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Mathematics Education in Africa

The Fourth Industrial Revolution

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Foreword

In 2009, I worked with teachers first in South Africa and then in Tanzania later that year. In both contexts, I was struck by the widespread teacher use of smart 'cell-phones', which at that time were used largely for social purposes. In contrast, in England, I had come from, yes, many teachers owned a mobile phone of some sort, but smartphones were not common: for many, the preferred mode of remote communication was via a landline telephone or email supported by a laptop or desktop computer. The general population in at least sub-Saharan Africa, abetted by canny marketing approaches by multinational corporations, has largely 'leapfrogged' landline and desk-based technology in favour of mobile technologies and so has developed a quite distinct emerging digital culture.

Excitingly, that has brought with it many teachers confidence to engage professionally and creatively with WhatsApp and other phone software, though data charges, and related taxes, remain an issue. By 2019, the teachers in rural Uganda with whom I had the pleasure to work had only intermittent access to electricity, but they were avid users of their phones for social and professional purposes. However – and here is a key challenge for education – many had no digital technology available to their students, and the coronavirus pandemic has shown very starkly that that remains the case across large swathes of Africa. Inevitably, twin challenges of reliable access to electricity and student access to appropriate digital technologies serve to constrain what teachers can sensibly aim for in terms of education for a 'Fourth Industrial Revolution (4IR)' – but this remarkable book shows that at present, that has not dampened the aspirations, nor the creativity and solution-focus, with which many are approaching that task.

The 4IR is a term that has been in circulation for some decades now, but this book adopts the usage made by Schwab (2016) for the World Economic Forum. It represents a paradigm shift in how technology is becoming embedded within human societies, an amalgamation of digital, biological, economic, and physical-digital affordances impacting all industries and economies simultaneously. The World Economic Forum narratives suggest the 4IR offers a panacea to contemporary development problems, comprising the driving force behind the future AI economy. However, there are clear threats arising from an apparently inexorable march to such aspirations: Jones and Ng

(2021, p. 1) point very lucidly to 'The dangers of this so-called "revolution" …. already evident in terms of cyber-attacks, fake news, threats to data security, and the detrimental effects of social media use on mental health and well-being.' There are bigger questions about who is shaping these technologies and why?

As a species, we live in, and our education systems should arguably be educating for, a world in which even current levels of consumption have given rise to global-level 'wicked problems' of depletion of resources, massive and growing health and economic inequalities, and environmental degradation. Nevertheless, can Africa, already in many ways 'left behind' in terms of economic wealth, afford not to chase the same goals of increasing digitalisation and unbridled consumption? The 'global north' needs to reduce consumption, and while equity would suggest much of Africa should have access to greater material wealth, there is still in Africa rampant consumption that threatens the health of the planet: for example, there is a need for access to clean and affordable piped water, rather than ever-increasing volumes of single-use plastic bottles. Africa is already the site of some of the most major impacts on the environment of growing electronic waste disposal and increasing rates of deforestation to feed the fourth industrial revolution: Jones and Ng (2021) argue that 'a sustainable mathematics education is much more than learning about numbers and shapes but noticing the world differently through mathematics to make responsible decisions to improve the well-being of not only human communities but also the environment and the eco-system at large'.

So it is becoming increasingly important that we educate our young people – and wider populations – to understand and challenge both the potential of rapidly developing digital technologies and their constraints. With economic power very much in the hands of the already-rich nations and invested in ever-expanding economies and consumption, that will take wisdom and integrity, as well as a hitherto-unrealised profound commitment to equity of opportunity and access to resources.

This book is an aspirational one that attempts to characterise the preparedness of mathematics education in Africa to educate, in the fullest sense of the word, for the 4IR, to evaluate its potential impact and ask difficult questions about what is really required for mathematics education in the next 50 years. Further, and ambitiously, the book takes a pan-African lens on that endeavour.

Authors cite a range of prior evidence that much of Africa has still not engaged productively with the Third Industrial Revolution – and that that extends to mathematics classrooms across schools and universities where, as well as challenges of reliable access to the internet and appropriate hardware, many teachers frequently remain under-equipped mathematically and pedagogically to harness digital technologies for mathematical and wider purposes. National and international assessments show effective education for mathematical thinking at scale remains elusive. Most chapters in this book, in some way, underline, and often evidence, such challenges. For example, Luneta in Chap. 2 says, 'the ability to represent ordinary market artefacts into living mathematics illustrations require teachers that are comfortable in mathematics and have the skills of problem formulation and solving'; Kamina and Ochieng in Chap. 3 argue that 'Challenges to address with implementers of a competency-based curriculum involve (i) for experienced teachers, making the shift to explicitly integrate soft skills during instruction, from long experience of teaching mathematics procedurally with independent seatwork to learning conceptually and collaboratively in small groups with discussions; (ii) inclusive and safe classroom management that permits learner's creativity, self-efficacy, innovation and much more self-directed and thus deepened learning.'

However, that situation is, in fact, broadly similar across many materially richer regions of the world (e.g. Golding & Lyakhova, 2021), and many jurisdictions in Africa have a comparatively short recent history of formal mass education for mathematics, often beginning from colonial models of both broad and mathematical education ill-suited to the mathematics of place where the young most naturally build mathematical meaning, as eloquently argued in Chap. 2.

So **a key theme in this book** is that, across phases, many teachers in Africa continue to struggle with both digital and mathematical equipping that would enable them to better harness technology for an enlightened and genuinely empowering mathematical education. There are no 'quick fixes' to such a situation, though several case studies included in this volume point to the ongoing need for teachers of mathematics at all levels to have ready access to subject-specific, including pedagogical, professional development that starts where they are, responds to their needs, and is interspersed with the building of renewed classroom practice.

However, in common with much of the globe, **the coronavirus pandemic** has catalysed the rethinking of what is currently possible for teachers and learners while also exposing key limitations. This book offers evidence that many teachers in Africa have had limited opportunity to develop either digital or digital pedagogy skills and that many African jurisdictions currently lack a coherent digital education policy to address that. In terms of digital equipping, there are challenges around both infrastructure – stable and broad bandwidth is a pervasive challenge – and equity of access to established, let alone emerging, digital tools and software: smartphones are widespread, though not uniformly, but laptops and tablets much less so, and emerging technologies such as 3-D printers are widely beyond the experience of teachers currently in the classroom. Recent experiences, though, have shown that with policy organisation and will, exposure to technical advances for educational purposes can take place remotely: teachers very often do not need personal experience of supercomputers or robotic manufacturing, but they benefit from a horizon-feeding that gives them, and so their learners, access to understanding the potential, but also the limitations, of such tools.

For mathematics education purposes, and still more for any possibility of reaping the rewards of integrated STEM education, teachers also need a deeply robust and conceptual grasp of mathematics and its pedagogy, as well as of digital tools, digital pedagogies and their potential for mathematics education. Several chapters in this book show that curricula in many African jurisdictions are, as elsewhere in the globe, little changed from those in place a century ago and unfit for purpose in a world where creativity, problem-solving, collaboration, imagination, and core literacies that include data and digital literacies are likely to be of more value than reproductive procedures – as well as not capitalising on the contextual and cultural knowledge that learners bring with them. What a waste of potential!

However, in mathematics education, this book particularly evidences growing **African confidence and imagination** to tread pathways unfamiliar to historic colonial systems, their successors, and modern empires. For example, we see a developing embrace of place-based mathematics and of competency-based curricula, which, for all their limitations and unexpected challenges, offer opportunities for schools and teachers to re-focus on core purposes of education, and with that, on the development of key affective and process characteristics, as well as the core cognitive skills, though not necessarily the same types of procedural knowledge, needed in a pre-digital age.

This book provides ample evidence of the potential of educators across Africa for creativity, imagination, and solution focus in education. Because of its geo-location, Africa has comparatively low domestic energy needs and a high potential for, e.g. solar energy. When many Western societies are experiencing fragmentation of family and neighbourhood structures and inter-dependabilities, and global super-economies, e.g. China and India are leading the production of the material wealth that underpins such aspirations, much of Africa enjoys strong family- and roots-based cultures that can lend stability to a society that has enough to sustain life. Does Africa also have the wisdom, and the capacity, to resist 'first world' narratives of ever-increasing consumption and people-centredness? This book offers hope that, if policymakers let them, teachers and teacher educators in Africa have the imagination and the determination to develop mathematics and wider education that will equip for wisdom to discern what is needed for human flourishing and the courage to pursue and even to lead, that.

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Preface

The Fourth Industrial Revolution (4IR) is a process and an event that has evolved into utilising new technologies. It is characterised by the fusion of the biological, physical, and digital worlds. The 4IR has created an interconnected, unpredictable, disruptive, and complex world full of new digital technologies. It embodies a new era of innovation in mathematics education, leading to the rapid emergence of new mathematics teaching and learning technologies. Thus, this edited volume focuses on mathematics teaching and learning in Africa in the 4IR.

The 4IR in mathematics education is happening in various parts of Africa at varied levels, locations, and entities. The authors of the various chapters in this volume have positioned the 4IR research in their respective local contexts. They have addressed numerous interests, concerns, and implications regarding Mathematics Education and 4IR in Africa. A number of authors have positioned their work in the context of the COVID-19 pandemic that has gripped the world, while others have situated their discussions on the implications of inequalities in Africa on mathematics education in the 4IR. Some of the chapters have hinged on arguments, observations, and suggestions on how mathematics teaching and learning in Africa can be conducted, improved, and transformed by 4IR, especially its Internet of Things (IoT). There are also chapters that have focused on challenges associated with 4IR in mathematics education in Africa.

In bringing together the chapters in this book, we provide a research-based resource for graduate students, mathematics teachers, mathematics education curriculum developers and policymakers, research enthusiasts, and everyone interested in Mathematics Education and the 4IR at various levels of education in Africa. The book is a rich source of information on research in mathematics education and the 4IR for the international communities interested in education issues in Africa.

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Contents

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About the Editors

Brantina Chirinda is a lecturer of Mathematics and Mathematics Education in the Faculty of Education at Cape Peninsula University of Technology (CPUT), a research associate at the University of Johannesburg (UJ) and a visiting research fellow at the University of California at Berkeley. Dr Chirinda holds a PhD in Mathematics Education from the University of Witwatersrand. She is interested in the teaching and learning of mathematics in contexts of disadvantage, specifically focusing on mathematical problem-solving and equitable access to content in the mathematics classroom. She has taught Mathematics and Mathematics Education courses at various Southern African institutions for over 20 years. She has published several conference proceedings, book chapters, and articles in accredited local and international journals. She has presented her work at several national, regional, and international conferences. Dr Chirinda leads a community project, Southern African Mathematics Empowerment Network (SAMENET), envisaged to empower female mathematics teachers and learners in contexts of disadvantage.

Kakoma Luneta is an associate professor of Mathematics Education in the Faculty of Education at the University of Johannesburg, South Africa, where he has been a faculty member since January 2005. Dr Luneta holds a PhD in Mathematics Teacher Education from the University of Witwatersrand and a Master's in Mathematics Teacher Education from the University of Sussex in England. His research interest is in mathematics teacher education at secondary and elementary school; mathematics/numeric cognition, and professional development and mentorship of mathematics teachers. He has taught mathematics and physics in various Northern and Southern African countries, the UK, and the USA. He has supervised to completion several master's and doctoral students from Sub-Saharan Africa, Greece, and South Korea. He has published four books, over 30 book chapters, and articles in accredited journals. Dr Luneta was recently appointed a visiting scholar at the University of Cambridge Centre for Neuroscience in Education (CNE) and was once a visiting associate professor of Mathematics Education at the University of British Columbia in Canada and a visiting scholar at Stanford University in the USA. He has also been an invited Keynote Speaker at various academic meetings.

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About the Contributors

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Conilius Jaison Changwiza is a lecturer in the Science and Mathematics Education Department at the Bindura University of Science Education. Dr Changwiza holds a PhD in Mathematics Education from the University of Kwa-Zulu Natal. He has served as a high school mathematics teacher in Zimbabwe from 1991 to 2008. He has been a lecturer at the Bindura University of Science Education for 13 years. He has published several articles in various local and international journals. His research interests include finding ways to improve the teaching and learning of mathematics at high schools and university levels.

Aline Dorimana is currently a PhD student in mathematics education at the African Center of Excellence for Innovative Teaching and Learning Mathematics and Science (ACEITLMS) based at the University of Rwanda College of Education (UR-CE). She has produced many scientific articles in reputable science education journals and reviewed more than 20 research articles. Her research focuses on supplementing daily mathematics instruction with a problem-solving focus. She uses problem-based learning as an instructional intervention to enhance learners' problem-solving abilities.

Faaiz Gierdien works as a mathematics educator in the Department of Curriculum Studies at Stellenbosch University. His research interests include mathematics education and teacher education, with a particular focus on aspects of continuing professional development and the use of information and communications technology in relation to school mathematics reform. He is also a research associate at the University of the Western Cape (UWC), where he studies university-school collaboration, a key design principle of the UWC-based Local Evidence-Driven Improvement in Mathematics Teaching and Learning Initiative (LEDIMTALI) project.

Fraser Gobede is currently a postdoctoral fellow in Mathematics Education at the University of Malawi. His career in pre-service teacher education started in 2014 after joining the University of Malawi as a lecturer in mathematics education. In his professional practice, he looks for educational technology applications in teacher preparation, partly due to his background in mathematics and computer science. The unexpected global events that have affected the education sector from 2020 have made his interest in educational technology applications in mathematics education grow further.

Jean Claude Habimana is currently a Master's student at the University of Rwanda. His research focuses on designing low-cost materials for effective teaching of mathematics and science.

Shemunyenge Taleiko Hamukwaya is a doctoral candidate under the Exact Sciences Doctoral Programme at the University of Turku, Finland. She holds a Master's degree (2010), Bachelor's Honours degree (2007), Advanced Certificate (2004), and Basic Education Teacher Diploma (2000) in Mathematics Education. Hamukwaya has 22 years of teaching experience in mathematics and physical science at both secondary and primary levels. She has headed mathematics and science departments and serves as a School Principal in Namibia. Her PhD study focuses on educators' and students' beliefs about mathematics learning difficulties. From her PhD, Hamukwaya aims to draw conclusions related to education policies, teaching, and learning, decrease negative beliefs about MLD, and, ultimately, how teachers teach and students learn.

Aloys Iyamuremye holds a Master of Education in Chemistry Education. He is currently an Educational researcher and CEO of Universal Chemistry Network. His research focuses on technology in science education. He is looking at bridging the gap between policy and practice in teaching and learning mathematics and sciences.

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Zingiswa Jojo is a full professor in the Department of Mathematics Education at UNISA. She serves as the South African Women in Mathematical Sciences Association president. Zingiswa Jojo is also an academic member of the Athens Institute for Education and Research (ATINER) Education Unit. She conducts and leads community projects (a) Bizana Teachers' Journey with FET Mathematics and (b) the Role of Mathematics Education in Women Empowerment (RMEWE), envisaged to empower mathematics teachers with both content knowledge and pedagogy and improve mathematics teaching at all levels together with technology integration in the teaching of the subject.

Cyril Julie holds a master's degree in Applied Mathematics from the University of the Western Cape and a doctoral degree in Mathematics Education and Computer-based Education from the University of Illinois at Urbana-Champaign. He is currently the FirstRand Foundation and NRF Chair in Mathematics Education at the

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Penina Kamina is a professor of Mathematics Education at the State University of New York at Oneonta, where she is responsible for teaching and advising pre-service elementary education majors. Annually whenever there is an opportunity, she gives back to Kenya by working with peers, graduate students, early-career researchers, pre-primary and primary school children and their teachers. Her current collaborative research interest with Kenya colleagues is on the primary level competency-based curriculum. Presently, in 2022, she is visiting as a Fulbright scholar at Strathmore University to continue ongoing research on CBC instruction in primary mathematics classrooms within Nairobi County, among other scholarly tasks.

Mercy Kazima is a professor of Mathematics Education at the University of Malawi. Her work includes teaching mathematics and mathematics education, supervising postgraduate students, and leading research and development projects. Mercy has substantial experience in mathematics education research in mathematical knowledge for teaching, teaching and learning mathematics in multilingual classrooms, and mathematics teacher education and professional development. She was president of the Southern Africa Association for Research in Mathematics Science and Technology Education (SAARMSTE) and has contributed to research in mathematics education in Southern Africa and beyond. Mercy is currently a member of the International Commission on Mathematical Instruction (ICMI) executive committee 2021–2024.

Jean de Dieu Kwitonda holds a Master of Education in Chemistry Education. He is currently an education researcher. His research focuses on pedagogy in teaching mathematics and sciences.

Justina Longwe is a mathematics teacher educator at Machinga Teachers' Training College and currently a postdoctoral fellow at the University of Malawi. Her research interest is in pre-service mathematics teacher education with a special focus on teaching mathematics in the early primary school. She recently completed her doctoral studies at the University of Malawi. She explored the area of pre-service mathematics teacher education in relation to how mathematics teacher educators help pre-service primary school teachers learn to teach mathematics to early years learners. She believes that one way of strengthening the teaching and learning of mathematics in Malawian schools is by paying attention to how pre-service teachers are trained for their teaching work.

France Machaba is an associate professor at the University of South Africa. He has worked as a Senior Education Specialist in Mathematics at Gauteng North District. He has received awards in mathematics education, including the best Honours research award in mathematics education from Wits University and the Vision keeper grant from the University of South Africa. He supervised master's and doctoral students. His research interests are varied, including the integration of mathematics and everyday context, mathematics and mathematical literacy and misconceptions in mathematics. He has published several academic articles and presented several national and international papers.

Tšhegofatšo Makgakga holds a Doctor of Philosophy (PhD) in Mathematics and Science Education and is a lecturer in the Department of Mathematics Education at the University of South Africa (UNISA). He offers undergraduate programmes and supervises master's and doctoral students in Mathematics Education. Prior to joining UNISA, he was a mathematics teacher in three secondary schools and received excellent awards for grade 12 final mathematics results. Dr Makgakga has presented conference papers nationally and internationally. Furthermore, he has published articles and papers in conference proceedings books since he joined academia. He is the chairperson of the postgraduate students' forum in the North-Eastern Region at UNISA and a chairperson for the UNISA Research Community of Practice. He organises and facilitates the workshops, seminars, roundtable discussions, colloquia, and conferences in the two committees. He is also an external examiner for master's and doctoral students' theses.

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Hlamulo Mbhiza obtained his BEd, BEd Honours, Master of Education by Dissertation degrees, and PhD at the University of the Witwatersrand. Rurality, mathematics education, and teaching practice form the basis for Dr Hlamulo Mbhiza's research. He has held lecturing and tutoring positions at the University of the Witwatersrand, Independent Institute of Education (Rosebank College), Instil Education, and the University of Limpopo. He is currently a lecturer of Mathematics Education at UNISA. He has held several prestigious scholarships, including the NIHSS-SAHUDA and NRF Scholarships. Dr Mbhiza is a Golden Key International member. Over the course of his research career, he has authored/co-authored book chapters and articles.

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Chapter 1
Is Africa Ready for the Fourth Industrial Revolution?

Fidele Ukobizaba, Ezechiel Nsabayezu, and Alphonse Uworwabayeho

Introduction

The rapid industrial and technological growth over the last 100–150 years caused an integral shift from traditional ways of living society to the modern society of knowledge and globalisation (Bayode et al., 2019). To cope with this shifting, the education system from elementary to higher education faced challenges of how students should be prepared to fit into the forthcoming era; the Fourth Industrial Revolution (4IR) era which is still in its infancy and not well known, especially within many African countries (Voskoglou, 2020). Only South Africa on the continent came after countries like Brazil, Russia, India, and China for having identified a significant role in addressing the gap that could hinder the policy of Industry 4.0 or 4IR (Lambrechts et al., n.d.). Indeed, the World Economic Forum (WEF) reported that South Africa seems to be the only country on the continent facing the requirements of the 4IR era (Uleanya & Ke, 2019).

The 4IR is characterised by fusing digital, physical, and biological worlds, including the use of new technologies known as artificial intelligence (AI), cloud computing, robotics, 3D printing, the Internet of Things (IoT), and advanced wireless technologies (Fomunyam, 2020). Since it is evidenced that technology is inevitably needed in our daily lives' practices, internationally, countries should be prepared for this 4IR era. For instance, more than half of the population around the world is connected to the internet. It is also expected that by 2022, about 60% of the global economy will be digitised. However, the literature shows that Africa is

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unprepared to embrace 4IR, despite the potential opportunities for Africa to harness the 4IR (Uleanya & Ke, 2019).

The education system within learning institutions and training is being adjusted for many developed countries in preparation for 4IR. However, developing and underdeveloped countries still struggle with the Third Industrial Revolution (3IR), which prevents them from preparing for the 4IR. For instance, countries like Niger, Togo, Lesotho, Nigeria, Ghana, and Ethiopia, among others, have been considered unprepared for 4IR (Uleanya & Ke, 2019). Nevertheless, it is believed that the integration of Information and Communication Technology (ICT) in teaching and learning will play a crucial role in countries' development as the world is shifting from knowledge to a digital economy (Bayode et al., 2019). To this end, there is an urgent call to use ICT to support the education sector in achieving the 4th Sustainable Development Goal (Quality education) within developing countries (Akintolu & Uleanya, 2021).

Drawing from historical background, it is agreed that the failure of many African countries to drive industrialisation agenda during the second and third industrial revolutions was denoted by policy structure, cultural and institutional obstacles like corruption, racial and familial divisions, political instability, insecurity, mismanagement and poor leadership (Ayentimi & Burgess, 2018). Considering the speed at which technological revolution is in Sub-Saharan Africa, it can be argued that this speed of 4IR may impose challenges to many countries in terms of developing appropriate physical, social, and economic infrastructure (Ayentimi & Burgess, 2018). For instance, in higher learning institutions, the massive spread of mobile devices, access to network broadband connectivity, and rich education open a trend on how education should be delivered (Mncube & Olawale, 2020).

Although some teachers in Africa have access to digital tools, teachers still teach based on paper, pen, chalk, and blackboard. Teachers do not integrate e-learning into their teaching. One of the major challenges teachers reported that hinders them from integrating e-learning is the unavailability and unstable internet connectivity (Mncube & Olawale, 2020). However, teachers nowadays should be aware of the 4IR demands. The methods of teaching need to move to Education 4.0 as a response to 4IR.0 (Akintolu & Uleanya, 2021; Ilori & Ajagunna, 2020). It is expected that technologies like AI, robotics, and IoT will replace human jobs. Thus, students should be prepared in ways that allow them to possess the skills needed in the workplace (Grinshkun & Osipovskaya, 2020; Mudaly, 2018). Therefore, students will not adapt to 4IR demands if teachers do not possess sufficient knowledge and skills to prepare them for the 4IR (Kayembe & Nel, 2019; Mpungose, 2020; Sikhakhane et al., 2021). Although computers and other clever AI will not replace teachers, it is logical that the role of teachers in the classroom will change significantly in the coming years (Naidoo & Singh-Pillay, 2020; Voskoglou, 2020). To this end, we claim that teachers' professional development will play a key role in embracing the new pedagogy and innovative teaching practice to enhance learning (Naidoo & Singh-Pillay, 2020).

Inadequate teachers' knowledge negatively affects the implementation of the new 4IR curricula within Africa (Mpungose, 2020). Thus, the 4IR will dictate what and how students must be taught. To make students access jobs and have skills reflecting the 4IR, teachers and students must keep up with the pace of technological advancement; otherwise, they will be left behind (Ilori & Ajagunna, 2020). Suppose teachers in Africa are called to be ready for the use of digital tools and resources. In that case, governments should establish directive policies governing the use of digital technology and then provide a strong bandwidth that will ensure fast and stable internet connectivity to facilitate the readiness for the 4IR (Mhlanga, 2021). In this regard, some specific areas in education like Science, Technology, Engineering, and Mathematics (STEM), should be envisaged (Uleanya & Ke, 2019). Particular attention should be paid to Mathematics Education since mathematics was reported to be weak in African countries at all levels, reducing the number of students who choose mathematics as a professional carrier (Ukobizaba et al., 2020). Hence, integrating ICT into the education system is a vital solution (Kayembe & Nel, 2019).

Purpose and Research Questions

This chapter aims to review the studies conducted in Africa that show to what extent Africa is ready to embrace the 4IR era in mathematics education. The authors of this chapter also wanted to explore different challenges and barriers that prevent African countries while teaching different subjects and mathematics, particularly when integrating technology in their teaching to meet the 4IR requirements. Further, the authors of this chapter wanted to look at the potential solutions that can help African education stakeholders prepare for the 4IR. Hence, this review intended to answer the following research questions:

1. To what extent are African countries ready to teach mathematics to cope with 4IR requirements?
2. What challenges hinder African countries from getting ready for the 4IR in teaching mathematics?
3. What potential solutions are needed for Africa to prepare for the 4IR in teaching mathematics?

It was anticipated that answers to the listed research questions would provide insight into the readiness of mathematics education for 4IR and challenges met in teaching with technology, thus, drawing further policy implementation for African countries' readiness for the 4IR.

Literature Review

ICT Integration in Mathematics Education in Some African Countries

Educators in Africa are positive about using ICT to advance mathematics teaching and learning in secondary schools. This is because ICT increases learners' learning, encourages learners' interest in learning mathematics, and improves classroom interactions during the lessons (Bhattacharjee & Deb, 2016; Netsianda & Ramaila, 2021; Nihuka & Bussu, 2015). Although, the ICT integration in mathematics education in Africa still meets with challenges. For instance, in their study, Netsianda and Ramaila (2021) found that teachers are unable to utilise ICT in the mathematics instruction process. In addition, effective ICT integration in mathematics education was hindered by poor access to technology resources and a deficiency of proper training on digital tools.

While researching ICT use in Science and Mathematics Teacher Education in Tanzania, Kafyulilo et al. (2015) showed that there is an inappropriate use of ICT in schools due to poor educator's understanding of the ICT integration Education process, lack of applied skills with technology, and absence of teamwork between teachers. While conducting a study on ICT use in the teaching of mathematics, Agyei and Voogt (2011) indicated that mathematics instructors in Ghana do not incorporate ICT in their mathematics teaching. The main barriers recognised were the absence of awareness about ways to incorporate ICT in object lessons and the lack of training prospects for ICT integration knowledge attainment.

The Benefits Gained for Learners Taught Through Technology-Based Instructions

The study conducted on the use of ICT in mathematics instructions in secondary schools of Vhembe District, South Africa, Netsianda and Ramaila (2021) demonstrated that the integration of ICT in mathematics education creates an attractive learning environment and increases the students' retention. Similarly, Bhattacharjee and Deb (2016) confirmed that ICT encourages changes in employed conditions, handling and exchanging of information, and teaching-learning methods. ICT improves the instruction methods and the ways learners are learning. ICT-based learning environment enables students to be active in learning, promotes student-teachers, and student-student collaboration enhances innovation and creativity. ICT improves the environment of education and the roles of learners and instructors in the teaching and learning process.

As it was also argued by Ratheeswari (2018), ICT use in teaching gives students chances to study and use the required twenty-first-century talents and skills and helps teachers to deliver a lesson more attractively. With ICT, students become able to involve themselves in collaborative tasks with a broader range of facts and knowledge during their education, and students improve their understanding of mathematics concepts (Agyei & Voogt, 2011; Barakabitze et al., 2019). Through the use of ICT, learning can happen anytime and everywhere and assists learners in accessing digital information. ICT upkeeps learner-centred and self-directed learning, produces a creative learning situation, promotes cooperative learning in a distance-learning condition, helps students develop critical thinking skills, and promotes lifelong learning (Fu, 2013; Netsianda & Ramaila, 2021).

Methodology

This study employed a desk review, whereby thematic and systematic analysis was used to collect data relevant to the research questions (Braun & Clarke, 2012) about African countries' readiness for the 4IR, focusing on mathematics education. To this end, peer-reviewed journal articles/papers were downloaded from Education Resources Information Center (ERIC), Research Gate, Google Scholar, and Academia databases. These databases were deemed valid, relevant, and reliable based on their high-quality contribution to the academic sphere. Thus, 'teaching mathematics within the 4IR era in Africa, the readiness of Africa towards the 4IR in teaching and learning mathematics, challenges of teaching mathematics linking it with ICT in Africa, Integration of ICT in teaching and learning mathematics within African countries, and the role of mathematics curriculum in Africa to cope with 4IR', were used as keywords to obtain 204 article papers.

Based on our study construct, we filtered out most of the downloaded resources (Moher et al., 2009). Among the downloaded articles, 39 were duplicated and thus, deleted. By considering articles discussing the 4IR and Africa, 88 articles were filtered out. We, therefore, remained with 77 articles. Through deep analysis, we found 57 papers talking only about African readiness for the 4IR. Though our focus was mathematics, it was realised that few articles or papers were concerned only with mathematics. However, the readiness of a country in any subject, especially STEM subjects, can inform us of the readiness for its readiness in mathematics education. As an implication, we remained with 20 articles that discuss African education for all subjects in general and mathematics in particular, ready for the 4IR, from elementary education through higher learning education.

Besides the above criteria, we only considered papers published after the launch of the 4IR in 2011. Thus, the reviewed articles were those published within the 2014–2021 period. The reviewed articles are presented in the table, showing the author(s), the article title, the findings, the country, and the observation (see Appendix). For more clarifications about the article's exclusion and inclusion process, Preferred Reporting Items for Systematic reviews and Meta-Analyses (PRISMA) are provided (Moher et al., 2009) (see Fig. 1.1).

The next sections present research findings.

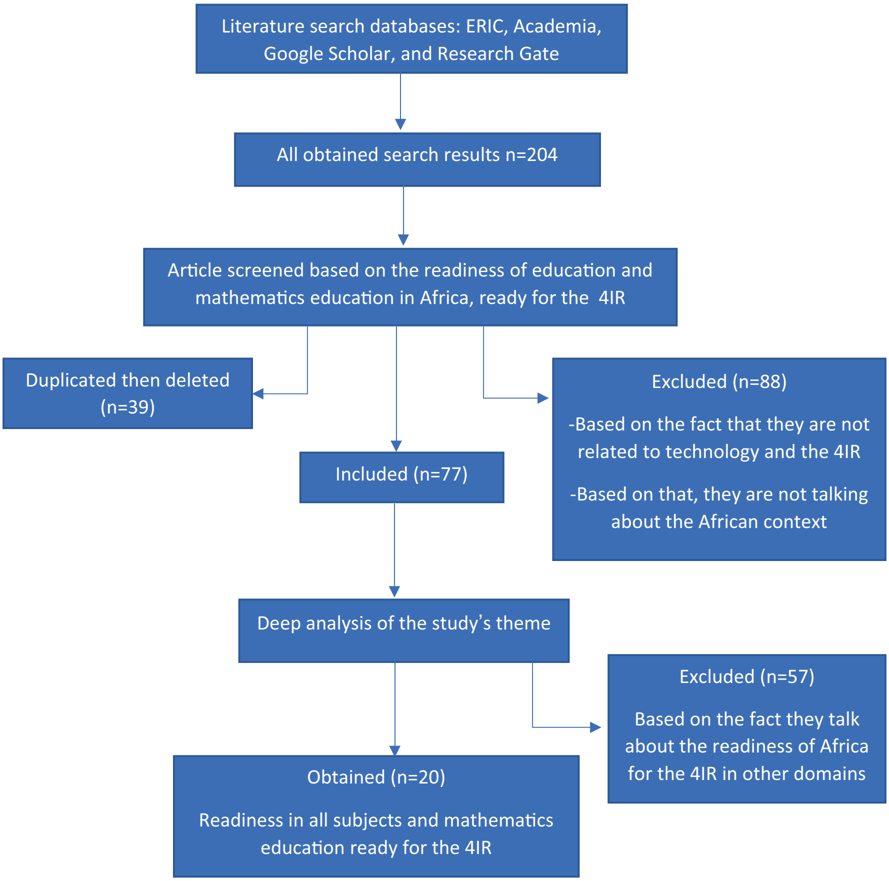

Fig. 1.1 The PRISMA diagram shows the selection process of the reviewed articles

Readiness in Teaching and learning STEM Subjects and Other Subjects During the 4IR

The literature showed that STEM subjects in general, and mathematics in particular, are not taught effectively to embrace the 4IR era. For instance, Naidoo and Singh-Pillay (2020) used a qualitative study that involved 24 participants during a workshop to explore mathematics teachers'professional development to embrace the 4IR. Their findings revealed that the majority of the participants employ technology-based tools during teaching and learning mathematics. However, while investigating the challenges and opportunities regarding the usage of computers in the teaching and learning of mathematics in five performing schools in the Tambo district, South Africa, Tachie (2019) revealed that mathematics instruction in schools was challenged by the lack of computer tuition opportunities, attitudes shown by school managers, and limited resources. Tachie added that the lack of teachers' access to computers prevented them from helping their students do mathematics-related tasks. However, in the study exploring the flipped learning approach within a mathematics higher education milieu in the era of the Fourth Industrial Revolution, Naidoo (2020a) showed that lecturers used flipped learning while teaching mathematics. Naidoo argued that this technology-based pedagogy could help students interact with their lecturers since they showed a willingness to shift from traditional pedagogy and use collaborative learning by engaging in technology-based enriched lecture rooms.

While exploring the accounting teachers' readiness to implement e-learning in their classrooms during the 4IR in the case of schools in the Eastern Cape province of South Africa, Skhephe et al. (2020) found that the classrooms were not designed in a way that supports the e-learning. In addition, accountant teachers are unaware of the importance associated with e-learning classrooms. In the investigation carried out on the postgraduate mathematics education students' experiences of using digital platforms for learning within the COVID-19 pandemic era, Naidoo (2020b) showed that lecturers face difficulties in using digital platforms and are not provided with enough time and space to practice online mathematics examples before interacting within the online community. Furthermore, while investigating the importance of mobile learning (m-learning) for STEM classrooms, Mutambara and Bagaya (2020) found its acceptance for the use of e-learning among rural high school STEM teachers. However, Mutambara and Bagaya reported that South Africa is still at a low rate.

In a study on the challenges of blended e-learning tools in mathematics, the University of Uyo, Nigeria, Umoh and Akpan (2014) found that ICT tools are not available, and even when available, they are not accessible. It was reported that students do not have adequate skills for using blended e-learning in teaching and learning mathematics. Similarly, the study conducted about opportunities and challenges with a focus on the integration of ICT in teaching and learning mathematics in secondary schools in Nairobi, Kenya, Amuko et al. (2015) revealed that there is a lack of capacity-building since teachers were faced the challenges of developing their technical skills and knowledge. Teachers are themselves striving for self-training. The lack of capacity building was one factor that discouraged teachers from integrating ICT into their teachings.

While investigating the harnessing the 4IR in Southern African Development Community (SADC) countries, with a sample of South Africa, Malawi, and Lesotho, Markowitz (2019) found gaps for the region in terms of technology-related policies that should be established to support access to the internet in rural areas, education, skills policy reform, and the creation of political and public buy-in for strong national visions of digital societies. In the study entitled "the fourth Industrial Revolution: Challenges and Opportunities in the South African context", Bayode et al. (2019) argued that adopting technologies like advanced analytics, cloud computing, and mobile devices is still at a lower level. In the study about South African technological readiness and implications for the South African Connect Broadband Policy, Mwapwele et al. (2019) found that the majority of teachers involved in the study are ready to use ICT. However, the participants of the study reported that financial, technical, and digital skills are challenges within their schools. In addition, the report showed that school leaders set policies preventing students' digital devices except for calculators as one of the policies that hinder the implementation of the South Africa Connect policy.

Different studies have been conducted on other subjects to see to what extent students use technology to prepare them for the 4IR era. For instance, in the study conducted on the readiness of public Technical and Vocational Education and Training (TVET) for the Fourth Industrial Revolution in the case of South Africa, Denhere and Moloi (2021) showed that the majority of public TVETs noticed the importance of readiness of 4IR with the experience got as they were pushing during COVID-19 period. However, teachers were not completely ready for the 4IR due to the absence or inadequate teaching and learning technologies, lack of training for the staff about the use of technologies, lack of ICT equipment, lack of ICT strategies, and lack of policy directives. Similarly, while exploring the contemporary employability skills needed for learners to succeed in the civil technology field in the 4IR era, Mtshali and Ramaligela (2020) found that civil technology teachers are aware of the industry needs in the 4IR era. However, teachers are unaware of the value that civil technology may bring to companies, entrepreneurs, and the economy.

Challenges That Hinder African Countries from Getting Ready for the 4IR in Teaching Mathematics

Identified main issues that hinder Africa from being ready for the 4IR include insufficient infrastructure, lack of continuous professional development, and Mathematics curricula that are not designed to accommodate changes for 4IR.

The Lack of Training on Professional Development Towards ICT to Embrace the 4IR Era

Most of the reviewed literature showed that teachers lack training for their professional development. The study by Mpungose (2020) found that student teachers have significant content, pedagogical, and technological knowledge, but there is a lack of advanced knowledge and inadequate training within institutions that leave students unprepared for the 4IR era. Similarly, Ajani (2021) showed that teachers in South Africa clearly understand the relevance of the 4IR in teaching and learning, only that they face barriers such as the lack of computer knowledge and skills and adequate resources. In the study conducted on challenges and opportunities for education in the 4IR, Kayembe and Nel (2019) argued that there are insufficient funds, infrastructure, and the required skills for teachers to train graduate students to fit into the 4IR. Furthermore, Sikhakhane et al. (2021) found that teachers used computers superficially and irregularly, thus insignificantly due to scarcity of computers, lack of incentives, and training about the use of computers. These challenges were identified as threats for the African countries to prepare for the 4IR in teaching mathematics.

Mathematics Curricula That Are Not Designed to Accommodate Changes for the 4IR

The introduction of smart communication systems involving artificial intelligence, the internet, virtual reality, robotics, and digital textbooks has opened a new vista about how and what is learned in schools (Ilori & Ajagunna, 2020). However, the African curricula are not preparing the students to fit the 4IR demands. These curricula contain content that does not sharpen African citizens' knowledge to achieve a good future (Fomunyam, 2020). Expressed that the curriculum used in the Education of African is not allowing the African citizen a better life; thus, there are no required skills to apt to computerised organisational sceneries (Iruonagbe & Egharevba, 2015). Therefore, the changes produced by the industry 4.0 revolution necessitate the adoption of new industry models, manufacturing processes, and new curriculums in academic programs (Mudaly, 2018).

The Potential Solutions for Mathematics Education Ready for the 4IR

Different scholars have suggested various solutions depending on their points of interest.

Most African nations' future stability will likely depend on how efficiently these needs are met. To meet these needs, it is vital to ensure integration among disciplines as in the STEM approach and incorporation between the educational policy, instructors, curricula, learning environment, learners, and industrial sectors in the community (Elayyan, 2021). Likewise, Mncube and Olawale (2020) suggested that to fit into the present industrial revolution; higher learning institutions should double their effort in providing training in STEM subjects to educate students since these subjects are the main pillars to stand against the challenges of the 4IR.

In his views, Mpungose (2020) proposed that Advanced Signal-Technological Pedagogical and Content Knowledge (AS-TPACK) should be a useful framework to strengthen the student teachers' knowledge in the era of 4IR. Mpungose added that student teachers should not only focus on three components of knowledge known as Teacher Knowledge, Content Knowledge, and Pedagogical Knowledge (TK, CK, PK), but they should also reflect on the needs of the discipline, society, and their needs. In addition, Naidoo (2020a) suggested that to integrate flipped learning within lecture rooms, there is a need to provide professional development workshops to enhance, in turn, students' learning. Later, Naidoo (2020b) suggested that for lectures to use digital platforms, it is important first to note that students are equipped with digital devices and internet connections. Naidoo added that it is important to ensure that the resources used are readily available, easy to use, cost-effective, and data-efficient.

Concerning mobile learning, Mutambara and Bagaya (2020) suggested that effort should be put into learning how to use m-learning platforms and be aware of their adoption and usefulness. There should be programs related to the awareness of m-learning to enhance teachers' attitudes and change their behaviours towards the adoption of the 4IR era. Similarly, Sikhakhane et al. (2021) argued that there should be a Technological Pedagogical Educational Psychology and Content Domain (TPEPCD) model in all teacher training institutions to train teachers who can teach students in agreement with the 4IR. Furthermore, Umoh and Akpan (2014) concluded that to provide practical support to students, such as the provision of a Virtual Learning Environment (VLE) to diversify students learning of mathematics, institutions and instructors should identify students' challenges and opportunities.

There should be a transformation in teachers' pedagogy for effective entry into the 4IR era, together with the curriculum and assessment policy (Naidoo & Singh-Pillay, 2020). Thus, the curriculum needs to be re-conceptualised and made relevant to national and global development aspirations and interventions. Also, the curriculum should be implemented in teaching and assessment as a competence-based curriculum to meet twenty-first-century requirements (Mncube & Olawale, 2020). Curricula development and review should be dynamic and must be aligned with the technological advancement and skills required in the twenty-first century (Ilori & Ajagunna, 2020). Curricula and instructional undertakings should be giving students a chance to have future work because problems of the future could not be the shortage of jobs but the skills needed for IR 4.0 thoughts (Grinshkun & Osipovskaya, 2020). Thus, supporting policies and regulations need to be implemented to adopt Industry 4.0 technologies, whereby human capital and infrastructure development need to be prioritised. It is believed that investing in ICT would improve the quality of education and innovation within African countries (Bayode et al., 2019). These improvements will only happen if mechanisms of pre-and in-service training are offered for teachers continuously to keep up to date with technological and pedagogical developments are put in place (Uworwabayeho, 2016).

Conclusion

The reviewed literature showed that the African continent is not ready to cope with the demand of the 4IR for all subjects in general and mathematics in particular. South Africa maximised the reviewed literature showing that the country is ready for the 4IR and at the level of dealing with the challenges brought about by the 4IR in schools. Generally, the analysis revealed that African countries do not have sufficient infrastructure. Teachers on the continent are not ready for the 4IR due to absence or inadequate teaching and learning technologies, lack of training for the staff about the use of technologies, lack of ICT equipment, lack of ICT strategies, lack of teachers' skills, lack of teachers' awareness and their negative attitudes towards teaching mathematics following the 4IR requirements, and the lack of policy directives. In addition, mathematics curricula are old and irrelevant, and they do not prepare African students to face the 4IR demands.

Some potential solutions were provided, such as for African leaders to establish and support policies and regulations needed to adopt Industry 4.0 technologies. To this end, the Advanced Signal-Technological Pedagogical and Content Knowledge (AS-TPACK) can be a useful framework to strengthen the student teachers' knowledge in the era of 4IR. In addition, the Technological Pedagogical Educational Psychology and Content Domain (TPEPCD) model in all teacher training institutions can develop positive attitudes towards using technological tools in their teaching. Higher learning institutions are proposed to double their effort in providing training in Science, Technology, Engineering, and Mathematics to educate students since these subjects are the main pillars to stand against the challenges of the Fourth Industrial Revolution. Last but not least, it was suggested that the curriculum needs to be re-conceptualised and made relevant to the national global development aspirations to meet the 4IR expectations.

Appendix: The Reviewed Studies on the Readiness of Education and Mathematics Education in Africa for the 4IR

SN	Author(s) and date	Article title	Subject(s)	Country	Findings	Observation
1	Ajani (2021)	Exploring the teacher professional development in the Fourth Industrial Revolution: In pursuit of social justice	All subjects	South Africa	Teachers clearly understand the relevance of the 4IR in teaching and learning, only that they face barriers such as the lack of computer knowledge and skills and adequate resources, among other systems	Not ready

(continued)

Appendix (continued)

SN	Author(s) and date	Article title	Subject(s)	Country	Findings	Observation
2	Amuko et al. (2015)	Opportunities and challenges: integration of ICT in teaching and learning mathematics in secondary schools, Nairobi, Kenya	Mathematics	Kenya	There is a lack of capacity building since teachers were found facing the challenges of developing their technical skills and knowledge. Teachers are themselves striving in self-training	Not ready
3	Denhere and Moloi (2021)	Readiness of public TVET for the Fourth Industrial Revolution: The case of South Africa	All subjects	South Africa	Teachers were not completely ready for the 4IR due to absence or inadequate teaching and learning technologies, lack of training for the staff about the use of technologies, lack of ICT equipment, lack of ICT strategies, in addition to lack of policy directives	Not ready
4	Fomunyam (2020)	Deterritorializing to reterritorializing the curriculum discourse in African higher education in the era of the Fourth Industrial Revolution	All subjects	South Africa	The current curriculum in Africa is obsolete and does not capture the changes ushered in by the Fourth Industrial Revolution	Not ready
5	Iruonagbe and Egharevba (2015)	Higher education in Nigeria and the emergence of private universities	All subjects	Nigeria	The curriculum used in Education of Africa is not allowing the African citizen a better life; there are no required skills to apt to computerised organisational sceneries	Not ready

(continued)

Appendix (continued)

SN	Author(s) and date	Article title	Subject(s)	Country	Findings	Observation
6	Kayembe and Nel (2019)	Challenges and opportunities for education in the Fourth Industrial Revolution	All subjects	South Africa	There are insufficient funds, infrastructure, and the required skills for teachers to train graduate students to fit into the 4IR	Not ready
7	Markowitz (2019)	Harnessing the 4IR in SADC: roles for policymakers	All subjects	South Africa, Malawi, and Lesotho	There are gaps for the region in terms of technology-related policies that should be established to support access to the internet in rural areas, education, skills, policy reform	Not ready
8	Mpungose (2020)	Student teachers' knowledge in the era of the Fourth Industrial Revolution	All subjects	South Africa	Student teachers have significant content, pedagogical, and technological knowledge (CK, PK, and TK) and inadequate training within institutions that leave students unprepared for the 4IR era	Not ready
9	Mtshali and Ramaligela (2020)	Contemporary employability skills needed for learners to succeed in the civil technology field in the 4IR era	Civil technology	South Africa	Teachers are not aware of the value that civil technology may bring to companies, entrepreneurs, and the economy in general	Not ready

(continued)

Appendix (continued)

SN	Author(s) and date	Article title	Subject(s)	Country	Findings	Observation
10	Mudaly (2018)	Decolonising the mind: Mathematics teachers explore possibilities for indigenising the school curriculum	Mathematics	South Africa	It is difficult for teachers to depart from the comfort of the factual mathematics they taught to include indigenous links from contexts around them	Not ready
11	Sikhakhane et al. (2021)	South African teachers' perspectives on using the computer as a tool for teaching and learning	All subjects	South Africa	Teachers used computers superficially, irregularly, thus insignificantly due to scarcity of computers, lack of incentives and training about the use of computers	Not ready
12	Mutambara and Bagaya (2020)	Predicting rural STEM teachers' acceptance of mobile learning in the fourth industrial revolution	STEM subjects	South Africa	The acceptance for use among rural high school STEM teachers in South Africa is still at a low rate	Not ready
13	Mwapwele et al. (2019)	Teachers' ICT adoption in South African rural schools: A study of technology readiness and implications for the South Africa connect broadband policy		South Africa	School leaders set policies preventing students' digital device use except for calculators	Not ready
14	Naidoo (2020a)	Exploring the flipped learning approach within a mathematics higher education milieu in the era of the Fourth Industrial Revolution	Mathematics	South Africa	Lecturers used flipped learning while teaching mathematics. It was observed that this technology-based pedagogy has the potential to help students interact between them and their lecturers	Ready

(continued)

Appendix (continued)

SN	Author(s) and date	Article title	Subject(s)	Country	Findings	Observation
15	Naidoo (2020b)	Postgraduate mathematics education students' experiences of using digital platforms for learning within the COVID-19 pandemic era	Mathematics	South Africa	Lecturers face difficulties using digital platforms and are not provided with enough time and space to practice online mathematics examples before interacting within the online community	Not ready
16	Naidoo and Singh-Pillay (2020)	Exploring mathematics teachers' professional development: Embracing the Fourth Industrial Revolution	Mathematics	South Africa	The majority of the participants employ technology-based tools during teaching and learning mathematics	Ready
17	Skhephe et al. (2020)	Accounting teachers' readiness for e-learning in the Fourth Industrial Revolution: A case of selected high schools in the Eastern Cape, South Africa	All subjects	South Africa	The classrooms were not designed to support e-learning, and teachers are not aware of the importance associated with e-learning classrooms	Not ready
18	Tachie (2019)	Challenges and opportunities regarding the usage of computers in the teaching and learning of mathematics	Mathematics	South Africa	Lack of computer tuition opportunities, attitudes shown by school managers, and limited resources	Not ready
19	Uleanya and Ke (2019)	Review of preparedness of rural African communities nexus formal education in the Fourth Industrial Revolution	All subjects	South Africa	The curriculum is not implemented in a way that ensures the desired development	Not ready

(continued)

SN	Author(s) and date	Article title	Subject(s)	Country	Findings	Observation
20	Umoh and Akpan (2014)	Challenges of blended E-learning tools in mathematics: students' perspectives University of Uyo	Mathematics	Nigeria	ICT tools are not available, even when available are not accessible	Not ready

Note: The readiness and non-readiness categories were confirmed simply because the authors showed one or more factors as challenges that hinder the effectiveness of embracing the 4IR era

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Chapter 2
Can the Fourth Industrial Revolution Resolve Why the Teaching of Mathematics in the Current Paradigm Continues to Be Decontextualised and Ineffective

Kakoma Luneta

Introduction

This chapter explores why mathematics is still underperformed, how place-based mathematics can support effective instruction and the challenges of 4IR in addressing mathematics curriculum changes in communities. In this chapter, place-based mathematics and its relevance to the curriculum is the mathematics that takes cognisance of the community and the relationship that the community has to the land and its people. The mathematics that addresses place-based curricula interrogates teachers' and learners' understanding of the community's role in mathematising the daily discourses. Connecting mathematics, community, place, and culture are complex and challenging undertakings that should be addressed (Nicol et al., 2021).

The Fourth Industrial Revolution (4IR), with its primary characteristics of the Internet of Things, 3-D printing, virtual reality, Artificial intelligence, robotics, and cloud computing, will be pivotal to the next mathematics classrooms (Engelbrecht et al., 2020). The challenges are that most mathematics classrooms in Sub Sahara Africa barely operate on the Third Industrial Revolution (3IR) – Communication, technology, and information revolution.

This chapter poses questions that address the difficulties of mathematics curriculum change in contexts that have not engaged and challenged the status quo of the prevailing curricula.

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Historical Implications of the Mathematics Curriculum in Southern Africa

At the dawn of the twenty-first century, most mathematics curriculum review, innovation or total reform has been guided by the need to alter teaching and learning to 'focus more on the conceptual meaning underpinning the mathematics that they teach students rather than relying solely on practising procedures and skills in the classroom' (Johnson et al., 2019, p. 1). Fundamentally, the national mathematics curriculum is content that ought to be covered and assessed, the envisaged instructional approaches and expected skills of the implementers, as well as the materials to be used in disseminating the curriculum in the classroom (Johnson et al., 2019; Anderson, 2009; Bhatt & Koedel, 2012, 2013; Koedel et al., 2017; Luneta, 2013). Effective curriculum changes, evaluations or modifications have always taken cognisance of the role teaching materials (test books and manipulatives) play in learning outcomes (Bhatt & Koedel, 2012; Bhatt et al., 2013). The mathematics curriculum is one of the most identical curricula in the world in the sense that every country covers the strands of number, algebra, geometry, measurements, space and shape and data handling or probability and statistics in one form or another. The main difference is largely in the number system used (Hindu-Arabic, Chinese, Roman, Egyptian, Amharic) and the content coverage at a given level (Agodini & Harris, 2010).

From the onset, it is also critical to acknowledge that every curriculum is politically informed. In Southern Africa, the mathematics curriculum of all previously colonised countries was informed and modelled on the curriculum of the former coloniser. Nicol et al. (2013) identified it as the 'settlers' curricula'. Southern African countries such as Botswana, Lesotho, Malawi, Zambia and Zimbabwe were modelled on the British mathematics curriculum, with examinations being set and marked at the University of Cambridge under the then Local Examination Syndicate (Cambridge Assessment, 2005). The countries only localised their mathematics examinations much later. In South Africa, not only was the curriculum determined by the then Nationalist government but also manipulated and watered down to undermine further and cognitively oppress the African learners. The Minister of the then Native Affair, Dr. HF Verwoerd, openly said in a classic often cited quote:

> People who believe in equality are not desirable teachers of Natives. What is the use of teaching the Bantu mathematics when he cannot use it in practice? (House of Assembly Debate, Vol. 78 August–September 1953, p. 3585).

This openly uttered statement was in another part of the world covertly adhered to in unison with the great master-chord of curriculum reform, for instance, in the Caribbean islands, among the First Nations in Canada and the Aborigines in Australia (Nicol et al., 2013). In the Caribbean, Ramsook (2017) posts that the changes to the curricula after the independence of most of the British colonies were in line with the preexisting curricula. The changes were merely made in names to appease the masses as independent states and not for conceptual and intellectual enhancement. In Nigeria, for instance, soon after independence in 1960, the introduction of Basic Science in 1962 and Social Studies in 1963 was 'to build a strong and dynamic nation' Ogunyemi (2010, p. 6), so the people were informed by the state.

In recent discourse, Parker (2006, p. 59) admitted that 'Radical curriculum change in South African schooling over the past decade has been driven by a need for social, economic and political transformation'. The unfortunate outcome of most reforms in Southern Africa has been the lack of critical questions of the settler-colonial impositions, taking assumptions for granted and not in any form disrupting or challenging the existing mathematics curriculum more consciously. The learner-centredness and autonomy have been at the centre of the curriculum changes around the world and more so in Sub-Saharan Africa and South Africa in particular (Johnson et al., 2019; Chisholm, 2007; Doorman et al., 2012; Reiss & Torner, 2007). While curriculum change is almost inevitable for all countries and has a political bearing, most of the curriculum reforms in mathematics are due to the difficulties students encounter and the assessment targets, as well as the failure rate associated with the targets not being met (Johnson et al., 2019; Gurlen, 2015; Koedel et al., 2017). The low performance becomes a public agenda and usually causes a revolt in the governing parties as the opposition political parties to take advantage of the situation. For instance, in the US, Stanic and Kilpatrick (2004, p. 12) wrote 'It was in this context that attempts to reform the mathematics curriculum took on more urgency, especially since mathematics had become one of the greatest sources of failure in school' and this was in 1904. In 2010, Ireland's 5-year mathematics curriculum reform named 'Project Math' was out of the 'unfortunate consequence that the state examinations have often come to define the goals of the curriculum' (Johnson et al., 2019, p. 1). The various Trends in International Mathematics and Science Study (Reddy 2005) reports in South Africa have all pointed to low performances in mathematics by South African primary learners, prompting ministers to request mathematics curriculum reforms.

Decolonising the Mathematics Curriculum Through the Place-Based Mathematics

By definition (Nicol & Luneta, 2018, p. 1), place-based mathematics education is an approach to critical mathematics education that engages students, teachers, and communities (their backgrounds and foregrounds) around interests of importance to students and their communities. Nicol et al. (2013) explain that the rationale for place-based mathematics education is that learners should be able to see the relevance and usefulness of mathematics by relating it to their community, culture, and context. Place-based mathematics addresses and attempts to bring classroom mathematics and educational experiences into the real world (Southwest Educational Development Laboratory, 1996). It is also assumed that by using place-based materials and teaching resources, teachers can connect instruction to context and local issues. In their study Nicol and Luneta (2018) show how each learning area of mathematics can be taught using teachers' context and the materials around them:

Number, Operations and Relationships

Counting using pebbles and using local language for terms such as addition, subtraction, division, and multiplication is fundamental to making mathematics a local entity. Figure 2.1, a picture of Mrs. Mhlanga, an artist and weaver in South Africa, shows basic principles of number operations, the addition of a particular number of stitches and straws to form patterns, number of rows to complete a basket, the diameters of the circles and use of knowledge of properties of both two-dimensional and three-dimensional shapes for the patterns to be symmetrical and the structure to be complete. Further scrutiny of Fig. 2.1 reveals that the two handles of one of the baskets are parabolic, while the handle of the cuboid-shaped basket is a semicircle and the hole that provides the passage for the handle is the diameter of the circle. While Mrs. Mhlanga used the terms parabolic and diameter in our conversation, her work spoke of the knowledge of properties of shapes and the criticality of abiding by those affordances to represent the shapes correctly.

Fig. 2.1 Mrs Mhlanga showing her knowledge of shapes and measurements in the making of baskets and mats

Patterns, Functions and Algebra

Patterns from the way natural and whole numbers are conceptualised can make melodic patterns of music and dance. The South African township gumboots dance is one example that follows a pattern of two footsteps and claps 1, 2, clap, 1, 2, clap, 1, 2, clap, and this pattern can be increased to form other series. Songs at primary school are made of patterns for children to follow sound and rhythm, making a sequence. Figure 2.2a, a dodecagon (12-sided shape), shows the mathematics in the thatched roof shown that can be used to illustrate basic concepts of various polygons (triangles, trapeziums) concept of parallel lines and a regular dodecagonal pyramid. At an advanced level, Fig. 2.2b shows using calculus an answer to the volume of the dodecagonal cone. In the 4IR, using a 3D printer in a classroom, a teacher can virtually illustrate the components of the dodecagonal prism and its properties.

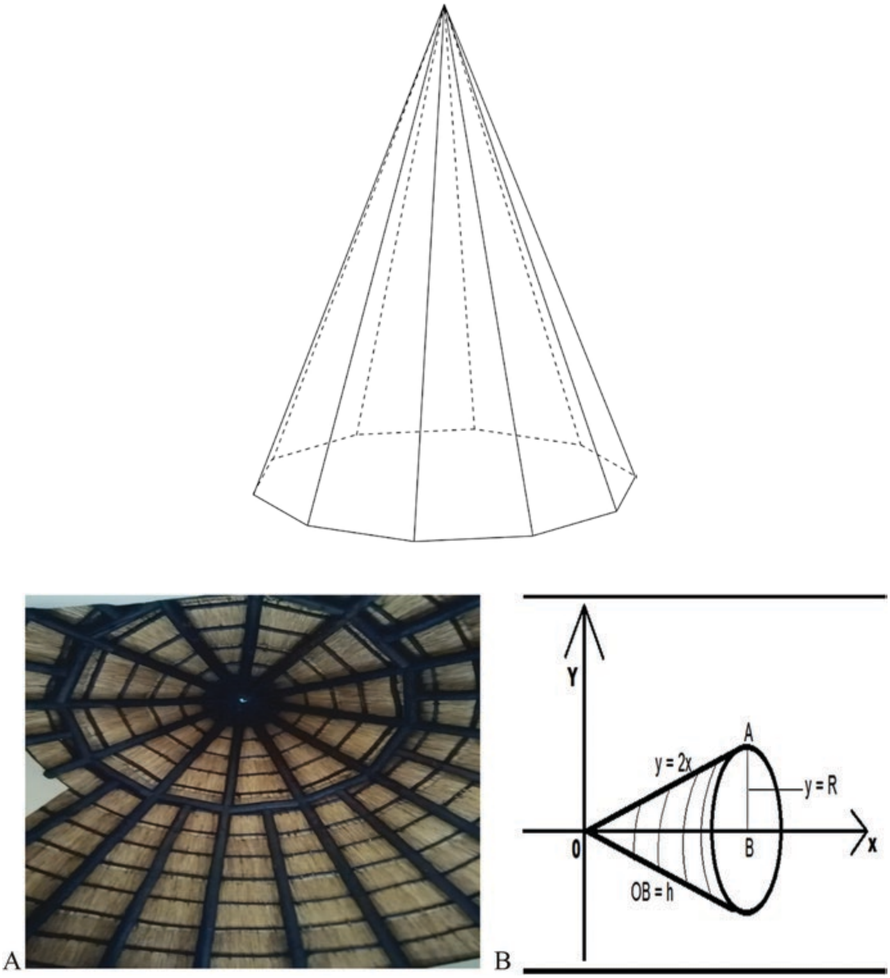
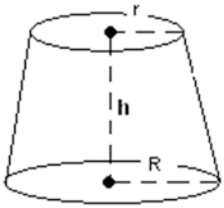


Fig. 2.2 (**a**) Thatched house. (**b**) Higher-order problems from the thatched house



$$\text{Area} = \pi(R + r)\sqrt{(R - r)^2 + h^2}$$

$$\text{Volume} = \frac{\pi}{3}h(R^2 + r^2 + R * r)$$

Use simple line $y = 2x$

1. Rotating line generates surface

$$A = \int_0^h 2\pi y dx = \int_0^h 2\pi 2x dx$$
$$= \frac{4\pi x^2}{2}$$
$$= 2\pi x^2 \int_0^h 1\pi h^2$$

For frustum from C to B

2. Rotating shaded part

$$\text{Volume} = \pi R^2 dx$$
$$= \pi Y^2 dx$$
$$= \pi (2x)^2 dx$$
$$4\pi x^2 dx$$
$$\int 4\pi x^2 dx \frac{3}{4}\pi x^3 \ \ l_0 h$$

For parabolic then $y = ax + bx^2$

$$\text{The volume of a cone is} = \frac{1}{3}(\text{Area of the base}) \times (\text{The perpendicular height})$$

BUT for our dodecagonal prism

Area of the base=The base of a decagonal prism base is the area of 12 isosceles triangles.

$$\text{Volume of a dodecagonal cone} = \frac{1}{3}(\text{Area of 12 is osceles triangles})h$$

This is the mathematics that can be derived from communities if the teachers are knowledgeable in place-based mathematics and how to bring their communities and land into the mathematics classroom (Nicol et al., 2021).

Space and Shape

Many examples in African rural and urban communities show examples of knowledge of mathematics and its applications. Examples of shapes such as circle – Africa drum used as an example of the cylinder and the top as a cycle; Quadrilaterals – squares, rectangles, parallelograms, trapezium, kite and rhombus are abounded in local markets and village structures. Figure 2.3 shows triangles – equilateral, isosceles, lines of symmetry – local structures showing lines of symmetry.

The artefacts, Fig. 2.4, further provide examples of how mathematics is woven in African communities. To recognise and put such artefacts to mathematical use in the classroom, teachers must be well established in content and pedagogical content knowledge of the mathematics they teach, in this instance, cylindrical and circular shapes.

Measurement

Figure 2.5 show how her knowledge of mathematics (measurement and shapes) provides examples of shapes and patterns that are only possible if one knows the properties of shapes and the right measurements, both linear and angular.

Figure 2.6 shows beds made by local carpenters at a market in Tanzania. The picture shows the use of various skills of measurements and how the different shapes (Rectangles, Rhombuses) are represented. The carpenters use crude but effective ways to convert units from meters to centimetres. Some carpenters use imperial units (yards, feet, inches) and convert them into metric systems (meters, centimetres, and millimetres). The carpenter displayed knowledge of the properties of two-dimensional shape; for a rhombus, for instance, the carpenter displays that – all the sides are equal, the opposite angles are equal, and diagonals bisect each other at right angles.

Data Handling

There are many examples that are community-based that teachers could use to illustrate and teach data handling, and these include a reading of water and electric meters, data from ages of children in the community, their shoe sizes, animals kept, eggs laid by chickens or car owned by people in the community. From Fig. 2.6,

Fig. 2.3 Various shapes and their properties illustrated in a thatched roof

Fig. 2.4 Cylindrical clay pots that teachers can use to teach various shapes

teachers can formulate questions around the value of beds, the number of trees that have been cut, the different implements used to make the beds, different colours that the beds can be painted and how many learners chose the various colours of the beds. Such knowledge is what Boaler et al. (2021) call data science. However, such questions and the ability to represent the ordinary market artefacts into living

Fig. 2.5 A display of baskets and mats made with mathematical precision measurements

mathematics illustrations require teachers that are comfortable in mathematics and have the skills of problem formulation and solving. Figure 2.6 can be used in other subjects such as commerce; one can teach about the value of beds, the cost of making a bed, and the demand and supply of beds and link that to the population and employment. Those teaching geography can discuss matters of deforestation, soil erosion and forestry. The point is that only a mathematically versatile teacher can visualise, appreciate, and use the mathematics displayed in Figs. 2.1, 2.2, 2.3, 2.5, and 2.6 effectively to the satisfaction and benefit of the learners.

The Influence of Problem-Solving Skills and Competences in Decolonising the Mathematics Curriculum

The mathematics curriculum can be decolonised by recognising the critical role of common cultural knowledge of mathematics matters. Problem–solving is highly embedded in African culture (Gerdes & Djebbar, 2007). In their seminal bibliographical representation of African mathematics and cultural scholars, Gerdes and Djebbar (2007) acknowledge from the onset that mathematics in Africa influenced cultures and how they lived and solved problems. Mathematics was used as a cognitive tool to engage with and solve social and communal problems. Pólya's 1945 problem-solving construct's influence on the mathematics curriculum has been fundamental in most mathematics curriculum reforms. The South African Department of Education (2001, p. 18) regards problem-solving as one of the *unique features of*

Fig. 2.6 Beds made at a market in Tanzania by a local carpenter

learning and teaching mathematics. The Singapore mathematics curriculum is solely based on problem-solving as a driving force behind mathematics instruction and knowledge acquisition (Ministry of Education, Singapore, 2012). The USA, Dutch and the British all had some problem-solving component engraved in the mathematics curriculum or as part of the mathematics curriculum in some form or another (Doorman et al., 2007; Romberg, 2001). Gurlen (2015) acknowledge that a problem-solving informed curriculum has become part of the knowledge society. In the US, all recent initiatives that involved mathematics curriculum reform have been informed by Schoenfeld (2014a, b) and Schoenfeld and Floden's (2014, p. 2) framework of "Teaching for Robust Understanding in Mathematics (TRU Math) analytic scheme".

In this framework, dabbed mathematics for a powerful and productive classroom Schoenfeld and Folden (2014, p. 2) proclaim five dimensions of an effective lesson that capture an essential component of "productive mathematics classrooms – classrooms that produce powerful mathematical thinkers". The five dimensions of mathematics curricula that produce powerful mathematics thinkers are: *Cognitive demand which* refers to classroom instructions and the environment that are intellectually demanding and conducive to students' mathematical development; *Access to Mathematical Content*, which is regarded as instructions that support active mathematical engagement by learners; and *Agency, Ownership and Identity* which refers to the engagement of most learners in mathematical opportunities and arguments, ability to conjecture, explore mathematics, and finally *use of assessment*, the extent to which teachers solicit students' thinking and address misconceptions, and *equitable access to content*, where the teachers ensure that all learners have equal opportunity to access and use mathematical content (Schoenfeld, 2014a, b). Gerdes and Djebbar (2007) declare that the mathematics reforms addressed in most of the African studies advocate for the constructivist approach to mathematics teaching geared toward learner-centredness and problem-solving. Hiebert et al. (1996, p. 14) further asserted that allowing mathematics to be problematic would result in classroom activities that encourage learner participation. During such activity, the teacher is responsible for developing a social and intellectual community of learners that problematises mathematics and shares in searching for solutions through examination and discussion of the methods used to achieve solutions. Teachers' content and instructional knowledge of mathematics, as well as knowledge of learners' thinking, in order to select appropriate tasks linked to learners' experience, ideas and skills are of paramount importance. The lack of problem-solving skills by teachers of mathematics in South Africa has been at the centre of ineffective instruction resulting in low learning outcomes. Hinging teachers and mathematics curriculum reforms around problem solving has also been pivotal in other countries. Anderson (2009, p. 1) recounts from the National Curriculum Board (NCB) of Australia that the 'fundamental aim of the mathematics curriculum is to educate students to be active, thinking citizens, interpreting the world mathematically, and using mathematics to help form their predictions and decisions about personal and financial priorities' (NCB, 2009, p. 5). The Australian Mathematics Curriculum, revised in 2011, was written on the strength of covering the generally five mathematics curriculum strands into three content strands, namely: Number and algebra, measurement and geometry, and Statistics and probability. Kilpatrick et al. (2001) mathematics teaching report played a part in the Australian curriculum review. As a result, the four proficiency strands – understanding, fluency, *problem solving* and reasoning recommended in their report were adopted by the NCB (Anderson, 2009).

Difficulties of Learning Mathematics in the Current Curriculum

The mathematics curriculum and its implementation in the classroom determined the learning outcome (Charalambous & Phillippou, 2010). The learning of mathematics is predominantly influenced by the students' experiences of the mathematics exposed to them. Students learning outcomes are defined by the skills, knowledge, and values they acquire as they interact and expose themselves to the experiences with mathematical constructs (Boser et al., 2015). The exposure and experience are a result of interactions with the teachers, the connections with the materials provided, such as textbooks and other learning guides, as well as educational contacts with their peers (Bhatt et al., 2013; Charalambous & Phillippou, 2010). The learning outcomes are influenced by these various experiences, which are predominated by teachers' mathematical instructions. At the centre learning outcome is the implementation of the curriculum by the teachers. The teacher implements the curriculum by using various strategies. According to Sherin and Drake (2009), curriculum implementation is how the teacher reads, evaluates and disseminates the curriculum to the learners by using all the available resources. The difficulty has been that most mathematics teachers are in dire need of *continuous* professional development programmes that address effective instructional approaches that lead to high learning outcomes (Luneta, 2012). The professional development programmes provided by the Department of Education are scanty, truncated and hardly address teachers' content or pedagogical content knowledge needs. The reasons for ineffective intervention programs are that no needs analysis is carried out to identify specific teachers' needs and zoom in on the professional development programmes in these areas of content and instructional need. The professional development programmes provided by the department of education for mathematics teachers are not based on research-informed teachers' conceptual and instructional needs.

Most curricula (Schoenfeld, 2014a; National Curriculum Statement, 2010) will prescribe the teachers the strategies to implement the curriculum. The curriculum implementation strategies influence the teacher's choice of materials, including textbooks and learning guides. The implementation of the curriculum hinges on the instructional approaches that the teacher adopts. The teacher's instructional approaches, sometimes referred to as instructional strategies or teaching strategies, are influenced by the teachers' content and pedagogical content knowledge that professional development initiatives for teachers ought to hinge on (Shulman, 1987; Luneta, 2013).

The impact of COVID 19 on teaching, teachers and learners cannot be underestimated, considering the sudden change it brought to modes of both learning and teaching. Teachers and learners had to adapt to different ways of instruction and learning. The use of technologies such as computers, androids, and cell phones (Internet of Things (IoT)) has altered learning and the role of teachers and schools (Cox, 2021). While the pandemic has ignited the Internet of Things (IoT), it has not accelerated the development of the digital age in Sub Sahara Africa (Engelbrecht et al., 2020). The pandemic has exposed the digital divide between developed and developing countries, between the rich and poor rural communities worldwide but more so in Sub Sahara Africa due to poor technological infrastructure. Advocacy has been for the use of blended learning where teachers use a variety of instructional approaches, including audio recording, zoom, online group discussion forums, and face-to-face sessions (Hodges et al., 2020).

Difficulties of Teaching Mathematics in the Current Curriculum

Stein et al. (2008) offer a pedagogical model that illustrates five key principles or practices designed for whole-class discussions after learners' have worked on cognitively challenging tasks. The first principle is 'Anticipating Students' Mathematical Responses', where teachers envisage how learners will approach a problem task and anticipate their responses. The second principle is 'Monitoring Learner Responses', where the teacher identifies the strategies or representations used by the learners. In the third principle, the teachers 'Purposefully Selecting Student Responses' and uses learners' particular aspects of mathematics or methodologies to reinforce whole class learning. The fourth principle is 'Purposefully Sequencing Student Responses', where the teacher maximises learners achieving their mathematical goals by making purposeful choices about the order in which their work is presented. The fifth and final principle is 'Connecting Student Responses', where the teacher draws connections between learners' different responses and expands on underlying mathematical ideas. In what he describes as "six key principles for effective teaching of mathematics", Sullivan (2011, p. 26) identifies the principle of "making connections" as the key.

The role of the teacher in empowering students to develop their problem-solving skills cannot be disputed. Nevertheless, Verschaffel et al. (2000) assert that teachers only address learners' problem-solving skills superficially by not providing them with problems that engage them at a high cognitive level but rather those that address the simple strategies of recall and memorisation. Regarding mathematics teachers in Australia, Sullivan (2011) alludes to their limited range of problem-solving strategies and the fact that most of the teaching requires the application of routines that lacks complexity and rarely involves reasoning. In the UK, Swan (2005) argues that most mathematics teaching consists of low-level tasks that could be completed by the mechanical production of procedures without deep thinking. The complexity of the problem posed dictates whether the students engage with the problem at the superficial level of simple recall or a deeper cognitive level of analysis, synthesis, and evaluation. Reusser and Stebler (1997), Schoenfeld (1991), and Verschaffel et al. (2000) conclude that students' inability to solve higher-order problems is due to the types of mathematics teaching they receive. According to Murray et al. (1998, p. 171), problem solving is regarded as the "*vehicle* for learning". They contend that it is essential to distinguish sharply between learning *to* solve problems and learning *through* solving problems. This is the essence of teaching problem solving as a *process* and as a *skill*. The teaching process through problem solving implies that the teachers provide the learners with tasks to work through before they are provided with the computational or algorithmic applications to the task. The learners attempt to solve the problem using their approaches and strategies. The problem is the starting point, and as a result of working on these problems, learners would be left with a "residue of mathematics" (Murray et al., 1998, p. 171). Cai (2003, p. 12) asserts that in teaching through problem solving, the focus is on "conceptual understanding, rather than on procedural knowledge". What the learners acquire in this approach is the *skill* of problem solving.

Hiebert et al. (1996, p. 14) assigned this approach to the principle of "problematising mathematics". They argue that learners' knowledge acquired in the classroom does not transfer well to the world of work because problem-solving conceptions are characterised by "a distinction between acquiring knowledge and applying it". The distinction suggests that computation procedures should be acquired first and then applied to solve problems. By making the distinction, most South African mathematics teachers have separated mathematical activity into two artificial categories and created equally artificial methods to bring them back together (Naroth, 2016).

Addressing Errors and Misconceptions Through Curriculum Review

Most mathematics curriculum interventions result from low mathematics achievements due to learners making errors or misconceptions (Cortes et al., 2015; Domina et al., 2015; Jackson & Makarin, 2016). The mathematics curriculum reforms have focused on teachers grounded in mathematics content knowledge and pedagogical and pedagogical content knowledge (Johnson et al., 2019). Luneta (2015) asserts that effective mathematics curriculum reform hinges on the materials and content to be taught and learnt on teachers' knowledge bases, including their content knowledge, pedagogical content knowledge (PCK), conceptual knowledge and procedural knowledge. Shulman (1986) regards content knowledge as the main knowledge base a teacher must possess to be effective. Schneider and Stern (2010, p. 178) describe conceptual knowledge as "providing an abstract understanding of the principles and relations between pieces of knowledge in certain domains". Zakaria and Zaini (2009) describe procedural knowledge as a learner's ability to provide mathematical answers without explaining why certain steps, methods, operations or formulae were used. PCK includes knowledge of how learners learn specific content, the misconceptions and errors associated with certain concepts, and the assessment tasks, remedial activities and enrichment tasks needed to challenge learners. Hill et al. (2008) expanded PCK to include SCK, which they related to mathematics knowledge for teaching (MKT), the essential knowledge that good mathematics teachers ought to possess to be effective. As "the experiential knowledge and skills acquired through classroom experience" (Lee & Luft, 2008, p. 18).

