after careful review and discussions of the transcripts.

Once the codebook was established through this iterative process, we independently coded the 287 highlighted passages using our agreed-upon codes. Our interrater reliability for each code and sub-code was 90% for level 1, 88% for level 2, and 75% for level 3. We suspect that the reliability measures were high because of the extent of our discussion before we began coding. The lower levels of agreement for level 2 (when compared to level 1) were expected, since a disagreement in the level 1 code assignment necessarily led to different level 2 codes. The same explanation

**Table 14.1** Level 1 codes and descriptions

| Level 1 code | Code description |
|---|---|
| *Person* | Describes oneself or student as featuring a trait of creativity<br>Focus on self or others with no mention of the process |
| *Process* | Describes a situation that called for creativity |
| *Product* | Describes something that was produced—lesson plan, students' written work, etc. |
| *Place* | Mentions setting as the focal point leading to a constraint or affordance |

**Table 14.2** Frequency of codes from interview transcripts

| Level 1 code | Level 2 code | Level 3 code |
|---|---|---|
| Person (50) | Instructor (34)<br>Student (13)<br>Both (3) | Flexibility (23)<br>Connections (8)<br>Open-mindedness (6)<br>Adaptability (2)<br>Enthusiastic (2)<br>Rigidity (1)<br>Resourceful (1) |
| Process (95) | Instructor (78)<br>Student (12)<br>Both (2) | Planning (24)<br>Implementing (23)<br>Engaging (9)<br>Grading (8)<br>Assessing (7)<br>Completing assignments (6)<br>Communicating (3)<br>Collaborating (2) |
| Product (16) | Instructor (9)<br>Student (7) | Lesson (7)<br>Homework (5)<br>Assessment (2)<br>In class (1) |
| Place (140) | Online (124)<br>In person (7)<br>Course (6) | Online, constraint (71)<br>Online, affordance (52)<br>In person, constraint (1)<br>In person, affordance (6) |

applies to the decreasing levels of agreement on level 3 codes when compared to level 2 codes. All disagreements were resolved through discussion.

To determine overall trends in the interview data, we conducted a frequency count of all codes. Level 1 codes (person, process, product, or place) helped us describe the ways in which creativity played a role in the transition to online teaching. Level 2 codes identified who was being creative (student, teacher, or both), and Level 3 codes illuminated the ways in which creativity was called upon.

### *14.4.2 Surveys*

In addition to the interviews, we collected data through online surveys before and after the semester (the text from the pre- and post-surveys is provided in Appendices 14.B and 14.C). Because the completion rate of these surveys varied, we consider these data secondary and supplementary to our interview data. Of the seven instructors, three completed both pre- and post-semester surveys (Michael, Shannon, and Stephanie), one completed the pre-semester survey only (Charles), and the other three only completed the post-semester survey (Caitlin, Shonda, and James). All names were converted to first name pseudonyms.

The survey responses were converted into an Excel sheet, and relevant passages were highlighted. As with the interview data, we highlighted passages that mentioned or alluded to the role of creativity in the transition to online teaching. The responses to the surveys are presented in this paper only when they exemplify, enhance, or clarify something that was said in the interviews by the same instructor.

## 14.5 Findings

Our findings suggest that constraints led both instructors and students to be creative in the implementation and completion of assignments. Instructors noted affordances of the online learning environment and how this transition redefined what it means to teach and learn mathematics.

Table 14.2 shows the frequency of each code. Of the level 1 codes, *place* was the most frequent. Of the remaining level 1 codes, the most frequently mentioned code was *process*, followed by *person* and *product*. When combined, it is clear that the interviewees most often recalled instances where they (the instructor) were creative, rather than situations where the students were asked to be. Of course, this may be an outcome of the interviewee's perspective and the nature of the questions being asked of the interviewees.

The level 2 codes were related to the actor of focus (when it was a *person*, *process*, or *product*) or the setting (when referring to *place*). For example, for *product* and *process*, either the instructor or the student (and in some cases both) was the actor. Similarly, *place*—online, in person, or the course in general—indicated the setting where creativity took place. The level 3 codes identified what was creative (e.g., flexibility, assessment, planning). *Flexibility* was the most frequent Level 3 code. This personal characteristic played a key role in the planning and implementing of the lessons as the online environment was at times unpredictable. Instructors spoke often of the constraints related to online teaching and learning, although they also described the affordances that online teaching supported.

With this understanding of the overall trends in the data, we analyzed the utterances that accompanied the quantitative data to better understand participants' experience transitioning to an online-teaching environment. The following are the findings based on this qualitative analysis, organized by the themes we identified.

## 14.6 How Traits of Creativity Were Called Upon in the Transition

Sternberg and Williams (2001) proposed six traits that help teachers encourage students' creativity. These include the ability to (a) model creativity, (b) repeatedly encourage idea generation, (c) cross-fertilize ideas, (d) build self-efficacy, (e) question assumptions, and (f) imagine other viewpoints. These traits came into play as teachers thought of how they could encourage creativity in mathematics in an online setting.

There was consensus that instructors were called on to be creative throughout the transition. As Shannon put it in her survey, “We need to be creative in order to learn how to best teach online.” Many instructors also mentioned their ability to be flexible. For James, previous life changes gave him the confidence to be flexible under these circumstances: “I don’t know explicitly but it kind of drove home that if I can bounce back and work through all of that, I can handle having to switch a course online” (Interview, December 2020). Michael relished the opportunity to be creative: “So, being able to go out and actually find content that’s relevant and reflects the topics from our course…I guess that entire process…I could consider creative” (Interview, December 2020). Similarly, Shannon described herself as liking change, and this allowed her to be flexible and creative when things did not go as planned in the online environment.

