Some Results on the Dominance Relation Between Conjunctions and Disjunctions

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Abstract. The dominance relations on the class of aggregation operators have the vital application in various areas of science, including fuzzy set theory, probabilistic metric space. The dominance relations between conjunctions and disjunctions are studied in this paper. We characterize the conjunctions (disjunctions) which dominate all triangular conorms (triangular norms). Moreover, as a generalization of the dominance relation, the weak dominance relation between conjunctions and disjunctions is also discussed.

Keywords: Dominance · Conjunction · Disjunction · Triangular conorm · Triangular norm

1 Introduction

The concept of the dominance relation was introduced in [1]. Then, Schweizer and Sklar [2] discussed the dominance relation for the class of associative binary operations. The domination plays a very important role in constructing Cartesian products of probabilistic metric spaces. Moreover, the domination of t-norms is used in the construction of fuzzy orderings [4], fuzzy equivalence relations [5], the open question of the transitivity [6, 7], inflexible querying, game theory and preference modelling. These applications initiated the study of dominance relation in the broader context of aggregation functions [5, 8, 10, 14]. Especially, the domination between aggregation operations is also vital to the aggregation procedures preserving T-transitivity of fuzzy relations [3, 9].

Besides these applications above, dominance is still an interesting mathematical notion. Due to the fact that all t-norms have common neutral element and their associativity and commutativity, dominance of t-norms constitutes a reflexive and antisymmetric relation. Moreover, some classical inequalities and equations [11] are related with the dominance relation such as the Minkowski inequality and bisymmetry equation.

In [13], the dominance relation between two quasi-overlap functions was dis-cussed. In [16, 17] the researchers studied some special problems of the dominance relation for conjunctions and triangular conorms. Sarkoci [18] examined the characterization of all t-seminorms dominating every triangular conorm. Bentkowska et al. [19] deals with some properties of dominance between binary operations defined in partially ordered set. Furthermore, the notion of weak dominance on the set of all two binary operations was

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introduced in [11]. It can be viewed as a generalization of the dominance relation. On the other hand, the weak dominance can also be treated as a generalization of modularity equation [15] which is related with the associativity of aggregation operations and is usually used in fuzzy theory. So, it is interesting to discuss the weak dominance relation for conjunctions and triangular conorms.

The structure of the paper is as follows. Firstly, we recall some basic definitions of binary operations which will be used in the sequel and the notion of (weak) domination concerning two binary operations. In Sect. 3, we characterize the class of conjunctions which dominate each triangular conorm, and through the duality we obtain the class of disjunctions which dominate each triangular norm. Section 4 is devoted to the weak dominance case. Finally, we will close the contribution with a short summary.

2 Preliminaries

We recall here definitions of some binary operations which will be used in the sequel.

Definition 1 [20]. A conjunction (disjunction) is any increasing binary operation $C(D)$: $[0, 1]^2 \to [0, 1]$, fulfilling

$$C(0, 0) = C(0, 1) = C(1, 0) = 0, C(1, 1) = 1$$

$$(D(1, 1) = D(0, 1) = D(1, 0) = 0, D(0, 0) = 1).$$

If a conjunction C (disjunction D) has a neutral element $e \in [0, 1]$ (i.e. $C(x, e) = C(e, x) = x, \forall x \in [0, 1]$) such that $e = 1$ $(e = 0)$, then it is called a triangular seminorm (triangular semiconorm).

Definition 2 [20]. A t-seminorm (t-semiconorm) is any increasing binary operation $T(S) : [0, 1]^2 \to [0, 1]$ with neutral element 1 (0).

Remark 1. Any t-seminorm T (t-semiconorm S) fulfils

$$T(x, y) \le \min(x, y) \quad (S(x, y) \ge \max(x, y))$$

for all $x, y \in [0, 1]$.

Example 1. The operation $T : [0, 1]^2 \to [0, 1]$ given by

$$T(x, y) = \begin{cases} 0 & (x, y) \in [0, \frac{1}{2}]^2, \\ 2(x - \frac{1}{2})(y - \frac{1}{2}) + \frac{1}{2} & (x, y) \in]\frac{1}{2}, 1]^2, \\ \min(x, y) & otherwise. \end{cases} \quad (1)$$

is a t-seminorm.

A triangular seminorm (semiconorm) is called a triangular norm (triangular conorm) if it is associative and commutative.

