



# Geometrical Parameter Identification for 6-DOF Parallel Platform

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**Abstract.** The positioning accuracy of the 6-degree-of-freedom (DOF) parallel platform is affected by the model accuracy. In this paper, the parameter identification of the 6-DOF parallel platform is carried out to obtain the accurate kinematic model. The theoretical inverse kinematic model is established by closed-loop vector method. The rotation centers of each spherical joint and hooker joint are selected as geometric parameters to be identified. The inverse kinematic model with geometric error is established. The geometric parameters are identified by iterative least square method. Simulation results show that the parameter identification method is correct.

**Keywords:** Parallel platform · Kinematics · Parameter identification · Iterative least squares

## 1 Introduction

Parallel mechanisms are widely used in industrial and medical fields due to the high stiffness, fast response and good stability [1, 2]. Parallel mechanisms can be divided into less degrees of freedom [3, 4] and six degrees of freedom [5, 6]. However, the actual model of parallel mechanism is often different from the theoretical model. The model errors will lead to the decrease of positioning accuracy and affect the working performance of the parallel mechanism. Therefore, the study of error calibration is of great significance to improve the accuracy of parallel mechanism.

The error sources of parallel mechanisms are mainly divided into geometric error [7, 8] and non-geometric error [9]. Geometric error is caused by machining error in manufacturing process and assembly error in assembly process. Non-geometric error is caused by friction, hysteresis and thermal error. The proportion of non-geometric error does not exceed 10% of the total error source of parallel mechanism. It is important to study the calibration of geometric parameters.

The geometric errors of the parallel mechanism cause some differences between the real kinematics model and the theoretical kinematics model. Kinematics model calibration can effectively solve the defect, and the economic cost is low. The purpose of robot kinematic calibration is to identify the error between the real model parameters and the theoretical model parameters to correct the theoretical kinematic model [10]. The

calibration process can be divided into four steps: error modeling, error measurement, error identification and error compensation.

As for the 2-DOF translational parallel manipulator, two types of kinematic calibration methods are proposed based on different error models [11]. A practical self-calibration method for planar 3-RRR parallel mechanism is proposed in literature [12]. On the premise that the structure of the mechanism remains unchanged, the line ruler is used to measure and record the output pose of the mechanism. A kinematic calibration method for a class of 2UPR&2RPS redundant actuated parallel robot is proposed in reference [13]. The geometric error model of the robot is established based on the error closed-loop vector equation, and the compensable and uncompensable error sources are separated. The error Jacobian matrix of 54 structural parameters for 6PUS parallel mechanism is established in literature [14]. The kinematic parameters are calibrated by separating position parameters and angle parameters [15]. There are few literatures about the calibration of 6-DOF platform.

The structure of this paper is as follows. Section 2 presents the kinematics of 6-DOF parallel platform. Section 3 provides the parameter identification algorithm. In Sect. 4, the simulation is conducted. Section 5 gives the conclusion.

## 2 Kinematics of 6-DOF Parallel Platform

### 2.1 6-DOF Parallel Platform

The 6-DOF parallel platform is shown in Fig. 1. It consists of an upper platform, a lower platform and six chains. Each chain is composed of a prismatic joint (P), a hooker joint (U) and a spherical joint (S). The upper platform can achieve the motion of six degrees of freedom.

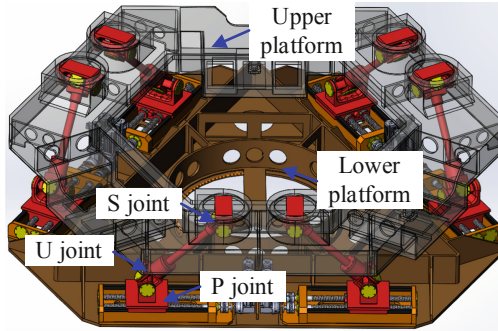


Fig. 1. 6-DOF parallel platform

### 2.2 Kinematic Model

The diagram of 6-DOF parallel platform is shown in Fig. 2. The Cartesian coordinate system  $o_1-x_1y_1z_1$  is established on the center of lower platform. The Cartesian coordinate

system  $o_2-x_2y_2z_2$  is established on the center of upper platform. The moving coordinate system is  $o_2-x_2y_2z_2$ , while the fixed coordinate system is  $o_1-x_1y_1z_1$ . According to the closed-loop vector method, the equation can be obtained as:

