



Automatic Parameter Calibration of Two Advanced Constitutive Models

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Abstract. This paper presents the automatic parameter calibration of two advanced constitutive models for sand: Hypoplasticity with Intergranular Strain and Sanisand. The application of the software is demonstrated by automatically calibrating the parameters for Karlsruhe Sand using data of oedometric compression and drained monotonic triaxial tests. The quality of the obtained parameter sets is demonstrated by the comparison of the experimental data with simulations, using 1) the automatically calibrated material parameters and 2) two reference parameter sets calibrated “by hand”. It is shown that the developed calibration software outperforms the “by hand” calibration in terms of accuracy, simplifies the parameter calibration and lowers the entry hurdle for the use of the advanced constitutive models.

Keywords: Automatic parameter calibration · Soil models · Hypoplasticity · Sanisand

1 Introduction

Numerous constitutive soil models have been developed in the past and continue to be developed today. Many of the constitutive models in use today are formulated either in the hypoplastic or in the elasto-plastic framework. Many new models extend and improve the predictive quality of the older models. Typically, this is accompanied by an increase in the number of material parameters which have to be calibrated. The quality of the simulation strongly depends on an appropriate choice of these parameters. However, their calibration is often time consuming and requires a high level of experience in handling the soil model to be calibrated, since many of the parameters cannot be determined by specific experiments or empirical equations. This makes the application of such advanced models a very challenging task, not only for beginners but also for experienced engineers and researchers and hinders the application of advanced soil models in practice and boundary value problems. Automatic Calibration (AC) aims to simplify and speed up the calibration process and, in particular, to reduce these hurdles. In addition, AC helps to reduce the “human factor” in the calibration of model parameters

(e.g., unconscious personal preferences, result-based calibrations, and the experience of the person performing the calibration).

A well-established tool for automatic calibration of constitutive soil models is ExCalibre¹ (Kadlíček et al. 2016, 2018, 2022), which allows, for example, the calibration of the parameters of two hypoplastic models for sand and clay, respectively. However, the software currently only supports calibration of a complete set of parameters (calibration of single parameters is not possible) and, in the case of the hypoplastic model for sands, provides parameters that underestimate dilatancy (Mendez et al. 2021). In this paper, an Automatic Calibration Tool (ACT) for constitutive soil models is presented. The calibration is based on heuristic optimization methods and offers access to different similarity measures to quantify the differences between experimental data and simulation results. The software is presented by means of the calibration of the parameters for two advanced constitutive models for sand: Hypoplasticity and Sanisand. The developed ACT, including the definition of the cost function as well as the optimization method applied in this work are introduced in Sect. 2. In Sect. 3, the experimental data set, and the corresponding reference parameter sets are presented. Based on these data, the performance of the ACT is evaluated in Sect. 4.

2 Numerical framework

The ACT is developed as part of the finite element (FE) software numgeo² (Machaček and Staubach, see e.g. (Machaček 2020, 2021; Machaček and Staubach 2021; Staubach et al. 2022a, b)), which gives access to several advanced constitutive soil models such as Hypoplasticity or Sanisand – whose calibration is the focus of this paper. The optimization library is implemented in Python and uses numerical libraries such as numpy (Harris et al. 2020) and scipy (Virtanen et al. 2020). A database has been developed to import different types of tests, such as oedometric compression tests or (drained monotonic) triaxial tests. Multiple tests per type can be included in the calibration. Functions are available for pre-processing (filtering, data reduction or interpolation) of the raw data. Besides the global heuristic optimizer, the software also offers the possibility to automatically estimate the parameters based on simplified calibration methods, such as the one proposed in (Herle 1997) for the hypoplastic model. In case of the global heuristic optimizer, the optimization algorithm operates on a set of model parameters, comparing the results of the numerical simulation with the constitutive model to the experimental results stored in the database until a set of parameters is identified that characterizes best the material behavior. Different algorithms are available for this purpose. For the present study, a Differential Evolution (DE, (Storn and Price 1997)) optimization algorithm was used. DE, is a population-based metaheuristic search algorithm that optimizes a problem by iteratively improving a candidate solution \mathbf{x}^j (where j is the candidate number) based on an evolutionary process. In this work, the DE implementation of (Virtanen et al. 2020) is used. The solution of the problem is described by a vector $\mathbf{x} = [x_1, x_2, \dots, x_{nD}]$ with the dimension of the search space nD (number of parameters of the constitutive model

¹ <https://soilmodels.com/excalibre/>.