Definition 3 [21]. A t-norm is a two place function $T : [0, 1]^2 \to [0, 1]$, such that for all $x, y, z \in [0, 1]$ the following conditions are satisfied:

(a) $T(x, y) = T(y, x)$.
(b) $T(T(x, y), z) = T(x, T(y, z))$.
(c) $T(x, y) \leq T(y, z)$ whenever $y \leq z$.
(d) $T(x, 1) = x$.

The four basic t-norms T_M, T_P, T_L, and T_D are usually discussed in literature. They are defined by, respectively:

$$T_M(x, y) = \min(x, y),$$

$$T_P(x, y) = x \cdot y,$$

$$T_L(x, y) = \max(x + y - 1, 0),$$

$$T_D(x, y) = \begin{cases} 0 & (x, y) \in [0, 1[^2, \\ \min(x, y) & \textit{otherwise}. \end{cases}$$

By duality we can obtain the definition of t-conorm and the four basic t-conorms S_M, S_P, S_L, and S_D [21].

Now we recall the notion of domination concerning two binary operations.

Definition 4 [1]. Consider two binary functions $F, G : [0, 1]^2 \to [0, 1]$. We say that F dominates G, and denoted by $F \gg G$, if

$$F(G(a, b), G(c, d)) \geq G(F(a, c), F(b, d)) \tag{2}$$

for all $a, b, c, d \in [0, 1]$.

Remark 2. It is obvious that t-norm T_M dominates every increasing operation. Every increasing operation dominates t-conorm S_M. Moreover, every t-norm dominates itself and T_D.

3 The Characterization of Conjunctions Dominating All Triangular Conorms

In this section, along the study line in [18], we offer the characterizations of the t-seminorms and conjunctions which dominate all triangular conorms.

For the t-seminorm which dominates all t-conorms [18], the following result holds.

Theorem 1. The t-seminorm C dominates the class of all t-conorms if

$$C(x, y) \in \{0, x, y\} \tag{3}$$

for any $x, y \in [0, 1]$.

Proof. For the completeness of this paper. We provide the proof here. By the Remark 1, we know that $C(x, y) \leq \min(x, y), \forall(x, y) \in [0, 1]^2$.

Let S be any t-conorm and $x, y, u, v \in [0, 1]$. If $C(x, y) \in \{0, x, y\}$ for all $(x, y) \in [0, 1]$ then the following cases are considered.

– If $C(x, u) = C(y, v) = 0$, then we get

$$C(S(x, y), S(u, v)) \geq 0 = S(0, 0) = S(C(x, u), C(y, v)).$$

– If $C(x, u) = \min(x, u) > 0$ and $C(y, v) = 0$, we have

$$S(C(x, u), C(y, v)) = S(\min(x, u), 0) = \min(x, u)$$

and

$$\min(x, u) = C(x, u) \leq C(S(x, y), S(u, v)).$$

– If $C(x, u) = 0$ and $C(y, v) = \min(y, v) > 0$. The proof is similar to the above case.
– If $C(x, y) = \min(x, y) > 0$ and $C(y, v) = \min(y, v) > 0$. then

$$C(S(x, y), S(u, v)) > 0$$

and

$$C(S(x, y), S(u, v)) = \min(S(x, y), S(u, v)).$$

Moreover, by the monotonicity of t-conorm S, we have

$$S(x, y) \geq S(C(x, u), C(y, v))$$

and

$$S(u, v) \geq S(C(x, u), C(y, v)).$$

Hence, $C(S(x, y), S(u, v)) \geq S(C(x, u), C(y, v))$.

Note that one cannot replace a t-seminorm by an arbitrary conjunction in Theorem 1.

Next, we consider the case of the increasing operator dominating any triangular conorm.

Theorem 2 [22]. Let $C : [0, 1]^2 \to [0, 1]$ be an increasing operation. If $C \leq T_M$ and dominates every t-conorm, if and only if

$$C(x, y) \in \{0, \min(x, y)\} \tag{4}$$

for any $x, y \in [0, 1]$.

Directly from Theorems 1 and 2 we obtain the following result.

Theorem 3 [22]. If C is a conjunction fulfilling the condition (4), then it dominates every t-conorm.

Note that there exist conjunctions such that (3) hold but do not satisfy (4), for example, C defined by

$$C(x,y)=\begin{cases}\max(x,y) & (x,y)\in[\frac{1}{2},1]^2,\\ \min(x,y) & otherwise.\end{cases} \tag{5}$$

is a conjunction which does not dominate t-conorm S_P.

Example 2. By Theorem 3 the operation $C:[0,1]^2\to[0,1]$ given by

$$C(x,y)=T_M(x,y)=\min(x,y) \tag{6}$$

dominates any t-conorm. This result also appears in Remark 2.