For some, the transition to the online platform gave them the opportunity to question their assumptions about teaching. In the excerpt below, Michael recalls how he changed his instructional mode partway through the semester after viewing videos of previous classes:

> I felt [it] was very monologue-esque and…I don’t want it to feel like that…like, you know, a Calc IV class where just the professor [is] doing whatever and people are taking notes and then who knows what’s going on? So, I think that allowed me to re-edit the lessons in that way because the medium of the course, the online class itself. (Interview, December 2020)

In this case, Michael was able to use the recordings as a tool to be a reflective practitioner. By viewing the videos of his own classes, he was able to recalibrate his instruction. For example, he looked for applets and interactive platforms that would allow students to explore: “I’d send a link out and I go, ‘Play with this. See what happens’” (Interview, December 2020). His use of the word “play” in this excerpt exemplifies the importance of play in both mathematics and creativity.

Students were also called on to be creative as learners. Instructors recounted how students had to be adaptable and think in ways that are not always asked of them during in-person mathematics classes. Resourcefulness was another trait of creativity that students exemplified. Caitlin noted that students “were more resourceful and sought out references to other things more by doing it virtually and turning in a virtual product…having the access to being on a computer, I think, was helpful in them pulling on meaningful resources” (Interview, December 2020). In addition to finding resources on their own, students needed to be creative in thinking about how to present their work, much in the same way that instructors were. For instance, James reflected that students had to be creative in the presentation of their work or answering questions online:

> Particularly with drawing [responses] because like they could have a picture in their head, but then they have to figure out exactly how they’re going to get it on Zoom with a mouse. Or, how they’re going to get it in the notebook to like maybe put up to the camera for me or for the rest of the students. (Interview, December 2020)

## 14.7 Constraints Leading to Creativity

Before the semester, some instructors were unsure of how they would transfer in-person activities to online. For example, Charles realized that the course "might become limited to what can be done with household resources" (pre-survey). The acknowledgment that there would be limitations—but that it wouldn't be impossible—led Charles to be creative in how he presented the material. When asked to describe an especially creative lesson, Charles recounted how he demonstrated the Pythagorean theorem using Cheez-It crackers. Similarly, instructors asked students to bring household materials to class, such as color pens, coins, and strips of paper. Caitlin noted that she tried to make the activity similar to "what we would be doing in person without me being able to provide the counting chips or the cubes or anything else" (Interview, December 2020). The constraint of materials led to creativity as instructors and students found ways to engage in similar activities in an online setting.

These adjustments are in contrast to some instructors who abandoned lessons that they thought impossible in an online setting. Michael lamented that "lesson plans that involve physical manipulatives such as paper, scissors, rulers, blocks, Legos, beach balls cannot be readily made available to students who do not wish to come to campus" (post-survey). This statement implies that Michael replaced lessons that required manipulatives with ones that didn't require materials. As another example, James decided not to implement a module on non-Euclidean geometry that required students to find the distance between two points on campus that was not easily measurable in order to motivate the distance formula. Although the setting for this activity could have been adapted, James described how he had to abandon the lesson (as well as one involving a beach ball). In his case, practicalities impeded his implementation of "creative" lessons, exemplifying a missed opportunity for instructor creativity.

## 14.8 Affordances of the Online Environment: More Higher-Level Thinking Allowed

One outcome of the online transition described by several instructors was that the online platform encouraged higher-level thinking in their students. Several attributes of the online platform were mentioned, including the way the transition forced the instructor to reevaluate their objectives and the "safety" that the online setting afforded the students. Shonda described how revising her tasks helped promote higher-order thinking:

> I felt a need to design distance learning experiences that have very clear instructions and utilize only one or two resources…Tasks with few instructions often lead to the greatest amount of higher-order thinking, as students figure out what to do within defined parameters (post-survey).

Shonda recognized that the need to give fewer instructions left more flexibility for students to consider different approaches. This contrasts with problem sets typically given by mathematics instructors. She also acknowledged her role as a learner in the process: "Distance learning has pushed me to think about how I can be a learner and more concise with the delivery of new information" (Interview, December 2020).

Some instructors saw "being at home" as an affordance that promoted creativity. In particular, Shannon considered how she thought more creatively in a comfortable, nonintimidating setting and wondered if this could be true for her students as well:

> For me personally, and maybe this is true for others, being at home allows you to think about things a little bit differently…you feel comfortable where you are, you know you're not in a classroom setting. So, I think for some students that might bring out some ideas that they wouldn't feel otherwise if they were in a less comfortable setting. (Interview, December 2020)

This is an example of how a constraint can also be seen as an opportunity. In this particular case, the constraint of being online provided a low-stakes environment for students to think more freely.

The online platform also impacted how instructors used electronic resources. For example, Stephanie had to "think a little bit harder about the questions that I pose when I start class or think a little bit…harder about making my lessons more engaging" (Interview, December 2020). Another way in which the online platform was seen as an affordance was in how the electronic resources made students more creative. For example, Shonda noted, "Online makes them more creative, because they are able to explore [websites], they are able to use them" (Interview, December 2020). Caitlin recalled an infographic assignment, which she had reworked for the online setting. In person, students collaborated mainly using poster paper and markers. However, because of the nature of being online, she opened submissions to PowerPoint or other technologies. Allowing different formats when thinking about student products, in her view, included more resources and was more in-depth, thought-out, and creative than submissions from previous semesters.

