An Interdisciplinary Model-Based Study on Emerging Infectious Disease: The Curse of Twenty-First Century

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1 Introduction

We have stepped in early days of 2021 and have carried with us the curse of 2020- COVID-19. It is wreaking havoc on the whole world at present, after its emergence in Wuhan in December 2019 and then global spread since February 2020 [1]. It has been declared as the Public Health Emergency of International concern in January 2020 and a pandemic in March 2020 by WHO. The first case was reported in Wuhan city of Hubei Province in south China on 31, December 2019 as unidentified pneumonia [2]. It is caused by severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2). As of 12 January 2021, more than 90.9 million cases have been confirmed, with more than 1.94 million deaths attributed to COVID-19. The disease has been able to put the world into a halt. The disease affects individuals in different ways ranging from no, mild, moderate to even severe symptoms requiring hospitalization. The most common symptoms include fever, dry cough, tiredness and less common symptoms are aches and pains, sore throat, diarrhoea, conjunctivitis, headache, loss of taste or smell, a rash on skin, or discolouration of fingers or toes. The disease mainly spreads by airborne transmission. When an infected person coughs, sneezes or speaks the infectious droplets are emitted and can enter another individual by mouth, nose or eyes [3]. It can also spread via fomites when an healthy individual comes in contact of them and the virus reaches their mucous membranes. There is no particular drug available for treatment. Only symptomatic treatment is recommended as per countries policies. Thus, preventive

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measures becomes more important than ever. Recommended preventive measures include social distancing, wearing face masks in public, ventilation and air-filtering, hand washing, covering one's mouth when sneezing or coughing, disinfecting surfaces, and monitoring and self-isolation for people exposed or symptomatic [4].

In this scenario, the awareness and adherence to the preventive measures becomes utmost important. Media plays a key role in this aspect. In today's era where media literally dictates one's decisions, it is essential that they play a fruitful role in such pandemic situations. During sudden outbreaks, the public needs access to timely and reliable information about the disease symptoms and its prevention [5]. Nowadays, social media are often seen as fast and effective platforms for searching, sharing, and distributing health information among the general population [6]. Media, thus becomes an essential weapon in our fight against COVID-19. The beneficial guidelines for preventing COVID-19 were reinforced among people through prominent advertisements on commonly used social media platforms. Facebook, Instagram, and television media posted the importance of 'social distancing' and 'stay at home' through free of cost and frequent, widespread ads. The printed media was utilized by supermarkets to promote their stores following the social distancing protocols. During road and air travel, there is continuous mention of ads like 'Stay home, stay safe,' 'Face covers mandatory in public,' 'COVID-19: less is more, avoid gatherings', 'give extra space with each other and on the road,' and 'wash your hands, stay healthy, avoid COVID-19'. This repetition is essential to consolidate the role of them in preventing the disease spread. This campaign was run extraordinarily by the media using all resources and its subtypes [7]. In our study, we have focused on this role of media and how it helps in reducing the spread of COVID-19.

In the present article, we formulated and analyzed a 4-compartment epidemiological model to study the impact of media on the spread of COVID-19, in a variable population with immigration. In the modeling process, we have assumed a population N which is the summation of susceptible unaware, susceptible aware and the infected classes respectively. The susceptible class (both aware and unaware) becomes diseased only by direct contact with the infected class. A part of the susceptible class will make conscious efforts to avoid being in contact with the infected under influence of media. The probability of contracting infection for individuals in aware class is less than those who are in unaware class. Further, we assume that a proportion of individuals recover and a fraction of these recovered individuals will join the aware susceptible class while the others will join the unaware susceptible class (may be due to ignorance, lacunae on their parts etc.). It is also assumed that the growth rate of the cumulative density of media coverage is proportional to the mortality caused by diseases in the infected population. Our study finds that when immigration is increased, the system becomes unstable. Also we found that the use of face masks and the efficiency of face mask, both are vital for maintaining a stable equilibrium. Further, we find that by increasing the implementation of media coverage above a threshold value, the system undergoes from stable to unstable through Hopf-bifurcation. Also, the proportion of infected individuals always decreases with an increase in the density of media coverage. In the next section, we formulate a mathematical model and examine the equilibrium point and stability of the system. Numerical results are given in later section. Finally, the paper ends with a brief discussion.