² The finite element software numgeo is freely available for download from www.numgeo.de. The ACT can be obtained from the corresponding author upon reasonable request.

to be calibrated). The solution space is investigated by multiple candidate solutions \mathbf{x}^j simultaneously. The set of solutions is called population $\mathbf{p} = [\mathbf{x}^1, \mathbf{x}^2, \dots, \mathbf{x}^{nC}]$, with nC individuals (candidate solutions). For the present work, the overall population size is $nC = \max\{15 \cdot nD, 2p\}$ (the next power of 2 after $15 \cdot nD$). The initialization of \mathbf{p}_0 is based on Sobol sequences, wherein each parameter varies within user provided ranges. These ranges depend on the constitutive soil model to be calibrated and remain fixed for all candidates during the optimization process. At each iteration I of the DE, a “mutation operator” is used for producing a so-called donor vector $\mathbf{v} = [v_1^j, v_2^j, \dots, v_{nD}^j]$ for each individual \mathbf{x}^j in the current population \mathbf{p} . In the present work, the mutation follows the so-called “best/1/bin” strategy³, see e.g. (Georgioudakis and Plevris 2020).

The basis of any optimization is the existence of a (scalar) objective function (“cost function”) $\in (\tilde{\sigma}, \tilde{\varepsilon})$, which has to be minimized by changing the parameters in the course of the optimization. The cost function in this work is defined based on a comparison of the experimentally measured and simulated relationships in the stress and strain spaces. For oedometric compression tests (*oc*) the contribution to the cost function \in^{oc} is evaluated in the axial stress - axial strain $(\tilde{\sigma}_1 - \tilde{\varepsilon}_1)$ plane, whereas for the drained monotonic triaxial tests (*dmt*), the contribution \in^{dmt} considers both the deviatoric stress - axial strain $(\tilde{q} - \tilde{\varepsilon}_1)$ and the volumetric strain - axial strain $(\tilde{\varepsilon}_v - \tilde{\varepsilon}_1)$ planes. Notice that instead of the actual values, scaled stress and strain measures are used to ensure that the variables used for the error calculation have a comparable influence on the optimization - despite their different value ranges and units ($\varepsilon \in [0 - 0.3]$ and $q \in [0 - 500]$ kPa). The scaling is performed according to Eq. (1):

$$\tilde{\zeta}^{\square} = \frac{\zeta^{\square}}{\max(\max(\zeta^{exp}), \max(\zeta^{sim}))} \text{ where } \square = \{exp, sim\} \text{ and } \zeta = \{\sigma_1, q, \varepsilon_v, \varepsilon_1\} \quad (1)$$

To quantify the discrepancy between the experimental curves and the simulated ones, a discrete Fréchet distance (Jekel et al. 2018) is used. The Fréchet distance (Eiter and Mannila 1994; Fréchet 1906) is a measure of the similarity between two curves taking into account the location and ordering of the points along the curves.

When parameter optimization is performed for advanced material models, it is necessary to limit the search space of possible parameters by defining bounds and/or constraints. The bounds used in the present work are summarized in Table 1. Therein, $\varphi_{c,lab}$ refers to the angle of repose from loosely pluviated cones of dry sand and $e_{c0,lab}$, $e_{d0,lab}$ are the maximum and minimum void ratios determined based on index tests in the laboratory.

3 Experimental Data and Reference Parameter

The performance of the ACT is investigated by calibrating the parameters of the hypoplastic and the Sanisand models for the so-called Karlsruhe Sand: a medium

³ The best vector in the population is mutated by a weighted difference of two randomly chosen vectors of the current population. In this work, the weight F randomly varies after each iteration in the range of $0.5 \leq F \leq 1$.

Table 1. Parameter bounds for the optimization

Hypoplasticity	φ_c	h_s	n	e_{c0}	e_{d0}	α	β
Min.	$0.9\varphi_{c,lab}$	10^{-4} GPa	0.1	$0.9e_{c0,lab}$	$0.9e_{d0,lab}$	0.0	0.0
Max.	$1.1\varphi_{c,lab}$	60 GPa	1.0	$1.1e_{c0,lab}$	$1.1e_{d0,lab}$	1.0	5.0

* e_{i0} has been fixed to $1.15e_{c0}$

Sanisand	e_0	λ_c	ξ	M_c	M_e	c	m	G_0	v	h_0	c_h	n_b	A_0	n_d
Min.	$0.9e_{c0,lab}$	0.01	0.1	1	1	0.1	0.01	50 kPa	0.0	0.01	0.9	0.01	0.5	0.1
Max.	$1.1e_{c0,lab}$	0.5	0.9	2	2	1.0	0.1	500 kPa	0.4	50.0	1.1	1.5	1.0	3

*The parameters controlling the cyclic mobility effect have been chosen based on (Machaček et al., 2021) to $z_{max} = 60$ and $c_z = 2000$

coarse sand which has already been the basis for numerous model tests (Vogelsang 2017) and simulations, see e.g. (Chrisopoulos and Vogelsang 2019; Machaček 2020; Machaček et al. 2021). The parameters have been calibrated “by hand” in a previous work (Machaček et al. 2021), where the detailed calibration procedure can be found. A comparison of the experimental data and the simulation results is shown in Fig. 1 for the oedometric compression tests and in Fig. 2 for the drained monotonic triaxial tests.

