Chapter 4
Rotorcraft Flight Performance

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Abstract This chapter introduces rotorcraft steady-state flight performance and stability, and we explain key concepts of the conventional helicopter as well as other rotorcraft types (tandem helicopter and compounds). Numerical models of performance estimations are provided for level flight, climb-out, and descent. Stability issues presented include longitudinal/lateral trim and speed stability. Different take-off procedures are illustrated, alongside the certification requirements (Category A and B rotorcraft). There is further discussion of ground effects, such as lift augmentation and ground resonance. We provide examples of methods used to estimate the direct operating costs, which are one of the major limiters to the use of rotorcraft. We complete the performance analysis with fuel planning methods, payload-range assessment, and speed augmentation concepts (compound helicopters).

Nomenclature

AEO	all engines operating
AI	autorotation index
OEI	one engine inoperative
SAR	specific air range
SEP	specific excess power
SFC	specific fuel consumption
TAS	true air speed
A, B	empirical coefficients in ground effect equation
A	rotor disk area
c	wing or blade chord
C_{DV}	vertical-flow drag coefficient
C_{D_g}	drag coefficient in ground effect
$C_{L\alpha}$	lift-curve slope

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C_M	pitching moment coefficient
C_T	thrust coefficient
$C_{T\sigma}$	effective blade loading coefficient, C_T/σ
$C_{y\beta}$	side force derivative
D	drag force
D_L	disk loading, T/A
I_b	blade's polar moment of inertia
$k_1 \cdots k_4$	stability derivatives, Eq. 4.25
k	blockage factor
k_g	ground effect factor, Eq. 4.16
h	rotor height above ground
L	lift force
M	true Mach number
n	number of operating engines
P	power
q	dynamic pressure
Q	torque
R	rotor radius
$\mathcal{R}$	rolling resistance
t	flight time , s
T	rotor thrust
T_p	propeller thrust
V	flight speed
W	gross- or take-off weight
W_1	corrected weight, due to rotor downwash
W_{f6}	fuel flow per engine , kg/s
x_{cg}	distance between main rotor and centre of gravity
x_{ht}	distance between H-stabiliser and centre of gravity
x_{tr}	distance between main- and tail rotor shafts
X	flight range, n-miles
Y_{vt}	side force of the vertical stabiliser
z	flight altitude (m or feet)
$\overline{z}$	normalised ground clearance, z/D

Greek Letters

α	inflow angle
α_r	rotor disk tilt angle
β	sideslip angle
δ	differential
γ	ratio between specific heats
λ_i	inflow velocity ratio
φ	tail rotor cant angle

μ rotor advance ratio
μ^* normalised rotor advance ratio, $\mu\sqrt{2/C_T}$
μ_r ground rolling coefficient
σ rotor solidity
θ collective pitch angle
ρ air density
Ω rotor angular speed

Subscripts/superscripts

$\overline{(.)}$ mean value
$(.)_a$ airframe
$(.)_{cr}$ critical value
$(.)_i$ induced
$(.)_j$ iteration count
$(.)_{ige}$ in ground effect
$(.)_{oge}$ out of ground effect
$(.)_{gbox}$ gearbox
$(.)_{mr}$ main rotor
$(.)_p$ payload
$(.)_{ht}$ horizontal stabiliser
$(.)_{ref}$ reference value
$(.)_{tr}$ tail rotor
$(.)_{vt}$ vertical stabiliser
$(.)_w$ wing
$(.)^*$ corrected value

4.1 Introduction

The value of the helicopter is determined by what an operator can do with it. Typical questions are: How fast can it travel?—How far can it go?—How long can it stay airborne?—How much payload can it carry?

The issue of rotorcraft speed has been around since the first generation of rotorcraft, and it has been emphasised that speed is not the best virtue of the helicopter.

The ability to stay airborne motionless (hover) is indeed a virtue, since it allows the vehicle to perform operations such as search and rescue, precise delivery or collection of payload, raising and lowering sling loads, and a variety of other missions. Where the helicopter falls short, is overall endurance, e.g. the amount of time it can remain airborne, since this is seldom longer than 2 h. In the following sections we will review the performance characteristics at the most important flight segments, which are identified as follows: hover, vertical flight, climb-out, loiter, cruise, descent, weight-drop, weight-load, accelerate, decelerate, turn, etc. Figure 4.1 shows an example of

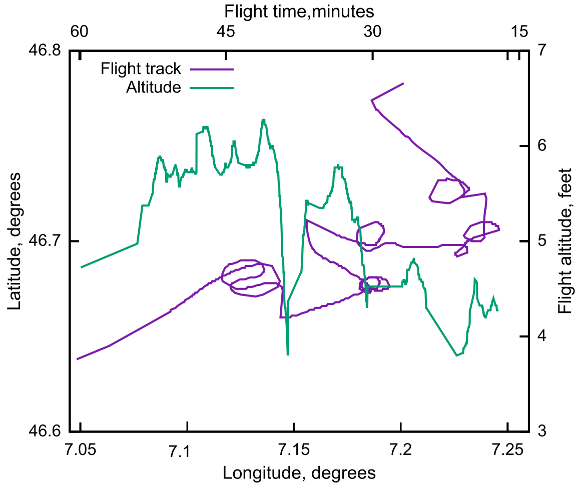


Fig. 4.1 Flight track and altitude of a search or recoinnassaince helicopter operation

flight operation for a certain helicopter on a typical day. The data have been extracted from an ADS-B database, and show show in this instance the helicopter performs several loops around a search area and changes altitude almost continuously.

When we look at the rotor performance, the tip speed must maintain a range between a minimum of stored kinetic energy (for autorotative performance requirements) and noise limitations—at all flight speeds, as explained in Fig. 4.2. As the flight speed increases, the rotor suffers one of two problems: either compressibility effects, due to high transonic Mach numbers of the advancing blade, or dynamic stall limitations, due to stall of the retreating blade, severe stall on the advancing blade, both compounded by large pitch oscillations. At point B in the graph, both aerodynamic compressibility and dynamic stall contribute to constraining the flight speed. Note that all these limitations are reached even before we involve issues of engine power. In fact, in most cases engine power is not a limiter to helicopter speed.

There must be ways to make rapid assessments of the helicopter capability by using first-order analysis. For example, the disk loading W/A is an indicator of the gross rotor loading. At maximum weight, the thrust delivered must be well in excess of the weight. High disk loading is related to the strength of the rotor downwash, which dictates what kind of ground operations are feasible from a safety point of view. Light utility helicopters have the lowest disk loading, and tilt-rotors (convertiplanes) have the largest loading.

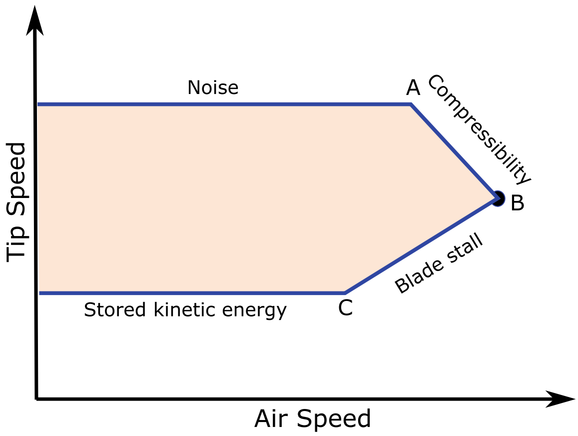


Fig. 4.2 Tip speed versus helicopter speed

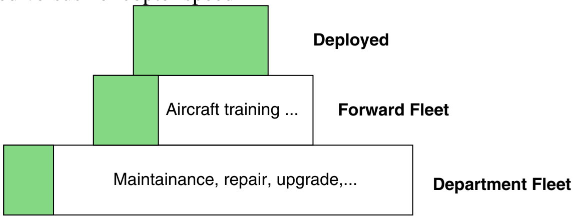


Fig. 4.3 Example of military rotorcraft reliability, availability and maintenance map