2 Literature Review

The World is now facing one of the biggest health challenge in human history in the form of COVID-19. The authors in [7] analyzed the role of mass media and public health communications from December 31, 2019 to July 15, 2020. They reviewed that the media played a dual role in this pandemic situation. They proved advantageous for spreading essential health information, health guidelines, helped in adherence to hygienic practices through repeated advertisements. The media ran the COVID-19 data through live update dashboards which played a big role for providing current situation reports. A trend among people to use telehealth and telemedicine was also noted. But at the same time, various misinformation like unscientific cures, unverified medicines, etc. were also spread using various media platforms. Fear and panic among the general population was also promoted by various media platforms. The authors in [8] concluded that social media has both advantage and disadvantage. The proper use of this will lead to the spread of essential information while misuse will lead to the spread of false information, myths, etc. So, the author advised that to be responsible while disseminating information through social media. Study of the influence of social media on public health measures of COVID-19 via public health awareness and public health behavioral changes in Jordan [9] through quantitative approach was adopted. A web questionnaire was used and 2555 social media users were sampled. The findings revealed that there is a positive influence of media on public health protection against COVID-19 as a pandemic. The analysis of a mathematical model [10] to study the impact of awareness programs by media on the prevalence of infectious disease revealed that by increasing the rate of implementation of awareness programs by media, the number of infected individuals decline and the system remains stable upto a threshold value, after crossing which the system oscillates. The scientists in [11] developed a three-dimensional mathematical model to study the impact of media coverage on the spread and control of infectious diseases. Stability analysis of the model revealed that the disease-free equilibrium is globally-asymptotically stable when the basic reproduction number (R_0) is less than unity. When $R_0 > 1$, the media influence is found to be strong enough. A mathematical model was developed and used to assess the efficacy of face masks, hospitalization and quarantine on COVID-19 [12]. The results revealed the above-mentioned interventions efforts should be high to control the outbreak in a short period of time. It also revealed that the interventions strength should be increased to eliminate the disease but only the sole use of face mask may not be enough in doing so.

3 Basic Assumptions and Model Formulation

***B*$_1$**: Let N(t) be the total population at time t the region under consideration. Here we consider that the total population is divided into three classes like susceptible unaware population (S_w), susceptible aware population (S_a) and infective population (I).

***B*$_2$**: Let the rate of immigration of susceptible is A. Also, we consider that M be the cumulative density of media coverage driven by the media in that region at time t which is related to the infective. We assume that diseases spread due to the contact between the susceptible and the infective only.

***B*$_3$**: It is assumed that susceptible avoid being in contact with the infective due to awareness through media coverage and forms a another class with a proportion λ called the aware susceptible. We assume that after treatment, a proportion of infected individuals recover and join susceptible class. After recovery, a fraction p of recovered people will join aware susceptible class whereas (1−p) will join unaware susceptible class.

***B*$_4$**: It is notified that the growth rate of the cumulative density of media coverage is proportional to the disease induced mortality rate of the infected population. Here β represents the contact rate of unaware susceptible with infective class and λ be the dissemination rate of awareness through media among susceptible due to which they form a different class. Here β_1 is a fraction which denotes the reduced probability of contracting infection and its value lies between 0 and 1.

***B*$_5$**: A proportion c_n of population wear face masks correctly and consistently in public places. Let ϵ_n be the efficacy of the face masks. Therefore, $Fn = 1 - \epsilon_n c_n$ represents the fraction which enters the infected class. A proportion h of aware population maintain social distance. The proper use of face masks reduces disease transmission effectively.

***B*$_6$**: The parameters d, γ and α denote the natural death rate, recovery rate and disease induced death rate respectively. Here, λ_0 represents the transfer rate of aware individuals to unaware susceptible class. The implementation of the awareness through media is proportional to the number of disease induced deaths.

***B*$_7$**: Let, the density of media coverage increase with increase in disease related death rate α. Here k be the proportionality constant which governs the implementation of awareness through media. We assume that μ_0 is the depletion rate of the media coverage due to ineffectiveness, social and psychological barriers in the population, etc. The parameter m represents the density level of media coverage on the disease from other region.