Now, we characterize the binary operations dominating every triangular conorm.

Theorem 4. Let C be a binary operation with $C(1,1)=1$. Then C is conjunction satisfying condition (4) if and only if there exists a decreasing function $h:[0,1]\to[0,1]$ such that

$$C(x,y)=\begin{cases}0 & y<h(x)\\ \min(x,y) & y>h(x)\\ 0\ or\ \min(x,y) & y=h(x)\end{cases} \tag{7}$$

and in intervals of constant values of function h with

$$C(x,u)=\begin{cases}0 & y<a_u\\ \min(x,u) & y>a_u\\ 0\ or\ \min(x,u) & y=a_u\end{cases} \tag{8}$$

Moreover, $u\in[0,1], E_u=\{x:h(x)=u\}, m_u=\inf E_u, n_u=\sup E_u, a_u\in[m_u,n_u]$.

Proof. (Necessity) Define a set $O_x=\{y\in[0,1]:C(x,y)=0\}$, and let $h(x)=\sup O_x$. According to the definition of conjunction, we have $C(x,0)\le C(1,0)=0$, so $0\in O_x$, O_x is non-empty.

Next we prove h is decreasing. First we note $h(0)=1$, because $C(0,1)=0$.

Let $x<y$. If $h(x)=1$ then $h(x)\ge h(y)$. If $h(x)<1$ then for all $l>h(x)$, $C(x,l)=\min(x,l)$. By the monotonicity of C we have

$$C(y,l)\ge C(x,l)=\min(x,l)>0.$$

Therefore, for all $l>h(x)$ with $C(y,l)=\min(y,l)$. It means $h(y)\le h(x)$.

So, h is decreasing.

Now, we prove that $a_u\in[m_u,n_u]$. Let $u\in[0,1]$ and $m_u<n_u$. Define $a_u=\sup\{x:C(x,u)=0\}$. We divide the proof into two parts:

- If $a_u < m_u$, then $m_u > 0$ and $u < 1$. Let $x \in (a_u, m_u)$. According to the definition of the set E_u and the monotonicity of the function h, we have

$$h(a_u) \geq h(x) > h(m_u) = u.$$

By (3) we get $C(x, u) = 0$, which leads to a contradiction.
- If $a_u > n_u$ then $n_u < 1$ and $u > 0$. Let $x \in (n_u, a_u)$. According to the definition of the set E_u and the monotonicity of the function h, we have

$$h(a_u) \leq h(x) < h(n_u) = u.$$

By (3) we obtain $C(x, u) = \min(x, u) > 0$, which leads to a contradiction.

So, $a_u \in [m_u, n_u]$.

By the definition of the point a_u and (7) we obtain (8).

(Sufficiency) Directly by (7) and (8) we deduce (4). Moreover, for all $x \in [0, 1]$, because $\min(x, 0) = 0$ we have $C(0, 0) = C(0, 1) = C(1, 0) = 0$, $C(1, 1) = 1$.

And then we prove C is increasing.

First, we prove the monotonicity with respect to the first variable, we consider a few cases.

- If $C(x, v) = \min(x, v)$, then $v \geq h(x)$. By monotonicity of h we have $v \geq h(y)$ and by (7) and (8) we have

$$C(y, v) = \min(y, v) \geq \min(x, v) = C(x, v).$$

- If $C(x, v) = 0 \leq C(y, v)$. It means that operation C is increasing with respect to the first variable.

Next, we prove operation C is increasing with respect to the second variable. Let $x, y, v \in [0, 1]$ and $x < y$. We prove it in the following cases.

- If $y \leq h(v)$, then $x < h(v)$ and $C(v, x) = 0 \leq C(v, y)$.
- If $x \geq h(v)$, then we have

$$y > h(v)$$

and

$$C(v, y) = \min(v, y) \geq \min(v, x) \geq C(v, x).$$

- If $x < h(v) < y$, then we have

$$C(v, y) = \min(v, y) \geq 0 = C(v, x).$$

So, C is a conjunction and satisfies (4).

The t-norms satisfying (4) can be characterized in the following result.