## 14.9 Redefining What It Means to Learn Mathematics

Because being online made it easier for students to share their work in real time, it encouraged them to interact with each other and to co-construct the mathematics. Being able to draw on a shared screen led students to experience mathematics differently. For example, in the following excerpt, James recalls how the drawing feature helped students see that they had a stake in the mathematics being explored:

> Something I've never really done before is save their work and make it accessible to everyone. So, typically…everyone would have a copy of whatever they did in their notebooks and they'd go home with it, but, now, I would tend to put up the whiteboard and let them draw on it. So, when I saved the notes, publish them, they had stuff that they did. So, I think

> it kind of gives a good impression that, oh, it's not just the professor that can do all this. It's other…people in my class that's doing it too. (Interview, December 2020)

Teaching online also expanded the instructors' views of what it means to collaborate. Whereas in person, students typically sit around a table or form a cluster with individual desks, the online platform motivated different ways to collaborate during class. As Michael described:

> When we talk about collaboration, I'm not sure if we're talking about two people sitting in a room looking at a board and really breaking it down. We're talking about maybe people texting or chatting by email or something like that. So, it's a different, it is a form of collaboration, but it does seem like they're more isolated…So, collaboration is certainly welcomed and especially if they're sending emails to each other about what they have and I still think that's good because that allows them to come up with their own example. (Interview, December 2020)

In his comment, Michael juxtaposes collaboration with isolation, highlighting the key difference in collaboration between an online and in-person class.

The breakout rooms were another feature that encouraged collaboration. It gave the students "a room of their own" and Stephanie noted the following:

> It was kind of nice that they had this space that was theirs, virtually, knowing [that] they were with their other classmates and they had this freedom of expression and they could come together and express their ideas…It definitely helped them be more creative in terms of communicating with their classmates. (Interview, December 2020)

This "freedom" was an outcome of the online environment, where it was possible for students to meet on their own away from the instructor. As Stephanie noted, this may have encouraged students to speak more freely, as opposed to in a classroom where the instructor is present.

## 14.10 The Need to Be Creative in Assessments

Instructors were called on to be creative in adapting assessments to an online setting. Almost all of the interviewees mentioned developing different forms of assessments—whether informal or formal—given the constraints and allowances of online teaching. For example, whereas having internet resources available to all students was seen as an affordance for projects, it constrained the types of questions instructors could ask on online tests. In many cases, this forced the instructors to be creative in how they assessed as they also thought about how to encourage their students to be creative in their responses. As Caitlin described:

> The focus of assessments became more about the process and thinking behind [the] topics rather than an answer that students could Google…So, I was trying to find out more about the relationships or, "What is the meaning of your…x- and y- axes…what's the relationship?" I think, to get away from a straightforward, "I can Google this and give you an answer, but still have no understanding of what it means." (Interview, December 2020)

Others encouraged the use of the internet as a resource, even on exams. This altered the kinds of questions they asked on assessments. For example, in the interview (December 2020), Shannon noted the following:

> Even for the final exam, I just said, use the internet. I even put a link to a web page on it as like, "Look at this web page, what do you see?" Trying to just adjust the exam to the [online] setting. It was interesting.

She reflected that allowing the use of web-based resources required students to "interpret available information and us[e] resources and information appropriately and also promoted an open-ended approach" (Interview, December 2020). Similarly, Caitlin reflected that she "wanted [assessments] to be something that was a little more open ended and there wasn't just one correct answer…questions that required a little bit more thinking" (Interview, December 2020).

Although some forms of formative assessments were curtailed during online teaching, others were newly discovered. Instructors acknowledged that it was more difficult to gauge students' engagement or comprehension without being able to see them. For example, James noted, "Typically, when I was in person, I would walk around and see what everyone's doing and I could course correct there, like, 'oh, that's not quite right, try this,' or 'you're on the right track—keep going'" (Interview, December 2020). Similarly, Shannon inquired, "How are you going to monitor their work, because again, you can't do that the way you've always done so it certainly needed some creativity with how to do that" (Interview, December 2020). She also reflected on student engagement and how students interacted with one another: "we're used to how that happens in the classroom, I think, and it's a little bit harder online." Instructors cited online surveys such as Mentimeter and the polling feature on Zoom as "help(ing) me have more access to students' ideas and what their thoughts would be" (Stephanie, Interview, December 2020).

## 14.11 Supporting the Creative Process

Because instructors were more flexible with their deadlines and with their availability outside of class, students were given more opportunities to be creative. Michael recounted how he rethought the submission process for his students. Whereas he did not allow for resubmissions when he taught in person, this semester, he realized the merits of allowing students to seek feedback and resubmit after receiving feedback. As he said in his interview (December 2020):

> I think doing this allowed me to…instill the attitude that mathematics is, "You have to fail,"…sometimes, you have to fail a bunch. There are some theorems that haven't been proven yet that were stated decades ago. So, people have been failing at it for decades and still haven't gotten to the solution.

The idea that failure is part of learning mathematics aligns with creativity because a central tenet of creativity is combining new ideas in novel and unfamiliar ways (Guilford, 1968).

The online platform also provided more time for the student and instructor to meet outside of class, contributing to the notion that mathematics is a process rather than a final solution. Several instructors mentioned that more students were reaching out to them than usual and instructors made themselves more available to students than they had in the past. Shonda and Stephanie discussed their increased availability for students, and Michael made the following comment:

> [I] liked that (online) approach to taking the course and this type of environment to be able to meet with students, whenever, of course I was free when they were free, being able to meet online and talk about their criticisms I think was beneficial. That was something I don't think I could have done during a normal semester where meeting times are restricted, and I don't have enough time before or after class to talk with this student and that student and this student. (Interview, December 2020)

Instructors were able to support students' creative approach to mathematics as they had more time available for students and could provide space for students to think about the material.