With these above assumptions our model system (Fig. 1) is

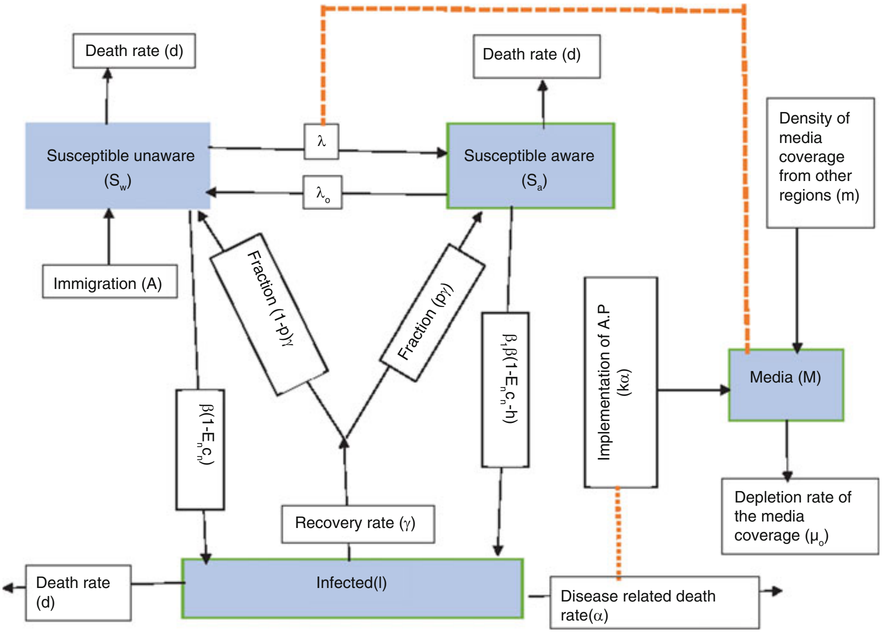

Fig. 1 Schematic diagram

$$\left.\begin{aligned}\frac{dS_w}{dt} &= A - \beta F_n S_w I - \lambda S_w M - dS_w + \lambda_0 S_a + (1-p)\gamma I \equiv G_1(S_w, S_a, I, M)\\ \frac{dS_a}{dt} &= \lambda S_w M + p\gamma I - \beta_1\beta(F_n - h)S_a I - dS_a - \lambda_0 S_a \equiv G_2(S_w, S_a, I, M)\\ \frac{dI}{dt} &= \beta F_n S_w I + \beta_1\beta(F_n - h)S_a I - \gamma I - \alpha I - dI \equiv G_3(S_w, S_a, I, M)\\ \frac{dM}{dt} &= k\alpha I - \mu_0 M + m \equiv G_4(S_w, S_a, I, M).\end{aligned}\right\} \tag{1}$$

The system (1) has to be analyzed with the following initial conditions,

$$S_w(0) > 0,\ \ S_a(0) \geq 0,\ \ I(0) \geq 0,\ M(0) \geq 0. \tag{2}$$

Using the fact that $N = S_w + S_a + I$, the system (1) transform to the following system:

$$\left.\begin{aligned}\frac{dN}{dt}&=A-dN-\alpha I\equiv G_1(N,S_a,I,M)\\\frac{dS_a}{dt}&=\lambda(N-S_a-I)M+p\gamma I-\beta_1\beta(F_n-h)S_aI-dS_a-\lambda_0S_a\equiv G_2(N,S_a,I,M)\\\frac{dI}{dt}&=\beta F_n(N-(1-\beta_1)S_a-I)I-\beta_1\beta hS_aI-(\gamma+\alpha+d)I\equiv G_3(N,S_a,I,M)\\\frac{dM}{dt}&=k\alpha I-\mu_0M+m\equiv G_4(N,S_a,I,M).\end{aligned}\right\}\tag{3}$$

Now it is sufficient to discuss system (3) rather that system (1). Here the region of attraction which is given by the set $\Gamma=\{(N,S_a,I,M)\in R_+^4:0\le S_w,I\le N\le\frac{A}{d},0\le M\le\frac{k\alpha(\frac{A}{d})+m}{\mu_0}\}$. According to existence and uniqueness theorem, the trajectories can not approach to unfeasible domain from positive octant which indicates that solution remain in positive octant. This ensure that the system is well defined.

Explicitly, the jacobian matrix at $\overline{E}=(\overline{N},\overline{S_a},\overline{I},\overline{M})$ can be defined as

$$\overline{J}=\begin{bmatrix}-d&0&-\alpha&0\\\lambda\overline{M}&m_{22}&m_{23}&\lambda(\overline{N}-\overline{S_a}-\overline{I})\\\beta F_n\overline{I}&-\beta F_n(1-\beta_1)\overline{I}-\beta_1\beta h\overline{I}&m_{33}&0\\0&0&k\alpha&-\mu_0\end{bmatrix},\tag{4}$$

where $m_{22}=-(\lambda\overline{M}+\beta_1\beta(F_n-h)\overline{I}+d+\lambda_0)$,
$m_{23}=-\lambda\overline{M}+p\gamma-\beta_1\beta(F_n-h)\overline{S_a}$,
$m_{33}=\beta F_n\overline{N}-\beta F_n(1-\beta_1)\overline{S_a}-2\beta F_n\overline{I}-\beta_1\beta h\overline{S_a}-(\gamma+\alpha+d)$.

4 Some Preliminary Results

4.1 Equilibria

The system (1) possesses the following equilibria: Disease free equilibrium (DFE) $E_0=(\frac{A}{d},\frac{m\lambda A}{d(m\lambda+(d+\lambda)\mu_0)},0,\frac{m}{\mu_0})$ and endemic equilibrium $E^*=(N^*,S_a^*,I^*,M^*)$.

4.1.1 Disease Free Equilibrium

E_0 is always feasible. The eigenvalues evaluate from (4) at E_0 are $-d<0$, $-d-\lambda_0<0$, $-\mu_0$ and (R_0-1). Thus, it is clearly indicates that E_0 is asymptotically stable if

$$R_0=\frac{\beta AF_n}{d(\gamma+\alpha+d)}<1\tag{5}$$

hold. Here R_0 is the basic reproduction number of system (3). Clearly, E^* exists for $R_0 > 1$.