4.2 Reliability, Availability and Maintenance

No single air vehicle can operate continuously over a long period of time (measured in hours or days or weeks), due to the complexity of the logistics that is required to supply routine maintenance, spare parts, etc. This can be particularly critical for the military. Only a small fraction of the helicopters in the fleet are operational at any given time. This is indicated as "deployed" in the graph of Fig. 4.3. The remaining large portion of the fleet is either a "forward fleet", mostly used for training purposes, or unavailable, because under maintenance, upgrade, repair, testing, in storage, etc. The case shown is typical of many military forces, with only $\sim$1/5 rotorcraft in the fleet that can be deployed at any given time. This case indicates a very complex and costly supply chain. Thus, efforts are required to make the pyramid steeper (or even revert it), by increasing the number of deployable vehicles, reducing the the vehicles used for training purposes (through flight simulators), reducing the need for maintenance and repair However, the problem of affordability remains, since very often the installation of every kit of equipment (avionics, systems, weapons) on every vehicle is not affordable, and perhaps not even needed.

4.3 Certification Types

There are two certification types, noted as Category A and Category B. Transport rotorcraft are governed by well established regulations, such as the Federal Aviation Regulations, Chap. 14, Part 29, which deal with the *airworthiness standards of transport category rotorcraft*.

Category A rotorcraft are multi-engine vehicles that are designed operate in demanding environments, even in cases of One Engine Inoperative (OEI). Specifically, they can take-off and land from/to challenging heliports, including offshore platforms, ship decks in presence of shear winds, tall buildings, hospitals. They can operate in narrow valleys, and in severe weather conditions, although the level of severity depends on the avionics systems available on the aircraft and the training levels of the pilots. All rotorcraft with MTOW $>$ 20,000 pounds/9072 kg, and 10 or more passengers, must have a *Type A* certification, regardless of the number of engines (FAR §29.1(c)).

Category B rotorcraft are single-engine vehicles that can operate from relatively safe areas, where a landing option is always available in case of engine failure. All rotorcraft with MTOW $<$ 20,000 pounds/9072 kg, and 9 or less passengers, can have a *Type B* certification (FAR §29.1(f)). Provisions are given by the FAR §29 for cases not listed here for other weight/passenger combinations.

4.4 Point Performance Parameters

A point-performance index is a parameter that depends on the instantaneous operational conditions of the rotorcraft: flight altitude, air speed, climb rate, gross weight and atmospheric conditions. As one of these operational conditions change, so doe the point performance.

The specific air range (SAR) is the distance that can be flown by burning one unit of fuel. This is either expressed in unit volume or unit mass (or weight, to add confusion). In this instance it is defined as the true airspeed divided by the fuel flow

$$\mathrm{SAR} = \frac{V}{W_{f6}} = \frac{1}{\mathrm{SFC}} \frac{V}{P_{shaft}} \tag{4.1}$$

Note that in this case we use the fuel flow in kg-weight, as this is common engineering practice. A preferred unit for the SAR is [m/kg], or [n-mile/lb] in imperial units. Often a hybrid unit is used, such as [n-mile/kg]. This parameter is only useful when the helicopter flies in level flight. In other flight conditions, its use is inappropriate, and in hover it is meaningless, because SAR $= 0$.

To determine hover performance we use is the specific endurance, defined as the time required to burn one unit of fuel. Again, we use the unit of mass, so we have

$$E = \frac{1}{W_{f6}} \tag{4.2}$$

Specific endurance is expressed in seconds, because fuel flows are of the order of a fraction of a [kg/s].

The specific excess power (SEP) is

$$\text{SEP} = \frac{P_e - P_{total}}{W} \tag{4.3}$$

e.g. the ratio between the net excess power and the gross weight. The excess power is the difference between what the engines can deliver at the required operational point and the power required to fly at at that point. Note that the physical dimensions of this quantity are [m/s], and thus it is a velocity, not a power. This parameter will be used in the discussion of climb performance, Sect. 4.8.

The specific fuel consumption (SFC), discussed in Chap. 3, is also a point performance, because its value changes with all the operational parameters listed.

4.5 Take-off and Landing Procedures

We discuss a number of ground operations intended to comply with the safety regulations of Category A rotorcraft. The actual take-off depends on whether there is a landing alternative in case of emergency requiring the flight to be aborted. For cases where such an alternative does not exist (take-off from helipads on off-shore bases, building tops, etc.), a short vertical climb is followed by a backward climb-out to a target altitude leaving the helipad slightly in front of the rotorcraft. At this target altitude, the rotorcraft pitches down, loses some altitude whilst accelerating and then climbs-out to its target cruise altitude and speed. If an engine failure occurs, the rotorcraft must demonstrate the ability to clear a minimum height of 35 ft above ground/sea in the most unfavourable conditions. This operation is displayed in Fig. 4.4.

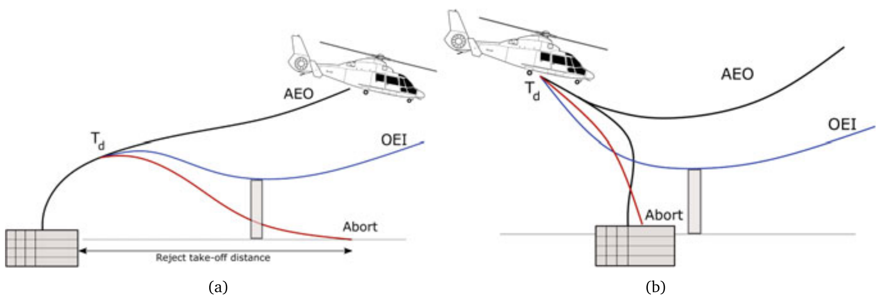


Fig. 4.4 Take-off procedures of Category A rotorcraft with and without alternative (emergency) landing option

When an alternative landing site exists, the same rotorcraft will perform a forward climb-out, following a small vertical climb, so that if the flight has to be aborted, it may be able to land safely. In any case, it must be able to demonstrate the same capability, e.g. clearing a minimum height of 35 ft with one engine inoperative in unfavourable conditions.

4.6 Emergency Landing Operations

Emergency landing operations occur for a variety of reasons, which include on-board emergencies, a fire, or a minor mechanical failure. In this instance we discuss the consequences of losing engine power, partially or totally. On a twin-engine rotorcraft, one engine failure is compensated by the transmission system, and the remaining engine is required to meet an increased torque demand. Aside from the control issues around the actual process of disengaging one engine from the main power train Chap. 3, there are flight mechanics issues and flight control response time.

In case of total loss of engine power, with any type of rotorcraft configuration, the default recovery strategy relies on using autorotation to slow down the loss of rotor rpm and the loss of flight altitude. *For a given gross weight and flight altitude, recovery and landing in autorotation depends critically on the relationship between flight altitude and airspeed.*

FAR Part 27, concerned with light helicopters ($W < 2{,}741$ kg), establishes performance criteria in autorotation. In §27.87, it is reported that *if there is h-V combination for which landing is not possible with engine failure, the flight envelope of unsafe h-V combination must be determined.*

Thus, the onus is on the helicopter design authority to demonstrate such control requirements. A typical h-V chart looks like the one in Fig. 4.5.

In this chart we display two shaded areas, one in low speed and the other in high speed, where helicopter recovery in autorotation is *highly unlikely* ("Avoid"). In fact, in the most up-to-date versions of this chart we have confidence levels that depend on the reaction time of the pilot. This time is measured in seconds, with 1 s being rapid, and 3 s being dangerously slow. The shorter the response time, the wider is the chart in the V-axis. Also shown is an ideal take-off flight path, denoted by a thick blue line that avoids both danger areas in the diagram: this is the safe take-off trajectory at the safe take-off speed, V-Toss.