Ding and Jones (2006) posted that the mathematics curriculum initiatives required teachers to develop instructional strategies and knowledge of useful teaching materials and activities. In addition to the content and instructional knowledge, the curriculum also emphasised teachers' conceptual and procedural knowledge. Mathematics curricula reforms have further picked up on research that has demonstrated teachers' knowledge to include procedural fluency, which according to (Star, 1999) and McCormick (1997), is the 'know-how-to-do-it knowledge' of mathematical knowledge. Procedural fluency involves the capacity to recollect mathematical facts swiftly and precisely implement procedures, and Danley (2002) asserts that procedural knowledge includes engaging in practice with procedures and the operation of numbers. Hill et al. (2008, p. 78) have defined content knowledge as knowledge of mathematics, and they split it into 'common content knowledge' (CCK), shared mathematical knowledge that anyone who professes to know mathematics would possess, and 'specialised content knowledge' (SCK), which is mathematics knowledge specific to the teaching of the subject. It is being revealed in current mathematics curricula reviews that effective teachers, through instruction, dissemination of this knowledge not in a compartmentalised but rather in an intricate combination of the conceptual and procedural knowledge (Johnson et al., 2019). Table 2.1 shows the three pillars of standard mathematics curriculum – Envisaged Teachers' Instruction - Curriculum Content, and the Assessment of the curriculum.

Table 2.1 Envisaged mathematics curriculum for effective instruction

Three Pillars of Mathematics Curriculum	Maths Curriculum input by Teacher	Maths Curriculum output by Learners
Teachers Instructions	CK - Content Knowledge PCK - Pedagogical Content Knowledge PK - Pedagogical Knowledge CCK - Common Content Knowledge SCK - Specialised Content Knowledge MKT - Mathematical Knowledge for Teaching	Acquire conceptual + preceedural knowledge content + change in behaviour + skills
Curriculum Content	- Number + Number relationships - Functions + Algebra - Space + Shape - Measurement - Data Handling	Mathematical content acquired + change in behaviour, skills + value + use of maths
Assessment	Summative and Formative assessment	Learners provide answers as a result of acquired knowledge + skills

Teaching and Learning in Mathematics Curriculum

The final goal of a good curriculum points to the achievements of the learning outcomes (Luneta, 2018).

Luneta (2015) accentuated that effective mathematics teachers reflect on their connected mathematical knowledge bases and fluidly combine them with their expertise, experience and understanding of mathematics when teaching. The problem with mathematics curriculum interventions in Southern Africa has been the lack of contextualised instruction that uses illustrations such as below when teaching parallel lines, triangles or any other measurement topic (Luneta, 2018). The problem is a by-product of the teacher conveyer belt, the institutions of higher learning, the universities and teacher training colleges. Institutions of higher learning have failed to attract the best mathematics students that schools produce into the teaching profession. The average mathematics mark in grade 12 examinations of the students that have taken up teaching mathematics as a career is 52%, whereas for those who have taken up engineering is 68%. This was across all 21 universities in South Africa. The grades are even lower for that trainee at the Foundation phases, where learners' effective introduction to mathematics is critical (Luneta, 2015). As a result of teacher trainees being weak in the mathematics content knowledge, their ability to change to be instructionally effective is very hard to achieve.

What Is the Fourth Industrial Revolution Mathematics Curriculum?

The 1990s to 2000 saw an influx of mathematics curriculum reform initiatives influenced by the new world order hinged on the technology and digital boom (Wang et al., 2017). The new knowledge, innovative technology, socialisation, and global citizenry injected a new perspective into mathematics and its societal influence. In most developed countries, especially China, social and economic development, particularly the development of information and digital technology, has raised the need for mathematics literacy and new demands for modern citizens (Wang et al., 2017). The digital and information age development has been relatively slow in Southern Africa. It is correct to point out that many parts of Sub Sahara African rural communities are still grappling with the Third Industrial Revolution (3IR), the digital revolution, which moved from analogue to digital and mechanical to electronic. Many rural areas in Sub Sahara Africa have not been digitally touched and are still using analogue to communicate and physical means of getting and purifying water.

The common academic definition of the 4th Industrial Revolution (4IR) is the world in which computer-informed communities are guided by overwhelmingly fast occurring disruptive innovative technologies. These include robotics (machines or robots that work as humans but a lot faster with fewer mistakes), artificial intelligence (AI) (The intelligence exhibited by machines to mimic human behaviour), internet of things (IoT) (the everyday electronic appliances, phones, camera, computers, cars and homes) that are all connected or can be connected to the internets will become vital to the mathematics teachers. Virtual reality (VR) is the bringing together of the digital, the physical and biological systems. There is no subject that the new industrial revolution (4IR) will call upon more than mathematics (Engelbrecht et al., 2020). These technologies are assumed to shape communities' future and schooling systems (Cox, 2021). Virtual reality, the reality of anything created by the user using the computer, will be critical to teachers of mathematics but more so to conceptually-grounded teachers that will be able to imagine mathematical situations and bring them to life in the classroom virtually. The critical role of place-based mathematics would be enhanced by virtual reality as teachers could bring virtually into class the mathematical existence of their communities using artificial intelligence (AI) and robotics (Cox, 2021).

Boaler et al.' (2021) work on data science further points to the essence of place-based mathematics. They argue that all learners from the foundation phase to further education and training need to learn 'the mathematics that will help them develop data literacy to make sense of the data-filled world in which we all live' (p. 508). The data revolution, the rate at which the world produces and records data, is insurmountable. Data awareness and literacy and allowing learners to understand and visualise data will open new avenues for them in the growing field of data scientists and make them less vulnerable to people who are misrepresenting issues and data. Sub-Sahara Africa curriculum developers should pay attention to Data Science (www.youcubed.org) and its affordances for 4IR.

Conclusion

The mathematics curriculum reform addresses decolonisation, and the 4IR is centred around teachers embracing their communities and being conceptually and instructionally informed as mathematics teachers. The chapter has addressed the importance of problem-solving skills and ability in mathematics teachers and learners' curriculum reforms. The final goal of a good mathematics curriculum is to address and achieve high learning outcomes through effective continuous professional development programmes informed by research-based teachers' conceptual and instructional needs. The 4IR will be difficult in Sub-Sahara Africa, where most schools are barely rising out of the Third Industrial Revolution.

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Chapter 3
Alignment and Gaps of the Competency-Based Mathematics Curriculum and the Fourth Industrial Revolution

Mary A. Ochieng and Penina Kamina

Introduction

According to Schwab (2016), the fourth industrial revolution (4IR) is building on the third digital revolution and "is characterized by a fusion of technologies that is blurring the lines between the physical, digital, and biological spheres" (p. 1) and is evolving exponentially. Jones and Ng (2021) argue that the 4IR is hype created by economists and marketers. However, the change being experienced as a result of the 4IR cannot be denied (Sutton, 2020). It is changing production processes in industry, business, healthcare and many other facets of our lives. Some potential negative impacts, such as job losses, are already being felt. At the same time, it is creating inequalities as most job losses will affect lower levels of skilled labour, which are common in developing countries. The nature of the 4IR jobs requires skills often lacking in developing countries such as those in sub-Saharan Africa. Maxim Shashenkov, a renowned businessman, in an interview published by Sarpong and Ofori (2020), characterized the calibre of employees required in equity markets to address the changing terrain in business brought about by the 4IR as people with a combination of skills which include interdisciplinary knowledge and skills; innovative critical thinkers who are well-informed about international relations.

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The successful future of 4IR calls for a people that will cope with the changing nature of technological advances, automation, artificial intelligence, robotics, and other 4IR requirements. Therefore it demands the field of education to shift in what it offers. Accomplishing the 4IR-related tasks, activities, or projects will require varied expertise and interdisciplinary work. Therefore, there is a need to prepare citizens who are not only knowledgeable in their areas of specialization but have a plethora of varied soft skills. The World Economic Forum report (2015) points out 16 educational skills that support the 4IR and classifies them into three categories:

- Six foundational skills in six broad content areas—literacy, numeracy, scientific literacy, Information Communication Technology literacy, financial literacy and cultural and civic literacy
- Four competencies—creativity, communication, critical thinking/problem solving, and collaboration
- Six personal character qualities—curiosity, initiative, grit, adaptability, leadership, social and cultural awareness

Thus, the present generation of school-age children needs explicit preparation in these competencies and character qualities—collaboration, communication, creativity, critical thinking, problem-solving, flexibility, resilience, initiative and adaptability—skill sets that have been subtle and implicit in past mathematics curricula. The question we seek to answer in this chapter is: *How is the implementation of CBC in Mathematics classrooms likely to contribute to the 4IR in Africa?*

The Fourth Industrial Revolution in Africa

According to Ndoye (2010), Africa is simultaneously going through three ages of revolution: pre-industrial, industrial, and post-industrial. As Schwab (2016) stated, the 4IR is building on the third digital revolution, whereas some parts of Africa—with the exception of technologies such as the mobile phones, which have penetrated even remote regions—are yet to experience the digital revolution fully. This means that different countries, and in some cases different regions within the same country, are at different stages of industrial development.

As Africa is simultaneously dealing with the different stages of development, it also needs to catch up with the rest of the world. Doing so sustainably requires efforts beyond fostering innovation to address social and ethical dimensions drawing on indigenous foundations for development (Ndoye & Walther, 2012). The 4IR has the potential to accelerate development or cause developing countries to lag if they do not leverage the opportunities it presents (Asghar et al., 2020). Due to the low labour cost in developing countries, most of them depend on offshoring and outsourcing jobs from developed countries. With the 4IR, this may change as developed countries may choose to bring home their industries as the 4IR enables cheaper and quicker production (Asghar et al., 2020).

Thus, developing countries may lose even what they currently have in terms of employment and investments by the developed world. Challenges to leveraging the 4IR include lack of infrastructure, expertise, and collaboration between academia and industry. There are efforts in some countries, for example, Ghana, to leverage the 4IR. Ghana is preparing to set up an Artificial Intelligence (AI) research centre. To get started, many countries in Africa must set up infrastructure, a major one being internet connectivity. The Central and East African regions have internet penetration rates of 26% and 24%, respectively, while for West Africa, the rate is 42%. South Africa's rate is comparable to the rest of the world at nearly 60%. (statista.com).

Beyond infrastructure, governments should prioritize education. Simbanegavi et al. (2018) emphasize that priority should be given to improving the quality of education to address the mismatch of skills between the graduates of the education systems and the rapidly changing workplace. They propose that STEM education be implemented to lay the foundation for the knowledge and skills required for the workplace and innovation in Africa. This is in the hope of addressing the challenges to implementation of the 4IR, which include lack of infrastructure, lack of highly skilled manpower and predominance of an informal economy.

Rationale for Prioritizing Education

Education is the gatekeeper of progress and development, despite technological innovation revolutionizing economies and industries. Peters (2017) poses the question about the role of higher education in the digital age when technological employment becomes the rule rather than the exception. We, too, ask how several African countries will survive if this is the rule? Should these countries care if the 4IR heavily leans on digital tools and technological advances, which these nations may lack? This is a question that needs to be addressed not only in higher education but at all levels of education if countries are going to respond in ways that would enable them to leverage the 4IR for economic growth. The question of the role of education in equipping students with skills for careers that align with the 4IR has been addressed by various researchers, with some studies exploring instructional techniques (e.g., Peters, 2017; Pinto et al., 2019) and others on the readiness of teachers for preparing learners for the 4IR (e.g., Akgunduz and Mesutoglu, 2021; Weinhandl & Lavicza, 2019). The United Nations (2015) sustainable development goals (SDGs) are being executed such that none is left behind come 2030. It states that "the creativity, know-how, technology and financial resources from all of society are necessary to achieve the SDGs in every context" (fourth para, https://www.undp.org/sustainable-development-goals). In general, education should provide students with knowledge and skills for this 4IR era. Education is a broad field, and although several articles and research work are dealing with engineering, robotics or similarly related areas of studies with respect to the 4IR, there is little in the area of mathematics education.

Our discussion focuses on mathematics education at the foundational, primary level of schooling. Additionally, we explore mathematics education in a few selected African countries where inequities in technological advancements abound. The lack of resources such as electricity or internet, especially in remote locations where the majority of the primary learners live—poses the impossibility of their education meeting the demands of 4IR. As such, we examine curricula of selected African countries that have adopted the CBC, a curriculum with a robust reputation and measurable learning outcomes aligned with SDGs, to understand its potential to contribute to the 4IR.

In the chapter, we seek to explicate how CBC is likely to contribute to building the skills required for the 4IR. We want to note that the availability of resources is not the most significant piece but rather the development of quality education and core 4IR skillsets. The 4IR calls for flexible, versatile and adaptable people to the changing nature of technological advances, automation, artificial intelligence and the like. Many developed nations have resources, but unless an effort is made for students to be knowledgeable, the resources go to waste or are underutilized due to a lack of in-depth understanding. A simple illustration of this is using a calculator, which is among the rudimentary oldest technology, where a student is asked to calculate 0.002023×0.000103. Often the majority of the class would simply copy what is displayed on the screen as 2.083690E-7 instead of the appropriate mathematical or scientific way of communicating the product as 2.083690×10^{-7}. Similarly, if all one has is resources and a procedure to go by, then if one step is forgotten, the entire project comes to a halt due to the inability to make sense of the issue.

We make the case that the selected African countries can meet the demands of the 4IR and surpass it by empowering their citizens with quality education. Being grounded in theory, in justifications, and in the "why" are crucial so that when resources are available, the usage, the practice and how to apply the resources can happen smoothly. Additionally, conceptual and relational understanding enables creativity, critical thinking and problem solving, which are high-impact skillsets. Furthermore, experience with a lack of resources enhances one's improvisation capability, resilience, adaptability and flexibility—all the necessary qualities to excel in the 4IR.

Why Mathematics Education?

What role does mathematics education play in the 4IR? Mathematics is the subject that all students are required to take in school, especially at the primary level. In comparison to school disciplines like science, social studies, and history, note that mathematics gets allotted more time in basic education. For example, in a week, mathematics is scheduled for seven to eight class periods or sessions while science has three. As such, mathematics is a key school subject for developing the 4IR proficiencies; teaching and learning mathematics ought to cover more competencies since ample time is apportioned relative to other subjects.

Mathematics education has many topics of interest, and our chapter addresses the school mathematics competency-based curriculum (CBC). Since 2010, several countries in South, West, Central and East Africa (e.g., Ghana, Tanzania, Kenya, Rwanda, Cameroon and Zambia) have adopted CBC for their basic education. We will focus on basic education, particularly preschool to grade 6, because this is where many of these countries have successfully implemented CBC to date. CBC emphasizes minimum knowledge, skills and attitudes that each learner must achieve at each grade level in all subject matter.

One may ask, how is CBC different from the previous curriculum? Regarding mathematical content knowledge, the CBC mathematics and earlier mathematics curricula are almost the same except for high-stake assessments. Nevertheless, when it comes to skills and attitudes, there is a huge difference—with CBC, there is a need for skills and attitudes to be explicit, and feedback ought to be given on a student's performance. This chapter will dwell on these aspects, which are little understood about CBC.

Broadly, the desired competencies in the teaching and learning of CBC mathematics can be classified into two categories: cognitive and non-cognitive competencies, where the cognitive loosely deals with mathematics content and the non-cognitive concerns skills and attitudes. We acknowledge that some skills overlap in this dichotomy. For this work, we generally define cognitive competencies as mathematical knowledge, concepts, facts, and understanding. These are the what-and-why of mathematics and must mainly be learned in a mathematics classroom. On the other hand, non-cognitive skills pertain to learning mathematics content, such as the processes of reasoning, representation, connection, problem solving, and communication (National Council of Teachers of Mathematics, 2000). The National Research Council (2001) also outline skills which constitute mathematical proficiency. These include flexibility, adaptability, critical thinking and creativity. We point out that non-cognitive skills can be learned elsewhere in other subject matter classrooms besides mathematics. We also use the terms non-cognitive skills and soft skills interchangeably.

Globally, the cognitive aspect of mathematics is the same; that is, mathematical facts, concepts, procedures, algorithms, principles, and relationships are the same everywhere. Context, language and method of presentation of the notion may differ, but the idea inculcated is the same. A fraction is a fraction regardless of the geographical region. The Trends in International Mathematics and Science Study (2015) provides a list of content knowledge from many countries or states on different continents. The mathematical content knowledge competencies in Central, West and East African schools are the same as in other countries. Table 3.1 shows two examples of measurement content competencies in four countries on three continents.

From Table 3.1, the concepts are the same despite the differing words of the competencies and therefore are not the competencies of our foci in this chapter.

In prior curricula, the cognitive content-oriented portion often tended to overshadow the non-cognitive aspects, yet both are crucial. This is an aspect that CBC mathematics may overcome. Besides the cognitive, the non-cognitive skills are

Table 3.1 Sample of measurement content competencies from four countries (Rwanda Education Board, 2015; Curriculum Development Center, 2013; Common core state standards for mathematics (CCSSM); National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010; Pedagogical Development Unit, 2012)

	Rwanda	Zambia	USA	England
Perimeter	Explain the perimeter of a shape as the distance around it	Finding the perimeter of simple plane figures	Finding and estimating perimeters	Measure and calculate the perimeter of composite rectilinear shapes in centimeters and meters
Unit conversion	Accurately convert different capacity measurements	Relate seconds, minutes, hours and days	Converting like measurement units within a given measurement system	Understand and use approximate equivalences between metric units and common imperial units such as inches, pounds, and pints

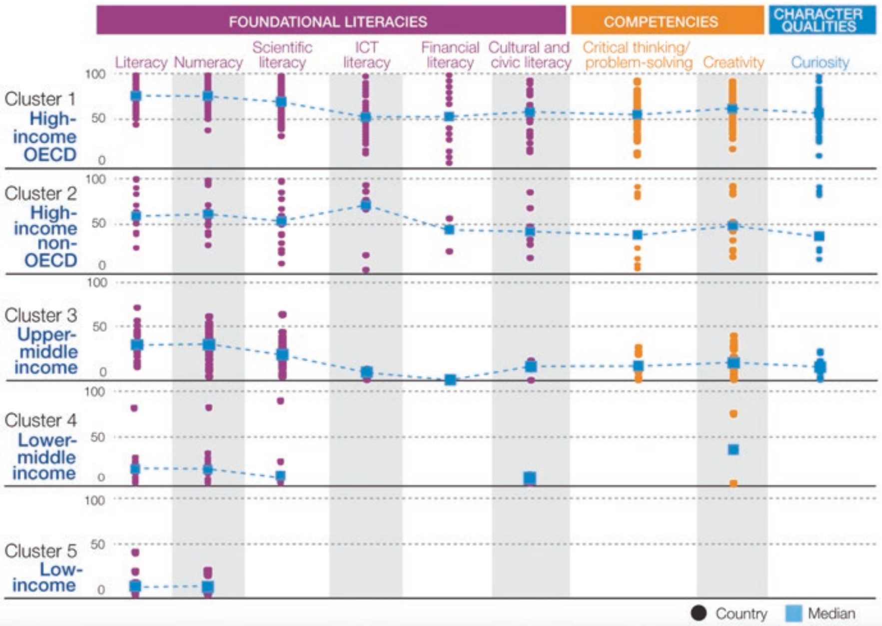

Fig. 3.1 Percentile ranking of skills per cluster of countries. (Source: https://widgets.weforum.org/nve-2015/chapter2.html)

important skills needed by all to succeed in the 4IR era, regardless of the discipline. We herein explore the non-cognitive soft skills since they are mainly the newest addition that makes CBC curricula different from prior ones. Also, the CBC soft skills align well with the required skill sets needed in the 4IR age. Figure 3.1 shows how 91 countries faired with regard to 16 twenty-first-century skills (WEF, 2015).

Figure 3.1 shows that for numeracy—what we have referred to so far as content knowledge of mathematics—there is some level of numeracy in all the 91 countries regardless of the cluster. A look at the soft skills (e.g. creativity, problem-solving, critical thinking) in the chart shows that countries in cluster 4 and especially cluster 5 have zero percentile rank. Based on how this data was collected, this is quite a steep gap compared to other countries in competencies and character quality skills (WEF, 2015). Our selected African countries for examination of their CBC mathematics curricula fall in these low-ranking clusters.

Soft Skill Competencies in Select African Countries

In this section, we explore Kenya, Zambia, Tanzania, Rwanda, Ghana, and Cameroon's soft skill competencies by examining their curriculum framework and mathematics syllabi. A brief summary of the findings is shared per country. Note that all these countries have adopted CBC, and the latest country to implement it was in 2017, with the sixth grade as the highest class of students taking CBC to date. The summary does not contain all core competencies spelt on the frameworks but only those that are obviously soft skills.

Zambia

The Zambia curriculum framework (2013) lists the soft skills to be integrated with teaching and learning as "critical, analytic, strategic and creative thinking; problem-solving; effective use of language; symbols and text; self-management; relationships with others; participation and teamwork; innovation, entrepreneurship and productivity; life skills and civic competencies" (p. 13). Life skills behaviours are further identified and are derived from various school subjects, including psychology and sociology. Beyond primary school, communication and information

Table 3.2 Highlighted soft skills in Zambia (Curriculum Development Center, 2013)

Country	Generic soft skills	Sample task or activity in math syllabus
Zambia	Communication Writing	Use of appropriate set symbols
	Collaboration-Teamwork	Through a shopping activity OR in collecting data
	Values	
	Appreciation	Appreciate usefulness of line graphs
	Curiosity	Explore nature or in making different right-angled triangles using squared paper
	Creativity	Combining basic shapes to make pictures or molding basic solids

technology are taught as standalone courses. The soft skills that stand out in Zambia's primary mathematics syllabus are shown in Table 3.2.

Rwanda

In Rwanda, soft skills are integrated into all school subjects and are referred to as generic competencies. According to the Rwanda curriculum framework (2015), the soft skills are creativity and innovation; critical thinking; research skills and problem solving; communication; cooperation; interpersonal relations and life skills; and lifelong learning. They are seen as significant skills in deepening knowledge, useful in the workplace and adaptability. Equally, ICT and digital competencies are incorporated with other subjects or taught independently as core courses with mature students. The soft skills emphasized in Rwanda's primary four to six mathematics syllabus are shown in Table 3.3.

Kenya

The basic education curriculum framework for Kenya outlines seven core competencies: communication and collaboration; critical thinking and problem solving; creativity and imagination; citizenship; digital literacy; learning to learn and self-efficacy. These competencies are to be developed through the different learning areas. The curriculum has also incorporated pertinent and contemporary issues

Table 3.3 Highlighted soft skills in Rwanda (Rwanda Education Board, 2015)

Country	Generic soft skills	Sample task or activity in math syllabus
Rwanda	Collaboration- Play games and cards Work in pairs and Work in small groups Work as a whole class	Each learner in a group of five gets a little slip of paper with his or her own large number. The learners arrange themselves in order, and read their numbers out loud
	Communication- Dictation Discussion Talk and share Explanation—verbal and written	Learners will discuss among themselves the difference between quantitative and qualitative data and also talk about the meaning of data in the broader context
	Attitude and values- Perseverance and grit i.e., show the spirit of hard work Appreciation i.e., the importance of money in our daily life	Learners should be given tasks in their groups to set priorities when making budgets. They should differentiate between wants and needs as well as understand why budgeting is important so that they don't waste money

Table 3.4 Highlighted soft skills in Kenya (Kenya Institute of Curriculum Development, 2019)

Country	Generic soft skills	Sample task or activity in math syllabus
Kenya	Collaboration	Pairs, groups or as individuals to identify and share information on where decimals are used in real life
	Communication Discuss and share	Learners may work out area of tables at home and report to the teacher.
	Critical thinking and problem solving	Round off numbers and estimate sums
	Creativity and imagination	Make patterns involving addition
	Digital literacy	Play digital games involving mass
	Self-efficacy	Make reports in their groups
	Citizenship	
	Patriotism	Learners discuss the importance of paying taxes

designed to address emerging needs in society and learners' safety. They include life skills development through storytelling, living values, moral values and social responsibility. The soft skills emphasized in Kenya's primary mathematics syllabi are as shown in Table 3.4.

Cameroon

The Cameroon curriculum framework (2018) has seven national core skills and four broad-based competencies. The seven national core skills are to a large extent related to subject content except for lifelong learning, spirit of autonomy, sense of initiative, creativity, and entrepreneurship, which in our view are already reflected in the four broad-based competencies below:

1. Intellectual competencies which include exploiting; solving; acquiring logical thinking and a sense of observation; exercising critical judgment; practising creative and innovative thinking.
2. Methodological competence, which includes giving oneself efficient working methods; exploiting information and communication technologies; organizing one's learning; arousing the desire to learn each subject.
3. Personal and interpersonal competencies enable the learner to develop his/her personality, acquire abilities in view of his/her socio-cultural integration and individual fulfilment and cooperate with others.
4. Communication competencies enable the learner to communicate appropriately in the two official languages and at least one national language. The four broad-based competencies are to be developed in all the five teaching and learning domains (Table 3.5).

Table 3.5 Highlighted soft skills in Cameroon (Ministry of Basic Education, Cameroon, 2018)

Country	Generic soft skills	Sample task or activity in math syllabus
Cameroon	Intellectual competencies Methodological competence Personal and interpersonal competencies Communication	To support the development of the soft skills, the curriculum outlines the following three pedagogical approaches 1. Project based learning 2. Cooperative learning 3. Integrated learning themes

Ghana

The national pre-tertiary education curriculum for Ghana (2019) highlights the following values: respect, diversity, equity, commitment to achieving excellence, teamwork/collaboration, truth and integrity. It also outlines the following core soft skill competencies whose development should be supported by instruction at all levels: critical thinking and problem solving; creativity and innovation; communication and collaboration; cultural identity and global citizenship; personal development and leadership; and digital literacy. See Table 3.6 for examples of soft skills and accompanying activities extracted from the sample lessons in the teachers' resource pack.

Tanzania

Tanzanian curricular soft skills (2019) fall under three categories: learning, literacy and life skills. Learning skills include critical thinking, complex problem solving, creativity, collaboration and communication. Literacy skills, which focus on how students can discern the accuracy of facts and publishing outlets and appropriate use of technology, include information, media and technology. And the life skills which enable students to acquire interpersonal relationships include emotional intelligence, customer focus/service orientation and personal skills. Table 3.7 gives examples of soft skills and their related classroom activities.

Opportunities and Challenges

In this chapter, we have examined the non-cognitive aspects of mathematics in the CBC curriculum in six African countries. Developing such soft skills in a classroom setting until they become socio-mathematical norms and habits is core learning that school graduates can take away and apply in any context. Despite one's location on the globe, be it rural Kenya where technological advances are wanting or in the US where the Silicon Valley resides, there is a need to garner expertise in non-cognitive

Table 3.6 Highlighted soft skills in Ghana (National Council for Curriculum and Assessment, Ministry of Education Ghana, 2019)

Country	Generic soft skills	Sample task or activity in math syllabus
Ghana	Communication and collaboration	Learners work together in their groups to order a given set of numbers in ascending or descending order verbally and in writing. E.g. 1020, 1025, 2673, 2873
	Problem solving, critical thinking, collaboration, personal development and leadership	Learners make the formation of sets of equal objects from a given quantity. For example, with 12 straws learners make groupings such as: Learner's pair grouping and the number of equal objects for each formation as factors. Learners collect from the pairs and form a set as the factors of the given number. For example, the set of factors of 12 = 1, 2, 3, 4, 6, 12 Assessment Form a set of factors for the following (a)16 (b) 6 (c) 25 Can someone tell me what we have learnt today? We learnt about factors, multiples and squares of numbers. Can someone tell me the factors of 4? {I,2,2,4} Independent activity/homework Write the factors for the following numbers (a) 15 (b) 12 (c) 24 (d) 32

skills. As mentioned earlier in this chapter, having conceptual understanding and persevering in problem solving in most cases is better than possessing digital tools with instructions.

The non-cognitive skill sets are what a learner carries away from the mathematics classroom to contexts outside the school—work, home, real-life situations, etc. The ability to persevere in solving posed mathematics problems to its end builds one's stamina and grit in the long run. Thus, primary school children in Rwanda, Kenya, Tanzania, Ghana, Cameroon and Zambia need to be effectively taught content and soft skills competencies to succeed in the 4IR age despite technological challenges. All the curricula we reviewed contain not only content but soft skills as well. This is an important first step. We call for concerted efforts in the implementation of soft skills.

As such, there might be a need for some of the syllabi to be revised to (a) Deepen each soft skill and enhance the strategies. For example, there is much more to communication besides talking with a partner, in small groups or sharing with the whole class, including structured journaling, paraphrasing, structured debates, critiquing each other's work, reasoning by explaining or justifying verbally or in written form, oral presentations, argumentative writing and restating what one said. (b) Unpack the soft skill competencies using specific language or give concrete examples where appropriate. For example, digital tools can be used in varied ways in addition to playing games which mainly enhances drill and practice of mathematics content (e.g., https://www.ixl.com/math or https://www.imaginelearning.com/en/us/

Table 3.7 Highlighted soft skills in Tanzania (Tanzania Institute of Education, 2019)

Country	Generic soft skills	Sample task or activity in math syllabus
Tanzania	Reasoning and proof	(a) Identifying things which show patterns (b) Identifying missing numbers in a sequence (c) Arranging numbers in ascending order (d) Arranging numbers in descending order.
	Problem solving	(a) Solving word problems involving addition (b) Solving word problems involving subtraction
	Communication	(a) Counting from 1000 to 9999 (b) Reading numbers from 1000 to 9999 (c) Writing numbers in numerals from 1000 to 9999 (d) Writing numbers in words from 1000 to 9999

products/math/math). Aside from calculators and scholarly websites, there are free mathematics software that can be used for problem solving, tutorials, simulations, and generally hands-on activities that support students' exploration. (c) Streamline the expected competencies per topic area or lesson to avoid overloading in content areas where the syllabus lists many competencies.

Challenges to address with implementers of CBC involve (i) for experienced teachers, making the shift to explicitly integrate soft skills during instruction given a long experience of teaching mathematics procedurally with independent seatwork to learning conceptually and collaboratively in small groups with discussions; (ii) inclusive and safe classroom management that permits learner's creativity, self-efficacy, innovation and much more self-directed and thus deepened learning.

A question that may arise is, how are soft skills taught in a mathematics class and assessed? Soft skills are not taught as standalone topics, but their teaching is integrated during instruction with mathematics content. For example, the mathematics content enables students to discuss the embedded mathematics principles, concepts, facts, strategies, etc. Given a problem task as follows:

> Three friends accidentally meet at a Diner on October 1. Tom is at the Diner every other day, Sue is at the Diner every third day, and Brooke visits every fifth day. When will they meet at the Diner again?

Students can then solve the problem task in small groups, and since the problem can be solved in several ways, groups can share their various strategies. The soft skills of collaboration, communication, and critique arguments of others are strengthened as students work together, as they explain, elaborate and justify their strategies.

The success of integrating soft skills in a mathematics classroom effectively depends on the classroom teacher's choices, including the choice of the mathematics problem tasks. Figure 3.2 shows an excerpt of two problem tasks from Smith's (2020) webinar. Both tasks will support the achievement of content-related objectives, but the one on the left (about the hexagon pattern) will allow for the integration of learning of soft skills as learning of content takes place.

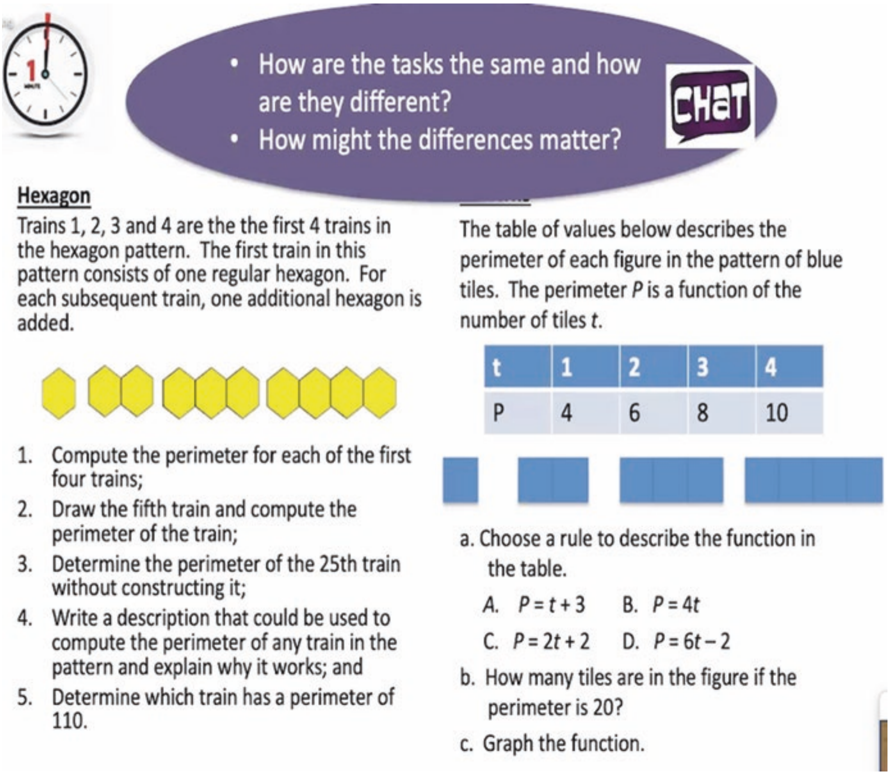

Fig. 3.2 Compare and contrast the two mathematics problem tasks. (Source: https://www.nctm.org/online-learning/Webinars/Details/520)

Conclusion and Way Forward

The examination of the competency-based curricula for the select countries revealed how to a large extent, the curricula have incorporated the skills needed for the Fourth Industrial Revolution. The four broad competencies—critical thinking and problem solving; creativity, communication; and collaboration—the World Economic Forum (2015) reports as the twenty-first-century skills are addressed in all the curricula that we examined. Foundational literacies, though not explicitly addressed in all the curricula, are likely to be achieved through the achievement of content objectives. Under character qualities, all the curricula examined have a values component that addresses different aspects of character qualities. The challenge is that some of these skills are not addressed explicitly, and even where they are addressed explicitly, they are not expounded on in a way that would allow teachers to leverage opportunities in the classroom to support their development.

As already stated, there is a need to unpack skills for teachers to support them in identifying opportunities for developing those skills and how they can leverage those opportunities during instruction. Currently, the skills are generically defined in the curriculum framework and syllabus but generally not elaborated on in the context of mathematics classrooms. Unpacking would involve elaborating on what each skill looks like in a mathematics classroom, such as defining collaboration beyond stating that children will work in groups. For example, a teacher looking out for collaboration in their class would ask themselves whether students have opportunities to consider mathematical ideas collaboratively and look for evidence of students discussing each other's ideas (Stockero et al., 2020). The teacher resource pack provided for the Ghanaian curriculum is a good initiative in providing sample tasks and activities relevant to the development of skills that certain content standards may provide opportunities for. More can be done to explicate what each skill would look like in a classroom and provide examples of tasks (see Fig. 3.2 above) and activities that would support the development of the skills.

Additionally, jobs in the 4IR require soft skills, often lacking in developing countries. Thus the development of soft skills must be integrated into the teaching and learning of mathematical content. Such integration calls for change in teaching approaches, which is acknowledged by all the curricula frameworks examined in this chapter. This requires professional development that is designed to support teachers in the implementation of the CBC. Most countries have offered training for teachers in the initial stages of implementation of the CBC, but some challenges remain regarding implementation. Studies have shown that traditional instructional approaches continue to prevail in classrooms (e.g., Paulo & Tilya, 2014). Some of the challenges include teachers' inadequate understanding of the curriculum (Jansen, 2009); and contextual issues such as large class sizes and inadequate learning materials (Paulo & Tilya, 2014; Savard & Cyr, 2018). Studies have also highlighted the inadequacies of the training offered to teachers in preparation for the CBC (e.g., Ndihokubwayo et al., 2019), most of which have been short-term training programs at the start of the CBC. However, for training to have a significant impact on teachers' skills, it should be continuous and long-term (Savard & Cyr, 2018).

Finally, as a way of preparing learners who will be able to contribute to the 4IR in their countries, we suggest the incorporation of content that would directly support the connections between mathematics and computer science. Discrete mathematics provides the foundations for the study of computer science. Topics in discrete mathematics, such as sets, and functions, are already addressed in some of the curricula examined here. However, all curricula should address these topics from an early age as children are capable of functional thinking right from kindergarten (Blanton & Kaput, 2004). Curriculum designers should also consider incorporating other discrete mathematics topics such as graph theory into the curriculum, which has many applications in computer science. Holliday (1991) describes graph theory as a non-threatening topic. It is accessible even to students struggling with traditional mathematics topics and allows them to engage with mathematical ideas. Graph theory also has activities that can be used with students right across the curriculum from kindergarten to eighth grade (Althoen et al., 1991). Problems in graph theory provide opportunities for the development of skills such as problem solving because they require students to make sense of problems, abstract the problems from the context and identify an approach for solving the problem. Graph theory problems also lend themselves easily to group work and discussion of the different approaches students may use, therefore allowing for the development of collaboration and communication skills.

We acknowledge that learners' development of the skills needed for the 4IR will be supported collectively by instruction in all the subject areas. However, mathematics provides the foundations for information technology, one of the main drivers of the 4IR. Therefore its contribution to the preparation of learners for the 4IR cannot be emphasized enough. The recently implemented competency-based curricula in the countries examined here hold great potential for preparing learners to contribute to the 4IR. However, that potential can only be realized if teachers are clear on how to integrate the development of the competencies into their lessons. Instruction in mathematics classrooms that incorporates the development of competencies will go a long way in complementing other subject areas with regard to preparing learners for the 4IR.

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Chapter 4
Mathematics Educators' Readiness for Online Education in the Fourth Industrial Revolution: A Case of Two Selected Universities in Ethiopia

France Machaba and Tola Bekene Bedada

Introduction

The online education system, mainly referred to as E-learning, uses technology to deliver information for education and training in the 4IR era. E-learning can best be defined as the science of learning without using paper-printed instructional material. E-learning provides an alternative to traditional classroom education. It enables students to access course information (course content, reading materials, quizzes) without time restrictions or geographical constraints in the fourth industrial revolution era (4IR). However, it may reduce physical contact between teachers and students. Since May 4, 2021, when the government media announced that three out every four people tested daily were positive, COVID-19 has brought tension to Ethiopians. This statistic was dire, so the need for acquiring 4IR skills was not negotiable for any education sector in Ethiopia. Accordingly, Ethiopian higher education had to organise the information and communication technology (ICT) workforce, invest in ICT infrastructure, and attempt to use ICT for education during COVID-19. Ethiopia's government planned to introduce 4IR skills in all sectors of the country, as was stated in the fifth Partnership for skills in Applied Science, Engineering and Technology (5th PASET) forum on the discussion of 4IR in 2019. As a result, most universities in Ethiopia implemented the new ICT workforce structure with a better salary scale, which may help universities implement online education systems(e-learning). The establishment of the higher education forum by the Ministry of Science and Higher Education to exchange information and resources among

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universities is another important move that is helping universities to use ICT for education.

One example of technology that enables universities to share educational resources is the e-learning platform (Ho & Dzeng, 2010). This platform is a fully online application where the teaching and learning process is ongoing. E-learning is emerging as the new paradigm of modern education (Sun et al., 2008). Moreover, e-learning is getting very popular these days, as many universities offer degrees (first, master's, and PhD degree programmed) and diploma programs through the e-learning mode. The University of South Africa (UNISA) is an excellent example.

In Ethiopia, efforts were made by the Ethiopian Education System to realign itself to the changing global landscape in e-learning systems (Semela & Ayalew, 2008). Moreover, during the COVID-19 pandemic, the Ministry of Science and Higher Education's (MoSHE) website and e-learning platform have been more like a web-assisted provision of course materials that teachers provide as part of their classroom instruction in Ethiopian University since its introduction. Nevertheless, the speed of integrating teaching and learning in e-learning platforms varies from university to university in Ethiopia. Wachemo university was one of the Ethiopian universities aiming to start an e-learning platform in 2021 as it was training teachers on how to upload the course contents and other information into the e-learning platform at the beginning of 2021. In e-learning platforms, resources are offered to students without opportunities for them to engage with their tutors or peers. It may seem these ideas oppose Vygotsky's concept of scaffolding in education. At the very least, this is an uninspiring method of teaching. More importantly, if students complete their academic requirements through the current arrangement, it will undermine the quality of education and make our universities digital slaves.

The online classes were forced to be conducted from home or the students' respective homes since the COVID-19 pandemic quarantine was instituted in March 2020 worldwide. Even though COVID-19 vaccines have been developed and distributed throughout the world, Ethiopia still faces many challenges from COVID-19. Scholars mostly agreed that the countries that are well equipped with 4IR skills might easily resist the negative effect of COVID-19 on the education sector by applying 4IR skills. Accomplishing this is up to teachers lobbying for integrating technology into the teaching and learning process, whether they have been trained or not with the technology, especially in Ethiopia. After the universities reopened, Ethiopian universities ran their teaching and learning by keeping the protocol of COVID-19; however, e-learning implementation was hardly ever occurring. The Ethiopian Ministry of Education (MOE) for elementary to secondary level students and the Ministry of Science and Higher Education (MoSHE) for university-level advised teachers and students to run their teaching and learning processes during the pandemic with the help of technology. More specifically, the concerned body advised the users to deliver their teaching and learning materials using social media such as Telegram during COVID-19 (Machaba & Bedada, 2022).

A comprehensive approach to e-learning indicates that the new e-learning system has unique benefits for individuals, organisations, and educational institutions. In the last century, the definition of educational technology has undergone fundamental changes based on the ongoing developments in epistemological perspectives, psychological approaches to learning, and other affiliated science of communications, systems, and education. To respond to online teaching and learning advantages, private and public organisations worked day and night to migrate conventional teaching and learning methods to online platforms, especially during the COVID-19 pandemic. However, several factors affect the satisfaction of using e-learning in education, leading to the acceptance and rejection of the technology starting from the e-readiness (Chapnick, 2000). Most of the time, the factors are categorised into six categories: student, teacher, course, technology, system design, and environmental dimension (Sun et al., 2008).

The current availability and political will for trying new approaches mean that there is currently much interest in and expenditure on technology for education. Ethiopia is one such country that currently needs technology for education as the government wants to lead countries to prosperity. For instance, the recently (October 1/2020) inaugurated Artificially Intelligence (AI) Center, which is one 4IR project, has signed Memorandums of Understanding with five institutions (Zewditu Memorial Hospital, Addis Ababa Institute of Technology, Addis Ababa Science and Technology University, National Meteorology Agency, and Oromia Cooperative Bank) to cooperate on artificial intelligence products and services developments. The project hopes to enhance AI products and services' health, education, agriculture, transport, and public protection and safety sectors. Without research, it is clear that environmental disasters such as COVID 19 force the world to use the appropriate technology. With this in mind, the experts from individual fields created the right technology that individuals could access by purchasing it or using it freely from the internet. The considerations raised in the study's introduction led to the statement of the problem as set out below.

Problem Statement

Integrating computers in education, even for training teachers, will never be possible without reconciliation between teachers and computers (Marcinkiewicz, 1994). However, Machaba and Bedada (2022) reveal that teachers' motivation toward using technology is high. In Ethiopia, integrating technology and some infrastructures in the teaching and learning process are not prominent in higher education. However, to come up with a solution to the COVID-19 risk to the educational system in Ethiopia, the government aims to change the teaching and learning protocol from the conventional method of teaching to the modern way of teaching (one of these methods is the e-learning method). Besides the aims of the government to implement the e-learning teaching and learning methods in higher education, Wachemo university, one of the Ethiopian public universities, trained the teachers at the university in the last month of January 2021 for the first-round trainer, and the training will continue until all the teachers on the campus are trained. E-learning should be given attention and priority by providing training for teachers in higher education in Ethiopia to benefit fully from e-learning (Anberbir, 2015). According to Anberbir's findings, despite the teachers' awareness of the benefits of e-learning in Ethiopian higher education, its growth is at the initial stage. Teachers' attitudes are a critical factor influencing e-learning satisfaction (Sun et al., 2008). Another scholar discusses that course design and contents, the facility for accessing and visualising information on the teaching platform, and the possibility of interaction were key aspects to be considered as the factor affecting the level of satisfaction and readiness of individuals to accept the e-learning platform (Martín-Rodríguez et al., 2015). So, the study aimed to investigate the mathematics teachers' readiness to use and accept e-learning in the 4IR era.

In the 4IR, one of the main questions that currently may be facing teachers is what might prompt them to adopt and integrate the new technologies available in their classrooms in the 4IR. There has been increasing interest in the use of Fourth Industrial Revolution (4IR) technologies, such as artificial intelligence (AI), the Internet of Things (IoT), blockchain, robotics, data analytics and others, to help achieve Sustainable Development Goals (SDGs) of the country, even when struggling with the COVID-19 pandemic (Ally & Wark, 2020). In the twenty-first century, chalk and talk is a key characteristic of an outdated way of delivering content in the classroom in 4IR. This idea was implemented more in COVID-19, but developing countries like Ethiopia still stick to the talk and chalk approach to delivering courses.

So, the need to study teachers' readiness towards integrating technology into education in this country is unquestionable. Some scholars cited that both students and teachers may have various levels of readiness for succeeding within the 4IR; teachers and students may be "digital natives" or "digital immigrants" (Wang et al., 2013). According to Wang et al., "digital natives are assumed to be inherently technology-savvy, digital immigrants are usually assumed to have some difficulty with information technology" (Wang et al., 2013, p. 1). "Digital natives" have always been involved and associated with technology-based tools (Helsper & Eynon, 2009) and are more likely to use technology-based tools successfully when learning. Although digital immigrants may become proficient in technology-based tools during their lifetime, they may typically print out information to review rather than utilising the internet for assistance or information (Helsper & Eynon, 2009). These individuals may need additional support with using technology-based tools.

Bedada (2020), Bedada and Machaba (2022) stated that lecturers must know their students and their abilities concerning using technology-based tools when learning, which is the principle of scaffolding. This knowledge will assist them in introducing the notions of the 4IR in the lecture room. However, teaching and learning in Ethiopia is still traditional and not aligned with 4IR. In short, according to Ally and Wark (2020), the teaching and learning paradigm experienced a significant shift away from the c*ustomary education* of the person reading during the First Industrial Revolution as it was necessary to educate more people better. This shift continued through the second and third industrial revolutions until the mass training required in the 4IR. Students supported the customer education ideal during the third industrial revolution. Still, now, with the 4IR, technology distorted the appearances concerning physical and numerical skills and encompassed more *interaction of teachers and students* with computers in education.

Developing countries (e.g., Ethiopia), because of their location, government structure, and lack of technical expertise and infrastructure, have not been able to take full advantage of the first three industrial revolutions. However, in this era, developing countries are moving into the 4IR. They have the chance to use smart technologies to improve quality and efficiency and thereby reduce the disparity between developed, developing, and underdeveloped populations (Ally & Wark, 2020).

To address this problem, the authors explored the teachers' readiness for e-learning in the 4IR, where Ethiopian Universities attempted to mitigate the effect of COVID-19 on the education sector in the academic year of 2021, leading teachers into the era of 4IR. Thus, the main research question to be addressed in this study is: To what extent are mathematics teachers ready, ***Psychologically, Sociologically, Environmentally, financially, and regarding human resources and content,*** for online education (e-learning) in the Fourth Industrial Revolution era.

Literature Review

The Stages of Education Versus Industrial Revolution World Views

Countries can use 4IR technologies to deliver education to students regardless of their status and location, especially in locations that are remote or require online education (Ally & Wark, 2020). For Alakrash and Razakb (2020, p. 163), the use of 4IR technology in the classroom is very important to provide opportunities for learners to learn and operate in the age of technology with "the arrival of digital natives and the inclusion of technology in teaching and learning, which are common nowadays". These researchers state that "the internet applications of educational tool mode[s] like *e-learning, blended learning, and Massive Open Online Courses (MOOCs)* have resulted in a re-evaluation of the transmission-based system where the teacher serves as the source of knowledge" (Alakrash & Razakb, 2020, p. 163). Online learning and e-learning allow for learner-centred education anywhere and anytime, rather than requiring learners to attend a physical location to learn. The intention of this study was bounded by one of the elements of 4IR, known as e-learning, which is reviewed in the next section.

The traditional education in Ethiopia started in the 6th century when the Sebean alphabet was introduced along with Christianity (Getaye, 2012). But Gemechu (2016) claims that associating traditional education with Christianity in Ethiopia is wrong, stating that before the religions came to Ethiopia, there was a way of imparting and acquiring knowledge from elders. For instance, in Oromo culture, there is a dialogue in which elders transfer knowledge to their children by saying *Hibbo Hibbaka* at night, which started before Christianity came to Ethiopia. Also, in mathematics, Oromo elders teach their children by saying *"Tokko maali*"? to mean, "What is one?" The answer to this is "*Takken takkituma*", to mean one is one, to teach their child the number one (1) in mathematics. For number two (2), the elders ask children, "*Lama maali*"? to mean, "What is two?". The answer to this is "*Laaman muucha ree'e*", meaning two is the breast of a goat, so that the children recall the number two (2) in the real world in a simple way. They continue in this way to teach numbers to their children at nighttime. The emergence of modern education in Ethiopia started in 1908. The development of modern education in Ethiopia can be divided into two stages (Woldeyes, 2017). This scholar describes the two stages as:

- The first stage took place from 1908 to 1935, when modern education started as a curious experiment toward westernisation and rapidly expanded during the period of Haile Selassie until the invasion by Italy.
- The second stage started with the occupation by Italy and encompassed the entire period since then as a complete Eurocentric education system was implemented in the country.

Between 1908 and 1935, education in Ethiopia aimed to master different languages such as French, Italian, Geʿez, Arabic and Amharic (Bishaw & Lasser, 2012). In general, understanding the educational revolution in Ethiopia requires careful consideration, even though it should be studied very well. However, the Ethiopian educational road map tries to address some problems of the country's education system to enable Ethiopia to participate in the fourth industrial revolution by clearly viewing the three past international education revolutions as depicted in Table 4.1.

Table 4.1 Industrial and education revolution

Stages	**Firts industrial revolution**	**Second industrial revolution**	**Third industrial revolution**	**Fourth industrial revolution**
	1750–1900	**1901–1960**	**1961–2000**	**2001–Present**
Activities	Steam engine Mechanical systems	Combustion engine Mass production	Large computers Micro-computers	Artificial intelligence Robotics Internet of Things
Stages	**First education revolution**	**Second education revolution**	**Third education revolution**	**Fourth education revolution**
Activities	Small classrooms (20–30 students) Use of blackboard	Large classrooms (30–40) Use of blackboard and whiteboard	Large classrooms (30–800 students) Use of blackboard, whiteboard, overhead transparencies, and internet.	There is a need to reinvent education in the 4IR

Adapted from Ally and Wark (2020)

Ethiopian Education During the Period of COVID-19

According to Allen and Seaman (2013):

> Online courses are those in which at least 80 percent of the course content is delivered online. Face-to-face instruction includes courses in which zero to 29 percent of the content is delivered online; this category includes traditional and web-based courses. The remaining alternative, blended (sometimes called hybrid) instruction has between 30 and 80 percent of the course content delivered online. (Allen & Seaman, 2013, p. 7)

Meanwhile, MoSHE and the Universities in Ethiopia directed students to access website repositories to download academic resources during COVID-19. It was still difficult for all users to benefit from the online databases because of many barriers that can be considered standard in online learning. By considering the barriers, Ethio-Telecom waived costs for access to these (online) sites, which was admirable. However, unless students had their devices for online access, they had to frequent internet cafes, which charge per minute without regard for the sites accessed. Ethio-Telecom's contribution thus, in some cases, benefitted internet cafes rather than students. Indeed, this method is not a long-term solution for teaching and learning. Integrating the technologies of Industrial Revolution 4.0 (4IR) into classroom teaching practices is necessary by inquiring about teachers' level of knowledge and readiness (Avelino & Ismail, 2021). MoSHE aims to implement a new platform, known as an e-learning platform, that mostly helps teaching and learning in higher education. Bearing this in mind, as the e-learning platform is the result of technology evolution, the searchers wished to review the literature on the acceptance model of technology in 4IR.

Use of e-Learning Platforms in 4IR

This section covers how some countries integrate e-learning platforms into teaching and learning. E-learning requires technology, including appropriate hardware, software, and network infrastructure. Most e-learning environments are *Web-based* and can be accessed via the internet or intranet (e.g., via a university campus network) by Web browsers based on the HTTP and TCP/IP protocols. The offering on e-learning platforms typically includes offerings for the administrators, the students, and academic staff. Students' offering includes access to their learning content, previously provided by textbooks, and tools and communication software to facilitate collaborative learning. Lecturers and teachers can use the e-learning platform for course management, assessments, and tests. The e-learning platforms can also offer campus management systems for administrators to manage student admissions and enrollment (sometimes termed "student life cycle management") and do resource planning and accounting.

We tested an e-learning system during the training given at the Wachemo university on e-learning. For activating the procedure, we first google Wachemo university on the e-learning platform and hit the enter key, and this site appears in the window. The trainer had to register and sign on using their e-mail to log in as a student to use this procedure. The e-mail confirmation was immediately sent to an individual registered trainer as set up by the ICT administrator. The administrator upgraded the participants from students to instructors on their own. Thus, the e-learning platform contains an administrative system. In addition, the university offering e-learning only needs standard server hardware and Internet connectivity.

Elements of 4IR

The Fourth Industrial Revolution is characterised by innovative approaches where technology is integral to societies and human bodies (Schwab, 2016). Within this 4IR era, technology that includes digital tools, robotics, and artificial intelligence transforms our existence. Students' ability to solve problems must be improved during the 4IR era. (Naidoo, 2020). According to Schwab, "the rapid pace of this transformation is unsettling society since the 4IR has transformed the way nations subsist. The 4IR is epitomised by merging the physical and virtual world, shaping a universally connected and advanced society" (Schwab, 2016, cited in Smit et al., 2020, p. 87). It is imperative during the COVID-19 pandemic that we, as educational professionals, support this transformation and engage and interact effectively with our students.

Students' preparedness for succeeding within the 4IR may vary from being considered "digital natives" to "digital immigrants" (Wang et al., 2013). "Digital immigrants" are those "who learnt to use computers at some stage during their adult life; whereas digital natives are assumed to be inherently technology-savvy, digital immigrants are usually assumed to have some difficulty with information technology" (Wang et al., 2013, p. 1). Digital natives are more likely to use technology-based tools successfully when learning. Conversely, digital immigrants may become proficient in using technology-based tools during their lifetime. However, instead of going online for information or working online from the outset, they may print out information to review rather than utilising the internet for assistance or information (Helsper & Eynon, 2009). Teachers must provide additional support for students who are digital immigrants when they need to use technology-based tools for learning. Therefore, lecturers must know who their students are and their abilities to use technology-based tools when learning. This knowledge will assist lecturers when they introduce the concept of the 4IR in the lecture room. As pointed out earlier, in the former societal revolutions, the main emphasis of teaching was altered by the First Industrial Revolution (Schiuma, 2017). Education initially concentrated on traditional education, later changing to become a mass offering. In the Second Industrial Revolution, education was accomplished through facility learning through information transfer to the Third Industrial Revolution. Students supported the customer education ideal but now, with the 4IR, the application of technology distorted the appearances concerning physical and numerical skills and encompasses more interaction of teachers and students with computers in education. In particular, according to Schwab (2016), the elements of 4IR consist of four pillars:

- Artificial intelligence (more and more machines will be performing tasks that people have routinely done. We already see many applications of this kind of artificial intelligence in our daily lives, from customer service to autonomous cars. IoT can be seen as the technological revolution help, providing solutions for computations and analytics;
- Data and information (data and information flow continuously from what we say and do when we use the internet or our cell phones and other devices, as well as from information gathered from loyalty clubs, billing lists, and many other sources);
- Robotics (robotics is the discipline that develops machines that can do the work of people by replicating what we do); and
- The Internet of things (IoTs) (embraces the communication of machines, devices, and systems with each other, without human intervention).

Initially, people connected with people through the help of the internet. Then, the Internet of Things came, connected with 4IR technologies through machines. Moreover, now, there is the Internet of Everything, which connects people with people, people with machines and machines with machines. These interactions allow a clear interaction between machines and people for teaching and learning in the education sector, aligning with Vygotsky's theories in education. Accordingly, for 4IR, the scholars, Ally and Wark (2020), point out that the method of teaching and learning may need to be reinvented in this revolution.

Theoretical Model

The Chapnick Readiness Model was used as the theoretical framework that underpins this study (Chapnick, 2000). Chapnick (2000) developed this model to determine organisations' e-learning readiness. In this study, Chapnick's Readiness model was used to assess teachers' readiness for online education at two universities in Ethiopia. Teachers must be aware of the upcoming technologies to decide what is to be done to stay relevant and serve the global community in the coming years. The most prominent domains of the users of e-learning are schools, colleges, and universities, which have paid special attention to e-learning, especially during thin order to expedite the learning procedures (Olatokun & Opesade, 2008). An E-learning readiness assessment helps organisations such as schools, colleges and universities, and even other sectors design e-learning strategies comprehensively and effectively implement their ICT goals (Olatokun & Opesade, 2008). A model for measuring the e-learning readiness of universities that groups different factors into eight categories was designed by Chapnick (2000). Chapnick identified the following factors as playing a role:

- *Psychological readiness* measures the role players' state of mind-affecting their effectiveness in e-learning initiatives;
- *Sociological readiness* considers the interpersonal aspects of the context where the technology will be implemented;
- *Environmental readiness* considers the internal and external macro forces influencing stakeholders in the organisation;
- *Human resource readiness* considers the availability and appropriateness of the human-support system;
- *Financial readiness* measures whether the necessary funding has been put in place to pay for the process;
- *Technological skill (aptitude) readiness* qualifies the visible technical skills to implement the ICT goals;
- *Equipment readiness* determines whether suitable and adequate equipment is available; and
- *Content readiness* considers the subject matter and the objectives of the envisaged e-learning.

Using this model, the e-readiness of the teachers implementing digital learning devices/online education learning in participating universities was assessed.

Methodology

The study used a quantitative research method based on a post-positivist paradigm that assumes that social facts allow the pursuit of the study's objective to form a reality apart from the beliefs of individuals participating in the survey (Firestone, 1987).

Research Design

The researchers used a descriptive research design as one of the quantitative research aspects. They are interested in knowing participants' readiness for e-learning at the university level to analyse the data obtained. The researchers cited that the design is suitable for collecting data that can be statistically analysed and described (Creswell & Poth, 2018; Mills & Gay, 2019). Some researchers cited that descriptive studies are "useful for investigating various educational problems and assessing attitudes, readiness, opinions, preferences, background, practices, and procedures" (Mills & Gay, 2019).

Participants and Sample Size

The participants of this study were university lecturers in two different universities in Ethiopia's 2021 academic year, as the researchers can easily exchange e-mails with the individual participants of the study. The study participants were purposively selected from the mathematics departments in the two universities. About 34 teachers out of 56 participated. The educational background and age of those participant teachers are presented.

Data Collection Instruments

We conducted a series of in-depth interviews with various experienced e-learning learners to examine the validity of our research study. After that, we developed questionnaire items based on the previous literature and comments gathered from the interviews. The questionnaires were revised with help from experts (including academics and practitioners) with significant experiences in e-learning. Thus, the study instruments were validated by Ethiopian higher education experts. The reliability was calculated during a pilot study and found to be 0.885, based on the Cronbach alpha after one item was deleted because it affected the overall internal consistency of the data. So the constructed questionnaire was reliable for the main data collection. The questionnaire contained two sections.

- The first section contained demographic information that included sex, age, work experiences, and device ownership (smartphone, tablet, laptop, and desktop).
- The second section contained a questionnaire concerning the technology readiness scale. This section is divided into eight categories depending on the used model, comprising 51 items partitioned into eight categories. The questionnaire integrated the five-point Likert scale with the endpoints of "*Strongly agree=5/ Strongly disagree=1*" for the respondents to select the level of their e-readiness on the provided items accordingly depending upon the model used to assess their E-readiness in terms of the *psychological view* consisting of eight items, *sociological view* consisting of three items, *environmental view* consisting of seven items, *human resources view* consisting of four items, *financial view* consisting of three items and *content view* consisting of seven items. Finally, the researchers used codes to assess the teachers' technological skills. For assessing the teacher skills for the device and internet application, the researcher used: *Daily=4, Weekly=3, Monthly=2, I do not use=1,* for evaluating the teachers' skills for twelve devices and internet applications. To assess teachers' skills and the device and programs they used, the researcher used *Daily=5, Occasionally =4, Rarely=3, Hardly=2,* and *Never=1.* Eight devices and programs were included.

Data Analysis

Descriptive statistics were run to analyse the collected data from the participants. Statistical Package for Social Sciences (SPSS)version 20.0 software was used to analyse the data descriptively and measure the mean score and standard deviation values. The range determined was based on the concepts of Yasin et al. (2016). According to their articles, mean scores of responses ranging from 1.00 to 2.33 are taken as low. Mean scores of responses ranging from 2.34 to 3.67 are moderate and mean scores of responses ranging from 3.68 to 5.00 are taken as high. The researchers used Analysis of Variances(ANOVA) statistics to investigate the differences according to the teachers' characteristics – sex, age, work experience, educational qualification, owning of electronics devices (Desktop, tablet, laptop, and smartphone), t-test, and analysis of variances.

Results

The background information of participants and questionnaire answers are presented. About 34 university lecturers from different educational backgrounds and ages participated in the study. The age of participants ranged from 26 to 56 years old, with the mean being 38.85 years old, and of these participants, only two were females. The education level of the participants varied from a degree to an assistant professor. Of these participants, about 24 (66.67%) had M.Sc. qualifications, 3 (8.33%) had M. Ed qualifications, 5 (13.89%) were assistant professors, and 2 (5.6%) had B.Sc. degrees. As discussed below, the respondents provided their opinions in response to the questionnaire, allowing the researcher to answer the study's research questions.

Mathematics Teachers' Backgrounds Influence e-Learning/Online Education in Their Classrooms in the 4IR Era

The Kruskal Wallis test analysis of sex with owning technology was found to be:

Smartphone: $H(1) = 2.595$, $p = 0.107 > 0.05$;
Tablet $(1) = 1.141$, $p = 0.285 > 0$;
laptop$(1) = 3.857$, $p = 0.50 = 0.5$; and
Desktop: $H(1) = 1.141$, $p = 0.285 > 0.5$.

The findings indicated that for all devices except for the laptops, the value of $p > 0.05$ and concluded that the differences are not statistically significant for owning technology besides sex.

With the work experiences of the teachers and owning of the technological devices, the Kruskal Wallis was calculated and found to be: -

Smartphone: $H(4) = 0.897$, $p = 095 > 0.05$;
Tablet: $H(4) = 18.508$, $p = 0.001 < 0.05$;
Laptop: $H(4) = 2.778$, $p = 0.596 > 0.5$, and
Desktop $(4) = 18.508$, $p = 0.001 < 0.05$.

These findings indicate that through work experiences, statistical significance occurs in owning tablets and desktops. Still, there are no statistical differences between the teachers having a smartphone and laptop.

Using the age of the teachers and owning of technological devices, the Kruskal Wallis value was calculated and found to be:

Smartphone: $H(13) = 4.802$, $p = 0.979 > 0.05$;
Tablet: $H(13) = 24.819$, $p = 0.024 < 0.05$;
Laptop: $H(13) = 16.000$, $p = 0.249 > 0.5$; and

Desktop $H(13) = 24.819$, $p = 0.024 < 0.05$. These findings indicate that through the age of the teachers, there is statistical significance in owning a smartphone and laptop, but there are no statistical differences between having a desktop and a tablet between the teachers. These findings support the study by Yeşilyurt (2021).

Mathematics Teachers' e-Readiness to Implement e-Learning

In this section, the researchers used the model developed by the author Chapnick (2000) to investigate the e-readiness of the teacher for accepting e-learning in the era of 4IR with constructed items by authors presented as follows:

Psychological Readiness

This factor measures the role players' state of mind, affecting their effectiveness in e-learning initiatives. The participants' responses concerned with the psychological readiness of university teachers are presented in Table 4.2.

The findings in Table 4.2 indicate that the psychological readiness of university teachers towards accepting the technology is moderate, which means teachers are psychologically ready but need a few improvements, as the overall mean of items was 3.02 (Yasin et al., 2016). This finding aligns with the finding of Bedada and Machaba (2022). They studied teachers in Ethiopian Universities on their knowledge of using technology during COVID-19 and found that they knew using technology in the classroom, but some factors hindered them.

Table 4.2 Psychological readiness to use e-learning

Psychological Iiems	SA	A	N	DA	SDA	Mean	SD
I enjoy using e-learning (online aspect)	8 (22.2%)	8 (22.2%)	9 (25.0%)	1 (2.8%)	8 (22.2%)	3.206	1.473
I have adapted to the new e-learning environment	14 (38.9%)	5 (13.9%)	11 (30.6%)	3 (8.3%)	1 (2.8%)	3.823	1.167
I see high value in e-learning for our students	0%	13 (36.1%)	8 (22.2%)	8 (22.2%)	5 (13.9%)	3.853	1.105
I feel e-learning in university is a waste of time	15 (41.7%)	2 (5.6%)	7 (19.4%)	2 (5.6%)	8 (22.2%)	3.412	1.654
Use of e-learning consumes a lot of time for class preparation	0%	8 (22.2%)	4 (11.1%)	14 (38.9%)	8 (22.2%)	3.353	1.098
I have poor attitude towards using e-learning	0%	14 (38.9%)	2 (5.6%)	0%	18 (50.0%)	3.882	.977
There is a poor attitude to e-learning among members of the University administration	5 (13.9%)	15 (41.7%)	4 (11.1%)	2 (5.6%)	8 (22.2%)	3.206	1.431
There is resistance to change among staff implementing e-learning	5 (13.9%)	5 (13.9%)	20 (55.6%)	3 (8.3%)	1 (2.8%)	3.294	.938
Overall						3.02	

Table 4.3 Teachers' sociological readiness to use e-learning

Sociological items	SA	A	N	DA	SD	Mean	SD
Teachers have shown interest in the use of the Digital Devices	4 (11.1%)	13 (36.1%)	10 (27.8%)	7 (19.4%)	0%	3.059	1.043
Teachers in my university are very supportive of the e-learning	3 (8.3%)	1 (2.8%)	19 (52.8%)	9 (25.0%)	2 (5.6%)	3.294	.906
There is no staff support with each other on e-learning because of Covid-19	9 (25.0%)	7 (19.4%)	8 (22.2%)	9 (25.0%)	1 (2.8%)	2.588	1.234
Overall						2.98	

Sociological Readiness

This section considers the interpersonal aspects of the context where the technology will be implemented. Their responses are presented in Table 4.3.

Table 4.3 indicates that the teachers' sociological readiness overall mean is somewhat moderate, thus needing more improvement than psychological readiness (2.98 < 3.02). In the case of sociological readiness, one of the factors is COVID-19. The teachers are not supported by each other because of the pandemic. The mean score of the statement that contained COVID-19 was 2.59, which is the smallest of all.

Environmental Readiness

This factor considers the internal and external macro forces influencing stakeholders in the organisation. Bedada (2021) considered this the first baseline of a cycle model in which one should consider the environment to integrate any instructional technology into the classroom teaching and learning. The Ethiopian government is ready to implement e-learning in education and other country sectors, as discussed in the study's literature review. At the university level, the study assessed environmental and equipment readiness (Laboratory class). The participants' responses are presented in Table 4.4.

Table 4.4 indicates that the overall environment and equipment in the university are only moderately sufficient, so it needs improvement by a concerned body. The findings indicate that the factors from all items with the lowest readiness are the training of teachers in the university with a mean of 1.68, which is low, and readiness to integrate e-learning in teaching with a mean of 1.94. In Table 4.2, the psychological readiness of teachers using e-learning is moderate, but the teachers are not ready on these two items, which should be considered as they may reduce the psychological readiness of the teacher to use e-Learning.

Human Resource Readiness, Financial and Course Content Items

According to Alakrasha and Razakb (2020), "for a nation to generate professionals that fulfil the demands of the workplace, the first necessity is to prepare the human resources". To set up these resources, the teachers must first be aware of the global transformations and understand the changing demands on teaching and learning strategies that must be adapted to remain competitive in the global marketplace. Teachers' responses toward Human Resource, Financial, and Course Content readiness items and knowledge of course design are presented in Table 4.5.

Table 4.4 Environmental and equipment items

Environmental and equipment items	Mean	SD
Community members of the University support e-learning	3.588	1.048
Teachers not involved in e-learning will get support at my university	3.382	1.129
We have ICT support staff linked to e-learning	3.794	.845
There is a mathematics laboratory at the university	3.559	.927
There is an electric power problem in the university	4.333	.777
I am trained for e-learning adoption for my teaching (University)	1.677	1.065
I am ready to integrate e-learning into my teaching because the university lacks connectivity	1.941	1.369
Overall	3.2	

Table 4.5 Human Resource readiness, Financial, and course Content items [HFC]

Human Resource, Financial, and Course content readiness items, and knowledge of developing course design for e-learning	Mean	SD
There is a technical support for e-learning	2.000	1.435
I'm educated on e-learning because the training was given by the ICT coordinator at my university	1.735	.666
The course I was trained in my degrees enables me to understand e-learning in the era of this fourth industrial revolution	1.735	.710
The University I was teaching at is ready to implement e-learning	3.059	1.434
My university can afford the budget to use e-learning in my department	4.735	5.224
My university gives the opportunities to experience sharing (Budget)with other University	3.618	1.015
I get a chance to take experience sharing with other universities concerned on e-learning	4.059	.694
The course was designed for the use of e-learning in my university	4.088	.866
I know the steps of designing a course for e-learning platforms	2.559	.504
I need help in designing my course to be used in e-learning	2.353	.950
The course prepared by MoSHE is best suited for using e-learning	2.000	1.000
I know 3'C model course design	2.059	1.013
I know we are in the fourth industrial revolution (4IR), thus expected from me to design the course for e-learning	2.353	.849
I know the strength, weakness, opportunities, and threats (SWOT) principle in 4IR	1.849	.870
Overall	3.17	

Table 4.5 indicates that the Human Resource, Financial, and Course Content items and knowledge of the course design for e-learning were moderate even though some individual items' mean was low and needed much attention. These items are the lack of knowledge of 4IR, the deficiencies in the teachers' training courses during their degree studies, a lack of training on e-learning, a lack of technical support on e-learning, and a lack of knowledge of designing a course for using e-learning. On these items, the teachers' readiness is low in using e-learning; in other words, they are not ready and need much work from concerned bodies. In the 4th IR, human skills (in this study, it can be technological skills and course content knowledge), infrastructure, and resources (budget), are factors of concern for employing the intended principles of the fourth industrial revolution in Africa (Kayembe & Nel, 2019)

Technological Skills

In these sections, we divide the skills into two parts: Technological skills readiness of the teachers in terms of Device/Internet application used and Technological skills readiness of the teachers in terms of Device/program used in March of 2021, and the responses of the participants are presented in Table 4.6.

Table 4.6 Technological skills readiness of Device/Internet application teachers used the previous month (March 2021)

How often have you used in last month (March) of 2021?	Mean	SD
Section A		
Desktop	1.647	.812
Laptop	1.647	.774
Tablet	2.667	.488
Smartphone	1.824	.834
Twitter	3.000	.000
Facebook	1.941	1.013
Google search	2.735	.666
Google Scholar	4.382	.493
Yahoo search	4.471	.507
e-mail	3.735	1.442
WhatsApp	3.265	1.657
Telegram	2.103	1.012
Overall	2.785	
Section		
Section B: Technological skills readiness of device/program teachers used last month (March 2021)	Mean	SD
Computer	1.0000	.00000
Internet	2.3824	1.20641
Word processor	2.9412	1.45521
Spreadsheet	3.1176	1.32035
Mathematica	1.0357	1.23175
Geometric sketch pad	1.0000	.00000
GeoGebra	2.2059	1.32068
Math lab	3.4118	1.45888
Overall	2.14	

In Table 4.6, Section A, we used a four-point Likert scale and coded it as *Daily = 4, Weekly = 3, Monthly = 2,* and *I do not use = 1* that consisted of twelve devices and created intervals for analysing the result in the table by applying the formula 4−1 = 3/5 = 0.75, and adding this value on to 1 until we get 4. Thus, we grouped it into four intervals, as shown in Table 4.7.

Using Table 4.6, the overall mean scores of Technological skills readiness of the Device/Internet application they used the previous month (March 2021) is 2.785, indicating that teachers use the technological devices weekly. They knew of the benefits of technology usage in education at the university level.

In Table 4.6 Section B, the teachers' technological skills were tested on the time they used the technological devices, and thus the finding shows that the programs/software usage in the previous month (March of 2021) was infrequent as the mean score was 2.14, they also never used computer (Mathematica, and Geometry) sketch pad. Participants occasionally used GeoGebra, Internet, and Word processing in the previous month (March 2021).

Table 4.7 Interpretation of mean scores

Groups	Range	Level (measurement)
1	1.00–1.75	I have not used
2	1.76–2.5	I used monthly
3	2.6–3.25	I used weekly
4	3.26–4	I used daily

Conclusion and Future Direction

The digital revolution is the widespread diffusion of information and communication technologies and transformation into an entirely digitised society. The study aimed to investigate Mathematics educators' readiness for online education during the Fourth Industrial Revolution and the COVID-19 pandemic in the case of selected Universities in Ethiopia. The Chapnick Readiness Model measured teachers' psychological, equipment, and technological readiness for online education. The study participants were university lecturers in two different universities in Ethiopia during the 2021 academic year. The study participants were purposively selected from the mathematics departments in the two universities. About 34 teachers participated. The age of participants ranged from 26 to 56 years of age, and their educational background ranged from BSc to assistant professor.

The study investigated the e-readiness of the teachers at the university level in this 4IR era. 4IR technology enables educators to understand the strength, weaknesses, opportunities, and threats in the university environment to teach their students using online systems to combat the COVID-19. In this way, the pandemic has brought another dimension to the teaching and learning process across all levels of education and the fourth industrial revolution. The lockdown imposed in many countries to fight the pandemic forced many learning institutions to shift to the online mode of teaching and learning. The study used the quantitative research method to investigate the universities' e-readiness for e-learning in the 4IR. The study findings revealed that in the 4IR, most of the teachers are far from the level of knowledge required in the 4IR, which supports the findings of El Nahrawy (2020). The study found that, while teachers psychologically, sociologically, environmentally, financially, and regarding human resources and content only had a moderate readiness, they lacked knowledge of 4IR in all these categories.

The study used the Chapnick model to determine the factors affecting teachers' e-readiness toward technology in general. These factors include the deficiencies in the training courses the teachers studied during their degree studies and a lack of knowledge of the first, second, third, and fourth industrial revolution, lack of training and support on e-learning, and lack of knowledge of designing a course for using e-learning. The teachers very seldom use these items and e-learning. In other words, they are somewhat not ready in all model dimensions. Therefore, they need training on e-learning, which aligns with the findings of Anberbir (2015). Moreover, attention to course content, presentation methods, software, hardware, and financial issues can create a suitable basic environment (Yazdanfar et al., 2021).

Future Direction

The study recommends the following main future directions:

The concerned body may need to forward the principles of the 4IR and the previous three education revolutions in the education sectors to bring Ethiopia to prosperity. In this regard, the universities may need to support teachers at all levels of education systems. If these are done, the teachers by themselves can design a course for e-learning to combat COVID-19 and other related issues by running their teaching and learning online system, as we do not know when COVID-19 will disappear from the world.

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Chapter 5
Mathematics Education and the Fourth Industrial Revolution: Are the High School Mathematics Teachers Ready?