4.1.2 Endemic Equilibrium

The endemic equilibrium at $E^* = (N^*, S_a^*, I^*, M^*)$ are $N^* = \frac{\frac{\beta}{d}F_n[A-(\alpha+d)I^*]-(\gamma+\alpha+d)}{\beta(1-\beta_1)F_n+\beta\beta_1 h}$, $M^* = \frac{k\alpha I^*+m}{\mu_0}$ while $I*$ is ensured by solving

$$A_1 I^{*2} + A_2 I^* + A_3 = 0, \tag{6}$$

where $A_1 = \frac{\beta_1\beta^2(F_n-h)(\alpha+d)}{c} - \frac{\kappa\lambda\alpha^2}{d\mu_0} - \frac{k\alpha\lambda}{\mu_0} - \frac{k\alpha\lambda}{\mu_0}\frac{\beta}{dc}F_n(\alpha+d)$,
$A_2 = \frac{A\alpha k\lambda}{d\mu_0} - \frac{\lambda\alpha m}{d\mu_0} - \frac{m\lambda}{\mu_0} + p\gamma - \frac{\beta_1\beta^2}{dc}(F_n - h)F_n A + \frac{\beta_1\beta(F_n-h)(\gamma+\alpha+d)}{c} - (d + \lambda_0 + \frac{\lambda m}{\mu_0})\frac{\beta}{dc}F_n(\alpha+d) + \frac{k\alpha\lambda}{\mu_0 c}[\frac{\beta}{d}F_n A - (\gamma+\alpha+d)]$,
$A_3 = (d + \lambda_0 + \frac{\lambda m}{\mu_0})F_n\frac{\beta A}{dc} + \frac{Am\lambda}{d\mu_0} - (d + \lambda_0 + \frac{\lambda m}{\mu_0})(\frac{\gamma+\alpha+d}{c})$, where $c = \beta(1 - \beta_1)(F_n) + \beta\beta_1 h$. Now for $I^* > 0$, solving (6) we get $I^* = \frac{-A_2\pm\sqrt{A^2-4A_1A_3}}{2A_1}$.

At E^*, the jacobian matrix of system (3) can be written as

$$J^* = \begin{bmatrix} n_{11} & 0 & n_{13} & 0 \\ n_{21} & n_{22} & n_{23} & n_{24} \\ n_{31} & n_{32} & n_{33} & 0 \\ 0 & 0 & n_{43} & n_{44} \end{bmatrix},$$

where $n_{11} = -d < 0$, $n_{13} = -\alpha < 0, n_{21} = \lambda M^* > 0$, $n_{22} = -\lambda M^* - \beta\beta_1(F_n - h)I^* - d - \lambda_0 < 0$, $n_{23} = -\lambda M^* + p\gamma - \beta_1\beta(F_n - h)S_a < 0$, $n_{24} = \lambda(N^* - S_a^* - I^*) > 0, n_{31} = \beta F_n I^* > 0, n_{32} = -\beta(1-\beta_1)F_n I^* - \beta_1\beta h I^* < 0, n_{33} = -\beta F_n I^* < 0, n_{43} = k\alpha > 0, n_{44} = -\mu_0 < 0$.
Now the corresponding characteristic equation is

$$\omega^4 + Q_1\omega^3 + Q_2\omega^2 + Q_3\omega + Q_4 = 0,$$

where the coefficients $Q_I, I = 1, 2, 3, 4$ are $Q_1 = -(n_{11} + n_{22} + n_{33} + n_{44}) > 0$,
$Q_2 = n_{11}n_{22} + n_{22}n_{33} + n_{33}n_{11} + n_{11}n_{44} + n_{22}n_{44} + n_{33}n_{44} - n_{23}n_{32} - n_{13}n_{31}$,
$Q_3 = n_{13}n_{31}n_{44} + n_{23}n_{32}n_{44} + n_{11}n_{23}n_{32} + n_{13}n_{31}n_{22} - n_{11}n_{22}n_{44} - n_{11}n_{33}n_{44} - n_{22}n_{33}n_{44} - n_{11}n_{22}n_{33} - n_{13}n_{21}n_{32} - n_{24}n_{32}n_{43}$,
$Q_4 = n_{11}n_{22}n_{33}n_{44} + n_{13}n_{21}n_{32}n_{44} + n_{11}n_{24}n_{32}n_{43} - n_{11}n_{44}n_{23}n_{32} - n_{13}n_{22}n_{31}n_{44}$.

Now, $Q_2 > 0$ if $n_{23}n_{32} > (n_{11}n_{22} + n_{11}n_{33} + n_{22}n_{33} + n_{11}n_{44} + n_{22}n_{44} + n_{33}n_{44} - n_{13}n_{31})$.

Also, $Q_3 > 0$ if $(n_{13}n_{31}n_{44} + n_{13}n_{31}n_{22} - n_{11}n_{22}n_{44} - n_{11}n_{33}n_{44} - n_{22}n_{33}n_{44} - n_{11}n_{22}n_{33} - n_{24}n_{32}n_{43}) > n_{13}n_{21}n_{32} - n_{11}n_{23}n_{32} - n_{23}n_{32}n_{44}$. Then $Q_1Q_2 - Q_3 > 0$ if $Q_1Q_2 > Q_3$ as well as $Q_3(Q_1Q_2 - Q_3) - Q_1^2Q_4 > 0$ if $Q_3(Q_1Q_2 - Q_3) >$

$Q_1^2 Q_4$. Then, by the Routh-Hurwitz criterion, E^* is locally asymptotically stable which depending upon system parameters.