Recovery and control is more problematic in hover. At the lowest point of the chart, with $V = 0$, there is a sudden vertical descent which can only be slowed down by quick pitch control and and aircraft flare to soften a crash landing. At the highest point, there is the possibility of gaining some horizontal speed before attempting to regain control. In any case, the entry point into autorotation is critical. As pointed out by Prouty [1], *a bad autorotation is usually survivable, but a bad beginning of an autorotation is usually not.*

The analysis of autorotational performance begins with consideration of the autorotative index, defined as the kinetic energy stored in the rotor mathematically, there are several definition. The physical definition is $\mathrm{AI} = I_b\Omega^2/2W$ (I_b is the polar moment of inertia of the blade); in practice some engineers prefer to use a practical definition:

$$\mathrm{AI} = \frac{I_b\Omega^2}{2\,W D_L} \tag{4.4}$$

where D_L is the disk loading, and the numerical value expressed in [ft^3/lb]. Light helicopters like the Bell 206 and the Robinson R-22 have a high autorotation index ($\mathrm{AI} \simeq 35-40\,\mathrm{ft}^3$/lb, or $2.2-2.5\,\mathrm{m}^3$/kg), and heavy-lift helicopters, such as the CH-53E have a relatively low AI ($\mathrm{AI} \simeq 10$ ft^3/lb), which makes an emergency landing more problematic. However, the CH-53E is a three-engined rotorcraft, rather than a single engine, and the odds of total engine failure are considerably lower.

By defining a critical height h_{cr} and a critical speed V_{cr}, Pegg [2] demonstrated that it is possible to correlate test points corresponding to a variety of gross weights and flight altitudes into a single curve. The critical speed is the largest speed in the low-speed danger area in Fig. 4.5, and h_{cr} is its corresponding flight altitude. The normalised plot is V/V_{cr} versus h/h_{cr}. One such chart is shown in Fig. 4.6 for selected flight data points.

Within an autorotative landing operation, there are five distinct phases.

The problem of autorotation is a subject that has vast coverage in the technical literature, from the very beginning [3, 4] to this day. The most recent research demonstrates that physiological aspects, pilot training and workloads are important in the determination of the final outcome of an autorotative manoeuvre.

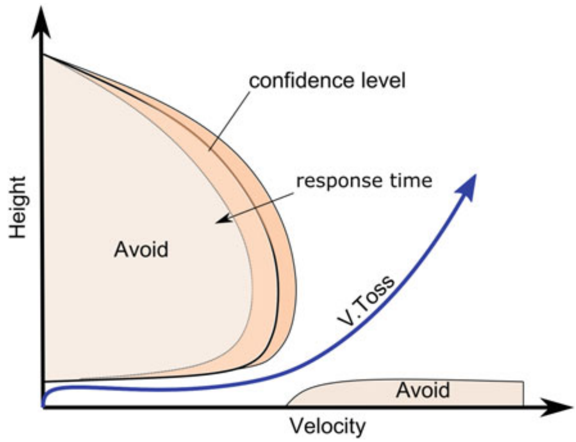


Fig. 4.5 Height-velocity chart

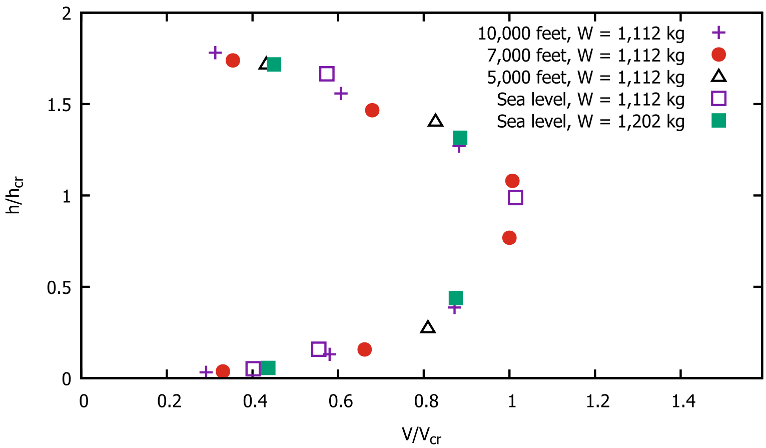


Fig. 4.6 Height-velocity chart in normalised format. Data from Ref. [2]

4.7 Vertical Flight Performance

Hover is unique capability of the helicopter, but it is performed sparingly by most helicopters. Only a few helicopters specialise in low-speed and hover operations, such as the Kaman K-1200 K K-Max, which is a single-engine (Honeywell T53-17A-1) synchropter with high-weight sling load capability.

There are safety issues, as discussed in the previous section, and fuel burn rates are high; thus, hover endurance is relatively short. A typical hover chart looks like the one in Fig. 4.7. The solid lines A, B $\cdots$ denote hover ceiling curves at constant gross weight. In this case, curve A denotes the highest weight and D is the lowest weight. The dashed lines display the standard day, a cold and a hot day. As the temperature decreases, the hover ceiling increases. This is the result of improved aero-thermodynamic performance of the gas turbine engine, alongside favourable air density effects.

The endurance is calculated from the integration of the specific endurance, up to a point when the fuel remaining reaches the minimum regulatory levels. If the fuel burn rate is dW_{f6}/dt, then the instantaneous fuel burn is $dW_{f6} = W_{f6}dt$. To calculate the time T_e required to burn a target amount of fuel W_f^* (endurance), we take the time step $dt = dW_{f6}/W_{f6}$ and carry out an integration

$$T_e = \int_o^{W_f^*} \frac{dW_{f6}}{W_{f6}} \tag{4.5}$$

At each point we need the fuel flow, which is calculated by a complete power analysis of a trimmed rotorcraft in hover. Note that in many cases the endurance quoted in the *Flight Crew Operating Manuals* refers to a loiter speed rather than hover.

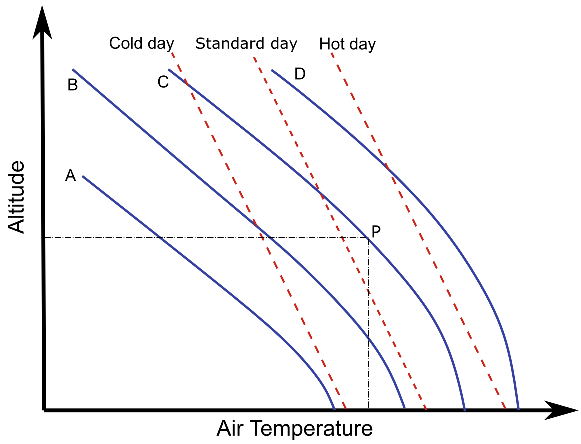


Fig. 4.7 Typical hover chart

4.8 Climb Performance

In Sect. 4.5 we discussed climb-out procedures. Now we calculate the power required to climb. This is done using *excess power*, defined as

$$v_c = \frac{P_e - (P_{mr} + P_{tr} + P_a + P_{gbox} + \cdots)}{W} \tag{4.6}$$

where the total power depends on the rotorcraft configuration. In this case, we have indicated the presence of a tail rotor. The gross weight requires a further clarification. In high-powered climb, the rotor wakes have a low skew angle and flow around the airframe, which may have a considerable amount of blockage, for example due to landing gear, sponsons, external stores, etc. This blockage generates a so-called *vertical drag* that must be overcome along with the gross weight. The net result is an increase in *apparent* weight. Although the inboard sections of the blades do not produce much thrust, there is some interference that must be accounted for, and in some heavy rotorcraft this additional download can be of the order of 7−12%. Figure 4.8 shows an example of a heavy-lift helicopter in such a situation. The wake flows around the sponsons and may interfere with the tail-rotor inflow, point B.