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Introduction

The roles played by education are essential to any society, and its influence is mirrored in a society's strengths and weaknesses locally and internationally (Moloi & Matabane, 2020). Every nation strives to ensure its citizens get a good education for self-sustenance. Kehdinga and Fomunyam (2019) argue that for a society to compete economically globally, education is foundational and conditioned by a wide range of interconnected elements, including economics, advances in scientific technology, and industrial knowledge. Education changed during the fourth industrial revolution, which calls for people who will not just be comfortable with being self-sustained but citizens who can creatively solve problems as they emerge both on the local and global scenes (Fomunyam, 2020). This is one of the significant objectives of education in the Fourth Industrial Revolution, also denoted as the 4IR or 4.0 era. The industrial revolutions happened to be the essential singular developments in human history over the past three centuries. 4IR encompasses major developmental transformations, which rewrite human functionality in the form of inventions and radical changes in the way humans live and work in their environments due to disruptive technologies and trends such as robotics, Internet of Things (IoT) (smartphones, smartboards, smart tv, smart fridges, smart watches, etc), Artificial Intelligence(AI), Mobile Devices (Huawei, iPad, IPhone, Samsung etc.), location detection technologies (GPS, Google), Advanced Human-machine interface, Authentication and fraud detection (Blockchain), 3D printing (furniture, components, houses), Smart Sensors (CCTV) and Nanotechnologies, Big Data Analytics (Voice detection, Facial recognition, econometrics, Google Translate, Facebook, etc.), multilevel customer (user/student) interaction and profiling, virtual and

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augmented reality (DGSs, Virtual labs, Simulations, YouTube videos), Emergency Remote Teaching and Learning (ERTL), Online teaching and learning (OTL), Blended or Hyperflex Learning, Digital Literacy/Resources/Classroom/Schools, Cloud pedagogy (Blackboard, Moodle, Sakai, Google Classroom, Google Meet, MS Teams, Zoom, WhatsApp, etc), computational thinking, coding (Scratch) and ubiquitous computing (Ndlovu, 2021).

These changes have transcended from the use of machines to telecommunications, from electricity to new developments in what has been denoted as technology (Schwab, 2016; WEF & ADB, 2017) (See Fig. 5.1). These technologies and trends have blurred the physical, digital, and biological spheres and have impacted all disciplines, industries, and economies. These revolutionary digital technologies are reshaping the world, so mathematics teachers in sub-Saharan Africa should embrace them to enhance their teaching pedagogies. They include mobile internet, automation of knowledge work, Internet of things, cloud, advanced robotics, autonomous vehicles, nanotechnology, energy storage, 3D printing, advanced materials, advanced oil and gas exploration, and renewable energy.

Around the world, these changes are transforming business models, educational systems, and production processes daily (Butler-Adam, 2018). As noted in the 4IR, the unprecedented changes are not limited to industries and businesses alone. The field of mathematics education is not left out. With far more powerful and clever technologies, the 4IR era will undoubtedly lead to a great deal of change in mathematics education (Nadkarni & Prügl, 2021).

Education 4.0 involves an educational system that aligns with the emerging 4IR era, which focuses on smart technologies that are currently part of our daily lives (James, 2019). As Education 4.0 with it is big data and advanced digital literacy will undoubtedly affect the cognitive, affective, and psycho-productive domains of

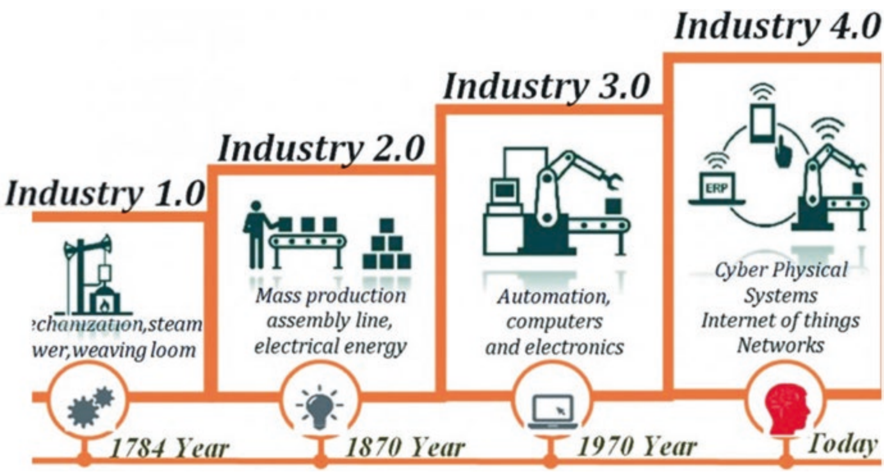

Fig. 5.1 The Dawn of 4IR. (Source: What is Industry 4.0 or the fourth industrial revolution? – TechFormation (cksimblog.com))

students' learning, it, therefore, becomes important that mathematics teachers should be ready to embrace relevant digital and data literacies that will improve their teaching practices and globally compete favourably in 4IR era. This calls for mathematics teachers who are willing and prepared with sufficient skills to take up the challenges in education in the 4IR. The concept known as *Mathematics Teacher 4.0* is adapted for future mathematics teachers who are versatile in handling and implementing new and modern technologies in their respective classes (Abdelrazeq et al., 2016). Since education is foundational to the workforce, the 4IR era consequently needs a new set of skills that must be impacted on the students, who will drive and lead the changes of the 4IR.

From the foregoing, previously acknowledged changes must be carried out in the current teaching strategies, and the contents are taught to students. And by extension, mathematics teachers saddled with this responsibility must be prepared through professional development programmes to acquire necessary and relevant skills in the 4IR era. This also calls for a new and more dynamic curriculum, in addition to using appropriate latest technologies in education (Junid et al., 2019). These processes cannot be achieved without preparing mathematics teachers, the educational sector's major human resources, and drivers. Additionally, the implications of 4IR for mathematics education vis-à-vis the readiness to embrace necessary skills in Nigeria need urgent attention. Mathematics teachers in Nigeria need to be responsive to the 4IR to ensure that students are well prepared and equipped to thrive in the future world of work. Hence, mathematics teachers' behaviour must change as modern and advanced classrooms emerge. They must be ready to adapt their roles and responsibilities as the processes of education change in the 4IR era.

The Fourth Industrial Revolution and Implication for Mathematics Education

The Fourth Industrial Revolution (4IR) is rooted in the integration of networked technologies to facilitate automation with the onset of artificial intelligence (AI) and the Internet of Things being used in our everyday lives. The application of such automation is also found in mathematics education. Teachers must be technology-oriented or digitally literate and responsible not only for teaching but also for learning. To adapt to the 4IR, students need to be equipped with innovative capabilities, life-long learning, and access to digital technology. In the digitalised society, mathematics is important in everyday life and the demands of the 4IR. Mathematical technology is needed to complement industry 4.0. Our everyday lives require quantities and numbers such as weight, length, area, and time, but the 4.0 industry requires us to understand quantities and numbers in terms of air pressure, humidity, population growth rates, and profits (Gravemeijer, 2017). As part of the Industry 4.0 initiative, we can understand big data analytics via mathematical knowledge such as statistics, space-geometry – which helps us understand 3D imaging for 3D printing – and covariation and functions. Since mathematics is the core of this digital era, students' mathematics competencies should be nurtured due to education 4.0 (Gravemeijer, 2017). Thus, mathematics teachers must have problem-solving skills, be creative, communicate properly, and be dynamic, resilient, flexible, and globally aware of the 4IR era. The discipline of mathematics education in 4IR offers to transform the teaching and learning process into a more effective strategy. Since 4IR is powered by artificial intelligence, we should prioritise changing how we teach. This part explores the impact of 4IR on mathematics education, including its curriculum and teaching and learning process (Butler-Adam, 2018). Considering the dynamic changes in society, mathematics education has to change. A revolution in teaching and learning methodologies is necessary to adopt a learning outcome based on competencies, blending academic and vocational education to answer the market need (Dabbagh, 2018).

The mathematics education curricular should be decolonised, and artificial intelligence should be integrated. Teaching and learning should reflect lifelong learning pathways, digital fluency, and mathematical skills for twenty-first-century curricula and educational innovations (Moloi & Matabane, 2020). This curriculum change should total reorientate mathematics in education and practice. Curricular should reflect technological sustainability and focus research on promoting technologies that immediately impact mathematics education. Kehdinga and Fomunyam (2019) further state that the curricular should also embed teaching and learning methodologies that would adapt to the market needs by becoming student-centred through task-based, project-based, problem-based learning, competency-based learning, and case study methods. Butler-Adam (2018) reports that teachers from various specialisations, including mathematics teachers, need to understand the different factors necessary to implement the 4IR successfully. This is because successful implementation of the 4IR in education will require vital skills to implement, manage, and work with new technologies and each other. Hu and Garimella (2014) also argued that a high percentage of teachers require support in integrating emerging technologies into their teaching pedagogies. Newhouse (2002); Schwab (2016) earlier submitted that a lot of teachers lack the knowledge and skills to use new technologies, in addition to not being enthusiastic about the changes being brought by twenty-first-century technologies.

Teaching Mathematics in the 4IR Era

Teaching Mathematics in the 4IR can be associated with Teaching 4.0 to 4.0 IR. The First Industrial Revolution brought a paradigm shift in mathematics teaching away from conventional methods toward future learning of mathematics (Gravemeijer, 2017). Therefore, Mathematics 4.0 can be formed to respond to the need for Teaching 4.0 and 4.0IR. Some individuals in the education sector say 4IR is hype and resist using 4IR technology for teaching and learning. Why not use 4IR technology in education to enhance pedagogical skills? (Ally & Wark, 2020). Digital technologies, open-sourced content, and creating a framework for future learning have high educational potential. There is an opportunity for teachers to revolutionise their teaching methods by learning, re-learning, and unlearning. Teachers' preparedness should not be limited to the current technology, and any new technologies must be accepted so that they stay current with 4IR's existing tools to enhance their teaching and learning (Jalil et al., 2022). Technology is an opportunity for them to differentiate instruction to modify information for the appropriate learning capabilities of their students. According to Oke and Fernandes (2020), 4IR could transform teaching and learning opportunities by changing learners' engagement. Learning materials and teaching are accessible to students irrespective of location, considering many African cities or towns' remoteness. Their role will change from teacher to mentor to facilitate students' learning, and they are still pertinent to academic performance, despite the changes in the education climate during 4IR.

Theoretical Framework

This study is anchored on the Unified Theory of Acceptance and Use of Technology (UTAUT). This theory was developed by Venkatesh et al. (2003). The scientific significance cannot be overestimated, and arguably the most recent model since Davis's Technology Acceptance Model was first propounded (Davis et al., 1989). Four dimensions were suggested by this model, which according to this study, are required to explore the complexity and ascertain whether mathematics teachers are ready to embrace the changes of the 4IR in mathematics education. Venkatesh et al. (2003) itemised these dimensions as performance expectancy (this is the extent to which a user believes a system use will help achieve gains in task performance), effort expectancy (this is the extent to which the user acknowledges that the system will be easy to use), social influence (this is the extent to which the user believes that significant others think they should use the system), and facilitating condition (this is the extent to which the user accepts that an organisational and technical infrastructure exists to support system use).

This theory is appropriate for this study because it is a technology acceptance model which explains user intentions to use an information system and integrates eight prominent models from many philosophical views (Venkatesh et al., 2003; Venkatesh & Davis, 2000; Davison & Argyriou, 2016). Education is centred on learners, yet the smart and innovative technologies of the 4IR will turn learning into something adaptive and more individualised in the future since there will be more use of smart instructional techniques (Gros, 2016; Hwang, 2014). Thus, the earlier the mathematics teachers key into the skills required for mathematics education in the 4IR, the better for their students and the nations. Teachers and learners must become literate in the different types of 4IR technologies to function effectively in the 4IR era. They do not need to be experts in the 4IR but require enough knowledge to protect themselves and use the technology responsibly. This study seeks to establish the influence of mathematics teachers' demographic variables such as age, and

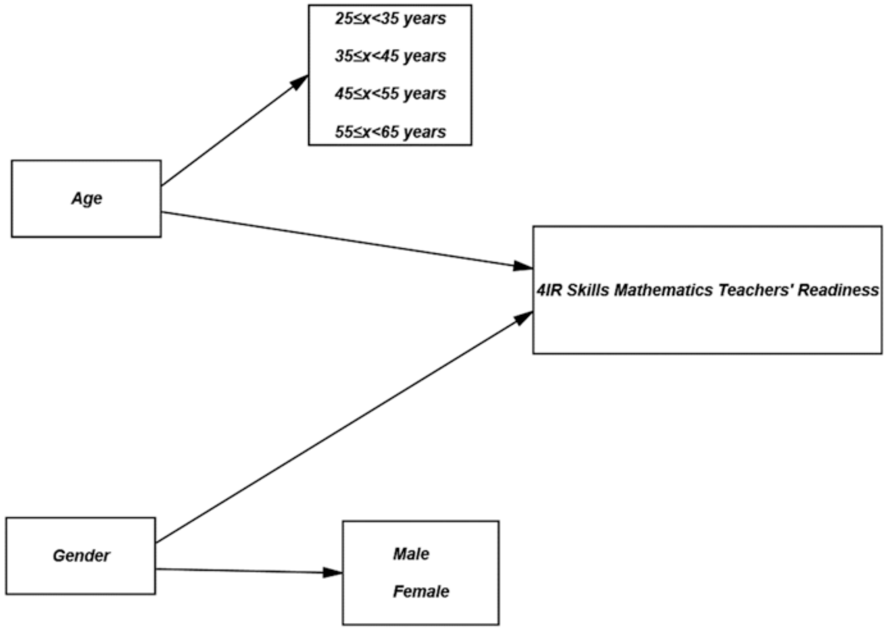

Fig. 5.2 Conceptual model

gender in embracing 4IR relevant skills to enhance their instructional pedagogy (See Fig. 5.2).

Gender and 4IR Tools Adoption

In a study of this nature, issues around teachers' gender are imperative. Since gender is one factor that affects the use of technology in teaching and learning (Zhou & Xu, 2007). Gender differences and the adoption of relevant 4IR tools have been reported in several studies. Some of these studies asserted that male teachers used more technology in their teaching and learning processes than their female counterparts (Kay, 2006; Wozney et al., 2006). Moreover, Markauskaite (2006) examined gender differences in self-reported information and communication technology (ICT) experience and ICT literacy among first-year graduate trainee teachers. The study remarked significant differences between males and females in technical ICT capabilities, while males' scores were higher. In another study on gender and computer anxiety levels, female teachers reported greater anxiety than male teachers. This connotes that female teachers are more likely to lag in embracing emerging technologies than male teachers (Mahdi & Dera, 2013).

Results from a study of 929 teachers' ICT integration in Queensland State indicated that female teachers' integration of technology into their teaching was less frequent than male teachers' (Jamieson-Proctor et al., 2006). In addition, Zhou and Xu (2007) suggested in their study that females had lower confidence and were less experienced in using computers in teaching. They tended to learn how to use technology from others, whereas males were more likely to learn from their own experiences. Based on these findings, their paper recommends that female professional development involve more showcasing and interaction. At the same time, training for males would be more appropriate when it provides many hands-on activities. Lescevica et al. (2013); Mazman et al. (2009) indicated that females are more inclined to adopt technological innovation through social influence rather than by a personal decision, whereas, in the case of males, the personal decision to adopt innovation is much stronger than social influence. Orser et al. (2019) submitted that gender influences technology adoption as men are more technologically adept than women. On the contrary, Davis and Davis (2007) reported no statistically significant difference in the overall perception of technological adoption in the teaching and learning process based on teachers' gender. Also, Goswami et al. (2015) asserted that technology adoption is consistent across gender.

Teachers' Age and 4IR Tools Adoption

The age group of teachers concerning their readiness to embrace 4IR tools for instructional methodology is considered in this study. For instance, Morris et al. (2005) studied the reaction and used behaviours among 342 teachers introduced to a new technology application for over a half year. They found that the adoption and use of technology differed based on age. Also, (Buabeng-Andoh, 2012) found that age was negatively associated with technology, pedagogy, and content knowledge. Czaja et al. (2006) indicated that the older teachers were less likely than younger teachers to use technology in general. Although more senior teachers in the United States are increasingly using technology, data indicate that they typically have more difficulty than younger ones learning to use and operate current technologies. Furthermore, Tacken et al. (2005) have found that older teachers expressed less comfort in using the technology and less confidence in their ability to use these emerging technologies successfully.

While there is an increasing emphasis on integrating technology into the teaching and learning of mathematics to produce digitally literate teachers that will function effectively in twenty-first-century mathematics education. In sub-Sahara Africa, there is evidence of a severe lack of innovative talents and skills among mathematics teachers to fully capitalise on the opportunities 4IR to enhance their instructional methodologies (Ayode, 2019). Consequently, based on the challenges raised in the background, the study is poised to answer the following research questions. (i) What is the level of mathematics teachers' readiness to acquire relevant skills for mathematics education in the 4IR, (ii) is there a statistical difference in mathematics teachers' willingness to develop relevant skills for education in the 4IR based on gender and age group respectively.

Methodology

In this study, a non-experimental design of survey research type was adopted, and a non-probability sampling technique of purposive method was used to select the samples for analysis. Mathematics teachers in government-owned schools were chosen to complete an online survey. Their consent to participate in the research was sought, and they were assured that their responses would be treated with the utmost confidentiality. However, before administering the scale items, ten research assistants were recruited to collect contacts of mathematics teachers in Education districts one, two, and three of Lagos State, Nigeria, for seamless dispatch of the survey link to their mails. The survey used a Likert response scale of 20 statements about the readiness of mathematics teachers to embrace relevant skills for 4IR that would enhance their teaching methodology. The survey took approximately 10 minutes to complete and comprised of two sessions.

The first session gathered information on teachers' demographics such as age, gender, and years of experience. The second session bothers teachers on their readiness to embrace the 4IR skills. The scale items were phrased as statements to which respondents were asked to indicate their extent level, using a four-point Likert scale of '4=Very large extent', '3= Large extent', '2= Some extent' '1=Not at all', respectively. Twenty-five pools of items were developed after reviewing literature and interacting with some mathematics teachers on their readiness to embrace 4IR skills. These items were subjected to face and content validity and reliability. After reviewing by three experts in computer science, their suggestions in terms of relevance, readability, language use, and rendition were used to come up with 20 items. The survived items had a content validity index (CVI) proposed by Lawshe (1975); Baghestani et al. (2019), after rating by five panellists in terms of "essentiality and non-essentiality", returned an index of 0.96 (i.e., 0.96 index implies that items of the scale adequately measured the variable of 4IR skills readiness) and MacDonald Omega reliability implemented in "user-friendly" package of R programming language gave an index of 0.84 (i.e., 0.84 index implies a measure of internal consistency and strength of association between items) respectively.

A goggle form containing scale items was developed, and the link was sent to all contacts. In all, 211 responses (61 females and 150 males) ranging in age from 26 to 58 years were received out of 302 emails sent out. The obtained data were subjected to descriptive statistics, an independent sample t-test, and one-way ANOVA implemented in Jamovi software version 2.2.3 (Jamovi Project, 2021). Jamovi is a new "3rd generation" statistical spreadsheet open-source software built on the R statistical language (R Core Team, 2021). The software is a compelling alternative to expensive statistical products such as Statistical Package for Social Sciences (SPSS) and SAS. Moreover, negatively worded items were recoded before the analysis was conducted.

Institutional Review Board Statement

The study was conducted according to the guidelines of the Declaration of Helsinki and approved by the Ethics Committee of the University of Johannesburg (protocol code Sem 2-2021-160 and date of approval: 10 November 2021).

Results

Preliminary analysis was conducted to check a few underlying assumptions on the adopted statistical tools for the study. Mathematics teachers' responses were subjected to normality and homogeneity of variance assumptions. The assessment revealed that the dataset followed a normal distribution with kurtosis and skewness values falling within the advanced benchmark by Hair et al. (2010); Bryne (2010) of −2.58 to +2.58, and Shapiro Wilk's test returned a non-significant value ($p > 0.05$). Furthermore, the test of homogeneity of variance using Levene's test yielded a non-significant value ($p > 0.05$). This implies that the participants used in the study do not statistically vary, i.e., they have similar attributes as mathematics teachers irrespective of their gender. Now that the two major assumptions are met, further analysis was conducted. To answer research question one, descriptive statistics were shown to assess mathematics teachers' readiness to embrace 4IR relevant skills. This was achieved using the median value, although there are other methods (frequency count, mode, quartile, etc.) since the dataset is in the ordinal measurement scale. These methods incorporate the variables' natural ordering to avoid power loss. It is imperative to reiterate that establishing a mean and standard deviation for ordinal data is often discouraged and not tenable statistically (Agresti, 2013). Table 5.1 presented the median value for each item, and for easy interpretation, the scale was classified into two, namely; 1–2.49 as low and 2.5–4.0 as high.

Table 5.1 showed that most of the items were above the cut-off that described their level of readiness. However, I feel apprehensive about *acquiring relevant 4IR skills to teach,* and *the 21st century Curricula is too ICT-oriented. As an educator, I am not prepared for its roles* had divergent views from the respondents. The overall implication of this result is that mathematics teachers have a positive disposition towards availing themselves to harness ample opportunities for 4IR tool uses to improve their classroom instruction. Also, an independent samples t-test was conducted to test the statistical difference in mathematics teachers' readiness to acquire relevant skills for education in the 4IR based on gender. This independent samples t-test was established at the item level and on the overall scale to see the difference in adopting 4IR skills between male and female mathematics teachers. The result is presented in Table 5.2.

Table 5.2 demonstrated t-test statistics for each item on the adoption of 4IR skills across mathematics teachers' by gender. Table 5.2 shows that 12 items showed a non-significant value at $\alpha = 0.05$ between male and female teachers. This implies no

Table 5.1 Descriptive statistics on the level of mathematics teachers' readiness to embrace 4IR relevant skills

Statements	Median	Skewness	Std. error skewness	Kurtosis	Std. error kurtosis
I am ready to learn about Artificial Intelligence, one of the technologies of the 4IR, to enhance my pedagogy.	3	−0.63	0.17	−0.50	0.33
My interaction with students would be clear and understandable with relevant 4IR skills.	4	−1.51	0.17	1.06	0.33
Embracing 4IR tools enables me to complete tasks more quickly.	4	−1.72	0.17	1.92	0.33
I am ready to learn modern pedagogies that will be the norm in the 4IR era.	4	−1.19	0.17	−0.06	0.33
Having 4IR relevant skills will increase my productivity.	4	−1.73	0.17	1.60	0.33
4IR skills would enhance career development.	4	−1.64	0.17	1.78	0.33
I am ready to acquire the skills of creativity and complex problem solving needed by educators of the 4IR.	3	−0.65	0.17	−0.49	0.33
I am ready to upgrade myself to fit into the teaching and learning pedagogies of the 4IR.	3	−0.58	0.17	−0.62	0.33
I am prepared to become the educator of the 4IR due to the many technological innovations involved.	3	−0.26	0.17	−0.42	0.33
The 4IR demands much learning. I am not prepared for the Lifelong Learning Pathways.	3	−0.88	0.17	1.44	0.33
I am ready to key into the opportunities of the 4IR by taking preparatory steps henceforth.	3	−0.94	0.17	1.41	0.33
I am ready to key into the transformation emerging digital technologies and innovations would cause in education in the 4IR era.	3	−0.75	0.17	0.64	0.33
I am ready to join the progressive educators preparing for the 4IR skills.	3	−0.14	0.17	−1.31	0.33
I would find 4IR skills useful in my instructional strategies.	3	−0.82	0.17	−0.55	0.33
The twenty-first century Curricula is too ICT-oriented. As an educator, I am not prepared for its roles.	2	0.01	0.17	−0.89	0.33
The use of smart boards scares me.	3	−0.65	0.17	0.09	0.33
I feel apprehensive about acquiring relevant 4IR skills to teach.	2	0.17	0.17	−1.45	0.33

(continued)

Table 5.1 (continued)

Statements	Median	Skewness	Std. error skewness	Kurtosis	Std. error kurtosis
I am ready to learn the Internet of Things (IoT) in preparation for 4IR	3	−0.09	0.17	−0.52	0.33
I am ready to key into Edutech Services and Education Innovation of the 4IR.	4	−1.01	0.17	−0.21	0.33
I am ready to use every available opportunity to update my knowledge to fit into the roles expected of educators of the 4IR.	3	−0.89	0.17	−0.58	0.33

difference in their disposition towards these statements on their readiness to embrace 4IR skills to improve their instructional pedagogy. However, when the independent t-test compared male and female responses on items such as *my interaction with students would be clear and understandable with relevant 4IR skills* ($t = 3.27$, $p < 0.05$), *embracing 4IR tools enables me to complete tasks more quickly* ($t = 4.71$, $p < 0.05$), *4IR skills would enhance career development* ($t = 3.52$, $p < 0.05$), *I am ready to join the progressive educators who are preparing for the 4IR skills* ($t = -3.53$, $p < 0.05$), *the 21st century Curricula is too ICT-oriented, as an educator, I am not prepared for its roles* ($t = -4.14$, $p < 0.05$), *I feel apprehensive in acquiring relevant 4IR skills to teach* ($t = -6.32$, $p < 0.05$), *I am ready to key into Edutech Services and Education Innovation of the 4IR* ($t = 2.10$, $p < 0.05$) and *I am prepared to use every available opportunity to update my knowledge to fit into the roles expected of educators of the 4IR* ($t = -3.61$, $p < 0.05$) and returned statistically significant gender differences. These results imply that male and female mathematics teachers had distinctly different opinions on these items. More so, to compare male and female mathematics teachers' overall disposition toward adopting 4IR skills to enhance their pedagogy and classroom learning. The t-test was conducted, and Table 5.3 presented the results.

Table 5.3 shows the estimated means of adopting 4IR skills between male and female teachers. The table reveals that male teachers had the highest mean score ($\bar{x} = 61.00$) than their female counterparts ($\bar{x} = 59.40$). The independent samples t-test (see Table 5.3) statistics showed that the mean difference was not statistically significant ($t = -1.81$, $df = 209$, $p = 0.07$). Consequently, the null hypothesis was accepted, which stated no statistical difference in the level of mathematics teachers' readiness to acquire relevant skills for education in the 4IR based on gender. This implies that mathematics teachers' readiness to embrace 4IR skills does not differ between males and females. They are equally ready to use these tools to enhance their teaching. Although, their readiness is subject to professional development and the provision of a conducive environment to acquire the necessary skills by the government.

Additionally, ANOVA was conducted to test the statistical difference in the level of mathematics teachers' readiness to acquire relevant skills for education in the 4IR based on age group. This test of ANOVA was established at the item level and

Table 5.2 Item level t-test statistics on the adoption of 4IR skills based on gender

Statements	Group	Mean	SD	t	p
Item1	Female	3.17	0.85	0.07	0.94
	Male	3.16	0.80		
Item2	Female	3.57	0.87	3.27	0.00
	Male	3.11	1.00		
Item3	Female	3.70	0.71	4.71	0.00
	Male	3.13	0.97		
Item4	Female	3.23	1.15	−0.17	0.86
	Male	3.26	0.93		
Item5	Female	3.54	0.97	1.55	0.12
	Male	3.31	0.98		
Item6	Female	3.60	0.84	3.52	0.00
	Male	3.15	0.87		
Item7	Female	3.19	0.82	−1.56	0.12
	Male	3.38	0.66		
Item8	Female	3.09	0.86	−1.77	0.08
	Male	3.31	0.79		
Item9	Female	2.57	0.75	−0.66	0.51
	Male	2.66	1.00		
Item10	Female	2.91	0.61	0.11	0.91
	Male	2.90	0.89		
Item11	Female	2.94	0.63	0.05	0.96
	Male	2.93	0.95		
Item12	Female	2.88	0.66	−0.46	0.64
	Male	2.93	1.01		
Item13	Female	2.23	1.06	−3.53	0.00
	Male	2.79	0.95		
Item14	Female	2.92	1.11	−1.31	0.19
	Male	3.13	0.94		
Item15	Female	2.27	0.84	−4.14	0.00
	Male	2.84	1.05		
Item16	Female	2.83	0.80	−1.84	0.07
	Male	3.07	0.91		
Item17	Female	1.95	1.06	−6.32	0.00
	Male	2.95	0.99		
Item18	Female	2.57	0.77	0.59	0.55
	Male	2.49	0.96		
Item19	Female	3.29	1.05	2.10	0.04
	Male	2.97	0.88		
Item20	Female	2.91	1.16	−3.61	0.00
	Male	3.49	0.79		

Table 5.3 Independent samples t-test on the readiness to adopt 4IR tools by gender

	Group	Mean	SD	t	p
4IR relevant skills	Female	59.4	5.49	−1.81	0.07
	Male	61	6.57		

on the overall scale to see the difference in the adoption of 4IR skills by different age groups of mathematics teachers. The result is presented in Table 5.4.

Table 5.4 depicts the one-way ANOVA statistics for each item on adopting 4IR skills across mathematics teachers' age groups. Table 5.4 revealed that 13 items showed a significant value at $\alpha = 0.05$ ($p < 0.05$) among the different age groups of the teachers. This implies that teachers' age contributes to their readiness to embrace 4IR skills to enhance instructional pedagogy and classroom activities. However, when the means were compared across the age groups on items such as *I am ready to learn about Artificial Intelligence which is one of the technologies of the 4IR to enhance my pedagogy* ($F_{(3,207)} = 0.37$, $p = 0.77 > 0.05$), *my interaction with students would be clear and understandable with relevant 4IR skills* ($F_{(3,207)} = 1.43$, $p = 0.24 > 0.05$), *I am ready to learn modern pedagogies that will be the norm in the 4IR Era* ($F_{(3,207)} = 1.34$, $p = 0.26 > 0.05$), *having 4IR relevant skills will increase my productivity* ($F_{(3,207)} = 0.59$, $p = 0.62 > 0.05$), *I am ready to acquire the skills of creativity and complex problem solving needed by educators of the 4IR* ($F_{(3,207)} = 1.48$, $p = 0.22 > 0.05$), *the 4IR demands much learning. I am not prepared for the Lifelong Learning Pathways* ($F_{(3,207)} = 0.20$, $p = 0.90 > 0.05$) and *I am ready to learn Internet of Things (IoT) in preparation for 4IR* ($F_{(3,207)} = 1.67$, $p = 0.18 > 0.05$), showed a statistical non-significant difference. These results imply that mathematics teachers did not differ on these items irrespective of their age group to embrace the necessary 4IR tools to better their instructional skills. Moreover, age group mathematics teachers' overall disposition toward adopting 4IR skills to enhance their pedagogy and classroom learning was conducted using ANOVA. Table 5.5 presents the results.

Table 5.5 shows the estimated means and analysis of the variance of adoption of 4IR skills across teachers' age groups. The Table revealed that age bracket $25 \le x < 35$ years had the highest mean score of ($\bar{x} = 0.02$, SD = 9.20), followed by age group $35 \le x < 45$ years with ($\bar{x} = 61.40$, SD = 4.94), next is $45 \le x < 55$ years with mean score of ($\bar{x} = 59.00$, SD = 4.00), and $55 \le x < 65$ years had mean score of ($\bar{x} = 56.20$, SD = 8.47) respectively. The results indicate that teachers in less age group are more ready to key into opportunities surrounded by the 4IR era and use the skills to better their teaching and learning process more than the older teachers. Moreover, the observed mean difference was examined using a one-way ANOVA. Table 5.5 showed that there is a statistically significant difference in the adoption of 4IR skills across mathematics teachers' age group with ($F_{(3,207)} = 10.8$, $p < 0.05$). Therefore, the hypothesis that "there is no significant difference in adopting 4IR skills across the mathematics teachers' age groups" was rejected. More importantly, since the difference is significant, assessing where the observed significance lies is inevitable (post hoc test). This feat was achieved using a pairwise comparison of the turkey method. The result is presented in Table 5.6 as follows:

Table 5.4 One-way ANOVA for item-level statistics on the adoption of 4IR skills by age

Statement	Age group	Mean	SD	F	p
Item1	$55 \le x < 65$ years	3.11	1.02		
	$45 \le x < 55$ years	3.17	0.85		
	$35 \le x < 45$ years	3.11	0.69	0.37	0.77
	$25 \le x < 35$ years	3.32	0.85		
Item2	$55 \le < 65$ years	3.39	1.04		
	$45 \le x < 55$ years	3.52	0.88		
	$35 \le x < 45$ years	3.37	0.79	1.43	0.24
	$25 \le x < 35$ years	3.12	1.24		
Item3	$55 \le x < 65$ years	3.28	1.02		
	$45 \le x < 55$ years	3.69	0.70		
	$35 \le x < 45$ years	3.08	1.00	6.39	0.00
	$25 \le x < 35$ years	3.60	0.82		
Item4	$55 \le x < 65$ years	3.00	1.24		
	$45 \le x < 55$ years	3.18	1.16		
	$35 \le x < 45$ years	3.37	0.88	1.34	0.26
	$25 \le x < 35$ years	3.56	0.77		
Item5	$55 \le x < 65$ years	3.33	1.03		
	$45 \le x < 55$ years	3.44	1.04		
	$35 \le x < 45$ years	3.53	0.80	0.59	0.62
	$25 \le x < 35$ years	3.68	0.80		
Item6	$55 \le x < 65$ years	2.78	1.40		
	$45 \le x < 55$ years	3.78	0.55		
	$35 \le x < 45$ years	2.97	0.89	18.34	0.00
	$25 \le x < 35$ years	3.12	1.01		
Item7	$55 \le x < 65$ years	3.17	0.86		
	$45 \le x < 55$ years	3.18	0.77		
	$35 \le x < 45$ years	3.32	0.78	1.48	0.22
	$25 \le x < 35$ years	3.52	0.77		
Item8	$55 \le x < 65$ years	2.50	1.15		
	$45 \le x < 55$ years	3.05	0.83		
	$35 \le x < 45$ years	3.45	0.50	10.99	0.00
	$25 \le x < 35$ years	3.72	0.54		
Item9	$55 \le x < 65$ years	2.00	1.03		
	$45 \le x < 55$ years	2.64	0.62		
	$35 \le x < 45$ years	2.66	0.94	3.64	0.01
	$25 \le x < 35$ years	2.72	1.21		
Item10	$55 \le x < 65$ years	2.83	1.20		
	$45 \le x < 55$ years	2.94	0.45		
	$35 \le x < 45$ years	2.87	0.70	0.20	0.90
	$25 \le x < 35$ years	2.88	1.20		

(continued)

Table 5.4 (continued)

Statement	Age group	Mean	SD	F	p
Item11	$55 \le x < 65$ years	2.33	1.09		
	$45 \le x < 55$ years	2.88	0.57		
	$35 \le x < 45$ years	3.24	0.59	8.31	0.00
	$25 \le x < 35$ years	3.20	1.04		
Item12	$55 \le x < 65$ years	2.50	1.15		
	$45 \le x < 55$ years	2.93	0.60		
	$35 \le x < 45$ years	2.82	0.80	2.55	0.04
	$25 \le x < 35$ years	3.12	1.09		
Item13	$55 \le x < 65$ years	2.61	0.98		
	$45 \le x < 55$ years	2.15	1.03		
	$35 \le x < 45$ years	2.79	0.91	6.45	0.00
	$25 \le x < 35$ years	2.88	1.17		
Item14	$55 \le x < 65$ years	3.22	0.81		
	$45 \le x < 55$ years	2.74	1.15		
	$35 \le x < 45$ years	3.37	0.82	6.55	0.00
	$25 \le x < 35$ years	3.48	0.71		
Item15	$55 \le x < 65$ years	2.39	1.20		
	$45 \le x < 55$ years	2.26	0.79		
	$35 \le x < 45$ years	2.68	0.96	5.28	0.01
	$25 \le x < 35$ years	2.96	1.17		
Item16	$55 \le x < 65$ years	2.56	1.15		
	$45 \le x < 55$ years	2.82	0.73		
	$35 \le x < 45$ years	3.05	0.90	4.08	0.00
	$25 \le x < 35$ years	3.32	0.85		
Item17	$55 \le x < 65$ years	2.67	1.19		
	$45 \le x < 55$ years	1.86	1.01		
	$35 \le x < 45$ years	2.74	0.92	16.73	0.00
	$25 \le x < 35$ years	3.16	1.11		
Item18	$55 \le x < 65$ years	2.28	1.07		
	$45 \le x < 55$ years	2.50	0.72		
	$35 \le x < 45$ years	2.74	0.64	1.67	0.18
	$25 \le x < 35$ years	2.68	1.28		
Item19	$55 \le x < 65$ years	2.67	1.09		
	$45 \le x < 55$ years	3.39	1.00		
	$35 \le x < 45$ years	2.89	0.73	4.98	0.00
	$25 \le x < 35$ years	3.00	1.16		
Item20	$55 \le x < 65$ years	3.56	0.78		
	$45 \le x < 55$ years	2.85	1.15		
	$35 \le x < 45$ years	3.34	0.88	5.08	0.00
	$25 \le x < 35$ years	3.48	1.05		

Table 5.5 One-way ANOVA on the adoption of 4IR skills across teachers' age groups

	Age	Mean	SD
4IR relevant skills	$55 \leq x < 65$ years	56.20	8.47
	$45 \leq x < 55$ years	59.00	4.00
	$35 \leq x < 45$ years	61.40	4.94
	$25 \leq x < 35$ years	64.50	9.20
		$F_{(3,207)} = 10.8$	$p < 0.05$

Table 5.6 Pairwise comparison of the adoption of 4IR skills across teachers' age group

		$55 \leq x < 65$ years	$45 \leq x < 55$ years	$35 \leq x < 45$ years	$25 \leq x < 35$ years
$55 \leq x < 65$ years	Mean difference	–	−2.82	−5.20	−8.35
	p-value	–	0.18	0.01[a]	0.01[a]
$45 \leq x < 55$ years	Mean difference		–	−2.38	−5.54
	p-value		–	0.09	0.01[a]
$35 \leq x < 45$ years	Mean difference			–	−3.15
	p-value			–	0.12
$25 \leq x < 35$ years	Mean difference				–
	p-value				–

[a]The mean difference is significant at the 0.05 level

Table 5.6 shows that there was a significant difference in the adoption of 4IR skills between $55 \leq x < 65$ years and $35 \leq x < 45$ years age group with (mean diff = −5.20, $p < 0.05$) as well as, $25 \leq x < 35$ years with (mean diff = −8.35, $p < 0.05$). Furthermore, there was a significant difference between the age bracket of $45 \leq x < 55$ years and $25 \leq x < 35$ years (mean diff = −5.54, $p < 0.05$). This result implies that teachers with less age bracket were positively disposed to embrace the 4IR tools for their instructional strategies. These results might allude to the fact that young teachers are more ICT compliant and have the technical know-how to use the tools of 4IR effectively compared to the older teachers.

Discussion

Technology has become an essential part of everyday life and an integral component of most daily activities. The 4IR is inevitable and happens fast compared to other previous revolutions. The current paper presents data from a diverse sample of mathematics teachers on their readiness to embrace 4IR skills to enhance their pedagogy and learning process. This article is poised to address the level of mathematics teachers' readiness to acquire relevant skills for mathematics education in the 4IR and to determine if there is a statistical difference in the level of mathematics teachers' willingness to develop relevant skills for education in the 4IR based on gender, age group, and teaching experience.

The results revealed that mathematics teachers in Lagos, Nigeria, are ready to embrace 4IR skills to enhance their pedagogy and learning process. A large percentage showed a positive disposition towards the items describing their readiness to adopt 4IR skills. Finding credence to Butler-Adam (2018) that teachers from various specialities need to understand the different factors necessary to successfully implement the fourth industrial revolution. Successful implementation of the 4IR in education will require vital skills to manage and work with new technologies. In contrast, the finding against the submission by Oke and Fernandes (2020) is that the education sector, especially in Africa, is unprepared for 4IR. However, there are indications for opportunities to harness the potential of the much-anticipated 4IR.

The findings showed no statistically significant mean difference between male and female mathematics teachers. This finding agreed with Davis and Davis (2007) study that no statistically significant difference was found in the overall perception of technological adoption in the teaching and learning process based on teachers' gender. Additionally, Goswami et al. (2015) support this finding that technology adoption is consistent across gender. Meanwhile, the results from this study disagreed with Mazman et al. (2009); Zhou & Xu (2007) study that females are more induced to adopt technological innovation through social influence rather than by a personal decision, whereas, in the case of males, the personal decision to adopt innovation is much stronger than social influence. In addition, these findings are against the submission of Mahdi and Dera (2013), Orser et al. (2019), and Jamieson-Proctor et al. (2006) that gender acts as an influencing factor in technology adoption as men are found to be more technologically adept compared to women.

Furthermore, at the item level, there was a significant difference in the 13 items describing teachers' readiness to embrace 4IR skills based on the age group, while seven items remarked a statistically non-significant difference across the various age brackets. Furthermore, mathematics teachers within the age bracket of $25 \leq x < 35$ and $35 \leq x < 45$ years were more ready to embrace the 4IR skills than older ones. In addition, the finding showed a statistically significant difference across the different age groups when examined through one-way analysis of variance, and post-hoc test analysis also confirmed this position. It indicates that the age group differs in their readiness to key into 4IR relevant skills to enhance their classroom methodology. The submission here laid credence to the work of Czaja et al. (Czaja et al. 2006); Tacken et al. (2015) that older teachers expressed less comfort in using technology and less confidence in their ability to use these emerging technologies successfully. This finding is against Buabeng-Andoh's (2012) position that age was negatively associated with technology knowledge and interest.

Conclusions and Recommendations

The study examined the mathematics teachers' readiness to embrace 4IR skills to enhance their pedagogy and learning. We conclude that mathematics teachers are ready to embrace the diverse skills and opportunities of the 4IR era. Furthermore, embracing 4IR skills did not differ across gender, although the reverse is the case among the teachers' age group. The study has implications for school administrators, mathematics teachers, and stakeholders in the education sector to aid policy design in the direction of 4IR. This would enhance the successful implementation of the 4IR in mathematics education that requires creativity, complex problem solving, social and systems skills (Schwab, 2016; Davis, 2016) or to implement, manage, and work with new/emerging technologies individually and collaboratively. Nevertheless, this study is without limitation, as the data was obtained from just three education districts in Lagos State, Nigeria, government-owned schools. This limits the generalizability of the findings as the scope of the study is geographically bounded, and mathematics teachers from private schools were not considered.

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Chapter 6
Transforming the Zimbabwean Secondary School Mathematics Curriculum to Align It with the Demands of the Fourth Industrial Revolution

Gladys Sunzuma, Brantina Chirinda, and Conilius Chagwiza

Introduction

Globally, various sectors, including the education sector, have embraced the Fourth Industrial Revolution (4IR) to address the challenges faced by African citizens. These challenges include climate change, drought, poverty, inequalities and the need to equip learners with twenty-first-century skills required in an ever-changing society. Zimbabwe is undergoing a socio-economic transformation where mathematics is key to development. Learners must acquire the necessary mathematical knowledge and skills and develop a positive attitude towards the learning area. To equip the learners with skills needed for the future industry and be ready for the 4IR, they need access to the mathematics curriculum that promotes technology-driven pedagogies. Technology-driven pedagogies are essential in fostering learners' problem-solving skills, critical thinking, collaboration and effective communication. The chapter focuses on how the Zimbabwean secondary school mathematics curriculum can be transformed to align it with the 4IR requirements. We formulated the following research question: How should the Zimbabwean mathematics curriculum be transformed to align it to the requirements of the fourth industrial revolution?

The chapter is organised in seven major sections: introduction, the 4IR; 4IR and mathematics education; focus and purpose of the chapter; methodology; findings and discussion; transforming the Zimbabwean secondary school mathematics curriculum to align with the 4IR requirements; and conclusion.

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The Fourth Industrial Revolution

People's lives and work have changed due to the 4IR (Schwab, 2016). The industrial revolution has undergone several stages, from the first to the fourth industrial revolution. Figure 6.1 shows the four industrial revolution stages.

The first industrial revolution (1IR), which stretched from 1760 to 1830, was characterised by the cotton gin, continent-wide railroad, electricity, and steam engines (Moloi & Mhlanga, 2021; Fomunyam, 2020). The systems mechanisation that characterised the 1IR led to the development of educational technologies, such as the graphite pencil, mechanical printing, typewriter, ballpoint pen, and paper-making machine. The teaching and learning of mathematics and other subjects were rooted in behaviourism and essentialism philosophies (Miranda et al., 2021). Teachers were at the centre of education, while learners were passive recipients.

The second industrial revolution (2IR), also recognised as the technological revolution, occurred between 1870 and 1914. The 2IR was characterised by technological innovations which focused on the expansion of electricity, and the production of steel and petroleum, which resulted in the manufacturing of aeroplanes and automobiles (Fomunyam, 2020). Some of the fundamental developments of the 2IR were the telephone, internal combustion engine, and light bulb (Schulze, 2019). The open-source resources from the libraries were the primary information sources. The technological developments during the 2IR resulted in educational electronic devices such as computers, calculators and printers. The 2IR was based on educational philosophies such as constructivism and andragogy. The role of the teacher changed from sagacious to being the source of information to help develop the materials for professional application. The role of the learner remained that of a

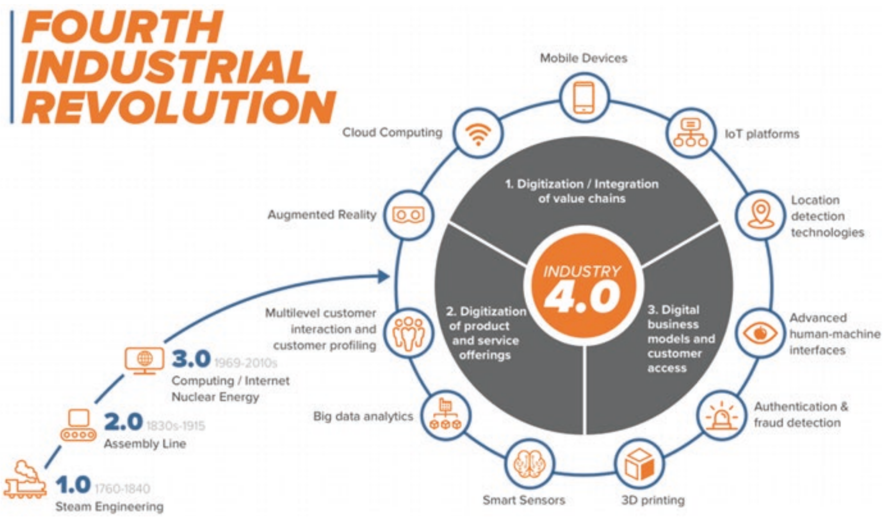

Fig. 6.1 The Dawn of 4IR (Schwab, 2016)

passive recipient. Learning was teacher-centred with encouragement on peer assessment and practices, for instance, broadcast education and correspondence education.

The third industrial revolution (3IR) began in the 1950s and is well-known as the digital revolution. The significant features of 3IR were personal computing, mainframe computing, semiconductors, and the internet used for phones, tablets and laptops (Marwala, 2020). Modern 3D printing, known as additive manufacturing, was another component of the 3IR. The 3IR marked the transition in which the teachers and the learners did not have to be involved sorely in synchronous teaching and learning sessions. Various resources reinforced new teaching and learning procedures, e.g., virtual laboratories, online tools and multimedia. The teaching and learning methodologies were more connectivist and heutagogical (Miranda et al., 2021). The teacher was regarded as a curator, collaborator and orchestrator, and the learner was empowered to construct knowledge.

Developing specific technologies that change society is a common feature of industrial revolutions (Moloi & Marwala, 2021). According to Schwab (2016), the 4IR is principally different from the preceding revolutions. The new revolution amalgamates the technologies and their interaction across the biological, digital and physical domains to nurture an all-encompassing, human-centred future. It focuses on innovative approaches to attaining the utmost good for the majority of the people, societies and organisations; it concentrates on how to improve life for the world's populace (Finley-Moise, 2019). The 4IR is a distinctive stage characterised by extraordinary scope, velocity and system impacts of technological innovations (Schwab, 2016).

Features of the Fourth Industrial Revolution

To understand the significance of 4IR in expediting teaching and learning practices, it is vital to have adequate knowledge of the various components of 4IR. The 4IR has led to disruptive technologies and trends, for example, Artificial Intelligence (AI), 3D printing, Virtual Reality (VR), Computational Thinking (CT), Augmented Reality (AR), robotics, and the Internet of Things (IoT) (Schwab, 2016). Embracing the components of 4IR implies that teaching mathematics is not limited to using a computer, particularly in the education sector, where opportunities such as developing an ecosystem might help share learning materials and data analytics to meet the learners' needs. In addition, biotechnology, nanotechnology, quantum computing and renewables are aspects of the 4IR (Schwab, 2016), which go beyond using computers and smart technology when teaching mathematics.

Artificial Intelligence (AI)

Artificial intelligence is the technology development that reproduces and develops human intelligence (Kambria, 2019). Technological advancements have resulted in an increased role AI plays in teaching mathematics. Gadanidis (2017) argues that AI affords numerous opportunities to design intelligent artefacts for mathematics teaching and learning. These interactive platforms explain mathematical ideas and theories, help solve problems and let students respond to each other.

Robotics

According to Perez et al. (2018), robotics resulted from improvements in computing, electrical engineering, and mechatronics. Robots are automated motorised teaching and learning materials that allow learners to comprehend logical and engineering concepts in action and enact mathematics concepts (Leoste & Heidmets, 2019). Educational robotics, which have become prevalent, comprises various modular educational robotics kits for developing programmable robots with intertwining plastic parts, such as Robotics Dream, VEX robots, LEGO MINDSTORMS and LEGO WeDo (Sisman et al., 2021). Educational robotics involves learners in interactive and hands-on activities and enables them to work together to understand the association between science, technology, engineering and mathematics (STEM) focused activities (Sisman et al., 2021).

Three-Dimensional (3D) Printing

Designing solid 3D objects from a digital file is known as 3D printing or additive manufacturing (Chow-Miller, 2018). It is cheaper and faster to design objects using 3D printing. In mathematics education, learners make 3D objects from geometrical designs instead of being provided with physically-build 3D objects. This allows them to be knowledge producers and not consumers of knowledge (Ng & Ferrara, 2020). The making of 3D objects requires incorporating skills and knowledge from diverse areas such as mathematics, computer science, and the science of materials and design (Budinski et al., 2019). 3D printing enhances learners' visualisation skills by designing and observing the finished objects. In addition, 3D printing enables the teachers to integrate project-based learning in mathematics teaching, whilst the learners experience the amalgamation of STEM subjects (Ng, 2017).

Computational Thinking (CT)

Computational thinking (CT) skills comprise the breakdown of complex problems into sub-problems using a set of rules that provide a sequence of stages in solving problems, investigating how a solution may be applied to similar problems through abstraction, and finding out whether computers can assist in solving such problems more efficiently through the use of automation (Yadav et al., 2016). An essential benefit of CT is automation, which involves programming environments, software modelling, and spreadsheet apps that help computers execute repetitive tasks (Hsu et al., 2018). CT involves data analysis, system thinking, programming, models, simulations and algorithmic thinking. It may be integrated into the mathematics curriculum through problem-solving.

Augmented Reality (AR) and Virtual Reality (VR)

Augmented reality (AR) is a technology used in overlaying reality using virtual objects in real-time and with the probability of interaction (Schutera et al., 2021). The way of comprehending is changed by integrating visual and sensory information using AR. AR uses 3D visualisations in real space, directly connecting them to the learning content in mathematics, such as geometry, which requires spatial imagination (Schutera et al., 2021). Virtual Reality (VR) is a simulated exercise where the learner can explore a collaborative computer-generated environment (İbili et al., 2020). VR and AR require a visualisation component that might be a traditional screen, such as computer monitors, mobile devices, and a head-mounted display (HMD). VR and AR are used in mathematics to explore spaces inaccessible because of physical restrictions, carry out experiments that would be intolerable due to risks or costs, and develop visualisation skills and spatial abstraction (İbili et al., 2020).

The Internet of Things (IoT)

Technological advancement has resulted in new notions of IoT. The IoT has made it possible for various electronic devices such as mobile devices, television sets and computers to connect to one another as well as the internet. The connection through the internet allows the sending and receiving of information from different devices. IoT is the network of physical objects or systems such as cloud technologies, wireless connections, embedded sensors, and actuators that communicate, sense, or interact with their interior environments or the exterior setting (Aheleroff et al., 2020). According to Karabegovi and Husak (2018), the digital interaction among and within devices and systems is central to the 4IR. Using actuators and sensors through IoT, collecting chemical, physical and biological data by learners from their environment allows the amalgamation of STEM disciplines. This leads to learners' awareness of the association between sciences and mathematics. One of the benefits of the IoT is immersing learners in mathematically-meaningful settings that enable them to value mathematics learning. IoT encourages inquiry-based learning that involves learners integrating mathematics with everyday situations (Kusmin, 2019).

The Fourth Industrial Revolution and Mathematics Education

Education in the 4IR incorporates new Infrastructure and evolving technologies. It goes beyond pedagogy and andragogy – towards an approach that merges *cybergogy*, *peeragogy and heutagogy* (Miranda et al., 2021). Cybergogy, which has resulted from technological advancements and the evolutions of the internet, is defined as learning approaches supported by ICTs that provide learning experiences that outdo space and time restrictions. Peeragogy talks about collaborative learning and encourages teaching methods that support learning amongst peers. Heutagogy encourages self-learning grounded in constructivist and humanist ideologies. It is centred on the learner whose understanding of the learning process is encouraged. Education during the 4IR promotes a transition from teacher-centred to learner-centred approaches and from passive to active learning.

Miranda et al. (2021) propose four principal components of education during the 4IR era, which are: (a) Competencies (training and development of desirable critical competencies in today's learners), (b) Learning Methods (incorporation of new learning methods), (c) Information and Communication Technologies (ICTs) (implementation of current and emerging ICTs), and (d) Infrastructure (use of innovative facilities, services, and systems to improve learning processes). Table 6.1 shows the four major components of 4IR education used to analyse the data in this study.

The above 4IR features allure to experiential learning, making mathematics concepts more accessible to learners. The amalgamation of knowledge from diverse subjects or topics allows mathematics applicability to be visible, including the opportunities for creativeness, invention, and problem-solving skills in realistic situations – the 4IR advocates for a project-based component and realistic learning experiences in the mathematics curriculum. The 4IR is a medium for altering mathematics teaching and learning from the accrual of principles, conventions and facts, which is usually the case in schools to mathematical applications and innovative activities. For the learners to be prepared to meet the ever-changing societal demands and problems, they should be educated differently from the past.

The 4IR has brought various changes in all sectors, fields and spheres of life, including education and industry. Technological changes such as AI and the IoT in the 4IR have varied implications for education and skills development. For example, the revamp of the education systems and tactical methods to increase creativeness and innovation. Zimbabwean education needs to be responsive to the 4IR so that learners are adequately prepared and armed to succeed in their future workplace. Responsiveness means that the curriculum can produce tech-savvy learners that can manage the workplace's digitisation in the 4IR era.

According to Schwab (2016), it has been established that societies and people who benefit more from the 4IR are those that can meet the expense of buying data and have access to the digital world. In Zimbabwe, digital resources are limited in most schools, yet the curriculum recommends using digital tools. The Zimbabwean curriculum has been described as too academic and insufficient to develop the needed human capital for the present 4IR (Moyo, 2022). The Zimbabwean curriculum does not specify how the technologies should be included in the teaching and learning of mathematics. For the 4IR to be extended to schools, the technology must be embedded into the education system to scale up the effectiveness of technology use in the education system in Zimbabwe. In terms of how the content is supposed to be delivered, the curriculum does not equip learners with the essential skills that

Table 6.1 Four components of Education 4.0

Categories		
Competencies	Transversal competencies	(a) Critical thinking, (b) cooperation, (c) collaboration, (d) communication, (e) creativity
	Disciplinary competencies.	(a) Training and developing functional, technical, and technological knowledge and successful workplace performance skills. (b) The capacity to research, design, create and implement new technologies. (c) The use of emerging technologies and best practices to propose technology-based solutions.
Learning methods	Learning delivery modalities.	(a) Face-to-face. (b) Learning is based mainly on active learning. (c) Online distance learning. (d) Hybrid learning.
	Learning strategies	Pedagogical approaches such as challenge-based learning, problem-based learning, learning-by-doing, and gamification-based learning.
Information and communication technologies (ICT) categories	Technology-based	Artificial intelligence and machine learning, high-data-processing applying data science, data analytics and cloud computing, and virtual image, processing for virtual and experiential environments.
	Tools and platforms	Synchronous online sessions to support student learning through web conference technologies (e.g., ZOOM, Meets, Webex, M-teams) and asynchronous learning platforms (Learning Management Systems, (LMS)).
Infrastructure levels	Classroom level	Innovative furniture; connected tools; classrooms, rooms, and libraries adapted with specific architecture, colours, illumination, sounds, and temperature to improve learning; virtual and augmented reality; and hologram systems.
	Institutional level	Includes recreation, comfort, sustainability, and accessibility; services such as online libraries, instant messaging systems, remote laboratories enabled and widely used; access to internet services.

Source: Miranda et al. (2021)

integrate mathematics with other subjects required for the 4IR. Teachers and learners are required to adapt to the era of digital technologies in 4IR that requires expertise and qualitative skills (Moyo, 2022), and yet Zimbabwe has been undergoing several economic challenges that have major negative effects on the education sector.

Methodology

The chapter focuses on how the Zimbabwean secondary school mathematics curriculum can be transformed to align it with the requirements of the 4IR. We used a curriculum analysis matrix to examine the four Zimbabwean secondary school mathematics curriculum documents to achieve this goal. The analysis was done in line with the four major components of 4IR presented in Table 6.1. The Zimbabwean competence-based curriculum has four different secondary school mathematics syllabi: the mathematics syllabus for Forms 1 up to 4 and the pure mathematics syllabus, statistics syllabus, and additional mathematics syllabus meant for Forms 3 and 4.

Findings and Discussion

We analysed the four Zimbabwean secondary school mathematics curricula to identify their relevance to the 4IR. The findings from the analysis are presented in Table 6.2 using the following categories: Content, Competencies, Teaching-learning methods, Technologies and Infrastructure.

Content

The content of the mathematics syllabus at the secondary school level has not changed since Zimbabwe gained its independence in 1980. It focuses on Numbers, Algebra, Geometry, Measurement, Vectors, Matrices, Probability and Statistics. Nonetheless, significant changes are noticeable in the teaching and learning resources; for example, there is a shift from using logarithm tables to calculators. The content in the four syllabuses is taught separately; however, teachers are encouraged to integrate mathematics into other learning areas and real-life situations. The 4IR advocates integrating STEM subjects, implying the four Zimbabwean mathematics syllabuses should not be taught separately. The topics such as vectors, matrices and sets are important in aspects of the 4IR such as artificial intelligence (AI). However, the syllabuses do not guide what is to be taught, when it will be taught, and the level to integrate it.

Competences

For learners to compete in the 4IR era, they need good communication skills, self-confidence, critical thinking skills, the ability to work together, and innovative and creative abilities (Sumar et al., 2019). Problem-solving and critical thinking are critical to the 4IR and are recommended in all four syllabuses. However, the challenge is how Zimbabwean learners can attain these skills in the classroom. The statistics syllabus is the only one that recommends that learners should have gained technology and innovation skills after completing Ordinary Level studies. Although creativity and innovativeness are indicated in the additional mathematics syllabus

Table 6.2 An analysis of the four Zimbabwean competence-based mathematics curricula in line with the components of 4IR education

Syllabus	**Mathematics syllabus (Forms 1–4)**
Content	(1) Real numbers (2) Sets (3) Financial Mathematics (4) Measures and mensuration (5) Graphs (6) Variation (7) Algebra (8) Geometry (9) Statistics (10) Trigonometry (11) Vectors (12) Matrices (13) Transformation (14) Probability
Competencies	It desires to produce learners with the ability to communicate effectively…p. 5. This will enable learners to be creative thinkers and problem-solvers ……p. 5.
Teaching-learning methods	The teaching and learning of mathematics must be learner-centred. The following are some of the suggested methods of the teaching and learning of mathematics guided discovery, discussion, interactive e-learning, exposition, demonstration and illustration, problem-solving, individualisation, simulation, visual-tactile, educational tours and expert guest presentation. p. 6 Six periods of 40 min each per week ….p. 6.
Technologies	Use ICT tools in problem-solving. p. 6 Use ICT tools to solve mathematical problems. p. 6
Infrastructure	None was specified.
Syllabus	**Pure mathematics syllabus (forms 3–4)**
Content	(1) Indices and irrational numbers (2) Polynomials (3) Identities, equations and inequalities (4) Graphs and coordinate geometry (5) Vectors (6) Functions (7) Sequences (8) Binomial expansions (9) Trigonometry (10) The logarithmic and exponential function (11) Differentiation (12) Integration (13) Numerical methods
Competencies	The syllabus intends to produce a citizen who is a critical thinker and problem solver. p. 5. Communicate mathematical ideas successfully. p. 6
Teaching-learning methods	The teaching and learning of pure Mathematics must be learner-centred and ICT driven. The following are suggested methods for teaching and learning pure Mathematics guided discovery, group work, interactive e-learning, games and puzzles, quiz, problem-solving, simulation and modelling and experimentation. p. 6 Six periods of 40 min each per week were allocated.
Technologies	Use ICT tools for learning and solving mathematical problems. p. 5 Use ICT tools for learning through problem solving. p. 6.
Infrastructure	None was specified.
Syllabus	**Additional mathematics syllabus (forms 3–4)**
Content	*Pure mathematics* (1) Indices and irrational numbers (2) Polynomials (3) Algebraic identities, equations and inequalities (4) Sequences and series (5) Coordinate geometry in two dimensions (6) Functions (7) Quadratic functions (8) Logarithmic and exponential functions (9) Trigonometrical functions (10) Differentiation (11) Integration *Probability and statistics* (1) Probability (2) Data collection and presentation (3) Measures of central tendency and dispersion (4) Discrete and continuous probability distributions (5) Normal distribution (6) Sampling methods (7) Estimation *Mechanics* (1) Kinematics of motion in a straight line (2) Forces and equilibrium (3) Newton's Laws of motion (4) Energy, work and power

(continued)

Table 6.2 (continued)

Competencies	Enhance confidence, critical thinking, innovativeness, creativity and problem-solving skills for sustainable development. p. 5–6.
Teaching-learning methods	The teaching and learning of mathematics must be learner-centred and practically oriented. The following are suggested methods guided discovery, collaborative learning, project-based learning, group work, interactive e-learning, problem-solving, simulation, visual-tactile, and educational tours. p. 6.
Technologies	Use ICT tools effectively to solve mathematical problems. p. 5. Use ICT tools responsibly in problem solving. p. 6.
Infrastructure	None was specified.
Syllabus	**Statistics syllabus (forms 3–4)**
Content	(1) Introduction to statistics (2) Data collection and presentation (3) Measures of central tendency (4) Measures of dispersion (5) Sampling (6) Probability (7) Random variables (8) Errors (9) Index numbers (10) Time series (11) Linear regression
Competencies	The statistics syllabus enables learners to develop skills in problem solving, critical thinking, decision making, leadership, self-management, communication, technology and innovation, and enterprise. p. 5
Teaching-learning methods	The following learner-centred and participatory methods are recommended in teaching statistics: Demonstrations, discovery, experimentation, group work, question and answer, problem solving, discussion, research and presentations, project-based learning, simulation and modelling. p. 6 Statistics is allocated five periods of 40 min each per week.
Technologies	Effectively use ICT tools to solve statistical problems. p. 5 Use ICT tools in statistical analysis. p. 6
Infrastructure	None was specified.

meant for those learners gifted in mathematics, communication skills are not emphasised.

Teaching-Learning Methods

The teaching and learning methods encouraged in the four Zimbabwean mathematics syllabuses are learner-centred and active learning. Learners need an education that offers opportunities to communicate, collaborate, solve problems, and think creatively and inventively. These skills and competencies are usually realised through blended learning - which combines face-to-face and e-learning. Blended learning entails optimising technology to obtain the competence and skills required to face the 4IR. All the four Zimbabwean mathematics syllabuses at the secondary school level are mainly anchored on the face-to-face mode of delivery and encourage learner-centred approaches to the teaching and learning of mathematics. Three of the syllabuses, except the statistics syllabus, list interactive e-learning as one of the teaching methods. However, the other three syllabuses do not indicate how interactive e-learning should be implemented in the Zimbabwean context.

Forty minutes are allocated for a mathematics lesson in the four syllabuses implying teachers have inadequate time to implement learner-centred approaches such as problem-solving and project-based learning, which are key to the 4IR. The success of using ICT tools hinges on the design of the ICT platforms and tools and the allocated time required to complete the detailed content in the curriculum (Sahal & Ozdemir, 2020). The component of formative assessment in the four syllabuses might imply spending more time on such assessments leaving inadequate time for learners' opportunities for creativity, innovativeness, problem solving and critical thinking.

Mathematics teaching and learning in Zimbabwe is confined to the four walls of the classroom where the teacher is in charge. This alone does not allow learners' creativity and innovativeness as required in the era of the 4IR.

Technology

The four syllabuses encourage the use of ICT tools in the teaching and learning of mathematics. However, it is not specified where and how the ICT tools should be used. Zimbabwean learners must memorise the geometry theorems and proofs for regurgitation during examinations despite the vast geometrical software freely available. This is because ICT tools are not emphasised as part of the teaching and assessment methods in the four syllabuses. The statistics and mathematics syllabus do not point out the use of Graphical software, Computer Algebra Systems and even the freely available Microsoft excel for teaching and learning statistics.

Regardless of technology innovations and advancements such as robots in science, technology, engineering and mathematics, the education sector has been unwilling to accept technology to facilitate teaching and learning (Oke & Fernande, 2020). The use of technology has been primarily limited to a didactic method of teaching and learning where teaching and learning are facilitated using a computer. Nonetheless, the technological approaches supporting the 4IR go beyond using a computer and electronic teaching resources and should be attuned to learner-centred approaches to be in a position to improve learners' learning experiences (Oke & Fernande, 2020).

Infrastructure

The Infrastructure that supports ICTs is one of the fundamental components of the education 4IR era. The Infrastructure required for teaching and learning mathematics is not indicated in Zimbabwe's mathematics four syllabuses. Infrastructure is one of the challenges that learning institutions must address, particularly in developing countries such as Zimbabwe.

Transforming the Zimbabwean Secondary School Mathematics Curriculum to Align with the Fourth Industrial Revolution Requirements

Our analysis of the Zimbabwean secondary school mathematics curriculum revealed that it does not provide learners with the necessary knowledge and competencies to perform in the 4IR. The rapid emergence of the 4IR digital technologies such as the IoT, CT, 3D printing, robotics, AR, VR, 5G networks, and AI has raised the urgency to adopt them in Zimbabwean mathematics education. However, there is no clear direction on how they should be integrated or how they should be implemented in the Zimbabwean mathematics curriculum. The curriculum needs to be transformed rather than reformed for the Zimbabwean mathematics curriculum to meet the requirements of the 4IR. Curriculum transformation focuses on generating new structures, yet reform focuses on existing systems (Akala, 2021). Zimbabwean mathematics curriculum transformation will help learners develop the skills and knowledge they need in the 4IR and establish thriving careers that are not endangered by the new technologies (Gwata, 2019). In the next sections, we propose how the Zimbabwean secondary school mathematics curriculum can be transformed to align with the 4IR requirements.

Collaboration Activities

Collaboration is a critical competence required for the 4IR. Employers in the 4IR era are looking for employees with collaboration skills. Collaboration skills are important in any workplace because teamwork drives productivity and promotes motivation and healthy employee relationships. As a result, collaboration skills need to be integrated into the Zimbabwean secondary school mathematics curriculum. Collaboration in the mathematics classroom requires learners to work together in pairs or groups to solve mathematical problems, develop mathematical ideas or complete projects. The Zimbabwean secondary school mathematics curriculum should provide exemplar activities that allow learners to work together and help each other. Zimbabwean mathematics teachers can adapt the exemplar activities according to their context or create similar activities that foster collaborative learning in their classrooms. The curriculum should require learners to work collaboratively on daily mathematics problems, activities, and projects. Teachers should check that each time learners work on tasks collaboratively, they agree or decide on the steps, strategies, processes and solutions as a group.

Collaborative learning help learners to organise and connect their mathematical thinking through communication, communicate their logical and clear mathematical thinking to their group members and teachers, and analyse and assess mathematical thinking and solution strategies other learners use (NCTM, 2000). As a result, working collaboratively in the mathematics classroom will help Zimbabwean learners develop interpersonal skills, communication skills, and knowledge-sharing techniques, which are necessary competencies to succeed in the 4IR. Learners who acquire these competencies from the classroom can easily apply them in their workplaces. Learners with collaboration skills are guaranteed to function seamlessly within workplaces of the 4IR.

New Literacies

Mathematics teaching and learning in Zimbabwe should focus on integrating digital literacy so that learners know how to use the recently developed technologies in the 4IR. Digital literacy is "the ability to find, evaluate, utilise, share, and create content using information technologies and the Internet" (Cornell University, n.d.). The Zimbabwean secondary school mathematics curriculum needs to add new literacies that require learners to read, collect, manage, evaluate, manipulate and use Big Data in their mathematics learning. The new literacies should also require learners to apply coding and AI technology in everyday mathematics. Zimbabwean mathematics learners must have a basic understanding of coding and AI concepts to interact efficaciously with AI systems in their future workplaces.

Mathematics Learning Factories

The Zimbabwean secondary school mathematics curriculum needs to integrate mathematics learning factories. Mathematics learning factories are collaborative spaces where practitioners benefit from the latest theory, research, and technology advances that mathematics learners bring. At the same time, mathematics learners learn from the practitioners and realise the practical advantages and disadvantages of implementing these theories, concepts and technologies in actual industrial settings. ICTs and VR technologies can create mock industries, allowing learners to visit industrial settings without interrupting processes or experiencing safety problems (Block et al., 2018). Mathematics learning factories help combine mathematics and real-life industrial settings to provide learners with a challenging and rewarding environment. Abele et al. (2015) observe that learning factories are an innovative way to simulate the industry settings for experimental and educational applications. In turn, this provides practitioners and learners with the opportunity to experience the industrial settings with the goal of understanding and integrating mathematical knowledge into contextual circumstances – a skill required in the 4IR.

Problem-Based Learning

Problem-based learning (PBL) should be integrated into the Zimbabwean secondary school mathematics curriculum. It is a teaching and learning approach that enables learners to learn mathematics while engaging actively with meaningful, ill-structured problems. This approach empowers learners to develop the necessary skills to solve real-world problems. PBL begins with an unstructured problem that the learners must solve using existing knowledge and new information they need to learn. If mathematics learners are confronted daily with ill-structured problems, they learn to apply new information to real-life problems, structure mathematical knowledge for future use in the 4IR, develop cognitive skills, and become lifelong learners. Moon and Seol (2017) observe that in the 4IR era, the idea of lifelong learning is high on the agenda. Dealing with ill-structured mathematical problems also helps learners to participate in learning actively and constructing knowledge rather than passively receiving information. As a result, learners acquire the necessary knowledge and competencies to solve future problems they will face at their workplaces in the 4IR era.

Our analysis revealed that the Zimbabwean secondary school mathematics curriculum predominantly focuses on structured problems. As a result, mathematical teaching and learning in the Zimbabwean classrooms are frequently guided by the assumption that each problem has one correct answer acceptable to the teacher. Consequently, learners' creativity is limited to replicating already known solution strategies and solutions (Renert, 2011). We recommend that the curriculum be revamped to make room for more ill-structured mathematical problems that require problem-based learning.

Infrastructure Relevant to Mathematics Teaching and Learning in the Fourth Industrial Revolution

We believe that technology on its own does not transform the mathematics curriculum; nonetheless, we recommend that the Zimbabwean government aim for interactive mathematics classrooms equipped with tablets that learners and teachers can use. The Egyptian government has done this where over 2000 schools have high-speed internet access, and 1.5 million tablets have been distributed to schools. The Egyptian government has also launched solar-powered smart classrooms, which are being built in rural areas, and the Zimbabwe government can follow the same route. Rwanda has endeavoured to electrify schools, allocate computers to all university students, and implement one laptop per child in schools (Munyengabe et al., 2017). This can also be done in Zimbabwe, a developing nation like Rwanda.

We recommend that the Zimbabwean government acquire technological gadgets like 3D Printing Pens to be used for teaching and learning mathematics in the Zimbabwean classrooms. The 3D Printing Pens can produce small, flat strings of molten thermoplastic on a surface or in the air. As the material hardens, it forms a volume of ink which simultaneously shapes into a 3D model. In contrast to a dormant 2D diagram drawn on paper with a pen or pencil, the 3D model can be manipulated. In this way, the Zimbabwean teachers and learners can interrelate with mathematics in a manner that is impossible when using computers, papers, pens or pencils. 3D models are essential for meaning-making in mathematics because they can be physically touched, transformed, or manipulated, thereby facilitating learners to learn abstract mathematical concepts physically through a trivial channel (Papert, 1980).

Conclusion

In the past couple of years, we have witnessed the emergence of various technological innovations from the 4IR, which have transformed the rules of the game regarding teaching and learning mathematics. Our analysis of the four Zimbabwean mathematics syllabuses revealed that learners are not being equipped with knowledge and skills that embrace 4IR technologies. Without the necessary knowledge and skills to compete in the 4IR era, Zimbabwean learners will not be prepared for the new jobs created. The Zimbabwean government needs to prepare learners with the knowledge and competencies that permit them to become productive employees and employers in the 4IR era. In this chapter, we have suggested how the Zimbabwean mathematics curriculum should be transformed so that learners can develop mathematics knowledge and competencies applicable in the 4IR era. Teachers' ability to teach in the 4IR was not the focus of this chapter; nonetheless, we recommend that the key stakeholders in Zimbabwe focus on developing the competencies, confidence, and desire of teachers to integrate technology into the teaching and learning of mathematics.

We are aware that mathematics education in 4IR requires investment, which can be challenging for a developing nation like Zimbabwe, still facing poverty. Nonetheless, like in Egypt, solar-powered classrooms can be constructed which make use of natural resources. We urge the Zimbabwean government to collaborate with non-governmental organisations and private companies to prioritise mathematics teacher and learner development in the 4IR. Zimbabwean private and public companies are already facing a dearth of 4IR skills, implying they need to invest in mathematics learners while still in school.

Future research should include teaching and learning systems that use AI, 3D printing, AR, robotics, IoT, and other technologies to learn more about the 4IR technology and how it affects the teaching and learning of mathematics in Zimbabwe. To better understand the various aspects of the 4IR technology and its impact on mathematics teaching and learning in Zimbabwe, future research should involve case studies, mixed methods research, and large sample surveys.