Theorem 5. Let $C : [0, 1]^2 \to [0, 1]$ be an increasing operation. Then C is a t-norm satisfying (4) if and only if there exists a subset I of $]0, 1[^2$ with the following properties:

i. I is symmetric, i.e., $(x, y) \in I$ implies $(y, x) \in I$.
ii. For all $(x, y) \in I$ we have$]0, x]\times]0, y] \subseteq I$.

such that C is given by

$$C(x, y) = \begin{cases} 0 & (x, y) \in I \\ \min(x, y) & \textit{otherwise}. \end{cases} \quad (9)$$

Proof. (Sufficiency) The commutativity (a) and the boundary condition (d) are satisfied by definition. And (c) is obvious. Then, we prove the associativity (b) of C. We divide the proof into three parts.

– For all $x, y, z \notin I$, in this case, let $x \leq y$ and $z \leq y$ then we have

$$C(C(x, y), z) = C(\min(x, y), z) = C(x, z) = C(x, \min(y, z)) = C(x, C(y, z)).$$

– For all $x, y, z \in I$, in this case, we obtain

$$C(C(x, y), z) = C(0, z) = 0 = C(x, 0) = C(x, C(y, z)).$$

– If at most one of the values x, y and z is contained in I, then

$$C(C(x, y), z) = \min(x, y, z) = C(x, C(y, z)).$$

(Necessity) If C is a t-norm satisfying (4), then C is a conjunction and satisfies (4).

By the proof of Theorem 4, the set $I = \{(x, y) : C(x, y) = 0\}$ and $h(x) = \sup\{y : (x, y) \in I\}$. By the commutativity of t-norm C and Theorem 4, the set I satisfies the conditions (i) and (ii) and C has the form (9).

By Proposition 9 and Remark 10 in [12], we can characterize continuous binary operations satisfying condition (4).

Theorem 6. Let $C : [0, 1]^2 \rightarrow [0, 1]$ be a continuous binary operation satisfying $C(0, 0) = C(0, 1) = C(1, 0) = 0$, $C(1, 1) = 1$ and (4). Then $C = T_M$.

By duality we may obtain a similar characterization of disjunctions which are dominated by any t-norm.

Theorem 7 [22]. If $D : [0, 1]^2 \rightarrow [0, 1]$ is a disjunction satisfying

$$D(x, y) \in \{1, \max(x, y)\} \quad (10)$$

for any $x, y \in [0, 1]$, then D dominated by every t-norm.

Theorem 8. Let D be a binary operation with $D(0, 1) = D(1, 0) = D(1, 1) = 1$. Then D is a disjunction satisfying condition disjunction dominates all t-norms if and only if, there exists a decreasing function $g : [0, 1] \rightarrow [0, 1]$ such that

$$D(x, y) = \begin{cases} 1 & y > g(x) \\ \max(x, y) & y < g(x) \\ 1 \textit{ or } \max(x, y) & y = g(x) \end{cases} \quad (11)$$

in intervals of constant values of function g with

$$D(x,u)=\begin{cases}1 & y>a_s\\ \max(x,s) & y<a_s\\ 1 \text{ or } \max(x,s) & y=a_s\end{cases} \tag{12}$$

Moreover, $s\in[0,1]$, $E_s=\{x: g(x)=u\}$, $m_s=\inf E_s$, $n_s=\sup E_s$, $a_s\in[m_s,n_s]$.

Theorem 9. Let $D:[0,1]^2\to[0,1]$ be an increasing operation. Then D is a t-conorm satisfying (10) if and only if there exists a subset I of $]0,1[^2$ with the following properties:

iii. I is symmetric, i.e., $(x,y)\in I$ implies $(y,x)\in I$.
iv. For all $(x,y)\in I$ we have $[x,1[\times[y,1[\subseteq I$.

such that D is given by

$$D(x,y)=\begin{cases}1 & (x,y)\in I\\ \max(x,y) & \textit{otherwise}.\end{cases} \tag{13}$$

Theorem 10. Let $D:[0,1]^2\to[0,1]$ be a continuous binary operation satisfying $D(0,0)=0, D(0,1)=D(1,0)=D(1,1)=1$ and (10). Then $D=S_M$.

4 The Characterization of Disjunctions Weakly Dominating All Triangular Norms

We recall the definition of weak dominance about two binary operations.

Definition 5 [1]. Consider two binary functions $F,G:[0,1]^2\to[0,1]$, we say that F weakly dominates G, and denoted by $F>>G$, if

$$F(G(a,b),c)\geq G(F(a,c),b) \tag{14}$$

for all $a,b,c\in[0,1]$.

For the relation between dominance and weak dominance, the following statement holds.

Proposition 1. Consider two binary functions $F,G:[0,1]^2\to[0,1]$ having a common neutral element $e\in[0,1]$. If $F\gg G$ then $F>>G$.