A low-order method for the estimation of this vertical drag is shown by Stepniewsky & Keys [5], and is based on using the strip theory. The result is that the gross weight is corrected as

$$W_1 = W + qA_1C_{DV} \tag{4.7}$$

where q is the dynamic pressure based on the vertical flow, A_1 is the wake blockage area, and C_{DV} is a vertical drag coefficient. Based on flat plate theory, $C_{DV} \sim 1$.

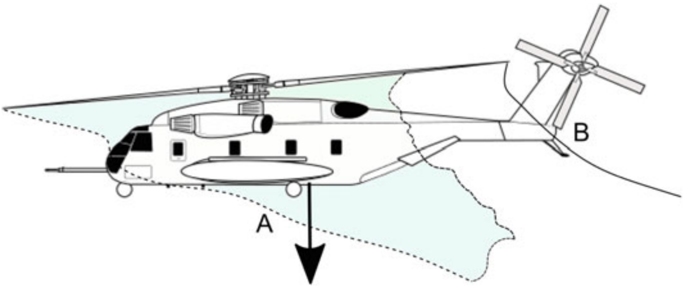


Fig. 4.8 Wake interference of the CH53 in high-powered climb-out.

The climb rate reaches a maximum a intermediate speeds, around the loiter speed. This result is easily justified, because at the loiter speed minimum power is required to fly at constant altitude, and more excess power is available for climb. At the maximum speed, there is no climb margin.

4.9 Ground Effects

The wakes considered so far are free from external constraints, but in reality when the rotorcraft operates near the ground there is strong interference, due to the fact that the wakes cannot penetrate solid walls and are thus forced to bounce back and spread outwards. There is a variety of situations, out of which we select a few for discussion. The simplest case is a rotor in hover near the ground. It has been demonstrated that at constant shaft power, the required hover thrust decreases. Vice versa, at a fixed thrust, the required power decreases. Ground effect only materialises when the rotor has a ground clearance of one rotor diameter or less. The more widespread empirical correlations are those of Cheeseman and Bennett [6] and Hayden [7], the latter one based on more recent ground-effect tests. The former equation for ground effect in hover is written as

$$\left(\frac{T_{oge}}{T_{ige}}\right)_P = \frac{1}{1-(\overline{z}/4)^2}, \qquad \left(\frac{P_{oge}}{P_{ige}}\right)_T = \frac{1}{A+B\overline{z}^2} \tag{4.8}$$

where $\overline{z} = z/R$ is the ground clearance normalised with the rotor radius, A and B are appropriate empirical coefficients.

When the rotor advances in ground effect, the wake takes complicated shapes that depend on the rotor advance ratio. An extended theory, also due to Cheeseman, indicates that at advance ratios $\mu \geq 0.1$ the ground effect is effectively negligible, because the wake is rapidly convected downstream past the aircraft. The extension of this theory yields the following:

$$\frac{T_{oge}}{T_{ige}} = \frac{1}{1-(\overline{z}/4)/(1+\tan^2\chi)} \tag{4.9}$$

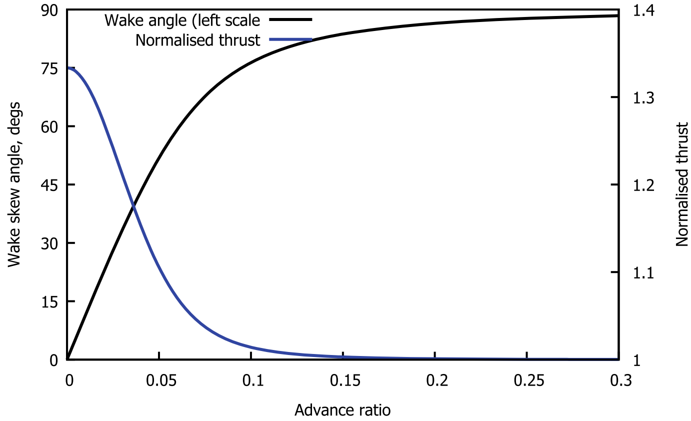


Fig. 4.9 Wake skew angle in ground effect, $C_T = 0.01$, $\bar{z} = 0.5$.

where χ is the wake skew angle. This skew angle is related to the advance ratio μ and the mean inflow velocity ratio λ_i

$$\cos\chi = \frac{\mu}{\lambda_i} = \frac{2}{C_T}\left(\sqrt{\mu^4 + C_T^2/4} - \mu^2\right) \tag{4.10}$$

displayed in Fig. 4.9, which shows no practical effects at $\mu > 0.15$. It is noted that the result of Eq. 4.10 depends on the C_T. A normalised advance ratio is defined as $\mu^* = \mu\sqrt{2/C_T}$.

Experimental data have been published, for example Ref. [8], which provide evidence two distinct flow regimes, at low- and high rotor advance ratios. At normalised advance ratios $\mu^* > 1$ the ground vortex below the rotor disappears, and so does the ground effect. This event corresponds to $\mu \simeq 0.007$, Fig. 4.9.

Another important ground effect problem occurs when operating to/from unprepared surfaces, which means there is loose ground below the aircraft. In this instance we have examples of brownout, white-out and a plethora of other problems caused by strong downwash.

The brownout problem arises when the rotorcraft attempts to land onto very loose ground, causing the raise of dust and other particulate that generate a large bowl of cloud enveloping the complete aircraft and obscuring the field of view of the pilots.

The disk loading is limited by ground operations. In fact, large disk loadings correspond to large downwash velocities, which may create hazards for personnel on the ground. Some practical limits are given in Table 4.1.

Table 4.1 Summary of disk loading limits

		Firm ground	*Loose ground*
Personnel limits	Over-turning moment	400 Nm	300 Nm
	Force	450 N	330 N
Surface limits	Disk loading	245 kg/m^2	73 kg/m^2

4.9.1 Ground Operations

Rotorcraft with landing skids can only touch down and lift off. However, if wheeled landing gear are available, it is possible (in principle) to manoeuvre on the ground at small speeds. In extreme cases, helicopters with conventional landing gear can using their ground rolling capability to take off from airfields at altitudes above their certified altitude. In fact, ground rolling takes the rotorcraft to a speed that corresponds to a lower forward flight power. The taxi speed is found from

$$kT_{ige} \sin\alpha_r = D + \mu_r \mathcal{R} \tag{4.11}$$

where α_r is the rotor disk tilt angle, T_{ige} is the thrust in ground effect, k is a blockage factor, due to fuselage and undercarriage interference, D is the aerodynamic drag of the rotorcraft, μ_r is a rolling coefficient depending on the runway conditions, and $\mathcal{R} = W - L$ is the rolling resistance. By expanding all the terms, Eq. 4.11 becomes

$$kT_{ige} \sin\alpha_r = \frac{1}{2}\rho A C_{D_g} U^2 + \mu_r \left(W - kT_{ige}\cos\alpha_r\right) \tag{4.12}$$

In Eq. 4.12, A is the rotor disk area, used as a reference for the drag coefficient in ground effect, C_{D_g}. The rotor downwash has a limit, usually specified so that there is no harm to ground personnel. Data from Table 4.1 can be used. Actually, by specifying the limit downwash (or limit disk loading), we can calculate the limit thrust in ground effect, and hence the limit taxi speed. Now divide Eq. 4.12 by the disk and solve for the ground speed V

$$V^2 \simeq \frac{2}{\rho C_{Dg}}\left[k\frac{T_{ige}}{A}\sin\alpha_r - \mu_r\left(\frac{W}{A} - k\frac{T_{ige}}{A}\cos\alpha_r\right)\right] \tag{4.13}$$