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Chapter 7
Mathematics Teachers' Self-Efficacy in Using Problem-Based Learning for the Fourth Industrial Revolution

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Introduction

The United Nations call upon scientists and policymakers to set principles that help achieve the Sustainable Development Goals (SDGs) by the 2030 agenda for sustainable development. The increased focus is on the use of the Fourth Industrial Revolution (4IR) technologies such as Artificial Intelligence (AI), the Internet of Things (IoT), robotics, data analysis and others in all sectors such as e-commerce, business, industry, healthy and social media and more importantly in education (Alakrash & Razak, 2022; Elayyan, 2021; Naidoo & Singh-Pillay, 2020). These 4IR technologies are advancing an integral shift in all aspects of human life (Bayode et al., 2019), education included. Education systems from elementary to university should transition to Education 4.0 to prepare learners to fit into the 4IR (Voskoglou, 2020). Increased attention is on integrating Information and Communication Technology (ICT) in teaching and learning (Akintolu & Uleanya, 2021). However, due to their location, government structure, lack of expertise and infrastructure, many African countries have not even been able to take full advantage of the first three revolutions (Ally & Wark, 2020). Their readiness for 4IR is still unknown, except for South Africa reported ready by the World Economic Forum (WEF), given insufficient infrastructures and lack of professional development on top of using curricula that do not accommodate changes of the 4IR (Voskoglou, 2020). In addition, some educators view the 4IR era as the hype and resist using it, making it remain at the infant stage and not well known (Ally & Wark, 2020; Voskoglou, 2020). So, there is a need to rethink how various technologies associated with 4IR can equip learners with twenty-first-century skills to evolve the job market and address challenge African citizens encounter, including exacerbating poverty, famine, and inequalities (Naidoo & Singh-Pillay, 2020).

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At the same time, educators are called upon to implement teaching approaches that are likely to enable learners to be equipped with skills for the 4IR, especially in mathematics, since it plays a fundamental role in building the mental discipline and reasoning (Baum et al., 2011; Lekwa et al., 2019; Scepanovic, 2019). These skills should involve learners going beyond remembering facts to how such facts can be used, for example, integrative problem solving that draws on multiple disciplines and critical and creative thinking (Elayyan, 2021).

In addition, cognitive flexibility is based on broader perspectives and the ability to see the connectivity of things (system view), the technological skills to understand links on the one hand and human skills on the other. These skills are needed to understand and relate to AI (robotics) products and how they were created. Although the literature describes this change and its apparent benefits, Katz (1996) observes that teachers often feel that there is little mention of how innovations might be implemented in an existing educational setting. Studies (Baum et al., 2011; Gravemeijer et al., 2017; Osman & Kriek, 2021; Scepanovic, 2019) that attempted to respond to this challenge encourage the promotion of students' learning of concepts and principles in mathematics through investigations of complex real-world problems.

In his presentation at the AFRICME6 conference (25–27 October 2021), Luneta expressed that while the 4IR describes the exponential change to the way the communities will live and communicate due to the internet of things and the cyber-physical systems, its direct influence on mathematics curriculum is yet to be debated and realized. In this line, the Problem-based Learning (PBL) model has received considerable attention in mathematics education as one of the promising teaching approaches to help learners embrace the 4IR. With PBL, skills will be developed to promote innovation in the context of 4IR. In addition, with PBL, learners acquire critical thinking, creativity, communication and collaboration skills to adapt to the 4IR through problem identification and proposing solutions.

In this book chapter, we intend to answer the following research question: to what extent do mathematics teachers acquire self-efficacy in using a problem-based learning approach to equip learners with skills for the 4IR? This is achieved through (i) analyzing mathematics teachers'knowledge and teaching practices change due to an intervention on PBL and (ii) teachers' experiences and challenges encountered while implementing PBL. The findings of the present book chapter are drawn from a study that explored participant teachers' experiences and changes in their practices through an intervention of problem-based learning approaches.

Literature Review

This section presents a brief theoretical background of the fourth industrial revolution, problem-based learning, and teacher self-efficacy.

The Fourth Industrial Revolution

Like the first, second and the third revolutions, the 4IR informs humans about crucial changes in daily activities. It forces humans to think creatively and to embrace changes in the workplace where everything will need to be reexamined and connected. In this regard, the educational sector is required to think about how it can prepare young generations to embrace changes and challenges ahead to transform society for the better (Scepanovic, 2019).

The 4IR affects the roles of educators in preparing youth for the workplace of this new world. Drawing on Effoduh (2016), the 4IR refers to the technological revolution that has altered how we live and think and who we are and is characterized by artificial intelligence. It is a revolution where machines have or intend to replace humans. These revolutions redesigned every aspect of human life today, apparent in the labour market. Indeed, Scepanovic (2019) predicted the loss of jobs due to the emerging technology and shortcomings of education systems due to emerging Artificial Intelligence (AI), robotics, the internet, nanotechnology, quantum computing, biotechnology, 3D printing, etc. This means there will be a shortage of skills to navigate this future. Schools, especially teachers as crucial drivers of educational systems, are therefore urged to apply the teaching approaches that prepare youth for the workplace and face the challenges of this new world. This approach is discussed in the next section.

Problem-Based Learning

Problem-based Learning (PBL) is learner-centred teaching and learning strategy. It involves learners in solving ill-structured problems to develop skills such as problem-solving, critical and creative thinking (that include imagination, creativity, flexibility and lifelong learning, which are all based on extreme curiosity) through collaboration and teamwork (De Graaff & Kolmos, 2003; Savery, 2006; Surif et al., 2013). The PBL approach allows learners to pose problems, make questions, think critically, present their creative ideas and communicate with peers. The learning process within PBL starts by introducing the problem, reading the problem critically and making discussions in small groups: research, brainstorming and proposing solutions, sharing the findings to the whole class in order to get reflections from peers; teachers monitor the discussions and orient them to the learning objective, and then the learning goal is evaluated.

The PBL approach has various advantages, including constructing extensive and flexible knowledge, developing practical problem-solving, self-directed and lifelong learning skills, becoming effective collaborators, and being intrinsically motivated (Barrows & Kelson, 1995 cited in Himelo-Silver, 2004). It supports learners in understanding the interconnection of different knowledge applied to the real world to create something for the 4IR. It integrates the learning content with real-life applications and offers a diversified way of thinking through the context of a particular problem (Savery, 2019). Scepanovic (2019) argued that education has to focus on learners' collaboration and teamwork that will enable them to solve real problems and create diverse educational reforms for the 4IR, which led to PBL. Thus, the PBL approach is a promising teaching and learning strategy that mathematics educators can adopt to equip learners with skills of the 4IR (Baum et al., 2011).

Although the PBL approach has been implemented in schools to facilitate the development of twenty-first-century skills and competencies for decades, teachers encounter several difficulties (Savery, 2019). Among them are a limited time for generating meaningful problems and content knowledge, limited equipment and infrastructure, trouble collaborating with other teachers and managing large class sizes during small group meetings, trouble getting IT resources, and internet connectivity to search information (Osman & Kriek, 2021). The difficulty of getting learners to work together and stay connected to work has also been reported (Aksela & Haatainen, 2019). Consequently, teachers fail to shift towards more specific teaching practices (Al Said et al., 2019; Lekwa et al., 2019). Therefore, there is a need to scaffold teachers' teaching practices within the PBL process to overcome difficulties. This would mean getting continuous shortcomings in the education system.

Chapman (2016) indicates that during the intervention, teachers need to understand what they have been doing to do what they are proposed to do and be confident to implement it successfully. Thus, by implementing PBL, we intend for in-service mathematics teachers to experience implementing an innovative teaching approach and help them equip learners with twenty-first-century skills. The study focused on grade eleven learners. Participant teachers are qualified to teach mathematics at the secondary level. They have received Continuous Professional Development (CPD) in teaching mathematics with specific teaching practices aligned with the teaching to develop competencies. We also note that these teachers have considerable difficulties communicating in the language of teaching and learning due to the shift from French to English in the Rwandan education system.

Teacher Self-Efficacy

Teacher self-efficacy refers to the capability of teachers to deliver a lesson effectively. According to Morris et al. (2016), teacher self-efficacy is defined as the beliefs or capacities associated with the teaching of a subject matter effectively in order to bring about desired learning outcomes. In line with this definition, the current study defines teacher self-efficacy as the sets of perceptions and interpretations developed over time to teach a subject matter effectively due to experience. Researchers (e.g., Nie et al., 2013) have argued that teacher self-efficacy is associated with classroom behaviours that affect instructional practices. Choi et al. (2019) argued that teacher self-efficacy is developed by implementing innovative teaching practices. Therefore, changing instructional practices may result in a change in teacher self-efficacy. It has been argued that teacher self-efficacy determines the use of constructivist instructional practices (Nie et al., 2013). Therefore, it is essential to look into this through the implementation of PBL to prepare for the 4IR.

Theoretical Framework

Constructivists' teaching and learning theory reflect the processes within which PBL is conducted in the classroom. The theory states that learners construct knowledge by building upon experience (Vygotsky & Cole, 2018). Remarkably, Vygotsky's social constructivism theory states that learners reconstruct knowledge and learn from one another due to interaction (Vygotsky, 1978). Learners benefit from learning socially and interactively as they obtain knowledge easily and get opportunities to develop skills, concepts, and understanding. Teamwork and collaboration characterize the PBL classroom, where the teacher works as a facilitator and catalyst of the learning process while learners actively engage in the learning process by making interpretations and constructing new knowledge (De Graaff & Kolmos, 2003; Savery, 2006; Surif et al., 2013). In addition, we believe that if learners are enabled to learn new mathematical concepts based on their prior knowledge, it would not be difficult for them to embrace the 4IR.

Research Methodology

This section presents the research design and paradigm, participants and sampling techniques, research instruments, data collection procedures, and data analysis.

Research Design and Paradigm

The study used an exploratory design. The study is qualitative and was guided by an interpretivist philosophical orientation. The choice was based on the fact that the study aims to understand the world from the subjective experience of secondary mathematics teachers after participating in the instructional intervention, PBL, after 14 weeks of practice.

Participants and Sampling Techniques

The study used a purposive nonprobability sampling technique to select key informants for the research. Two out of five mathematics teachers of one school in the Kayonza district of Rwanda were selected. These teachers were qualified to teach mathematics at the secondary level. One teacher had 33 years of experience in teaching mathematics, while another had only two years of teaching experience. Both were teaching grade 11 learners in different combinations. Before conducting this research, teachers consent to participate in the study to ensure voluntary participation. The researchers explained the purpose of the study and the probable duration of the intervention prior to commencing the research to the participant teachers.

Research Instruments

An interview guide and a classroom observation protocol were used to collect qualitative data. Teachers were interviewed before and after the intervention. The pre-intervention interview consisted of six questions that explored teachers' knowledge and practices of the PBL strategy before the intervention. After the intervention, a post-interview was given to explore changes in the teaching knowledge and challenges encountered while implementing the PBL strategy. Four questions covered the changes, while two questions covered the challenges. A Reformed Teaching Observation Protocol (RTOP) was used to capture the teaching practices of participant teachers before, during and after the intervention. The protocol was appropriate for this study since it has five primary constructs similar to the PBL behaviours, as Colburn (2000) cited in Osman and Kriek (2021) argued. For instance, in PBL, teachers explain, monitor the classroom discourse, talk when students work in small groups, engage students in the learning process, and challenge them to develop their problem-solving skills (Surif et al., 2013).

According to Sawada et al. (2002), the five main constructs are: *lesson design and implementation*, where the lesson observation focuses on the teacher acknowledging and respecting ideas that students bring in the classroom; *propositional knowledge*: the lesson observation focuses on how well the lesson is organized and presented to learners, does the lesson promotes conceptual understanding, does it make connections with other disciplines? Do real-life phenomena valued; *procedural knowledge*: the focus was on how do teachers apply reasoning and understanding to help students make predictions, approximations, hypotheses and ways to test them); *communicative interactions*: the lesson observation values students' communications/ teamwork and how the teacher asks questions to foster diverse thinking, and *the teacher-student relationship* focuses on how do teachers help learners to develop skills and engage them in activities in order to encourage active participation and being responsible for their learning. Our Reformed Teaching Observation Protocol (RTOP) focused on these constructs.

Procedures of Data Collection

Before commencing the intervention, the lessons of participating teachers were observed to collect database information on their teaching practices. Teachers were interviewed after each classroom observation to collect information on their PBL knowledge prior to the intervention. A two-day workshop was conducted with mathematics teachers of the school of intervention to develop their skills of PBL in Algebra, specifically, sequences, solving equations and logarithms and exponential equations. The intervention focused on structuring good ill-structured problems using available and accessible school books and processes involved when teaching using the PBL strategy. Participants in the workshop were five in total and worked together to scaffold each other. After the workshop, two teachers implemented the PBL strategy for 14 weeks. Since the workshop took a few days, researchers assisted teachers during the intervention period. Data were again collected during implementation using RTOP to assess changes in the teaching practices. After the intervention, teachers were again observed to establish changes in the teaching practices before, during and after the intervention. In addition, interview with teachers was conducted to explore teachers' experiences while implementing the PBL strategy.

Data Analysis

Thematic analysis and descriptive statistics were used to analyze the collected data. Interview responses were recorded and transcribed. Narration methods were used to describe identified themes for two teachers. A particular focus was given to the knowledge of the PBL strategy and teachers' experiences while implementing the PBL strategy. Data from RTOP were coded on ratings 0 = no, 1 = low, 2 = moderate, 3 = high and 4 = very high for the five primary constructs of PBL. Each construct has five items. A high score would indicate alignment with specific teaching practices. The classroom observations were performed in three blocks: before, during and after the intervention. Every teacher was observed at least three times in each block, as Sawada et al. (2002) recommended. The ratings for each teacher were compared before, during, and after the intervention.

Results and Discussions

The question guided the present book chapter: to what extent do mathematics teachers acquire self-efficacy in using a problem-based learning approach to equip learners with skills for the 4IR? The question was answered by analyzing mathematics teachers'knowledge, and teaching practices change due to an intervention on PBL. The two aspects are discussed in the present section that ends with some authors' thoughts on how the 4IR might enhance mathematics teaching through a problem-based learning approach.

Teachers' Knowledge

Data from the interview were analyzed to identify teachers' knowledge of the PBL before and after the intervention. Six themes were identified, including a good understanding of PBL concepts, processes of PBL, conditions to formulate a good problem, learning objectives of a lesson, assessing understanding, and willingness to implement PBL. The data indicates that participating teachers were unaware of the PBL strategy prior to the intervention. After the intervention, both teachers reported a good understanding of the processes involved in implementing the PBL strategy.

> Identification of the problem; the problem is given to students to read critically to understand it. Then students are given time to work in small groups to search for information and propose solutions; teachers can also support students through questioning. After that, we afford students time to present their products to the whole group. Reflections from peers are welcomed. Our role is to monitor the classroom discourse, remind them to speak after another, respect ideas from peers, and so on. Then, we provide a conclusion by linking the product with the learning goal.

In addition, one of the teachers demonstrated a good understanding of what is required to formulate a good ill-structured problem that is in line with Savery (2019) as follows.

> The problem must be challenging and exciting to the level of students and should be designed to make students want to solve it. The problem must stimulate students' willingness to stay connected to the task. This problem is framed to real-life situations and connected to the content we want to teach in the class. This means that the learning objective of a particular topic should be incorporated into the problem. The problem is open-ended and welcomes different pathways to the solution. This requires students to make decisions based on the correct information and facts since they may have collected information from different sources.

From the description above, it is evident that teachers have developed a good knowledge of the PBL after participating in the PBL intervention and collaboration between researchers and teachers.

Teaching Practices

To explore changes in the teaching practices of participant teachers, we captured the classroom behaviour using the RTOP observation tool before, during and after PBL intervention. The overall performance of the observed teachers on the 25 items of RTOP indicators out of 100 shows that the intervention was continuously digested towards more specific teaching practices aligned with PBL characteristics (see Fig. 7.1) with 36% of average change.

However, evidence shows that Teacher 1 changed the teaching practices than Teacher 2. This can be attributed to the teaching experience of Teacher 1 with 33 years old. It was easier for Teacher 2 to adopt the teaching strategy than for

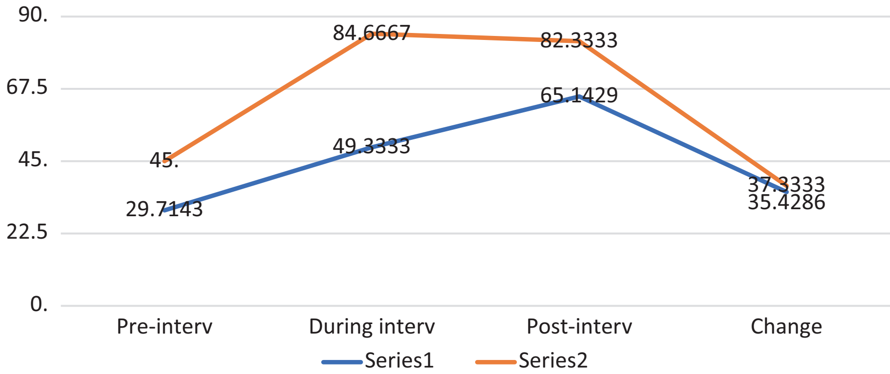

Fig. 7.1 Teachers' teaching practices before, during and after the intervention

Teacher 1, who had been accustomed to the traditional teaching strategy for so long. This finding aligns with the study by Kini and Podolsky (2019), who found that more experienced teachers improve their effectiveness through continuous professional development intervention.

To explore changes in the teaching practices of in-service mathematics teachers more specifically, we calculated the average scores of each item of the RTOP over 20, as illustrated in Table 7.1, before, during and after the intervention of two teachers. Results indicate a significant difference among five primary constructs of RTOP for both Teacher 1 and Teacher 2 in the pre, during and post-intervention (see Table 7.1). The most significantly improved communicative interaction with learners (43% of change). The comparison of pre and post-intervention percentages indicates that both teachers (Teacher 1: before 32% and after 73% and Teacher 2, before 43% and after 90%) focused on or valued the teamwork spirit during and after the intervention as they did before intervention.

Both teachers have triggered divergent modes of thinking to activate learners to focus on their discussions. For instance, one of the Teachers1's questions was:

> What could you be focusing on if you are to make the general formula (conjecture) of this arithmetic sequence? Why?
>
> How would you conclude for the graph of $y = 2^x$for$-2 \leq x \leq 3$ and the graph of $y = -2$ and $y = 3$ lying on the same plane?

According to the classroom evidence, participant teachers respected students' prior knowledge, learning communities, voices, and ideas while preparing lessons during and after the intervention. This finding indicates that the intervention has improved their lesson plan and implementation level. The finding is consistent with the view of Osman and Kriek (2021) that teachers gain enough knowledge after participating in the professional development PBL course that leads them to substantive change in the teaching practice.

Classroom observation also shows that although teachers demonstrated a good understanding of mathematics concepts prior to the intervention, but have strengthened the way a lesson is well organized and better presented to learners. This was

Table 7.1 Five themes (lesson design and implementation, propositional knowledge, procedural knowledge, Communicative Interactions and Teacher-Student Relationship) of the 25 RTOP statements

		Teacher 1			Teacher 2		
SN	RTOP statements	B	D	A	B	D	A
1	The instructional strategies and activities respected students' prior knowledge and the preconceptions inherent therein.	50.00	60.00	75.00	66.67	90.00	91.67
2	The lesson was designed to engage students as members of a learning community.	28.57	73.33	82.14	41.67	100.00	91.67
3	In this lesson, student exploration preceded formal presentation.	7.14	63.33	42.86	58.33	65.00	83.33
4	This lesson encouraged students to seek and value alternative investigation or problem-solving modes.	10.71	28.33	64.29	58.33	90.00	50.00
5	The focus and direction of the lesson were often determined by ideas originating from students	21.43	35.00	42.86	33.33	90.00	75.00
6	The lesson involved fundamental concepts of the subject.	57.14	76.67	85.71	50.00	100.00	91.67
7	The lesson promoted strongly coherent conceptual understanding.	32.14	65.00	60.71	33.33	90.00	83.33
8	The teacher had a solid grasp of the subject matter content inherent in the lesson.	64.29	76.67	89.29	33.33	85.00	91.67
9	Elements of abstraction (i.e., symbolic representations, theory-building) were encouraged when it was important to do so.	14.29	35.00	39.29	33.33	90.00	66.67
10	Connections with other content disciplines and/or real-world phenomena were explored and valued.	25.00	21.67	39.29	41.67	30.00	33.33
11	Students used a variety of means (models, drawings, graphs, concrete materials, manipulatives, etc.) to represent phenomena.	17.86	40.00	53.57	41.67	70.00	75.00
12	Students made predictions, estimations and/or hypotheses and devised means for testing them.	0.00	26.67	42.86	50.00	50.00	41.67
13	Students were actively engaged in thought-provoking activities that often involved the critical assessment of procedures.	46.43	43.33	67.86	33.33	95.00	100.00
14	Students were reflective about their learning	28.57	46.67	64.29	58.33	95.00	91.67
15	Intellectual rigour, constructive criticism, and the challenging of ideas were valued.	28.57	45.00	64.29	33.33	95.00	91.67
16	Students were involved in the communication of their ideas to others using a variety of means and media.	28.57	46.67	82.14	50.00	90.00	66.67
17	The teacher's questions triggered divergent modes of thinking.	7.14	45.00	53.57	33.33	95.00	91.67

(continued)

Table 7.1 (continued)

SN	RTOP statements	Teacher 1			Teacher 2		
		B	D	A	B	D	A
18	There was a high proportion of student talk, and a significant amount of it occurred between and among students.	35.71	71.67	89.29	41.67	100.00	100.00
19	Student questions and comments often determined the focus and direction of classroom discourse.	42.86	26.67	57.14	33.33	95.00	91.67
20	There was a climate of respect for what others had to say.	50.00	56.67	82.14	58.33	100.00	100.00
21	Active participation of students was encouraged and valued.	25.00	53.33	64.29	58.33	95.00	100.00
22	Students were encouraged to generate conjectures, alternative solution strategies, and ways of interpreting evidence.	21.43	28.33	64.29	50.00	90.00	58.33
23	In general, the teacher was patient with the students.	53.57	65.00	75.00	33.33	100.00	100.00
24	The teacher acted as a resource person, working to support and enhance student investigations.	28.57	55.00	75.00	50.00	100.00	100.00
25	The metaphor "teacher as listener" was very characteristic of this classroom.	17.86	48.33	71.43	50.00	95.00	91.67

B before intervention*, D* during intervention *and A* after intervention

reflected in the demonstration of teacher content knowledge, the value provided to the link between academic content and real-life, demonstrating the understanding of the interconnection of math concepts. Furthermore, teachers have improved strategies for cultivating scientific reasoning and understanding of abstract concepts amongst learners after the intervention. Teachers encouraged students to make predictions/hypotheses/conjectures and engaged students in intellectual conversation and critical self-reflection.

Moreover, participant teachers have significantly increased productive dialogue with students. Students were given room to communicate their ideas differently, and teachers' questioning strategy triggered divergent modes of thinking. This finding aligns with the social constructivism view of collaboration (Kivunja & Kuyini, 2017). In line with this, teachers allowed students to interact when engaged in meaningful activities that allowed them to rely on their own experiences. In this learning process, learners are placed in the active role while teachers act as facilitators to maintain high-quality lesson interaction (Sawada et al., 2002).

The semi-structured interview with teachers identified five themes: interest and motivation, self-directed learning, easy and autonomous learning, retention of knowledge, and problem-solving and critical thinking skills.

The two participating teachers believed that PBL creates positive attitudes for learners. They claimed that learners were more interested and engaged in finding things independently than in the traditional lecture classroom. They indicated that learners were motivated to do the work and share what they could do with peers. This has boosted their interest in learning mathematics, which they usually see as tricky since most learners chose the MEG (Mathematics, Economics and Geography) combination because they are interested in learning either Economy or Geography. Also, teachers argued that learning mathematics using the PBL strategy supported learners to visualize the usefulness of mathematics in their subject of interest. Other researchers reported that when learners learn with ill-structured problems, they show positive attitudes and are motivated to learn (Osman & Kriek, 2021); PBL creates self-directed learning (Savery, 2019; Himelo-Silver & Barrows, 2006). Indeed, Dolmans and Schmidt (1996) pointed out that PBL encourages self-directed learning and develops transferable critical thinking, teamwork, and problem-solving skills. Also, the independent learning that PBL offers increases students' interactions, motivation, and interests (Dolmans & Schmidt, 1996).

Additionally, the two teachers indicated that PBL provided learners with the ability to learn independently. They explained that during research to find the solution to the problem individually or in a group, learners did not seek help from teachers because they knew exactly what they wanted to look for to get the needed information. This has shown that learners can learn things themselves. Teachers also argued that learners started being responsible for their learning, which they met to discuss after school. This practice of making learning self-directed aligns with constructivists' view that learners construct and reconstruct knowledge even when the information is incomplete (Vygotsky & Cole, 2018). As Vygotsky (1978) stated, learning is student-centred, and new knowledge is constructed due to social interactions. Students are engaged in the activity; they get facilitations from their teacher through appropriate tasks and monitoring productive talks. Thus, students' experience when solving problems in small groups during learning improves thinking and mental function.

Desi et al. (2019) reported similar findings that PBL provides an outstanding opportunity to develop critical thinking skills than traditional teaching. They claimed to see learners going beyond the expected learning objective while analyzing the problem, assessing the solution strategy, and checking if the solution makes sense to the problem (Desi et al., 2019). Furthermore, one teacher confirmed that learning within PBL supported learners in developing their critical thinking skills due to the association between classroom learning activity and real life.

Difficulties While Implementing PBL

Although teachers liked implementing the PBL strategy, they complained about several difficulties, including time, content knowledge, learning resources, and classroom management associated with their workload. The two teachers indicated that the PBL strategy consumes enough time. It requires them to take enough time preparing an appropriate ill-structured problem and provide learners with enough time to do research and present their findings than they have in the allotted time. One of the teachers claimed that he could not complete the content he had to cover in 9 weeks of the trimester if he continued applying PBL (Kolmos, 2017) reported that the completion of the PBL module is often too short, given that learners research to find the solution. Consequently, they claimed they might not continue implementing PBL since it is time-consuming.

Also, the two teachers complained about a lack of content knowledge. They claimed that implementing the PBL strategy requires them to master the content and its application in real life to design an appropriate content task that engages interest and intellect than they already know since the way they were prepared to teach was different from the PBL strategy. Consequently, they may need additional time and technological knowledge to search for information on the internet which is problematic given their workload. Usually, teachers in Rwanda teach between 36 and 40 of 45 working hours per week. Thus, they do not get time to prepare their lessons. This may be attributed to the low number of teachers in schools.

Furthermore, teachers complained about the class size, which prevented them from better managing the classroom conversation within the PBL setting. One of the teachers reported difficulty keeping one conversation of 40–46 students in the room; even facilitating them in their small group was complex in the 40 min of a period since they shouted. Klegeris and Hurren (2011) affirmed that the application of PBL in large classes consisting of 45–85 students is often possible and positively impacts students' motivation and engagement than in the traditional lecturer classes.

On the side of learners, they complained about getting access to the internet to search for information even though their school has internet facilities. Searching for information in PBL is fundamental.

Moreover, teachers reported the language issues that prevent them from designing good problems. Consequently, they copy problems from textbooks and paste them onto the chalkboard because they fear changing the question's meaning. This was raised by Teacher 1 with 33 years of teaching experience. Again, as a result, he fails to engage students in the discussion or trigger a divergent thinking mode.

Those who favour the integration of technology in teaching and learning would argue that technologies could have enabled learners to explore and deepen their understanding of solving mathematical problems in the present study. In particular, the effective use of technologies associated with 4IR could increase motivation and engagement and help teachers improve their lesson plans and teaching. Moreover, the 4IR technologies can help students build essential twenty-first-century skills. However, teachers' lack of infrastructure and expertise constitutes barriers to the 4IR-enabled teaching and learning. This leads us to suggest continuous professional development to help teachers gain competencies to enhance learning through technologies associated with 4IR.

Conclusion and Recommendation

Problem-based Learning is one of the innovative teaching approaches that supplement mathematics teachers to equip learners with competencies of the 4IR. It has been widely used to develop students' abilities to apply knowledge in real-life settings by working collaboratively on meaningful problems. The study's finding reveals that participants' mathematics teachers implemented a problem-based learning approach to effectively teach and learn mathematics in their classrooms. The study illustrated how problem-based learning approaches equip learners with skills for the fourth industrial revolution. The authors argue that the 4IR technologies can support this approach to accelerate learning and equip learners with skills such as critical thinking, creativity, communication and collaboration to adapt the Education 4.0. The data analysis and 14 weeks of intervention reveal that mathematics teachers transformed their teaching pedagogy towards specific teaching practices that prepare learners of today and the future. In particular, participant teachers demonstrated self-efficacy that may likely influence future practices to bring about desired requirements for the students to be skilled for the 4IR.

Findings also highlight that mathematics teachers value the promotion of problem-based learning intervention instead of presenting information intervention. This finding has implications for policymakers about what quality intervention looks like. The policy can help support evidence-based intervention described in this study; some continuous professional development interventions can be looked at since they contribute to the teacher teaching effectiveness. Furthermore, participants indicated that they require support to use the problem-based learning approach effectively within their educational settings. This implies that interventions focusing on content and technology is necessary.

Participant teachers have appreciated the adoption of PBL. However, some difficulties aligned with previous authors were identified, including time, lack of enough content resources, language issues, especially for more experienced teachers, and limited technological content knowledge. Difficulties that previous authors did not report include a lack of access to e-resources due to limited internet connectivity while searching for information. This implies policymakers.

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Chapter 8
Computer Adaptive-Based Learning and Assessment for Enhancing STEM Education in Africa: A Fourth Industrial Revolution Possibility

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Introduction

The fourth industrial revolution (4IR) ushers the world into a technological revolution that is fast changing the way people function with unprecedented effects already experienced. Based on this reality, its response must be integrated and comprehensive, involving all stakeholders of the global polity, from the public and private sectors to academia and civil society (Schwab, 2016). From the first industrial revolution characterised by water and steam power to mechanised production, 4IR is an improvement that engages smart technologies that blear the virtual, visible, and botanical spaces in time (Marwala, 2020). While 4IR is seen as building on the third, velocity, scope, and systems impact make it distinct with no historical precedent. Aikman (2017) reiterates that the fundamental transformation comes with new technologies and powers, and responsibilities while stressing the need for frameworks to manage the empowering 4IR waves of transformation systems.

The developing world is still exploring the second and third industrial revolutions with disparages in access to electricity and while more than 50% lack internet access, respectively, showing that the region still lags in terms of digital indicators of use, access and preparedness essential for a successful digital revolution (Marwala, 2020; Ndung'u & Signe, 2020; Oladele et al., 2021; Schwab, 2016). The bright side is that while the first industrial revolution took over a decade to extend the shores of Europe, the third premised on the infobahn penetrated the world within the last ten years. Like none else, the speed of 4IR was described as exponential rather than linear (Schwab, 2016; Yue, 2018). Worthy of mention is the fact that the

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proliferation of 4IR might have been aided by the Covid-19 Pandemic, which occasioned lockdowns in Africa as experienced in other continents (De Giusti, 2020; Haider et al., 2020; Herman et al., 2020; United Nations, 2021) and forced all sectors to leverage on available technological solutions that could enable them to remain functional amidst the lockdowns (Li & Lalani, 2020; Oladele et al., 2021; Patrinos & Shmis, 2020; Zalat et al., 2021).

These digital solutions have been tagged disruptive considering that they have blurred the lines between the previously distinct spheres that impact all economies, disciplines, and sectors, fast reshaping the world (Manyika et al., 2013). Frontier technologies significantly alter how people, industries, or businesses operate, eroding and replacing existing systems due to their superiority (Bower & Christensen, 2021; Manyika et al., 2013; Millar et al., 2018; Smith, 2020). They include cloud computing, hyperspace, geothermal power, additive manufacturing, AI (robotics, self-driving cars, tech assistants, chatbots found on websites, digital travel agents, and translation software, among others), Global Positioning Systems (GPS) systems and next-gen storage solutions (Indeed, Editorial Team, 2021; Schwab, 2016; Wong, 2018). Koutroumpis and Lafond (2018) stressed that inter-firm relationships individually and institutionally determine disruptive technologies‘emergence, diffusion, and impact. As such, relevant stakeholders in the education sector should focus on integrating these technologies into teaching and learning, as developing countries cannot be apathetic to information communications technology (ICT) for global relevance (Jhurree, 2005; Schwab, 2016; The World Bank Group, 2021). Failure to recognise and capitalise on 4IR possibilities in education will impose considerable risks, which calls for urgent interventions from African stakeholders (Ndung'u & Signe, 2020). This chapter seeks to assess the efficacy of Computer Adaptive Learning CAL as a digital intervention for implementing assessment as learning for enhanced STEM education relevant for higher education in Africa.

Literature Review

The abrupt suspension of physical interactions in educational institutions has impacted most student populations across the globe (ECLAC & OREALC/UNESCO Santiago, 2020; Pokhrel & Chhetri, 2021). While there is no gain-saying the fact that the education sector, like other sectors in the developed world, experienced close to a seamless adoption of these technologies to aid the continuity of teaching and learning, the case was different with developing countries struggling to integrate these technologies in teaching and learning (Ed Hub, 2020). Studies by EdTech Hub (2020), Okeji and Alex-Nmecha (2021) and Ebohon et al. (2021) revealed that many African countries relied on television and radio for instruction at the basic school level, while some teachers in West and East Africa (Nigerian, Kenya and Liberia) used WhatsApp, Zoom Edmodo, Google Classroom and Microsoft Teams. Moodle for teaching and learning using smartphones. E-learning was mostly experienced in the private education sector, leaving students in the public sector (Iseolorunkanmi et al., 2021). Mhlanga and Moloi (2020) reported that the education sector in South Africa is one of the foremost African countries that leverage 4IR tools across the board in the education sector.

Whether the 4IR risk reverses the narrowing of the gaps between economies was a question raised considering the reality of disruptions and their inevitability impact on the common good (Schwab, 2016). Therefore, values-driven policy choices are germane for 4IR relevance. It becomes necessary for them in developing countries to harness development and hasten leapfrogging. In light of the 4IR, with recent digital solutions offering new teaching and learning avenues, teachers should devise appropriate pedagogies to work with emergent technologies (Arroyo et al., 2014).

Assessment of Learning (AoL- evaluating performances at the end of learning), Assessment for Learning (AfL- learner-centred classroom, where students learn from mistakes) and Assessment as Learning (AaL- premised on formative assessment) are three widely recognised assessment approaches in the literature (Berry, 2013; Chiappe et al., 2016; Clark, 2008; Dann, 2014; Daramola et al., 2019; Earl, 2003; Gardner, 2012). According to Jones (2005), AfL translates assessments from being a requirement for achieving a qualification into feedback on the quality of students' learning. With AaL, students couple the assessment and learning process for meaningful internalisation of the curriculum, also referred to as the "assessment as learning to learn paradigm" (Berry, 2008; Earl, 2003). While AaL, AfL, and AoL relate to students' cognitive processing activities and how they make sense of their learnings, AaL provides self-regulation, which enhances the skill of metacognition germane to mathematical thinking (Dann, 2014; Schoenfeld, 2016). AaL marks a departure from AfL being learner-centred approaches which can be deployed using Computer Adaptive-based Learning and Assessment platforms for Enhancing STEM Education in Africa as a way of leveraging on 4IR for teaching Mathematics (Schellekens et al., 2021).

Therefore, the AaL approach and STEM education have common grounds that make the approach apt for teaching and learning Mathematics. Mathematics has been described as a precursor of scientific discoveries and inventions while providing problem-solving opportunities (Klerlein & Hervey, 2019). Mathematics is frequently also referred to as the language of engineering construed as techno-mathematical literacies for bridging the discrepancy between school and work mathematics, where computers often perform most calculations (van der Wal et al., 2017). The importance of Mathematics to society accounts for its inclusion in the school curriculum as a compulsory subject with implications for producing a versatile and resourceful workforce (Gravemeijer et al., 2017). It, therefore, means that Mathematics is important for scientific development. Most African universities insist on at least a credit in Mathematics as a prerequisite for admission into any course of study except for law and art-related courses. Efforts should be geared toward pedagogies that enhance deep mathematical and conceptual understanding relevant to the twenty-first century (Mynbayeva et al., 2018).

The Problem

Education "is one of Africa's largest and most consequential government activities, and policymakers and aid agencies ignore it at the continent's peril. Indeed, by continuing to support education during the pandemic, governments can strengthen their countries' immediate Covid-19 response and long-term recovery. As one survey participant from Liberia put it, the "Covid-19 crisis has only pointed out the country's weakness in the educational sector". Marwala (2020) reported a 2018 Accenture study that revealed a threat to jobs in South Africa due to automation by 2025. Another report shows that frontier technologies which already represent a $350 billion market, could grow to $3.2 trillion by 2025 (United Nations, 2021). These findings call for an urgent need for AI-integrated education using AaL with CAL to properly equip the next generation to suit the emerging world of work with an appropriate skill set which must be long-termed and sustainable (EdTech Hub, 2020; Schwab, 2016). Marwala (2020) stressed that the African continent is uniquely positioned to lead the charge. New horizons should be charted to address the learning crisis to inform leveraging solutions previously considered difficult or impossible to implement (De Giusti, 2020). Moreover, the investments that countries have made in remote learning could be improved to address existing educational inequalities (Chirinda et al., 2021).

Many countries are now considering the importance of remote learning for mitigating future calamities. While this direction indicates progress in the digital age, learners and instructors need a more interactive and collaborative teaching approach (Ayanwale & Oladele, 2021; Osadcha et al., 2020). This stance leaves the upcoming workforce looking to higher institutions to fit into the rapidly changing job market and 'top up' required twenty-first-Century relevance. Therefore, learning systems need to provide tailor-fit content to learners based on progressive assessments-integrated learning approaches, premised on the Machine learning type of AI being the most dominant technology in the 4IR (Marwala, 2020). An adaptive learning model that enhances self-direction was developed while raising concern about its gains' certainty (Kruger, 2020). Also, Murray and Pérez (2015) sounded a cautionary note that arises from relying on adaptive systems for online teaching and learning without paying attention to quality. Furthermore, Osadcha et al. (2020) presented a territorial distribution of the adaptive learning systems, revealing non-implementation in Africa. These needs and concerns create a gap that assessment as learning with adaptive systems is poised to fill.

While the concepts of assessment as learning have been in discussion for some time, there is little information on how the concept can be transferred into actions (Osadcha et al., 2020). This manuscript informs the direction for implementing CAL as a teaching method to leverage technology intervention through assessment as learning for enhanced Mathematics education relevant for higher education in Africa.

Methodology

This study adopted the theory of adaptation grounded on the learning theory of connectivism (Jaakkola, 2020). This theoretical, together with the conceptual framework, informed the direction of literature reviewed with 97% (71 of 73) from online sources, which informed the discussions raised to close the assessment gaps as learning with adaptive systems for quality Mathematics education. Recency was also ensured in selecting the literature published between 2003 and 2021, with 89% (65 of 73) in the last two decades. Therefore, informed on the most recent literature, this chapter aims to reiterate the need to implement mathematics education in Africa using a scalable model for computer adaptive learning systems applicable at higher learning institutions. All citations were duly referenced, providing access links where applicable. Ethical clearance was not necessary, considering that no human participants were involved.

Theoretical and Conceptual Frameworks

This study adopts the learning theory of connectivism, which propagates a meaningful combining of thoughts, theories, and facts. Students' constant technological connectedness enhances learning (Western Governors University-WSU, 2021). Described as a new learning theory for the digital age, the appropriateness of this theory to this study is premised on the availability of smart solutions for teaching and learning, such as Google Assistant, Siri, and Alexa, among others, fast gaining relevance for seeking answers and finding information (Herlo, 2016; Kop & Hill, 2008). This state is a sharp contrast to the realities twenty years ago when students might go to an encyclopedia for answers, but now they can ask their smartphones or type the question into Google with split seconds of response time (Siemens, 2004).

With connectivism, learning is more than knowledge assimilation. Instead, content from external networks is also considered relevant in the teaching and learning process. From this theory, nodes and links are terms commonly employed to describe how information is internalised within a network (WGU, 2021). A node refers to any object; in this case, the student is connected to another object (a book, webpage or person). Connectivism; is based on the theory that learning occurs when a connection or "links" are made between various "nodes" of information sustainably to bring about expertise (Downes, 2007). This theory is apt considering the volatility of the twenty-first Century career requirements. Furthermore, learning experiences are mostly informal and multi-modal as technology rapidly alters how work is being done. Therefore, applying the learning theory of connectivism to examining the effectiveness of CAL technology can potentially redefine and shape human thinking in the teaching and learning process. AaL with CAL can be conceptualised within the learning theory of connectivism, as shown in Fig. 8.1.

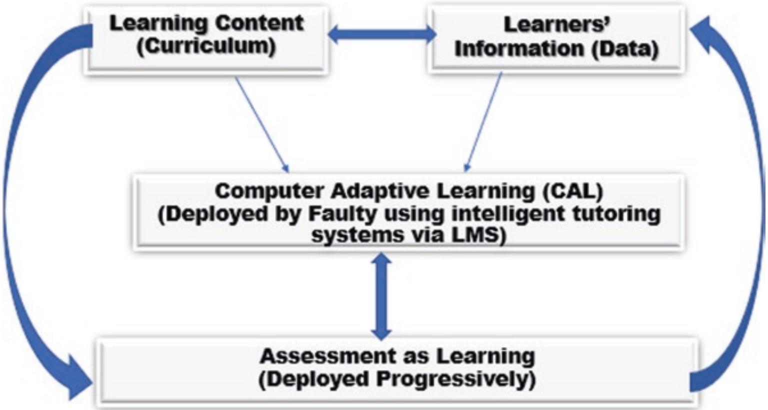

Fig. 8.1 Computer Adaptive Learning (CAL) with assessment for learning connectivism framework

As shown in Fig. 8.1, the framework depicts the connectivity learning framework for implementing adaptive systems for AaL using CAL. The figure also shows CAL driven by faculty members using intelligent systems deployed using Intelligent tutoring systems integrated with existing LMS with a feed into the process from AaL. Using a bi-directional arrow, a connection is made between learning content and learners' information for adapting the curriculum to individual learning needs. Similarly, a bi-directional arrow shows a connection between learners' information and the learning content, with feedback from assessments given progressively. Assessments are driven by the learning content drawn from the curriculum adapted to the needs of individual learners. This framework considers the content, learner and instructional models, which can be premised on either the simplest or complex rule-based or algorithm-based systems described by Sjaastad and Tømte (2018) for use in virtual Mathematics classrooms at the higher level of education.

Assessment as Learning with Computer Adaptive Learning as a 4IR Application in Education

Earl (2003) described AaL as a "preferred future" for assessment, which is now with the 4IR wave, characterised by the combination of the biological, physical and digital spaces and resulting in new technologies. Gravemeijer et al. (2017) revealed a growing demand for a versatile workforce with relevant skill sets. Education has a crucial role in this regard relevant to mathematics education. Ndlovu et al. (2020) lend a voice to ICT-enhanced mathematics education for teachers-in-training. This submission was supported by Kruger (2020), who presented the New Media Consortium Higher Education Horizon 2018 Report, where adaptive technology ranked high as relevant for learning, teaching and creative inquiry in higher education (Becker et al., 2018). One such pedagogies is computer adaptive learning (CAL), which leverages technology for educational gain (Smith, 2020).

As a digital learning system, CAL is adaptive when it dynamically adjusts to suit students' learning needs premised on data collected during learning (Murray & Pérez, 2015; Osadcha et al., 2020; Sjaastad & Tømte, 2018). Adaptive learning enables a student-centred approach to learning and responds to a student's performance (Chen et al., 2018; Dyro, 2017; Kruger, 2020). Learning in higher education is usually one-size-fits-all centric and is described as a "static" learning environment (Murray & Pérez, 2015). Kruger (2020) posed some salient questions on this model to education, bothering on how to accommodate learners with varying entry knowledge while allowing for multi-modal context. CAL is a pedagogy that uses computer algorithms and artificial intelligence to orchestrate the interaction with the learner and deliver customised resources and learning activities to address the unique needs of each learner (Kurt, 2021). CAL enables well-defined and personified learning goals while progressively gauging the extent of knowledge through formative assessments enabling a student-centred learning path premised on student-specific data such as achievements scores, demographics, attendance records, awards, behaviour categorisation/observations, benchmark assessments courses, data from partners (internships) among others (Kruger, 2020; Spacey, 2021). With the advent and integration of technology in educational assessment, CAL leverages the advantages of assessment as learning while engaging metacognitive strategies of self-analyses, self-references, self-evaluations, and self-corrects through a personalised learning process that promotes individual talents development (Andrade, 2019; Lubbe & Mentz, 2021; Osadcha et al., 2020).

An informed decision guides an adaptive learning system's working on presenting learning contents while maximising information about both the learner and the training materials, enhancing intelligible decisions relevant to the entire learning trajectory (Chen et al., 2018; Murray & Pérez, 2015). This process is accomplished through a sophisticated data-driven mechanism. In some cases, a non-linear approach to instruction and remediation is adopted, which adjusts to each learner's interactions and demonstrated performance level and subsequently anticipating what types of content and resources meet the learner's needs at a specific time (Kruger, 2020). Adaptive systems help address learning challenges while considering the varying student learning ability, backgrounds, and resource limitations. Machine learning is broadly an approach to building intelligent systems by leveraging data science to improve functionality (Marwala, 2020). Concerning education, machine learning systems intend to use proficiency to determine student entry knowledge and move students accurately and logically through a sequential learning path to prescribed learning outcomes, enabling knowledge construction required for mastery (Pugliese, 2016; White, 2020). These specific features will transform first-generation digital learning systems and a sure way to alleviate phobia experienced by students in Africa with online teaching and learning, which is fast becoming a new normal at the higher level of education.

Also, adaptive learning provides a blended learning environment with the advantages of close self-monitoring, independence and autonomy relevant for effective online learning in higher education (Clark et al., 2021; Kruger, 2020). Bosch et al. (2020) reiterated that self-directed learning is germane for acquiring twenty-first-century skills needed to cope in the interconnected, global society. This assertion was proved to be true by a study by Bakker et al. (2021), which pointed to the need to monitor students and their work and promote self-regulation. This submission was proved to be true by a study by Bakker et al. (2021), which pointed to the need to monitor students and their work and promote self-directed learning in mathematics. Adaptive learning is touted as a potential game-changer in higher education, a panacea with which institutions may solve the riddle of the iron triangle: quality, cost, and access (Murray & Pérez, 2015). AaL premised on intelligence and adaptive systems of learning while teaching Mathematics can be achieved through testing for monitoring the quality of acquired knowledge to achieve the highest level of intellectual development according to students' natural abilities and inclinations (Osadcha et al., 2020). Adopting CAL for the teaching-learning process would benefit accessibility, collaboration, communication, value diversity, active and social learning, self-direction, content engagement, project learning, and global exposure in Mathematics virtual classrooms.

Maintaining that adaptive learning environments promote self-directed learning and lifelong learning needed for work-life, Osadcha et al. (2020) gave a plethora of adaptive learning systems, one of which was the Intelligent tutoring system (ITS). ITS, referred to as Cognitive Tutors, is premised on artificial intelligence, which leverages students' performance gathered progressively and based on their specific propensities to learn applicable to Mathematics classrooms and can be integrated into existing learning management systems (Arroyo et al., 2014). Other platforms are Realizeit, Barnes and Noble Education (BNED), Muzzy Lane Software, Straighterline, CogBooks, and SoftChalk Create, which are available on a subscription basis and some with demo versions. Also, they are highly customisable and available on blackboard. The most rudimentary systems incorporate straightforward rule-based architectures.

Murray and Pérez (2015) described an implementation whereby a student is given a mathematical problem to solve. If answered correctly, a more difficult problem is presented to reinforce basic math skills being a rudimentary adaptive learning system. In contrast, a more sophisticated system adjusts the presentation of instructional materials based on assessing the student's understanding of the concept by incorporating complex diagnostics and data-driven algorithms deployed using a plethora of instructional objects such as videos, animations, simulations, and case studies. Arroyo et al. (2014) studied Wayang Outpost tutors, which used scaffolds or hints as key components to aid the mastery of mathematics problems; while providing an adaptive selection of problems with increased/decreased difficulty depending on recent students' successes gauged by progressive assessment. The study revealed that the Wayang Outpost tutor was effective within cognitive, affective and metacognitive dimensions and regarded as a novel adaptive learning technology that provided teachers with valuable information about student's performance in mathematics classrooms. Another study by Rihák (2015) studied the system for adaptive practice of foundations of mathematics premised on response times to provide learners with the most useful tasks for ensuring subject mastery. Similarly, empirical findings from Yilmaz's (2017) study revealed that mathematics instruction via ALEKS has a statistically significant positive effect on students' math achievement. Also, the need for computational thinking using digital game-based learning for mathematics, considered an effective educational tool for improving education in future classrooms, was stressed (Bertram, 2020).

These are illustrations of adaptive systems that hold the potential of reshaping students' beliefs of Mathematics as a collection of isolated procedures with one correct solution, requiring memorisation of the correct rule presented by the teacher and applying it as fast as possible, even if this can hardly be remembered once the huddle of assessments has been scaled. However, the "ideal Mathematical mindsets" should be about creativity, sense-making and communication while engaging in an exploration of patterns, systems and quantitative phenomena, and developing a Mathematical agency while ensuring efficient collaboration and communication of Mathematical ideas has been identified as germane learning Mathematics achievable with CAL and skills required in the era of the 4IR (Sjaastad & Tømte, 2018). This submission further strengthens the need to implement CAL for mathematics education in Africa to achieve 4IR compliance.

The impact of the 4IR has been well captured by Marwala (2020) on structure safety, aerospace, mining and electricity industries which are dominantly in the engineering and technological fields. Similarly, major AI users are retail, banking, and discrete manufacturing (United Nations, 2021). However, Kara and Sevim (2013) stressed that aside from these fields, technology-based applications with computerised adaptive systems applicable in the education industry have great importance in today's online teaching and learning world. Gravemeijer et al. (2017) reported a growing demand for employees with complex communication and expert thinking skills. Adopting CAL for deploying mathematics instruction in higher education would aid the development of such skills required for relevance in the era of the 4IR.

Conclusion and Recommendations

Education technology itself is not a panacea, evident from unchanged learning outcomes despite increasing EdTech investments. While teaching and learning remotely are not the same as face-to-face pedagogy, many teachers use e-contents in physical classes. This practice is a pointer to the fact that many countries, especially the least developed and those in sub-Saharan Africa, are unprepared to equitably use, adopt and adapt to the ongoing technological revolution; there is a need for developing nations to prepare for a period of deep and rapid technological change that will profoundly affect societies. Developing countries will need to adopt frontier technologies while diversifying their production bases by mastering existing technologies such as CAL in the education sector to catch up and forge ahead. This suggestion will mean strengthening innovation systems while aligning STI and industrial policies, building basic digital skills, and closing gaps in ICT infrastructure.

Seeing that remote education is fast becoming the new normal at higher institutions, it must be deployed correctly to ensure that students' outcomes are achieved and that the universities are producing graduates that fit the requirements of emerging worlds and further the frontiers of change the only constant thing. Using CAL, an Industry 4.0 technology, would further build the need for complex problem-solving, critical thinking, and creativity required for Mathematics education. As a response to this, countries must advance through science, technology and innovation policies appropriate to their development stage and economic, social and environmental conditions by strengthening and aligning science, technology and innovation systems and building digital skills among students to increase productivity.

Considering that South Africa is one of Africa‘s foremost transitions and developing economies, with an ICT ranking of 69 and an overall readiness towards the use, adoption and adaptation of frontier technologies ranking 54, the country can be regarded as properly positioned to lead adaptive technologies in education by implementing CAL research. This position would align the country with developed counterparts such as Australia, Italy, Japan Republic of Korea, and the United Kingdom using AI to improve public sector services efficiency. Worthy of mention is that all the empirical studies concerning implementing CAL in Mathematics classrooms available to the researcher when writing this chapter were at the basic and secondary school levels. This observation creates a need for replicating these studies at higher levels of education and a direction for further research through randomised controlled trials, standardised procedures, and methods.

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Chapter 9
Exploring the Challenges of Teaching Mathematics During the Fourth Industrial Revolution in Selected Rwandan Secondary Schools

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Introduction

Industrial revolution (IR) refers to the transformation of the economy based on agriculture and handcraft into an economy based on industry and machines (Mohajan, 2019). The first industrial revolution started in 1760 with the discovery of the steam engine machine for the transformation of agricultural products; textile and steel were the dominant industries (Agarwal & Agarwal, 2017). The second industrial revolution started in 1900 and was characterized by rapid industrialization using oil and electricity to power mass production (Xu et al., 2018). During the second industrial revolution, the steel and chemicals industries dominated (Agarwal & Agarwal, 2017). The third industrial revolution began in 1960 with the development of the computer and internet (Taiwo, 2020). During the third IR, industries were based on science and technology (Brian, 2015). Computerization and web-based learning started in the third IR (Gleason, 2018). The fourth industrial revolution (4IR) boost Mathematics, science, and technology. The fourth industrial revolution started in 2000 and was characterized by the development of green energy industries, artificial intelligence, machine learning, robotics, and 3-D printing. In recent times, the internet is becoming faster, more accessible, and more mobile. At the same time, information can easily be manipulated within a short time through the increase in data storage and processing capacity (Taiwo, 2020).

Education in the 4IR emphasizes that students learn science and technology that enable them to live in society and be successful workers. People must have the skills required to implement, manage and work with the new technology and with one another (Butler-Adam, 2018). Now days use of technology help students to become

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problem solvers, Critical thinker, and decision-maker in society. 4IR requires changing the method of teaching mathematics. Teachers must adopt a new teaching methodology, be creative and innovators and create new learning opportunities (Naidoo & Singh-Pillay, 2020). It was found that the practice of teaching and learning mathematics remains dominated by the manipulation of symbols with the paper-and-pencil medium (Ng & Tsang, 2021).

Teaching methods of mathematics include lecture, inductive, deductive, heuristic or discovery, analytic, synthetic, problem solving, laboratory, and project methods. Teachers may adopt any method according to the specific unit of the syllabus, available resources, and the number of students in a class (Baig, 2015). Effective teaching and learning mathematics requires an effective method that helps students think and develop competencies. Rwanda has adopted a new curriculum called the Competence-Based curriculum from the Knowledge-Based curriculum in 2015, which focused on a learner-centred approach. It was found that Rwandan mathematics teachers were aware of learner-centred pedagogy. Some methods used by Rwandan mathematics teachers are question-answer, group discussion, and experimentation. The common method used by Rwandan mathematics teachers are question-answer and group discussions (Nsengimana, 2017).

In 4IR, the research shows that effective teaching and learning science, technology, engineering, and mathematics (STEM) is encountering different challenges. According to Ng and Tsang (2021), school teachers have often expressed that they have inadequate knowledge and resources to integrate STEM concepts and practices in their classrooms. Rwanda mathematics teachers encounter some challenges in their work. Research conducted in Rwanda showed that teacher knowledge and skills in the learner-centred approach are insufficient. It was found learner-centred approach is limited to simple oral questioning, group discussion, and experimentation or doing exercises (Nsengimana, 2017). It was also found that there was a lack of teaching materials to accommodate a Competence-Based Curriculum, a large number of learners in a classroom, low level of learners, and English language problems for teachers and learners (Ndihokubwayo & Habiyaremye, 2019). More research on the challenges of teaching mathematics was conducted. However, this study tends to investigate challenges uncounted by Rwandan mathematics teachers in the teaching and find potential solutions to those challenges to meet the needs of students in the 4IR. The study's results will help teachers design instructional activities that qualify students for future jobs. In the future, the problem could not be the lack of jobs but the skills shortage that will depend completely on IR 4.0 ideas.

Literature Review

Philosophy of Mathematics Education

Mathematics education enables people to see beyond the stories of society as it helps us imagine different possibilities. The aims, goals, purposes, and rationales of teaching mathematics should be related to society (Ernest et al., 2016). Ernest

(1998) proposed two mathematics philosophies, absolutist and fallibilist. Fallibilist philosophy of mathematics believes that mathematics is a human construct. In the fallibilist philosophy, mathematics is no longer knowledge that is simply memorized in a rote fashion. It is societal knowledge that must be interpreted in a manner that holds meaning for the individual. According to Ernest (1998), the other mathematics philosophy is absolutism. Absolutists believe that mathematics is a divine gift. Mathematics is considered a cultural product or a human activity complete with cultural, historical, and humanistic aspects, that is, "a dynamic, continually expanding field of human creation and invention, a cultural product" (Ernest, 1998). Knowledge and skills in mathematics can be constructed socially, meaning that knowledge is constructed through interaction with others (Izmirli, 2020). According to (Anthony & Walshaw, 2009), they are ten effective methods that stimulate effective teaching and mathematics, including classroom community, arranging for learning, mathematical communication, mathematical language, worthwhile tasks, making connections, tools and representation, and teacher knowledge. The study conducted by Mazur (2011) found that mathematical tasks as an important method in shaping the system of basic mathematical knowledge, abilities, and habits in students. It was also found that independent learning and research work as methods that help students' problem-solving skills and develops their creative thinking processes and skills. In addition, the research conducted by Khan et al. (2021) among 33 government colleges in Punjab, Pakistan, found that most teachers use teaching methods that connect theory and practice, question and answer techniques to explain mathematical concepts.

An Overview of Mathematics Education and the Fourth Industrial Revolution

The fourth industrial revolution is known as the assemble of physical resources, innovative digital technologies, non-natural aptitude, robots, drones, autonomous vehicles, 3D printing, cloud computing, and more that interconnect, investigate, and enable civil service and society to be malleable (Xu et al., 2018). It involves how technologies like artificial brainpower, self-governing vehicles, and the internet are integrated with individuals' physical lives (Fomunyam, 2020). Mathematics education is one of the most important subjects which can help to fit into 4IR goals, not only to provide sustainable education but also to promote core values and qualities of social justice (Moloi & Matabane, 2020). The teaching and learning of mathematics is an approach to teaching that seeks to permit students to gain the required skills for world challenges. Nowadays, there is an emphasis on incorporating technology into the teaching and learning of mathematics to produce educated citizens who can function in 4IR. The sustainability of mathematics classroom practices is enhanced when 4IR realizes human possibility and promotes core values and qualities of social justice. For mathematics classroom practice to be active and sustainable, cultural practices and surviving experiences of learners must be incorporated into our teaching and learning. 4IR orders the way students have to be taught.

Learners must experience work places and have the knowledge and think about the 4IR. Instructors and students have to keep up with the pace of industrial advancement. If not, they can be left behind (Ilori & Ajagunna, 2020).

The current curriculum utilized in African education is ancient and irrelevant to the needs of the 4IR. Many topics of this curriculum do not prepare learners with the required skills to perform the upcoming civil service tasks (Fomunyam, 2020). Mathematics Education is not strong, and this impacts the number of learners who choose mathematics as a specialized carrier. It is attributed to the low number of mathematics teachers in Africa. It is paramount to implant in the young generation the ability to study mathematics and equip them with the necessary skills.

Method of Teaching Mathematics in Rwanda in 4IR

According to Rwanda Basic Education Board (REB), a new curriculum called Competence-Based curriculum launched in 2015 proposed different teaching methods in teaching mathematics in ordinary or advanced levels in Rwandan secondary schools, including practical, group discussion, mental task, presentation, research work, questioning, and demonstration (REB, 2015). They are limited literature about mathematics education in Rwandan secondary schools. This section highlighted teaching methods used in teaching mathematics in Rwanda based on available literature. According to (Nsengimana et al., 2017), most teaching methods of teaching mathematics at ordinary levels are peer learning, group work and expository methods. Most activities done in the classroom are doing homework, doing exercises, taking a summary of the lesson notes, and reading math books (Ukobizaba et al., 2019). Nsengimana (2017) found that Rwandan mathematics teachers use traditional teaching methods, classroom activities, research, problem-solving, practicals, group discussion, and questioning methods. The research conducted by Uworwabayeho (2009) found that using Geometer's Sketchpad improves students learning. Gichuru et al. (2016) found that ineffective teaching methods result in poor performance in math. This research found that group discussion, learning in pairs, and demonstration are mostly used by Rwandan teachers.

Challenges Encountered by Teachers During Teaching Mathematics in 4IR

Mathematics is a subject that has a significant contribution to society. However, teaching mathematics has been uncounted by different challenges. This section discusses the challenges mathematics teachers face in the teaching and learning process. The study of Assessing the Challenges of Learning and Teaching of Mathematics in Second Cycle Institutions in Ghana conducted by Peter et al. (2014) indicated that lack of teaching and learning materials, poor attitude towards teaching by teachers, fear of mathematics, abstractness nature of mathematics concepts, limited mathematics periods, incompetent teachers, poor teaching methods, and inconsistent curriculum as major challenges that hinder effective teaching and learning mathematics. These challenges agree with Mutarutinya et al. (2020), who found that inadequate skills for conducting practical science and a lack of permanent friendly professional development program are challenges encountered by Rwandan mathematics teachers.

The research conducted by Gafoor and Kurukkan (2015) in Malappuram and Kozhikode districts in Kerala, India, found that big class size, lack of relevant prerequisites, the reluctance by students to seek support from others, poor motivation of students, and difficulty in the speedy grasping the concepts as major challenges that hinder effective teaching and learning of mathematics. They also found that planning for teaching mathematics, executing mathematics lessons, and assessing are obstacles mathematics teachers face in their teaching. The study by Darkis (2020) showed that it is difficult to integrate technology in mathematics classrooms for mathematics teachers in rural areas. In addition, teachers responded that mathematics does not respond to the needs and interests of students in rural areas. Insufficient teaching aids and poor teaching methods lead to mathematics difficulty (Sakilah et al., 2018). Using the traditional methods in teaching mathematics makes students inactive and not creative (Tanujaya et al., 2017).

Research Questions

- What challenges do teachers and students encounter while teaching mathematics in Rwandan secondary schools during the fourth industrial revolution?
- What are challenging mathematics topics to teach and learn in Rwanda in the fourth industrial revolution?

Methodology

The target population was all 328 mathematics teachers from ordinary level and advanced levels from three districts in Rwanda, namely Kayonza, Rwamagana, and Gatsibo districts, Rwanda. However, a total of 109 mathematics teachers comprised, 76 males and 33 females, were randomly selected from the government, public and government-aided schools. A simple random sampling was used where the participant was selected after the third participant. Simple random sampling is useful because it allows all the units in the population to have an equal chance of being selected (West, 2016). A Likert scale questionnaire composed of 68 rating multiple-choice questions was used to explore challenges and their potential solutions and investigate challenging topics uncounted by Rwandan mathematics teachers in their teaching.

The questionnaire was developed by the researcher in close collaboration with experts in Mathematics education from the University of Rwanda, College of Education. Descriptive statistics such as mean and standard deviation was used to analyze data. To confirm the content validity and reliability of the research instrument, the questionnaire was tested and approved by an expert in research and mathematics education from the University of Rwanda, college of education (UR-CE). The research instrument was piloted on 37 secondary school mathematics teachers. This was done to ensure the internal consistency of the instrument. The reliability coefficient was calculated by using Cronbach's and was found to be 0.86. The anonymity and confidentiality of the respondents were preserved by not revealing their identification in the collection of data, data analysis, and presentation of findings. All individuals were willing to participate in this study.

Findings and Discussion

Challenges of Teaching Mathematics in 4IR

A Likert scale questionnaire with five levels was used to measure challenges faced by mathematics teachers in 4IR. The levels were: very low extent = 1, low extent = 2, moderate extent = 3, high extent = 4 and very high extent = 5. Findings were descriptively analyzed in terms of mean, standard deviation and variance. Among 21 statements related to the challenges faced by teachers in teaching mathematics in secondary schools, only 15 showed that there were difficulties uncounted by mathematics in their teaching. Teacher's responded at most that abstractness nature of mathematics ($\overline{x}$ = 4.1009, SD = 0.8271), time for planning lessons ($\overline{x}$ = 4.0275, SD = 0.92755), teaching materials ($\overline{x}$ = 4.0275, SD = 0.85482) and classroom size ($\overline{x}$ = 4.2661, 82,398), at most that make mathematics difficult to teach. The teacher's agreements are illustrated in Table 9.1 below.

The result from the above Table 9.1 revealed that mathematics teachers are faced many challenges in their teaching. Teachers reported 15 challenges faced by them in their work, including abstractness of nature of mathematics, time allocated and planning lessons, lack of CPD, Change in the curriculum, lack of technological tools, Uninterested students, lack of teaching materials, poor work habits of students, lack of support from parents, negative peer influence and poor motivation of students, too much content and class size. Other authors support the above results and found that the nature of mathematics and the negative perception of students make mathematics difficult to understand. The research conducted in secondary schools in Bangladesh showed that most students fail mathematics because of a lack of teaching materials and most teachers are not adaptable to modern methods of teaching mathematics (Khaleduzzaman, 2020). A study conducted by (Koji et al., 2016) in the four selected secondary schools in Mufulira district, Zambia, found that a lack of pre-requisite knowledge by pupils; lack of conceptual, procedural, and

Table 9.1 Descriptive analysis of challenges faced by teachers in teaching mathematics

Statements	N	Mean	Std. deviation
Large classes size	109	4.2661	0.82398
Mathematics is primarily an abstract subject	109	4.1009	0.82710
Teaching materials are not enough to teach mathematics	109	4.0275	0.85482
Time for planning lessons is not enough	109	4.0275	0.92755
Too much content in the curriculum	109	3.9174	0.94407
Lack of continuous professional development (CPD)	109	3.9083	0.95783
I have difficulty keeping up with all of the changes to the curriculum	109	3.8624	1.18218
Poor work habits of students	109	3.8532	1.13713
Lack of student effort/motivation	109	3.8440	1.09014
Teachers do not have adequate technological resources	109	3.7156	1.09791
Uninterested students	109	3.6606	.98338
Negative peer influence	109	3.6514	1.00348
Lack of support from parents/guardians	109	3.3853	1.21646
Time allocated to teach mathematics is not enough	109	3.2752	1.21618

strategic knowledge and skills required for solving linear equations; and inappropriate approaches and methods as challenges that hinder effective teaching of algebraic linear equations. Low tests or exam scores, teachers' harshness, and carelessness were reported among the factors demotivating students to like Mathematics (Ukobizaba et al., 2021). The study by Nsengimana et al. (2017) showed that Rwandan mathematics teachers have limited knowledge of learner-centred pedagogy.

Potential Solutions to the Challenges of Teaching and Learning Mathematics

To investigate the potential solutions to the challenges associated with teaching mathematics, ten rating multiple-choice questions and one open question were used. The mean value, standard deviation and variance were calculated on agreement of a three-point scale (*slightly valuable = 1, somewhat valuable = 2, and very valuable = 3*). Potential solutions are classified into two categories, including teaching and learning materials and teaching methods. Table 9.2 shows the solutions to the teaching and learning materials, while Table 9.3 shows solutions to the teaching method. Teachers indicated that computer software, calculators, web-based resources, and manipulative materials are potential tools in teaching and learning mathematics. They also pointed out that using class discussion, problem-solving, group work, practicals, and alternative methods of finding solutions are potential teaching methods of mathematics that can help them in their teaching.

Evidence shows that the continuous profession of in-service mathematics teachers helps students understand mathematics concepts (Bingolbali, 2011). Those results agree with practising specific methods for solving mathematical problems. Students could learn a way of thinking to approach and solve problems successfully in a broader context in life (Szabo et al., 2020). The study by Darkis (2020) suggests that integrating technology in teaching is the best solution for effective teaching and learning of mathematics. Darkis proposes solutions to the problems for effective mathematics teaching, including Strengthen Mathematical Basis, attracting student interest, doing extra classes, and using the Easy-math model and cut-stop-solve model to effectively teach mathematical problem-solving.

Mathematics Challenging Topics Difficult to Teach

A Likert scale question with four levels of agreement (*very difficult = 1, difficult = 2, easy = 3, and very easy = 4*) was used to explore challenging mathematics topics difficult to teach. Before data collection, the researchers, in close collaboration with 15 ordinary and advanced level mathematics teachers from the Kayonza district, sit together and choose challenging mathematics topics. Among 40 challenging topics proposed to secondary school mathematics teachers. During the analysis, we combined very difficultly and difficult as a challenging topic while easy and very easy as a not difficult topic. From this rating, only 16 were selected as challenging topics difficult to teach in secondary schools. Quadratic equation, complex number, integral, Thale's theorem inverse and composite transformation, trigonometry, differentiation of polynomial, and the rational and irrational function were found to be at most challenging topics difficult to teach in secondary schools. Table 9.4 below shows a descriptive analysis of challenging mathematics topics difficult to teach.

From Table 9.4 above, during analysis, the mean of 1–2.9 was considered difficult while 3–4 was considered easy. The challenging mathematics topics identified were: collinear point and orthogonal vectors, point, line, and angles, solids, probability, logarithm, and exponential function, circle theorem, the limit of a polynomial, rational and irrational function, conics, sequences, Thale's theorem, integral, differentiation of polynomial, rational and irrational function, trigonometry, inverse and composite transformation, complex numbers, and quadratic equations.

Those results are in agreement with other authors. According to (Olubukola, 2015) found that trigonometry, arithmetic, geometry, probability, inequalities,

Table 9.2 Potential solution for teaching and learning materials of mathematics

Statements	N	Mean	Std. deviation
Use of web-based resources	109	2.5963	0.52914
Computer software	109	2.5596	0.51696
Manipulatives (e.g., ten base blocks, colour tiles, geometric solids	109	2.4495	0.60071
Use of calculator	109	2.3119	0.64832

Table 9.3 Potential solutions to effective teaching methods of mathematics

Statements	N	Mean	Std. deviation
Working in groups	109	2.6147	0.48892
Doing extra exercises	109	2.5742	0.5732
Practising	109	2.5688	0.64359
Presenting alternate methods of finding solutions	109	2.5413	0.50059
Class discussion	109	2.5046	0.58727
Problem-solving	109	2.4495	0.60071

Table 9.4 Descriptive analysis of challenging mathematic topics difficult to teach

Descriptive statistics			
Topics	N	Mean	Std. deviation
Collinear point and orthogonal vectors	109	2.3761	1.01643
Point, line, and angles	109	2.3211	0.76836
Solids	109	2.3211	1.06180
Probability	109	2.2936	1.01199
Logarithm and exponential function	109	2.2844	2.26524
Circle theorem	109	2.1468	0.88009
Limit of a polynomial, rational and irrational function	109	2.1009	0.91231
Conics	109	2.0550	0.95097
Sequences	109	2.0092	0.99532
Thale’s theorem	109	1.9908	0.86598
Integral	109	1.9633	1.30474
Differentiation of polynomial, rational and irrational function	109	1.9541	0.82085
Trigonometry	109	1.9541	0.91677
Inverse and composite transformation	109	1.9174	0.90399
Complex numbers	109	1.7890	0.73375
Quadratic equation	109	1.7798	0.83175

numbers, and numeration are challenging topics to teach senior secondary school students. (Festus & Orobosa, 2013) showed circle and geometry theorem, Application of Trigonometry to the 2nd and 3rd Dimensional Problems, Drawing and making inferences from statistical diagrams, and Probability theorems as challenging topics difficult to teach in high school schools in Belize. The study by Batanero and Díaz (2012) highlighted probability as a challenging topic to teach in secondary schools. Batanero and Daiz proposed an action that teachers should consider, including teachers’ collective analysis and discussion of the students’ responses, behaviour, strategies, difficulties, and misconceptions when solving probability problems. Also, planning a lesson to teach students some content using a given instructional device to develop teachers’ probability and professional knowledge of working with technology. Evidence from (Ibrahim & Ibrahim, 2018) indicates challenging topics in secondary schools, such as sequences and series, Quadratic Equations, Logarithms, Volumes, Circles, Linear Inequalities, and Angles of elevation and depression. The research showed that integral is a difficult topic, and GeoGebra is an effective method to overcome difficulties.

Conclusion and Implication

Based on the finding and discussion of the study, the analysis showed that the abstractness nature of mathematics, time allocated and planning to mathematics, lack of CPD, curriculum changes, inadequate the technological tool, uninterested and negative peer influence of students, poor work habits, lack of teaching materials, lack of support from parents or guardian, too many contents and classroom size as challenges faced by mathematics teachers in their work. However, the study suggests potential solutions to overcome those challenges, such as using computer software, calculators, web-based resources, and manipulative tools in the teaching and learning materials category. In addition, use of problem-solving, class discussion, working in groups, Presenting alternate methods of finding solutions, practical work, and doing extra exercises in the category of teaching method. Furthermore, the study investigates challenging mathematic topics difficult to teach, which are: point, line and angles, solids, probability, Thale's theorem, quadratic equation, circle theorem, Collinear point, and orthogonal vectors, Inverse and composite transformation, trigonometry, Limit of polynomial, rational and irrational function integral, Differentiation of polynomial, rational and irrational function, sequences, logarithm and exponential function, complex number, and conics. Discussion results propose some teaching methods that can be used to teach highlighted challenging topics: project work, work with technology, doing extra exercises, discussion method, planning lessons, using the easy-maths model, and the cut-stop-solve model.

Based on the results, the study implies the following actions for effective mathematics teaching. Before starting a new topic, teachers should revise the previous content and give students more exercises that promote problem-solving skills. Teachers should also motivate their students to study mathematics and show them its application in everyday life. Teachers are also encouraged to integrate technology into their teaching because technology provides additional opportunities for learners to see and interact with mathematical concepts. The students are also encouraged to use their effort and explore more while studying as a method that will help them to succeed in mathematics. Students should change methods of learning during 4IR by embracing technology. Nowadays, in 4IR, where there is the rapid growth of technology, mathematics teachers and students are encouraged to join the platforms for teachers and students, such as web-based graphing calculators, Demos, and Mathspace. Teachers can also join online platforms for sharing, discussing, and testing mathematics concepts.

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Chapter 10
Emerging Realities from COVID-19 and the Fourth Industrial Revolution: Mathematics Education Lecturers' Collaborative Autoethnographic Experiences

Hlamulo Wiseman Mbhiza, Motshidisi Masilo, Zingiswa Jojo, and France Machaba

Introduction

It is both frightening and thrilling to learn and teach in a time of pedagogical transformation in higher education. As with the rest of society globally, higher education in South Africa is in the throes of transformations and shifts towards using the Fourth Industrial Revolution (4IR) tools for teaching and learning. Davis (2016) described the 4IR as "the advent of cyber-physical systems involving entirely new capabilities for people and machines." Although the capabilities of people and machines are reliant on the technological tools and infrastructure of the previous revolution, the 4IR entails completely new and different ways in which technology is utilised within societies (Shava & Hofisi, 2017). Moreover, Melville and Robert (2020, p. 1) stated that 4IR refers to the "productivity-enhancing industrial innovations enabled by rapid technological progress and combinations of digital technologies." This suggests that clearing the hurdle of 4IR necessitates advanced integration of various technological tools for teaching, learning and assessment in education. This makes it essential that both students and lecturers learn and own new skills and accept new interactional approaches. It also becomes important that higher education institutions continually adjust to prepare students to succeed within a technologically advanced society (Butler-Adam, 2018).

To remain globally competitive, the University of South Africa periodically reinvents the teaching, learning and assessment approaches to enable effective learning and teaching as well as provide students with authentic higher education learning

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experiences using the appropriate tools. The emergence of 4IR necessitates that, for higher education institutions in general, lecturers need to be aware of how students utilise educational technologies and, in turn, adapt learning opportunities and experiences to meet the needs of their students (Mbhiza, 2021). This also equally addresses the need to equip lecturers with skills and technological knowledge to guide students competently and effectively. With UNISA positioned as a leading provider of higher education opportunities through open distance e-learning (ODeL) nationally, on the African continent, and internationally, it was suitable for guiding other institutions on ODeL processes, practices and systems that favour online teaching. With the advent of Covid-19 in South Africa and the subsequent lockdown restrictions, traditional universities looked to the University of South Africa (UNISA) for best practices regarding online teaching and learning (Maringe, 2020). However, there is a dearth of studies from UNISA that offer accounts of lecturers' experiences, views, and attitudes towards online content delivery and assessment, although this would have been one way of sharing knowledge with institutions that were thrust into online teaching.