Remark The system could have a Hopf-bifurcation at the coexistence equilibrium if the following two conditions are satisfied,

$$Q_1(A_c)Q_2(A_c)-Q_3(A_c)=0, \quad Q_1'(A_c)Q_2(A_c)+Q_1(A_c)Q_2'(A_c)-Q_3'(A_c)\neq 0. \tag{7}$$

4.2 Hopf Bifurcation at Coexistence

Theorem (Hopf-Bifurcation) *If $\psi_1(A)>0$, then the equilibrium E^* of system (3) is locally asymptotically stable. If there exists $A_c \in R$ such that $\psi_1(A_c)=0$ and $(\frac{d\psi_1}{dA})\mid_{A_c}\neq 0$, then as A passes through A_c, a Hopf-bifurcation occurs at E^*.*

For positive equilibrium $E^*=(N^*, S_a^*, I^*, M^*)$, the characteristic equation is

$$\omega^4+Q_1\omega^3+Q_2\omega^2+Q_3\omega+Q_4=0.$$

Define

$$\psi_1(A)=Q_1(A)Q_2(A)Q_3(A)-Q_3^2(A)-Q_1^2(A)Q_4(A). \tag{8}$$

Let $\omega_i\,(i=1,2,3,4)$ be the roots of above characteristic equation. Then we have

$$\begin{aligned}
\omega_1+\omega_2+\omega_3+\omega_4&=-Q_1,\\
\omega_1\omega_2+\omega_1\omega_3+\omega_1\omega_4+\omega_2\omega_3+\omega_2\omega_4+\omega_3\omega_4&=Q_2,\\
\omega_1\omega_2\omega_3+\omega_1\omega_3\omega_4+\omega_2\omega_3\omega_4+\omega_1\omega_2\omega_4&=-Q_3,\\
\omega_1\omega_2\omega_3\omega_4&=Q_4.
\end{aligned} \tag{9}$$

If there exists $A_c \in R$ such that $\psi_2(A_c)=0$, then by the Routh-Hurwitz criterion at least one root, say ω_1, has real part equal to zero. From the fourth equation of (8) it follows that $Im\ \omega_1=\omega_0\neq 0$, and hence there is another root, say ω_2, such that $\omega_2=\overline{\omega}_1$. Since $\psi_2(A)$ is a continuous function of its roots, ω_1 and ω_2 are complex conjugate for A in an open interval including A_c. Therefore, the equation in (8) have the following form at A_c,

$$\begin{aligned}
\omega_3+\omega_4&=-Q_1,\\
\omega_0^2+\omega_3\omega_4&=Q_2,\\
\omega_0^2(\omega_3+\omega_4)&=-Q_3,\\
\omega_0^2\omega_3\omega_4&=Q_4.
\end{aligned} \tag{10}$$

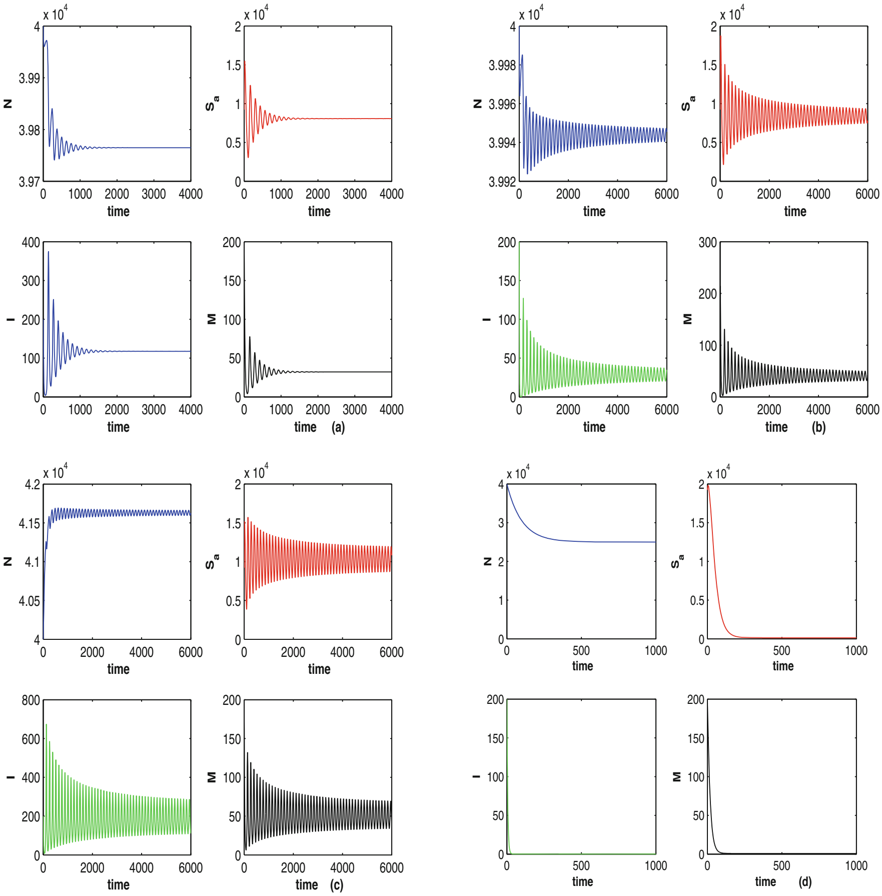

Fig. 2 **(a)** The equilibrium point E^* is stable for the parametric values as given in (11). **(b)** The figure depicts oscillatory behavior around the coexistence (endemic) equilibrium point E^* of system (1) for $k = 4.2$. **(c)** The figure depicts oscillatory behavior around coexistence (endemic) equilibrium point E^* of system (3) for $A = 450$. **(d)** The figure depicts disease free equilibrium E_0 for $A = 250$