This expression contains the nominal disk loading W/A and the equivalent disk loading in ground effect T_{IGE}/A, for which we need to set a limit $T_{A_{max}}$. Note that the vehicle is capable of taxiing only if

$$W < \frac{k}{\mu}(T/A)_{max}\left(\sin\alpha_r + \cos\alpha_r\right) \tag{4.14}$$

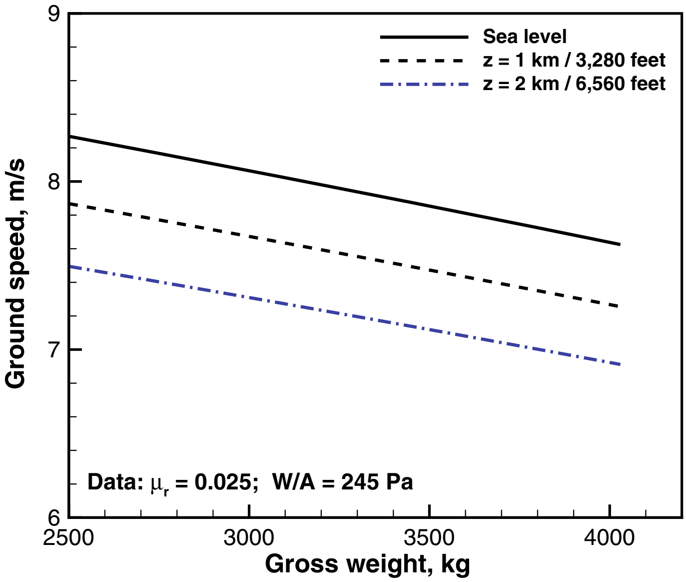


Fig. 4.10 Calculated ground speed of reference helicopter at selected altitudes

Since is the speed is generally very small, changes in thrust with the speed can be neglected. The thrust T_{IGE} will be calculated from the condition of constant power derived by Cheeseman and Bennett [6]

$$\left(\frac{T_{ige}}{T_{oge}}\right)_P = \frac{1}{k_g} \tag{4.15}$$

with k_g the ground effect factor,

$$k_g = 1 - \frac{(R/4z)^2}{1 + (\mu/\lambda_i)^2} \tag{4.16}$$

having neglected the effects of blade loading on the rotor; In Eq. 4.16 h is the height of the hub on the ground; $z/2R$ is the ground clearance of the rotor.

A solution of Eq. 4.13 is shown in Fig. 4.10. The problem's parameters are: $\mu =$ 0.025 (dry hard ground); $W/A = 245$ Pa (limit rotor downwash), or $P = 100$ kW.

An interesting case is that of the autogyro. Since this rotorcraft cannot hover and fly vertically, take-off and landing must be achieved through a ground run. Therefore, this type of rotorcraft *must have wheeled landing gear* in all cases.

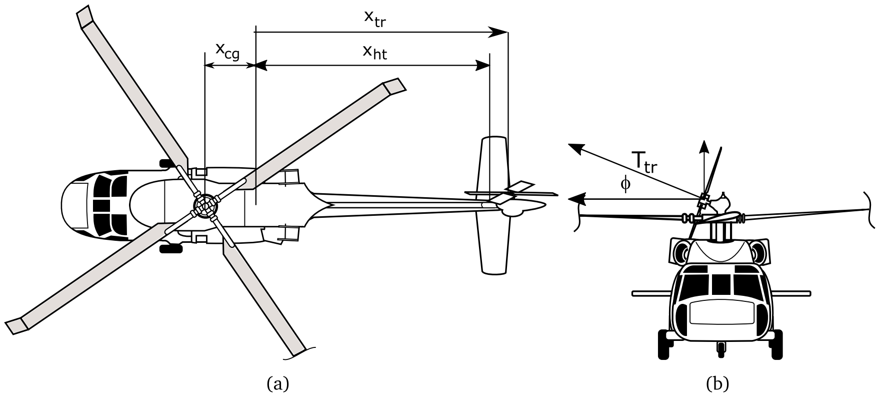


Fig. 4.11 Orthographic view of a UH60 model for trim static trim calculations

4.9.2 Ground Resonance

In the early days of rotorcraft development, some helicopter rotors exhibited violent vibrations when on the ground. In some cases these vibrations were disastrous. At first this problem was attributed to rotor blades flutter, but later it was understood to be caused by a hitherto new phenomenon: the conversion of rotational energy into vibration energy in the presence of the ground. Hence the phenomenon was called *ground resonance*. The problem was solved mathematically by Coleman and Feingold [9] in the 1940s, but it is quite interesting to this day to observe video recordings of rotorcraft running to self-destruction in some ground tests.

Ground resonance is an instability of the rotor that is placed on a flexible surface. An imbalance or perturbation on a rotor blade causes a forcing to be transferred to the airframe. The airframe is on the ground standing on flexible landing gear The perturbation can be transferred to the landing gear and back (amplified) to the airframe, and onward back to the rotor. When these oscillations self-amplify, there is resonance.

4.10 Static Trim Conditions

A helicopter is said to be trimmed if all forces and moments are balanced, and the helicopter can fly steady-state level flight. The solution of the trim equations, easy in principle, requires considerable mathematical treatment, and in most cases is requires numerical solutions. The terms in trim equations depend on the specific rotorcraft.

Consider a helicopter that has a tail rotor with a cant angle φ, a horizontal stabiliser, such as the UH60. The vertical trim and the pitching moment are written

$$T\cos\alpha_r + T_{tr}\cos\varphi + L_{ht} = W$$
$$x_{cg}T\cos\alpha_r - x_{tr}T_{tr}\cos\varphi - x_{ht}L_{ht} = 0 \tag{4.17}$$

where α_r is the forward tilt angle of the main rotor, L_{ht} the stabiliser lift and the x-distances are indicated in Fig. 4.11. There are too many unknowns in this equation to be solved, and thus we need further elaboration. The stabiliser lift is

$$L_{ht} = q\,(AC_{L\alpha}\alpha)_{ht} \tag{4.18}$$

and requires the effective inflow angle α_{ht}, which may be dissimilar between the two sides of the stabiliser, due to the rotor downwash and the presence of the fin; q is the dynamic pressure based on the flight speed, A_{ht} is the area of the stabiliser used to calculate the aerodynamic derivative $C_{L\alpha ht}$. The tail rotor thrust depends on the yaw trim condition, and is written

$$T_{tr}\sin\varphi = \frac{1}{x_{cg} + x_{tr}}\frac{P_{mr}}{\Omega} \tag{4.19}$$

The main tilt angle of the thrust α_r depends on the total drag of the helicopter; thus, we require a trim equation in the horizontal direction

$$T\sin\alpha_r = D \tag{4.20}$$

unless the tail rotor also has a tilt in the forward direction that provides a contribution to the total thrust (now excluded to avoid further complications). The helicopter drag is unknown, and can only be estimated, as it depends on the airframe and its inflow/yaw angles, the rotor head, the rotor systems and any ancillary elements.

Hence it is clear that the equations, although algebraically simple when written on their own, suddenly become complicated because of the cross-correlation effects.

The next case is that of a helicopter with an asymmetric empennage, in particular two vertical stabilisers at a yaw angle with respect to the longitudinal axis of the aircraft, such as the Eurocopter (now Airbus) AS365, Fig. 4.12.

The lateral trim equation is

$$Q_{mr} + Q_{tr} + Q_{vt} = 0 \tag{4.21}$$

where we have included the effects of the vertical stabilisers. These contribute a side force Y_{vt}, which depends on the design yaw angle β and the flight yaw angle β_∞:

$$Y_{vt} = qA_{ref}C_{y\beta}(\beta + \beta_\infty), \qquad Q_{vt} = x_{vt}Y_{vt} \tag{4.22}$$

where $C_{y\beta}$ is the side force derivative of the stabiliser with respect to the yaw angle β; A_{ref} is a reference area. This coefficient is calculated with aerodynamic considerations and a separate demonstration is given through a *video tutorial*.