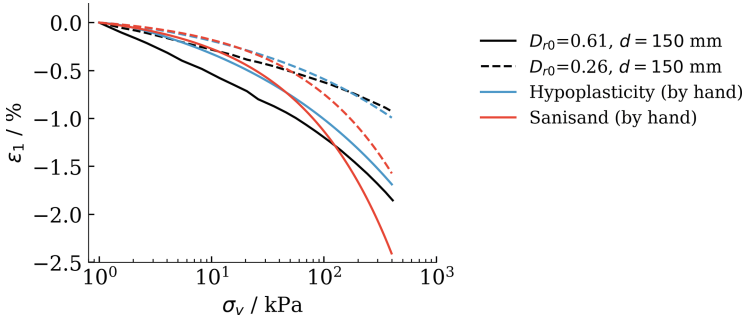


Fig. 1. Comparison of experiments (black) and simulation results (colored) for oedometric compression tests on two samples of Karlsruhe Sand with different initial relative density

For the oedometric compression test, a good agreement between the hypoplastic model and the experiments is observed – which is expected, since the parameters h_s , n and β are calibrated to reproduce the first loading curves of the oedometric tests. For the Sanisand model, the comparison with the experimental results reveals that the oedometric stiffness is underestimated. The poorer performance of the Sanisand model under oedometric conditions, however, is known (see e.g. (Wichtmann et al. 2019)) and is expected to be improved by using a Sanisand version with closed yield-surface.

Comparing the results of the drained monotonic triaxial tests with the simulation, one may note that the peak strength is noticeably underestimated by both models for the test with $D_{r0} = 0.62$ and $p_0 = 100$ kPa, see Fig. 2.

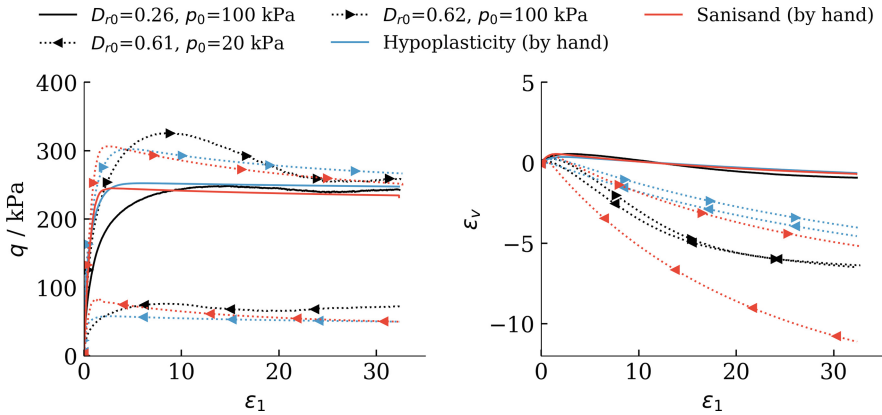


Fig. 2. Comparison of experiments (black) and simulation results (colored) for drained monotonic triaxial tests on samples of Karlsruhe Sand with different initial densities and different initial mean effective stresses

Further, both models show an initially too stiff response but reproduce the residual strength fairly well. In terms of the $\varepsilon_v - \varepsilon_1$ behavior, both models are in good agreement with the experimental results of the test on the loose sample. The inability of the hypoplastic model in predicting the development of volumetric strain of the dense samples is clearly visible. Notice that in (Machaček et al. 2021) undrained cyclic triaxial tests were part of the calibration, which, in favor of an overall better agreement, forced to accept a slightly worse performance of the Sanisand model in the monotonic tests.

4 Results of the Automatic Calibration Tool

The results of the ACT are presented by means of comparison of simulation results and experimental data of the oedometric compression tests in Fig. 3. For the drained monotonic triaxial tests, the simulation results with the hypoplastic model and the Sanisand model are shown in Fig. 4 and Fig. 5, respectively. The comparison given in Fig. 3 shows that the simulation results obtained by the ACT for the oedometric compression tests are in satisfactory agreement with the experimental data. Compared to the simulation results obtained with the reference parameter sets, the ACT can be attested a better agreement with the experimental data. This applies especially for the simulations with the Sanisand model and for the hypoplastic model in case of the dense sample.