If ω_3 and ω_4 are complex conjugate, from the first Eq. (9) it follows that $2Re\ \omega_3 = -Q_1 < 0$. If ω_3 and ω_4 are real, from the first and fourth equations of (9) it follows that $\omega_3 < 0$ and $\omega_4 < 0$. Also after some calculations it follows that $\frac{d}{d\gamma_2} Re(\omega_1)_{A=A_c} = -\frac{Q_1}{2[Q_1^2 Q_4 + (Q_1 Q_2 - 2Q_3)^2]} \frac{d\psi_1}{dA} \ |_{A_c} \neq 0$. Thus, we have the following result.

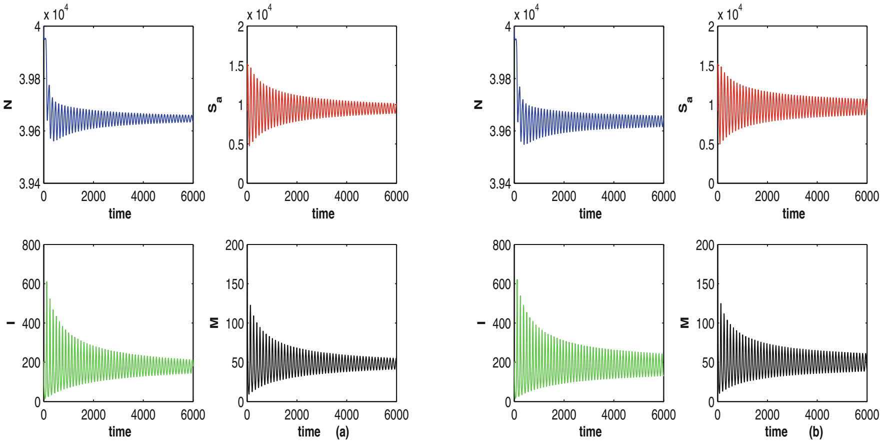

Fig. 3 **(a)** The figure depicts oscillatory behavior around coexistence (endemic) equilibrium point E^* of system (3) for $\epsilon_n = 0.1$. **(b)** The figure depicts oscillatory behavior around coexistence (endemic) equilibrium point E^* of system (3) for $c_n = 0.01$

5 Numerical Simulations

In this section, we study the impact of awareness programs with the help of numerical simulation. Here we investigate the effects of the various parameters on the qualitative behavior of the system, by using MATLAB. We begin with a parametric values[10, 13, 14]

$$A = 400, \beta = 0.00002, \beta_1 = .2, \lambda = 0.0002, \lambda_0 = .02, \gamma = .6, \alpha = 0.02,$$
$$d = 0.01, \mu_0 = 0.06, \epsilon_n = 0.5, c_n = .1, h = 0.02, \ k = 0.8, \ p = 0.05, m = .05. \tag{11}$$

Dealing with above set of parametric values, we note that the system is locally asymptotically stable at endemic equilibrium $E^* = (39637, 7834, 182, 37)$ in which $R_0 = 1.2698$(cf. Fig. 2a). Taking $k = 4.2$, the system exhibits oscillations around E^* (cf. Fig. 2b). Figure 2c illustrate the oscillatory behavior of each population for high value of A ($A = 450$). Analytical, we see that endemic equilibrium E^* exists if $A > \frac{d(\gamma+\alpha+d)}{\beta}$. We obtain the critical value of immigration rate $A = 333$, above which the endemic equilibrium exists. Taking $A = 250$, we observe that the system exhibits disease free equilibrium E_0 which satisfy our analytical finding (cf. Fig. 2d). It is interesting to see that low value of $\epsilon_n = 0.1$ and $c_n = 0.01$ play a big impact to destabilize the whole system respectively (cf. Fig. 3a, b). Now for clear understanding of dynamic change, we plot a bifurcation