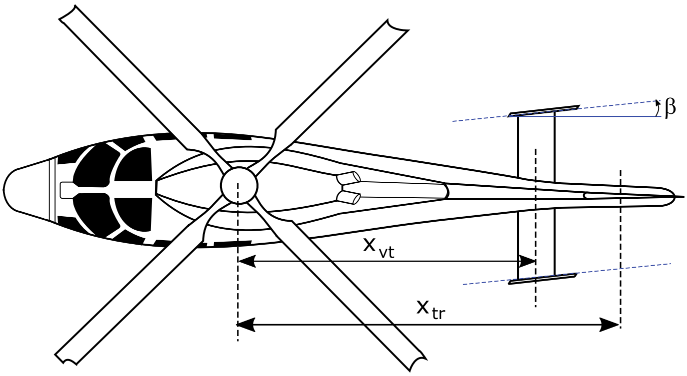


Fig. 4.12 Top view of an AS365 model for torque balance calculations. This helicopter has a fenestron, which is not visible in this graph

Assume $\beta_\infty = 0$ to simplify the analysis. The fenestron thrust required is

$$T_{tr} = \frac{1}{x_{tr}} \frac{P_{mr}}{\Omega} - q A_{ref} C_{y\beta} \beta \tag{4.23}$$

which demonstrates that with a well-designed stabiliser, *for some flight conditions*, the stabiliser contributes to the overall side force and thus offsets the requirements on the tail rotor power.

For more general analyses, we need a variety of stability derivatives, as demonstrated in earlier literature [10–12]. Modern examples are shown in Chap. 5 for the convertible rotor.

4.11 Helicopter Speed Stability

The speed stability problem is related to the change of pitching moment with increasing speed. The helicopter can have an inherent nose-up or nose-down pitching moment. With reference to Fig. 4.13, using the control stick fixed, if the rotorcraft has a nose-up C_M, it will decelerate, because its thrust line tilts backward. If the rotorcraft has nose-down C_M, it accelerates further, because of its nose-down attitude. The nose-down pitching moment is destabilising and the helicopter is said to be unstable with speed. A nose-up pitching moment is stabilising. The flight control system must be able to compensate this pitch tendency.

A particularly interesting case is that of the tandem helicopter, a case studied by Tapscott and Amer [13] who provided a simplified analysis for the collective differential required to maintain longitudinal stability of this type of helicopter. The key assumptions of this model include equal-diameter rotors, equal rotor solidity, and equal angular speed. Critical to the whole procedure is that the fore rotor is

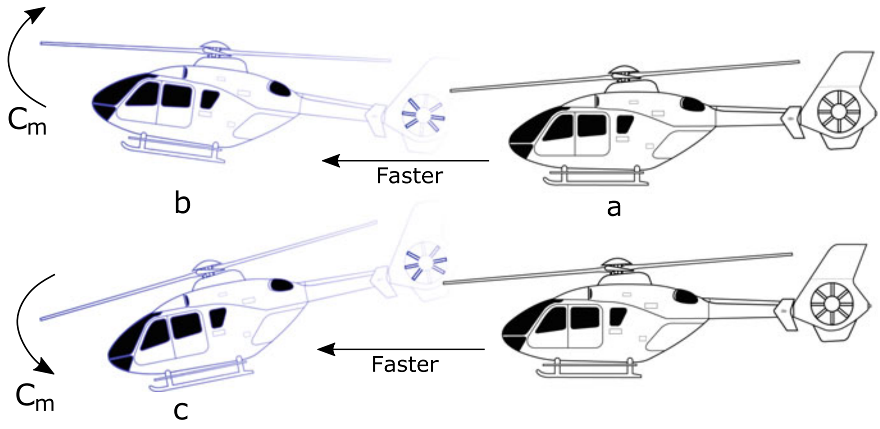


Fig. 4.13 Speed stability of a conventional helicopter

unaffected by the aft rotor; therefore, the aerodynamic interference is neglected, just as the contributions of the airframe lift and pitching moment. With these assumptions, it is possible to show that the collective differential between the two rotors (or swashplates) is

$$\frac{\Delta T}{W} = \frac{1}{2}\frac{1}{C_{T\sigma}}\left[\left(\frac{\partial C_{T\sigma}}{\partial \alpha}\right)\Delta\alpha + \left(\frac{\partial C_{T\sigma}}{\partial \theta}\right)\Delta\theta\right] \tag{4.24}$$

where ΔT is the thrust differential, $\Delta\theta$ is the difference in collective pitch, $\Delta\alpha$ is the difference in mean inflow angle on the rotors and $C_{T\sigma} = C_T/\sigma$ is the mean blade loading. The derivative are mean values between the two rotors. Equation 4.24 must be solved for $\Delta\theta$. To begin with, we differentiate with respect to the advance ratio and set $\partial \Delta T/\partial \mu = 0$ to eventually find the following expression:

$$\frac{\partial \Delta\theta}{\partial \mu} \simeq k_1 \overline{C}_{T\sigma}\frac{\Delta T}{W} + k_2 f(\Delta\sigma, \Delta V_{tip}) + k_3 \Delta\alpha_d + k_4 \overline{C}_T \tag{4.25}$$

where k_1, k_2, k_3, k_4 are stability derivatives depending on the blade loading and the advance ratio; $\Delta\alpha_d$ is a swashplate *dihedral effect*, one effect that is difficult to estimate, but discussed in the original work. The second term with stability factor k_2 can be neglected if there is no difference in tip speed and rotor solidity.

A plot of these functions is shown in Fig. 4.14. When applied to a model of the CH-47 helicopter, the collective differential increases rapidly at high speeds from a negative value. The predicted collective differential $\Delta\theta$ is displayed in Fig. 4.15.

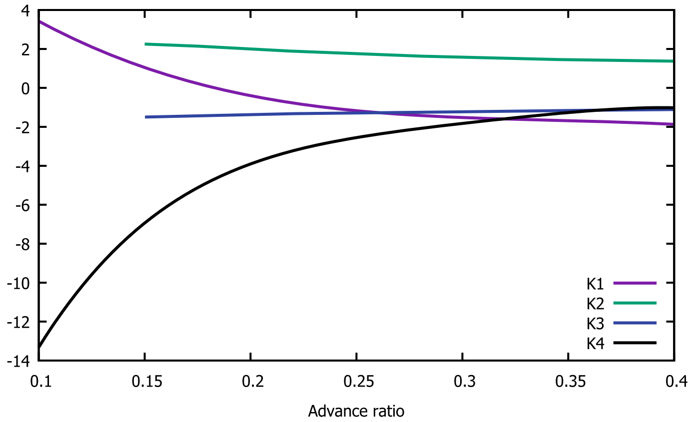


Fig. 4.14 Stability coefficients in Eq. 4.25

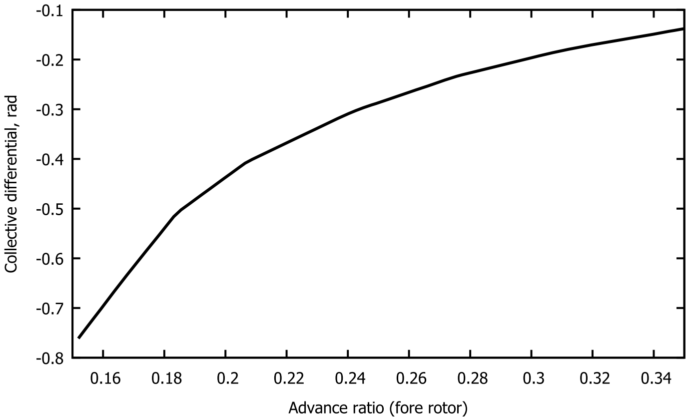


Fig. 4.15 Predicted collective differential for the CH-47

4.12 Mission Planning

Mission planning is the series of calculations of fuel requirements for a specified operation. In the process, we define the take-off gross weights and analyse the mission limitations, depending on required payload, atmospheric conditions and other external factors.