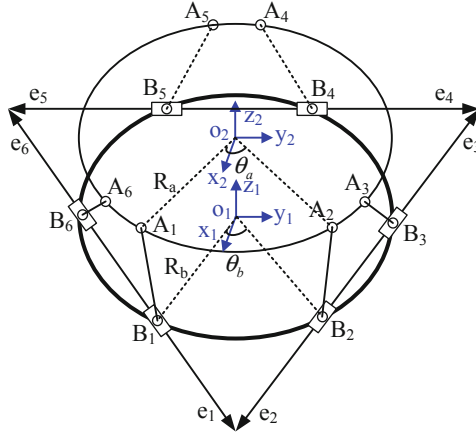


Fig. 2. The diagram of 6-DOF parallel platform

$$p + Ra'_i = b_i + q_i e_i + l_i \tag{1}$$

where  $p$  is the position matrix,  $R$  is the rotation matrix,  $i$  is the  $i$ th chain,  $b_i$  is the coordinates of  $B_i$  in the coordinate system  $o_2-x_2y_2z_2$ ,  $q_i$  is the driving displacement of the  $i$ th chain,  $e_i$  is the unit vector of the  $i$ th chain.

### 3 Parameter Identification

The rotation centers of each spherical joint and hooker joint are selected as geometric parameters to be identified. Assuming that the coordinates of  $A_i$  in the moving coordinate system are  $(x_{A_i}, y_{A_i}, z_{A_i})$ . The coordinates of  $B_i$  in the fixed coordinate system are  $(x_{B_i}, y_{B_i}, z_{B_i})$ . Then the following equation can be obtained as:

$$\delta q = K \delta \Theta \tag{2}$$

where  $K$  is the coefficient matrix.

$$\delta q = [\delta q_1, \delta q_2, \dots, \delta q_i]^T, \quad (i = 1, \dots, 6)$$

$$\delta \Theta = [\delta x_{A_1}, \delta y_{A_1}, \delta z_{A_1}, \delta x_{B_1}, \delta y_{B_1}, \delta z_{B_1}, \dots, \delta x_{A_i}, \delta y_{A_i}, \delta z_{A_i}, \delta x_{B_i}, \delta y_{B_i}, \delta z_{B_i}]^T$$

Taking the chain 1 for example, it can be obtained as:

$$\begin{bmatrix} {}^{mea}q_{11} - q_{11} \\ {}^{mea}q_{12} - q_{12} \\ \vdots \\ {}^{mea}q_{1j} - q_{1j} \end{bmatrix}_{j \times 1} = \begin{bmatrix} M_{11} & M_{21} & M_{31} & M_{41} & M_{51} & M_{61} \\ M_{12} & M_{22} & M_{32} & M_{42} & M_{52} & M_{62} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ M_{1j} & M_{2j} & M_{3j} & M_{4j} & M_{5j} & M_{6j} \end{bmatrix}_{j \times 6} \cdot \delta \Theta_1 \tag{3}$$

where  $\delta\Theta_1 = [\delta x_{A_1}, \delta y_{A_1}, \delta z_{A_1}, \delta x_{B_1}, \delta y_{B_1}, \delta z_{B_1}]^T$ ,  ${}^{mea}q_{lj}$  is the driving displacement obtained by the  $j$ th measurement.