## 14.12 More Time to "Stew"

One characteristic of the creative process is that it benefits from an incubation period (e.g., Dodds et al., 2003; Ritter & Dijksterhuis, 2014). A solution that is automatic or takes little time to generate is typically associated with an "exercise" rather than a problem-solving situation that requires creativity (Schoenfeld, 2014) and disjoints from what mathematicians would describe as mathematics. In adjusting their instruction for online teaching, some instructors allotted more time for each assignment. For example, James noted, "Instead of making everyone do everything in the hour they were with me, I gave them the whole week to do it. And then they could come in and work through stuff with me if they wanted" (Interview, December 2020). He thought that this additional time gave students the opportunity to "stew" and "use the time outside of class to think about the problem". James recognized that ideas or different approaches often occur when students are not directly focused on the problem at hand. The online transition made James realize this and led him to adapt his instruction accordingly.

Similarly, Michael identified that in person, students are quick to agree with one another (or with the instructor) and proceed. However, in an online setting that incorporates independent work, students are:

> Taking it and they're trying to own it. Whereas in class a student might say, "Is this fine?" And a bunch of other students will overhear them and then go, "Oh, well now it's got to be this." So…you're not creating an atmosphere where it allows them to think about this type of problem the way it should be thought out. (Interview, December 2020)

The online setting afforded Michael's students a chance to think on their own before being influenced by more vocal students. In addition to providing them the time to think, it is also possible that students became accustomed to being asked to think through ideas on their own, rather than relying on their classmates to verbalize an answer.

## 14.13 Features of the Course that Played a Role in the Transition

Given the deliberate attempts to encourage students and teachers to be creative in their approach to teaching and learning mathematics, we were interested to see what elements of this particular course helped or hindered the transition to an online environment. As mentioned above, many of the instructors had taught this course in person prior to the pandemic and had thought deeply about the role of creativity in mathematics.

In describing the particular characteristics of the course, the instructors seemed to agree that MATH 106 is a course that emphasizes process over the answer and encourages multiple approaches. In describing his class, Michael said, “We want to see students have a unique thought process when it comes to interpreting some type of problem, especially if they’re not a science or a math major” (Interview, December 2020). Caitlin echoed this idea when she said, “I think, the course also, forcing or encouraging students to think about it from a different perspective maybe got them thinking about other things outside of the class in those ways that maybe they wouldn’t have otherwise” (Interview, December 2020). In the post-semester survey, Michael also recounted how he tried to convey his enthusiasm to students. He stressed the importance of the instructor embracing creativity: “I believe creativity to be contagious—especially if one is enthusiastic about the creative process in general. Although it could be much more difficult to convey this enthusiasm through a tiny webcam, I believe the impact is the same.” The open-endedness and process-oriented nature of the course, as well as the instructors’ instructional philosophy, were brought up by others as being conducive to an online platform.

Instructors’ conceptions of what it means to be creative helped them seek creativity in their students in an online setting. When asked about the role of creativity in online teaching, Stephanie replied that creativity gave her a mindset that helped her approach online teaching. She noted, “[The course] brings with it a sense of fun, freedom, and flexibility to try new things” (post-survey). The course itself also had an impact on instructors’ conceptions about mathematics. When asked how teaching a course on creativity in mathematics influenced his view of mathematics, Charles replied that “it forces me to generate original ideas.” This mindset of being open to new ideas and being flexible came through in Stephanie’s analogy of being in an experiment. She reflected, “We’re like in this experimental mode. You know, we’re experimenting with new technologies; we’re experimenting with different class formats; we’re experimenting with all these different things” (Interview, December 2020).

The course objectives related to creativity in mathematics also offered the instructors a language to use when conveying the challenges and opportunities afforded by the online transition. When communicating with her students, Stephanie conveyed the flexibility inherent in creativity to the students in presenting the online transition as a challenge:

> It wasn't always the same kind of format like every week. It was kind of just, you know, when I told them I was like, "we're going to see what works." Like this is new to me too, you know, like we're going to be creative with it. We're going to be open with it. We're just going to kind of like change it up and see how it goes. (Interview, December 2020)

The course objectives also provided an opportunity for students to learn in a different way. In his pre-course survey, Michael recognized the significance of creativity in the context of students' multiple online courses:

> Creativity is essential in all modalities of learning. However, for the student that is stuck in their house all day taking five classes—having the chance to be creative allows the student to not just escape their perhaps mundane schedule, but allows them to think about the world in a new way.

## 14.14 Discussion

Our findings illuminate the role of creativity in the transition to online teaching among instructors teaching a course on creativity in mathematics. As this group of instructors had been engaged in discussions centered on creativity research before the pandemic, they offered a unique perspective on the experiences of educators as they faced unfamiliar territory—one that came with constraints and affordances. Some of their experiences highlighted the need to be flexible, open-minded, and responsive to the students and to the realities of the online classroom. Many of the instructors spoke of how the experience of teaching online changed their perspectives about both mathematics and teaching. Some of these lessons can inform future online offerings as well as in-person courses to aid instructors as they prepare for the many unknowns that can arise when teaching a mathematics course online.

The flexibility of the curriculum offered instructors leeway in deciding which of their previous (in-person) lessons and activities they would implement in the online setting. Some instructors abandoned lessons that required materials, while others adapted those lessons and incorporated household items (e.g., crackers, string, coins). Related to the flexibility of the curriculum, because MATH 106 does not serve as a prerequisite for any other course, instructors were able to be creative in how they assessed their students. There was no departmental final exam or set of common questions that spanned sections. As such, instructors were able to ask questions that emphasized process over answer and became creative about how they assessed student understanding through open-ended instead of calculation questions. The ease with which students were able to Google answers also necessitated a shift to more open-ended questions that prompted different responses from each student.