Proof. Taking $d=e$ in (2), we get the result.

Contrary to Theorem 1, we have the following result about the t-seminorms weakly dominating all t-conorms.

Proposition 2. There exists no t-seminorm F and t-conorm G such that F weakly dominates G.

Proof. On the contrary, suppose that there exist a t-seminorm F and t-conorm G such that $F >> G$. Then for arbitrary $x \in [0, 1]$, taking $a = 1$, $b = c = x$ in (14), we have

$$x = F(1, x) = F(G(1, x), x) \geq G(F(1, x), x) = G(x, x).$$

Since $G(x, x) \geq S_M(x, x) = x$, $G(x, x) = x$ for arbitrary $x \in [0, 1]$. Hence, $G = S_M$ by the dual result of Proposition 1.9 in [20].

Taking $a = 0$,$b = c = x$ in (14), we have

$$F(x, x) = F(G(0, x), x) \geq G(F(0, x), x) = G(0, x) = x.$$

Since $F(x, x) \leq F(1, x) = x$, $F(x, x) = x$ for arbitrary $x \in [0, 1]$.

However, taking $a = c = \frac{1}{4}$, $b = \frac{3}{4}$, we have

$$F(G(a, b), c) = F(\frac{3}{4}, \frac{1}{4}) \leq \frac{1}{4} < \frac{3}{4} = G(F(a, c), b).$$

Hence, the result holds.

Corollary 1. There exists no t-seminorm which weakly dominates every t-conorm.

Proposition 3. Let D be a t-semiconorm. If D weakly dominates every t-norm T then $D(a, c) \in \{1, max(a, c)\}$ for all $a, c \in [0, 1]$.

Proof. On the contrary, suppose that there exists $a \leq c \in]0, 1[$ such that $c < D(a, c) = d < 1$. Then by Proposition 3.63 in [21], we can construct a t-norm defined by

$$T(x, y) = \begin{cases} 0 & (x, y) \in]0, c]\times]0, 1[\cup]0, 1[\times]0, c], \\ \min(x, y) & otherwise. \end{cases}$$

Taking $b \in]c, d[$ in (14), we have

$$c = D(0, c) = D(T(a, b), c) \geq T(D(a, c), b) = T(d, b) = b,$$

a contradiction with the assumption.

Remark 3.

i. If a t-semiconorm D satisfying $D(a, c) \in \{1, max(a, c)\}$ for all $a, c \in [0, 1]$ then D may not weakly dominate any t-norm T which can be compared with Theorem 1 in Sect. 3 (see Example 3 below).
ii. If the t-semiconorm D weakly dominates every t-norm T then D has the form (13) by Theorem 9 and Proposition 1.

Example 3. Let the t-semiconorms $D : [0, 1]^2 \to [0, 1]$ given by

$$D(x, y) = \begin{cases} 1 & (x, y) \in]0, 1] \times [0.5, 1] \cup [0.5, 1]\times]0, 1], \\ \max(x, y) & otherwise. \end{cases}$$

It is obvious that $D(a, c) \in \{1, max(a, c)\}$ for all $a, c \in [0, 1]$. Consider t-norm $T_L(x, y) = \max\{0, x + y - 1\}$. Taking $a = 0.1$, $b = 0.6$, $c = 0.5$, we have

$$D(T_L(a, b), c) = 0.5 < 0.6 = T_L(D(a, c), b).$$

Hence, t-semiconorms D does not weakly dominate t-norm T_L.

Moreover, by Proposition 3 and Example 3, we also know that the similar results about disjunctions do not hold comparing to Theorems 2 and 3 in Sect. 3.

Remark 4.

i. Let $D : [0, 1]^2 \to [0, 1]$ be an increasing operation. If $D \geq S_M$ and weakly dominates every t-norm, then

$$D(x, y) \in \{0, \max(x, y)\} \tag{15}$$

for any $x, y \in [0, 1]$. The converse statement is not true (see Example 3 above).

ii. If a disjunction D satisfying $D(a, c) \in \{1, max(a, c)\}$ for all $a, c \in [0, 1]$, then D may not weakly dominate any t-norm T (see Example 3 above).

5 Conclusion

The dominance relation between binary operators is an interesting relation that arises in some mathematical problems, such as the retention of certain properties of fuzzy relations which is important in fuzzy decision. In this paper, we characterize the conjunctions dominating all triangular conorms in different cases and offer the dual results. Furthermore, we also partially provide the characterization of t-semiconorms which weakly dominate all t-norms.

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