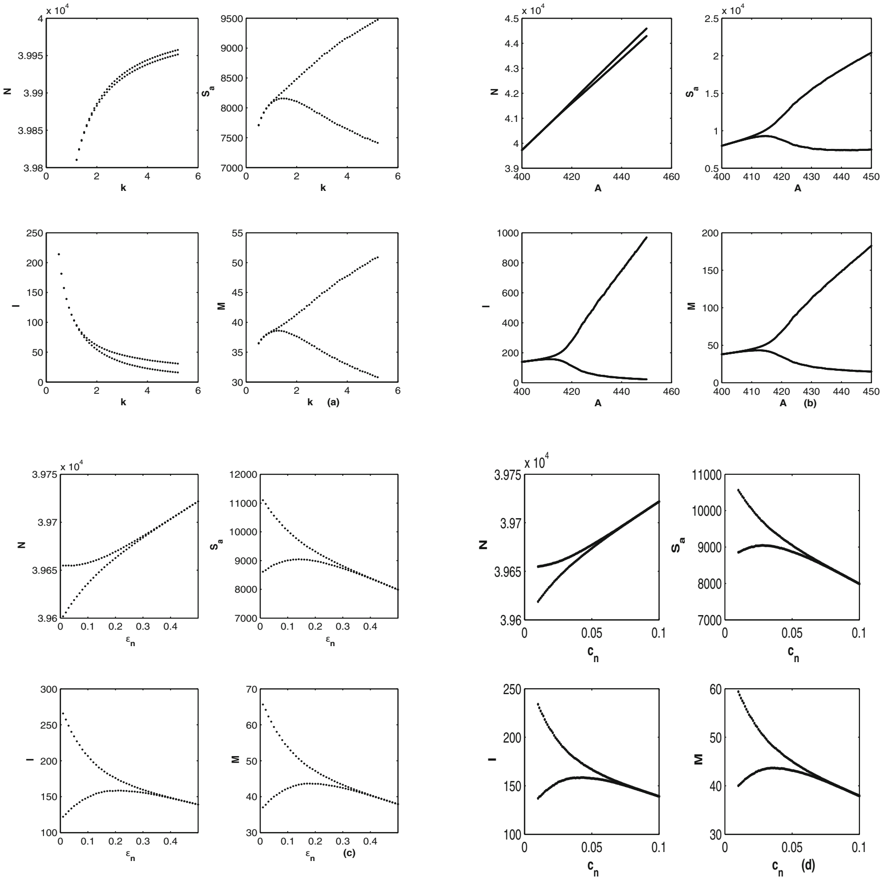

Fig. 4 **(a)** Bifurcation diagram for k. **(b)** Bifurcation diagram for A. **(c)** Bifurcation diagram for ϵ_n. **(d)** Bifurcation diagram for c_n

diagram with respect to k. Form Fig. 4a, it follows that lower values of k, the system is stable but above a threshold value of $k = k_c$, the system losses its stability and periodic solution arises through Hopf-bifurcation. Further, we also vary A as a free parameter, a bifurcation diagram (cf. Fig. 4b) indicates that the system looses its stability for high value of A after it crosses the critical value. Further, we plot another two bifurcation diagrams for efficacy of the face masks i.e. ϵ_n and masks compliance, c_n respectively. It is clear to see that the system looses stability for low value of these two parameters (cf. Fig. 4c, d). Figure 5a illustrates the different steady state behaviour of infected class in the system (3) for the parameter A. Here, we see a Hopf bifurcation points at $A = 419$ (denoted by a red star (H)) with eigenvalues -0.103689, -0.001029, $\pm 0.52642i$ and first Lyapunov coefficient being $-1.881669e^{-10}$ and generates a family of stable limit cycle bifurcates from the H and loses its stability. Here $A = 333(BP)$ denotes the branch point of the

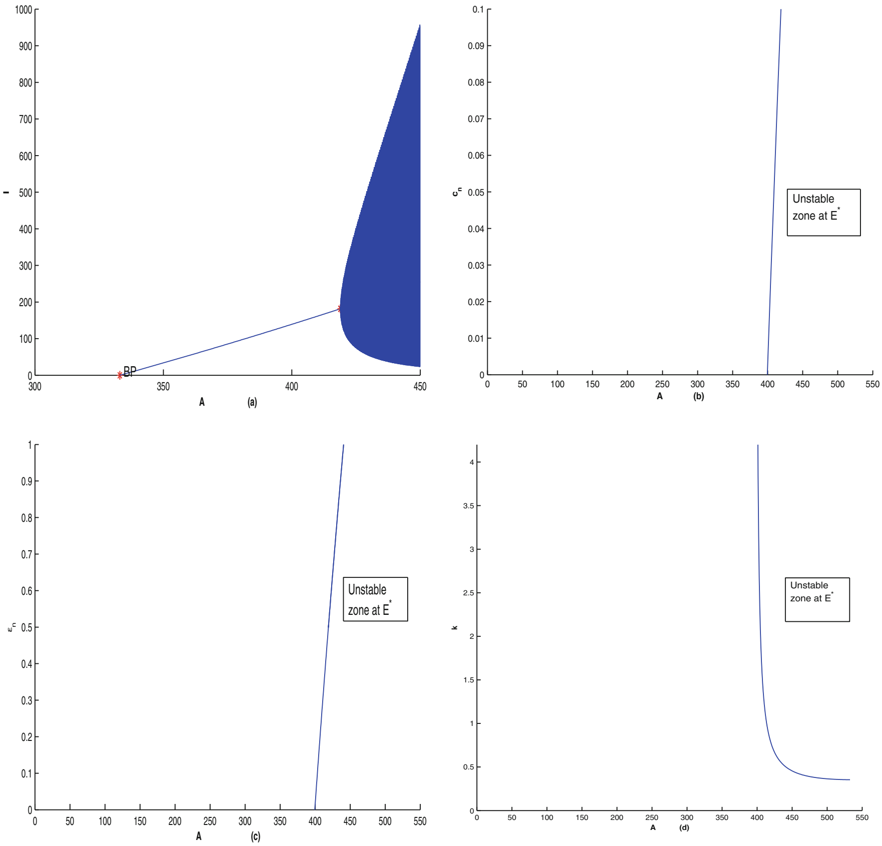

Fig. 5 **(a)** The figure depicts different steady-state behaviors of infected population for the effect of A. **(b)** Two parameter bifurcation diagram for $A - c_n$. **(c)** Two parameter bifurcation diagram for $A - \epsilon_n$. **(d)** Two parameter bifurcation diagram for $A - k$