The explicit goal of the course—to promote creative thinking through mathematics—also offered a context that eased the transfer to an online setting. Instructors spoke of how students and instructors were in experimentation mode, exemplifying the importance of play (Monahan et al., 2020), risk-taking (Glover & Sautter, 1977; Tyagi et al., 2017), and collaboration (Paulus & Nijstad, 2003) in creativity. This mindset was consistent with what students were called on to consider. In this way, the online environment served as a meta-example of how a constraint can lead to creativity. Also, because learning through experimentation was emphasized in the course, instructors came to believe that this mindset opened the door for more students to participate than perhaps would be expected in an in-person environment. The shift from in-person to online teaching also changed the instructors' thinking about teaching and about mathematics more generally.

The implications of our study inform professional development, course design, and assessment. We saw the important role that constraints (even when caused by a pandemic) play in the instructors' creative and reflective practices. In light of our results and considering the course retrospectively, our results indicate that it may be beneficial to base professional development meetings on the role of constraints in teaching and learning. As Peterson et al. (2013) noted, instructors need support in managing and thriving within constraints. An explicit focus on constraints and the sharing of experiences through this lens may provide instructors with additional ideas and may help them shift their mindset to see constraints as affordances. For example, we saw evidence that some instructors abandoned lessons they initially thought of as impossible in an online environment. Discussing how these obstacles can be considered as constraints to overcome and encouraging instructors to be creative in how they restructure a lesson to fit the context could benefit teaching and learning mathematics. In this way, we could provide a language to discuss issues related to teaching and to encourage creativity in teaching.

Through this research, we identified how the transition to the online environment opened up ways of thinking about teaching creatively, teaching for creativity, and teaching and learning mathematics. Although in the case of this research, the context of a course on creativity helped instructors through the transition, other courses could use creativity as a lens to approach teaching and assess learning. Lessons that we learned during the pandemic related to teaching for creativity can be carried over into future online or in-person offerings. For example, the availability of information online led to more creative assessment items and student products. Similarly, students were encouraged to doodle and to show mathematics using household items. Both of these tasks (doodling and finding creative uses for everyday items) have been associated with creativity (Guilford, 1968; Torrance, 1966). Another lesson learned is that students had to be creative in how they presented their thinking. Through the act of creating a video or a drawing, students were able to merge mathematics and communication in creative ways. Nuanced changes in how we ask questions and how we ask students to participate may encourage deep thinking about mathematics both in in-person and online settings. These added emphases of creativity in course design, professional development, and instruction can be the basis for further research on the role of creativity in higher education.

Lastly, it is impossible to discuss the transition to the online environment without acknowledging the impact of the pandemic that spurred it. It was a devastating year filled with uncertainty. It was a time when the world as a whole was forced to be creative: in managing day-to-day tasks, in how we socialized, and in how we faced life changes. Teaching and learning online was just another way in which instructors and students alike were called on to be creative. This common goal often led to a sense of community, which spurred on more creativity. As Stephanie reflected:

> I think [being online] kind of opened up more communication which contributed to the creativity of the class because we just were able to have richer, deeper discussions…how everyone was feeling and what their thoughts on math…the community that was built in the class. And…knowing that their thoughts were valued and they could kind of just like say what was on their mind and speak their mind and it was very non-judgmental, and it was like "oh yeah like I think that too." Or, "oh yeah…I see what you're saying. But I don't know if I agree with that."

The students' sense that they were a part of a community of learners, where learning was put in the context of their lived realities, was central to Stephanie's teaching. Students felt safe to voice their ideas and agree or disagree with one another. Instilling a sense of community among students was especially important during the pandemic, and it will undoubtedly remain just as important in the years to come.

## 14.15 Conclusion

This study reported on the role of creativity in the teaching and learning of mathematics in a course with a special emphasis on creativity. The context of the course provided us with a lens to characterize the shifts in teaching and learning prompted by the transition. An extension of this study would be to analyze class recordings to understand how creative teaching and teaching for creativity appear in an actual online class. While the particular course provided a fitting context to study creativity in teaching online, it also highlights a limitation of the study. A study using a similar framework on a more conventional course would help extend our findings more broadly. We acknowledge that this course was unique from most mathematics major courses. This course did not have a set curriculum and offered more flexibility than would have been possible for a course in a sequence. However, there are elements of this study that could be considered for more typical courses. For example, the online platform and, more specifically, the access to answers and solutions through the internet called for more creative approaches to assessment.

Undoubtedly, one outcome of the pandemic year will be that more attention will be paid to online teaching and learning. Further studies on practices to encourage student engagement and to promote mathematics as a creative enterprise would help us consider all of the possibilities of online learning.

## Appendix A

### *Interview Protocol*

1. Person
   - Teacher
     - Overall, would you identify yourself as creative? Why or why not?
       - Some traits of creativity are flexibility, adaptability, innovation, risk-taking, and making connections. Would you identify yourself as having any of these traits? Please explain.
     - What were some attitudinal and personal characteristics that you possess that helped you plan/implement this class online?
     - What is your general attitude towards change and uncertainty?
     - Is there another time you experienced uncertainty (aside from the teaching of this course) that helped you through this?
   - Student
     - Tell us about a really creative student you have.
       - What makes them creative?
     - Did online teaching and learning affect how their creativity was able to shine?
     - How did teaching online help students be creative?
2. Place
   - Online
     - Did the online platform help you be creative? Help your students be creative?
     - In what ways did you take advantage of the online platform in an unconventional way?
     - What was possible online that isn't possible in person?
   - Hawkmix
     - What were some affordances given by the Hawkmix modality?
     - What did you have to consider as you plan your classes for dual modality?
     - What was possible under Hawkmix that wouldn't be in a purely online or in-person setting?
3. Process
   - Teacher
     - What were some constraints that online teaching presented? How did you overcome them?