The main purpose of this chapter is to offer insight into UNISA lecturers' experiences and views of teaching and assessing mathematics education courses using online platforms. The following questions guided the study:

- What are UNISA mathematics education lecturers' experiences of online curriculum delivery?
- What are UNISA mathematics education lecturers' views of online learning and assessment during Covid-19?

In relation to the above questions, Maringe (2020) has identified two challenges that stand out in online curriculum delivery, online assessment and how to conduct practical work in an online mode. In this chapter, we heed Maringe's (2020) call that "UNISA has to come to the party to share its wisdom with traditional universities regarding those seemingly insurmountable matters." We believe that sharing our experiences of online teaching and assessment at UNISA is one way of propagating our expertise in online curriculum delivery. The following section reviews the literature on remote learning in higher education.

Literature Review: Understanding Remote Learning in Higher Education

Online learning and teaching entail sharing knowledge through digital tools, including the use of digital learning management systems. The interaction between students and lecturers occurs in physical isolation, unlike in traditional lecture spaces (Gherheș et al., 2021). Barr and Miller (2013, p. 2) asserted that online curriculum delivery focuses on "a wide range of technology-based learning platforms, delivery methods, and the integration of educational technology components into the learning environment" to facilitate students' epistemological access. This relates to how the 4IR has transformed how we conduct our teaching, learning and assessment in higher education (Schwab, 2016). In the 4IR teaching environment, university lecturers are expected to design, organise, and plan educational activities and students take responsibility for their individual learning through online platforms. UNISA is a champion of online curriculum delivery within the African continent, as the teaching and learning models at the institution are characterised by modern values of 4IR and lifelong learning in higher education.

In addition, it is important to note that online curriculum delivery and assessment offers pedagogical benefits, which include flexibility and reduction of financial costs related to travelling and accommodation of students, making UNISA and a few other private institutions in South Africa attractive (Naveed et al., 2017). While this is the case, uncertainties and anxieties from both lecturers and students have been cited in previous research (Aristovnik et al., 2020; Şahal & Ozdemir, 2020). For the students from impoverished backgrounds, the lack of electricity, poor internet connectivity, and computers, which are the prerequisites for successful online teaching and learning, are some of the impediments to the effective roll-out of online curriculum delivery (Grand-Clement et al., 2017; Mbhiza, 2021; Sahu, 2020). For lecturers, on the other hand, a lack of technological skills and competence and confidence to use the available digital infrastructure causes great anxiety and uncertainty when they are trying to teach using this mode (Grand-Clement et al., 2017; Jeffrey et al., 2014). Thus, although the digital world continues to change the educational environments and teaching and learning in higher education are being transformed using technology-based teaching resources, it is important that institutions are reflexive and responsive to the various challenges that both students and lecturers may face. As a result, prior to the era of the Covid-19 pandemic, UNISA operated its curriculum delivery in a blended mode to accommodate the shortcomings dictated by the lack of connectivity for students in rural areas.

Mathematics Education in 4IR

It is evident that the use of technological tools in mathematics pedagogy enables effective teaching and learning and greatly affects students' academic performance (Handayanto et al., 2018; Mlotshwa & Chigona, 2018). However, within the South African context, mathematics teachers' and lecturers' use of 4IR tools for pedagogical purposes is negatively affected by their deficient computer knowledge and skills, as well as how to integrate technology within their classrooms (Mbhiza, 2021). Accordingly, previous studies have reported that many mathematics lecturers and teachers are uncertain about transforming their pedagogical practices to include the use of technological tools during teaching (Maringe, 2020; Mbukusa, 2018). Considering this, it is important to note that various educational applications can influence students' effective learning and achievement, especially within the higher education space (Cheung & Slavin, 2013; Pope & Mayorga, 2019). We believe that the use of online Learning Management Systems (LMSs) in mathematics education facilitates students' learning by stimulating their interest in engaging with mathematical processes during problem solving, which subsequently has a positive effect on their academic performances (Handayanto et al., 2018; Lopes et al., 2008).

In a study conducted by Cheung and Slavin (2013), the findings indicated a strong positive correlation between students' mathematics achievement and the incorporation of technology for teaching and learning the subject. In addition, Mlotshwa and Chigona (2018) asserted that implementing digital platforms for mathematics teaching and learning enables supportive relationships to be formed within communities of practice, thereby increasing student performance. While studies on the use of technology for teaching and learning mathematics in higher education exist, there is a dearth of studies offering UNISA's mathematics education lecturers' experiences and views of using online platforms for teaching and assessment. This is a concern because many universities in the country look up to UNISA for best practices regarding online teaching (Maringe, 2020; Mbhiza, 2021). Therefore, as mathematics education lecturers at UNISA, we should strive to increase writing outputs that offer accounts of our experiences and views of online teaching to allow our traditional university counterparts to learn and adopt best practices from our experience of online curriculum delivery and assessment, especially during the ongoing Covid-19 pandemic. Critical reflections on using digital platforms to support mathematics pedagogy in higher education within the 4IR are crucial to unearth the constraints and enablers of effective teaching and learning.

Teaching mathematics within higher education in the 4IR context should uphold the transformative perspective that mathematics is a product of social skills and social processes (Moloi & Matabane, 2020). Therefore, as the increase in the use of LMSs such as Moodle and Blackboard for online teaching and learning is noted, mathematics education lecturers are tasked with ensuring that students are provided with extended opportunities to interact, connect with their peers and lecturers, and share content (Mershad & Wakim, 2018; Saadatmand et al., 2017). Institutions of higher learning also use online tools such as Moodle and social media such as WhatsApp and Facebook to transform online curriculum delivery and expand teaching and learning opportunities (Aldowah et al., 2017; Tsakeni, 2021).

The use of online educational platforms has been argued to be effective in facilitating teaching and learning as students have expanded opportunities to "create, edit, and share content and communicate and view content posted by instructors and other learners" (Tsakeni, 2021, p. 130). It can be said that the use of LMSs, online tools and social media platforms increases students' capabilities of self-learning and independent problem solving in which students are motivated to self-learn and collaborate with their peers (Makamure & Tsakeni, 2020). We argue that the use of LMSs, online tools and social media platforms has far-reaching implications for mathematics teacher education because the students do not only use online tools for their own learning but are equally being trained for their own future classroom practice. The following section details the theoretical framing we adopted.

Community of Inquiry and Online Learning

One of the theoretical frameworks extensively used to explore and understand online curriculum delivery is the Community of Inquiry (CoI) developed by Garrison et al. (2000). CoI comprises three key interrelated components: Social Presence (SP), Cognitive Presence (CP), and Teaching Presence (TP). This framework advocates that meaningful learning occurs when there is evidence of satisfactory levels of the abovementioned presences. In this chapter, we have adopted the CoI framework to construct meanings on how "deep and meaningful learning experience *is [emphasis added]* through the development of three interdependent elements – social, cognitive and teaching presence" in mathematics education courses (Garrison, 2011, p. 15). Through the connections between SP, CP, and TP, we can describe and critically discuss our lived experiences of teaching the modules online and produce a meaningful educational experience for our students, as depicted in Fig. 10.1.

Anderson et al. (2001) defined TP as the activities of content presenters in designing and facilitating the other two forms of presence to achieve educational outcomes. In other words, TP entails the lecturers' roles in designing, directing, and facilitating learning experiences and, in this chapter, the learning of mathematics education courses. Garrison and Arbaugh (2007) viewed TP as the primary determinant of students' satisfaction, perceived learning, and sense of community during learning. The TP is further compartmentalised into three dimensions: instructional

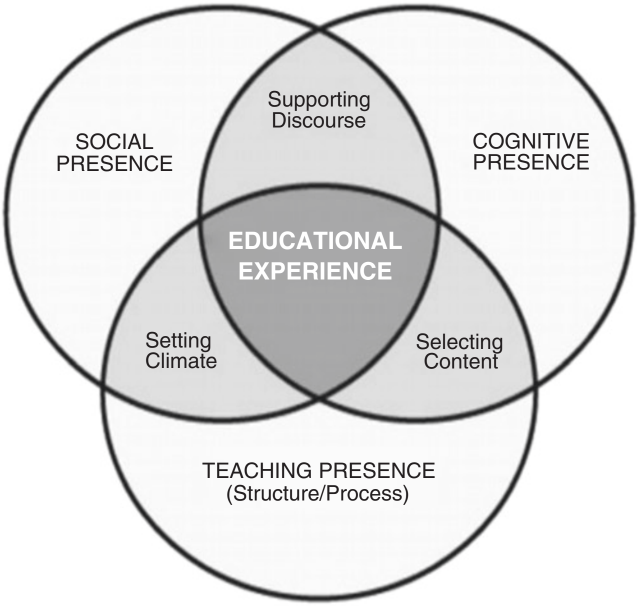

Fig. 10.1 The community of inquiry framework. (Garrison et al., 2000)

management, building understanding, and direct instruction (Anderson, 2008). The first dimension, the instructional management dimension, entails the lecturers' design and organisation of the online lessons. Secondly, the building understanding dimension focuses on the process through which lecturers facilitate the discourse for the subject to enable students' development of meaningful knowledge and skills. Lastly, direct instruction is about the teaching process in which lecturers use explicit instruction to teach the students specific skills and knowledge (Garrison & Arbaugh, 2007). Tsakeni (2021) explained that in the direct instruction dimension, "the instructor, tutor, or any participant in the discussion who is more knowledgeable in the subject articulates what is important to learn" (p. 132). In the current chapter, all three dimensions were adopted as the base for analysing the online teaching and learning processes that we critically reflect on.

CP is about the experiences of meaning-making which occur through the interactions in the online learning platforms. Thompson et al. (2017) averred that CP is where the essential work of knowledge construction occurs. CP is made possible and sustained through four progressive phases, beginning with a triggering event, then exploration, integration and resolution (Hosler & Arend, 2012). In this chapter, this form of presence allows us to interrogate and understand learning opportunities and constraints during the lessons online, helping the students construct meanings of mathematics concepts through sustained questions and answers between the students and us.

Lastly, SP is seen from the sense that the students and lecturers engaged in an online learning environment are part of a group, thereby constituting a community of practice for knowledge construction and sharing (Thompson et al., 2017). DeNoyelles et al. (2014) opined that, for SP, students become real people in an online learning space mediated by the available online tools by means of social interactions and communication. That is, within the CoI framework, it is believed that students who demonstrate some degree of active participation have better chances of being successful in online learning (Chang et al., 2021; Shea & Bidjerano, 2012). For example, Chang et al. (2021) found that students who engage actively in discussion forums are more likely to succeed in online learning. In this chapter, SP allows us to interrogate and understand how students' and lecturers' presence as group members in online learning facilitates and/or constrains knowledge construction and knowledge sharing of mathematical contents. This is one of the ways we become reflexive as lecturers about the quality of learning opportunities we create for our students and configure strategies to improve our pedagogical practices further to enable epistemological access. The following section presents the methodological approach we adopted for the paper.

Methodology

A collaborative autoethnographic research and writing approach focuses on describing and systematically analysing the lived experiences of multiple authors relating to a specific phenomenon (Lapadat, 2017). Hornsby et al. (2021) stated that collaborative autoethnography "is a multivocal methodology that supports a shift from an individual to a collective agency, thereby offering a path toward personally engaging, non-exploitative, accessible research that enhances the reflexive method" (p. 105). In this chapter, we employ this approach because it promises to challenge the canonical approaches to research. The collaborative autoethnographic approach views the research processes as socially just, political, and socially conscious acts of creating knowledge through group interaction. This suggests the importance of centring group discussions on individual subjective experiences to attain a collaborative voice about phenomena.

Our individual personal experiences of teaching mathematics education courses using online platforms at UNISA are the primary source of knowledge. As authors, we "look inward and outward, exposing a vulnerable self that is moved by and may move through, refract, and resist cultural interpretations" (Ellis, 2009, p. 10). We use a collaborative autoethnographic reflexivity approach to reveal the intersections between self and the university society, the particular and the general, the personal, and the politics of knowledge, pedagogy, and assessment.

In this chapter, we critically reflect on our experiences of online teaching and learning, present our views regarding some of the best practices that enable effective teaching and learning of mathematics in online classrooms, and reflect on some of the constraining factors for effective teaching and learning. Schmid (2019, p. 265) argued that autoethnography "facilitates inclusion and allows for multiple voice(s) and knowledge(s) and thus adds to our collective, multifaceted understanding of South Africa." As a "deeply personal research approach ... *it links [emphasis added]* identity and culture, as well as the individual and social, and so simultaneously contextualising the research and the researcher" (Schmid, 2019, p. 266). In this chapter, the collaborative autoethnography reflexivity approach brings into focus opportunities to express silenced stories, thus promoting inclusion by allowing many voices on the experiences of the effects of online curriculum delivery for mathematics education courses. The research focused on the subjective experiences of four UNISA lecturers as we taught mathematics education courses. In view of this, the interpretive paradigm was relevant for this study as it helped us understand each lecturer's utterances as subjective experiences (Kivunja & Kuyini, 2017).

Jensen-Hart and Williams (2010) suggested that it is further important that autoethnographic researchers and writers detail some background relating to their social positioning and/or worldview to help the audience have a better understanding of the context and identities that frame their writing. This means that for autoethnography, "critical reflection must incorporate an understanding of personal experience within social, cultural and structural contexts" (Fook & Askeland, 2007, p. 522). In the current chapter, we reflect on the four mathematics education lecturers' lived experiences of teaching and assessing courses on online platforms.

The first author (HM) is a middle-class Black African man who is a mathematics education lecturer at the University of South Africa. HM's research interests include teaching and learning mathematics, rural education, and curriculum issues generally and in mathematics.

The second author (ZJ) is a Black African female full professor in the Department of Mathematics Education. ZJ is interested in learners' conceptual understanding of calculus and geometric concepts together with meaningful and instructional design in mathematics teaching and learning.

The third author (MM) is a Black African female senior lecturer. Her research interest is in teaching and learning mathematics education based on maximal utilisation of digital resources and online platforms to advance students' mathematics cognition and pedagogy practices that align well with the 4IR and advance inquiring minds in learning mathematics education. MM believes that if prospective teachers could be empowered to be inquisitive and critical thinkers, they could easily contribute the skills and impart learning through inquiry in their respective mathematics teaching environments.

The fourth author (FM) is a Black African male associate professor whose research interests are in teaching and learning mathematical literacy alongside mathematics, investigation of learners' misconceptions in understanding mathematics concepts, together with the usage of everyday context in the teaching and learning of mathematics.

Data Collection and Analysis in Collaborative Autoethnography

Collaborative autoethnography researchers (Hornsby et al., 2021; Lapadat, 2017; Tombro, 2016) have suggested that multiple sources of evidence should be used to support the writers' opinions, pointing to the need for hard evidence to support soft impressions that authors may have about the phenomenon, to generate interpretations and make informed conclusions. This chapter presents our experiences of teaching mathematics education courses as stories. This allows us to offer insights into the patterned processes in our interactions with students and the university structures and how these structures facilitated and/or constrained effective teaching of mathematics education courses.

We position ourselves as insiders and outsiders in the system as we analyse our roles and the pre-service teachers' participation in learning online (Boylorn, 2015). We use extracts from our reflective conversations about teaching and assessing mathematics education students online as our data. Thus, we invite readers to allow their own lived experiences and thought processes to adopt the stories. As Frank (2013) explained, "to think with a story is to experience it affecting one's own life and to find in that affect a certain truth of one's life" (p. 23). The notes we wrote individually and the extracts from our utterances during reflective conversations have been analysed and re-analysed by deductive thematic analysis and employing processes of "rememory" to identify and discuss the commonalities and differences in the lived experiences of the four lecturers. The predetermined codes included remote student support, students' interactions in online spaces and assessment challenges, and these codes framed the analysis and discussions of our reflective experiences as lecturers within the context of 4IR and Covid-19 in mathematics education.

Ethics on Collaborative Autoethnography

As with any research that involves humans as subjects, Ellis (2007) posited that ethical considerations in personal narrative writings, especially when speaking of other people in the narratives of one's own story, should be observed. Lapadat (2017) contended that "autoethnographic stories of our experiences, therefore, are not wholly our own; they implicate relational others in our lives" (p. 593) to address relational ethics. In this chapter, while we speak of students in describing our experiences of online curriculum delivery and assessment, we do not use personal identifiers such as their names because we talk about them as groups. We adopted Tullis' (2013) request seriously to consider our "responsibilities to intimate others who are characters in the stories we tell about our lives" (p. 247), even though such stories do not involve others but ourselves.

Findings and Discussion: Emerging Realities

Much research has been conducted on barriers caused by online teaching and learning in non-pandemic educational situations (Ali & Magalhaes, 2008; Eady & Lockyer, 2013; Karasavvidis, 2010). However, existing studies did not offer insights into online learning, teaching and assessing mathematics within South African higher education institutions. In the current chapter, we focus on student support and assessment challenges to illuminate the experiences and views on challenges and obstacles we faced in teaching mathematics education during the pandemic.

Remote Student Support Through e-Tutoring in a Community of Inquiry

The e-tutoring programme is central to UNISA's online teaching and learning and is one of the key strategies to support students' learning (Sedio, 2021). Although student support through e-tutoring was practised at UNISA before the outbreak of Covid-19, the rise of the pandemic has accelerated fully online students' support.

Maré and Mutezo (2021) indicated that UNISA has adopted e-tutoring as a new approach to engage with the students and share educational materials. The university launched an integrated e-tutoring model in 2013 to enhance student support to ensure maximum student success, mitigate drop-out rates, and motivate life-long learning. In this chapter, student support encompasses diverse means to include students as active participants in learning the content of mathematics education. Tutors are said to be assistants facilitating learning in collaboration with lecturer efforts. However, in an ODeL education context, face-to-face tutoring does not apply anymore. It has been replaced by e-tutoring, where facilitators utilise online teaching and learning platforms to bridge the distance gap between tutor and students, students and their peers, as well as students and the module itself.

To ensure that the CoI is sustained in learning, the e-tutor, as the facilitator, should utilise the online platforms and resources to advance the teaching presence to promote social presence for collaborative learning to support the cognitive presence. It is imperative to consider the essence of a collaboration trio that encompasses a CoI, that is, the mathematics education lecturer, e-tutor, and students. As the content and pedagogy expert, the lecturer must collaborate with the e-tutor, who is also a content expert but is unlikely to be a mathematics education pedagogy expert. Reflecting on this, MM stated that:

> Most of the e-tutors appointed for mathematics education modules are Intermediate, and Senior Phase school teachers with an Honours degree in Mathematics Education assumed to have limited skills to facilitate the online pedagogy for prospective teachers as they mostly teach mathematics in conventional settings in a very abstract manner.

The above statement illustrates that, in some instances, the recruited e-tutors might not be able to train teachers due to limited pedagogical knowledge to help pre-service teachers learn the mathematics knowledge for teaching. The need to rethink and strengthen the role of e-tutors' content and pedagogical practices during online learning has increased within the UNISA ODeL (Halverson & Graham, 2019). This resonates with Sedio (2021, p. 335) that the "ability of online instructors in designing, organising, instructing, and facilitating via online platforms are essential in the successful implementation of the hybrid learning mode." Nonetheless, MM appreciates the process of orientation at UNISA: "*e-tutors, as teaching assistants, upon their recruitment, are trained by the centralised institutional unit to operate online and on the e-tutor site*" to ensure that they support students during the online teaching and learning process. This further points to the need for e-tutors to have specific skills to work with students and manage online learning environments, thus, enabling students' epistemological access (Pratiwi & Ariani, 2020). However, it would strengthen collaborative and productive online and mathematics teaching and learning skills for the e-tutors if the UNISA centralised unit that trains e-tutors worked with the mathematics lecturers. The package of student support offered by e-tutors would be both online and subject-specific.

In relation to the module content and how to engage pre-service teachers in learning, it has been the module lecturers' duties to engage the e-tutors appointed for their respective modules. The only challenge posed during the e-tutor orientation is that the institutional central training unit does not involve lecturers or the module co-ordinator when orienting the e-tutors. ZJ and HM concurred that this causes the e-tutor to embark on facilitation of learning without proper engagement with the module co-ordinator. HM stated, *"it is not sufficient for the lecturer to upload e-tutor guidelines but not to maximise interaction that is supposed to happen from the commencement of training."* Similarly, ZJ said:

> It is important that when e-tutors are trained, the university also involves the lecturers to ensure that the e-tutors fully understand the expectations relating to their roles during online learning … and these tutors should have adequate mathematical knowledge as well as pedagogical knowledge.

From these statements, an inherent assumption is that the role of an e-tutor becomes decisive during online learning in knowledge management practices, ensuring that effective learning is enabled (Vegliante & Sannicandro, 2020). Of importance to note is that the enhancement and legitimacy of the e-tutoring programme should shift beyond training in practical operational knowledge to disciplinary knowledge (Maphalala & Mpofu, 2020; Youde, 2020).

Furthermore, the authors experienced that e-tutors are not encouraged to introduce themselves first to the lecturer and plan with the lecturer; but the lecturer must learn through "*some communication popping up where the e-tutor introduces himself or herself to the students*" (MM). In some instances, lecturers have no knowledge that the e-tutor has been seconded and trained to assist in facilitating the module. This is a serious problem as the e-tutor starts operating without proper guidelines from the module co-ordinator. If mathematics education students are to learn effectively through e-tutor support, it means the CoI trio relatedness should be upheld. HM mentioned that "*these impacts on the teaching presence that alternatively impacts negatively on the social presence as collaboration and group cohesion cannot be maintained to support mathematical cognitive presence.*" We posit that if group cohesion could be maintained, group competency would be achieved, and, thus, individual cognitive competency and the effectiveness and success of online curriculum delivery would be maximised. This links closely with Sedio's (2021, p. 330) assertion that there is a "need for e-tutor support roles that might normalise the learning of content knowledge, especially of the design process by the students at a distance."

It becomes a challenge if guidelines are uploaded on the e-tutor site after the e-tutor has commenced engaging students. The e-tutors need material such as a study guide, prescribed and recommended books, tutorial letters and other resources to support students online successfully. It would help the e-tutor find all material ready on the site if the lecturer knew the e-tutor commencement time on the e-tutor module site. Consider MM's statement below:

> A handover from the institutional support team to the module co-ordinator would also alert the module co-ordinator of the skills possessed by the e-tutor and the gap that the module co-ordinator would close to make sure that upon engagement with students, the e-tutor will be fully equipped with online content delivery skills that align well with the modes applied by the module co-ordinator in question.

This statement suggests that when the e-tutor is involved in the module, a community of inquiry trio (the module co-ordinator or lecturer, e-tutor, and students) emanates, and the trio in the community of inquiry must align well to maximise students' support. In this chapter, this refers specifically to support in the learning of mathematics education. In the case of the didactic modules of mathematics education taught by MM (senior phase) and ZJ (Further Education and Training), the community of inquiry in relation to the CoI trio was not well aligned. In this sense, the effectiveness of a community of inquiry was compromised.

Notwithstanding the above, as module co-ordinators of mathematics education, we must ensure that high-quality student support is provided. The development of e-tutor guidelines is critical. However, the guidelines are submitted to the institutional e-tutor support team that uploads the guidelines on the module e-tutor site. Furthermore, it seems the e-tutor training unit retrieves the study material and provides it to the e-tutor. We argue that direct interaction between lecturers and e-tutors in sharing and discussing the content material was limited. Of importance to note is that this adversely affects the CoI as the cognitive presence domain is compromised when the content is not properly delivered by the e-tutor, which in turn affects the effectiveness of students' mathematics education learning online.

Students' Response to an Online Support System

While the emphasis of a CoI is more on the teaching presence as the main indicator of a thriving student support system, students are also challenged to play their part in full as members of the CoI trio. As much as they value the facilitation by the lecturer, it is of essence that they respond positively to the efforts of the e-tutor as a supporting facilitator. However, since the activities designed and posted by the e-tutor were not graded, students opened the tutorials posted on the e-tutor's site to read but did not participate in the assessment activities posted by the e-tutor. The students' response contributed to challenges in effective support through e-tutoring. In this sense, one of the e-tutors echoed words of concern as follows:

> The majority of students open the site to glance and read through activities. However, only a few complete assessments after reading the material posted. It is only a few students who respond to discussions and announcements.

In support of challenges expressed by the e-tutor in the above extract, FM indicated:

> One of the challenges encountered during discussing with students was the constant internet interruptions. You would find that students must relocate to an area with strong internet connectivity to join the meeting. Some of the students would tell you that they do not have a data internet bundle (FM).

In his observation of teaching the module, FM noticed that students often like to use non-specialised e-learning platforms such as WhatsApp rather than formal social media platforms such as Microsoft Teams and Zoom for interactions. He noted that:

> As a new phenomenon with the emergence of Covid-19, I would engage my students in my teaching on the WhatsApp platform to clarify certain key mathematical concepts and address our questions. Surprisingly, from my observations, it appeared that students would often like to use non-specialised e-learning platforms such as WhatsApp rather than using formal social media platforms such as Microsoft Teams and Zoom for interactions. These could be mainly because WhatsApp seemed to have quickly taken over most of the roles of the specialised platforms due to its affordability and user-friendly, lower cost of internet connectivity (FM).

In her concern, the e-tutor suggested a solution:

> The university should come up with a system that awards students who are participating on the myUnisa platform. For instance, the university/lecturer may indicate that students who participate consistently earn 5% or 10% of their year mark.

The onset of Covid-19 not only helped students learn how to use this online platform; it even helped lecturers who were not using it in their assessment and teaching. For example, MF said:

> Teaching and assessing during COVID-19 was when I learned to upload the audio and other documents online, on my UNISA and WhatsApp and other audio-visual means of teaching such as Google Meet and Zoom. However, even though I have tried my best to teach my module, the truth is that some topics were very difficult to teach using these online platforms. For instance, teaching ethnomathematics, which has more cultural artefacts using these kinds of platforms, was a very difficult experience. In some instances, students would constantly ask for technical support services from me on how to use these online gadgets; I would not be positioned to solve their problems (FM).

This calls for revised assessment practices where student support activities will be included in the assessment programme of the module. We might agree that the students who read the posts only are well conversant with the content. However, the e-tutor also needs the students' inputs in the online classroom for her to maximise the support. Including e-tutor assessment activities in the formal assessment of mathematics modules can strengthen the CoI and support students to respond positively to a thriving social presence and promote cognitive processes that can withstand the focus of assessment for knowledge, not only for examination or promotion purposes.

Assessing Students Online

One of the big challenges we faced during online teaching and learning in the context of Covid-19 was the evaluation and assessment procedures of students' learning. Considering that students were not allowed on campuses, we had to teach and assess them using myUnisa and Microsoft Teams as online platforms. The following subsection describes the nature of the challenges we experienced related to administering online assessments and the implications for pre-service teachers' learning of mathematical content.

Collaboration: Multiple Streams of Cheating

This section details how students saw an opportunity to take the easy way out by sharing answers on social media platforms such as WhatsApp and/or Telegram, requesting third parties to write the exams for them, and gathering under one roof to exchange answers during the assessments. The easy path paved by students contributes to digital citizenship that is not based on academic integrity. We define digital citizenship as students' ability to participate competently, honestly, positively, and critically in an online or digital environment. Previous research on online teaching and learning has suggested the importance of building online learning communities to allow students to collaborate with their peers for meaning-making (Espasa & Meneses, 2010; Kuo et al., 2014). Earlier, Cross (1998) stated that learning communities involve "groups of people engaged in intellectual interaction for the purpose of learning" (p. 4). Students' interactivity in online learning environments helps them improve their understanding of the subject matter contents and, in turn, academic achievements and persistence in their courses as this forms part of their SP. However, our experiences teaching mathematics education online and administering the assessments online revealed intentional collaboration to cheat. For example, in the module that ZJ handles on the mathematics subject didactics, the first assignment is usually multiple-choice questions that students must complete individually online. While ZJ was reflecting on this assessment, she mentioned that:

> Even before the pandemic, this assignment was fully online and marked online. On monitoring students' conversations on the discussion forums, one would encounter quotes like, "I have run out of time. Can somebody just send me the ten correct responses? My WhatsApp number is …

Similarly, HM reflected that although the university put in place the use of the Invigilator app during examinations as a monitoring mechanism, students continued to engage in academic dishonesty, which was detected by the application:

> Even though the students were made aware that the Invigilator app would detect any form of unethical and dishonest practices during the examination, the students continued to share answers on WhatsApp and Telegram, asking other people to write for them and even gathering under the same roof to share answers as evidenced by the GPS coordinates on the app. To maintain the institution's academic integrity, flagging the students for disciplinary processes was done for my module. My biggest concern is that these individuals will be teachers; they need to learn and own the mathematical skills for themselves to be able to transform the contents and teach their own learners once they are qualified. These cheating processes are problematic as these people will qualify for the degree they do not have mastery of the contents taught in the courses because they get other people to write for them or share answers with their peers just to ensure that they pass.

However, in his reflection, FM stated that:

> Although the Invigilator app has its challenges, such as students being unable to access and use it for uploading and downloading, I think if students could be thoroughly trained, this could be the best tool to use for monitoring online exams and maintaining the integrity of the university in general. Lecturers also complain that the Invigilator app is giving them a lot of work because they have to listen to what transpired when students are writing exams

and eventually write a report which, in my view, this a task to be given to the exam section but not to the lecturer.

When administering the exams online, with the aid of an Invigilator app, we realised that students engaged in discussions on the social instant messaging applications WhatsApp and Telegram, which were removed from our direct supervision and involvement, to exchange answers to examination questions. The analysis of the students' activities during the assessments through the Invigilator app revealed one drawback of online curriculum delivery and assessment was the intensification of plagiarism practices, as exemplified by the lecturers' statements above. We are concerned that as much as the 4IR tools provide opportunities for effective communication amongst students, they deter mathematics pre-service teachers' conceptual development and mathematical rigour as they depend on other people to complete assessments for them. This, we believe, constrains their learning and understanding.

The students in our courses not only used the different online communication tools to learn mathematical concepts and skills collaboratively and cooperatively within online learning communities but to cheat in assessments. The following excerpts are further illustrative of students' unethical practices during assessments:

> The rest of the assignments in my module were in the form of either mathematical teaching strategies and/or calculations together with a portfolio of evidence of professional development. Unfortunately, students would leave this portfolio till the last moment. Many times, some of them shared responses and examples required in some questions and submitted the same portfolio (ZJ).

> Of concern for me is whether these students care about learning and owning mathematical skills and the development of their identity as mathematics teachers. Our students view online learning as an opportunity to have other people write for them, which means they do not study for themselves. I have the biggest module in the department, and most of the students were found to have engaged in unethical practices during the exam, with some of them citing their difficulties in understanding mathematics concepts. The outsourcing of people to write the exam for the students or the exchanging of answers among students does not only weaken the integrity of the institution but the future of mathematics education in the country (HM).

> During COVID-19, as lecturers, we were obliged to assess and teach the student online, and this was an unusual task for me. However, through this productive struggle, I have learned to give out take-home exams to the students, considering the university's quality assurance principle. Even though I set my exam to avoid compromising the standard of assessment of the university; however, I was frustrated by students' evidence of copying answers from the internet and phoning each other during the exam. Some of the students, as we heard, could even hire other people to write exams for them. What is of concern to me is that many students obtain their qualifications when they don't deserve those degrees. Hence the university introduced the Invigilator app – a cellphone-based tool that allows non-venue-based assessments to be written in a more controlled and monitored environment and assists the institution in upholding academic integrity, which is to the benefit of all stakeholders, including you, the student (FM).

Church and de Oliveira (2013) stated that WhatsApp had gained prominence due to its advantageous features, such as being able to simultaneously send instant messages to an individual and/or groups, low cost, and the privacy it offers. The students in our course used these features to cheat in real-time as they continued to write the exams. Our experiences echo findings from existing research that has revealed that the new generation of university students hold a fluid perspective relating to unethical behaviour compared to students in the past (Gulli et al., 2007; Troop, 2007). We acknowledge that the use of the WhatsApp, Telegram and discussion forums is one way of creating learning communities to allow information sharing and continuous collaboration to learn challenging concepts. However, the cheating mechanisms revealed by the Invigilator app in our modules impede not only the reliability of the assessments but also the quality of their conceptual understanding as prospective teachers. On the topic of online cheating, Adedoyin and Soykan (2020, p. 5) iterated that "in online learning, assessments are often carried out online whereby instructors are limited to proxy supervision of learners, making it impossible to regulate and control cheating."

Given the CoI framework adopted, we acknowledge that the creation of the communication channels among the students could be regarded as Social Presence as it resonates with the dimensions of group cohesion, open communication, and emotional expression (Lowenthal & Snelson, 2017). However, in our case, we argue that online teaching and learning exacerbates academic dishonesty and lowers the academic rigour, development, and retention of mathematical knowledge in our courses. Thus, as with our experiences, the students failed to recognise the traditional conceptualisation of academic dishonesty as they exchanged answers to the assignments and examination questions as well as in requesting third-party assistance in answering the questions.

Whereas the above discussion addresses students' cheating in online assessments using WhatsApp and Telegram, we also realised from the analyses of the Invigilator app reports for our modules that students had gathered in one place to write the assessment. This further alerted us that the students were using "multiple streams of cheating" to ensure that "they tackle the difficult questions in groups." Considering that these students are pre-service teachers, our concern is that they are not making an effort to learn the mathematical skills, concepts, and processes to ensure that when they are qualified as teachers, they will have developed mathematical rigour, which is a prerequisite for the enablement of their own learners' epistemological access to mathematics knowledge (Lotz-Sisitka, 2009). Emerging from our experiences is the understanding that we cannot fairly administer mathematics assessments, which include affective and cognitive aspects, in an online teaching and learning environment, as students even continue to cheat with the Invigilator app put in place for monitoring and surveillance purposes. We still believe that the use of the Invigilator app to monitor students' activities during the exam is a good move by the university because students cannot continue with the violation activities, which include gathering in one room to answer the questions, having third-party assistance during the exam, or even accessing information online to answer the questions.

Conclusion

This chapter has shed some light on mathematics education lecturers' online teaching, learning and assessment experiences. The current chapter further emphasises the essence of a community of inquiry in ensuring quality teaching, learning and assessment in mathematics education modules. In a community of inquiry that fosters quality learning (cognitive presence), teaching presence is shown when facilitators such as lecturers and e-tutors display high competency in executing online facilitation focusing on essential skills. These include good planning and organisational skills, familiarity and competence in the module content and structure, enthusiasm, flexible online facilitation approaches, and effectiveness in setting up online learning platforms and resources. Such skills will influence social presence characterised by responsible digital citizenship, good classroom communication, free expressiveness, group cohesion and competency, ultimately leading to individual responsibility in learning, including individual student effectiveness in flexible learning and individual competency in learning mathematics education.

Recommendations

Mathematics education teaching at higher education institutions must provide education environments that train a teacher who is responsible and able to teach in a digitally developed setting where learners in diverse classrooms will be able to adapt to the 4IR needs. Maximum student support will need the inclusion of lecturers in the unfolding of institutional training sessions of additional support staff like e-tutors. To encourage the students' interest and participation in the support systems, especially e-tutoring, we recommend that the support assessment activities should be included in the main assessment plan in formative assessments. In addition, all facilitators involved must assess the students' ability to apply and create knowledge to develop students' high-order thinking skills in Mathematics.

Furthermore, we recommend that teaching on online lecture platforms must include all students as active participants and digital natives who can create knowledge and apply inquiry and critical thinking in solving problems mathematically. Our narrations in this chapter aim to recommend and contribute relevant post-Covid-19 strategies of adapting to transformation to fully online education systems. In all mathematics education settings that were characterised by conventional learning during the pre-Covid-19 era, we recommend a shift in facilitation, where mathematics education facilitators will apply the teaching presence strategies that will promote social presence among all participants in the educational environments; a social presence that is aligned with the high digital literacy skills needed by students to cope with the post-Covid-19 demands and 4IR needs. This type of social presence in an educational setting will promote cognitive presence and competency that will empower prospective teachers with global competency in mathematics teaching. Furthermore, we recommend that teaching mathematics education in the online lecture sessions should be supplemented by diverse strategies of student support encapsulated in a CoI. Such student support strategies, as outlined in this chapter, are critical and require improvement to ensure the eradication of students' academic dishonesty during online assessments.

Limitations

This chapter narrates the experiences of only four module facilitators in the Mathematics Education Department at UNISA. The experiences are based only on the participating lecturers' modules and the interaction of students and e-tutors as part of the trio in the respective modules reflected in this chapter. Even if the findings may not be generalised, the recommendations may contribute to the development of more online classrooms in mathematics education.

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Chapter 11
Fault Lines in Designing Learning Activities for Practising Mathematics Teachers: An Autoethnographic Account

Cyril Julie

Opening

I was worried. Only one meeting was held with teachers in 2020 (31 January - 1 February 2020). And then the country went into its first hard lockdown at midnight on 26 March 2020. Schools were closed. Teachers were not seeing their learners. Gatherings of large groups of people, including workshops with teachers, were out of bounds. “How do I go about with steering the CPD of mathematics teachers under these circumstances?”; was lingering in my mind. I envied my colleagues who were, in addition to being project collaborators, also teaching courses to prospective mathematics teachers and preparing to do their work under the “new normal”. Going through my mind was that “They are lucky”. They had to teach the enrolled students virtually. The teachers participating in the project did so voluntarily. But they were also caught up in organizing for working with their learners in a virtual manner.

I recalled the last institute we had with teachers in 2019. Amongst other issues, teachers were asked to respond to “Imagine that you can see **your teaching** of mathematics 10 to 15 years from now. Write down 2 images of what you see” One teacher responded as follows.

> I stand in my kitchen and work out mathematics. All learners sit and are in different places; McDonalds, Burger King and even in the park (Mathematics Teacher, 23 August 2019).

Was this response prophetic? Definitely not. However, 42% of the 60 teachers who completed the questionnaire in one way or another referred to forms of information communication technology playing a significant role in the teaching and learning of

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school mathematics. One responding teacher expressed the familiar Luddite-like teacher replacement response "*Educators replaced by robots*". Another one was more circumspect but proffered a robot-incorporated image articulated as "*A computer based tuition and the teacher assistant is a robot*."

I did not view the responses by the teachers as strange. There was much talk and effort going on at the time about the rolling out technology in the form of tablets and reasonable access to data in line with the United States' act of "No Child Left Behind". South Africa's president, the minister of basic education, and the members of provincial executive councils for education expressed high enthusiasm for this thrust. The teachers were aware of these developments. However, I still had trepidations about CPD provisioning virtually. Will the vibrancy that is present in our face-to-face sessions be there? What about the valuable insights that are gained in informal discussions during breaks? The gossiping? The sharing of joys and sorrows? And so forth, and so forth…I just could not come to grips with the virtual offering of CPD sessions. Contact with teachers was nigh non-existent. The primary activity was the project workers producing a set of booklets of activities dealing with grade 11 Mathematics that are examined in the National Senior Certificate Mathematics examination. The booklets were couriered to grade 12 teachers. It was well-received by teachers. My hopes were high that it would go better in 2021.

"Go better" meant that there will be meetings with teachers. At the first project meeting for 2021, it was agreed that physical (face-to-face) and virtual workshops with teachers for continuing development purposes were the most viable given Covid-19. It was decided that second school term in 2021 was deemed safe with little risk of spreading Covid-19 as long as strict protocols were adhered to. The first face-to-face workshop with teachers was held on 10 April 2021. Deciding that the Covid-19 situation was still at a safe level, a second 4-hour workshop with mathematics teachers was held on 4 June 2021. These workshops were primarily driven by my desire, shared to various degrees of commitment by fellow project team members, for teachers to consistently implement the project's signature notion of productive practice to enhance learners' achievement in high-stakes school-based and nationally-set external examinations.

As is customary, I devised an initial agenda of activities for the workshop. Fellow collaborators were presented with the proposed agenda to suggest changes, additions and the like. No major revisions to the 4-hour workshop were suggested. Customarily workshops are conducted with Biggs's (1981) dictum for CPD courses "that teachers should have first-hand experience of learning mathematics at their own level through investigations so that they would understand the exhilaration children experienced, as well as the problems they encountered, when working in this way" (Biggs, 1981, p. 59).

This chapter emerged from what transpired in the second workshop.

A Brief Note on the Fourth Industrial Revolution and Mathematics Education

Obviously, an extensive exposition on school mathematics and the 4IR is not possible within the confines of a book chapter. Much has been written about it from various philosophical, sociological, psychological, and political orientations. In a fair amount of these publications, mathematics (sometimes designated as numeracy) is propounded as necessary but not in its current form. The knowledge-based economy, digital society and twenty-first-century society are used in conjunction or synonymously with the fourth industrial revolution (Gravemeijer et al., 2017; Jones & Ng, 2021; Sahlberg, 2006). Regardless of the different terminology used, there is agreement about the competencies and skills, consonant the fourth industrial revolution, that learners must command. These competencies and skills include creativity, problem-solving, critical thinking, collaboration, communication and flexible working styles. The above skills and abilities are commonly highlighted in most international education systems. However, a consideration of high-stakes examinations, in South Africa at least, reveals that the weightings assigned to examination problems dealing with these competencies and skills are low. Learners can actually be successful in the examinations without even doing these problem types.

A bit removed, but connected to mathematics, are newer considerations such as robotics and coding.

Gravemeijer et al. (2017) delineate three mathematical competencies necessary for the fourth industrial revolution society. These are mathematical modelling, including the applications of mathematics, understanding and verification of results. The research described in this chapter focuses on understanding concepts and processes and verifying the results. How some competencies consonant with the fourth industrial revolution are built into the activities are described when they are given below.

A Perspective on the Research Canon—The Approach

I present a story of myself as a CPD provider. "[A] **story** is a detailed organization of narrative events arranged in a (story) structure based on time although the events are not necessarily in chronological order" (Kim, 2016, p. 33). The value of stories is captured by Achebe (1987, p. 35) as "It is only the story...that saves our progeny from blundering like blind beggars into the spikes of the cactus fence. The story is our escort; without it, we are blind." Stories, as a form of research involving the self, are situated within the domain of autoethnography. Adams and Herrmann (2020) unpack "Autoethnography" as a form of research comprising three interrelated components: "auto," "ethno," and "graphy." Thus, autoethnographic projects use selfhood, subjectivity, and personal experience ("auto") to describe, interpret, and represent ("graphy") beliefs, practices, and identities of a group or culture ("ethno")"

(p. 2). Autoethnography is thus an approach that investigates and reports on personal experience.

My Experience with Story-Telling

My first encounter with story-telling was in a doctoral course on qualitative research methods. Amongst other issues I was exposed to story-telling by the course presenter, professor Terry Denny. I was hooked and read Denny (1977, 1978) many times—even, for this chapter I reread the chapters. What grabbed me was Denny's way of bringing to life, in a journalistic fashion, people's stories of their experiences. He, for example, titles one of the sections of an evaluation report journalistically as "The Great Mystery" (Denny, 1977, p. 1–36). To exemplify the mystery, he uses quotations coming from teachers such as "Except for the "block la" (30 of 140 third graders) all third graders know one thing perfect in math one day and the next it is gone...I mean GONE (p. 1–37, underline in original). The title of the section and the actual words of respondents captivated me about the 'power' of story-telling. Denny does not refer to autoethnography but how he presents his reports epitomize the spirit story-telling in educational research.

As alluded to above I was hooked on story-telling and ventured into it a few times. None of them barring one—Julie (2019)—was explicitly labelled a story. One, Julie (1990), was not strictly autoethnographic but instead dealt with obtaining data when opportunities arose in strange places. The other two, Julie (1989, 1992), had a more story-like flavour in that I relay the struggles of my dealings with mathematical problems I tried to solve.

This chapter is about me as a CPD facilitator for mathematics teachers and an activity I gave them to do at the second workshop in 2021.

The Spark

Every story has a spark that ignites it—something that is observed, some reflection on what was read, some experience the author had, some comment along the lines of "you should write a story about it", a wonderment about something, and so forth. The trigger for this story was not so outlandish. Rather, it was in the context of my work. This was the design of an activity teachers had to do during the second CPD workshop on 4 June 2021. Approximately forty teachers from 12 high schools in the Cape Flats area in the Cape Peninsula attended. The Cape Flats is the area where indigenous people, their later descendants and descendants of slaves were firstly banished to by the early colonizers and later forcibly removed to by the Apartheid regime. Crime, gangsterism, drug peddling and addiction characterize the area. Some of the areas have the ignominious fame of having the highest murder rates in the country.

Teachers were teaching grades 8 to 12 Mathematics, with some teaching Mathematical Literacy. A $1\frac{1}{4}$ - hour session was devoted to the topic indicated on the programme as "Designing 'productive practice' activities related to work completed in the first term and to be "examined" in upcoming tests and examinations in phase groups." This task was presented to teachers teaching grade 8 and 9 Mathematics. The teachers teaching grades 10 to 12 had a different activity facilitated by another project worker. Prior to the design activity, a presentation was given on "a productive practice" activity of work completed in the first term in grade 8." The exemplifying activity was an "Always, Sometimes, Never (ASN)" one, given in Table 11.1.

ASN activities add a "sometimes" category to normal "true/false" activities. Regarding the competencies deemed desirable for the 4IR society, an ASN activity requires the use of underlying concepts, the generation of own examples and non-examples and, if needed, the execution of procedures. The generation of examples and non-examples is important for coding, which in many instances has to do with the identification of allowable and non-allowable values. During coding, precautions have to be taken to handle non-allowable values to avoid programming crashes.

For their designs, teachers had at their disposal a 4-page handout with the adjusted teaching plan for 2021 provided by the Western Cape Education Department. The teaching plan was adjusted to compensate for time lost due to the school year starting later because of Covid-19 concerns. The topics that had to be covered during the first term and with excerpts on integers from the teacher's guide of the recommended textbook were provided to teachers. The task presented to the GET group of teachers was to design an activity related to the 'practicing of integer operations'.

Amongst other activity types, teachers also chose to design ASN activities with some be exemplified below.

These activities were collected with the purpose of me refining them for classroom use in the form of teaching toolkits to post on the project's website. I describe how, by consideration of the designs the teachers produced at a workshop, led to the construction of a follow-up activity for a subsequent session with the participating teachers. As mentioned above, it is thus a personal kind of story where I recollect events, as far as I could extract them from the recesses of my mind, that drove the construction of the activity to its current form.

Table 11.1 Activity presented during the opening plenary

Fractions			
Indicate with a cross (X) which of always, sometimes or never fits the given statement. Motivate your answer with examples.			
Mathematical statement	Always	Sometimes	Never
When two positive proper fractions are divided the result is a positive fraction greater than any of the two fractions.			

Fault Lines of My CPD Facilitation Regarding Getting Teachers to Design an Activity

I reflected on the designs produced by teachers. This rendered:

(a) There were activities conforming to the tenets of an ASN activity requiring learners to engage with mathematics, albeit that one of the response categories was included in the mathematical statement as demonstrated in the excerpt of the activity in Fig. 11.1.
(b) Some of the mathematical statements were fairly specific, with the response incorporated in them requiring either direct recall or a simple way to obtain the required response (Fig. 11.2).
(c) The designs appeared to be highly influenced by the "true/false" question type. This question type figures strongly in resources teachers have available. Ostensibly the existence of a third category, sometimes true, is not that which teachers teaching in this phase have much experience.

These considerations made me realize that I missed a lot that I had to draw teachers' attention to when I prepared the activities for the workshop. This realization bugged me and made me decide to revisit ASN activity types at the next meeting with teachers. Acting on designing an ASN activity for teachers to address the above issues was not immediate. It was delayed due to other more immediate commitments such as supervision and guidance of post-graduate students, attending virtual meetings with management staff of the faculty and the human resources department to finalize arrangements for the project's upcoming phase, etc. The issue was, however, lurking in the back of my mind. Not focusing on a problem but it being ostensibly alive in the recesses of one's mind is a well-known aspect of problem solution-seeking. One of the popular ones is Poincaré's gaining insight into mathematical problems. Topolinski & Reber (2010, p. 402) give an account of Poincaré's gaining insight as follows:

> After working for weeks on new kinds of mathematical transformations, mathematician Henri Poincare´ stopped working and went on a geological excursion, during which he put the mathematical problem out of his mind. One day on that trip, he entered a bus: "Just as I put my foot on the step, the idea came to me, though nothing in my former thoughts seemed to have prepared me for it, that the transformations I had used to define Fuchsian functions were identical with those of non-Euclidean geometry... I made no verification.. but I felt absolute certainty at once" (Poincaré, 1913/1996, p. 53). Only days later, after having returned home, he verified this discovery.

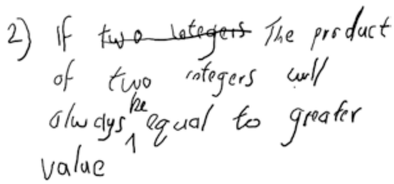

Fig. 11.1 Excerpt 1 of teacher-designed activity

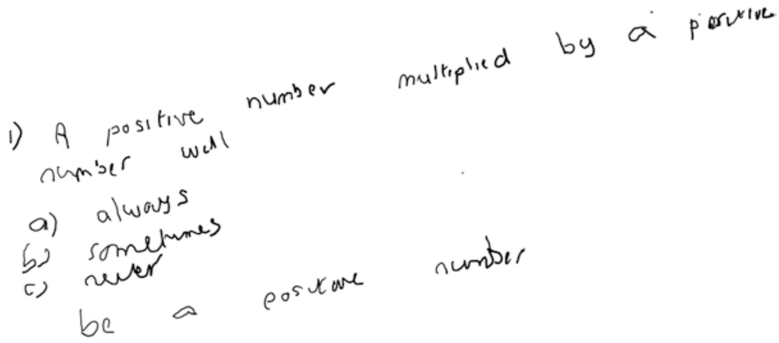

Fig. 11.2 Excerpt 2 of teacher-designed activity

Poincaré's experience in his domains of interest and the prior thoughts lurking in the recesses of his mind probably triggered the insight.

I cannot recall exactly when it happened, but my thoughts wandered to an activity anchored in palindrome numbers and divisibility by 11. My experience with this problem was linked to a course on mathematical problem solving and mathematical modelling I co-taught with a colleague when I was working at a university in Norway. The problem was "Is ABBA always divisible by 11?" This kind of problem, I thought, would be apt for teachers to engage with in the light of engaging teachers to "experience of learning mathematics at their own level", as suggested by Biggs in the quotation above.

Another issue that influenced my wondering was thoughts related to the features of effective CPD. These thoughts were around in my mind because I was engrossed with my CPD project entering a new phase in July 2021. For this I consulted the literature collection I had on the effectiveness of CPD. Eight studies were consulted to summarize these criteria. One study (Cordingley et al., 2015) is an ""Umbrella" review, i.e. a review of reviews of the evidence". Timperley et al. (2007), Walter and Briggs (2012) and Vangrieken et al. (2017) conducted systematic reviews. The studies by Huang and Bao (2006), Hiebert and Morris (2012), Cobb et al. (2003) and Cobb and Jackson (2015) were single research studies. Since there was a virtual absence of explicit theories of change in these sources, Fullan's (2006) work on theory of change was included. Thoughts about these issues were not at the forefront of my mind but they were around and not visibly acted on.

During the session I went for my second COVID-19 vaccination injection. I penned some ideas for a possible activity. During the first vaccination session I learned that after receiving the injection, one had to stay in a holding area for 15 minutes before leaving the vaccination centre. For the second vaccination session I took a notebook with me. I was more aware of writing thoughts down since they tend to disappear. Although when supervising post-graduate students, I impress on them the importance of writing ideas down when they pop up, I am not as disciplined in regularly doing it myself. However, during my reading of Roald Dahl's stories for leisure at this time the issue hit me head-on. Dahl's (2013, p. 2) work was on what he calls “not an autobiography”. In the closing section of the book, he gives writing tips. He writes: “‘A story idea is liable to come flitting into the mind at any moment of the day, and if I don’t make a note of it at once, right then and there, it will be gone forever.’” (Dahl, 2013, no page number).

These initial thoughts were heavily influenced by the criteria for effective CPD as pertaining to sessions with teachers. These criteria are:

- A trajectory for teacher learning must be known, and teachers’ current and ongoing practice and knowledge must be taken as starting points for the CPD content.
- The same norms suggested for teaching to teachers for use in their classrooms must underpin work with teachers during CPD sessions.
- Materials must be accompanied, “at a minimum, [by] the types of questions the teacher should ask students to spur their thinking” (Cobb & Jackson, 2015, p. 1027). Pre-prepared questions must be part of the toolkits. These questions are obtained through thought experiments, experience and elicitation from teachers and observations.

Regarding teachers’ “current knowledge and ongoing practice…taken as starting points”, I took the designs by the teachers as the starting point. A learning trajectory for teacher learning is also mentioned. Simon (1995, p. 133) describes a hypothetical learning trajectory as “The consideration of the learning goal, the learning activities, and the thinking and learning in which the students might engage….” I did not even think about the learning trajectory during the preparation of the presentation and its delivery in the workshop. I attended to the surface level features of the presented ASN activity. It was as if I expected that, through some osmotic process, teachers would get it.

Furthermore, teachers were left to design “in the wild”, with the only prescription being that previously taught work must be revised. Questions to spur the teachers’ thinking were glaringly absent and not given any thought. As facilitator, I took an indifferent-non-interventionist approach. The hope was that when the groups presented their designs in a plenary report-back session issues, such as those in my reflections above, would surface. They did not, and neither myself nor my co-facilitator alerted teachers to these issues. It was a “moment for engagement” lost. At best engaging teachers in the design of activities in line with ways of working they are expected when implementing activities in their classrooms was by allowing teachers to work in voluntary groups of not more than 4. These reflections pertain to me as a CPD facilitator and the fault lines are clear.

Towards an Activity

Based on the above considerations, the initial thoughts I penned in the vaccination centre waiting room about a teacher ASN activity for an upcoming workshop is presented in Fig. 11.3 with the cleaned activity given in Table 11.2.

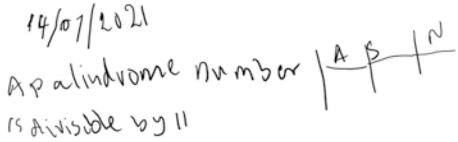

Fig. 11.3 Jotted down notes of a possible activity

Table 11.2 Cleaned version of the activity

Indicate with a cross (X) which of Always True, Sometimes True or Never True fits the given statement. Motivate your answer			
Mathematical statement	Always	Sometimes	Never
A palindrome number is divisible by 11			

A Mere Activity is Not Enough for Teacher Learning

Normally, I ended my design of activities for workshops with teachers with the cleaned activity as given in Table 11.2. The thoughts regarding effective CPD features were now in the foreground of my mind. Specifically, the teacher learning goal, anticipations and questions to spur teachers' thinking when they are engaging with the activity stood out.

The Teacher Learning Goal

I responded to the goals as follows (Fig. 11.4):

- Recognize the similarities and differences between a "true/false" and an ASN activity
- Recognize the formulating structure of the mathematical statement of an ASN activity
- Become conversant with the learner goals (Read and understand problems; generate appropriate examples of mathematical constructs; formulate conjectures; construct arguments to support the formulated conjectures; communicate conjectures and supporting arguments; critique the reasoning of others; carry out correct mathematical, including algebraic, procedures)

Thinking and Learning in Which Teachers Might Engage (Anticipations) and the Types of Questions to Be Asked to Spur Teachers' Thinking

The notes I jotted down are given in Fig. 11.5 and a cleaned, and expanded version is in Table 11.3.

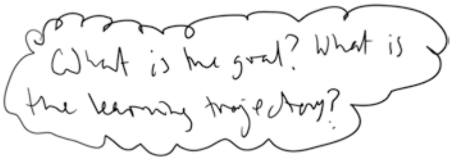

Fig. 11.4 Rough notes on goals

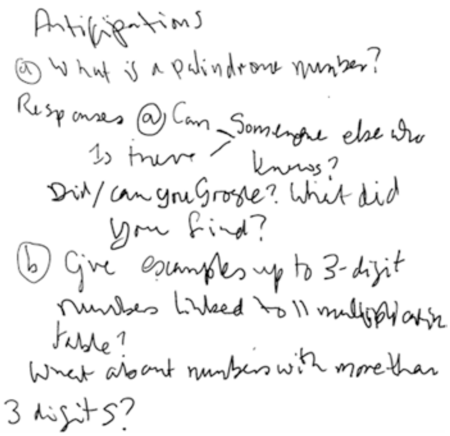

Fig. 11.5 Rough notes on anticipatory issues

Table 11.3 Cleaned list and prompting questions for use during workshop

Anticipatory issue	Suggested prompting questions
Not certain what a palindrome number is: What is a palindrome number?	Can someone else (another teacher) explain what it is? Did you Google it? If not, Google it quickly. What did you find?
Generate limited examples: Give examples of up to 3-digit numbers linked to the 11-multiplication table	What about numbers with more than three digits?
Insufficient variation of numbers to observe a pattern: Single number with more than three digits	Are there more palindrome numbers than the one you have given? What happens when you increase the number of digits by 1?
Provides a weak motivation: If the number is a palindrome with an even number of digits, then the sum of the digits in alternate positions will be equal to sum of the remaining digits. The difference between these sums will be zero, which is the rule for divisibility by 11	Can you do this with a general palindrome number with an even number of digits?

Recollecting the procedure suggested to teachers for the use of our toolkits with learners, I realized that there need to be some guidelines for the implementing the activity myself or another project worker. The developed guidelines are:

Guidelines for Implementing the Teacher Learning Activity

Hand out the activity and allow teachers to read through and discuss among themselves the activity. Observe and note what teachers are discussing. Note particularly whether the first issue surfaced (Not more than 5 min).

After the first anticipation is sorted out, collect some responses—a variety of examples (2-digit, 3-digit, etc.)—from teachers. It might be that teachers require a few minutes to generate responses (Record on newsprint, whiteboard or document-viewer). Lead a discussion on the responses keeping in mind the second anticipatory issue.

Ask some teachers for justifications (motivations) for their responses. Record it. Discuss justifications in the light of the anticipatory issue 3. Encourage teachers to construct a stronger justification using some generalized palindrome number based on their explorations.

I distributed in the "notes", basically starting from the section titled "the spark" up to the aforementioned last sentences, to fellow project workers. The comments were at the low end of positive. I concluded there was general approval for the activity and its implementation guidelines.

A Light Shock

The ASN activity still bothered me. I do not know what it was. I realized that as soon as palindromes with three digits are hit, then "sometimes true" will naturally emerge with counter-examples such as 111, 131, etc. "Well", I said to myself, the "suggested prompting questions" in Table 11.3 covered this. My anxiety did not disappear. The troubling was somewhat resolved when it dawned on me that the activity had more of a flavour of a mathematical investigation. Investigations require substantially more time than what is suggested for teachers to implement these activity types with their learners. I then decided that the mathematical statement should be "a palindrome number with more than four digits even number of digits is divisible by 11". I still have to adjust accompanying anticipations and "suggested prompting questions" to fit this revamped activity.

Conclusions

I presented my journey of designing an activity for practicing teachers. The activity is linked to the skills and competencies required for the fourth industrial revolution. As is evident from the initial activity in Table 11.1, the quest is to allow teachers to implement such activities in their classrooms. The exemplary activity is dealing directly with content that is prescribed in the intended curriculum the teachers follow. An obvious question that may be raised is why the nearness of the activity for learners to the operative curriculum and not one more directly aligned to competencies and skills for the fourth industrial revolution as, for example, reported by Ng et al. (2018). I believe, and it is my experience, that the likelihood that teachers will be less inclined to incorporate such activities in their normal teaching due to the accountabilities accompanying adherence to the intended curriculum. I thus followed a route of sufficient alignment with the curriculum teachers are teaching and included some elements consonant with the abilities desired for the fourth industrial revolution. My contention currently is that a viable way to link the fourth industrial revolution to current teaching is to exploit opportunities for such in the operative curriculum and sensitize teachers to such in continuing professional development initiatives. A fully fletched mathematics curriculum aligned with the fourth industrial revolution requires changes in de-emphasizing (and some instances, shedding) current content and using digital technologies with which to do mathematics (Gravemeijer et al., 2017). This requires policy changes which takes time.

Any story is interpreted by the reader of it. I thus do not offer any direct conclusions. Instead I close with Denny's (1978, 46-57) advice to "allow readers 'elbow room'… [to enable] the reader to draw reasoned conclusions".

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Chapter 12
Teaching Mathematics in the Fourth Industrial Revolution: Instances of Instrumental Orchestration When a Teacher Integrates GeoGebra in Solving Linear Inequalities

Charles Smith, Cyril Julie, and Faaiz Gierdien

Introduction

This chapter comprises two parts. The first part of the chapter recounts the work of a teacher participant who demonstrated to his peers how he integrates technology in his classroom. The second part of the chapter provides some guiding principles to teachers on making technology integration decisions.

The Fourth Industrial Revolution (4IR) is beginning to impact education in a way that cannot be ignored by teachers and teacher educators, with the advent of smart classrooms. The incorporation of digital technologies in mathematics teaching and learning has been an ongoing concern since the 1970s. However, according to Joubert (2013), the uptake of these technologies remains disappointing. For example, a recent report of the Joint Mathematics Council in the UK stated that "technology within mathematics is underused and where it is used, its potential is generally underexploited" (Clark-Wilson et al., 2011, p. 6).

Currently, a vast number of technologies for the teaching and learning of mathematics are available. Some of these technologies privilege a teacher-centred approach, whilst there are also technologies amenable to a learner-centred approach in the classroom. Out of these available technologies, Dynamic Mathematical Technology (DMT) haws received much attention. The reason is ostensible that the DMT-type technologies afford both teacher-centred and learner-centred pedagogical moves. DMT is defined "as a subset of technology with multiple, dynamically linked, mathematical representations that can be manipulated so users can engage with underlying concepts and relationships. Such technologies offer the potential for teachers and pupils to (re-)express their mathematical understandings"

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(Clark-Wilson & Celia Hoyles, 2019, pp. 333–334). GeoGebra, the focal technological tool in this study, is one such DMT and is widely used in the school setting.

The drive to integrate DMT in mathematics teaching and learning underpins the need for teachers to develop a repertoire of teaching skills to use available technologies to enhance the learning of mathematics in school classrooms. Teachers are critical agents of transformation in this regard. Hammonds et al. (2013, p. 36) assert that "For true change to take place in the classroom, the paradigm shift must begin with the teachers rather than the students."

Continuing professional development (CPD) is used, in many cases, as a vehicle to capacitate practising teachers with these technological skills (Alqahtani & Powell, 2017; Bu et al., 2013; Clark-Wilson & Celia Hoyles, 2019; Daly et al., 2009). CPD is currently a widely implemented intervention for skills development and involves all modalities of teachers' professional learning, whether formal or informal, within or out of school, self-directed or externally prescribed. Consequently, it is also a mechanism well suited for capacitating practising teachers to implement technologies for teaching mathematics.

Tamam and Dasari (2021, p.1) so aptly stated that "The most important thing is that teachers must have sufficient knowledge of insight and skills to operate the software." Belgheis and Kamalludeen (2018, p.113) concluded that:

> In short, should the teaching of mathematics require the integration of software or other technological tools, teachers should be given enough exposure and training in using the technological tool and the pedagogical skills to ensure the impactful delivery of classroom lessons.

Salinas-Hernández et al. (2018, p. 91) emphasised the need for targeted CPD to enable teachers to integrate technology in a meaningful way:

> [These] … results emphasise the need for finding ways to assist teachers in integrating [technology tools], meaningfully… [A] possibility to consider is the interaction between teachers, expert teachers and mathematics education researchers, engaging in lesson analysis, discussions, reflections, sharing of experiences – from the lesson planning stages to the implementation and lesson analysis.

This call is also pertinent for South Africa, as Mokotjo and Mokhele-Makgalwa (2021, p. 79) asserted that: "The key finding of this study revealed the need to strengthen the professional development of teachers concerning the integration of GeoGebra into the mathematics curriculum."

It is widely acknowledged that one of the factors contributing to the effectiveness of CPD is teachers sharing their experiences regarding implementing novel teaching strategies during CPD sessions with other colleagues (Desimone, 2011; Macias, 2017; Nooruddin & Bhamani, 2019). This article deals with one such sharing regarding the use of GeoGebra by a teacher during a CPD session. This was part of a series of CPD workshops of the Local Evidence-Driven Improvement Teaching and Learning Initiative (LEDIMTALI) (ledimtali@uwc.ac.za).

This approach in CPD is also predicated on what Bruner and Olson (1973) termed the principle of learning through modelling. They posited that "One of the more transparent instructional approaches is that of modelling or providing demonstrations… (p. 29).” They further elaborated:

> Observational learning is realised through the provision of a model. As we pointed out earlier, carrying out a performance for its own sake and carrying it out to instruct another are not identical. A good demonstration explicitly makes the decisions made in the course of the activity – thus, a good demonstration shows the child what not to do and what to do (p. 35).

They added to that principle the potential of technological media to enhance this process: “Technological media can greatly facilitate these processes by highlighting in various ways the critical points in the performance (p. 35)”.

In this chapter, we employ the instrumental approach (Rabardel, 2002) to tool use and the metaphor of instrumental orchestration (Trouche, 2004) as a theoretical framework to explore technology integration in mathematics classrooms. This approach recognises the complexity of the integration of technology in mathematics classrooms. The problem statement and the research question are presented in the next section. This is followed by a discussion on the theoretical stance taken in the research. The data collection and analysis methods are subsequently dealt with, and the discussion of results follows. The article concludes with recommendations for consideration for the implementation of GeoGebra in classrooms.

Problem Statement and Research Question

Only if adopted and appropriated in classrooms will the potential of ICT and other digital technologies to improve teaching and learning in Mathematics classrooms be realised in the South African context. This belief has led to increased government- or donor-funded projects to equip schools with the requisite infrastructure. For example, in the Western Cape Province, the provincial education department launched the Khanya project, equipping all public schools with ICTs and training educators on how to use the technology when delivering the curriculum. The project, which started in 2001, is an initiative of the Western Cape Education Department (WCED). According to the Khanya (2008) report, 59% of the schools in the province had computer laboratories, and 70% of educators had been trained to use ICTs in their pedagogy.

However, evidence suggests that even in cases where the ICT and computer infrastructure are provided, it is not effectively integrated with teachers’ everyday teaching practices (Chigona et al., 2010).

There is limited understanding of what happens in a classroom when teachers employ technological tools to teach a mathematical topic. Drijvers et al. (2010, p. 213) pointed out that when it comes to integrating technological tools in the classroom, the teacher’s role is critical and problematic.

> It is critical in the sense that how teachers approach the use of technology has major consequences for the effects of its use in the classroom. It is problematic, as teachers who do not

> perceive the use of technology in their teaching as valuable for their educational goals can avoid it unless explicitly required to do so by institutional or curriculum constraints. Also, teachers often experience difficulties adapting their teaching techniques to situations in which technology plays a role (p. 214).

In order to support teachers in integrating technology as a routine feature of their practices, we explore this issue based on the research question: What type of instrumental orchestration emerges in using GeoGebra in a technologically enriched mathematics classroom?

Theoretical Underpinnings

This section presents relevant literature regarding the use of technological tools in the classroom, specifically focussing on Mathematics teaching and learning. We will start this section by providing an overview of GeoGebra.

GeoGebra is a software tool developed by Markus Howenwater in 2001 as an outcome of his master's studies: GeoGebra – a Software System for Dynamic Geometry and Algebra in the Plain. This project aimed to develop a completely new kind of tool for mathematics education in secondary schools. In brief, it is a versatile tool for school mathematics and may be used differently. Hohenwarter and Fuchs (2004) listed the following uses of the software: as a tool for demonstration and visualisation; as a construction tool; as a tool for investigation and inductive discovery; and lastly, as a tool for designing and preparing teaching and learning material.

Using computer tools in education involves a complex process of *instrumental genesis*, where a tool or artefact is transformed into an instrument. By this, it is understood that the artefact becomes a so-called instrument when appropriated by the user to perform a specific task. GeoGebra software is regarded as an artefact and becomes an instrument when the teacher appropriates it for teaching and learning. Instrumental genesis involves two processes. These are referred to as *instrumentation* and *instrumentalisation* (Rabardel, 2002; Trouche, 2004). Boon et al. (2010) point out that instrumental genesis is a process during which the object or artefact, in this case, GeoGebra software, is turned into an instrument. This requires the artefact plus a scheme or implementation techniques. On the other hand, instrumentalisation is described as the teacher's adaption or transformation of the tool for specific use during the didactical performance. Rabardel (2002, p. 106) contended: "Instrumentalisation can be defined as the process in which the subject enriches the artefact's properties. This process is grounded in the artefact's intrinsic characteristics and properties."

Trouche (2004, p. 293) explains the process of instrumentalisation:

> Instrumentalisation can go through different stages: a stage of discovery and selection of the relevant functions, a stage of personalisation (one fits the artefact to one's hand) and a stage of transformation of the artefact, sometimes in directions unplanned by the designer:

modification of the taskbar, creation of keyboard shortcuts, storage of game programs, automatic execution of some tasks (calculator manufacturers' websites and personal websites of particularly active users often offer programs for certain functions, methods and ways of solving particular classes of equations, etc.).

The instrumental genesis process is predicated on the assumption that technical knowledge about the artefact and domain-specific knowledge are combined in a series of schemes and techniques for using the artefact in the mathematics classroom.

An associated construct in this approach to instruments is *instrumental orchestration*. Drijvers et al. (2010, p. 214) offer the following definition: "An instrumental orchestration is defined as the teacher's intentional and systematic organisation and use of the various artefacts available in a—in this case computerised— learning environment in a given mathematical task …". Furthermore, it may be intuited that "Instrumental orchestration [thus implies] the intentional and systematic organisation of the various artefacts available in a computerised learning environment by the teacher for a given mathematical situation ..." The term *learning environment* encompasses learning resources and technology, means of teaching, modes of learning, and connections to the school contexts (Drijvers & Trouche, 2008, p. 377). There are different types of instrumental orchestrations.

Drijvers et al. (2020, p. 1458) identified the following types of whole-class orchestrations: (1) Technical demo, (2) Explain-the-screen, (3) Link-screen-board, (4) Discuss-the screen, (5) Spot-and-show, and (6) Sherpa-at-work. Numbers 1 to 3 are predominantly teacher-centred, where the teacher stands in front of the class and projects the outputs of the technological tool on a big screen. Orchestration types 4 to 6 are more interactive. Briefly, these orchestrations imply:

1. Technical-demo orchestration: the demonstration of techniques by the teacher using a data projector and a large projection screen.
2. Explain-the-screen orchestration: whole-class explanation by the teacher, guided by what happens on the screen. The explanation goes beyond techniques and involves mathematical content.
3. Link-screen-board orchestration: explanation of the mathematics represented on the screen, using the writing board in a setting where both screen and board are visible. This normally happens at the onset of a task or through intervention to address issues observed whilst the learners are working.
4. Discuss-the-screen orchestration: a whole-class discussion on what happens on the screen. The goal is to develop an understanding of concepts or observation of patterns by the learners. Here the teacher facilitates learner discussions of the outputs of the technological tool.
5. Spot-and-show orchestration: student reasoning is made visible by interesting digital student work produced by students in solving the classroom task. The teacher may have the students whose work is shown explain their reasoning and ask other students or her−/himself to provide feedback on the student's work.
6. Sherpa-at-work orchestration: the student uses the technology to present his/her work or to carry out operations the teacher requests.

In addition to the six orchestrations mentioned above, we may also consider how these orchestrations manifest operationally. In this regard, three pedagogical moves have been described within a teacher's instrumental orchestration (Trouche, 2004; Drijver et al., 2010). Pedagogical moves refer to particular teacher actions to create learning opportunities for students. For the topic under investigation, these pedagogical moves are: (1) a *didactic configuration* which determines the gathering and arrangement of the tools in the mathematics classroom; (2) an *implementation scheme* which involves the teacher's lesson planning and choice of learning design and (3) the *didactical performance* meaning the delivery of the lesson and dealing with classroom contingencies.

Data Collection and Analysis

The data comprise a video recording of a teacher participant demonstrating to his peers how he integrates technology in his classroom. The teacher teaches at a school in a low socio-economic status area in the Northern Cape Town Metropole. His presentation was not merely a lecture. The demonstration moved between a PowerPoint presentation and the use of GeoGebra. The topic he dealt with was quadratic inequalities. During the presentation, he frequently posed questions to the other participating teachers.

A deductive analytic approach was applied where the first author identified instances of the theoretical underpinnings mentioned above. As a validity check, the other authors checked whether the assignment of the theoretical constructs was appropriate. No major differences were found.

Results and Discussion

The outcomes of the processes of instrumentation and instrumentalisation are reported in this section.

Figure 12.1 represents the basic affordance of the software. These graphs represent the outcomes of two inputs, $f(x) = x^2 - 4$ and $g(x) = x + 2$, which were used to address various inequalities.

However, the software possesses embedded features which are not easily available to the casual user. So in the process of instrumentalisation, the teacher exploited these features to achieve the intended purpose of the lesson.

The outcome of this process is shown in Fig. 12.2.

To achieve instrumentalisation, the teacher had to adapt the basic GeoGebra construction in the following ways:

1. Remove the gridlines so that the salient features are easily observed.
2. Create a slider that enables him to toggle between three different graphical displays—the linear graph (Graph = 1), the quadratic graph (graph = 2) and the combination of the two, as shown in Fig. 12.2 (Graph = 3).

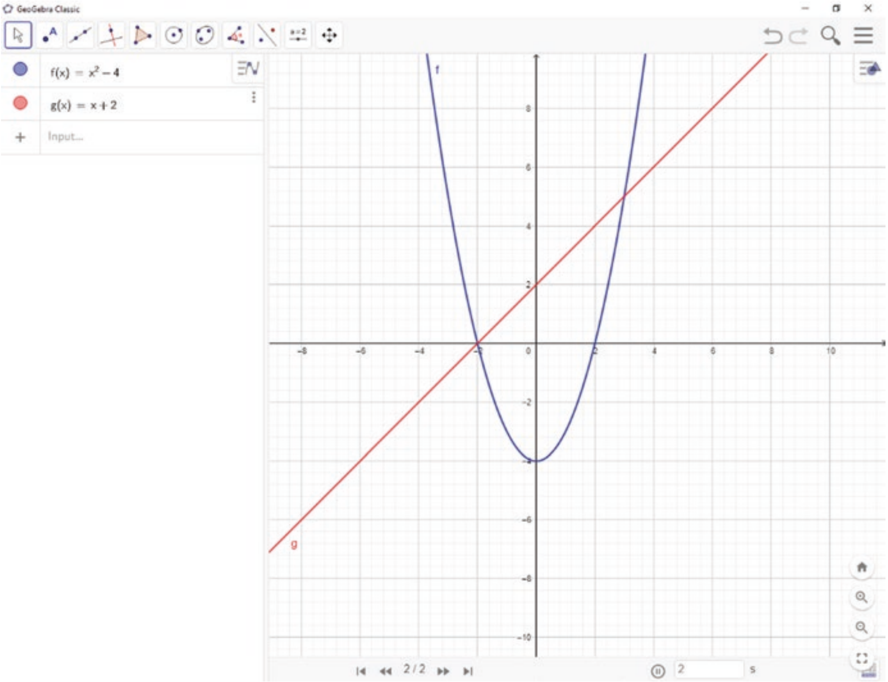

Fig. 12.1 Basic affordance of GeoGebra for sketching the initial graphs

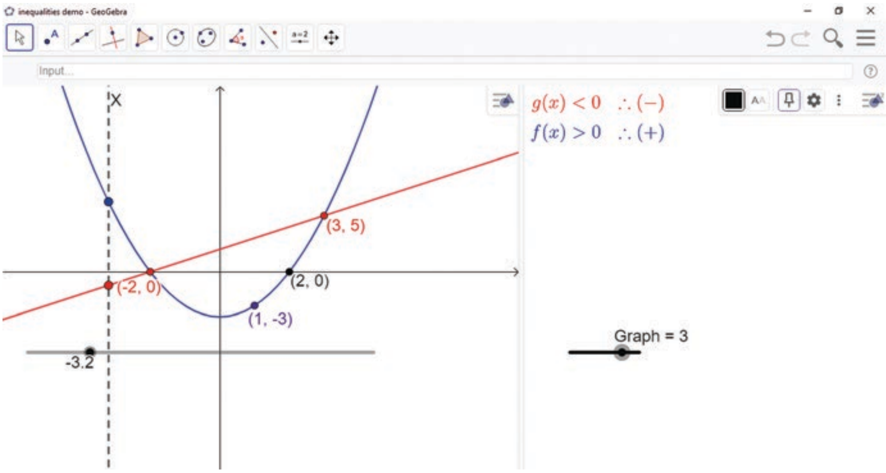

Fig. 12.2 Outcome of the use of embedded features

3. Create a horizontal slider that enables the teacher to explore different values of x.
4. Construct a movable point on the slider, which also shows the value of x.
5. Construct the vertical line, and construct the points of intersection with both graphs to explore different inequalities.