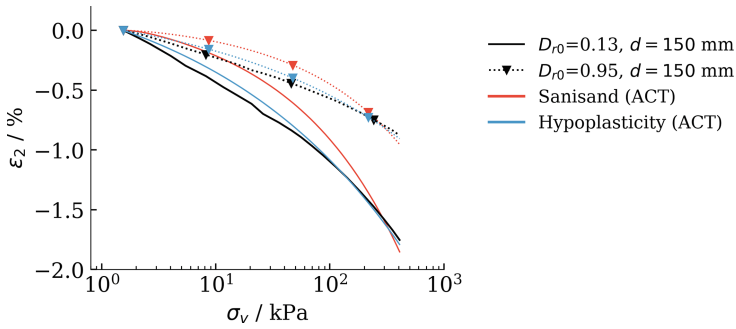


Fig. 3. Comparison of experiments (black) and simulation results (blue/red) with the hypoplastic and the Sanisand model calibrated by means of ACT for oedometeric compression tests on two samples of Karlsruhe Sand

For the drained monotonic triaxial tests, an overall acceptable agreement between the simulations using the hypoplastic model (see Fig. 4) and a good agreement for the Sanisand model (see Fig. 5) is observed. The largest deviations are observed for the recalculation of the triaxial test on the very dense sample ($D_{r0} = 0.97, p_0 = 100$ kPa) with the hypoplastic model. Here, the simulation strongly overestimates the maximum deviatoric stress q . However, the simulations of the other samples show a good agreement, and much improvement compared to the simulation using the reference parameter set, in terms of $\varepsilon_v - \varepsilon_1$ behavior.

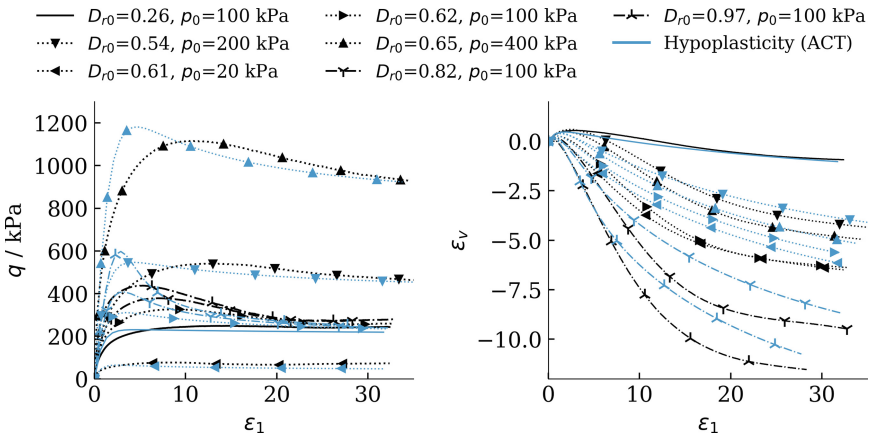


Fig. 4. Comparison of experiments (black) and simulation results (blue) with the hypoplastic model calibrated by means of ACT for drained monotonic triaxial tests on samples of Karlsruhe Sand

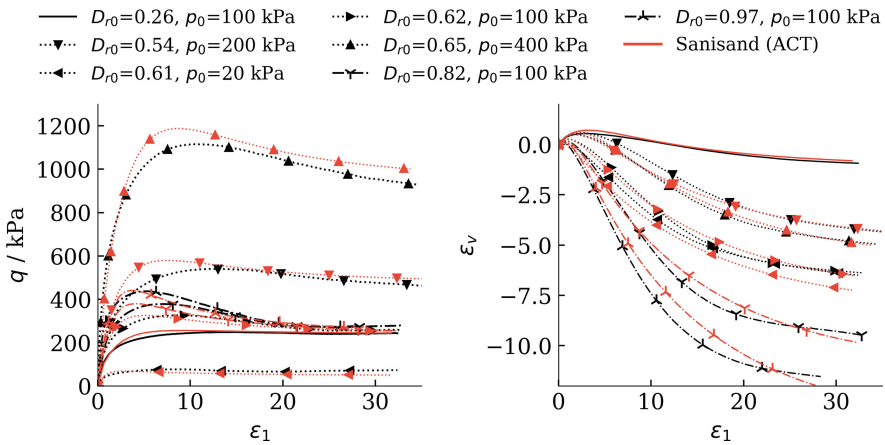


Fig. 5. Comparison of experiments (black) and simulation results (red) with the Sanisand model calibrated by means of ACT for drained monotonic triaxial tests on samples of Karlsruhe Sand

5 Conclusions

An Automatic Calibration Tool (ACT) has been developed, which performs the calibration of constitutive model parameters by minimization of a cost function that quantifies the discrepancy between experiment and simulation using a heuristic optimizer. The automatic calibration is able to successfully calibrate parameters for advanced constitutive soil models which was shown by calibrating parameter for Hypoplasticity and Sanisand. For both models, the ACT was able to determine parameter sets that led to better agreement with the experiments than simulations using reference parameter sets calibrated “by hand”

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