    - In what ways were you more creative because you were teaching online this semester?
        - As you planned or executed your ideas?
        - Was any of this made possible specifically through the online platform?
    - What elements of creativity did you have to rely upon as you taught your class?
- Students
    - How were you able to ask your students to be creative in your class, especially given the online setting?
    - Do you think there are elements of online learning that forced your students to be creative?

4. Product: Being that the focus of this class is on creativity in mathematics:
    - What was your most creative lesson?
    - What was your least creative lesson? Do you think it would have been more creative in an in-person setting? Was the creative element lost because we are online?
5. How does creativity figure into:
    - Pandemic
    - Online transition—different modalities
    - Nature of this course
6. Is there anything else you'd like to add about your experience teaching MATH 106 online?

## Appendix B

### *Pre-semester Survey*

1. What technologies (learning platform, programs, etc.) are you planning on using?
2. What went into the decision to use these technologies?
3. How will you adjust your in-person teaching practices to accommodate learning online?
4. For mathematics courses in general, what about in-person teaching that might not be possible (or is very difficult) in an online setting?
5. For MATH 106 in particular, what about in-person teaching that might not be possible (or is very difficult) in an online setting? (Please be specific about your past experiences if you've taught MATH 106 before.)

6. For mathematics courses in general, what do you think will be possible in an online platform that isn't possible in in-person teaching? What affordances does online teaching offer you that might not be possible in person?
7. For MATH 106 in particular, what do you think will be possible in an online platform that would not have been possible in in-person teaching? What affordances does online teaching offer that might not be possible in person?
8. What does creativity mean to you?
9. What role does creativity have in teaching mathematics online?
10. How is teaching a course on creativity in mathematics influencing your thinking of mathematics?
11. Please describe one lesson/module you're looking forward to facilitating.
12. Please add any other comments/suggestions/questions you have.

## Appendix C

### *Post-semester Survey*

1. What technologies (learning platform, programs, etc.) did you use to teach MATH 106 this semester? What went into the decision to use these technologies?
2. How did you adjust your in-person teaching practices to accommodate learning online?
3. For mathematics courses in general, what about in-person teaching that might not be possible (or is very difficult) in an online setting?
4. For MATH 106 in particular, what about in-person teaching that might not be possible (or is very difficult) in an online setting? (Please be specific about your past experiences if you've taught MATH 106 before.)
5. For mathematics courses in general, what was possible in an online platform that wasn't possible in in-person teaching? What affordances did online teaching offer you that might not have been possible in person?
6. For MATH 106 in particular, what was possible in an online platform that would not have been possible in in-person teaching? What about online teaching that made this possible?
7. What does creativity mean to you?
8. What role does creativity have in teaching mathematics online?
9. How did teaching a course on creativity in mathematics influence your thinking of mathematics?
10. Please describe one lesson/module you implemented that you are particularly pleased with. Please briefly explain why you chose this one.
11. Please add any other comments/suggestions/questions you have.

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# Part IV
# Research Application, Implications, and Future Directions

# Chapter 15
# Concluding Thoughts on Research: Application, Implications, and Future Directions

**Scott A. Chamberlin**

## 15.1 Introduction

One of the most intriguing aspects of mathematical creativity is that it is not identical in process or product for a 6-year-old grade-one student as it is for a 19-year-old student in second year at university. Hence, the practicality of the research is interpreted in a manner that enables stakeholders the opportunity to utilize the scholarly works and literature of peers in their research and/or teaching efforts to forge new paths and respond to areas of interest in creativity that are yet to be explored. In specific, the editors and authors of this book maintain that development is rarely considered in relation to mathematical creativity, and it deserves attention when conducting research and implementing activities designed to facilitate creativity. This is because not all students, across and within grades, reside at precisely the same place. Schools all across the world are based on the premise of chronological status (age) and not developmental status. In effect, having a classroom composed of myriad ability–level students is likely more typical than having one composed of students all on exactly the same ability level. More specifically, if you have a grade-five mathematics class with one student on a grade-three level, another on grade-four level, many students on grade-five level, one on grade six, and one on grade eight level, you have a perfectly *normal* classroom.

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S. A. Chamberlin et al. (eds.), *Mathematical Creativity*, Research in Mathematics Education, https://doi.org/10.1007/978-3-031-14474-5_15

## 15.2 General Overview of the Book

In the first section of the book, the editors each contribute a chapter, with a focus on development in relation to mathematical creativity. Each editor was rather informally assigned age band (5–12, 13–18, and 19–23). Much of the research in mathematical creativity coalesces around the elementary years (ages 5–12) and secondary years (ages 13–18). The tertiary years are typically less researched in mathematical creativity, as they are in many content domains. Thankfully, the only group dedicated to researching mathematical creativity at the tertiary level, the Creativity Research Group (http://www.creativityresearchgroup.com), agreed to actively participate in the finalization of this book (e.g., Chaps. 7, 12, and 13). Miloš Savić was instrumental in soliciting their assistance because without it, this book would have been absent with their presence. Peter Liljedahl was instrumental in spearheading the contributions in secondary education, and Scott Chamberlin initiated much of the elementary sections.