6. Construct dynamic text to describe the outcome for any value of x.
7. The teacher hid the algebraic view as this could distract from the results he wanted to demonstrate.

In the presentation, the teacher constructed a *didactic configuration* of a laptop connected to a data projector. The screen was placed in front of the venue to be visible to all participants. In addition to the GeoGebra architecture, the teacher also developed a PowerPoint presentation to explain specific mathematical concepts and procedures during the lesson presentation. This, together with the pre-designed worksheets, constituted his *implementation scheme*. Inevitably, questions arose, which he answered. Some of these questions related to the mathematical content and others to the didactic configuration. His responses were instances of his *didactical performance.* Hence, we observed the didactical moves described by Trouche (2004) and Drijver et al. (2010) manifested in clear terms. It is important to highlight these pedagogical moves as they are intrinsic to integrating digital technologies in the classroom. They have to be contemplated beforehand and planned thoughtfully and systematically. The ultimate successful execution of the intended teaching trajectory is contingent upon these moves.

In addition, we also observed the teacher-centred orchestrations, which we will explain in the following section. The teacher used a *technical-demo orchestration* to explain the techniques afforded by GeoGebra to configure the representation shown in Fig. 12.3. This led teachers to question him on the time he took to produce his final result. One participant asked: “Tell me, how long did you take did it take you to reach that point?” to which he responded: From 2 till 6 last night – that is roughly 4 h.” In this question, we notice the teachers’ reservation and hesitation, if not trepidation, in adopting the technology.

Other instances of instrumental orchestration taken from the presentation.

Figure 12.3 illustrates how the teacher explains the solution of $g(x) > 0$ using the instrumentalised technological tool, namely the GeoGebra construction and the slider. This is an instance of discuss-the-screen orchestration.

Figure 12.4 shows an instance of *screen-board orchestration*. The teacher reverted to the board to explain the solution to the inequality $f(x).\ g(x) \geq 0$. The teacher used his instrumentalised GeoGebra presentation to establish the critical values $x = 2$ *or* $x = -2$. These are the values where the function value changes from positive to negative or vice versa. After that, he establishes the signs of the two functions on the different sides of these critical values. Figure 12.4 shows the process completed for $g(x)$. The table becomes a tool for semiotic mediation (Mariotti & Maffia, 2018) of mathematical content. Here we use the term semiotic mediation to describe the support that the table method gives one in accomplishing a task through its use. The idea of mediation in this instance relates to the potentiality of the table method of fostering learning processes with respect to solving inequalities in general. The semiotic mediation perspective stresses that many teachers use the table method in the classroom for accomplishing a didactical task, as referred to in the preceding statement. The outcome of the table method is predicated on the facts that:

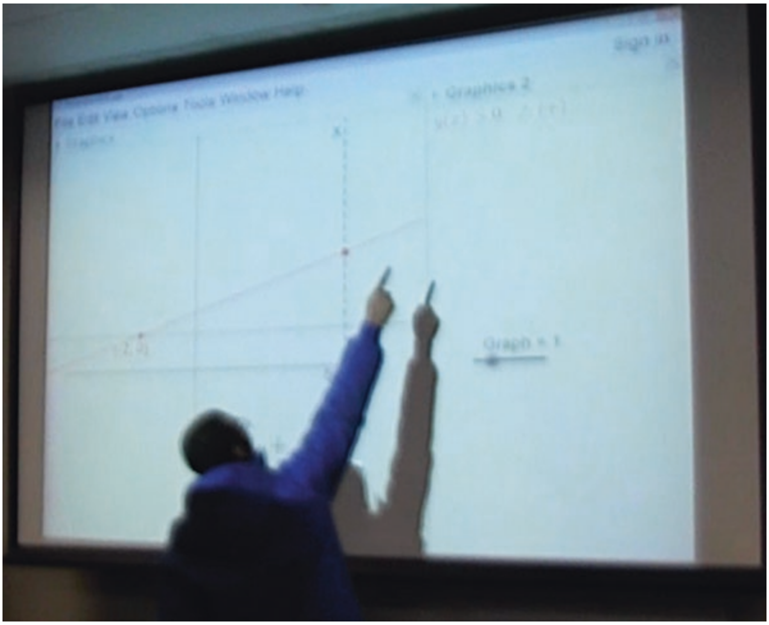

Fig. 12.3 Discuss-the-screen orchestration

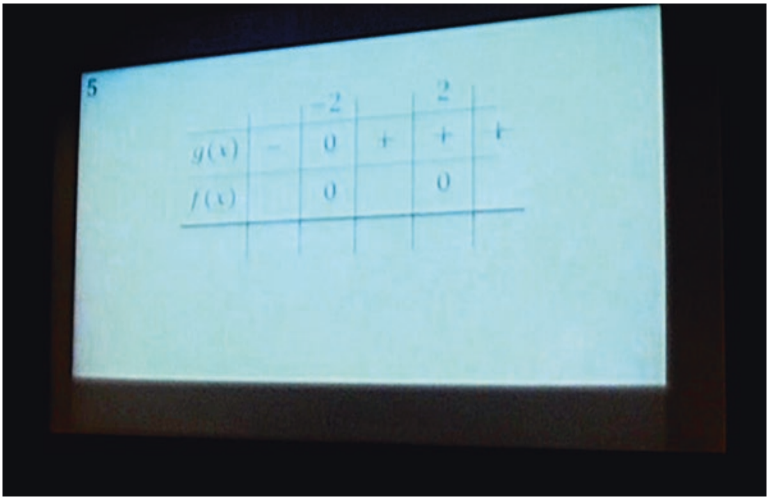

Fig. 12.4 Screen-board-orchestration

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$$\text{apositivenumber} \times \text{apositivenumber} = \text{apositivenumber}$$

Differently stated: to get a positive value (> 0), the product of $f(x)$ and $g(x)$ the signs of the functions must be the same in a given interval.

The observed implementation scheme indicated that in terms of teaching, sequence and strategy during the presentation closely resembled the traditional method. This may be interpreted that teachers in resource-deprived settings are more likely to employ Discuss-the-screen and Explain-the-screen orchestrations. It is self-evident that the technical-demo orchestration is a sine qua non of this didactical performance.

Guiding Teachers in Making Technology Integration Decisions

There are quite a number of studies describing the use of technology in mathematics classrooms. However, there is a dearth of studies investigating the factors that influence teacher decisions regarding deploying particular technologies in their thinking (McCulloch et al., 2018). Where teachers have made the transition to incorporate technology in their classrooms, attention should be given to the following questions:

1. Why do teachers choose to use technology to teach mathematics?
2. What tools do they choose to use and why?
3. What are the general criteria that guide teachers' choices?

Answers to the above questions will assist researchers and teacher educators, and educational administrators in providing guiding principles for teachers interested in integrating technology into their everyday routines. Teachers and their students may or may not have access to technological tools for deployment in the teaching and learning of mathematics.

However, a vast array of such tools are available, for example:

- The internet provides vast amounts of information
- Communication tools (different social media platforms and e-mail)
- Productivity tools (e.g., word processing, spreadsheets and dynamic geometry systems)
- Creativity tools (e.g., for generating multimedia presentations, producing digital videos, and computer-aided design)
- Here are three guiding questions for selecting technology to use for a particular mathematics lesson (McCulloch et al., 2018)

Loague et al. (2018, p. 3) describe the following continuum for technology use in the classroom:

- Low Acceptance/ Use: Teachers are unenthusiastic and sceptical about the benefits of technology. He or she generally avoids or dismisses conversations about technology if possible.
- Intermediate Acceptance/ Use: Teachers recognise the benefits of available technological tools, especially where they are compatible with existing practices; ease of use determines attitude daily. They are amenable to conversing about technology‘s merits in educational settings.

- High Acceptance/ Use: Here, we encounter very positive teachers who embrace learning new technology. They usually also encourage their peers by sharing information and resources and offering assistance. Such teachers will initiate conversations about technology. We also find that they are frustrated by the lack of technology in their classroom settings. Hence, they will sometimes, at their own expense, procure technological hardware and software for their classrooms.

Let us now focus on the principles guiding a teacher's deployment of technological tools. The following set of questions will elucidate these principles.

- **Question one: Should I use technology in this lesson?** The answer to this question depends on the availability of appropriate instruments and whether they will enhance understanding or practice by learners. If the answer to question one is yes, then follow up with:
- **Question two: If so, what type of technology should I use?** The answer is contingent on the hardware and software available. This will steer the instrumentation the teacher is afforded with.
- **Question three: What type of instrumentalisation is needed or possible?** This will depend on the teacher's knowledge of the content matter and the instrument's features.
- **Question 4: What exploitation mode will be most effective?** Here I will have to consider one or a combination of the instrumental orchestration types referred to earlier in this paper. In this regard, the teacher will be guided by the classroom ecology and his or her preferred teaching style.

We should heed the caution of Boileau (2012), p. 1):

> The use of technology cannot replace conceptual understanding, computational fluency, or problem-solving skills. In a balanced mathematics program, the strategic use of technology enhances mathematics teaching and learning. Teachers must be knowledgeable decision-makers in determining when and how their students can use technology most effectively.

Daniel et al. (2020, p. 260) proposed four phases to the process of integration of technology in the classroom:

> We specifically recommend a four-phase approach to help preservice teachers (a) build their knowledge and learn to value technology in physical education, (b)observe and explore through instructor modelling and integration, (c) experiment and collaborate with mentoring and scaffolding, and (d) discover through innovation and utilisation.

This approach has merit even for in-service teachers as it is predicated on a coaching and mentoring model supported by in-class experimentation.

A New Learning Ecology

Barron (2006, p. 195) defined a learning ecology as the "set of contexts found in physical or virtual spaces that provide learning opportunities." By a learning ecology, we refer to all the components that make up the learning environment that is

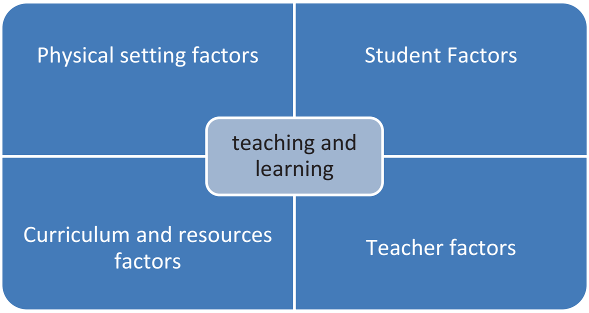

Fig. 12.5 Components of a learning ecology

created by the following factors: the physical setting factors, the curriculum and resources factors, the teacher factors and lastly, the student factors. Schematically the learning ecology may be represented as shown in Fig. 12.5.

Access to technological tools in the classroom may potentially change the learning environment, bringing about a new learning ecology. This may introduce novel ways.

for students to be engaged and educated." Spires et al. (2012, p. 234) envisage such a learning ecology with the following features:

- Immediate and constant access to information and a global community
- Intensity, relevance and personalisation of learning
- Highly developed teacher capacities
- Highly developed student dispositions

Teacher Concerns

In their review of research related to teachers' uses of technology, McCulloch et al. (2018, p. 10) identified some of the teachers' concerns about integrating technology. These concerns included personal concerns, managerial concerns, and technological concerns. In addition, barriers existed such as the amount of curricular freedom afforded to the teacher, previous teaching experiences with technology, adequate training and preparation of teachers, adequate planning time, preferred teaching style and the lack of appropriate software.

Recommendations

The role of preservice teacher education at universities is of utmost importance in preparing prospective mathematics teachers for the appropriate integration of technologies in mathematics classrooms. The ways in which technology may be incorporated into teacher education programs may include specific courses focused on technology integration. Some of the understandings which could be developed in such programs include:

- How to use technology to enhance the students' understanding and interaction with mathematics
- How to use technology to perform mathematical procedures with ease, fluency and accuracy
- Providing opportunities for students to practice mathematical concepts and procedures with a view to consolidation.

Conclusion

We have presented a general framework for instrumental orchestration, which is predicated on a process of instrumental genesis. We have shown the relevance of instrumental orchestration to integrating GeoGebra as a tool for teaching specific mathematical content. We have explained the process of instrumental genesis, which consists of two bi-directional activities, namely instrumentation and instrumentalisation. We demonstrated how this process is underpinned by a particular exploitation scheme described in three phases: a didactic configuration, an implementation scheme and a didactical performance.

In terms of the conceptual framework for this paper, teachers are amenable to instrumentation. However, the process of instrumentalisation requires specialised knowledge of the embedded features of the software and is also time-consuming and tedious. Thus, in the initial stages of adoption, it is important to provide teachers with pre-designed applets and tasks or worksheets. This will facilitate meaningful integration in normal classroom settings.

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Chapter 13
Integration of GeoGebra in Teaching and Learning of Mathematics in the Niger Republic Classrooms

Abdoul Massalabi Nouhou

Introduction

Information and communication technologies (ICT) are an opportunity in all activity sectors because they make many services and daily activities easier. It is undeniable that the education sector is not an exception. Therefore, the higher secondary education system should promptly adopt new ICT tools and know-how to harness their power to train learners (Grinshkun & Osipovskaya, 2020, p. 1). The Mathematics Learning Process through ICT can be a response to the needs of the fourth Industrial Revolution (4IR), where technological progress aligns with human needs, more specifically by improving students' skills to meet the changing demands of 4IR. The integration of new technologies in teaching and learning mathematics can help children in African countries prepare for lifelong learning and the ability to predict the upcoming changes.

Literature Review

The literature review of this research was built from three aspects. The first aspect is the Fourth Industrial Revolution (4IR). According to Schwab (2017), the 4IR is a technological revolution that has altered our very being in terms of how we live, work and relate to one another. The scope and complexity of this transformation will not resemble anything that humanity has experienced during the previous industrial revolutions. The 4IR is characterised by the fusion of the digital worlds and disciplines (biology, mathematics, physics …), nonlinearity, and the re-emergence of digital into material and physical domains. The key to the 4IR is that

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today's youth will no longer start a career path or grow in one role; roles will change regularly (Naidoo & Singh-Pillay, 2020, p. 2). Equipping African citizens with the skills needed in an evolving job market is necessary. They have to be prepared for lifelong learning and have the ability to predict change. In education, teacher training programs should improve the educational processes and outcomes of the educational system. It must be constantly reformed in the light of adjusting to the new technological changes and must be the object of continuous evaluation. Therefore, we can retain that a new form of mathematics education is emerging from the fourth industrial revolution.

The second aspect is the transformation of the mathematical learning process through new technologies (Grinshkun & Osipovskaya, 2020, p. 3). GeoGebra is a software created by Markus Hohenwarter to improve mathematics learning through ICT. The acronym GeoGebra derives from two words: ***Geo*** *of Geometry and* ***Gebra*** *of Algebra.* GeoGebra is an interactive geometry, algebra, statistics and calculus application for learning and teaching mathematics and sciences from primary school to the university. This dynamic mathematics software is free to access and can be used in several languages, such as French. It is available on multiple platforms and websites. GeoGebra is being used in other educational disciplines.

GeoGebra can be installed and used for learning about computers, tablets and smartphones. GeoGebra is software that allows an individual to work on multiple representations of a function object. It can be used as an environment for actively exploring the function of a mathematical object (Freiman et al., 2009, p. 39). The entry line is used to write algebraic and symbolic expressions for the function object. The algebraic window displays the algebraic, the numerical or the formal writings. The dynamic working window is used to display the graphic representation of the function (the curve and the graph). The formal calculation window allows you to enter symbolic expressions of the function and perform the usual calculations of the functional analysis (solving an equation or inequation, calculating the limits, calculating the derivative, calculating the integral, etc.). The spreadsheet window allows you to enter numbers, coordinates, functions or commands and establish a correspondence table (x; y). GeoGebra software can offer higher secondary students the possibility of mobilising several registers of semiotic representations of the mathematical function during the learning process. It can be learned from this literature review that the emergence of ICT tools such as GeoGebra can improve the process of mathematical learning in relation to the requirements of the 4IR in Africa.

The third aspect is the approach to teaching and learning mathematics concepts, such as the concept of mathematical functions in higher secondary school. The mathematical functions are tools of problem solving, and this acquires greater visibility by studying some of its properties, distinguishing between algebraic, numerical, geometric and analytical registers for the concept of mathematical functions. Each register of representation does not explain the same aspects of the concept of a function, and the absence of an interaction of the frames makes students experience difficulties coordinating the different contexts. Therefore, it is essential for the learning of mathematical functions to be able to mobilise several registers of semiotic representations (Duval, 2002, p. 83) and to insist particularly on the interaction between the registers to validate the mathematical properties of this concept (Bloch, 2003, p. 26). Dynamic mathematical software offers multiple representations of mathematical function and its visualisation, a potential that can contribute to the learning of higher secondary school students (Minh, 2012).

From this literature review, we examine the key challenges of mathematics learning to enable higher secondary school students to meet the demands of 4IR in Niger. We formulated the following research question: ***How does the integration of GeoGebra in the teaching and learning of mathematics in Niger enable students to accommodate the requirements of the 4IR****?*

Theoretical Framework

The theoretical framework of this research was built from two theories. The first is the theory of semiotic registers of representation (Duval, 2006). This approach provides a framework for studying the activity of mathematics as the activities of the students around the treatments of the registers of representation and conversions. Duval (1993) internalises the distinction between a mathematical (conceptual) object and its representation. It introduces the notions of semiosis and noesis to designate the apprehension, or production, of a semiotic representation and the conceptual apprehension of an object, respectively. He says that «noesis is inseparable from semiosis» to promote learning. The semiotic aspects of the mathematical activity propose an analysis of the activities of the pupils around the treatments of the registers of representation and conversion. According to Coppé et al. (2007), there are six main registers in teaching and learning numerical functions: the register of the natural language, the algebraic register of formulas, the graphic register of curves, the numerical register of table values, and the graphic register of variation tables and the symbolic register. This theoretical framework enables us to analyse the student's activities in solving problems with Geogebra tools (Jacinto & Carreira, 2017, p. 1118) and understand the strategies of treatment registration and conversions of the registers used during this process.

The second construct of the theoretical framework is the theory of semiotic mediation (Bussi & Mariotti, 2008). This approach provides a framework for studying the activity in mathematics as a mediated activity. The mediating potential of an artefact is related to the accomplishment of mathematical tasks through this artefact (Mariotti & Maracci, 2010). The Geogebra software can allow students to work simultaneously on the multiple representations of the function. For example, writing an algebraic expression of a function in the input line (register of algebraic writing), the algebraic expression of this function immediately appears on the algebra window (register of algebraic symbolic expression), and its representative curve is plotted in the dynamic worksheet (graphic register). We will analyse the mediation of the GeoGebra tools in the process of solving mathematical functions.

Methodology

The quasi-experimental approach was retained to examine the key challenges of mathematical learning through GeoGebra among higher secondary school students. It was built from five (5) points: the field of our research, the participants, the observation tools, analysis of secondary higher school mathematics programs, and the data processing tools collected.

The Field of Our Research

In our research field, this public higher secondary school welcomes students of all socioeconomic classes due to its position in the Agadez region (Niger Republic). The school has 627 students of the three levels of higher secondary school, divided into 15 classes, in 2017–2018, the year in which our experiment began.

Our research approach is quasi-experimental with a non-equivalent control group. That is to say; our non-probability sample is composed of experimental and control mathematics classes.

The Participants

The participants of this research are 65 students. Two Mathematics classes were randomly chosen, with the normal age of the students ranging from 17 to 18. The experimental group of 34 students of class mastered the features of GeoGebra related to mathematical functions (SC2), while 31 students of class 1 did not master them (SC1).

The Observation Tools

The observation tools have been developed and validated during the pre-experimental phase. Two (2) teachers checked the consistency of the content of the test scores before it was administered to the students of the experiment. At the end of the experiment, five (5) copies were drawn randomly from the sets of copies and photocopied in triple. The two teachers and the researcher corrected the five copies based on the initial correction grid. The comparison of the results gave a value of 0.99 for the Alpha of Cronbach (Less than 0.70). This confirms the internal consistency of the instrument for measuring the test scores.

Analyses were conducted at two levels. The first level of analysis examined the learning gains. The unit of analysis was *the cognitive gains in learning mathematical functions.*

The second level of analysis examines the mathematical tasks of the register of the semiotic representation of the mathematical functions in the base of the current mathematics syllabus in higher secondary school (MES, 2016). The unit of analysis was *an occurrence of each of the strategies of treatments of the register of the semiotics representation of the mathematical function, conversions of the registers and obstacles* identified related to GeoGebra tools.

Analysis of Higher Secondary School Mathematics Curriculum

The analysis of the higher secondary school mathematics curriculum will allow us to clarify the treatments of semiotic representation registers of mathematical functions (Table 13.1) and Conversions of registers (Table 13.2), which will be approached during the teaching and the learning process, and the mathematics learning with GeoGebra.

Learning Approach of the Concept of Functions in Higher Secondary School

Before the experimentations, the two teachers prepared two mathematics sessions of mathematics lessons according to the current ASEI / PDSI approach[1]implemented by the SMASSE[2] program in Niger, since October 25, 2006. In preparing for these lessons, they developed two mathematics lesson sheets. Each mathematical lesson sheet describes the learning scenario representing the a priori description of the progress of the learning situation of the numerical function. The aim was to describe all activities for the appropriation of a given mathematical concept by specifying, among other things, the role of the teacher and that of the learners, the activities to achieve, the resources available for achieving the learning objectives, the didactic tools and the forms of mediation necessary for the implementation of teaching and learning activities. The scenario follows the course of the pedagogical progression; therefore, the sequence of these steps only makes sense if the teacher achieves the set goals before moving on.

[1] https://www.jica.go.jp/niger/french/activities/activity01.html

[2] SMASSE = Strengthening of Mathematics And Sciences in Secondary Education

Table 13.1 Treatments of semiotic representation registers of mathematical functions

Semiotic representation registers	Code	Treatments of semiotic representation registers
The algebraic register of formulas	Ra	Determine the definition or study set of the mathematical function (Ta1) Calculate the image, the antecedent of a number (Ta2) Calculate algebraic equations (Ta3) Algebraic calculation of inequalities (Ta4) Calculate the limit of a function (Ta5) Deduce the expression of the equation or the inequality (Ta6) Determine the algorithmic expression of a function (Ta7)
The intrinsic symbolic register	Rs	Write $f < g$ (Ts1) Write $f = g$ (Ts2) Write $\lim_{x_0} f$ (Ts3) Write the sign $(+\infty\ , 0ou - \infty)$ (Ts4) Determine $C_f \cap C_g = \{A\}$ (Ts5) Determine $C_f \cap (xx')$ (Ts6)
The natural language register	Rl	Define the relative position of two mathematical functions (Tl1) Define the limit of a mathematical function at a point (Tl2) Verbal or written interpretation (Tl3) Interpretation by a mathematical language (Tl4) Interpretation of the relative position of the two curves (Tl5)
The graph register of curves	Rg	Place a point (Rg1) Draw the curve of a function (Tg2) Read the image graphically, the antecedent of a number (Tg3) Graphically read a function over a given interval (Tg4) Find the solutions of an equation or an inequality graphically (Tg5) Find the relative position of a curve with respect to a straight line with respect to another curve (Tg6) Graphically read the sign of a function (Tg7) Graphically read the limit of a function (Tg8) Graphic interpretation (Tg9) Graphically read the coordinates of a point (Tg10) Use zoom for graphical visualisation (T11)
The numeric register of tables of values	Rn	Read coordinates (Tn1) Read the trend of numerical values (Tn2) Determining the coordinates of a point (Tn3) Complete table of values (Tn4) Determine the numerical value corresponding to the solution of an equation (Tn5) Determine the interval for numerical values corresponding to the solution of an inequality (Tn5)
The graphic register of variation tables	Rv	Calculation of the comparison from the trends of the curves of two of the functions, f and g (Tv1) Draw up the table of the signs of the difference between f and g (Tv2)

Table 13.2 Conversions of semiotic representation of registers of mathematical functions

Semiotic representation registers	Direction	Conversions of semiotic representation registers
The algebraic register of formulas (Ra) and the graphic register of curves (Rg)	$Ra \rightarrow Rg$	From an algebraic expression of a mathematical function, draw a curve (Cag)
	$Rg \rightarrow Ra$	Find the algebraic expression of a mathematical function whose curve is known (Cga)
The algebraic register of formulas (Ra) and the numerical register of tables of values (Rn)	$Ra \rightarrow Rn$	From an algebraic expression of a mathematical function, establish a spreadsheet of values (Can)
	$Rn \rightarrow Ra$	From a table of values, establish an algebraic expression of a mathematical function (references functions) (Cna)
The algebraic register of formulas (Ra) and the register of variation tables (Rv)	$Ra \rightarrow Rv$	From an algebraic expression of a mathematical function, establish a variations table (Cav)
	$Rv \rightarrow Ra$	From a variations table, establish an algebraic expression of a mathematical function (references functions) (Cva)
The numerical register of the table of values (Rn) and the graphic register of curves (Rg)	$Rn \rightarrow Rg$	Plot a curve of a mathematical function from the table of values (Cng)
	$Rg \rightarrow Rn$	Find the table of values of a mathematical function whose curve is known (Cgn)
The numerical register of the tables of values (Rn) and the register of the variation tables (Rv)	$Rn \rightarrow Rv$	From the table of values of a mathematical function, establish a table of variations (Cnv)
	$Rv \rightarrow Rn$	From the table of variations of a mathematical function, establish a table of values (Cvn)
The graph register of curves (Rg) and the register of variation tables (Rv)	$Rg \rightarrow Rv$	From a curve of a mathematical function, establish a table of variations (Cgv)
	$Rv \rightarrow Rg$	Find a curve of a mathematical function whose table of variations is known (Cvg)

Presentation of the Problems on the Mathematical Functions and a Priori Analysis

Problems 1 and 2 were chosen by the mathematics teachers during the mathematics teaching unit (MTU) meeting according to the mathematics program in Niger (see appendix). This approach is part of choosing an ordinary math class experimentation. The teachers had chosen problem 1 to introduce the mathematical notion of comparing two mathematical functions. According to the mathematics program, the objective was "to compare two functions (algebraically and graphically)". Problem 2 was proposed to the students by the teachers during the introduction of the mathematical notion of function limits at a point x_0 (see appendix). According to the program, the objective was to "establish the behavior of the reference functions".

As for the resolution of problem 1, the students are called upon to work on the semiotic representations of functions in the GeoGebra environment. To answer the question (a), they can mobilise the algebraic expressions of the functions f and g (Ra) to graphically represent the curves of these two functions (Rg). This process involves converting the algebraic expression to a graphical framework (Cag). Students can use the input tool to input algebraic formulas, and GeoGebra's graph tool will allow students to plot representative curves for functions f and g. To answer question (b), that is to say, complete the correspondence table, the pupils can mobilise the graphic representations of the curves of the functions f and g of the curves (Rg) from the graph tool to place the coordinates of the points (x; f(x)) and (x; g(x)) then read the coordinates of each point and write the numerical value of the image of numbers in the table. It is in this process of passing from the graphic frame to the numerical expression of the coordinates (Cgn). Students can use the point tool to place the different points and read the ordinates. Students can also use the input tool to input the algebraic expressions of each function (Ra) for the values given in the table of values in the GeoGebra environment and then use the spreadsheet tool to calculate coordinates (x;f(x)) and (x;g(x)) (Rn). To answer question (c), students can use table of values (Rn) to conjecture that the functions f(x) = g(x) (Ra). This resolution process allows the pupils to pass from the numerical expression to the algebraic writing (Cna). Students can identify from the correspondence table the values of x having the same image by f and by g and make the conjecture algebraically. To answer question (d), i.e. to make a graphic comparison between two functions over a given interval, the pupils can mobilise trends of the curves (Rv) to interpret by sentences the relative positions of the curves of these two functions (Rl), then compare f(x) and g(x) (Ra) on a given interval algebraically, finally to be able to interpret it symbolically (Rs). The process requires a passage from the variation of curves over a given interval to natural language (Cvl), from natural language to algebraic writing (Cla), and from algebraic writing to symbolic expression (Cas). Students can use the graph tool to visualise and read the relative positions of the two curves over a given interval (Rg) and interpret this algebraically (Ra) (Cga). Students can use the zoom tool on the curve of the two functions (Rg) to make it easier to read the two curves on the interval (Ra). To answer question (e), students can mobilise the graphical representations of f and g (Rg) to read the coordinates of these points (Rn). It is in this process of the passage of the point of the intersections of the two curves to obtain the numerical coordinates (Cgn). Students can use the intersection tool to determine the intersection points of these two curves and read the coordinates of the intersection points with respect to the axes. They can use the zoom tool on the curves of the two functions to enlarge the image (Rg) and get approximations of the numerical values (Rn) (Cgn).

As for the resolution of problem 2, the students are asked to work on the semiotic representations of functions in the GeoGebra environment. To answer question 1., i.e. to complete the table of values, the pupils can mobilise the algebraic expressions of each of the reference functions (Ra) to represent these functions in the GeoGebra environment and then complete the table of values (Rn). It is explicitly a process of converting the algebraic expression to a numerical expression (Can). Students can use the input tool to input the algebraic expressions for each datum function (Ra) into the GeoGebra environment and then use the spreadsheet tool to calculate coordinates numerically (Rn) (Can). They can also use the graph and point tools to place the corresponding points (Rg) and then read and write the numeric value of the image in the table (Rn). This process will therefore require students to do a double conversion (Cag) then (Cgn). Students can also calculate the coordinates from the algebra tool by calculating the images of the given antecedents (Ra) and then read and write in the correspondence table (Rn). To answer question 2a, students can use the table of values (Rn) to read the trend of the numerical values of x^2 as a function of that of x to conjecture that "when x becomes infinitely large, x^2 becomes infinitely large" (Rl). It is a process that consists of the transition from digital to natural language (Cnl). The students can visualise the trend of the square function by visualising its representative curve (Rg) using the graph tool and then make a conjecture on the behaviour of the function in the neighbourhood of x (Rl). They can mobilise the computer algebra tool to calculate algebraically (Ra) and finally validate their conjuncture. To answer question 2b, students are asked to rely on the information stated, from the expression "when x becomes indefinitely large" to a suitable mathematical language expression "x tends to $+\infty$" (Rl) for deducing the appropriate "$+\infty$" sign (Rs). This is a shift from proper language to symbolic expression (Cls). To answer question 3), students can use the terms "infinitely large" or "infinitely small" to pass a more suitable expression "$+\infty$" or "$-\infty$" and deduce the behaviour of each of the reference functions in terms of "$+\infty$", "$-\infty$" or 0. This is a change from a formal language to a symbolic expression (Cls).

Findings and Discussion

Regarding the point of finding and discussion, we can retain two main results: *the cognitive gains scores for students in learning the mathematical functions* and *the strategy of treatments, conversions of register of semiotics representation of the mathematical function.*

The Cognitive Gains Scores for Students in Learning Mathematical Functions

Figure 13.1 shows that the mean scores of the pretest for students in the two classes of mathematics classes are less than the average scale of 100. The difference in the mean score of the two groups is not significant. The result shows that the students of these classes have a lower level. The graphic also shows that the means scores of the posttest are higher than the average of 100. The difference in the mean score of the two classes is significant.

Fig. 13.1 Means scores

Table 13.3 Comparison of the means at the pretests and posttests of the two class groups

Students of class	Number of students	Pretest			Posttest		
		Means	Standard deviation	Coefficient of Variation	Means	Standard deviation	Coefficient of Variation
SC1	31	32	10.80	0.34	57.26	20.62	0.44
SC2	34	36.35	15.81	0.43	76.21	16.76	0.34

Table 13.3 above shows an analysis of the level of knowledge at the start of the pupils. For SC1, the pretest mean is 32, the standard deviation is 10.80, and the coefficient of variation is 0.34. For SC2, the pretest mean is 36.35, the standard deviation 15.81, and the coefficient of variation 0.43. Kruskal Wallis' chi-square statistical test indicated that the difference in mean scores between the two groups is insignificant ($p > 0.5$). This result indicates that the pupils of these three classes are globally at the same level of knowledge before the teachers' interventions. However, the averages obtained by his students are below 50, a value for which it is considered that the group of students has globally reached the average threshold out of 100 in terms of knowledge of the mathematical concept of function. In the beginning, the participant students had a low level of knowledge of numerical functions with real variables.

Secondly, an analysis of the level of knowledge after the pedagogical intervention (see Fig. 13.2). For SC1, the pretest mean is 57.26, the standard deviation is 20.62, and the coefficient of variation is 0.44. For SC2, the pretest mean is 76.21, the standard deviation is 16.76, and the coefficient of variation is 0.34. Kruskal Wallis' chi-square statistical test indicated that the difference in mean scores between the two groups is significant ($p < 0.5$). This result indicates a statistically significant difference in knowledge acquisition between the students of these three classes after the teachers' interventions in favour of SC2. This result reveals that

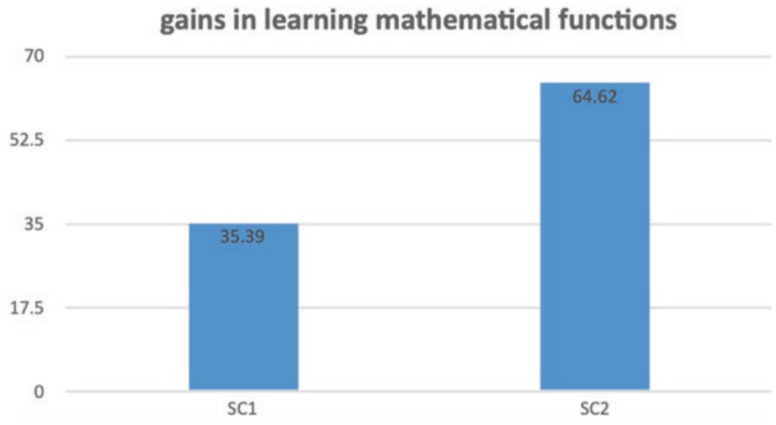

Fig. 13.2 Gains in learning mathematical functions

Table 13.4 Correlation between pretest and relative gain for the three-class groups

Students of class	Number of students	Correlation (r)	P-value
SC1	31	−0.49	0.010
SC2	34	0.098	0.581

using Geogebra enabled SC2 students to perform significantly on scores compared to the other two groups. It also appears that the averages obtained by its two groups are above 50, a value for which it is considered that the group of students has globally reached the average threshold out of 100 in terms of the acquisition of knowledge of the mathematical concept of function during learning. Students who have mobilised the potential of the software have reached three-quarters (3/4) of the maximum threshold of knowledge required according to the learning objectives of the mathematical functions.

Figure 13.2 shows a gain of 35.39 for the students of SC1 and 64.62 for the students of SC2. These results show that the students of SC2 made a 64.62% gain in learning mathematical functions while the students of SC1 made a 35.39% gain. This means that students who master the features of GeoGebra have significantly progressed in learning mathematical functions compared to those who do not master the features of GeoGebra. This corroborated with the research thesis of Vasquez (2015), who estimated that if students learned how to use GeoGebra before learning mathematics with the software, it would enhance the mathematics learning process through ICT. This illustrates the need to accommodate requirements for the 4IR in mathematics learning through ICT.

Table 13.4 shows the analysis of the correlations between pretest and relative gain. The results are negative and significant in SC1 ($r = -0.490$ and $p = 0.010 < 0.05$) while it is only positive and no significant in SC2 ($r = 0.098$ and $p = 0.581 > 0.05$). These results indicate that learning the functions made it possible for the weakest pupils at the start in SC1, while in SC2, all the pupils progressed in learning the functions whatever their starting level.

The observations of the students during the learning of mathematical functions show that students of SC1 had difficulties writing expressions for mathematical functions. In contrast, the students of SC 2 could overcome these obstacles related to the algebraic writing adapted to the tool seized from GeoGebra.

Writing square function on GeoGebra tools requires converting $f(x) = x^2$ into the expression $f(x) = x$ ^ 2 (caret) (Fig. 13.3).

For writing the square root function requires to convert f(x) = $\sqrt{x}$ into to the expression f(x) = sqrt (x) (Fig. 13.4).

These two expressions of the same mathematical function have the same denotation, but their algebraic representations are different. The first representation is an algebraic symbolic expression, while the second is a computerised algebraic expression. These algebraic algorithmic expressions of the mathematical function adapted

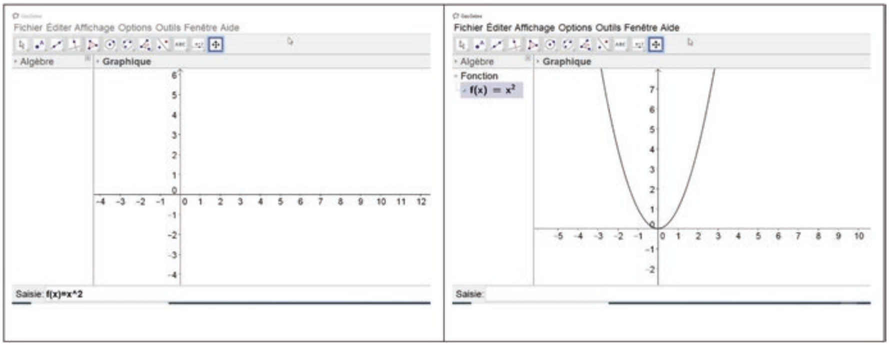

Fig. 13.3 Square function written on GeoGebra tools

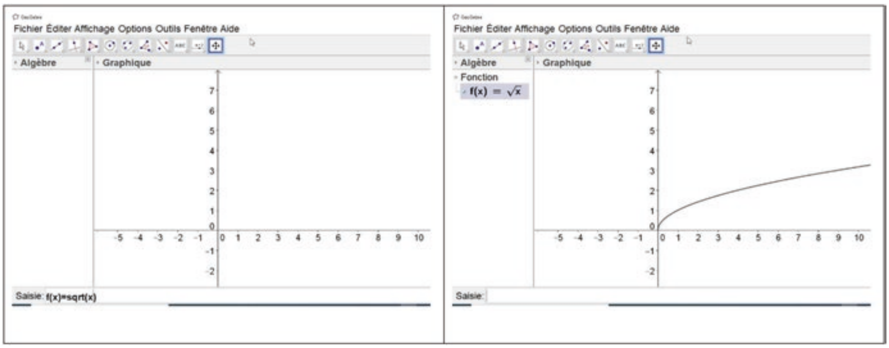

Fig. 13.4 Square root function written on GeoGebra tools

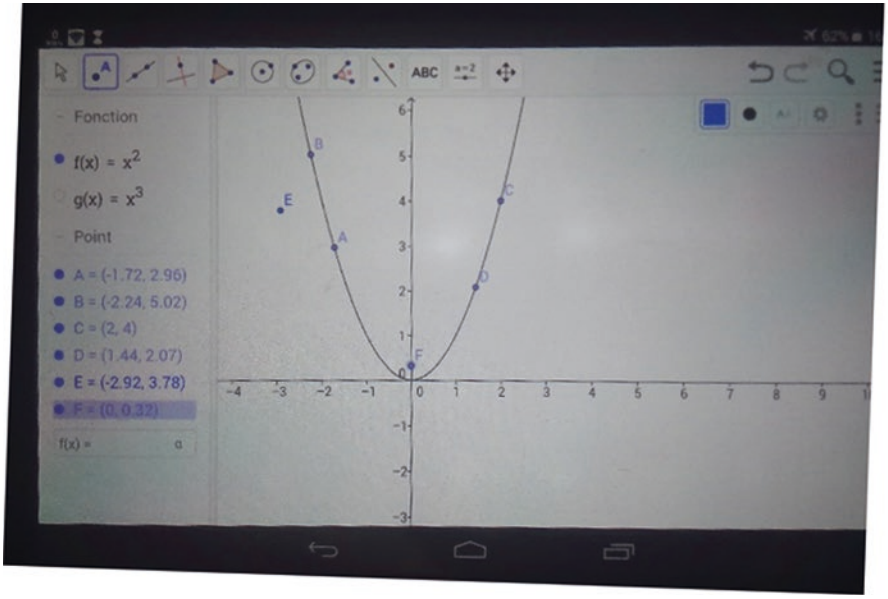

Fig. 13.5 Wrong use of the algebraic tool by students

to GeoGebra are external to the classical semiotic representation system of mathematical functions at a higher secondary level. These expressions are unfamiliar to students (Artigue, 2002). must learn all mathematical function expressions before the process of solving problems; otherwise, GeoGebra indicates that the expression is inappropriate.

The findings show that the students of SC1 encountered difficulties in making conversions of the algebraic register of formula into the numerical register of table values using the GeoGebra tools (Fig. 13.5). The students of SC2 also encountered the same difficulties, but they could overcome them personally by asking for help from the teacher. For example, to complete the table value of the algebraic expression of the mathematical function. The Students of SC1 used the algebraic tool of GeoGebra despite the difficulties instead of the calculus tool. On the other hand, those in SC2 quickly realised that this tool was inappropriate for this task. They, therefore, adapted the algebraic tool. This result shows that even among students who have mastered GeoGebra features, certain manipulations of dynamic software remain difficult (ibid, p. 252) without anticipating these obstacles or the teacher's intervention. These results show that integrating GeoGebra in teaching and learning mathematics will require greater student efforts. This show that mastering the features of GeoGebra in relation to the mathematics concepts can enhance the mathematical learning process through ICT and enable students to accommodate requirements for the 4IR.

Conclusion

The Fourth Industrial Revolution led to new technologies for teaching and learning mathematics. Mathematics learning processes must be constantly reformed in the light of adjusting to the new technological changes and must be the object of continuous evaluation.

The results of this research reveal that it is important for students to master the features of GeoGebra to make significant progress in learning digital functions in high school using this dynamic software. The results also reveal that introducing an algebraic algorithmic expression of the function adapted to GeoGebra makes the activity on numerical functions more complex (Artigue, 2002). The role of the teacher as a guide is necessary for particular semiotic tasks that are not taken into account by classical semiotic analyses.

The use of Geogebra in learning mathematical functions can be evident (Jacinto & Carreira, 2017). It offers the students multiple representations of semiotics registers and their visualisations, a potential that can contribute to the learning (Minh, 2012). This research suggests that the teacher should think, before any intervention, about structuring the learning environment of mathematical functions using GeoGebra to allow students to use treatment and conversations of semiotic registers strategies.

Our findings reveal that mastering the features of GeoGebra in relation to the mathematical notions plays an important role in the mathematical learning process through this ICT. It is one of the key challenges of mathematical learning through ICT to enable higher secondary school students to accommodate the requirements of 4IR in the Niger Republic.

Appendix

Problème 1

On considère les fonctions f et g définies par: $f(x) = x^2$ et $g(x) = x^3$. C_f et C_g sont les courbes représentatives de f et g.

(a) Représenter graphiquement les fonctions f et g à l'aide du logiciel GeoGebra.
(b) Compléter à l'aide de GeoGebra le tableau des valeurs suivant.

x	−3	−2	−1	0	1	2	3
$f(x)$							
$g(x)$							

(c) Identifier dans le tableau les points où f et g sont égaux?

(d) Comparer graphiquement les fonctions f et g sur l'intervalle $[-3; 3]$.
(e) Déterminer les points d'intersection de C_f et C_g avec l'axes des abscisses.

Problème 2

1. Compléter le tableau ci – dessous. On donnera les résultats sous forme d'une puissance de 10.

X	−1000	−100	−10		10	100	1000
x^2							
x^3							
$\sqrt{x}$							
$\frac{1}{x}$							
$\frac{1}{x^2}$							

2. (a) Que devient x^2 lorsque x devient indéfiniment grand?
   (b) Lorsque x devient « indéfiniment grand », on convient de dire que x tend vers $+\infty$. Remplacer alors les pointillés par $+\infty$.

$$\text{Lors}\,\textit{que}\,\text{x tend vers}\ldots;x^2\text{ tend vers}\ldots\ldots$$

3. Remplacer alors les pointillés par $+\infty$, $-\infty$ ou 0.
   (a) Lors que x tend vers $+\infty$; x^3 tend vers ……
   (b) Lors que x tend vers $+\infty$; $\sqrt{x}$ tend vers ……
   (c) Lors que x tend vers $+\infty$; $\frac{1}{x}$ tend vers ……
   (d) Lors que x tend vers $+\infty$; $\frac{1}{x^2}$ tend vers ……
   (e) Lors que x tend vers $-\infty$; x^2 tend vers ……
   (f) Lors que x tend vers $-\infty$; x^3 tend vers ……
   (g) Lors que x tend vers $-\infty$; $\frac{1}{x}$ tend vers ……
   (h) Lors que x tend vers $-\infty$; $\frac{1}{x^2}$ tend vers …..

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Chapter 14
Factors Influencing Preservice Teachers' Adoption of WhatsApp as an Interactive Social Media Platform in Mathematics Teacher Education in Malawi

Fraser Gobede, Mercy Kazima, and Justina Longwe

Introduction

Education systems have often been the last to adopt the Fourth Industrial Revolution (4IR) and its associated technologies (Oke & Fernandes, 2020; Strommen & Lincoln, 1992). Strommen and Lincoln (1992) critiqued how the classroom has maintained its traditional set-up for over a 100 years while other systems have been transformed tremendously through the use of technology. Some developed countries have made advancements in the application of 4IR technologies in education, whereas most developing countries have typically included the adoption of 4IR technologies in education in their long-term development agenda (El-Masri & Tarhini, 2017; Oke & Fernandes, 2020). The onset of the COVID-19 pandemic in March 2020 revolutionised the norms in the adoption of digital technologies in education at all levels of schools. Governments had to devise feasible strategies to continue offering education to children after implementing lockdowns. These abrupt global changes also affected the norms in preservice mathematics teacher education in Malawi. In this chapter, we start by discussing the norms in mathematics teacher education in Malawi, followed by how these norms were disrupted by the COVID-19 pandemic, which in turn, relates to the core issue of adoption of social media technologies discussed thereafter.

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Norms in Preservice Mathematics Teacher Education in Malawi

Over the years, teacher education in Malawi has traditionally been conducted using face-to-face teaching, which has, in most cases, resulted in overcrowded lecture rooms due to increased enrolments (Mambo et al., 2016). This section describes the preparation of secondary school mathematics teachers at one public university. Preservice mathematics teachers at this public university are offered pure mathematics courses by the Faculty of Science, while the Faculty of Education offers education foundation courses and mathematics education courses. The mathematics education courses concentrate on teaching and learning mathematics and have components specifically aimed at preparing the preservice teachers for teaching in secondary schools. The chapter focuses on these mathematics education courses. The teaching and classwork are mostly done through workshop mode, where the preservice teachers participate through discussions, working groups and presenting to others. The preservice teachers are given opportunities to practice teaching through peer teaching and school experience. Peer teaching is where they teach their fellow preservice teachers in class, while school experience is where they practice teaching a real secondary school class.

The school experience is organised in such a way that preservice teachers work in groups of four to five. Each group is assigned to one school class where they collaborate with the class teacher. Each group works together to plan a lesson, one of the preservice teachers teaches the lesson while the others observe, and then they all reflect on the lesson. Each preservice teacher is expected to teach at least one lesson during the semester. In the final semester of the programme, preservice teachers are allocated to schools for teaching practice for 10–12 weeks.

Disruption of Norms in Teacher Preparation Due to the Abrupt Onset of COVID-19

Malawi closed all schools, universities and other educational institutions due to the COVID-19 pandemic from March to September 2020. Plans were made for radio lessons for primary schools and online teaching and learning for secondary schools and universities. However, remote teaching did not reach all students. As such remote teaching was discontinued as it increased the inequalities between those that could and could not afford the necessary gadgets to access the lessons. All education institutions were reopened in September 2020, with strict guidelines on COVID-19 prevention measures, such as splitting large classes into smaller groups and taking shifts attending school.

Regarding teacher education, preservice teachers are required to visit neighbouring secondary schools to experience how teaching is done in a real classroom. After the reopening of schools in September 2020, ordinary school experience could not resume as normal because COVID-19 restrictions did not allow preservice teachers in schools. As such, teacher educators had to find ways of offering preservice teachers the school experience without physically going to the schools. One strategy was to capitalise on peer teaching and extend the peer teaching time from the normal 10 to 40 minutes which is the duration of secondary school lessons. The preservice teachers worked in groups of four or five, and each decided on a school topic to teach. Each group planned their lessons together, and one taught the lesson to the whole class. Afterwards, the group and the whole class reflected on the lesson.

Abrupt Transition to 4IR Technologies in Mathematics Teacher Education

In 2021, another wave of COVID hit Malawi, but educational institutions in Malawi were not closed. At the public University, this chapter focuses on remote teaching was encouraged, and attempts to shift to online teaching were made.

Ideally, the transition from traditional face-to-face to online teaching is expected to be gradual, sometimes starting with an interim shift to blended teaching, where technology augments the existing practice. However, this norm was disrupted, ending face-to-face teaching with an abrupt switch to online teaching (4IR technologies) before familiarisation with the existing e-learning infrastructure. For the mathematics education program discussed in this chapter, the licenced platforms made available to the preservice teachers were Google Classroom and Moodle for asynchronous teaching and learning, whereas Google Meet and Zoom were made available for real-time lessons. Despite the provision of these institutionalised platforms, it became evident that non-formal social media platforms, mainly WhatsApp, seemed to quickly take over most of the roles of the specialised platforms. This observed behavioural pattern in the use of technology among the preservice teachers thus raised the question: What influences preservice teachers' adoption of interactive social media platforms over specialised e-learning platforms?

Review of Related Literature and Theoretical Framework

The use of technology in mathematics teacher education in Malawi and other countries in the region is affected by several issues that positively and negatively affect people's choice to use it. Some of the key issues are related to the connectivity infrastructure, global technological developments in mobile technology and its applications. These contextual issues directly affect the implementation of the 4IR in Malawi and the region.

The Role of 4IR Technologies in Teaching

The 4IR refers to the fusion of different technologies blurring the distinction between physical, digital and biological spheres (Oke & Fernandes, 2020). The 4IR has had a disruptive influence in many sectors and has disrupted the norms of routine activities in sectors like communications, banking, and commerce. In the education sector, however, the uptake of 4IR technologies has not been comparable to other services. Computers and related ICT tools and platforms have sometimes just been used to augment teaching, such as providing notes or searching for information (Oke & Fernandes, 2020).

Advancements in computer hardware, software and connectivity have increased strategies for the utilisation of electronic media for teaching and learning (e-learning). Both open-source and proprietary e-learning systems have been developed and distributed in the form of learning management systems, video conferencing platforms, and mobile applications (Kirange et al., 2021), with popular platforms integrating all these features. Popular specialised e-learning platforms include Moodle, Blackboard, and Google Classroom (Kirange et al., 2021; Yau et al., 2009). These platforms allow educators to create content, enrol students, access student feedback, and assess students. The interaction with students can either be synchronous (real-time) or asynchronous. Asynchronous approaches provide room for self-paced learning. Self-paced learning is one of the key features of Massive Open Online Course (MOOC) platforms that provide online learning. In addition to the use of specialised educational software, video conferencing applications such as Zoom, Google Meet, Microsoft Teams, and Cisco Webex have also been used to enable real-time interaction between educators and students (Kirange et al., 2021). Conferencing applications often do not require expertise in setting them up compared to specialised e-learning platforms. However, since they are not meant for education, they may lack features that educators need, such as tracking student participation, due to privacy issues embedded in these platforms. Educators and students may sometimes augment their interactions with students through social media platforms that are already popular among them, such as Facebook, Twitter, and WhatsApp, which is the central idea of this chapter. The popularity of social media platforms for classroom interaction relates to the observation by Raman and Don (2013), who found that the preservice teachers in their study were not willing to use the messaging and chatting feature provided within the specialised e-learning platform that was being used at their university.

Challenges and Opportunities Associated with the Implementation of 4IR Technologies in Africa

Accessibility to 4IR technologies has been a typical challenge in many African countries. The access issues are often related to the quality of the connectivity infrastructure and the cost of devices and connectivity services such as the internet.

Extremes in the access to technology, typical for those in towns and cities compared to the rural majority, have resulted in the digital divide.

In Malawi, the connectivity divide is noted in terms of bandwidth distribution. As of 2019, only 16% of the Malawi population was using the internet compared to an average of 29% in sub-Saharan Africa (World Bank, 2021). However, there has been an exponential rise in internet users in Malawi, from about 0.38% in 2005 to the 16% in 2019 (World Bank, 2021). This can be attributed to the proliferation of smartphones, whereas before 2005, access to the internet was mainly through desktop and laptop computers using dial-up telephone connections. The implication is that most people who accessed their education during the dial-up era (just as most of the teacher educators did) may not have the aptitude compared to the current generation of preservice teachers, who may be more flexible with the use of technology. With the limited capacity to deliver 4IR technologies, there may be relative reluctance among the gatekeepers to promote the switch to 4IR alternatives in teacher education (Oke & Fernandes, 2020).

The cost of connectivity in Malawi has been extremely high (Igunza, 2015), which further affects the connectivity divide, with people in rural areas not being able to afford internet services. The integration of mobile phones and internet connectivity has made mobile phone providers become internet service providers, thereby increasing the competition in the market. As such, these providers introduced accessible services and innovations. Some of these include subsidised rates for apps such as WhatsApp and Facebook and lower off-peak rates. With COVID-19, some providers introduced subsidised connectivity for students, as long as they register their number through appropriate school authorities.

Regarding opportunities, Malawi has positively benefited from the mobile technology revolution and its associated services. For instance, there has been an exponential increase in the number of people acquiring and using mobile phones in Malawi (Igunza, 2015). The lowering costs of smartphones with brands tailor-made for the African market has also attributed to this increase. The proliferation of web 2.0 apps such as WhatsApp and Facebook has made many people in Malawi and the region find themselves online for the first time through these apps. The same applies to many preservice teachers who have had their first online experience through smartphones before exposure to a desktop or laptop computer, making them prefer mobile apps to desktop apps.

Theoretical Framework

Research on user acceptance of new computing innovations has resulted in various theoretical models explaining individuals' intention to use technological innovations. These frameworks theoretically originate from information systems, psychology, and sociology. We will briefly discuss a few of these frameworks as a background to the Unified Theory of Acceptance and Use of Technology framework (Venkatesh et al., 2012), which we have used in this study. Some of the frameworks that have been used to study people's adoption and use of technology include the Theory of Diffusion of Innovations (Rogers, 1995), the Technology Acceptance Model (Davis et al., 1989) and the Unified Theory of Acceptance and Use of Technology (Venkatesh et al., 2003). The Technological Pedagogical Content Knowledge (TPACK) framework was developed by Mishra and Koehler (2006) to address the absence of a theory to guide the effective integration of technology in education.

The Theory of Diffusion of Innovations

The Theory of Diffusion of Innovations by Rogers (1995) is among the oldest, tracing its origins to 1962. This theory was based on over 508 diffusion studies and explained that an individual's adoption of innovation happens after going through several stages, including understanding, persuasion, decision, implementation, and confirmation. Rogers (1995) classified people as innovators, early adopters, early majority, late majority and laggards based on their swiftness to adopt innovations. Applying the same classification, Parasuraman and Colby (2001) named these categories of technology adopters as explorers, pioneers, sceptics, paranoids, and laggards. Davis et al. (1989) tried to explain what determines an individual's acceptance behaviour towards the use of computing technologies, leading to the Technology Acceptance Model (TAM), which was conceptualised in 1986.

The Technology Acceptance Model (TAM)

TAM identified two specific beliefs that explained users' acceptance of technology: perceived usefulness and perceived ease of use (Davis et al., 1989). Perceived usefulness implies the degree to which an individual feels the new technology will enhance their job performance. On the other hand, perceived ease of use refers to the extent to which the individual expects the new technology to be free from effort. Venkatesh and Davis (2000) extended the original TAM to Technology Acceptance Model 2 (TAM2) by identifying social influence processes and cognitive instrumental processes that significantly influenced user acceptance. Still, the improved TAM2 failed to explain user acceptance behaviours related to newly developed systems, such as e-commerce. These new systems appeared to have new factors influencing their acceptance, such as trust and perceived risk. This led to the development of the Technology Acceptance Model 3 (TAM3) (Venkatesh & Bala, 2008).

The Unified Theory of Acceptance and Use of Technology (UTAUT)

Considering the proliferation of theories explaining user adoption and use of technology, Venkatesh et al. (2003) synthesised the propositions put forth by eight models of technology acceptance, including the Theory of Diffusion of Innovations and Technology Acceptance Model, and proposed the Unified Theory of Acceptance and Use of Technology (UTAUT). The UTAUT model incorporates four key determinants to individuals' acceptance and use of technology: namely, performance expectancy, effort expectancy, social influence, and facilitation conditions. The model also identifies four key moderators to users' adoption behaviour as gender, age, voluntariness, and experience. With the advancement of computing technologies such as gaming and social media, the original UTAUT model failed to explain the drive behind users' voluntary engagement on such platforms. As such, Venkatesh et al. (2012) extended the original four key determinants to user acceptance and use of technology to include three new constructs, which are hedonic motivation, price value and habit, leading to the development of the second version of the Unified Theory of Acceptance and Use of Technology (UTAUT2) shown in Fig. 14.1. The individual differences moderating the acceptance and use of technology were reduced to three (gender, age, and experience), leaving out voluntariness from the original version. This is the version we used as this study's analytical framework. We have discussed the elements of the UTAUT2 with respect to how we used them during data analysis in the methodology section.

Since our focus is on the use of social media in mathematics teacher education, we opted to be guided by the UTAUT2 model, as it addresses the motivating factors behind people's use of such platforms. Other studies have similarly used UTAUT and UTAUT2 to study the acceptance and use of technology in education. Oke and Fernandes (2020) used the UTAUT model to assess the readiness of the education sector for the adoption of 4IR technologies. The UTAUT2 model was applied by Raman and Don (2013) in their study on factors affecting preservice teachers' adoption of a learning management platform. Other studies have tried to extend the UTAUT2 model to address factors affecting e-learning systems (Ain et al., 2016; El-Masri & Tarhini, 2017).

Research Design and Methodology

We adopted a qualitative case study design (Creswell, 2014; Yin, 2016). The study participants were drawn from a class of 41 preservice secondary school mathematics teachers during their final year of study at a public university. These preservice teachers had been learning through the traditional face-to-face approach until the COVID-19 pandemic necessitated the switch to specialised e-learning platforms provided by the university. A technical glitch that disrupted the availability of the

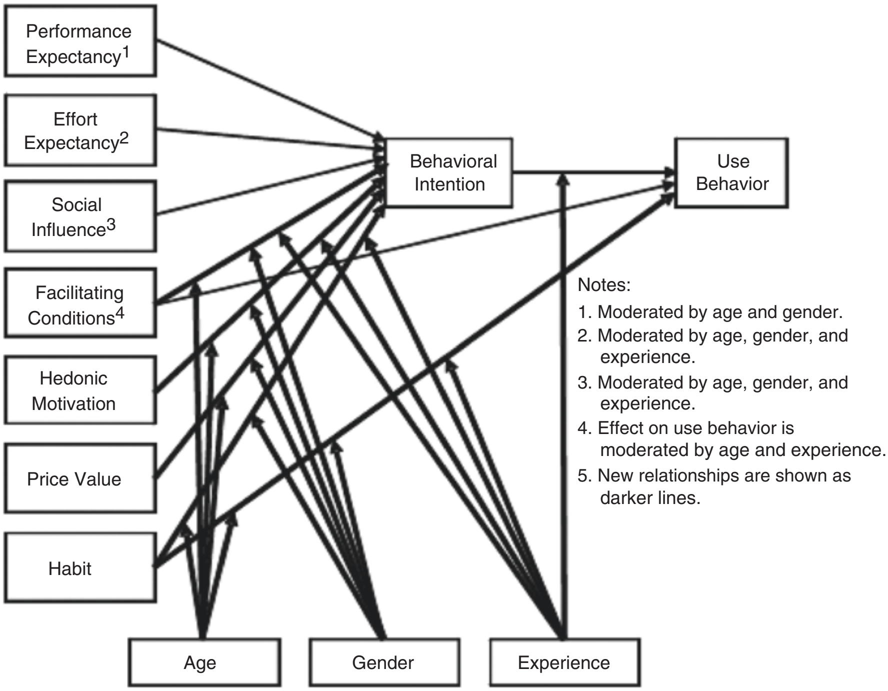

Fig. 14.1 The unified theory of acceptance and use of technology model 2 (UTAUT2) (Venkatesh et al., 2012)

official platforms for an extended period necessitated the exploration of alternative platforms that could be accessible to most of the preservice teachers in the class, hence this study.

Data Collection and Analysis

The primary data source was the transcripts of the online discussions done with the class during lesson preparation, lesson delivery, and post-lesson reflection. The online chats with the class were analysed thematically to identify persistent themes emerging from the data. The identified themes were then checked against the categories of usage behaviours reflected in the UTAUT2 model shown in Fig. 14.1. As such, the observed usage patterns among the preservice teachers were checked to whether they reflected the seven key determinants: performance expectancy, effort expectancy, social influence, facilitating conditions, hedonic motivation, price value, and habit (Venkatesh et al., 2012).

Performance expectancy refers to the extent to which technology users will benefit in the way they perform their routine tasks (Venkatesh et al., 2012). For the preservice teachers, this was related to the usual academic tasks in mathematics education. During data analysis, we checked whether the preservice teachers appeared to be influenced by the extent to which their use of social media platforms was advantageous to their academic work.

The second theme was effort expectancy. Venkatesh et al. (2012) define effort expectancy as the degree of ease associated with an individual's use of technology. Regarding social media, we checked whether the observed and proposed usage patterns had implications on effort expectancy.

Social influence was the third theme used during data analysis. This is the extent to which technology users perceive that people who are important to them believe that they should use a particular technology. In this study, the social influence was mostly perceived by classmates.

The fourth theme was facilitating conditions, which is an individual's perception of the availability of resources and support related to the use of a technology product. For the preservice teachers, this was associated with the availability of technical support from the university related to the use of a specific type of technology.

During data analysis, we also examined the presence of evidence pointing to hedonic motivation as a fifth factor associated with user acceptance and use of technology. Venkatesh et al. (2012) define hedonic motivation as the enjoyment associated with the use of a technology product. We checked whether relative enjoyment might have possibly influenced the preservice teachers' choice of a platform for academic work.

Price-value is the sixth aspect that informed data analysis. Users are sometimes influenced to adopt a certain technology depending on relative monetary cost or who bears the cost. We examined if there were any cost advantages or disadvantages associated with the technology options available to the preservice teachers.

The seventh and last theme examined during data analysis is habit, which is also related to experience. Habit is deemed to influence the relative ease with which an individual works with a particular type of technology due to routine use. During the study, we also paid attention to the extent to which the observed practices among the preservice teachers pointed to habit and experience with particular technology platforms.

Findings

We begin by giving a summary of the experiences faced by the class of preservice mathematics teachers during the transition from face-to-face to online teaching. We then proceed with a discussion of the findings related to the observed usage behaviours of the interactive social media platform.

Migration from Face-to-Face to Online Teaching

The class was in the final year of their mathematics education program during the study period. During the preceding years, the preservice teachers had been exposed to the use of Google Classroom and Moodle as official platforms for receiving handouts and assignments and submitting their completed assignments for assessment. The class also formed a WhatsApp group for instant communication amongst themselves, but they also used the same platform for sharing documents. The class had no prior experience with real-time teaching platforms such as Google Meet and Zoom.

During the second wave of COVID-19, the university opted not to close but rather let the students use the connectivity resources of the university and migrate to online teaching. The Google Meet platform was tried with the class during the week that the university set for online drill lessons. During that period, only 4 out of the 41 preservice teachers managed to connect with their laptops, whereas the majority reported connectivity problems with their devices. It is possible that the university's networking infrastructure failed to cope with the huge increase in data traffic as everybody tried to go online for their lessons.

Adoption of WhatsApp as an Alternative Platform for Teacher Education Activities

The preservice teachers proposed the use of WhatsApp for online lessons. This proposal seemed unusual considering that this platform was tailor-made for the demands of social interaction as compared to specialised e-learning platforms. Considering these limitations, the preservice teachers devised innovative ways of adapting the features of WhatsApp for moderating online lessons and even proposed how it can be used in peer teaching. As shown in Fig. 14.2, a few changes in class interaction were implemented during the first WhatsApp lesson. Slides were posted as images on the classroom chat. Communication was done using text such as "we are following" or emoticons carrying the same message. Verbal communication was done by posting voice notes.

Some decisions on the usage of the platform were made on the fly. For instance, after noting the possibility of unnecessary comments from the class, the class representative decided to assign an administrator role to the lecturer, as shown at the bottom right-hand corner of Fig. 14.2 (*"you're now admin"*), so that the lecturer should be able to mute comments from the class and enable the comments only when responses from the class were required.

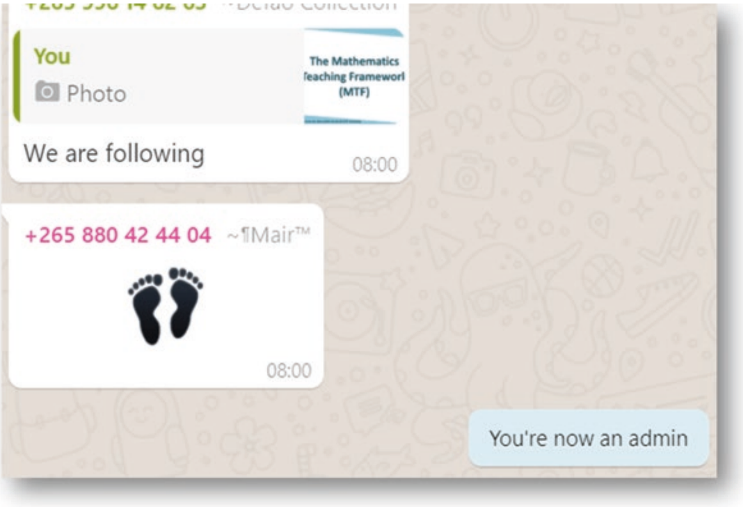

Fig. 14.2 Novel ways of classroom interaction during a WhatsApp lesson

Possible Factors Influencing the Adoption of the WhatsApp Platform

Using the UTAUT2 model (Venkatesh et al., 2012) as the analytical lens, we now present the possible factors that influenced both the proposed and the observed usage practices by the preservice mathematics teachers before, during, and after the first WhatsApp lesson.

Performance Expectancy

The preservice teachers differed on their perceptions regarding the usability of WhatsApp for a real-time mathematics education lesson. Those who were pessimistic felt that WhatsApp could not enable them to be engaged in the core tasks of the mathematics education course, such as workshop-style forums and peer teaching. One preservice teacher posted that *"I doubt, according to the nature of SCE [course code], if the lesson can be successfully done on WhatsApp. I don't know why we are so ridged [rigid] in using our maths room"* (see Fig. 14.3).

On the other hand, those who were optimistic felt that the WhatsApp platform could even be used for peer teaching, and one quickly prepared a sample video on how that could be done. One preservice teacher even indicated that *"…I think WhatsApp can be more productive than zoom and the like"* (see Fig. 14.4).

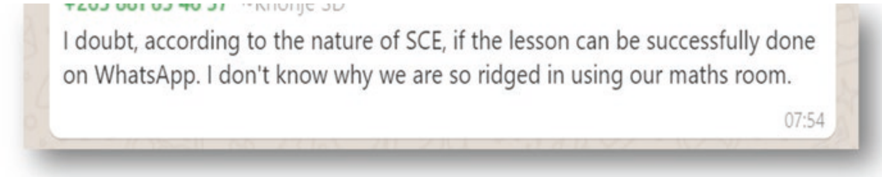

Fig. 14.3 Pessimism on the use of WhatsApp for mathematics education

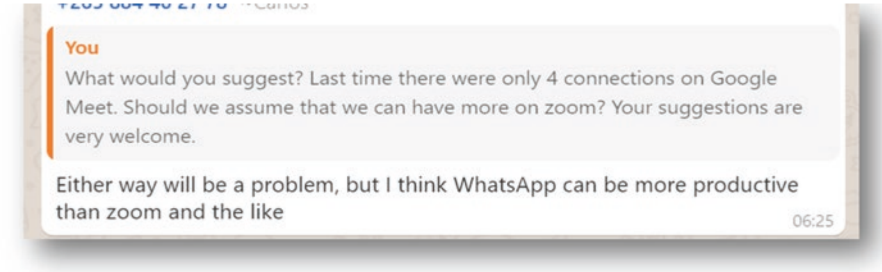

Fig. 14.4 Optimism on the use of WhatsApp for mathematics education

Effort Expectancy

Some preservice teachers indicated their dissatisfaction with their struggle to access the new eduroam infrastructure that had just been rolled out at the university for them to access the licenced e-learning platforms. One commented, *"ena ikumachita kulemba [for some, it is displaying the message] can't connect, so it doesn't matter or the whole day."* They possibly found it easier to work with WhatsApp, which was already accessible on their mobile phones and could access the connectivity through a mobile phone service provider.

Peer Influence

A collective view of the WhatsApp platform indicated that the class more favoured it over the official platforms. Those in favour of the WhatsApp platform received support from other classmates with comments like *"Gud [good] idea"* (see Fig. 14.4).

Facilitating Conditions

Later, some preservice teachers felt that they could now use the official e-learning platforms after receiving technical support from the university's computer technicians on connecting to the university's eduroam. One of these preservice teachers wrote: *"For some of us, our phones were not working that day, ICT assisted us, and I guess we can have more than 4 connections today."*

Hedonic Motivation

The positive attitude towards using WhatsApp (see Figs. 14.4 and 14.5) could possibly be attributed to the intrinsic pleasure associated with social media platforms compared to the official platforms. This could explain why the preservice teachers proposed innovative ways of adapting the features of WhatsApp for conducting and moderating online lessons instead of putting their effort into getting familiarised with a specialised educational platform.

Price Value

Mobile phone operators in Malawi offer a lower cost of internet connectivity for social media apps, including WhatsApp and Facebook. For the official e-learning platforms, connectivity would be deemed free if accessed through the university network. The preference for WhatsApp, therefore, indicates that the relative ease of

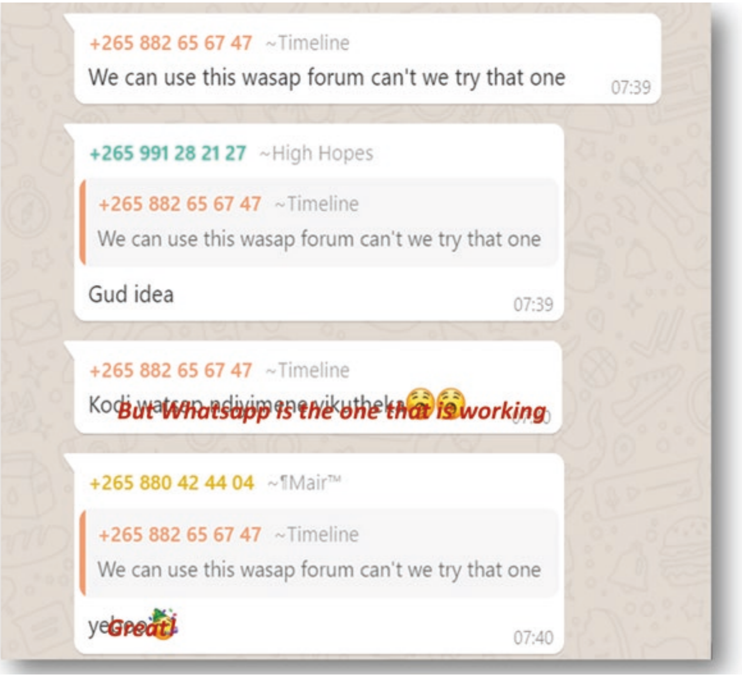

Fig. 14.5 Supporting posts

using the social media platform possibly outweighs the accessibility effort associated with the free official platforms.

Habit

The preservice teachers were already familiar and competent with the WhatsApp interface, using it for their daily communication needs. As such, learning new interfaces would bring additional challenges to them, hence their observed resistance to the adoption of the official platforms.

Concluding Discussion

Despite the internet connectivity challenges as discussed in the literature review, there seems to be some potential in the application of 4IR technologies in teacher education. Though having limited features for online teaching, the use of WhatsApp made it possible to continue teaching despite the ongoing COVID-19 pandemic. Unlike physical face-to-face sessions that ended when the preservice teachers had left the room, WhatsApp discussions on the day's topic informally continued after the lesson time. This is not the case with e-learning platforms with chat features. Raman and Don (2013) noted that despite having a chat feature, the preservice teachers in their study only used Moodle for educational purposes.

Regarding notes and handouts, the preservice teachers automatically received their notes and handouts on their devices by the end of the WhatsApp lesson instead of the wait time experienced with the distribution of physical resources. Likewise, students who would have likely missed face-to-face lessons were still able to join the class.

However, the main challenge with teaching on a social media platform was the need to be mindful of individuals who may have joined the chat later and hence derail the progress of the class discussion. Such new chat entrants start responding to issues that have been resolved by the other members. Official platforms such as Google Meet and Zoom had more features for regulating real-time interaction with the class, most of which were absent on WhatsApp.

Considering the proliferation of mobile phones compared to desktops and laptops, using mobile phone apps, such as WhatsApp, could minimise the effects of the digital divide experienced in Malawi and the region. Considering that the younger generation of preservice teachers is mostly "digital natives", the integration of mobile-phone-based 4IR technology would mean a lesser learning effort on their part. This suggests that the preservice teachers have the potential to use 4IR technologies even if they are not mainstreamed in the mathematics education curriculum.