system (3) with eigenvalues are o, -0.06, -0.03, -0.01. Figure 5b–d represent two parameters bifurcation diagrams for $A - c_n$, $A - \epsilon_n$ and $A - k$ respectively.

6 Discussion

The information and the awareness of the preventive strategy for COVID-19 is majorally emphasized through media coverage. So, in our paper we have analyzed a 4-compartment mathematical model. It is assumed that pathogens are transmitted via direct contact between the susceptible and the infective. The model exhibits two equilibria like the disease-free equilibrium and endemic equilibrium under certain conditions. Firstly, the model is studied analytically and shown that when the basic reproduction number $R_0 < 1$, the system exhibits disease-free equilibrium. For $R_0 > 1$, it leads to the existence of an endemic equilibrium.

Our study indicates that if we increase the density of media coverage, the number of infected individuals decline. But after crossing the threshold value, system becomes unstable. The constant immigration may be one of the possible reasons of such outcomes. Further, we observe that lower value of immigration rate the system becomes disease-free equilibrium. Also, the efficacy of face mask and it's usage in public areas helps in keeping the system stable.

References

1. M. Mandal, S. Jana, S. K. Nandi, A. Khatua, S. Adak and T. K. Kar. A model based study on the dynamics of COVID-19: Prediction and control. *Chaos, solitons, and fractals*, 136, 109889 (2020) .
2. H. Lu, C. W. Stratton and Y. W. Tang. Outbreak of pneumonia of unknown etiology in Wuhan, China: the mystery and the miracle. *Journal of medical virology,* 92(4), 401–402 (2020).
3. N. V. Doremalen, T. Bushmaker, D. H. Morris, M. G. Holbrook, A. B. N. Williamson Gamble, A. B. N. et al. Aerosol and surface stability of SARS-CoV-2 as compared with SARS-CoV-1. *New England Journal of Medicine*, 382, 1564–1567 (2020).
4. World Health Organization, Coronavirus disease 2019 (COVID-19). WHO situation report -73, 2020. Avaiable from: https://www.who.int/docs/default-source/coronaviruse/situation-reports/20200402-sitrep-73-covid-19.pdf.
5. P. Bastani, and M. A. Bahrami. COVID-19 Related Misinformation on Social Media: A Qualitative Study from Iran. *Journal of medical Internet research* (2020).
6. Y. Zhao and J. Zhang. Consumer health information seeking in social media: a literature review. *Health information and libraries journal*, 34(4), 268–283 (2017).
7. A. Anwar, M. Malik, V. Raees and A. Anwar. Role of mass media and public health communications in the COVID-19 pandemic. *Cureus*, 12(9) (2020).
8. D. A. González-Padilla and L. Tortolero-Blanco. Social media influence in the COVID-19 pandemic. *International braz j urol*, 46, 120–124 (2020).
9. H. Al-Dmour, A. Salman, M. Abuhashesh and R. Al-Dmour. Influence of social media platforms on public health protection against the COVID-19 pandemic via the mediating effects of public health awareness and behavioral changes: integrated model. *Journal of medical Internet research*, 22(8), e19996 (2020).
10. S. Samanta, S. Rana, A. Sharma, A.K. Misra and J. Chattopadhyay. Effect of awareness programs by media on the epidemic outbreaks: A mathematical model. *Applied Mathematics and Computation*, 219(12), 6965–6977 (2013).
11. J. Cui, Y. Sun, and H. Zhu. The impact of media on the control of infectious diseases. *Journal of dynamics and differential equations*, 20(1), 31–53 (2008).
12. A. K. Srivastav, P. K. Tiwari, P. K. Srivastava, M. Ghosh and Y. Kang. A mathematical model for the impacts of face mask, hospitalization and quarantine on the dynamics of COVID-19 in India: deterministic vs. stochastic[J]. *Mathematical Biosciences and Engineering*, 18(1): 182–213 (2021).
13. L. Zuo and M. Liu. Effect of Awareness Programs on the Epidemic Outbreaks with Time Delay. Abstract and Applied Analysis, *Hindawi Publishing Corporation*, Volume 2014, Article ID 940841 (2014).
14. L. Zuo, M. Liu and J. Wang. The Impact of Awareness Programs with Recruitment and Delay on the Spread of an Epidemic, Mathematical Problems in Engineering, *Hindawi Publishing Corporation*, Volume 2015, Article ID 235935 (2015).