According to the least square method, it can be obtained as:

$$\delta\Theta_1 = (M^T \cdot M)^{-1} \cdot M^T \cdot \begin{bmatrix} {}^{mea}q_{11} - q_{11} \\ {}^{mea}q_{12} - q_{12} \\ \vdots \\ {}^{mea}q_{1j} - q_{1j} \end{bmatrix}_{j \times 1} \tag{4}$$

where

$$M = \begin{bmatrix} M_{11} & M_{21} & M_{31} & M_{41} & M_{51} & M_{61} \\ M_{12} & M_{22} & M_{32} & M_{42} & M_{52} & M_{62} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ M_{1j} & M_{2j} & M_{3j} & M_{4j} & M_{5j} & M_{6j} \end{bmatrix}_{j \times 6}$$

Because the actual geometric error of the 6-DOF parallel platform is different from the differential component, the least square iterative method is introduced to identify geometric parameters. The identification process is shown in Fig. 3.

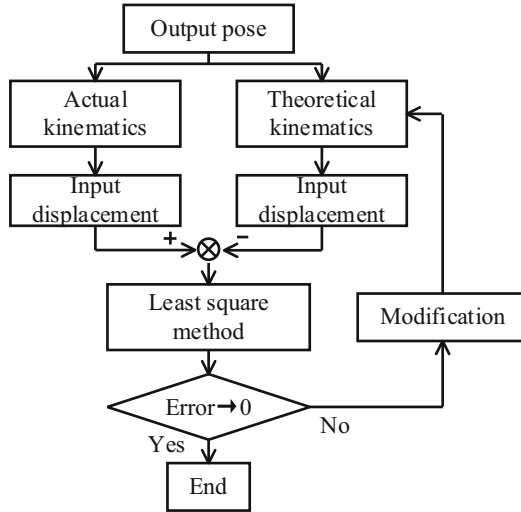
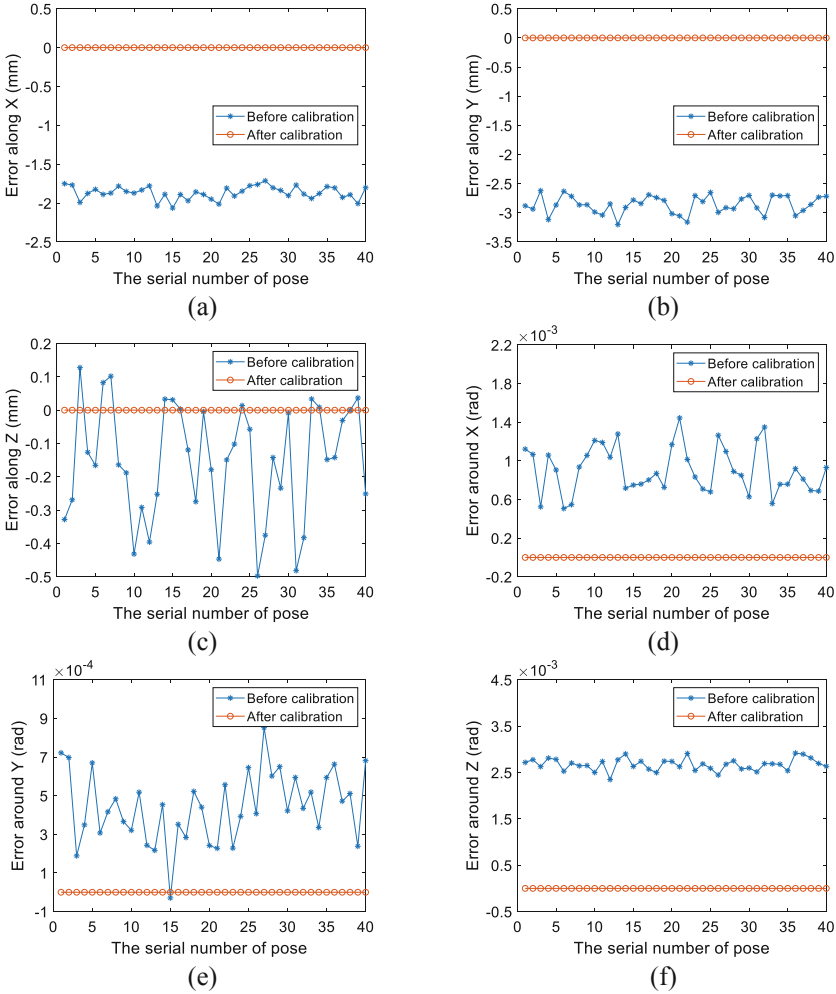