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Chapter 15
The Use of Animal Metaphors to Reveal Beliefs of Grade Three Namibian Learners Who Experienced Mathematics Learning Difficulties During the COVID-19 Pandemic

Cloneria Nyambali Jatileni and Shemunyenge Taleiko Hamukwaya

Introduction

Many countries across Africa are experiencing poor performance in mathematics (International Mathematics Union Report, 2014). Indeed, Sandefur (2018) points to the underperformance of African countries at the Southern Africa Consortium for Monitoring Education Quality (SACMEQ II and III) of 2000–2004 and 2006–2011, respectively and argues that they perform below the countries ranked in the Trends in International Mathematics and Science Study (TIMSS) in 2015. This has been corroborated by Saal et al. (2019), who concluded that mathematics performance among South African students was shocking. Similarly, (Hadi et al., 2018) state that students have difficulties solving mathematics problems, and many perform poorly due to low higher-order thinking skills. The situation is not different in Namibia, which obtained the lowest scores of all the countries that participated in TIMSS (Sandefur, 2018). Namibia was ranked the lowest among 15 countries in the SACMEQ test reports in mathematics (Ministry of Education, 2006, p. 12). However, Namibian learners managed to reach the benchmark of 500th in the fourth SACMEQ assessment, which was conducted in 2012–2014. This may suggest that learners did not perform well in the mathematics assessment; MLD may be one of the causes of low achievement.

As in many other countries, in Namibia, mathematics is a compulsory subject throughout the school curriculum (Ministry of Education, 2016). Mathematics is one of the requirements for passing to the next grade level, and learners perceive the

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subject as a gatekeeper to many careers (Ministry of Education, Sport, Art and Culture, 2015). (Naidoo & Kapofu, 2020), whose work focuses on the South African context, identifies mathematics as an entry requirement for many careers, including law, medicine, and science, and such jobs as a land surveyor, pilot, geologist, and architect. Meaning, learners may not have careers if they do not pass mathematics. Similarly, (Hadi et al., 2018) identify mathematics as a key compulsory subject because of its high relevance and practical value in everyday life applications. However, according to the Organisation for Economic Co-operation and Development (OECD), mathematics becomes a dead-end and a stumbling block for many students (OECD, 2014), especially those who experience mathematics learning difficulties (MLD).

Hadi et al. (2018) have further observed that a high percentage of learners at all school levels experience difficulties in comprehension, which worsen their mathematics learning and understanding. Sanders (2009) references mathematics as a subject that many students find hard to cope with, lose their interest in, and develop negative attitudes toward, and this is apparent in the Namibian context. Furthermore, Nambira et al. (2009) explore reasons for low performance in mathematics in grades six, seven, and eight in Namibia. These results showed poor performance in mathematics and learners' difficulties in attaining certain competencies caused by poor mathematics foundation at the lower primary school level (Nambira et al., 2009). Thus, the study also shows that learners' difficulties in mathematics at high grades were not eccentric. It keeps increasing as learners progress to higher grades (Nambira et al., 2009). Nambira et al. (2009) indicate that MLD is caused by the language of instructions and lack of parental support, among other issues.

Preliminary research shows little is known about Namibian lower and upper primary learners' experiences of MLD. Many studies have focused on high school learners, pre-service teachers, or in-service teachers (e.g., Ellion, 2016; Mukwambo et al., 2018; Nambira et al., 2009). Therefore, in this chapter, we use metaphors to examine the beliefs (about mathematics) of Namibian grade three learners whom their teacher had observed to experience MLD. Grade three is part of Namibia's junior primary education, consisting of grade zero (pre-primary) to grade three. These grades lay the foundation for childhood education. The language of instruction at the school participating in this study is the predominant local language, Oshindonga. Education focuses primarily on literacy, numeracy, broad knowledge of the learners' environment, and personal health (Ministry of Education, Arts and Culture, 2016). Learner-centred teaching is the adopted approach; teachers use a class-teaching model, whereby the teacher is responsible for seven subjects in their class. Repeating a grade is recommended if a learner, as observed by their teacher, parents, or guardians, does not master the required minimum basic competencies.

Our own experiences of teaching mathematics influenced the focus of this chapter, as we have observed many Namibian learners experience MLD during their early grades. Our focus is on grade three learners because we believe they could express their views and thoughts better than grade one or two learners. Our purpose is to contribute to a better understanding of the learners' beliefs about mathematics and learning mathematics during the COVID-19 pandemic that may suggest potential types of intervention in lower grades.

Impacts of COVID-19 on Teaching and Learning in Namibia

The Namibian education sector was negatively affected by the COVID-19 pandemic, and basic education suffered more than higher education (Evelina et al., 2020). Namibia reported its first two COVID-19 cases on March 14 2020: a Romanian tourist couple visiting the country. Thereafter, the Ministry of Education Arts and Culture (MoEAC) announced the closure of all public and private schools in Namibia on March 16 2020. Also, in response, the Namibian Head of the State declared a state of emergency on March 17 2020. On March 28 2020, the country went into a full lockdown. This brought an unimaginable disruption to the Namibian school calendar as no teaching or learning occurred during the state lockdown; all schools were closed. The closure imposed difficulties in formally assessing learners for the academic year (Evelina et al., 2020). Parents were called on to facilitate learning and support their children in studying all their subjects at home (Bubb & Jones, 2020). This arrangement was challenging due to a lack of eLearning platforms that could have helped teachers remotely facilitate learning mathematics (Angula & Mutelo, 2021). It was also unrealistic to expect schools, especially in rural areas with no internet access or electricity, to suddenly begin to use online learning platforms (Evelina et al., 2020).

In the absence of face-to-face teaching, parents were expected to take over the teachers' tasks, although some were illiterate and could not teach their children mathematics (Darragh & Franke, 2021). Of all the subjects that parents supported learners in during the COVID-19 lockdown, mathematics was the least enjoyed (Cahoon et al., 2021). Thus, there is a high possibility that some learners did not acquire any mathematics competency during the remote learning period. Even when grades 10–12 were permitted to proceed with face-to-face schooling, classes for pre-primary grades and grades 1–9 remained suspended. As of September 2020, Namibia was among Africa's top 18 countries with the most COVID-19 cases. Although the schools have resumed face-to-face teaching, two (Khomas and Erongo) of the 14 regions of Namibia stayed in lockdown for a longer time due to a high number of COVID-19 cases. Learners in those two regions were thus left behind in learning mathematics and other subjects. When face-to-face teaching resumed, outbreaks of COVID-19 cases among teachers and learners in the schools continued to cause disruptions in learning (Wakui et al., 2021; Spaull & Van der Berg, 2020). There were even some cases where a COVID-19-related death of a teacher or learner slowed down learning (Jacob et al., 2020).

The MoEAC issued circulars to guide basic education and everyday schooling in response to issues caused by COVID-19. The COVID-19 pandemic forced mathematics teachers to reduce or soften the content and slow the usual pace of teaching (Rodríguez-Muñiz et al., 2021). As Ferdyan et al. (2020) noted, the COVID-19 emergency curriculum involved several topics in learning being reduced from the syllabus and lessons focusing on the selected few. For instance, grades one to nine in Namibia were taught using a rationalised syllabus (Hamukwaya & Jatileni, 2022). The prolonged lockdown during COVID-19 widened the learning gap in school subjects, including mathematics (Khan et al., 2021). Thus, the MoEAC removed some basic competencies/content from the normal syllabi and issued a rationalised syllabus as a COVID-19 emergency curriculum. The rationalised syllabus or emergency curriculum aimed to reduce the subjects' content, which would result in shorter face-to-face teaching times so that the classes could meet the end of the school year calendar. Much of the content of the subjects had to be left out due to lost instruction time in the classroom (Kuhfeld et al., 2020).

In Namibia, instead of seven subjects at the junior primary level, the new syllabus focused only on four subjects: first and second languages, mathematics, environmental studies, and a list of suggested topics (MoEAC, 2020). Since some basic mathematics competencies usually taught were excluded, it could be argued that a gap developed in learners' mathematical knowledge and skills required for completing grades was created. Lepp et al. (2021) state that it is more difficult for learners to acquire the missed basic competence in mathematics than in other subjects. This means that missed content creates a mathematical knowledge gap. Unless learners continue with the same teacher the following year—then it would be possible for that teacher to know specifically what was not learned, and they may aim to fill that gap (Lepp et al., 2021). The grade three learners discussed in this chapter had missed some essential basic mathematical competencies in previous grades due to school closures, cancellations of face-to-face learning, and the implementation of a rationalised syllabus. This knowledge gap and limited time for learning new concepts could negatively affect learners' mathematical performance and lead to MLD.

The Study Rationale

Grade three learners were in the final grade of their junior primary phase, which is a transitional grade before the senior primary phase in Namibia. Each learner's mathematics foundation must be firmly laid at this phase before moving to grade four (MoEAC, 2016).

Identifying learners' beliefs about mathematics in early grades can help teachers support learners studying mathematics. Moreover, many studies have been conducted on beliefs about mathematics education and learning difficulties. Using animal metaphors, learners can express their beliefs about mathematics by describing an animal. Despite learners' different educational needs, few studies have used animal metaphors to address learners' beliefs. Studies of this nature are scarce in the mathematics education community, especially in Africa. Specifically, no such study has been conducted in Namibia. Therefore, in order to understand grade three learners' beliefs about mathematics as a school subject (their relationships with mathematics), this chapter addresses the following research question:

- Using animal metaphors, what do grade three learners whom their teacher has observed to experience MLD believe about mathematics?
- What do learners think about mathematics?
- What beliefs do learners have about mathematics?

Literature Review

Beliefs About Mathematics Education

In this chapter, we only discuss literature about students' or learners' beliefs about mathematics, the one aspect of beliefs classified by researchers (e.g., McLeod (1992).

Students hold beliefs about what mathematics comprises, for instance, who they are as mathematics students (Ndlovu & Ngcobo, 2018). Based on the literature, the students' main source of mathematical experience is the classroom, and what occurs in mathematics classrooms influences their beliefs. Their beliefs are developed during the teaching and learning processes and influence the students' construction of knowledge and academic performance (Steele & Ambady, 2006). This includes the way teachers teach (Black et al., 2018) and what both students and teachers do in class. This indicates that early experiences in mathematics are crucial in shaping students' beliefs.

One of the common categories of students' beliefs found in the literature is about mathematics as a subject. This refers to how students perceive/see mathematics (McGregor, 2014; Op't Eynde et al., 2002). Some students may find mathematics easy, while most believe it is difficult, and some perceive mathematics as being founded on rules, and that only bright students can succeed in the subject (Ndlovu & Ngcobo, 2018). Another common category of beliefs outlined by scholars is beliefs about oneself in relation to mathematics. These beliefs are related to the students' confidence, determination, and understanding of success or failure in learning mathematics (McGregor, 2014; Ndlovu & Ngcobo, 2018). These beliefs significantly influence students' mathematical learning and impact their interest and motivation in learning mathematics (Kele & Sharma, 2014).

Building on the literature, in this chapter, we consider beliefs about mathematics as individual learners' experiences in learning mathematics and how their thinking may impact the desire to learn the subject. Specifically, we look at how learners associated mathematics with animals. We focused on grade three learners who had been observed to experience MLD and how their beliefs (i) determined effort in learning mathematics (Kloosterman, 2002); (ii) influenced engagement in the learning of mathematics (Markovits & Forgasz, 2017); (iii) impacted interest and enjoyment towards the subject (Kloosterman, 2002); and (iv) contributed to academic achievement (Geisler & Rolka, 2020). Understanding learners' beliefs about mathematics may suggest a potential intervention to enhance their early grades' mathematics learning.

The Concept of Mathematics Learning Difficulties

Researchers define MLD as any limitation researchers believe to hinder a student's mathematical learning. For example, limitations can be academic: students' fears based on negative past experiences, insufficient opportunities for learning mathematics, a lack of adequate mathematics knowledge, or insufficient teaching practices or materials (Hadi et al., 2018). Hamukwaya (2021) and Hamukwaya and Haser (2021) investigated characteristics of learners experiencing MLD, such as lack of arithmetic skills, difficulty recalling arithmetic facts, and inability to build connections between mathematics concepts, which may influence the learners' beliefs about mathematics. Furthermore, researchers have made some of the following conclusions about learners experiencing MLD:

- Usually, learners believe mathematics is difficult (Zakaria et al., 2010).
- They have difficulty grasping concepts quickly (Gafoor & Kurukkan, 2015).
- They may relate mathematics to dangerous or poisonous animals or scary situations (Stanley et al., 2021; Haslam et al., 2011).

Moreover, the current chapter regards grade three learners experiencing MLD as struggling to grasp the basic mathematics knowledge and skills required to comprehend the subject effectively (Jitendra et al., 2013). However, if learners receive learning support, they can improve their understanding of mathematical concepts (Hamukwaya & Haser, 2021) and may change their beliefs about mathematics. Therefore, this chapter focused on grade three learners' beliefs about mathematics. It aimed to contribute to other studies about beliefs in mathematics education, especially in the Namibian context – a country under-represented in mathematics research studies. We aimed to align learners' beliefs with animal metaphors because if learners experience MLD at early grades, they may have difficulty understanding mathematics at high grades and consequently make inadequate progress in learning mathematics, which may negatively affect their career paths.

Metaphors in Mathematics Education

Over the years, educational researchers and practitioners have used a collection of quantitative and qualitative methods to interpret the world of mathematics education to understand better educational theory and practice (Jensen, 2006). The current chapter uses metaphor analysis as a possible means for qualitative educational inquiry. Types of metaphors include symbols, figures of speech, similes, images, and allegories. A metaphor occurs when a unit of discourse is used to indicate an object, animal, process, or concept (Woodside, 2008). Metaphors can depict aspects of who someone is or what they are like, and the animal that first comes to mind in different situations can be telling (Magaña & Matlock, 2018). Metaphors can also identify and select someone's inner animal companion (Dove & Fayard, 2020, April). Thus, we used animal metaphors to explore which animal learners depicted as relating to them or that they believed impacted their lives as mathematics did. We chose animal metaphors because the metaphors that learners form and encounter have been shown to exert a powerful influence on how they think about mathematics (Olsen et al., 2020). For instance, Hendriana and Rohaeti (2017) contributed to the role of metaphors in understanding mathematical concepts and how types of metaphors can provoke interpretations of mathematical problems for learners. Thus, when a learner participates in forming a metaphor representing themselves in the way they view mathematics. The source and the target domains can be dissected to determine the concepts, properties, and relationships from the source domain to the target domain. Hence, in the context of this chapter, the target domain of the animal metaphor is grade three learners who had been observed by their teachers to experience MLD.

Learners hold various beliefs about mathematics, which influence how they learn the subject (Ndlovu & Ngcobo, 2018); the systemic view of animal metaphors can reveal learners' beliefs about mathematics learning (Latterell & Wilson, 2016). Such beliefs can be associated with understanding the role mathematics plays in the lives of learners in and outside the classroom. While beliefs can influence learners' mathematics learning, the challenge is determining a specific learner's beliefs about mathematics. Thus, contemporary metaphor theories such as animal metaphors can provide a means of unpacking the beliefs that specific learners have about mathematics (Markovits & Forgasz, 2017). Detecting and understanding learners' metaphors for mathematics can give mathematics teachers a hint about learners' internalised views about mathematics (Olsen & Weber, 2020). For instance, there can be an underlying reason a learner is associating mathematics with a sweet than chilli. Thus, a better sense of learners' beliefs can inform teachers' understanding of how learners' beliefs influence their acquisition and practice of mathematics as a school subject (Latterell & Wilson, 2016; Markovits & Forgasz, 2017; Presmeg, 2020; Soto-Andrade, 2020; Schinck et al., 2008).

Research in mathematics education uses metaphors to gain insight into learners' beliefs about mathematics (Al Said et al., 2019; Cassibba et al., 2020; Olsen et al., 2020). Also, the metaphors that learners form and encounter have been shown to exert a powerful influence on how they think about mathematics (Olsen & Weber, 2020). Furthermore, Çetinkaya et al. (2018) state that metaphor assessments help reveal learners' attitudes on learning mathematics and can provide opportunities for learners to analyse their mathematics learning beliefs critically. According to Latterell and Wilson (2016), mathematics learning metaphors illuminate learners' beliefs about mathematics. Therefore, through animal metaphors, we can discuss learners' beliefs and perceptions about mathematics in unique ways in this chapter (Markovits & Forgasz, 2017).

Animal metaphors are a dominant type of metaphor in research and are mostly used in negative discourse (Hart, 2021). They can convey a wide range of meanings, such as those centred on depravity, insulting slurs, expressions of love, disagreeableness, and stupidity (Haslam et al., 2011). In their study, Haslam et al. (2011) found that the offensiveness of an animal metaphor is projected by the revolt felt towards the animal and by the dehumanising view of the target to which it was applied. Their study also revealed that the offensiveness of animal metaphors differs with the tone of expression and the gender of the target domains. These differences influence the offensiveness by altering the extent to which animalistic properties are attributed to the target (Haslam et al., 2011). As in the study by Stanley et al. (2021), metaphorical animal descriptions were used in the current study to reveal grade three learners' beliefs about mathematics.

This study only targets grade three learners experiencing MLD in the year 2021, as they learned during the COVID-19 pandemic. Results from research using animal metaphors in cross-cultural comparison studies may differ to a certain extent since many aspects, such as language, are culture-specific (Belkhir, 2021; Hijazo-gascón, 2020). However, similarities and differences in the understanding of the six animal metaphors in this study among the participants can be assumed to have not been influenced by culture or language as the participants all shared the same culture and language. Thus, the conceptual metaphors used in this study are grounded on learners' experiences and are not culture-dependent (Schaerlaeken et al., 2019).

Methodology

Research Design and Approach

We conducted qualitative research by employing in-depth semi-structured interviews to allow data to emerge from participants as they explained their beliefs about mathematics (Selvi, 2008; Sohn et al., 2007) in terms of animal metaphors. Also, we explored the data in detail to understand how the grade three learners experiencing MLD perceived mathematics.

The Context and Site

The study was conducted in 2021 at Yambeko Combined School (YCS is a pseudonym). The school accommodates grades zero to 11 learners in a rural area of Omusati, a northern region of Namibia. At the time of this study, the school had two grade three classes, one with 34 learners and one with 35. Learners were divided into two groups in each class, meaning fewer than 20 learners were in the class at a time. This arrangement was necessary to adhere to social distancing requirements and health protocols due to the COVID-19 pandemic. There were nine mathematics lessons per week at YCS before the pandemic. However, lessons were reduced to three per week during the pandemic, plus one lesson for learning support.

Grade three at YCS was unique compared to that of other schools in the region. The two teachers responsible for teaching grade three shifted from whole-class teaching to the subject teaching model: one taught mathematics and environmental studies, and the other taught language and reading. They were committed to supporting their learners to help them achieve sufficient understanding during the COVID-19 pandemic—a significant reason why we selected YCS for this study.

Participants

Six grade three learners (two boys and four girls) observed by their teacher to experience MLD participated in this study. Learners had not been diagnosed or assessed by a health professional to have MLD. Five of the learners were nine years of age, and one was 11 years of age (Iyaloo, pseudonym). Based on her parents' request, Iyaloo was repeating grade three. During the final grade three assessment in 2020, Iyaloo had performed below the average (30%) in all school subjects. Iyaloo's parents felt she would not benefit from being transferred to grade four. Although professionals did not clinically diagnose Iyaloo, her parents observed that she had learning difficulties in all content subjects: Oshindonga (native language), English as a second language, mathematics, and environmental studies.

Instrument

Analysing (i) learners' drawings and (ii) their interviews is a unique method for exploring young learners' beliefs about mathematics as a subject (Markovits & Forgasz, 2017; Batchelor et al., 2019). Hence, the learners' drawings using animal metaphors and semi-structured interviews were used as instruments to collect the data among the grade three learners. Learners expressed their feelings, beliefs, and imagination regarding mathematics by depicting an animal. The questions were designed to produce belief statements about mathematics, and the interview guide was based on the two themes that elicited learners' beliefs:

- The learners' beliefs about mathematics (e.g., Draw an animal you think mathematics is like and explain why.)
- The learners' beliefs about learning mathematics (e.g., How good are you at mathematics?)

Procedure

The Namibian MoEAC granted permission for the study, and voluntary participation was sought. Ethical issues were considered, such as anonymity, the participants' willingness, and signed consent by the parents of the participating learners.

In Namibia, in the first 6 weeks of the first semester, the junior primary grades one to three begin with readiness lessons to review the content previously covered. In 2021, after the readiness lessons, the grade three mathematics teacher at YCS designed a readiness test as part of the lower primary policy. Since mathematics teachers may vary in their understanding of why learners may experience MLD (Hamukwaya & Haser, 2021), we requested that the teacher identify learners they observed as experiencing MLD based on readiness; ten learners were identified. Permission requests were sent to parents for their children to participate in the study, and six parents responded positively. These learners were interviewed, and their drawings were used in this study.

Data Collection

We developed a common interview framework about beliefs based on other scholars in mathematics education (e.g., Karagiannakis et al., 2014; Lewis, 2014). We arranged the guided interview in a manner that promoted comfortability in the discussions and encouraged learners to share their beliefs about mathematics as a subject.

Due to the travel ban because of the COVID-19 restrictions at the time, the grade two teacher assisted with data collection. We provided the teacher with an interview guide on tailoring subsequent questions and the espoused interview technique. We requested that the grade two teacher assist us with data collection. The interviews were conducted in Oshiwambo (the learners' native language and the medium of instruction in grade three). They were recorded and then translated into English.

Data Analysis

We analysed the transcripts of the six audiotaped interviews based on (i) the learners' beliefs about mathematics and (ii) the learners' beliefs about learning mathematics. To reiterate, we were interested in grade three learners' feelings, imagination, and expressions of how they perceived mathematics using animal metaphors. For trustworthiness, all authors analysed the data, and the learners who participated in this study were treated as individuals. The analysis was conducted in the following order:

- The interviews were transcribed, and the recordings were replayed several times to ensure accuracy.
- We employed open coding that enabled qualitative data reduction to understand the learners' expressions and how they perceived mathematics (Creswell, 2009).
- The transcriptions were read thoroughly, and significant words and common phrases based on the aim of this study were highlighted. The learners provided

common expressions, and keywords were identified, including "mathematics is difficult", "struggle", "scary", and "fear".

- We selected data about each theme (The learners' beliefs about mathematics and the learners' beliefs about learning mathematics).
- We gathered quotations to ensure the accuracy of the coding process in the qualitative research, which helped preserve the learners' voices.
- For validity, feedback from fellow researchers in mathematics education was used to revise the data analysis, which increased the repeatability of the findings.

The following section presents a sample of the learners' statements to support our analysis. The names of the learners used in this paper are pseudonyms.

Findings

Metaphors Revealed by Learners

The animal metaphors were used to explore learners' beliefs about mathematics and learning mathematics. When the learners were asked to draw an animal that they thought mathematics was like, three learners (two boys and one girl) drew snakes, one (a girl) drew a lion, and another (a girl) drew a bee, as shown in Table 15.1. The drawing by Iyaloo differed from the others. Iyaloo did not draw an animal; she drew fire to represent mathematics.

Table 15.1 Learners' metaphors and their beliefs about mathematics

Learners' name	Learners' metaphor	Learners' beliefs about mathematics
Betty	Figure 15.1	"I am not good at mathematics because it's difficult. [...] I don't know how to do all the calculations."
Aina	Figure 15.2	"I don't know mathematics. Mathematics is difficult [...]. It's like a snake because the snake bites."
Tomy	Figure 15.3	"I struggle with mathematics. [...] Mathematics is like a snake because it bites."
Festus	Figure 15.4	"I am not good at mathematics. [...] I am scared of mathematics like I am scared of a snake."
Maria	Figure 15.5	"I don't know how to do the calculations. Mathematics is difficult [...]. It is like a bee because the bee stings people."
Iyaloo	Figure 15.6	"I am not good at mathematics. [...] I struggled with math. It's difficult. [...] Math is like fire."

Fig. 15.1 Lion

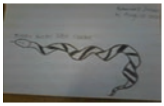

Fig. 15.2 Snake

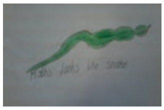

Fig. 15.3 Snake

Fig. 15.5 Bee

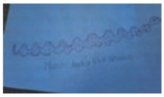

Fig. 15.4 Snake

Fig. 15.6 Fire

Learners' Beliefs About Learning Mathematics in Relation to Animal Metaphor

During the interviews, learners gave reasons for each drawn animal metaphor (as shown in Table 15.1) that included the fear of mathematics in the same way as they would fear snakes, lions, bees, or fire because of the danger associated with them. For example, Festus expressed his beliefs to the grade two teacher, who served as a Researcher Assistant (RA), that mathematics is like a snake: "I am scared of mathematics like I am scared of a snake." Another example is Maria, who thought mathematics was like a bee "because it stings people."

Nearly one-fourth of the learners' metaphors suggested negative views about mathematics and mathematics learning (Latterell & Wilson, 2020). There were no significant differences between girls and boys noted in the responses.

The following example shows how Iyaloo and Maria expressed their beliefs about learning mathematics to the RA. The follow-up questions in the excerpt build a conversation about their feelings, beliefs and views. In the following excerpt, Iyaloo characterises learning mathematics as difficult. The excerpt indicates that Iyaloo "struggled with math".

RA:	Iyaloo, can you please tell me how good you are at learning mathematics?
Iyaloo:	I am not good at mathematics.
RA:	Okay. Can you explain to me why you are not good at learning mathematics?
Iyaloo:	I struggled with math. It's difficult.

Another example is from Maria, who shared her belief that she was experiencing difficulty and did not know the subject:

RA:	Can you please tell me how good you are at learning mathematics?
Maria:	I don't know how to do the calculations.
RA:	You don't know how to do the calculations?
Maria:	Yes.
RA:	Why?
Maria:	Mathematics is difficult.
RA:	Okay. Maria, you drew a snake, that mathematics is like a snake. Can you please tell me why you drew a snake?
Maria:	It's like a snake because the snake bites.

All participating learners saw mathematics as difficult; they struggled when solving mathematics tasks. However, Aina stated that the difficulty with mathematics might occur because she "does not know how to write well". Interestingly, all the learners believed mathematics was an important subject that must be learned and indicated that they wanted to excel in mathematics but that mathematics is scary. For instance, Festus stated, "I am scared of mathematics".

Discussion

In this chapter, we explore six Namibian grade three learners experiencing difficulties in learning mathematics to understand better learner beliefs: what beliefs the grade three learners held about mathematics and what they thought about learning the subject. Our discussion sheds light on mathematics as a scary animal or subject.

Examining mathematics representations through pictures allowed grade three learners observed to experience MLD to express their beliefs about mathematics. Their descriptions of the animals play a significant role in the current study. It was surprising that one of the participants drew fire instead of an animal, but it is still an image often associated with danger. In relation to Fig. 15.6, Iyaloo's imagination aligns with the views of Ernest (2018), who states that different forms of imagery or symbols may be used in ways that make abstraction and generalised views of mathematics. Dangerous animals or situations learners associate with mathematics may determine their effort in learning mathematics. The way learners associate themselves with learning mathematics may influence their success or failure in this subject. For this reason, if learners have maths anxiety-if they are afraid to learn mathematics at such an early grade-this may negatively affect their academic performance and achievement in mathematics (Joseph et al., 2019; Luttenberger et al.,2018) as they progress with schooling. Consequently, this anxiety could lead to not participating in class, having difficulty in learning mathematics, and poor overall performance in the subject (Hamukwaya, 2021).

These findings further indicate that MLD may begin to pose a problem in primary or elementary grades. The types of beliefs shared by grade the three learners led us to believe that an effective and supportive mathematics system that aids grade three learners experiencing MLD is needed before they move to grade four (the first grade of the senior primary phase). If teachers do not take measures to decrease a learner's MLD in the early grades, the learner may have trouble understanding mathematics as they progress in school and may make inadequate progress in mathematics.

Based on the findings, learners felt that they did not want to associate themselves with mathematics. Their metaphors described mathematics and learning mathematics as something one must not engage in. All the figures created by the learners connected mathematics to external features or characteristics associated with negative connotations. The types of symbols that the grade three learners used in the metaphors revealed that they are afraid to learn mathematics because it is hard. This could also be taken to mean that the learners may not want to be associated with such animals and, therefore, not the subject either.

Moreover, the excerpt that mathematics is a difficult subject indicates that learners struggled to learn the subject. Although in some cases, for example, Maria's excerpt, saying she did not know how to do the calculations, may mean that such a learner finds mathematics difficult because she struggled when solving the mathematics tasks.

Our findings are congruent with those of Markovits and Forgasz (2017), who used animal metaphors to explore the beliefs of grades four and six learners about mathematics and themselves as mathematics learners. Their study showed that certain learners perceived mathematics as complicated, while others saw it as connected to wisdom. Although there is consistency between their results and ours, the grade three learners' beliefs indicated a need for serious learning support and a change in negative beliefs. According to Luttenberger et al. (2018), such negative beliefs about mathematics may arise from environmental factors such as teachers' or parents' attitudes towards the learners' ability in mathematics, societal stereotypes (male/female mathematics abilities), or personal factors such as traits or gender.

The beliefs revealed in the study indicate the role that mathematics plays among learners. The results may also reveal grade three learners' beliefs about mathematics or how they think about it. Furthermore, learners' beliefs in this study may guide teachers to conduct their teaching practice in ways that may change the non-availing beliefs, i.e., that mathematics is a difficult, dangerous, and scary subject. Hence, the way learners perceive mathematics may influence their construction of mathematical knowledge and their learning and performance (Joseph et al., 2019). Moreover, the beliefs shared may also impact learners' interest, engagement, and motivation in learning mathematics. Learners may lose interest and consequently fail mathematics.

Furthermore, COVID-19 has created some positive educational factors, such as a low teacher-learner ratio in the participating school (fewer than 20 learners per class). This has allowed teachers to reach every individual, leaving more time for teachers to provide learning support to those experiencing MLD (Zafarnejad & Griffin, 2021). We are suspicious about the reasons for the belief that "mathematics is difficult" and the negative connotations that related mathematics to dangerous animals or situations resulting from the study. It is possible that unstable school calendars, the implementation of a rationalised syllabus/COVID-19 emergency curriculum, and the school closure due to the lockdown have resulted in learners developing a mathematics learning gap and an increase in MLD. Without formal mathematics assessment during the remote learning period (Evelina et al., 2020), learners may not have acquired the necessary learning competency. Also, parents may not have taught them mathematics as expected because not all parents had that ability (Bubb & Jones, 2020).

Conclusion

Using the animal metaphor to understand the Namibian grade three learners' beliefs about mathematics revealed two themes: (i) the learners feared mathematics, and (ii) they connected mathematics to poisonous and dangerous animals or situations. These patterns may have arisen from negative experiences in the early years of schooling, school closure due to COVID-19 and prolonged periods of not learning mathematics or implementing the COVID-19 emergency curriculum. The beliefs held by grade three learners could have been influenced by the meanings they attributed to mathematics. Additionally, their choices of animals may have been influenced by their personal experiences and familiarity with particular animals. Furthermore, home and school factors shape learners' beliefs and how they view mathematics. The associated animal metaphors indicated that the young learners had a negative attitude towards mathematics education. This revealed a need for serious learning support and motivation for learners experiencing MLD that would help them develop a positive attitude towards mathematics in early grades.

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Chapter 16
Teaching and Learning Mathematics Using Technology in Rwanda Basic Education Schools Amidst the COVID-19 Pandemic

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The Fourth Industrial Revolution and Education

We live in a world marked by automation in many socio-economic sectors. The collection and handling of huge amounts of data that can be analysed and used to solve problems are done utilising machines with little human involvement. Artificial intelligence, where robots perform many of the activities that humans previously did but with more precision and efficiency, is picking pace. This era is referred to by many scholars as the fourth industrial revolution (Maynard, 2015), builds on the third industrial revolution, which was marked by, among others, the emergence of computers and automation (Xu et al., 2018). The fourth industrial revolution is marked by the convergence of digitalisation, automation, and the internet of things (Maynard, 2015). It is an era where many jobs are not lost but are changed. Xu et al. (2018) noted that each industrial revolution destroys many of the jobs of the preceding revolution. This means that each new generation comes with some jobs that may be not only new but also unique to the era. More routine jobs will be changed to new roles that require new competencies. While things are changing and it is becoming difficult to predict how the future jobs will be, it is vivid that the current jobs need new competencies (Maynard, 2015). This chapter is inspired by the fact that the workforce must adapt to new roles requiring new knowledge, skills, and attitude.

To develop the human resource for the fourth industrial revolution, socio-economic systems must change their strategies of human resource training. This is because no system seems to be spared. Schwab (2016) contends that the fourth industrial revolution is affecting nearly every sector and country, whereby the

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breadth and depth of these changes envisage the transformation of every socio-economic system. At the heart of these endeavours is the education system. Maynard (2015) argues that as the fourth industrial revolution pics pace, stakeholders of all levels will benefit from being aware of its potential implications as they make and influence decisions, and this requires educational initiatives that are highly innovative. It is the education system that has an audience with the most vibrant minds to train. The system is charged with such responsibility to produce products that possess the required knowledge, skills, and similarly important attitudes for continuous learning in the ever-changing world of work. The education system must meet the challenge of rapid breakthroughs whose rate of change has no historical precedent (Xu et al., 2018). Technology integration in various socio-economic sectors, including education, is central to the adaptation to the fourth industrial revolution. This adaptation in education includes not only automation of school activities but, more importantly, the instructional processes.

The fourth industrial revolution in education includes, among other things, the integration of digital technology. This involves digitising schools where many activities may be done by or with automated machines. The learning content is digitised and can be obtained electronically rather than the traditional print media. For example, learners are now exposed to enormous amounts of information from online libraries that house text, audio, and still and moving images. The learning activities can also be done with the help of technology. For example, learners can access learning resources electronically, including searching and retrieving, reading, and analysing the learning content on and by electronic gadgets, communicating and collaborating online using collaborative and conferencing digital tools (Salim et al., 2021; Umbara et al., 2021). Various online tools may sharpen the learning materials' presentation by providing electronic content, questions for practice, discussions, quizzes, modules, and summaries (Anggraini & Mahmudi, 2021). There are also digital tools, including learning management systems, that afford educational assessment entirely or partly electronically.

Although in the fourth industrial revolution, many instructional resources are being automated, and educational technologies are affording learning activities, the role of teachers remains vital (Umbara et al., 2021). However, the teaching and learning practices in the classroom must be adapted to strategies that subscribe to the development of the competencies needed in the fourth industrial revolution. Teachers must be trained to acquire teaching competencies that enable their effective integration of technology in ways that transform classroom practices. On the other hand, learners must have access to facilities and the ability to use technology for their learning (Mulenga & Marbán, 2020). This study focuses on the adaptation of educational technologies in the teaching and learning process as one but important means to embrace and develop learners' knowledge, skills, and attitudes for the fourth industrial revolution era.

Mathematics Education in the Fourth Industrial Revolution

Mathematics is one of the important and core learning areas in the school curriculum. In addition to developing numeracy skills for daily practical life, mathematics offers the abstraction of concepts relevant to unlimited socio-economic problem solving (Taleba et al., 2015). Mathematics does not only find value in disciplines that are heavily mathematics-dependent, like STEM subjects; it is central to the development of numeracy skills relevant to any human endeavours. The subject plays a key role in the material infrastructure of nearly every product and service, communication, and interpretation of concepts in STEM fields. As such, every product, including modern technology infrastructure and facilities, is highly dependent on, among others, the knowledge of mathematics.

Whereas mathematics finds importance in every area of human life, its learning has not been without challenges. In many countries, mathematics is a poorly performed subject in the school curriculum. Abstraction, students' attitudes, and ineffective teaching methods have been attributed to the difficulties in learning mathematics (Umbara et al., 2021). Subsequently, efforts are in place to improve mathematics learning by substantiating the abstraction, improving students' attitudes, and, as argued by Rahayu et al. (2020) and Umbara et al. (2021), enabling teachers to use more effective teaching methods.

Technology integration has been thought to be one possible way to address some of the problems that the teaching and learning of mathematics face. For example, some abstract concepts in mathematics may be visualised through multimedia. The use of technology provides opportunities for learners to develop mathematical knowledge through multiple and active senses (Murtafiah et al., 2020). Technology also allows multiple representations of concepts, allowing learners to acquire knowledge by interacting with different media for knowledge construction (Murtafiah et al., 2020; Umbara et al., 2021). Learning activities through technology are thought to affect learners' attitudes as well. With the decreased abstraction and increased interaction and engagement that technology may support, learners may develop positive attitudes towards the learning activities and the content.

To support mathematics instruction through the use of technology, multiple software have been developed. These include computer software, web 2.0 technologies, and mobile applications (Salim et al., 2021). In addition to general-purpose software such as spreadsheets that may be used for various mathematics activities, numerous mathematics-specific software have been developed. Software such as GeoGebra, Microsoft Mathematics, Geometry Pad, Math Editor, Maxima, Photomath, and many others have been developed, used in schools, and researched for their effect on learning. Also, the breakthrough of the second generation – web 2.0 technologies has allowed the contribution of electronic content that may be useful for mathematics teaching and learning (Murtafiah et al., 2020). For example, platforms such as YouTube, MUCOOBs, and OER that house thousands of relevant mathematics content and other disciplines may be increasingly important (Maynard, 2015). In addition, the recent development of mobile technology has allowed the creation of mobile applications that are useful for mathematics instruction (Salim et al., 2021). There are numerous, still increasing in number, mobile mathematics applications that may be suitable for various learning activities. Today, learners may have access to mobile applications stored in google play and the app store right from their mobile gadgets.

Rwandan Adaptation of Mathematics Education in the Fourth Industrial Revolution

Many countries are investing in technology integration in education, and Rwanda is not an exception. Although Rwanda is one of the developing countries, its dedication to technology integration has been remarkable (Rwanda Education Board, 2015). This is not only evidenced by its efforts to electrify schools and improve teacher training regarding technology integration. It is more substantiated by the distribution of computers to all university students and the implementation of one laptop per child in schools (Adam et al., 2016; Munyengabe et al., 2017). The one-child one laptop policy, for example, was intended to enable every school child in Rwanda to have access to digital gadgets that would be used in their learning. These efforts, although they do not necessarily mean taking active participation in the fourth industrial revolution. For example, the breakout of the COVID-19 pandemic caught everyone off-guard despite increased access to digital facilities. However, these efforts serve as a starting point in addressing challenges brought by the pandemic and the move towards the fourth industrial revolution.

To benefit from technology-assisted mathematics instruction, Rwanda has been training its teachers to be technologically savvy. It plans to use technology to enhance learning as the impact of science and technology increasingly affects all life aspects (Rwanda Education Board, 2015). Several efforts have been put in place to train mathematics and science teachers to adopt the competence-based curriculum, which recognises the importance of technology integration (Nsengimana et al., 2021; Rwanda Education Board, 2015).

The recent strike of the COVID-19 pandemic not only affected the provision of education in schools but also challenged the readiness to use electronic means for instruction (Mukuka et al., 2021; Rahayu et al., 2020). Devoted to ensuring learning does not stop completely, Rwanda shifted some learning online using learning management systems in universities. Primary and secondary schools benefited from local televised programs. To allow more access to learning content, more internet access with moderated prices was offered by mobile operators and controlled by the government. Nevertheless, the effectiveness of these efforts, particularly during the COVID 19 pandemic, has not been investigated. Although the one laptop per child policy has been in place since 2006 and university students have access to personal laptops, little is known about how effective is digital learning with these devices. In addition, during the COVID-19 pandemic, efforts to shift or blend with digital learning are made, including online teaching and learning of mathematics. Nonetheless, little is known about what is happening with students and teachers as they forge ahead in integrating technology in education. To this end, this study investigated mathematics learning during this period.

Conceptual Framework and Research Questions

We live in a technological revolution that inevitably affects every aspect of our lives, and this is not a temporary situation. Amidst the Covid-19 pandemic and its social distancing, parents are suddenly responsible for providing their children with access to computers, tablets, smartphones, radio, televisions, and the internet to study virtually. To be successful within this era, teachers and students need to transform teaching and learning by using pedagogy suitable for the Fourth Industrial Revolution. The literature shows a digital divide between rural and urban areas (Furuholt & Kristiansen, 2007; Otioma et al., 2019), suggesting that access to technology is inequitable. Technology access influences technology use (Daya & Laher, 2019) in the first place, although it is not a guarantee. The digital divide may also create differences in experience in integrating technology in the teaching and learning process. This is because technology integration experiences are gained through technology use (Njiku et al., 2020) which is unevenly distributed between rural and urban. Also, the level of education (Voithofer et al., 2019), years of teaching with technology (Yurdakul, 2018), or students' grade level may indicate the experience in terms of time one has integrated technology in education. As teachers are trained to use technology, coupled with experience in using (Uslu & Usluel, 2019) since the adoption of the one laptop per child policy (Munyengabe et al., 2017), they develop confidence that informs their continual use (Njiku et al., 2020). In addition, efforts to integrate technology have always been gender sensitive (Baturay et al., 2017; Markauskaite, 2006; Roussinos & Jimoyiannis, 2019; Vitanova et al., 2015). This literature on technology integration shows that females and males may develop different levels of self-efficacy in the use of technology.

Therefore, this chapter explores students' and teachers' self-efficacy, perceptions, and experiences on the use of technology in the learning and teaching of mathematics. The findings of this study will advance our understanding of technology integration issues in mathematics education amid the COVID-19 pandemic and fourth industrial revolution, especially in Rwanda. The present study adds valuable insights to the literature on teachers' and students' use of technology in mathematics teaching and learning, as well as important recommendations for improving mathematics teaching and learning using technology, especially during the COVID-19 pandemic. Accordingly, this study is designed to answer the following research questions:

- What are the students' and teachers' experiences (in terms of access and use), self-efficacy, and perceptions of learning and teaching mathematics using technology during the COVID-19 pandemic?
- To what extent do gender, class level, and school location affect students' technology integration in the learning of mathematics?
- To what extent does gender, teaching experience, educational level, and school location affect teachers' technology integration in mathematics?

Methodology

Research Design

In this study, we used quantitative research methods to examine how Rwandan students and teachers learn and teach mathematics during the Covid-19 pandemic and their perceptions and attitudes toward technology. A survey research design was used where participants were reached out through an online questionnaire. Teachers and students filled out self-reported questionnaires with Likert-type items that collected quantitative data.

Participants of the Study

The population for the study was mathematics teachers and students in primary and secondary schools in the 30 districts of Rwanda. Both primary and secondary school teachers were invited to participate in the study. In the analysis, questionnaires with less than 60% completion rates (24 teachers and 34 students) were discarded. A total of 91 teachers (52 males and 39 females) and 91 students (31 females and 60 males) voluntarily participated and completed the online questionnaire. In addition, most participants came from rural areas (62%), while the remaining 38% were from urban areas. Most (73%) of the teachers had teaching experience of between 3 and 10 years, and only 25% held a bachelor's degree. Very few teachers (9%) had teaching experience of more than 10 years. Students who participated in the study were between 6 and 19 years old, and the majority were from rural areas (62%). We had more male students than females. See Table 16.1 for further details.

Instrumentations

An online survey questionnaire was developed among students and mathematics teachers in elementary and secondary schools to reach a large number of respondents from rural and urban areas. Therefore, the study relies on students' and

Table 16.1 Teachers' and students' background information

Variables	Description	Teacher	Student
Gender	Male	52 (57%)	60 (66%)
	Female	39 (43%)	31 (34%)
Age (mean)		33	11
Class grade	Primary	48 (53%)	47 (52%)
	Secondary	43 (47%)	44 (48%)
School location	Rural	56 (62%)	56 (62%)
	Urban	35 (38%)	35 (38%)
Teaching experience	Less than 3 years	16 (18%)	
	Between 3 and 10 years	66 (73%)	
	More than 10 years	9 (9%)	
Educational level	Advanced level certificate	34 (37%)	
	Diploma	35 (38%)	
	Bachelor	22 (25%)	

teachers' self-reports. From the literature, various dimensions were identified regarding technology integration. As such, an item pool was generated and used to design questionnaire items that reflect the four perspectives—perceptions, self-efficacy beliefs, access, and use of technology. Items included Likert-type items developed from the literature. To ensure the questionnaires' validity and reliability, a pilot test was conducted on a group of participants. As a result of the participants' feedback, the piloted questionnaire items were improved.

The questionnaire was created with Qualtrics, an online system that facilitates digital signing for consent. Researchers complied with all privacy and data management guidelines. The online survey launched on November 2, 2021, and ended on December 20, 2021. Participants were invited via Facebook and WhatsApp groups and by email. Participants were promised a summary of the findings after data analysis.

A convergent and divergent validation approach was used to validate the questionnaire. Additionally, we used composite reliability and variance extracted values to assess reliability. The Cronbach's alpha coefficients presented in Table 16.2 are above 0.70, indicating that all items are likely reliable and measure the same concept.

Data Analysis

To answer the research questions, the data obtained from teachers' and students' responses to the instruments were analysed using descriptive and inferential statistics. Descriptive analysis was conducted using means and standard deviations of responses to all of the items of the study constructs (see Table 16.3). Using ANOVA, we examined significant differences in technology use across the categories of

Table 16.2 Cronbach's alpha coefficients of constructs

Constructs	Cronbach's alpha coefficient
Teacher's self-efficacy beliefs	0.78
Student's self-efficacy beliefs	0.79
Teacher's use of technology	0.79
Student's use of technology	0.81
Teacher's perceptions of technology	0.85
Student's perceptions of technology	0.87
Teacher's access to technology	0.81
Student's access to technology	0.82

teaching experience and educational level, and using an independent t-test, we examined differences in technology use according to the participants' gender, class level, and school location.

Findings

Teacher's Self-Efficacy Beliefs and Perceptions About Teaching Mathematics With Technology

As indicated by the results presented in Table 16.3, most teachers did not have adequate knowledge and skills to use technologies during the Covid-19 pandemic (*mean* = 2.76, *sd* = 0.91), and, as a consequence, e-learning technologies during the pandemic were not convenient for them (*mean* = 2.71, *sd* = 0.12). The majority of teachers did not have sufficient experience in using instructional technology (*mean* = 2.63, *sd* = 0.98). Additionally, teachers did not strongly believe that e-learning technologies were useful for teaching mathematics during the pandemic (*mean* = 2.33, *sd* = 0.72). Teachers were also not and would not feel confident to use technology during and after the Covid-19 pandemic period (*mean* = 2.60, *sd* = 0.29; *mean* = 2.53, *sd* = 0.09 respectively). Regarding teachers' perceptions about teaching mathematics using technologies, teachers prefer face-to-face interaction to teaching mathematics via technologies (*mean* = 3.84, *sd* = 0.09). Results also revealed that teachers find it very hard to teach mathematics content using educational technologies (*mean* = 3.04, *sd* = 0.65).

Teacher's Access and Use of Technology

Most teachers do not have a well-equipped classroom with access to technology facilities such as computers and tablets (*mean* = 2.80, *sd* = 0.15). In addition, schools do not have internet connection (*mean* = 2.84, *sd* = 0.82). Most teachers used

Table 16.3 Descriptive results of the study constructs and items

Items	Mean	SD
Teacher's self-efficacy beliefs (mean = 2.59; sd = 0.54)		
I have sufficient knowledge and skills to use technologies during the Covid-19 pandemic	2.76	0.91
I am confident that I can use technology during the Covid-19 pandemic	2.60	0.29
I have been using the educational technologies even before the covid-19 pandemic	2.59	0.50
I have sufficient experience in using instructional technology	2.63	0.98
I believe that the use of e-learning technologies in teaching is useful during this pandemic	2.33	0.72
The use of e-learning technologies during this pandemic is convenient for me	2.71	0.12
I would feel confident in using the educational technologies after the covid-19 pandemic	2.53	0.09
Teacher's use of technology (mean = 2.68; sd = 0.73)		
I used technology to teach mathematics during the pandemic	2.70	0.45
I used technology to prepare lessons during the pandemic	2.68	0.70
I used technology to collect students' work during the pandemic	2.65	0.58
I used technology to score students' work during the pandemic	2.76	0.62
I provided learning feedback to students using technology during the pandemic	2.49	0.36
I guided my students to attend lessons broadcasted in mainstream media such as television, radio, etc.	2.91	0.88
Teacher's perceptions of technology (mean = 2.98; sd = 0.94)		
I think mathematics content is difficult to be understood by students through technology	3.03	0.99
Teaching mathematics using educational technologies is time-consuming	3.02	0.96
Mathematics content cannot be taught using technology	2.95	0.95
I think it is very hard to teach mathematics content using educational technologies	3.04	0.65
Face-to-face interaction is more efficient than teaching mathematics using technology	2.84	0.09
I have no intention of using educational technologies in the near future	3.02	0.98
Teacher's access to technology (mean = 2.75; sd = 0.49)		
My classroom is equipped with technology facilities (e.g. computers, tablets, …)	2.80	0.15
Our school has facilities for computer-assisted instruction	2.62	0.78
Our school has an internet connection	2.84	0.82
Student's self-efficacy (mean = 2.74; sd = 0.82)		
I am confident that I can use a smartphone to study mathematics during the covid-19 pandemic	2.68	0.85
I am confident that I can use computers to study mathematics during the covid-19 pandemic	2.59	0.94
I have sufficient knowledge and skills to learn mathematics using technologies during the covid-19 pandemic	2.96	0.58
I am confident that I will be able to use educational technologies after the covid-19 pandemic	2.84	0.69
I am uncomfortable learning mathematics via e-learning technologies	2.52	0.81
I would feel confident in using the educational technologies after the covid-19 pandemic	2.76	0.87

(continued)

Table 16.3 (continued)

Items	Mean	SD
Student's use of technology (mean = 2.76; sd = 0.85)		
I have followed mathematics lessons through television during the covid 19 pandemic	2.79	0.09
I have used a smartphone to study mathematics during the Covid 19 pandemic	2.52	0.78
I have followed mathematics lessons through radio programmes during Covid 19 pandemic	2.75	0.90
I have used school computers to study mathematics after the Covid 19 pandemic	2.98	0.92
Student's perceptions about technology (mean = 3.01; sd = 0.68)		
Smartphones can not be used to learn mathematics	2.05	0.65
I think learning mathematics through radio and television programmes is very difficult	3.26	0.15
Learning mathematics via e-learning technologies is very hard	3.52	0.81
Learning mathematics using educational technologies is time-consuming	3.73	0.89
I find it difficult to learn mathematics using technology	3.12	0.69
I like face-to-face interaction since it is more efficient in learning mathematics	3.91	0.95
Student's access (mean = 2.64; sd = 0.60)		
My classroom is equipped with technology facilities (e.g. computer, tablets, internet, …)	2.86	0.76
The school computers have mathematics resources	2.38	0.65
We have a television at home	2.42	0.88
There is a computer at home	2.45	0.45
There is a smartphone at home	2.91	0.87
I have access to the internet at home	2.49	0.75

technologies in neither preparing nor teaching a lesson during the pandemic. However, to some extent, teachers could guide their students to attend lessons broadcasted in mainstream media such as television and radio (*mean* = 2.91, *sd* = 0.88).

Student's Self-Efficacy Beliefs and Perceptions About Learning Mathematics With Technology

In regards to the student's self-efficacy beliefs, the results suggested that most students were not sure that they could learn mathematics using technologies during the covid-19 pandemic (*mean* = 2.96, *sd* = 0.58), and they were not sure whether they would feel confident to use the educational technologies after the covid-19 pandemic (*mean* = 2.84, *sd* = 0.69). Majority of students were uncomfortable to learn mathematics via e-learning technologies (*mean* = 3.52, *sd* = 0.81). Regarding students' perceptions about learning mathematics using technologies, students like face-to-face interaction more than learning mathematics via technologies since it is more efficient (*mean* = 3.91, *sd* = 0.95). Results revealed also that students find it very hard (*mean* = 3.12, *sd* = 0.69) and time consuming (*mean* = 3.73, *sd* = 0.89) learning mathematics content using educational technologies.

Student's Access and Use of Technology

Similar to their teachers, most students do not have a well-equipped classroom with access to technology facilities such as computers, tablets, internet (*mean* = 2.86, *sd* = 0.99). In addition, students did not have access to television (*mean* = 2.42, *sd* = 0.67), computers (*mean* = 2.45, *sd* = 0.45) and internet connection at their homes (*mean* = 2.49, *sd* = 0.75). However, a number of students have access to smartphones at their homes (*mean* = 2.91, *sd* = 0.87).

The Use of Technology According to Teaching Experiences, Education Levels, Gender, Grade Level, and School Location

We used an independent t-test and One-Way ANOVA to examine differences in technology use according to participants' backgrounds. The results in Table 16.4 show that there is no significant difference in technology use between neither male and female students ($t_{89} = 0.777$, $p > 0.05$) nor between male and female teachers ($t_{88.991} = -2.885$, $p > 0.05$). Moreover, there was no significant difference in technology use between primary and secondary students ($t_{89} = -1.212$, $p > 0.05$). However, there was a significant difference between rural and urban students' and teachers' use of technology in mathematics ($t_{89} = 6.088$, $p < 0.001$, $t_{89} = 11.153$, $p < 0.001$ respectively).

As demonstrated by one-way ANOVA (see Table 16.5), there is no statistically significant difference in technology use according to teachers' teaching experiences or educational levels ($F(2,88) = 1.568$, $p = 0.214$ and $F(2,88) = 0.97$, $p = 0.908$ respectively).

Table 16.4 Results of independent t-test

Student's use of technology in learning mathematics	
Gender	*Mean* (Female = 2.855; Male = 2.708), *Sig*. = 0.439, *df* = 89, *t* = 0.777
Grade level	*Mean* (primary = 2.656; secondary = 2.872), *sig*. = 0.229, *df* = 89, *t* = −1.212
School location	*Mean* (rural = 2.507; urban = 3.595), *sig*. = 0.000, *df* = 89, *t* = 6.088
Teacher's use of technology in teaching mathematics	
Gender	*Mean* (female = 2.316; male = 2.958), *sig*. = 0.500, *df* = 88.991, *t* = −2.885
School location	*Mean* (rural =2.212; urban = 4.254), *sig*. = 0.000, *df* = 89, *t* = −11.153

Table 16.5 Results of one-way ANOVA

Teacher's use of technology in teaching mathematics	
Teaching experience	*Mean* (Less than 3 years = 2.316; Between 3 to 10 years = 2.958; More than 10 years = 2.57), *Sig*. = 0.214, *df* within groups = 88, *df* between groups = 2, $F = 1.568$
Educational level	*Mean* (advanced level certificate = 2.212; diploma = 4.254; bachelor =2.69, *sig*. = 0.908, *df* within groups = 88, *df* between groups = 2, $F = 0.97$

Discussion

In this study, teachers' and students' self-efficacy beliefs, perceptions, access, and technology use in mathematics were examined, as well as differences in technology use according to teaching experiences, educational level, gender, grade level, and school location.

The study suggests that a teacher's self-efficacy beliefs and perceptions influence his or her technology integration in mathematics. The majority of teachers said they had insufficient knowledge and skills to use technologies during the Covid-19 pandemic, and as a result of which, they found e-learning technologies inconvenient. Participants also agreed that they did not have enough experience with instructional technology. Despite the country's great dedication to integrating technology (Rwanda Education Board, 2015), findings reveal that students and teachers had not been prepared for technology use before this pandemic. Consequently, Rwanda must continue to enhance teacher training regarding technology integration.

In the present study, we found that participant teachers and students have limited access to facilities and the ability to use technology for teaching and learning mathematics. Interestingly, this finding contradicts Rwanda's one laptop per child program, which aimed to provide every school child with access to digital gadgets used in the classroom. In developing countries, limited access to technology and facilities has been one of the obstacles to the development of the competencies needed for the fourth industrial revolution.

This study found no significant differences between male and female students and teachers regarding technology use. Based on this finding, the gender differential in technology use by males and females is no longer valid (Mailizar, 2018). In contrast to earlier studies, these findings contradict the belief that technology-related activities are traditionally male-dominated (e.g., Markauskaite, 2006; Vitanova et al., 2015). It is often believed that a higher education level and more experience in teaching are essential components to developing the skills and knowledge required for effective teaching. The present study found that teachers with different educational qualifications and teaching experiences used technology similarly when teaching mathematics. However, there was a significant difference between rural and urban students' and teachers' use of technology in mathematics. Specifically, urban teachers and students use technology more frequently than rural teachers and students.

These findings suggest that as teachers and students get trained in technology integration, their use is likely to be similar across gender and education levels.

These findings suggest that the move towards the fourth industrial revolution may be without discrimination if appropriate training for technology integration is provided to all. However, the findings show that a digital divide exists between rural and urban teachers and students. This means that some members of society, especially teachers and students, may be left behind as Rwanda and the world enter the fourth industrial revolution. It may be important to note that education has a very important role in preparing people to participate in technological revolutions (Xu et al., 2018), where the teacher's role is indispensable (Umbara et al., 2021).

Conclusions

Research findings reveal that participant mathematics teachers and students in primary and secondary schools need to improve their knowledge and technology skills for teaching and learning. Additionally, participants had poor perceptions and self-efficacy beliefs about using technology in mathematics teaching and learning. Both teachers and students had limited access to necessary technology and facilities. The background of teachers and students did not affect the use of technology. The findings reveal that urban teachers and students use technologies more frequently than rural teachers and students.

In conclusion, this study demonstrates that teachers and students faced challenges using technology for instruction and learning during school closures caused by COVID-19. This suggests that students and teachers are yet to migrate into the fourth industrial revolution. This situates Rwanda's education provision, similar to other countries in the region, in a difficult position to fight the impacts of pandemics such as the COVID-19 pandemic. The present study contributes to the existing literature on the most significant barriers to technology integration in mathematics education during and after the pandemic.

The findings of the present study imply that there is still a need to invest in teacher training in addition to the provision of digital tools. Also, motivation in terms of supporting teachers' use of technology needs to be done to ensure that teachers not only utilise the available resources as supported by the one laptop per child policy but also the effective exploitation of digital learning affordances. The study also implies that students still have limited access to digital devices despite the implementation of one laptop per child. This does not only call for more investment but also attention toward the more accessible mobile gadgets such as smartphones, whose educational use and challenges have not been received enough attention in Rwanda.

In considering research results, it is very important to note that the data for this study were collected using online self-reported questionnaires, which are prone to social desirability (Creswell & Clark, 2017). Therefore, quantitative results should be complemented by qualitative results in future research to validate and enrich the study results.

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Chapter 17
WhatsApp as Social Media to Enhance the Dialogic Interactions in Mathematics: Grade 9 Teachers and Learners' Voices

Tšhegofatšo Makgakga

Introduction

Social media is becoming popular among individuals and institutions (Oyman, 2016). Social media's importance is connecting people via smartphones or computers for internal and external communication. Zarei and Fathi (2020) noted that social media facilitates social interaction and a sense of community. It was also noted that people communicate, reach information instantaneously, create a platform for discussion, and become transparent through social media (Doğan, 2019). WhatsApp, Instagram, Twitter, and Facebook are the most commonly used social media platforms involving smartphones or computers. These social media tools are now commonly used to facilitate learning to provide opportunities in the education landscape (Doğan, 2019).

WhatsApp is a social media tool that is gaining popularity. It can facilitate sending texts, videos, pictures, audios, GIFs, and stickers (Zarei & Fathi, 2020). Doğan (2019) said that WhatsApp is being used by more than a billion people in 180 countries in which WhatsApp groups formed by internal businesses are prevalent. WhatsApp groups (WAG) promote communication, save time and stationery, and allow quick decision-making and implementation to come to the fore. WAG can be effective in learning if monitored by teachers. In WAG, learners can explore solutions to problems via discussions with peers.

Schools are among the institutions that use WhatsApp to facilitate teaching and learning (Panah & Babar, 2020; Zarei & Fathi, 2020). Panah and Babar's (2020) study noted that WhatsApp could be used for group learning to support the various learning environments possible. Kufre and Abe (2017) added that WAG can increase and strengthen communication, allow the learners to share information, encourage

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creativity and critical thinking, and promote problem-solving skills among the learners. Nsabayezu et al. (2020) concurred that WAG enhances communication and collaboration between the teachers and learners by sharing content, videos, and activities. Some studies have reported drawbacks of using WAG in the education landscape, such as increasing the teachers' workload and privacy issues (Hew, 2011).

Several studies have been conducted on using WAG in the education landscape. For example, WhatsApp as a mediating tool in Higher Education institutions was examined by Motaung and Dube (2020), while Durgungoz and Durgungoz (2021) explored the use of WhatsApp in mathematics learning. Doğan (2019) analysed a school WAG using teachers and administrators, and Nsabayezu et al. (2020) explored how WhatsApp can be used as an educational communication tool in Higher Education.

However, little attention has been given to using WAG to enhance the dialogic interactions in teaching and learning mathematics in South African secondary schools in the Limpopo province. Therefore, this study reports how WAG has improved the dialogue in the teaching and learning of mathematics. Dialogue in the context of this study refers to the teacher-learner dialogic interaction (TLDI), the learner-content dialogic interaction (LLDI), and the learner-learner dialogic interaction (LCDI) when sharing mathematical content knowledge in their WAG. This study reports on the two Grade 9 mathematics teachers and their 25 learners' voices and their understanding of the utilisation of WAG to enhance the dialogue in mathematics. Furthermore, this study analysed the drawbacks of the use of WAG to enhance dialogue.

Theoretical Lens

The study used the Transactional distance theory of Moore (1997) and the Community of Practice of Lave and Wenger (1991) to underpin the study. These two theories have been used to understand the voices of both teachers and learners on how WAG enhances the dialogue between teacher-learner, learner-learner and learner content interaction and their understanding of the utilisation of WAG in mathematics. A transactional distance theory was used to design the teachers' and learners' interview questions to report on how WAG has enhanced the dialogue in mathematics, and (b) the community of practice theory of Lave and Wenger (1991) was used to interpret the factors that affect the smooth interaction between teacher-learner, learner-learner, and learner-content in the social network selected.

Theory of Transactional Distance

The theory of transactional distance is a pedagogical concept rather than referring to the physical distance between the teacher and learner from each other (Al-Mashaqbeh & Atef, 2018). It is noted that this theory is useful when using

WhatsApp as a tool to enhance the learners' learning (Benson & Samarawickrema, 2009). This distance requires the development of teaching and learning techniques and strategies to minimise it (Al-Mashaqbeh & Atef, 2018).

This theory of transactional distance (TD) considers two main factors: dialogue and structure (Moore, 1997). Dialogue refers to the communication between the teacher and learner where the teacher provides instructions, and the learner responds to the teacher (Saba & Shearer, 2018). Teacher and learner communication relies on educational philosophy, environmental factors, the course designers, course content, and the personalities of the teacher and learner (Panah & Babar, 2020). Scholars further note that success in teaching can be associated with the teacher-learner, learner-learner, and learner-content dialogues.

The second factor is the fundamental structure that describes the level of flexibility and rigidity of the pedagogical learning aims and objectives, teaching styles, and assessment to accommodate the learners' needs (Benson & Samarawickrema, 2009). These scholars note that TD is presumably lower in programmes that have more dialogue with pre-determined compositions, such as through WhatsApp and teleconferencing. Thus, the high levels of the learners interacting in groups with little pre-determined structure despite classroom learning is an effective and useful tool for supporting TLDI, LLDI, and LCDI in mathematics.

Community of Practice

As noted earlier, the community of practice (CoP) theory is used to complement the theory of TD to underpin this study. This theory is used to understand how WAG enhanced the dialogue for secondary school learners as a means of interaction in mathematics. Wenger-Trayner (2015) describes CoP as a group of people who have gathered to share a concern or passion for something they do. In CoP, practitioners take collective responsibility for managing the knowledge they need, recognising that, given the proper structure, they are in the best position to do so (Wenger-Trayner, 2015). Farnsworth et al. (2016) noted that learning is not an individual endeavour but a social practice situated within the cultural and historical context.

In addition, individuals in CoP interact through activities to solve problems, request information, seek experience, coordinate, find synergy, build an argument, develop confidence, discuss new developments, document projects, and map and identify gaps in the knowledge (Wenger-Trayner, 2015). Wegner (1998) described three principles: mutual engagement, joint enterprise, and a shared repertoire.

Mutual engagement (how it functions) refers to an engagement and how those involved establish mutual relationships in the community based on identity participation (Wegner, 1998). The joint enterprise is the accountability and the extent to which the participants contribute to the ongoing negotiations of meaning. The shared repertoire is the knowledge of practice acquired to engage in CoP.

The researcher used these theories as a framework for this study. TD was used to address the dialogue and structure components, and CoP was used to address mutual engagement, joint enterprise, and shared repertoire. Furthermore, they were used in this essay to develop open-ended questionnaires and the analysis of the data.

Literature Review

The literature review in this study has concentrated on TLDI, LLDI, and LCDI.

Teacher-Learner Dialogic Interaction (TLDI)

TLDI refers to the interaction and communication between the teacher and their learners. Mbwesa (2014) notes that during the dialogic interaction, teachers are important as they guide and reinforce the learners' understanding. The role of dialogue in TD, as measured via TLDI, impacts outcomes such as perceived learning and student satisfaction (Ekwunife-Orakwue & Teng, 2014). However, TLDI did not have an impact on learner grades.

Panah and Babar (2020) point out that TLDI is normally extended beyond the classroom limitations in terms of time and space. This type of dialogue can impact the learners' social, academic, and emotional development (Elhay & Hershkovitz, 2019). Elhay and Hershkovitz (2019) investigated the TLDI relationship through WhatsApp using a survey involving 155 teachers. Their study showed that TLDI through WhatsApp resulted in a better relationship and classroom environment with and for the learners.

Rosenberg and Asterhan (2018) discussed TLDI through WAG and a personal questionnaire using personal interviews and focus group interviews involving 88 learners. The scholars assert that WAG can be used to send and receive updates and manage activities. In their study, Best and Conceição (2017) found that learners have reported a sense of community and satisfaction regarding the in-person elements of the program. Panah and Babar (2020) suggested that some features like easy access, safeguarding personal privacy, creating communities, and communications are appealing in learning when engaging in WAG. Durgungoz and Durgungoz (2021) concur that WAG promotes TLDI and facilitates learning. However, these scholars have identified communication overload as a challenge for the teachers to monitor and influence the learners' dialogic interaction.

Learner-Learner Dialogic Interaction (LLDI)

LLDI is referred to as the interaction and communication between learners through social networks like WhatsApp. Smith and Erika (2016) used interviews and open-ended questions to explore the university students' perceptions of educational interactions through social media. The scholars reported the prominence of LLDI and LCDI rather than TLDI when using social media for education. Al-Omary et al. (2015) conducted a survey involving university students and found that WhatsApp can improve learners' learning skills. Kassandrinou et al. (2014) found a relationship between LLDI and learner satisfaction and perceived learning. Furthermore, Gasaymeh (2017) also conducted a survey and found WhatsApp integration in education easy, useful, and fun.

Learner-Content Dialogic Interaction (LCDI)

LCDI is associated with the learner and content posted by the teacher on social media like WhatsApp. Panah and Babar (2020) noted that this dialogic interaction could improve the learners' learning. Chen (2001: 462) defines LCDI as "the distance of understandings that learners perceive as they study the course materials and the degree that the materials meet their learning needs and expectations of the course." LCDI using technology, enables the learners to access and navigate the course content, and it influences how the content is arranged and presented (Best & Conceição, 2017). Gunter and Junia de Carvalho (2018) examined the cognition, social, and teaching presence of the posts made by teachers during the course through WhatsApp, involving a total of 80 participants. Their findings show evidence of these presences and further show that emojis have a leading role in WhatsApp communication to keep learners on the task. Ekwunife-Orakwue and Teng (2014) revealed that LCDI has a larger impact on learner satisfaction than other types of dialogue but has no impact on the learner's final grades.

Methodology

This study aimed to report on the utilisation of WAG to enhance TLDI, LLDI, and LCDI in mathematics. The study used a phenomenological design to describe the teachers' and learners' experiences when using WhatsApp as a communication means in mathematics. Polit and Beck (2017) note that the phenomenological approach involves careful descriptions of the ordinary conscious experiences of everyday life or the descriptions of things as people experience them.

This study was conducted in one of the secondary schools in the Polokwane district of Limpopo province. The school has one principal, one deputy principal, three heads of department, and 19 teachers, including four mathematics teachers. The school has an enrolment total of 821 learners. The rationale for choosing this school as the study context is that the teachers have widely used WAG when teaching Grade 9 mathematics. However, little is known about the teachers' and learners' voices regarding the utilisation of WAG to enhance the dialogic interactions in mathematics.

The current study participants are comprised of 25 Grade 9 learners (10 males and 15 females) and their two teachers, all of whom are males from the selected school in this district.

The researcher used standardised open-ended interview questions to evaluate the teachers' and learners' voices on the utilisation of WAG to enhance the dialogue in mathematics. The question items were asked in a specific order and exactly as worded (Johnson & Christensen, 2014). The question items were adapted from the study by Al-Mashaqbeh and Atef (2018). The selected question items were modified to align with this study. The teachers' and learners' voices were interpreted based on the utilisation of WAG to enhance the dialogue in mathematics. Due to the Covid-19 restrictions, it was difficult for the researcher to conduct the focus group interviews to stimulate new ideas and concepts.

Data Collection and Analysis

The researcher administered open-ended questions to 25 learners and their two teachers to obtain the data for this study. The participants were given 2 weeks to complete the questions and return them to the teachers. All questions used in the question instrument were designed in English, as English is an official language understood and preferred by the participants. However, the participants were free to use their mother tongue when completing the given questionnaires. Prior to the analysis of the data, the researcher repeatedly read through the completed questionnaires to make sense of the meaning of the data provided by the participants.

Thematic analysis phases were used to analyse the data by organising and preparing it appropriately (Crewel & Creswell, 2018). The transcripts were developed and coded, and the data was collected from teachers and learners in a Microsoft Word table. The researcher familiarised himself with the data by reading it several times. Open coding was done by identifying the meaning chunks in the data, which were merged into several categories. The categories were merged into themes. The rationale for using content analysis was to understand the teachers' and learners' voices on the utilisation of WAG to enhance TLDI, LLDI, and LCDI. This involved the identification of prominent themes by breaking the data down into smaller units and naming the units according to the content they represent.

Ethical Issues

Permission to conduct this study was sought from the Department of Basic Education and the school principal. The study was conducted according to the Department of Mathematics Education community engagement project of the University of South Africa. After permission to conduct the study was granted, informed consent was requested from the Grade 9 learners and the two teachers. The study's purpose and rationale were explained. In addition, the researcher established a rapport with both the learners and teachers by assuring them that their intention was not to evaluate their competency in mathematics. Their role and right to choose whether to participate in the study were also explained to both the learners and teachers. Participation was voluntary, and confidentiality was assured. The participants were permitted to withdraw from the study without punitive measures, and no personal details would be disclosed (Johnson & Christensen, 2014). Pseudonyms such as T_1, T_2, L_1, and L_2 were used instead of their real names throughout the study.

Methodological Norms

As this was a qualitative study, the researcher ensured that the study was conducted rigorously and methodologically to yield meaningful and useful results. Regarding the study's trustworthiness, the researcher ensured that the data analysis was precise and consistent and disclosed the analysis methods with enough detail to enable the reader to determine whether the process was credible (Nowell et al., 2017). As the researcher and respondents viewed the data with different eyes, member checking to strengthen the analysis and interpretation was done by checking whether what was captured was what they meant.

Furthermore, the study was sent to two of the researcher's peers for peer debriefing to provide an external check that may increase the credibility and examine the adequacy of the findings and interpretations of the raw data, as Lincoln and Guba (1985) suggested. Data source triangulation was achieved by using both the teachers' and learners' data to report on the utilisation of WAG to enhance the dialogue in mathematics.

Methodological Approach

The researcher used the main concepts from the TD and CoP theories in the methodological approach. The main concepts emanating from TD are dialogue and structure, while those of CoP are mutual engagement, joint enterprise, and shared repertoire. The definitions, descriptors and features of the main concepts are outlined in Tables 17.1 and 17.2.

Results and Discussion

The data analysis in this chapter followed a methodological approach to make sense of the participants' open-ended questionnaire responses. The data was read through repeatedly, and the participants were coded as T_1 and T_2 for the two teachers and L_1,

Table 17.1 Summary of the methodological approach (transactional distance)

Concept 1	**Dialogue**
Definition	It is communication between the teacher and learner during instruction.
Descriptors	TLDI LLDI LCDI
Features	Learners communicate to set the rules for WAG Learners interact with the teachers to learn mathematical concepts and seek clarity Learners interact among themselves to share ideas when solving mathematical problems
Concept 2	**Structure**
Definition	It is the level of flexibility and rigidity of the pedagogical learning required to achieve the aims and objectives, learning styles, and assessment to accommodate the learners' needs.
Descriptors	Teaching style Assessment
Features	Learners confirming the teaching styles used in WAG for understanding mathematical concepts Teachers successfully assessed the learners using WhatsApp to test their mathematical understanding

L_2, $L_{3,}$ etc., for the learners who participated. Twelve of the twenty-five learners who received the open-ended questions completed and returned the question responses. The question items were returned by the teacher, who was requested to collect, scan, and send them to me electronically using my email address. The teachers and learners were able to use the language of their choice. The following themes were developed after repeated reading through the data: understanding WhatsApp, communication of the teacher's instructions, peer-learning, getting instrumental support, and glitches and suggestions. All direct quotations in the analysis are used verbatim.

Understanding WhatsApp

WhatsApp App is a digital tool that reduces the physical distance between teachers and learners. Al-Mashaqbeh and Atef (2018) refer to it as a pedagogical tool. The teachers and learners described it as a digital platform that reduces distance and promotes interaction and communication between the teachers and learners in the teaching and learning of mathematics. T_1 and $T_{2,}$ when describing what WhatsApp App was, added that the app follows a blended approach to teaching in which the teachers communicate instructions to the learners, and the learners respond to the teachers' instructions, as Saba and Shearer (2018) suggested. The findings on WhatsApp from both the teachers and learners revealed a dialogic component of TD, as stated by Moore (1997). It *"reduce[s] the physical distance between the teacher and learners"* as the learners are used to face-to-face classroom interactions. According to the findings, the distance can be reduced, but physical distance

Table 17.2 Summary of the methodological approach (CoP)

Concept 1	**Mutual engagement**
Definition	It is the interaction between individuals that leads to the creation of shared knowledge and meanings on issues or a problem
Descriptors	Sustain mutual relationships Share ways of doing things together Knowing what others know and can do
Features	Learners respect each other's ideas during WAG discussions when creating knowledge in mathematics Learners communicate mathematical ideas, i.e., what they need to learn and share
Concept 2	**Joint enterprise**
Definition	It is the process through which people are engaged and working together towards a common goal
Descriptors	Knowing what others know and what they can do, and how they can contribute to an enterprise Shared ways of engaging and doing things together
Features	Learners know what they can do and what they can contribute to the WAG when solving mathematical problems. Learners sharing knowledge and skills to learn mathematical concepts. Teachers and learners working together to achieve a common goal when learning mathematics
Concept 3	**Shared repertoire**
Definition	It refers to the common resources used to negotiate and facilitate learning within the group.
Descriptors	Identifying specific tools, representations, and artefacts Ability to assess the appropriateness of actions and products
Features	Learners assess their own progress when learning mathematics Learners share teaching and learning in the WAG to solve mathematical problems. Learners develop ways to adapt the practise guidelines in the teaching and learning of mathematics.

can still exist, according to Saba and Shearer (2018). The issue of physical distance required the teachers to develop a WhatsApp group as a teaching and learning technique to minimise the distance (Al-Mashaqbeh & Atef, 2018).

Communication of the Teacher's Instructions

The findings show that the teachers interact and communicate instructions to the learners when creating mathematics concepts and that the learners respond to those instructions (Saba & Shearer, 2018). In other words, the teachers initiated a dialogic interaction between themselves and the learners (Moore, 1997) during mathematics through WAG. In addition, the two teachers appeared to have formed a CoP (Wenger-Trayner, 2015) where the teachers and learners engaged and solved mathematical problems. For example, T_2 said, *"In this platform (referring to WAP), we give instructions and mathematical problems to solve. We. We allow them to share ideas about those problems before they can write their solutions in their classwork* *books on their own time"*. This quotation supports what Panah and Babar (2020) pointed out that TLDI can be extended beyond the classroom anytime and anywhere. This shows that the teachers gave the learners mathematical activities to solve in the group and then individually in their classwork books.