Each section contains themes and foci in each age band. Most chapters represented either an empirical study or a comprehensive literature review. Hence, all chapters were some combination of research or reports on research in the form of literature reviews and theoretical contributions. In the first section, Chaps. 1, 2, 3, and 4, introductory discussions are provided so that readers are prepared to make senses of discussions in the book. Of note in this section is the third chapter in which the organizational framework for the book is provided. In this chapter, several operational definitions are shared so that all readers share common understandings. In the second section, Chaps. 5, 6, 7, and 8, research for each age band (5–12, 13–18, and 19–23) is shared to provide a basis for subsequent discussions of empirical work. In the third section, Chaps. 9, 10, 11, 12, 13, and 14, empirical work is discussed so that readers can appreciate efforts of scholars in an attempt to understand development and mathematical creativity research. Commentators provided opinions on the positive and negative attributes of each section, in the last chapter of each section, and a foreword was shared by Demetra Pitta-Pantazi. Chapter 15 is a culminating chapter for the book, designed to highlight what is known, what is not known, and the implications for researchers and practitioners.

## 15.3 Needed Research

For as much that is known about mathematical creativity, it appears as though volumes more is not known about it. As an example, instrumentation to carefully document the creative process is sorely lacking. This is not to suggest that no instrumentation exists, but such instrumentation is not particularly sophisticated. More specifically, instrumentation that documents mathematical creativity in relation to development does not appear to exist. Accepting the flawed premise that all learners develop at the same rate and that chronological age determines one's level of development, instrumentation of a high degree of precision could highlight the rather dramatic differences in ability levels. Data from studies could elucidate the disparities and encourage all parties to not view mathematical learning episodes as one-size-fits-all scenarios. Further, curriculum developers and teachers may be more inclined to take learning episodes seriously and differentiate activities so that they are low-floor/high-ceiling activities. In so doing, students may be more engaged than they otherwise might have been, and they may realize that learning facilitators (e.g., teachers, curriculum coordinators) have compassion for their abilities. Consequently, learners may be inclined to persist for teachers who illustrate that they care for their students (Lumpkin, 2007; Umarji et al., 2021).

To return to the point of maturational rate, it is an assumption of contributors that learners do not develop at the same rate in coordination with their chronological age. Hence, proactively investigating how learners could be best served would aid scholar-researchers and practitioners alike. Moreover, simply finding consensus on this idea through empirical work would be invaluable in motivating educational stakeholders to generate materials to serve a greater number of learners than are currently served.

Another focus that deserves attention pertains to creative output in mathematical learning episodes. In specific, one may wonder if it is predominately a result of development or if there are other factors at play. It would be logical to assume that other factors are involved. For instance, one's level of intelligence may well have some influence on the product delivered, and though Sheffield (2009) and Haylock (1997) suggest that a modicum of intelligence is necessary for creative output to be realized, the precise level has never been determined. Interestingly, many scholars have questioned what the lowest level of intelligence could be for creative output to register, but few have questioned whether the other end of the spectrum (i.e., advanced intelligence) may also preclude or more likely hinder mathematically creative output. Luchins and Luchins (1959) questioned if rather advanced states of mental ability may negatively influence creative output in the respect that individuals with an intimate acquaintance of a domain and an advanced state of understanding might be inclined to rely on preexisting conventions to solve problems, rather than invest time in pursuit of novel solutions. Nevertheless, investigating factors associated with creative output in mathematics represents an important line of research and inquiry.

Given the significance of development in the emergence of creative output, one may question whether maturation can be influenced, that is, intentionally accelerated or unintentionally decelerated pending external factors such as classroom environment, curricular choices, teaching/instructional decisions, and/or any other effects. This is an area in need of attention, and scholars would do well to critically look at how the factors help determine creative output. It is likely that factors other than simply development play a considerable role in creative output. The question remains: what specifically are the factors, and how do each of them load? It could be hypothesized logically that an overbearing teacher may impede creative pursuits (Chamberlin & Mann, 2021) by negatively influencing one's affect. Also, a curriculum that over-relies on review of materials and includes mathematical content in repeated years (e.g., 3–5 years) may provide a motive that discourages students from investing attention in mathematics. Once a student has turned off attenuation, it may be difficult to get them to return to productivity, even if activities are intellectually stimulating. Also, a classroom environment that is conducive to not participating can serve to discourage highly creative individuals. In total, there are likely many more factors that can negatively and positively influence creative output, and they deserve attention in empirical work.

Another consideration pertains to whether development in various subdomains of mathematics (e.g., number theory, geometry, algebra, measurement, and statistics and probability) can occur at various rates. It would seem logical that they can and most likely do. However, having an empirically based answer to this question would be most helpful for researchers and scholars in mathematical creativity. Counter to many discussions in this book, mathematical maturity often pertains to the tertiary level and not the elementary and secondary (Faulkner et al., 2019; Yani et al., 2019). In fact, Lew (n.d.) discusses components of mathematicians that lead college professors to believe that they are mathematically mature. Still, the rate at which students develop in various domains of mathematics is not known. It is not fully known if students develop in some domains earlier than others (e.g., perhaps number sense advancement appears before algebraic reasoning). Logically, some would say that it does, based on the premise that number sense is infused with very young learners (e.g., preschool, kindergarten), and algebra is not formally presented until around age 14 or 15. This alone is not proof that algebra is a domain that should come later than number sense; it is merely a comment on the arrangement of domains and disciplines in mathematics. After all, Carraher et al. (2006) illustrate that students as young as 8–10 years of age can reason algebraically. This is certainly not the first evidence of early-elementary students' ability to reason algebraically. Preschool teachers have been engaging students in activities relevant to (pre-)algebra for decades. For instance, the National Council of Teachers of Mathematics (2000) stated that pattern thinking is evidence of engagement in early algebraic thinking. Also, does students' progression of development transpire at the same levels if each domain has relatively similar amount of instructional time invested in it? Logic would dictate that this is a most unreasonable hypothesis. Nevertheless, relevant to the theory and studies provided in this book, it would behoove researchers to investigate this issue to resolve this issue.