Fig. 3. The flow chart of parameter identification

### 4 Simulation

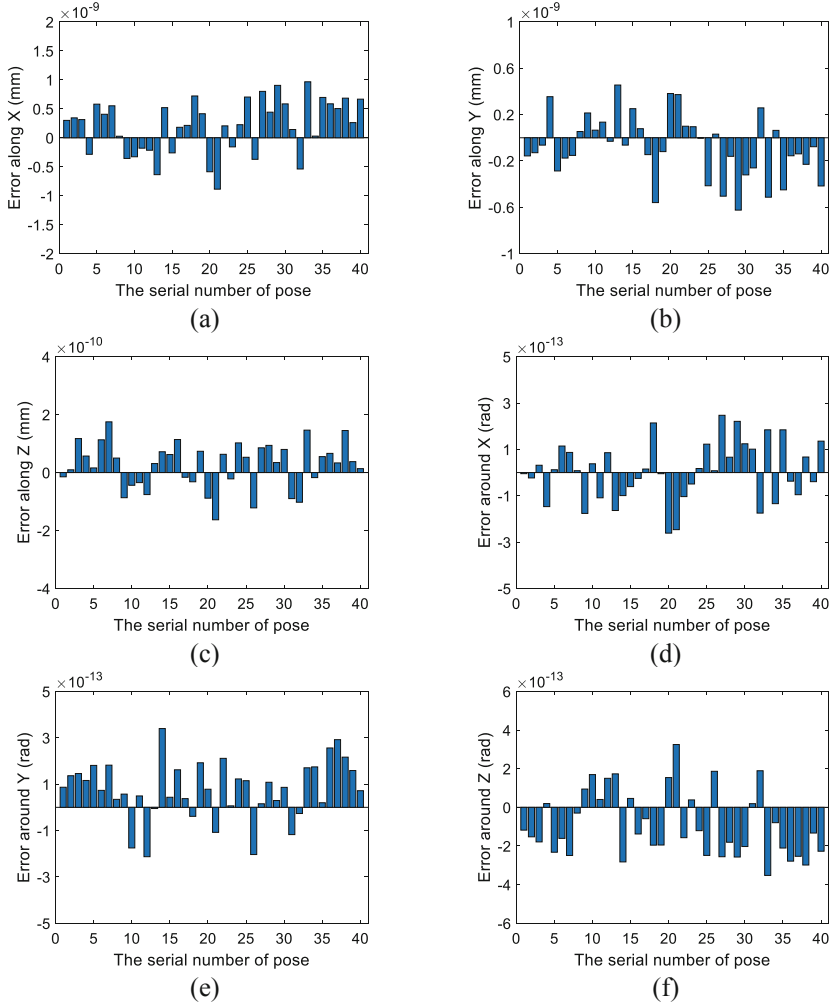
Given the output poses of the upper platform, the corresponding driving displacements can be calculated by theoretical inverse kinematics and actual inverse kinematics. The iterative least square algorithm is used to identify geometric errors.

The correctness of parameter identification method is verified. Given different output poses, the input displacements are calculated by the actual inverse kinematics model. The Newton iteration method is used to calculate the output poses of the identified model and the theoretical model. The comparison of errors before and after calibration is shown in Fig. 4. The pose errors after calibration is shown in Fig. 5.



**Fig. 4.** Comparison of errors before and after calibration

As can be seen from the figure, after calibration, the positioning accuracy of the 6-DOF parallel platform along the X direction is improved from  $-2.064 \text{ mm}$  to  $9.659 \times 10^{-10} \text{ mm}$ . The positioning accuracy along the Y direction is improved from  $-3.202 \text{ mm}$  to  $-6.242 \times 10^{-10} \text{ mm}$ . The positioning accuracy along the Z direction is improved from  $-0.4974 \text{ mm}$  to  $1.75 \times 10^{-10} \text{ mm}$ . The positioning accuracy around the X direction is



**Fig. 5.** Pose errors after calibration

improved from 0.00145 rad to  $-2.614 \times 10^{-13}$  rad. The positioning accuracy around the Y direction is improved from 0.00085 rad to  $3.398 \times 10^{-13}$  rad. The positioning accuracy around the Z direction is improved from 0.00292 rad to  $-3.525 \times 10^{-13}$  rad.