The process begins with the identification of the flight segments involved in the forecast mission. This is sometimes straightforward (flight from A to B to deliver a payload), but in many cases is a rather complicated undertaking that involves scenario forecasting, risk analysis and contingency planning. This is particularly true for search-and-rescue and most military operations. In these cases, we might not have an exact definition of flight segments, but we need to plan around an extension of the flight, the duration of a hover and refuelling, on the ground or airborne.

In the simplest case, we have a full list of flight segments, we calculate the fuel required for each segment, sum all these fuel contributions, add a contingency amount of fuel to reach a first-order estimate. The correct process is as follows:

- Establish the required payload, W_p.
- Establish the atmospheric conditions.
- Establish the flight segments, S_i.
- Make an initial guess of the gross take-off weight, $W_{j=1}$ (j is the iteration count).
- Calculate the fuel required for each segment, W_{fi}, each corresponding to S_i.
- Calculate the regulatory fuel reserves, W_{fr}.
- Calculate the sum of all fuel required: $W_f = \sum_i W_{fi} + W_{fr}$.
- Calculate the gross take-off weight: $W_{j+1} = W_e + W_p + W_f$.
- Calculate the relative difference in gross take-off weight: $\epsilon = |W_{j+1} - W_j|/W_j$.
- Establish a convergence criterion, for example $\epsilon < 10^3$.
- if $\epsilon > 10^3$, restart the process with the new gross take-off weight W_{j+1} until convergence.

In most cases, convergence is achieved in a few iterations, but in a very few instances, this numerical method does not converge. Upon convergence, it is possible that the final weight $W >$ MTOW, in which case, the mission cannot be fulfilled: the mission specification must be changed and/or the payload must be reduced.

Each flight segment must be specified by a few parameters which include: flight time, initial and final altitudes, initial and final air speed, climb rate (mean value), other specific data. Examples of mission calculations are shown below.

Mission Scenario #1: Medevac operation of a CH-47. We allow the rotorcraft to land, after search operations, slow down the rotors, load as many casualties as possible, take-off and start the return journey, with the listing of flight segments shown below.

A warm-up of 3 min at an altitude of 80 m ($\sim$260 ft) is followed by a climb-out to a target altitude of 305 m ($\sim$1000 ft), to a target air speed of 100 kt, with an average climb rate of 3 m/s ($\sim$600 ft/min). Then there is a cruise at 100 kt, at 305 m altitude to a target distance $X = 100$ n-miles. Loiter for search-and-rescue for up to 10 min at 1000 ft (305 m), then descend and land to a target altitude of 100 m (328 ft), and so on.

Note that in this case there is a segment called "weight-load", after landing, requiring the rotorcraft to load up to 3000 kg in 15 min (200 kg/min); then the rotorcraft makes the return journey.

The complete analysis is demonstrated in a *video tutorial*, alongside a number of variations on the medevac operation, including a case when landing is not possible and loading of casualties has to be done whilst airborne; there is also another case when both landing and refuelling are possible.

Listing 4.1 *Example of medevac operation.*

```
# FLIGHT              time     z      V       X     Other    Notes
#                     [min]   [m]   [kt]    [nm]    [-]
#---------------------------------------------------------------------

'Medical Evacuation'
'Payload'     100                                           ! initial payload weight
  'warm-up'            3      80      0       0      0      ! z = airfield altitude
  'climb-out'          0     305    100       0      3      ! climb rate (m/s)
  'cruise'             0     305    100     150      0      ! z = cruise altitude; V
        > 0
  'loiter'            10     305      0       0      0
  'descent'            0     305      0       0      0      ! z = initial descent
        altitude
  'landing'            0      80      0       0      0      ! z = airfield altitude
  'hover'            0.5      10      0       0      0      ! cargo loaded in hover
  'weight-load'       15      10      0       0   3000      ! cargo loaded on the
        ground
  'climb-out'          0     305    100       0      6      ! climb rate (m/s)
  'cruise'             0     305    100     150      0      ! z = cruise altitude; V
        > 0
  'descent'            0     305      0       0      0      ! z = initial descent
        altitude
  'landing'            0     100      0       0      0      ! z = airfield altitude
  'END'                                                     ! END PARSING OF DATA
```

4.13 Payload-Range

The most common way of demonstrating a mission performance is via a payload-range diagram. This is also the simplest possible mission of a rotorcraft, since it implies a flight from origin to destination with a fixed payload. Whilst this is the appropriate performance description of a fixed-wing aircraft, it is rather limiting for a rotorcraft, as a result of the peculiar flight characteristics of this vehicle. In any case, payload-range diagrams are produced by manufacturers to demonstrate range and payload capability as part of their overall marketing strategies.

A typical payload-range diagram appears as in Fig. 4.16. In graph 4.16a, the segment A-C indicates that the payload decreases as the amount of fuel increases, subject to the aircraft starting at MTOW. Upon reaching range X_C, any further increase in range can only be achieved by dropping payload, subject to the rotorcraft starting with full tanks. The gross take-off weight in the segment C-D is lower than the MTOW, and upon reaching distance X_D there is virtually no payload capacity left. Point D is called *ferry range*. The case of Graph 4.16b corresponds to a case when the rotorcraft has been equipped with additional fuel tanks. For any flight range $X < X_C$ because the additional tanks add to the structural empty weight of the vehicle. This is inevitable, but it also indicates that additional tanks are only to be fitted on rotorcraft specifically allocated to long-range or long-endurance missions.

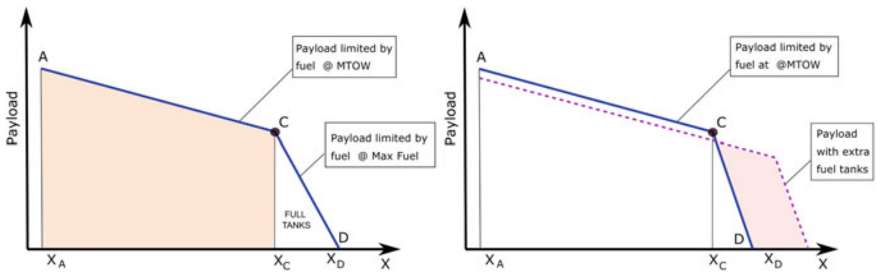
As demonstrated in the case of the medevac operation, Sect. 4.12, performance charts like the ones in Fig. 4.16 do not reveal the whole performance envelope of the vertical lift vehicle; therefore, alternative charts have to be used. For example, an alternative performance charts would account for a specified amount of time in hover and loiter. This amount of time can be somewhat arbitrary, but for the sake of discussion, let us consider the case of a rotorcraft requiring a minimum 10 min hover alongside 10 min of loiter for search-and-rescue. If we introduce such considerations in the flight operations, we still face some arbitrary decisions. For example, at what point into the flight is the rotorcraft required to hover and/or loiter?

4.14 Direct Operating Costs

Ownership and operation of helicopters is notoriously expensive, and the costs of ownership are possibly the highest barrier to entry for many potential customers, alongside flight safety. A good costs analysis highlighting the difficulty with helicopters is shown in Ref. [14]. In that work, it was demonstrated how acquisition costs are critically dependent on installed power, rather than empty weight. This result would translate into a relatively higher cost for a rotorcraft with a high design disk loading, and it was concluded that *designing for minimum empty weight does not equate to minimum helicopter purchase cost*.