## 15.4 Application of Research

In this section, the application of research pertains to (1) peer scholars and researchers and (2) practitioners. Often, educational stakeholders think of themselves as one or the other when in reality, the distinction in the two parties is not as disparate as some believe. In fact, some teachers engage in action-research to provide insight in assessment, instruction, learning, and curricular decisions. Moreover, researchers have been known to instruct students to investigate their reasoning (Lesh et al., 2000) through using curricular materials such as thought-revealing activities.

### *15.4.1 Application of Research to Scholars*

The most significant contribution of this book is to reveal what is known and what is not known about mathematical creativity and development. In a sense, this book (as are many academic books) serves as an update on the current status of literature. Throughout this book, empirical work is provided for reader consideration. Much of this research has not been publicly shared, and the research is supplemented by comprehensive literature reviews. Many of the research efforts have been conducted to specify indicators of creative output such as originality, fluency, flexibility, and elaboration (Imai, 2000), while other works have invested in stages of creativity (e.g., Wallas, 1926). The Wallas work has very much stood the test of time insofar as his four stages of creativity (preparation, incubation, illumination, and verification) are still considered to have accuracy in explaining the creative process. Naturally, in nearly 100 years since his theory was promoted, it has endured some criticism, and *experts* seem willing to modify it (Sadler-Smith, 2015). Nevertheless, a theory that is discussed 100 years after its design does warrant consideration. Also, the role of development in relation to creativity was discussed, historical contributions were considered, and an inclusion of creativity research in mathematics was provided in all three age bands (5–12, 13–18, and 19–24). Also, the value of theoretical lens in interpreting student research, the process(es) used to engage in creative thinking, the manner in which mathematical creativity can be promoted by tertiary students, and the consideration of creativity emerging in online mathematics settings was discussed. All researchers that utilize this book will have an expanded understanding of mathematical creativity, with emerging research that can positively influence the domain.

Of course, every time that a comprehensive overview of research and scholarly work is provided, it invariably reveals that a domain is not as advanced as it might desire, with respect to knowledge. Hence, additional efforts need to be invested in qualitative work, as was done by many contributors in this book, as well as quantitative efforts. Principally speaking, this is because qualitative efforts provide rather a microscopic look at why a phenomenon transpired as it did. Quantitative work, on the other hand, often helps researchers reveal what happened, and it can provide badly needed generalizability that qualitative efforts often lack. Moreover, in accordance with the domain's (creativity) emphasis on novel products in mathematics (Nadjafikhah & Yaftian, 2013), researchers in mathematical creativity must forge efforts in new paths to expand the domain's knowledge and to engage researchers' intellect. Further, scholars with an emphasis on generating new knowledge about a field should avoid replication studies, particularly in situations in which multiple studies have shown a consensus, for example, studies in which the value of divergent thinking as a mechanism to promote creative output is ubiquitous in mathematical creativity and that date to at least the mid to late 1960s (Bradfield, 1969; Guilford & Hoepfner, 1966; Madaus, 1967). In nearly 55 years of research, researchers have had ample time to investigate every conceivable facet of divergent and convergent thinking and its relationship to mathematical creativity and, likewise, discussions of indicators of creativity such as fluency and flexibility, often attributed to Krutetskii (1976), who initially investigated these characteristics in the 1950s and 1960s; originality, often attributed to Chassell (1916); and elaboration, often attributed to Guilford (1959). In much the same sense, ample time has been provided for research on the four main indicators of creativity (over 100 years in the case of originality). Whatever could have been learned about it has likely surfaced.

Hence, replication studies are no longer needed in areas with literally decades of research already accumulated. New areas of research must be pursued to provide a more complete picture of mathematical creativity in relation to other factors that already exist. For instance, an emerging area in the domain of mathematical creativity pertains to feelings, emotions, dispositions, attitudes, and beliefs and their effect on mathematical creativity (Chamberlin & Mann, 2021). Affective states, after all, have a considerable influence on cognitive efficiency (Clore et al., 2018), and cognitive efficiency is intricately intertwined with mathematically creative processes and therefore products (Doyle, 2016). Certainly, this book represents an emerging area of scholarly efforts in promoting the idea that development and maturation may play a large role in mathematical creativity. Other areas worthy of investigations by researchers are international comparisons of countries and their creative output. To compare mathematical learners in various countries, considerably more sophisticated instrumentation must be developed, relative to what currently exists. One such instrument that should be created is an instrument that helps researchers understand the relationship between affect and problem posing. Another area in which instrumentation should be developed pertains to instructional decisions and the environment created, relative to amenability to facilitate creative output. Critically analyzing curricular materials in relation to creative process also deserves research efforts.

### *15.4.2 Application of Research to Practitioners*

Of note in the previous section are several areas that could help complete the picture of mathematical process and product in relation to various factors. Near the end of the section, the call for increasing sophistication regarding instrumentation is issued. Several of these requests illustrate the earlier claim that the work of researchers and practitioners is difficult to disentangle, for instance, efforts relevant to investigating student emotions during problem posing, critically analyzing instructional and curricular decisions in relation to creative process (and ultimately product), and other classroom implications for mathematical creativity. For instance, investigating professional development in its effect on how mathematical creativity is promoted, including its importance, in the larger picture of mathematical learning is badly needed and originating scenarios in which situations to increase the likelihood of mathematical creativity are an important endeavor. Scholars can discuss mathematical creativity in a theoretical notion endlessly, but until increased output is realized, no real effect will occur. Overlaid with all of this is the central component of development and its role in enhancing the quality of creative-learning episodes. Teachers are often well acquainted with student developmental levels, if even only informally, and they could greatly enhance the quality of research. Hence, coordinated efforts between practitioners and researchers should occur.

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