## 5 Conclusion

In this paper, the kinematics model calibration of the 6-DOF parallel platform is carried out. The theoretical inverse kinematic model is established by closed-loop vector method. The rotation centers of each spherical joint and hooker joint are selected as geometric parameters to be identified. The iterative least square method is introduced to identify the

geometric parameters. Simulation results show that the parameter identification method is correct.

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## References

1. Sun, T., Zhai, Y., Song, Y., Zhang, J.: Kinematic calibration of a 3-DoF rotational parallel manipulator using laser tracker. *Robot. Comput. Integr. Manuf.* **41**, 78–91 (2016)
2. Dasgupta, B., Mruthyunjaya, T.S.: The Stewart platform manipulator: a review. *Mech. Mach. Theory* **35**(1), 15–40 (2000)
3. Ma, N., et al.: Design and stiffness analysis of a class of 2-DoF tendon driven parallel kinematics mechanism. *Mech Machine Theor* **129**, 202–217 (2018)
4. Song, Y., Zhang, J., Lian, B., Sun, T.: Kinematic calibration of a 5-DoF parallel kinematic machine. *Precis. Eng.* **45**, 242–261 (2016)
5. Feng, J., Gao, F., Zhao, X.: Calibration of a six-DOF parallel manipulator for chromosome dissection. *Proc. Inst. Mech. Eng. Part C* **226**(4), 1084–1096 (2011)
6. Dong, Y., Gao, F., Yue, Y.: Modeling and prototype experiment of a six-DOF parallel micro-manipulator with nano-scale accuracy. *Proc. Inst. Mech. Eng. C J. Mech. Eng. Sci.* **229**(14), 2611–2625 (2015)
7. Daney, D.: Kinematic calibration of the Gough Platform. *Robotica* **21**(6), 677–690 (2003)
8. Renders, J.-M., Rossignol, E., Becquet, M., Hanus, R.: Kinematic calibration and geometrical parameter identification for robots. *IEEE Trans. Robot. Autom.* **7**(6), 721–732 (1991). <https://doi.org/10.1109/70.105381>
9. Gong, C., Yuan, J., Ni, J.: Nongeometric error identification and compensation for robotic system by inverse calibration. *Int. J. Mach. Tools Manuf* **40**(14), 2119–2137 (2000)
10. Renaud, P., Andreff, N., Martinet, P., Gogu, G.: Kinematic calibration of parallel mechanisms: a novel approach using legs observation. *IEEE Trans. Robot.* **21**(4), 529–538 (2005)
11. Zhang, J., et al.: Kinematic calibration of a 2-DOF translational parallel manipulator. *Adv. Robot.* **28**(10), 1–8 (2014)
12. Shao, Z.: Self-calibration method of planar flexible 3-RRR parallel manipulator. *J. Mech. Eng.* **45**(03), 150 (2009). <https://doi.org/10.3901/JME.2009.03.150>
13. Zhang, J., Jiang, S., Chi, C.: Kinematic calibration of 2UPR&2RPS redundant actuated parallel robot. *Chin. J. Mech. Eng.* **57**(15), 62–70 (2021)
14. Fan, R., Li, Q., Wang, D.: Calibration of kinematic complete machine of 6PUS parallel mechanism. *J. Beijing Univ. Aeronaut. Astronaut.* **42**(05), 871–877 (2016)
15. Chen, G., Jia, Q., Li, T., Sun, H.: Calibration method and experiments of robot kinematics parameters based on error model. *Robot* **34**(6), 680 (2012). <https://doi.org/10.3724/SP.J.1218.2012.00680>