Before communicating the instructions in the WAG, the teachers set baseline rules for the learners to adhere to manage their interactions. This supports what Rosenberg and Asterhan (2018) highlighted in their study that WAG can be used to send and receive messages and manage activities. The finding shows that the learners are expected to have mutual respect, as Smith and Erika (2016) suggested for CoP. For example, T_2 said, *"All group members are not allowed to post texts or messages other than mathematical activities and also to respect each other's ideas"*. In other words, the learners should work harmoniously in the group by not posting unwanted messages and working within the set boundaries as Wenger (1997) suggested. One of the basic rules was to keep in mind the language used in the group during the interaction. T_1 said, *"We requested learners to use the language that can be acceptable"*.

The findings also revealed that TLDI offers ongoing feedback via this platform (Trenkov, 2014). The learners indicated that their teachers provided constant feedback for clarity when solving mathematical problems during the interaction. Moreover, the teachers were informed about their learners' progress via the group and knew how to support them to help them understand the mathematical concepts. For example, L_6 said, *"Our teachers respond to our queries ge re sa kwišiši (when we do not understand) to clarify some mathematical concepts"*. This finding shows that WAP promotes TLDI to facilitate learning, as supported by Durgungoz and Durgungoz (2021).

Peer-Learning

LLDI revealed a dialogue among learners (Al-Mashaqbeh & Atef, 2018; Moore, 1997), mutual understanding and joint enterprise in a CoP when responding to the instructions communicated by their teachers (Wenger-Trayner, 2015). The findings revealed that WAG facilitates LLDI sharing and constructing mathematical knowledge. For example, L_4 said, *"WhatsApp group provides us with opportunities to interact, share and construct knowledge of mathematical problems in given activities."* This shows that learning is not an individual endeavour but a social practice, according to Farnsworth et al. (2016). Wenger-Trayner (2015) adds that individuals should engage and work together when sharing knowledge and determining the meaning of problems. This study revealed that WAG enables learners to listen to and respect each other's ideas and points of view. For example, L_5 said, *"This platform gives us chance to listen to our peers' point of view during discussions"*. This shows that WAG in this study can sustain mutual engagement and positive relationships among the learners.

In LLDI, the structural component of TD that increases the level of flexibility of pedagogical learning is revealed (Moore, 1997; Al-Mashaqbeh & Atef, 2018). This shows that WAG has made the mathematical content more flexible, fun, and easy for the learners in LLDI. This supports what Gasaymeh (2017) found in their studies that WAG's integration into education is easy, useful and fun. For example, L_2 said, *"WhatsApp group makes mathematics content easy and fun through constructive feedback from other learners, as we are able to learn from each other."* This means that the constructive feedback that the learners give each other in WAG enables the learners to solve more complex problems in mathematics and makes mathematics more fun and interesting. In the same vein, the learners can measure their progress and the amount of knowledge acquired in different mathematics topics during the discussion. L_{10} said, *"In this group, we are able to see how far we can solve mathematics problems and ra tseba se re se kgonago in mathematics (know what we have acquired in mathematics)."*

Gaining Instrumental Support

The findings of LCDI showed a shared repertoire for CoP (Wenger-Trayner, 2015). The learners showed that they had access to mathematics activities through WAG and could easily access the content posted on the platform. This supports Best and Conceição's (2017) study showing that the use of technology enables learners to access and navigate the content of the subject or course. For example, L_7 said, *"In the WhatsApp group, we find mathematics activities posted by the teachers to solve, and we are used to attending classes."* This WAG appeared to meet the needs of the learners when learning mathematics as the learners are used to engaging in a face-to-face classroom.

The findings on LCDI revealed a structural component of TD (Moore, 1997; Wenger-Trayner, 2015). This shows that the more dialogic the learners' interactions are with the content, the more pedagogical the learning is to achieve the aims and objectives of the mathematical activities given by the teacher. L_{11} said, *"If we have a chance to discuss mathematical problems, we then understand maths topics."* Thus, learners with learning difficulties will be accommodated to help them deal with mathematical problems in WAG. This supports what Gunter and Junia de Carvalho (2018) said in that this type of dialogic interaction keeps the learners on task. Ekwunife-Orakwue and Teng (2014) found that this type of dialogic interaction impacts learner satisfaction and can improve the learners' learning (Panah & Babar, 2020).

Glitches and Suggestions

The glitches in this study oppose the mutual engagement of CoP (Wenger-Trayner, 2015). The scholar pointed out in CoP that the team members should have mutual respect to achieve their objectives. The findings show that the learners posted flood messages, wrong messages, and other information in WAG, such as adverts. L_8 said, *"some of the learners send wrong messages and unnecessary information in the group (e.g., Bitcoin adverts), which may lead learners to stop doing a follow-up on messages that came after."* Furthermore, most of the learners seemed not to participate in the WAG due to the socioeconomic background. This could affect the effectiveness of the dialogic interactions of TLDI, LLDI, and LCDI. The findings show the inaccessibility of smartphones and laptops in terms of connectivity during WAG.

Furthermore, the issue of data being expensive for the learners was raised. L_1 said, *"Ga re na smartphones goba di-laptop tsa go connecta (we don't have smartphones or laptops to connect); our families don't afford to buy us those gadgets."* Another glitch highlighted by the learners was a poor network connection in their area, hindering them from participating in the WAG. For example, $L_{3,}$ L_4 and L_9 supported this as they had a poor network connection, especially on their Vodacom line. T_1 and T_2 indicated that the use of WAG has increased their workload and is time-consuming. T_1 said, *"This group has increased our workload and needs more time to monitor participation and address learners' difficulties."* This is one of the drawbacks highlighted by Hew (2011) in the study showing that WAG increases the teachers' workload.

The teachers and learners both had suggestions for implementing WAG in teaching and learning. It is suggested that learners use the line that is most conducive to them, as poor connectivity seems to be a contextual factor. In addition, it is suggested that if the learners can be given data, their participation in the WAG may improve as some parents cannot afford to buy data for their children. L_5, L_6, L_9 and L_{12}'s responses supported each other on the issue of a lack of data. Maybe if the department can support them with data, they will be able to participate in the WAG. T_1 and T_2 said that learners could work together during dialogic interactions when solving mathematical problems, especially those who do not have smartphones. T_2 said, *"Learners who don't have smartphones may pair themselves with those who have them to avoid leaving them behind in mathematics."*

Conclusion

This study aimed to report on the teachers' and learners' voices regarding the use of WAG to enhance the dialogic interactions in mathematics. In this study, TD and CoP were used as the selected theoretical frameworks to report on the voices of the teachers and learners. TD addressed the dialogic and structural components but was unable to pay attention to the social aspect of the study, which is CoP. The two theories complement each other as part of developing an open-ended questionnaire and analysing the collected data. The findings of this study show the positive relationships of the TLDI, LLDI, and LCDI and how they are enhanced in mathematics. It additionally highlights the glitches and suggestions concerning using WAG in mathematics to enhance dialogic interactions. This study suggests that other teachers and learners can use WAG to enhance dialogic interactions in education. Another study can be conducted on the impact/effectiveness of WAG in the teaching and learning of mathematics.

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Chapter 18
Redefining Distance Learning for the Fourth Industrial Revolution: Lessons Learnt from Egyptian Educators

Mariam Makramalla

Introduction

The COVID pandemic has struck the world with a need to redefine social life as we know it. In an attempt to control the spread of the pandemic, many social activities were restricted. I second Carrigan (2021) in arguing that, looking back, what took place was not “social distancing” but rather “physical distancing” as socially, people were interacting on virtual platforms facilitating and easing social interactions. The practice of schooling was not an exception to the social limitations discussed above. Serious concerns about student socio-emotional development were raised by numerous educators around the world (Orben et al., 2020).

Nevertheless, in most Western contexts, the act of learning resumed online using a blended learning approach. Numerous research considered existing educational approaches, such as synchronous and asynchronous instruction (Johnson, 2006), hybrid model instruction (Delialioglu & Yildirim, 2008) and blended learning (Bonk & Graham, 2006). Focusing on mathematics education, the interdisciplinary approach of blending ICT in facilitating the teaching and learning of mathematics became even more attractive (Keong et al., 2005).

However, the approaches above assumed the existence of a solid virtual infrastructure, which could support a virtual learning environment. It also considered questions of equitable access to electronic devices to make possible this virtual learning reality. Additionally, it assumed a level of digital literacy amongst caregivers, educators and learners that would enable a virtual learning and knowledge exchange opportunity. Finally, it assumed a stable environment free of grief, anxiety and sickness, where learners and educators would have a free mind to think and exchange knowledge in a typical setting (Fig. 18.1).

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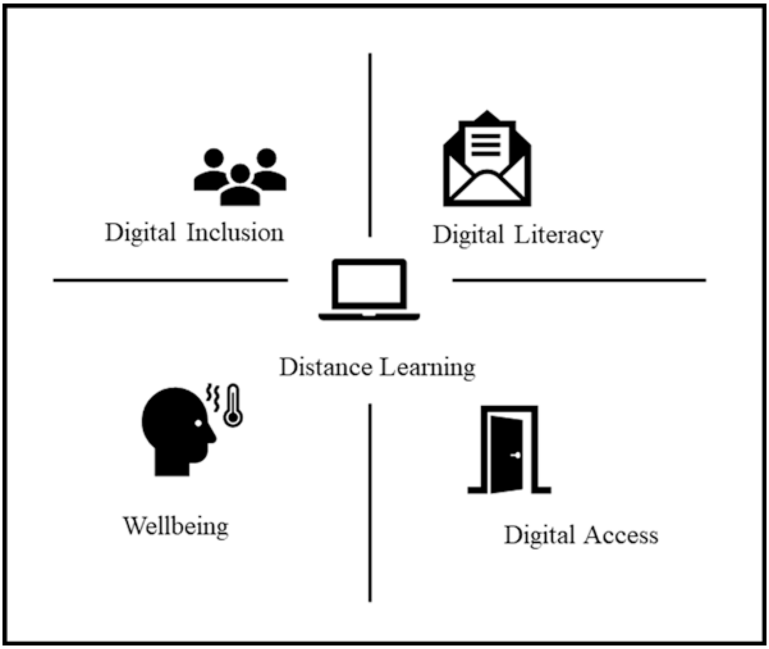

Fig. 18.1 Four notions associated with distance learning during COVID

I dedicate the following sections to discussing the illustrated notions (Fig. 18.1) related to distance learning, emphasising the Egyptian teaching and learning context. I use the community engagement framework lens as an underpinning theoretical stance to discuss these notions. These arguments are based on my extensive involvement with a group of marginalised female learners throughout the pandemic. I believe that the broader concept of virtual inclusion and the role of community ownership in provoking this dialogue is of interest across communities stricken by similar limitations in the African context.

I start by re-iterating the essence of the fourth industrial revolution. I present it based on the affordances imposed by the widespread pandemic. Then, I show the theoretical framework that underpins the study, namely the community engagement framework (Hastings et al., 2011). In light of this framework, I argue for a need to more widely envision the fourth industrial revolution by expanding the scope of viewing technology. I argue for pedagogies that became more visible due to the distance learning environments. I use the single case study (Yin, 2011) of a group of 30 female learners in an underprivileged set-up to support my argumentation. The specific research design of the case study is not elaborately presented in this chapter, as the focus is more on the affordances and lessons learnt. In the research design section, I briefly explain the case study as a research methodology, the sampling process, and the analytical lens. The same case study is then used to discuss the downside of distance learning in remote set-ups. Based on the community engagement framework, the chapter concludes with a discussion on how the discovered contextual affordances can be utilised to balance the reported limitations.

The Fourth Industrial Revolution: A Timely Correspondence with COVID

The term "Fourth Industrial Revolution" (Miller & Wendt, 2015, p. 3) was first announced by the World Economic Forum. It refers to a reality whereby members of a given ecosystem constantly flip between an online and an offline reality (Xu et al., 2018). As educators, it is essential to be aware of the times we live in and utilise the fourth industrial revolution in a way that is productive to the learning environment. It is also essential to be aware of the downside of technology in the classroom. Much research has looked into the downside of technology-aided learning, given learners' socio-emotional development (Gleason & Von Gillern, 2018). Waghid's (2019) work is particularly relevant for the African context, whereby the fourth industrial revolution is re-envisioned given its benefit to the African learner.

The pandemic outbreak has given rise to reconsidering much of the previous research related to the downside of technology-aided education. Around the world, the resilience of educators has been remarkable (Bhagat & Kim, 2020). Even more impressive was the creativity and the ability of educators, caregivers and learners to utilise different forms of digital technology to promote student wellbeing and student learning of mathematics (Viberg et al., 2020). In this chapter, I dig deeper into discussing some of the affordances and limitations of these approaches.

The Relational Framework for Community Engagement

In her study of the Australian context, Johnston (2010) lays forth a relational framework which enables the analysis of community collaboration and community engagement practices. She argues that her framework facilitates a systematically theorised approach toward community engagement. Figure 18.2 illustrates the relational framework for community engagement as a whole.

As presented in Fig. 18.2, we can view community engagement as existing in three separate spheres. Firstly, looking from a broader overarching policy point of view, the purpose and goal are viewed from the lens of the local ecosystem (the political environment). The social opinion is sought to ensure that the broader meaning and intent are locally bought into. Accordingly, resources and skills are channelled to the public. Building on the more general overarching policy, the engagement relationship strategies are set in place. These strategies mirror a typology of community engagement, which exists on a spectrum extending from participation on the one end to information on the other. The degree of community involvement in the dialogue mirrors the resulting engagement typology. As a result, the outputs and evaluation measures are implemented to ensure sustainability. For this chapter, I focus on the typology of engagement as an underpinning theoretical framework for the conceptual matter of community engagement. Figure 18.3 focuses on the typology of engagement in more detail.

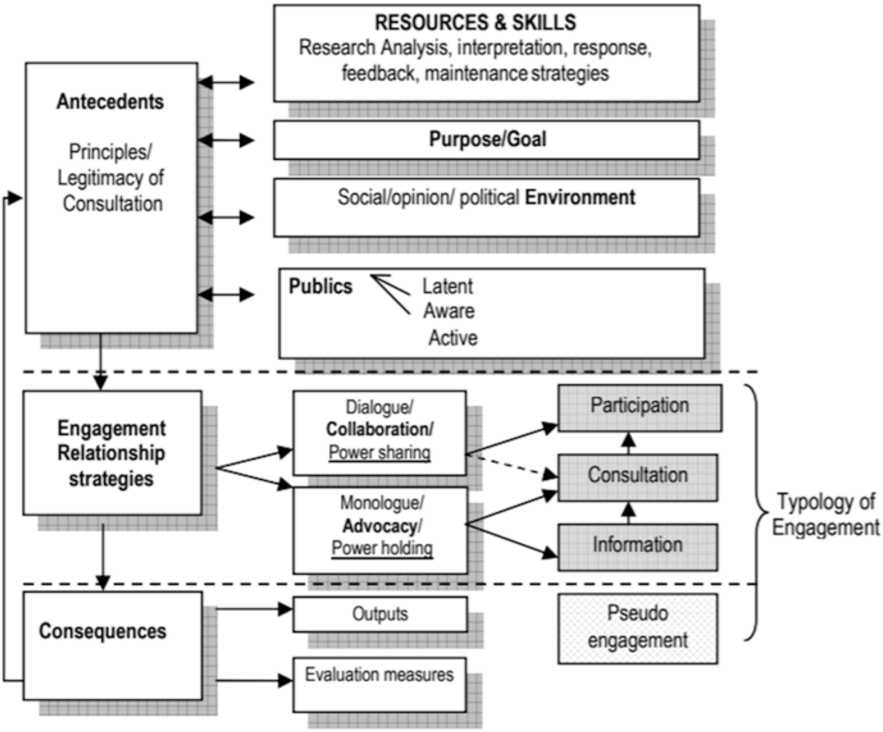

Fig. 18.2 Relational framework of community engagement (Johnston, 2010, p. 5)

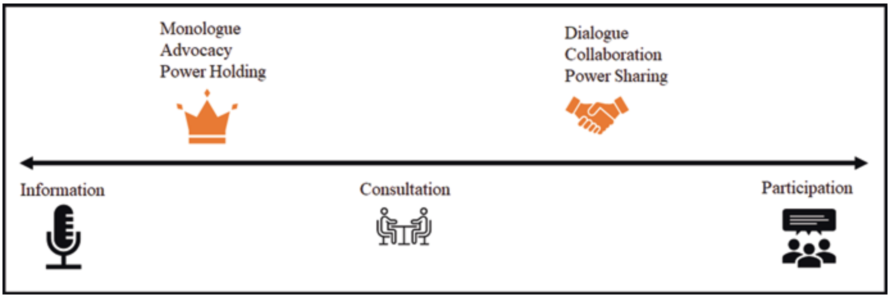

Fig. 18.3 Typology of engagement

As presented in Fig. 18.3, the engagement typology is split into multiple states, namely participation, consultation and information, with the former being the highest form of engagement. The act of participation results from collaborative social integration where community members are viewed as equal in power. Contrary to this is the position of a monologue (also referred to as power holding), where the community is not consulted on matters or where information is simply passed on.

As will be further discussed in the upcoming sections, the degree of perceived ownership, and hence the involvement of a community, to a very high degree determines how creative the respective community will be in utilising the given technological tools in a contextualised way fit for purpose.

This contextual and collaborative means of underpinning interventions, particularly in unprecedented circumstances, such as integrating digital learning in a COVID educational era, is better received by local communities and hence more sustainable (Bruning et al., 2006). Concerning Egyptian educators, the contextualisation of educational interventions and the community ownership of their underpinning ethos is of tremendous value to ensure a valuable educational experience for students (Makramalla & Stylianides, 2019).

Research Design

The affordances and limitations discussed in this paper are underpinned by the relational community engagement framework, focusing on the typologies of engagement. This framework was also adopted as an analytical framework to map the patterns that have been identified in my involvement with a group of 30 female learners (aged 6–11) that have been almost entirely deprived of distance learning opportunities, in its traditional definition of online classrooms. I present a single case study (Yin, 2011) of 30 marginalised girls living in a village with a precarious digital infrastructure. The girls were enrolled at a national school, which, in turn, was located in a very impoverished region in Upper Egypt. Neither caregivers, educators, nor learners can be classified as digitally literate (Poulsen & Roos, 2010).

Caregivers and educators have had to resolve contextualised and disruptive solutions to bridge the hurdles of digital access in the teaching and learning space where the learners were situated. Though I report on a particular sample of 30 learners, these findings have been cross-checked against more extensive sets of learners with similar contextual limitations. It is out of the scope of this chapter to present a comprehensive depiction of the research design and methodology. Instead, my focus is mainly on discussing the affordances and limitations in light of the framework mentioned above.

Distance Learning: The Need for a Contextual Redefinition

Even though the term distance learning has been mainly associated with online learning lately, let us agree to refer to the term distance learning as any form of learning that happens at a distance through any given channel. In the following, I present success stories in which local resources have been used to facilitate learning at a distance, despite the technical accessibility hurdles.

Reflective Practice

The COVID era has witnessed a lot of reflective practice amongst young learners. “With less emphasis on cramming as much knowledge as possible before the test, we can now focus on reflecting on what we know and how we can use it”, exclaimed one of the girls. Parents reported not feeling guilty about not covering the entire curriculum with their children. “I don’t even finish half of the curriculum, and I don’t even feel guilty about it”. Learning seems to have taken a larger dimension beyond knowledge acquisition. Throughout the COVID year, endless talk shows, seminars and informal chats have encouraged learners to engage in reflective practice activities. Many learners have even learnt to make reflective journaling a part of their everyday life, a means to life-long learning. Reflective practices can be encouraged, even without the stability of digital infrastructure. The digital infrastructure and the framework of the online lesson become the traditional mould, which seems to have been successfully disrupted by introducing young learners to reflective practice activities.

Play-Based Learning

Worldwide, multiple initiatives[1] have been launched to encourage the learning of mathematics at home through educational games. These games often did not require access to the internet. In fact, they usually were paper and pen-based and more focused on developing logical thinking while engaging learners in a given problem, a problem that could be presented in the form of song, arts (An & Tillman, 2014) or a vignette (Skilling & Stylianides, 2020). Similar to what has been said about reflective practice, play-based learning seems to present itself as a distance learning channel that does not necessitate the presence of a solid digital infrastructure.

Project-Based Learning

Through the work of Enactus[2] and other organisations involving the Higher Education Departments in societal debates, university students have been engaged in activities that enable them to design home-based learning projects, which, in turn, can benefit school-aged learners. The mark of these projects was that these were projects where everyday home utensils have been used in a creatively disruptive way to build new projects that teach students, for example, about urban design and planning. Students have learnt to sketch and design their city and have been

[1] https://www.legofoundation.com/en/

[2] https://enactus.org/

challenged to re-envision the city in a virtual world, thereby only including the buildings that are not replaceable by home offices. Home activities such as these do not necessitate the existence of digital infrastructure and are an innovative means of learning beyond the hurdles of digital inclusion.

Cross-Generational Exchange

Negotiations with caregivers and educators have revealed that, throughout the pandemic, the traditional structure of same-year group learning has been exchanged with the more natural structure of cross-generational exchange (Parr et al., 2020). Learners were no longer instituted in a system geared to learn in a certain way and within a specific ecosystem of peer-to-peer and peer-to-teacher power dynamics (Makramalla & El Deghaidi, 2020). Instead, learners could resort to more natural means of knowledge exchange. For example, learners would engage with their grandparents in story-based discussions to learn about history. To learn about mathematics, learners would engage with their siblings in real-life application situations, where they had to practically adopt what they would have otherwise theoretically practised in school. This kind of exchange was only minimally available in the usual schooling routine and turned out to be more natural, more engaging and more practically appealing to young learners. Similar to the reported types of home-based learning, this too is a type of knowledge exchange based in and owned by the community, and that does not require a solid online infrastructure.

Essence-Based Thinking

Caregivers often had to cater to several learners while toggling with more than one job to sustain the family in these uncertain times. Caregivers also often had to attend to caring duties as they looked after other members of the family, whom the social services could not support due to the shift in priorities during the high peaks of the pandemic outbreak. This toggling between roles, as discussed above, necessitated a minimalist operation. Educators in the community had numerous discussions about the essence of learners' journeys that could not be compromised. These conversations were of the qualitative sort that has not taken place during the times of regular schooling as we know it. As one caregiver puts it:

> *The real question is whether our kids will be missing on much, now that most of the curriculum has been cut out and the emphasis is only on the minimalist portfolio. To be honest, I don't think that the children that have learnt in the pandemic will be missing much in comparison to their counterparts [that were studying in the same year before the pandemic]. The only difference is that with these kids [studying during the pandemic] we have focused on what really matters, thus giving them more time to be part of normal life, which in my opinion counts as learning too. I wish this minimalist operation would have been adopted from the beginning.*

Qualitative negotiations about the essence of learning and providing a minimalist portfolio of the core learning values that must be formally studied alongside the regular participation in cross-generational debates and other life-long learning initiatives happened due to the missing online infrastructure. In my opinion, an added value was establishing a support network between caregivers and educators. It works well for the respective context of teaching and learning, a context which is quite remote and isolated.

Interdisciplinary Learning

A final note about the affordances of teaching and learning that resulted from the engagement of the community in the educational journey of learners during the pandemic has to do with the promotion of interdisciplinary learning approaches (Jones, 2010). More than ever, caregivers have experienced first-hand how learning can happen beyond the regular multidisciplinary approach (Chaudhry & Higgins, 2003) that was part of traditional schooling. "This natural exposure to interdisciplinary learning will make it easier to convince parents about the usefulness of interdisciplinary learning as a teaching and learning methodology, even when we are back to normal school operation", one of the teachers. In the years that preceded the pandemic, the Ministry of Education in Egypt had launched an educational reform that aimed to shift the teaching and learning framework away from the regular multidisciplinary approach that caregivers were familiar with as part of traditional schooling. Being foreign to them, this approach was highly resisted by caregivers. The pandemic allowed for a first-hand educator and learner community experience of interdisciplinary learning in a very natural way, which, in turn, allowed for a higher conceptual buy-in of interdisciplinary teaching and learning approaches.

Distance Learning: Some Contextual Limitations

Even though the mechanisms above have been channelled to support a conducive learning environment, it is still important to note how marginalised communities have repeatedly felt isolated due to their restricted access to a solid network infrastructure during the peaks of the spread of the pandemic. Teachers reported having no means to communicate with their students whatsoever, and young learners have repeatedly expressed feeling isolated, even from contacting their peers. Below, I present four main limitations that need to be discussed as I advocate for the marginalised communities in a COVID era in Africa.

Questions of Digital Inclusion

The question of digital inclusion needs to be discussed in-depth, especially in the case of Egypt. A wide range of regions has been strongly digitally excluded simply because a solid digital infrastructure was not a given reality at the time of the pandemic outbreak. Despite the attempts to disruptively find other means to educate their learners, the need for communication remains the same. This is a question worth advocating for, even beyond the case of Egypt.

In light of the relational community engagement framework, it is interesting to note how communities have taken ownership of their problem, still resorting to different means of communicating with each other. One of these means has been through re-envisioning the role of the television and radio stations. "As a means of broadcasting, television and radio stations can, arguably, also be considered vehicles for distance learning", declared one policy maker. Young learners that do not have stable access to the internet can always learn from a video that has been provided on the television or a lesson that has been broadcast over the radio. To that end, Discovery Education[3] have been intensely collaborating with the Ministry of Education to produce engaging educational content that maps against the curricular objectives of each year and that can be broadcast using television stations.

Questions of Access

Beyond questions of inclusion, it is also important to consider access questions. Many remote locations that had access to somewhat stable network infrastructure might not have been able to provide learners with digital devices that can solely be used for teaching and learning purposes. Many learners reported having to share digital devices amongst siblings or not having access to a reliable device to take an online lesson and, hence, having to resort to using a smartphone instead. The same concern was reported by educators who, in turn, have also often had minimal access to a reliable and stable device, wherefrom they could prepare and teach the mathematics curriculum of a given year group.

To that end, multiple initiatives have targeted the smart scheduling of home device use so that the same device could toggle between different users in the same household. Other initiatives have focused on recycling unused devices to ensure equitable access to digital devices. Unused devices were often lent or donated as a contribution to limiting the scarcity of available devices. Additionally, used devices were revamped to be fit for purpose and redistributed equitably amongst learners with limited access to digital devices.

Overall, it remained a reality that learners did not all have access to a reliable digital device to ease their daily learning encounters. Similarly, educators and

[3] https://en.discoveryeducation.ekb.eg/

caregivers struggled to find ways to have exclusive access to a digital device. In the next section, in light of the theoretical framework, I discuss the impact of this scarcity on redefining the question of learning beyond formal schooling.

Questions of Digital Literacy

While government initiatives to eradicate illiteracy (Kamaly, 2017) have been successful in the past years, it remains a reality that the notion of digital literacy has come to light more clearly during the COVID era in many parts of the world. Ongoing initiatives target the eradication of digital illiteracy, yet there is still a long way to go. The presence of a solid digital infrastructure is not the only facilitator of an online distance learning platform. The know-how that relates to this infrastructure is of vital importance.

Buckingham (2013) draws on a four-fold framework for media education as he defined digital literacy for educators. In his work, he goes beyond the technical definition of digital literacy, which is traditionally related to the skill of effectively operating essential computer software. He argued for the importance of educators to be aware of the online gaming industry and be familiar with basic concepts of web literacy and social media literacy to be fully submerged in the digital reality familiar to learners. Repeatedly, Egyptian mathematics teachers have expressed their incapacity to handle online platforms. Many of them also said they lack ease in teaching online, not knowing who their real audience is and how well-aligned they are with cyber security protocols.

Hence, there is a need to widen our understanding of distance learning beyond the framework of a virtual classroom. We need to see the teacher as part of this virtual community, to accommodate their needs as a learner to ensure a smooth learning experience for the learners, which, in turn, this educator is entrusted with.

Questions of Wellbeing

Finally, it is also important not to ignore the actual reality of the pandemic, which goes beyond the fact of lockdown restrictions, which included the distance learning arrangement for teaching and learning mathematics. While most Western education literature seemed to have focused on lockdown in discussing the educational repercussions of COVID, I choose to shed light on voices from Africa, where challenges to staying at home often strongly affected student mental health and wellbeing. The school as a community empowerment hub acts as a space where children can be safe, well-fed and well-nurtured. These realities are not transferable to the virtual space, no matter how technologically capable it is.

Conclusion

The Typology of Community Engagement Concerning Distance Learning

Having discussed the affordances and limitations caused by the pandemic, let us now reconsider the typologies of engagement depicted in Fig. 18.3 as we map the different levels of community engagement.

First, for the scope of this work, let us define the community as the direct stakeholders of the teaching and learning enterprise, namely the learner, the educator, and the caregiver. I choose to focus my argumentation on how the degree of community engagement resulted in contextual solutions that were disruptive and productive to the broader scholarly audience. Figure 18.4 maps out the different affordances and limitations.

Figure 18.4 maps the different trends outlined earlier as affordances against the typology of community engagement. It shows how locally, through productive dialogue and exchange, various initiatives have been sought to facilitate distance learning and redefine it as existing beyond an online learning platform. As a community, educators and caregivers engaged in productive conversations about the actual essence of learners' education journey. As a result, a minimalist portfolio was put in place that often was practised through practical local means, such as home-based projects and games or cross-generational exchange. This means of educating learners also exposed caregivers to interdisciplinary learning, enabling a smoother buy-in to a conceptual shift away from traditional multidisciplinary instruction.

In addition to these disruptive community-owned solutions that did not require an online presence to facilitate learning, policymakers also discussed alternative

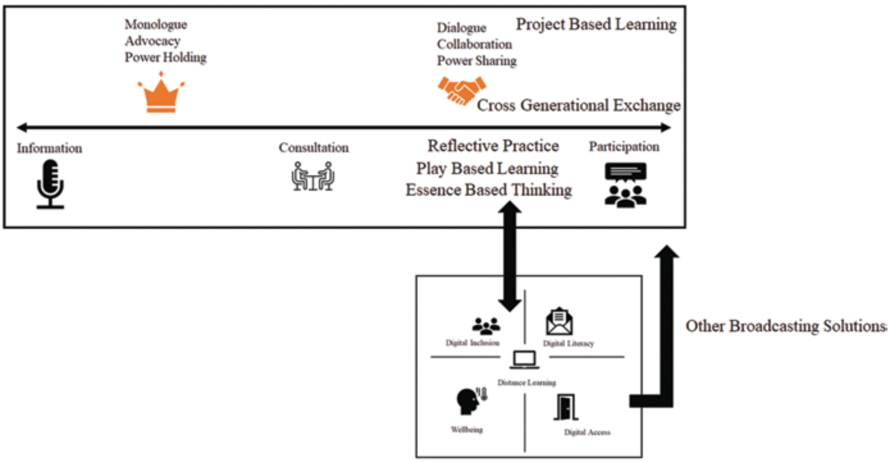

Fig. 18.4 A participatory approach to overcoming digital exclusion and digital literacy

broadcasting means – through radio and television stations- to enable learners to access more traditional means of knowledge dissemination. All of this occurred in quite turbulent times, where learners'' wellbeing was often at stake due to unhealthy domestic environments.

Implications

The Unfolding of the Relational Model for Community Engagement

In conclusion, I argue the need to re-envision the notion of learning and involve relational power structures in the community, thereby making possible the utilisation of local outreach assets as tools to transfer the agreed-upon community vision of a distance learning schooling space. Building on my presented argumentations, the implication of this study is a re-definition of the term 'distance learning'. Distance learning has proven to exist outside of the limited scope of a synchronous online classroom. Distance learning can hence be coined as: 'learning that happens at a distance from the traditional classroom'. Distance learning happens in cross-generational exchange in homes. It happens during play-based activities and as a result of reflective practice. It needs to be viewed from this broader lens, and it needs to be defined by the community of teaching and learning stakeholders.

I argue for the importance of local ownership of the redefined notion of learning and the importance of engaging local leadership as change agents in the local cultures. Locals become ambassadors of their stationed teaching and learning a reality. When empowered correctly, they discuss philosophical notions of impact and essence, where a contextualised local framework can emerge, a framework that stems from within the society and empowers teachers and educators alike.

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Chapter 19
How Current Literature on the Fourth Industrial Revolution Can Be Used to Enhance Learners' Self-Regulated Skills Required to Solve Circle Geometry Problems

Puleng Motseki

Introduction

The aim of the Technical and Vocational Education and Training (TVET) colleges across South Africa is to reduce the skills shortages of artisans, auto mechanics and electricians, among others (Department of Higher Education and Training (DHET), 2013). Basic knowledge of mathematics is a necessity, and Euclidean geometry is a part of that knowledge because these students are required to measure, to estimate, and to solve problems. Critical thinking skills and autonomy are important in mathematics education. However, students enter the TVET colleges with limited knowledge of Euclidean geometry, autonomy, and critical-thinking skills. The researcher therefore explored students' levels of autonomy and problem-solving abilities in terms of the following research question:

Research Question

How can self-directed learning skills enhance NC(V) level 4 students' ability to solve circle geometry problems and to what extent can the theories of modern educational technologies advanced by 4IR enhance those skills?

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Self-Regulated Learning in Mathematics Education

Scholars in mathematics education (De Corte et al., 2011; National Research Council, 2001) have argued that to become adaptively competent in mathematics, students must acquire a mathematical disposition. Self-regulation is one of many mathematical dispositions. Pintrich (2000) defined self-regulated learning as "an active, constructive process whereby learners set goals for their learning and then attempt to monitor, regulate, and control their cognition, motivation, and behaviour, guided and constrained by their goals and the contextual features in the environment" (p. 453). In line with the socio-constructivist approach to mathematics learning, Winne (1995) defined self-regulated learning as an inherently constructive and self-directed process that is characterised by coherent regulation of cognition, motivation, and affect (Boekaerts, 1999; De Corte, 2000; Pintrich et al., 2000).

According to De Corte et al. (2011), advancing and mastering the self-directedness process is central to the following five classifications of cognitive, affective and conative constituents:

(a) A mathematical knowledge base that includes facts, symbols, algorithms, concepts and rules that form part of mathematics as subject matter
(b) Heuristic processes that involve searching for problem-solving strategies that do not guarantee, but increase the chances of arriving at a correct solution
(c) Meta-knowledge, which is knowledge about individuals' cognitive potential that can be nurtured through learning and effort, and knowledge about individuals' motivation and emotions (for instance, being aware of one's fear of failure when engaging in complex mathematical activity)
(d) Self-regulatory skills, which include skills that relate to self-regulation of individuals' cognitive skills (for example, planning and monitoring individuals' problem-solving processes and skills that regulate individuals' motivation and emotions) and
(e) Positive mathematical affects that involve positive attitudes and emotions towards epistemic beliefs about mathematics, the self as a mathematics learner (motivational beliefs), and the social context of the mathematics classroom.

These views suggest that self-regulation is a multifaceted and comprehensive framework that relates to how students become active participants in their learning processes. In this context, students are seen as self-regulators – able to use different problem-solving strategies to conceptualise given content (Marcou & Phillippou, 2005), and to majke decisions on when, why and how to use the strategies in relevant contexts (Zimmerman, 2005).

In the context of learning mathematics, where students deal with complicated and challenging problems, their ability to deal with self-regulated strategies is seen to be an important predictor of their problem-solving performance (Howard et al., 2000). The reason is that self-regulated learning increases students' autonomy and personal agency over their problem-solving experiences (Zimmerman, 2001).

Victor (cited in Özsyoy & Ataman, 2009) argued that unsuccessful problem solving generally results from failure to: organise mathematical operations; choose the most appropriate strategy from a list of possible problem-solving strategies; analyse and understand the point of the problem; and monitor and control the problem-solving operations that need to be carried out. Additionally, research (Schoenfeld, 1985; Lester, 1994; Desoete et al., 2001) has revealed that students with high levels of metacognitive skills are more controlled; that is, they try to break down complex problems into simpler parts and ask themselves questions to clarify their thoughts. When one encounters failure during problem solving, control skills (metacognition) become helpful for applying problem-solving strategies successfully (Schoenfeld, 1985).

Metacognition plays a central role at each level of mathematical problem-solving (Goos et al., 2000). An inability to acquire control skills results in failure in mathematical thinking and problem solving (Özsyoy & Ataman, 2009). The problem-solving process involves the analysis of given information about the problem, organisations of the information possessed, preparation of the action plan and evaluation of all the problem-solving operations needed to be carried out (Goos et al., 2000). This problem-solving process requires students to arrange each level and step, and to make decisions simultaneously. These are the skills that constitute the nature of metacognition (Yimer, 2004).

Research suggests that metacognition is a necessary skill for becoming a successful mathematical problem solver. Montague (1992) outlined three commonly used metacognitive skills used in problem solving: self-instruction, self-questioning, and self-monitoring. According to him, self-instruction assists students in identifying and managing previously applied problem-solving strategies while working on a problem; through introducing internal dialogues, self-questioning enables students to systematically analyse the information given about the problem and to manage the relevant cognitive skills; while self-monitoring allows students to monitor their own performance during the problem-solving process.

Methodology

Eighty NC(V) level 4 students registered for a mathematics course at a TVET College in Gauteng Province participated in this study. Purposive sampling was used to select grade 12 learners studying towards the NC(V) certificate who had passed grade 12 mathematics, which is a prerequisite for this qualification. The Geometry Performance Test and focus-group interview questions required respondents to be skilled in problem solving and self-regulation (Darr & Fisher, 2005). According to Darr and Fisher (2005), a skilled problem solver is an active, self-regulated problem solver with the ability to analyse questions, choose from a range of problem-solving strategies, monitor his or her own progress, and reflect on the solution.

Research Instruments

Test items in the geometry performance test were adapted from a national assessment paper (DHET, Mathematics Paper 2, 2014). It was developed according to the assessment guidelines stated in the mathematics policy documents for the South African Higher Education National Certificate (Vocational) Assessment Guidelines (2013). The taxonomy levels were therefore considered to have been adhered to.

Two main data-collection instruments were used: a Geometry Performance Test and focus group interviews. Data was also extracted from a document analysis of articles (Ngwabe & Felix, 2020; Shadaan & Leong, 2013; Nisiyatussani et al., 2018) that explored the use of GeoGebra mathematical software in teaching geometry. Problem solving is one area in mathematics where the application of self-regulatory skills is evident (Zimmerman, 2001). The researcher developed the test items. In making sense of problem-solving contexts, self-regulated students apply a fully self-regulated problem-solving approach such as analysis, planning, exploring and reflection (Darr & Fisher, 2005). The learners were tested on their capabilities and the skills required to reason logically and prove theorems using a deductive approach. Further, the problems required respondents to recognise and strategically co-ordinate the features that make up proportional relationships, because self-regulated problem solving is highly associated with proportional reasoning (Darr & Fisher, 2005).

Euclidean geometry was chosen as the component of proportional reasoning to focus on in this study. When preparing the test items, the researcher took into consideration the cognitive demands of each test item, to ensure that low, moderate and high levels of complexity were represented. The cognitive complexity level of an item corresponds to the depth of knowledge required to understand it, rather than to participants' abilities (Webb, 1999). For instance, a test item with low complexity level may require students to recall previously learned knowledge; it may be a one-step problem (DBE, 2011; DHET, 2013). A question of moderate complexity level requires critical thinking, or a choice to be made from alternatives, which is not the case with low-complexity-level questions. Students are required to apply reasoning and problem-solving strategies, and to integrate previously learned knowledge (DBE, 2011; DHET, 2013). Questions with high complexity level require abstract reasoning, planning, analysis and judgement, and such skills are pivotal in self-regulatory engagement (Fadlelmula et al., 2015). Complex problem solving involves solving non-routine problems or having multiple steps or decision points (Fadlelmula et al., 2015; DBE, 2011; DHET, 2013). For this study, cognitive complexity was attained by choosing the following: two items of low complexity, two of moderate complexity, and five of high complexity, based on the classifications of Bloom's taxonomy.

The second data collection instrument was focus-group interviews, which pursued further enlightenment based on the research question. Questions for the interviews were developed in line with the geometry test. Focus-group interviews took place 2 days after the test was written. Nine participants took part in the interviews, chosen based on their performance (three high achievers, three average achievers and three poor achievers).

Items 19.1, 19.3, 19.6 and 19.9 focused on mastery goal orientation, centred on understanding the learning material as the main goal of learning (De Corte et al., 2011). These items included questions that required the respondent to display knowledge of algorithms, symbols, and rules as part of the subject matter. Items 19.4, 19.5 and 19.10 were central to the area of students looking at their own errors and misconceptions as a way towards self-regulated problem-solving. Students reflecting on their individual errors and misconceptions may be helpful to their self-awareness and self-regulation (Fadlelmula et al., 2015). Item 19.2 evaluated participants' performance expectations and judgements about their capabilities in accomplishing an assessment task (Fadlelmula et al. 2015). Items 19.5, 19.7 and 19.8 assessed participants' knowledge of problem-solving strategies; that is, making sense of a problem situation, analysis, planning, exploring, and reflecting (ability to identify and translate certain keywords in problems into equivalent mathematical sentences) (Pape et al., 2003).

A pilot study was undertaken with a group of NC(V) level 4 mathematics students, to ensure the consistency and stability of results obtained when administering the Geometry Performance Test and focus group interviews.

Confidentiality and voluntary participation was guaranteed. Consent was obtained before commencement of the study.

Data Analysis and Discussion

Qualitative data analysis using open coding was used (Corbin & Strauss, 1990). The focus was on creating a new theory resulting from the data (Creswell, 2013). It is important to note that respondents had all completed their grade 12 mathematics qualification with circle geometry as part of their final assessment. The researcher sought to understand their problem-solving strategies; that is, their ability to communicate their understanding of circle geometry, their justification on proof reasoning, and how their problem-solving was influenced by their self-conception.

The discussion hinges on four geometry questions that learners were required to answers. Learners' work is represented by the letter P for participant. In Question 19.1, O is given as the centre of the circle and passes through A, B and C, $C\hat{A}B = 48°$, $C\hat{O}B = x$.

Question 19.1
The cognitive complexity level for this question required a recall of previously learned knowledge of the theorem which states that the angle at the centre is twice the angle at the circumference, and this is a one-step problem (DBE, 2011; DHET, 2013) (Fig. 19.1).

Question 19.1 Respondents were asked to determine the value of x, with reasons.

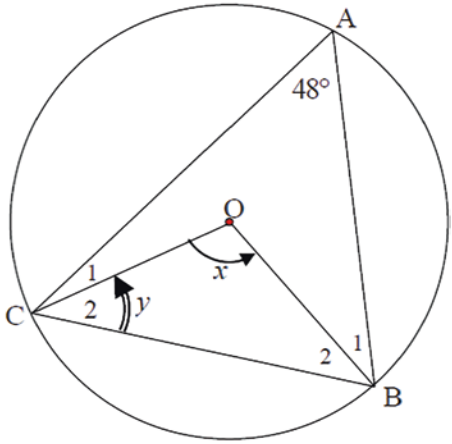


Fig. 19.1 Geometry Question 19.1

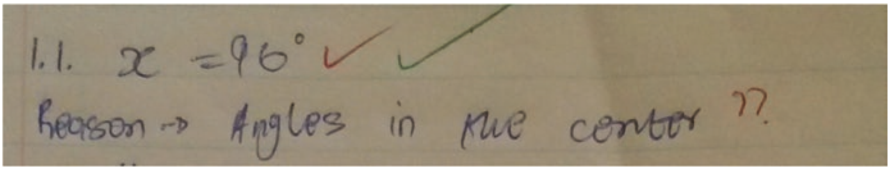


Fig. 19.2 P_17 Response to Question 19.1

Thirteen participants out of 80 gave the correct answer, which is that $x = 96°$, and the reason as 'angle at the centre', instead of 'an angle at the centre is twice the angle at the circumference'. The example below is a response provided by student P_17 (Fig. 19.2):

Several learners gave similar but wrong reasons for the answer. Participants might have understood the theorem but found it difficult to express themselves. Thirty respondents provided the correct answer $x = 96°$ without providing a reason. These learners may have failed to understand the relationship between the stated theorem and how they arrived at their answer. During the interviews, when participants were asked to explain the theorem that states 'the angle at the centre is twice the angle at the circumference', their responses were often incomplete. This suggests that respondents' thinking could not be clearly articulated unless they reflected on the problem and the problem-solving strategies that they used. As Fennema et al. (1999) argued, thinking cannot be articulated unless students reflect on the problem and on the strategies they use to solve it; articulation in turn increases reflection, which leads to understanding.

Question 19.1.2 required participants to determine the value of y and provide reasons. This questions was developed based on a moderate complexity level, requiring learners to display critical thinking skills (DBE, 2011; DHET, 2013). In answering this test item, participants were required to have gained problem-solving strategies and self-efficacy skills for problem solving. Participants were further required to apply previously learned knowledge on the concept that the sum of the angles of a triangle is 180°.

In order for learners to solve this problem they needed to apply self-regulation skills that adhered to:

(a) Analysis: analysing what is given, i.e. $A = 48°$and $(x = 96\,° \,, from 1.1 above)$

Planning: If $A = 48\,°$ and $x = 96°$, then explore the theorem that states that the sum of the angles of a triangle is equal to 180°.
Exploration: $y = \hat{B}_{2\,+}\, \hat{C}_2 = 180\,° \, - 96\,° \, = 84°; \therefore y = 42°$.

(b) Reflection: Review the problem-solving strategy used.

Thirty-one respondents provided the value of y as 78°, with the reason 'sum of a triangle'. 'sum of a triangle' which was an incomplete reason, and in this instance could not be accepted as correct. The example below is the response provided by respondent P_39 (Fig. 19.3):

Twenty-nine participants did not give an answer for *y* or a reason. This implies that respondents were not sure which problem-solving techniques to apply.

Understanding the problem is a key step to problem-solving. Marchis (2011) emphasised that students should read the question, identify the context of the question, rephrase the question in their own words, write down the known and the unknown information, draw diagrams or figures to help themselves better understand the question, and see the relationship between the known and the unknown information. The participants in this study may not have performed these actions successfully, judging by the incorrect responses given to the question. The responses to Question 19.1 also revealed that participants were unable to solve problems using the tan-chord theorem which is important prior knowledge for NC(V) level 4 learners.

Question 19.2
In Fig. 19.4 required participants to determine $\hat{F}_1$, as 19.2.1, with reasons.

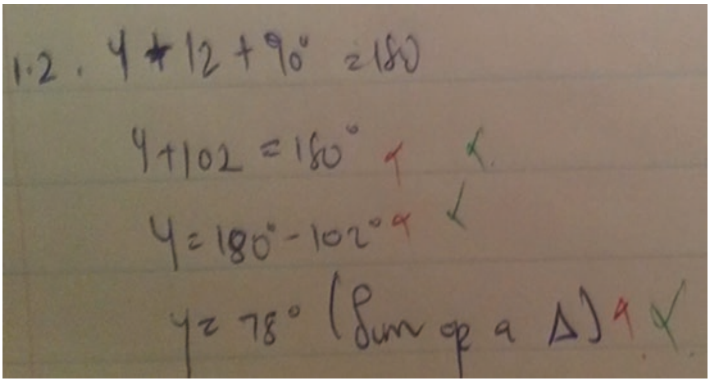


Fig. 19.3 P_39 response to Question 19.1.2

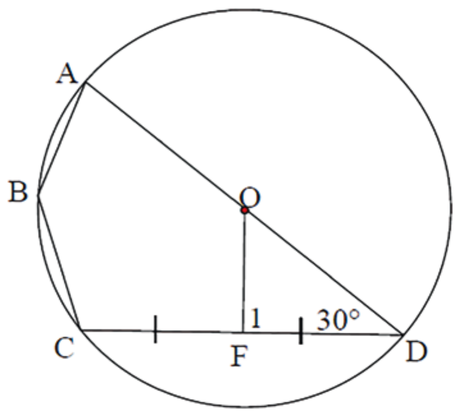
In the diagram, O is the centre of the circle that passes through A, B, C and D. AOD is the straight line and F is the midpoint of chord CD. $O\hat{D}F = 30°$. The cognitive complexity level for the question required critical thinking or choice among alternatives (DBE, 2011; DHET, 2013). In answering this question, participants were required to have gained problem-solving strategies and self-efficacy skills. They were also required to apply previously learned knowledge of the concepts of the midpoint chord theorem and the theorem on properties of a cyclic quadrilateral.

When answering Question 19.2.1 and determining $\hat{F}_1$, with reasons 'Self-regulatedness' could be shown in performing this tasks when respondents make sense of the question by adhering to following knowledge and skills:

(a) Analysis: Make sense of the question by recalling previously learned knowledge of the concepts of midpoint and chord.
(b) Planning: What is given: $O\hat{D}F = 30°$, *F is the midpoint of chord CD*, and *AOD is a straight line*
(c) Exploration: If F is the mid-point of CD, then OF is perpendicular to CD, and forms a right-angled triangle $O\hat{F}D$, then $\hat{F}_1 = 90°$

Responses to this Question 19.2.1 suggested that participants had learned and used an algorithm without understanding it purpose and use. Such lack of conceptual understanding reflects minimal mathematical thinking, problem-solving skills and reflective ability. Seventeen participants out of eighty gave the value of F_1 as *60°* and the reason as 'angles of triangle'. These 17 participants could not recognise that F_1 'looks' like *90°*. Their reasons might be attributed to weak visualization skills. Twenty-four participants gave the reason that $CF = FD$, and 19 gave the reason as 'an angle subtended by the diameter'. These responses were incorrect. The participants made unjustified statements, suggesting that they had confused the theorem with its converse, and guessed the reason as 'line from the centre to chord'. Learners could not deduce that line *OF* bisects *CD* in the middle and forms a right-angled triangle $O\hat{F}D$. As a result, $\hat{F}_1 = 90°$. Respondents could not explain the terms

Fig. 19.4 Diagram depicting Mid-Point theorem. (Source: DHET, 2014, p. 29)



chord, *perpendicular* or *midpoint chord* during the interview. Below is an example of one of the responses provided by a respondent.

Geometric concepts and ideas such as the introduction of lines, planes and spaces (CAPS, 2011) are presented to learners in early school grades. However, several geometric concepts and ideas in secondary school learning are presented in an abstract form, which is not as concrete as that used in primary learning (Luneta, 2015). Coordinate geometry requires fluency in working with coordinates on two-dimensional and three-dimensional sets of axes. The abstractness of geometric concepts may be one of the reasons students encounter learning difficulties when learning Euclidean geometry.

In addition, a study by Luneta (2015) reported that grade 12 learners are not able to solve Euclidean geometry problems that are of grade 7 standard; as a result, these grade 12 learners operate at Level 2 of the Van Hiele framework of geometrical understanding. Ngwabe and Felix (2020) asserted that the use of GeoGebra is one of the most effective ways to enable learners to understand and make Euclidian geometry meaningful. The introduction of GeoGebra into classrooms can be used to animate and visualise geometric concepts (Ngwabe & Felix, 2020). For instance, GeoGebra can 'bridge the gap' by allowing students to visualise and understand Euclidean geometry through exploration (Shadaan & Leong, 2013). As a result, GeoGebra can serve as an enabler for students who are experiencing learning difficulties in solving problems, since the students would not have to rely on the traditional approach to solving problems. The Fig. 19.5 show learners P 40 answer to Question 19.2.2 where participants were required to determine A$\hat{B}$C.

Self-regulated learning increases students' autonomy and personal agency over their problem-solving experiences (Zimmerman, 2001). However, 19 participants gave the value of $\hat{B}$ as 90° and the reason as 'angles in the semi-circle'. The value of $\hat{B}$ they gave is incorrect, and the reason is incomplete and cannot be accepted as correct. These answers suggest the participants were not aware of the configuration to which 'angle in the semi-circle' can be applied. During the focus group interviews, when respondents were asked to explain the properties of a cyclic quadrilateral, their responses were incomplete. This implies that participants may have experienced visualisation problems. Based on the responses, it can be concluded

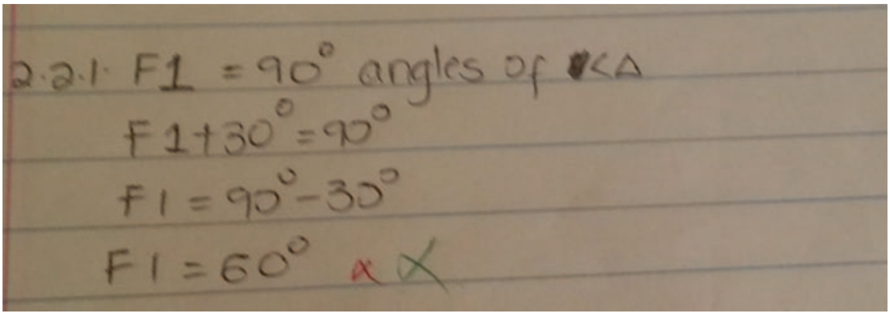


Fig. 19.5 P_40 response to Question 19.2.2

that respondents lacked both conceptual knowledge and procedural knowledge required to answer Question 19.2.2.

‘Self-regulatedness’ of the task would have required participants to make sense of the question and think of following:

(a) Planning: using what is given ($\hat{D} = 30°$); recall of previously learned knowledge of the properties of a cyclic quadrilateral
(b) Exploration: $30° + A\hat{B}C = 180°$ (opposite angles of a cyclic quadrilateral are supplementary); $180° – 30° = 150°$
(c) Reflection: $30° + A\hat{B}C = 180°$ (opposite angles of a cyclic quadrilateral are supplementary, therefore, $150° + 30° = 180°$

Figure 19.6 shows another answer by student P_45 to Question 19.2.2; it further suggests that respondents lacked self-regulatory problem-solving skills.

Thirteen participants did not answer the question. During the interview, when asked the questions ‘Explain the difficulties encountered in Question 2’, and ‘Explain the error(s) in 19.2.2’, participants provided responses such as “*I guessed the answers*; *hence I did not get any correct answer for the question.*” This suggests that most of the students were not aware of the importance of reflecting on difficulties that might be experienced during problem-solving. ‘Self-efficacy’ relates to students’ evaluation of their capabilities required to successfully complete a task, as well as their confidence in their skills for performing that task (Pintrich et al., 1993). Questions associated with such self-regulatory skills revealed that participants were not aware of the importance of checking their problem-solving strategies and the correctness of their solutions.

The responses to Question 19.2 indicate that participants lacked knowledge of problem-solving strategies, as well as knowledge of the concepts involved on proportionality theorem and the similarity of triangles. Questions 19.3 was an example of high order questions and was divided into five-sub questions.

Question 19.3

Question 19.3, a high-complexity-level question, required abstract reasoning, planning, analysis and judgement (Fadlelmula et al., 2015). Participants were required to analyse the task – that is, to understand the question; identify the given information, the unknown data and the relationship between the given information and the unknown; and recall prior knowledge related to the question. Eventually learners were required to solve the question – that is, to select, apply and evaluate plans and strategies; check the results, and apply problem-solving strategies and results; and revise and abandon unproductive plans and strategies (Marchis, 2011) (Fig. 19.7).

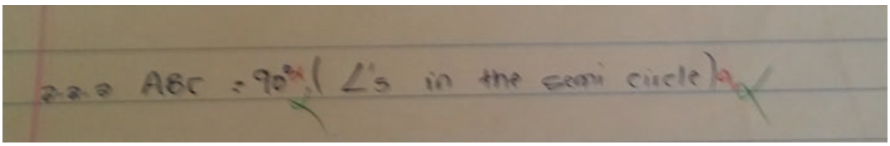


Fig. 19.6 P_45 response to Question 19.2.2

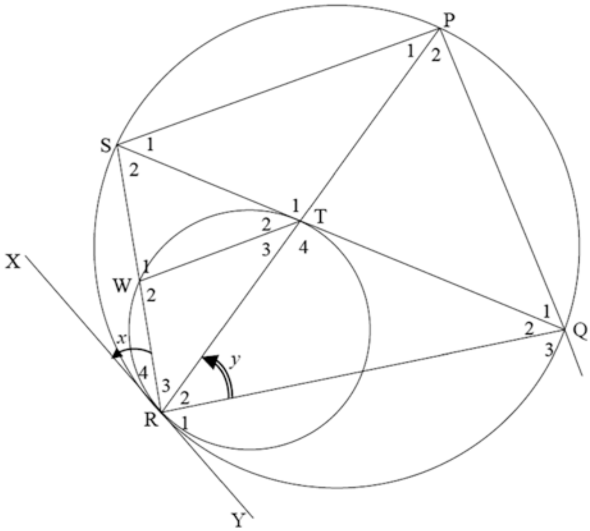


Fig. 19.7 An example of a high-order question

In order to answer the question, participants were required to have learned various problem-solving strategies and self-efficacy skills for problem-solving. According to Marchis (2011), these skills are important for self-regulated learning and successful mathematics problem solving. Questions were structured in the following way:

Question 19.3.1 Provide reasons for the following statements:

Statement	Reason	
$\hat{T}_3 = x$	_______________	(1)
$\hat{P}_1 = x$	_______________	(1)
WT ‖ SP	_______________	(1)
$\hat{S}_1 = y$	_______________	(1)
$\hat{T}_2 = y$	_______________	(1)

Thirty-five participants provided responses such as 'corresponding angle with $\hat{T}_3$'. The example below is the response given by learners P_57 (Fig. 19.8).

Fifty-five participants answered with irrelevant reasons such as 'corresponding angles with $\hat{T}_3$'. Such irrelevance to the questions meant that their answers incorrect. Participants' inability to recognise that T_3 and P_1 are equal corresponding angles might have been because of weak visualisation skills of geometry, because these angles are in a non-stereotypical position.

Thirty respondents provided the reason 'corresponding angles', rather than 'corresponding angles are equal'. During the interviews, their explanations of the properties of the tan-chord theorem were either incomplete or completely incorrect. Most of the learners did not recognise that $\hat{S}_1$ and y are subtended by chord *PQ*.

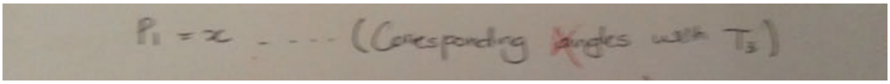


Fig. 19.8 P_57 response to Question 19.3.1

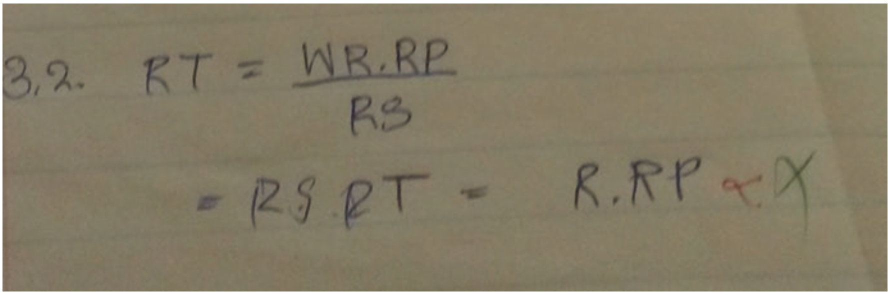


Fig. 19.9 P_47 response to Question 19.3.2

Question 19.3.2 Prove that RT = $\dfrac{WR.RP}{RS}$

Thirty-four participants were unable to reach the correct conclusion. Figure 19.9 is an example of one of the many responses to Question 19.3.2. It was provided by learner P_47:

Thirty-five respondents did not provide an answer. These respondents might not have known the steps to use to arrive at a solution and may have lacked conceptual knowledge of the proportionality theorem. In solving Question 19.3.2, participants were expected to apply different problem-solving strategies and previously learned knowledge of the similarity of triangles. According to Fadlelmula et al. (2015), self-regulated problem-solvers use more abstract reasoning, planning, analysis and judgement. For this question, participants were meant to analyse the two triangles WTR and SPR and to recognise that W_2 = S2, the reason being that corresponding angles are equal (WT // SP); or to explore the proportionality theorem ($\dfrac{RW}{RS} = \dfrac{RT}{RP}$. Reason: *Line parallel to one side of the triangle; proportionality theorem; WT II SP)*

Another way of solving the problem would involve recall of previous knowledge of the concept of congruency of triangles:

$$RTW\ III\ RPS\,(angle; angle; angle)$$

$$\frac{RW}{RS} = \frac{RT}{RP}\,(Triangle\ RTW\ \mathrm{III}\ Triangle\ RPS)$$

$$Therefore\ \ RT = \frac{RW.RP}{RS}$$

In answering Question 19.3.2, participants might have lacked the necessary problem-solving strategies and belief in their self-efficacy. Their responses are characterised by procedural knowledge without proper understanding of the underlying concepts of congruency, proportionality and parallel lines. Studies (Frank, 1988; Lampert, 1990; Schoenfeld, 1991; Schommer-Aikins, 2008) have shown that some students – in various grades, and at various levels of schooling – believe that mathematical knowledge is characterised by certain absolute facts, rules, and procedures that are approved and transmitted by authorities (teachers and textbooks) and are therefore split into 'wrong' and 'right' answers.

Darr and Fisher (2005) argued that problem-solving is one area of mathematics where application of self-regulatory skills should be more visible. Self-regulated students analyse the task by trying to understand the problem; to identify given information, the unknown, and the relationships between the given and the unknown; to recall prior knowledge about the problem; to apply heuristic problem approaches in trying to solve the problem; to check the outcomes; and to revise and abandon unproductive strategies (Marchis, 2011). Participant achievement in this question revealed a lack of the self-regulatory ability to apply different problem-solving strategies, and a lack of self-efficacy skills in problem solving.

Question 19.3.3 Determine, with reasons, the other two angles that are equal to *y*.

Thirty-seven participants provided incorrect solutions to this question. The example below is the response from P_11, and is representative of some of the incorrect solutions (Fig. 19.10):

For this question, the respondents might have read the question but could not make sense of what was required of them. Twenty-eight participants did not attempt to answer the question at all. Participants were expected to break down the complex diagram, and: identify that $S_1 = y$ (reason: angles subtended by the same chord are equal in a bigger circle); identify the parallel lines SP and WT; and from there, recognise that $S_1 = T_2$ (reason: alternating angles, WT//SP). The responses to this question imply that participants lacked the skills necessary to break down the complex diagram into its constituents.

Question 19.3.4 Prove that $\hat{Q}_3 = \hat{W}_2$.

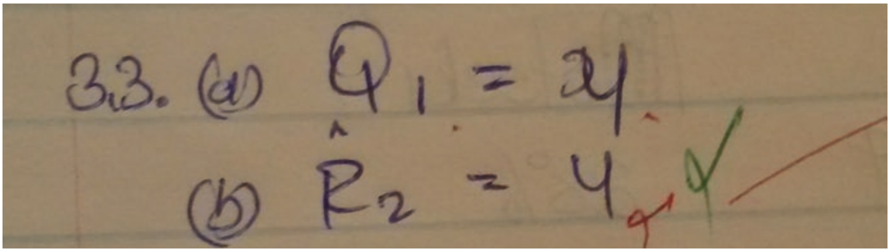


Fig. 19.10 P_11 response to Question 19.3.3

Thirty-nine participants were not able to establish relationships between angles, with some only stating that $\hat{Q}_3 = \hat{W}_2$ because 'the exterior angle of a cyclic quadrilateral is equal to the interior opposite angle'. Fig. 19.11 shows the response provided by learner P_48 which was common among many participants.

Eight participants did not answer the question. To answer this question, participants were expected to name and describe their problem-solving strategies and use various representations (Marchis, 2011). For instance, there were two ways in which this question could be solved:

$\hat{Q}_3 = P\hat{S}R$ *(ext angle of a cyclic quad)*
$P\hat{S}R = \hat{W}_2$ *(Corresponding angles; WTII SP)*

OR

$\hat{Q}_2 = x$ *(angles in the same segment)*
$\hat{Q}_3 = 180° - (x + y)$ *(angles on a straight line)*
$\hat{W}_2 = 180° - (x + y)$ *(angles of a triangle WRT)*
Therefore $\hat{Q}_3 = \hat{W}_2$

However, the responses given to this question suggest that participants were unable to recall previous knowledge related to the question; to solve the problem (select, apply and evaluate problem-solving strategies); or to check their outcomes and results (Marchis, 2011).

Question 19.3.5 Prove that $\frac{WR}{RQ} = \frac{RS^2}{RP^2}$.

Thirty-two participants did not provide a solution to the problem. Thirteen provided the answer with 'as proved above' as the reason for claiming a pair of angles to be equal, although they had not been proved equal anywhere. None of these answers were correct. Participants claimed that *R* is common, seemingly unaware that the two angles in question are actually in two different triangles and are thus not common.

Attempts to answer most of Question 3 showed that most of the participants were challenged by questions that required proof. For instance, students who were unable to link Questions 19.3.1, 19.3.3 and 19.3.4 found it difficult to answer Question 19.3.5. Makhubele (2014) argued that geometric shapes are class inclusions of shapes on their own; identifying a shape means identifying the type of a shape and its properties, while class inclusion means that participants should be able to sort out and classify different shapes based on their appearance and properties. This

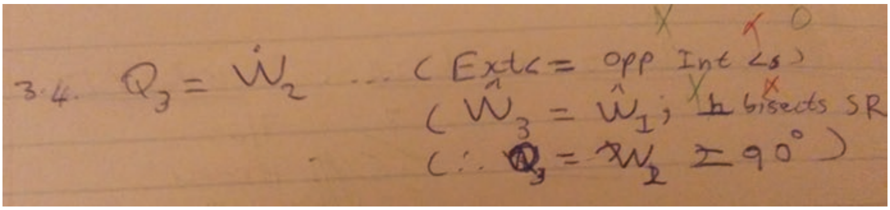


Fig. 19.11 P_48 response to Question 19.3.4

suggests that in a complex diagram, participants should be able to identify various shapes and their corresponding properties. Students who are unable to identify the different shapes in a complex diagram experience challenges with problem solving.

In response to these questions many participants failed to link logically different properties in a complex diagram. Focus-group interview questions such as “Point out the errors you made in solving Questions 19.3.2. and 19.3.4. What alternative approaches could be used to solve Questions 19.3.2 and 19.3.4?” and “Describe the strategy(ies) you applied when solving Question 3” prompted answers that indicated a lack of problem-solving strategies. For instance, P_18 gave the responses “I cannot think of any other approach. I answered the question incorrectly” and “I did not attempt to answer the question, but there is no other approach to the question”; these reveal a lack of problem-solving strategies.

The low achievement level of Question 19.3 can be explained by the fact that traditional mathematics learning is still widely practiced in most South African classrooms; the teacher presents a method, and students engage in drills and practice of the provided method (Luneta, 2015). This method of learning mathematics does not promote the development of mathematical thinking, problem-solving skills, or self-regulated learning (Marchis, 2011). When solving mathematical problems, “control has to do with the decisions and actions undertaken in analysing and exploring problem conditions, planning courses of action, selecting and organising problem-solving strategies, revising and abandoning unproductive plans and strategies, and reflecting upon all the decisions made and the actions taken during the course of working on a problem” (Lester et al., 1989, p. 4).

The answers given for Question 19.3 also suggest that participants were unable to break down the complex diagram shown into its simpler parts, meaning an inability to establish the relationships between Questions 19.3.1, 19.3.3 and 19.3.4. Further analysis indicates that participants were unable to identify different shapes and their properties within one complex diagram: and as a result, experienced challenges when problem solving.

The Integration of 4IR to Enhance Self-Directed Learning

The researcher in engaging with the literature on 4IR pointed out that incorporating technology in the mathematics instructional process is important in terms of enabling students to practice trial and error, make conjectures, test, and generalise based on mathematical instructional activities (Engelbrecht et al., 2020). Euclidean geometry is abstract in nature, and students are expected to apply high-level thinking skills (Celen, 2020). The use of GeoGebra and other 4IR technologies enables students to acquire high order thinking skills. Dynamic geometry programs include learning features such as the creation of geometric shapes, manipulating angles and edges of shapes, dragging of shapes, and dragging the quantities measured during the movement, and allowing the measurement of variables such as the length, area, and width (Ngwabe & Felix, 2020). Almost all Questions 19.1, 19.2, and 19.3 can be illustrated using various dynamic geometry programs in the classroom (Engelbrecht et al., 2020).

Kagiso and Nkhwalume (2019) assert that when dealing with geometric tasks such as the Questions 19.1, 19.2, and 19.3, digital technologies have generated new ways of instruction that support appropriate knowledge acquisition. The traditional instructional approaches for learning geometric constructions, using pen and paper – where students are required to internalise abstract geometric shapes, and grapple with them to connect shapes to theorems on which geometric concepts are based in order to acquire skills and knowledge – are no longer the only way to acquire knowledge necessary (Kagiso & Nkhwalume, 2019). "New technologies have raised expectations about their potential for innovating teaching and learning" (Mariotti, 2013, p. 441). This means that the integration of educational software such as GeoGebra into the learning of geometry enables students and teachers to maximally exploit its potential and capabilities for supporting classroom practices to overcome the teaching and learning challenges presented by traditional instructional approaches.

In addition, the National Council of Teachers of Mathematics (NCTM, 2000) prescribes technology as a key principle for enhancing the quality of mathematics: "Teachers should use technology to enhance their students' learning opportunities by selecting or creating mathematical tasks that take advantage of what technology can do efficiently and well – graphing, visualising, and computing" (NCTM, 2000, p. 10). In this era of rapidly evolving technology, it is important to keep up with the current innovations in technology-based pedagogical approaches to meet twenty-first-century learning needs.

Integration of technology in learning supports the epistemological gains of constructing, creating, discovering, and negotiating, rather than being told or being given support (Kagiso & Nkhwalume, 2019). Proponents of constructive learning believe that individuals acquire knowledge through engagement with the world; as a result, the world offers individuals the desire to know, which motivates them to solve problems (Surif, 2002). Surif (2002) proposes three epistemological concepts from Bruner's theory of representation, namely:

- Enactive: learning through action and manipulation.
- Iconic: learning through formation of images and organisation of learning, seeing and kinetic perception.
- Symbolic: learning through words and symbols.

Through technology integration, the theory of representation triggers active participation (as opposed to rote learning) to enable clear understanding of concepts (Omar, 2009), which gives students the learning experiences required to discover underlying geometric concepts.

Implications and Recommendations of the Study

Theories on self-regulation stress the importance of developing understanding of the problem as one of the most important steps towards successful problem solving. The focus of the study was on the role played by students, who should be nurtured to do mathematics with the belief that they must be able to understand the mechanisms and will gradually play a more active role in assuming responsibility for their own learning.

The Geometry Performance Test items were intended to certify participants' self-regulation skills. But the responses to these questions revealed that participants were held back by maladaptive academic behaviours such as lack of problem-solving skills and lack of knowledge of concepts relating to circle geometry. The incorrect or non-existent responses to Question 19.3 also suggested that participants tended to give up early when faced with challenging questions.

On the other side of the classroom, mathematics educators should facilitate a stimulating learning environment and provide scaffolding learning activities that engage students' cognitive and metacognitive activities, promoting good problem-solving strategies and producing good answers from problem solving. This suggests that educators must encourage students to understand that problems can be solved in different ways, and that procedures to find solutions should not be applied mechanically; meaning that the correct solution is not enough, if the student does not understand what lies behind the problem. Mathematics educators should present learning activities that allow multiple representations, numerous paths towards a solution, and problems that require realistic mathematical modelling through the application of real-life contexts.

Another area of interest is the role in problem solving played by self-efficacy beliefs. Self-efficacy is defined as students' convictions about their ability to complete mathematical tasks successfully. Fadlelmula et al. (2011) argued that students reflecting on their own learning has a desirable influence on building improved mathematical understanding. This implies that educators should encourage students to explain the challenges they have experienced in the process of unsuccessful problem solving and evaluate the correctness of their solutions. Mathematics educators must enable collaborative learning, where students' self-regulation competencies will be developed and provide them with opportunities to verbalise their thinking in explaining their reasoning.

Regarding unsuccessful problem solving, certain steps can be taken. For instance, Zimmerman (cited in Darr & Fisher, 2005) argued that self-regulation takes place when students "become masters of their own learning processes" (p. 1). Self-regulated students are active participants in maximising their abilities and opportunities to learn. For instance, asking students to explain how they solve a problem is important for developing their self-regulation skills. Questions such as 'What exactly are you doing?', 'Why are you doing it?', and 'How does it help you?' (Schoenfeld, 1992, p. 206) assist students to reflect on their problem-solving strategies and verbalise their reasoning. "When thinking is articulated regularly, patterns of thinking develop that are iterative. Thinking cannot be articulated unless students reflect on the problem and the strategies, they use to solve it; articulation, in turn, increases reflection, which leads to understanding" (Fennema et al.1999).

Mathematics instruction, especially for Euclidean geometry, should not only be done through the traditional teaching and learning approach. Teaching and learning should cover a variety of instructional approaches, including the use of scaffolding tools to help stimulate student interest in mathematics (Engelbrecht et al., 2020). GeoGebra software can be used as an instructional aid in the teaching and learning of Euclidean geometry (Ngwabe & Felix, 2020). The recommendation to integrate GeoGebra in learning is informed by the beliefs that 4IR presents a range of skills that can help students face the new demands of an evolving education (Engelbrecht et al., 2020). The easy availability of GeoGebra, as free educational software, facilitates performing the task of communicating mathematical knowledge in a way that is beneficial to students.

Current literature on 4IR reports that students lack cognitive and reasoning skills in understanding Euclidean geometry concepts can best be supported by modern digital technologies that facilitate effective instruction and optimise learning (Ngwabe & Felix, 2020). Shadaan and Leong (2013) mentioned that students encounter difficulties in learning geometry; as a result, many are unable to understand geometry content and associated theorems. GeoGebra can facilitate the learning of Euclidean geometry by helping students to visualise and understand introductory theorems such as those required in Questions 19.1, 19.2, and 19.3 'an angle at the centre of the circle is twice the angle at the circumference' through exploration (Shadaan & Leong, 2013). One-way GeoGebra is used in learning mathematics is by helping students to draw geometric shapes accurately and quickly (Nisiyatussani et al., 2018). In addition, GeoGebra helps students to manipulate Euclidean geometry concepts visually (Syahbana, 2016). Technology can change geometry learning by avoiding mere 'talk and chalk'by providing at teachers' our disposal anymore but also advanced digital technologies that provide virtual images and mathematical content to enable the understanding of concepts (Putra, 2012).

Conclusion

The study focused on an analysis of the problem-solving strategies participants use when answering questions on circle geometry. To answer the questions, participants were required to have problem-solving skills as well as the self-efficacy skills used in problem solving. But the analysis showed that NC(V) level 4 students were unsuccessful at circle-geometry problem solving. Participants were unable to solve circle-geometry problems at the expected level suggested by their academic progress, meaning that their ability to self-regulate was weak. As a result, they found the questions daunting.

Lack of basic problem-solving strategies and self-efficacy should be looked at in detail before students engage in circle-geometry problem solving at NC(V) level 4.

Mathematics educators at TVET colleges should look at the important aspect of self-regulated learning, including nurturing awareness of how individuals' thinking can be used to provide problem-solving options and strategies.

Literature and also the COVID 19 pandemic has taught the teaching world that modern technologies cannot only enhance effective instruction but is the new world order and teachers and learners must be exposed to new educational technologies.

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Index

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