In this section, we discuss the overall costs, which include – critically—the direct operating costs, beginning with ownership models. There are several ownership models; often the operators are not aircraft owners, but rather leasers of the vehicles. This poses further questions as to the structure of the costs. *Direct Operating Costs (DOC)* are the sum of all costs that are incurred by an owner/operator. Ultimately, it all comes down to a very important economic figure: DOC/flight hour.

A summary of cost items is provided in 4.2. In this case, we have three types of operations: transport, training and heavy lifting. For each type of operation, the data to inser in the table must be *pre-calculated*, and are intended as averages, otherwise



(a) (b)

Fig. 4.16 Payload range charts of a helicopter we would need a mission calculation for each specific mission. Thus, the data that must be pre-calculated include: block time and fuel burn.

Listing 4.2 Summary of Direct Operating Costs

```
 Item                                               Unit
-------------------------------------------------------------------
 aircraft price                                      any unit (dollars)
 fuel price                                          /kg
 fuel price inflation, estimated                     %
 insurance, based on aircraft actual value           %/year

 spares price1 (airframe, landing gear, tyres)       /flight hour
 spares price2 (engine, APU, lubricants)             /flight hour
 spares price3 (avionics/systems/pax_services)       /flight hour

 life time
 depreciation over life-time                         % of initial cost
 financing                                           % of initial cost
 interest rate                                       %
 years of repayment

 crew price, pilots                                  full time/pilot/year
 crew price, ground                                  full time/staff/year
 crew price inflation                                % year
 off station price                                   /person/night
 labour rate1, engines                               man-hour
 labour rate2, in-house maintainance                 man-hour
 labour rate3, contracted out                        man-hour
 man-hour1, power plant                              /flight hour
 man-hour2, in-house                                 /flight hour
 man-hour3, contracted out                           /flight hour
 landing charges, airfield services                  /operation
 ground handling costs                               /movement
 recurrent training costs                            crew training/year
 ground service costs: hangars                       /year

transport          type of operation
  300                  cycles in category this category     cycles/year
 1500.                 fuel burn in mission transport    [kg]
   80.                 block time                           [min]
  220.                 stage length, n-miles (transport)    [average]
 3000.                 cargo/freight (inside)               [kg]
 4000.                 cargo/freight (under-sling)          [kg]
 5.25                  cargo price (inside)                 [kg]
 7.50                  cargo price (under-sling)            [kg]
 1.0                   any other cost                       /flight

training           type of operation
   10                  cycles in category this category     cycles/year
  500.                 fuel burn in full mission training  [kg]
   60.                 block time                           [min]
 3000.                 cargo/freight (inside)               [kg]
 4000.                 cargo/freight (under-sling)          [kg]
```

```
  5.25           cargo price (inside)                  (currency/kg)
  7.50           cargo price (under–sling)             (currency/kg)

heavy-lift     type of operation
  10               cycles in category this category    cycles/year
 1000.             fuel burn in full mission heavy–lift [kg]
  60.              block time                          [min]
```

The use of this configuration table is demonstrated in a *video tutorial*.

4.15 Performance Augmentation

One of the most-often heard criticisms of the helicopter is that it is not fast, and for quite some time engineers have worked on concepts to increase the speed by using *compounding*. The criticism is unwarranted, because the helicopter can carry out missions that are not comparable with any fixed wing – just think of the ability to deliver large external loads, almost anywhere, with high precision.

A flight envelope is a safe operating domain in the z-V space. Generally, it refers to steady state flight, but accelerations and manoeuvres are possible in this domain. An expansion of the flight envelope requires higher speed and higher flight altitudes.

Let us start with the flight speed. Speed is good, but not at all costs. The *Westland Lynx* that achieved a world speed record [15] was an exception in that it was a conventional helicopter finely tuned for speed, but that was not a production version.

Compounds are meant to increase speed. There are two basic type: *thrust compounds*, that increase the net thrust via additional thrusters, which can be propellers (ducted or unducted), or jet engines. Examples in this category include the *Piasecki X-49* (with aft ducted propeller), the *Sikorsky X2* and *S-97* (coaxial rotor with aft pusher propeller), the *Sikorsky S-69* (coaxial with two auxiliary side-by-side turbojets), and the *Airbus X-3* (with two wing-mounted left-right propellers).

Then there is the *lift compound* which is capable of generating additional lift through lifting surfaces; at the same time they offload the rotor, which can be slowed down to limit retreating blade stall and high-Mach flows on the advancing blades (see also Fig. 4.2). Examples in this category include the *Sikorsky S-67* and the *Lockheed AH-56*.

This is a vast subject that needs its own chapter, but a number of references can help to start out [16, 17]. Only a few concepts are given below. The thrust compound helicopter requires a separate power plant or engines that are oversized to drive at least one more propulsor; they are the domain of military operations.

Another limiter in helicopter flight performance is the relatively low operational altitude limits. These are essentially due a combination of sharp increase in induced power ($P_i \sim 1/\sqrt{\rho}$) with a rapid decrease in net turboshaft power at high altitudes. Although the rotors can be optimised, and the weight can be reduced, the only way to fly higher is to increase the installed engine power, as explained graphically in

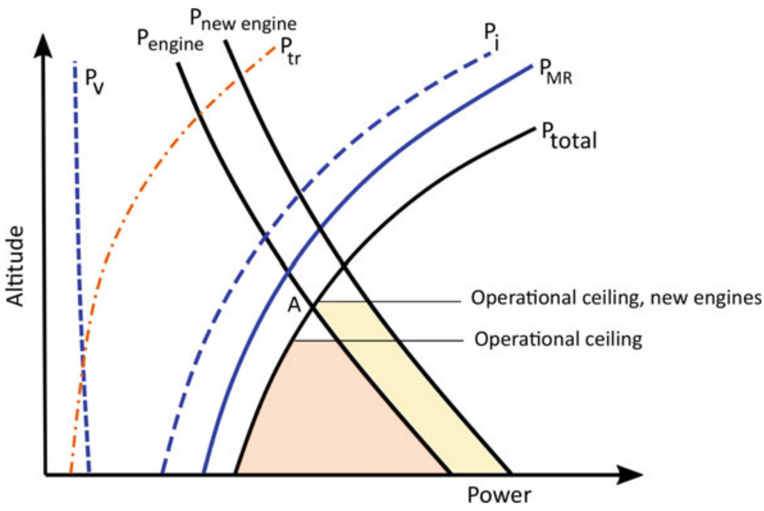


Fig. 4.17 Analysis of operational ceiling of a conventional helicopter; "A" denotes the absolute ceiling with the standard engine

Fig. 4.17. By further decreasing the gross weight, the required power shifts to the left, and the power margin increases, which allows the rotorcraft to fly higher.

For example, the *Eurocopter/Airbus AS350-B3*, powered by a *new* Turbomeca Arriel 2B turboshaft engine rated at 632 kW[1], would have an operational ceiling of 3415 m (11,200 ft) on a standard day and MTOW. With the weight reduced by 650 kg (by dumping payload), the ceiling increases to 6520 m (21,390 ft). By relaxing the constraint on the minimum climb rate to 1 m/s (200 ft/min), this lighter weight configuration can climb and operate at 7000 m (22,965 ft). Further engine upgrades, and a cold day, can take this helicopter up to mount Everest.

4.15.1 Lift Compound

Lift compounds have the problem of aerodynamic interference between the rotor downwash and the wing, which may cause off-design inflow conditions (large and inflow angles on the wing, asymmetric dynamic pressure, vertical drag, etc.), which makes them unsuitable for some flight conditions. However, they do not require an additional propulsor, which has some advantages in terms of complexity, weight and cost. Size and position of these surfaces is critical, because as mentioned they can cause interference in vertical- and low speed flight. Ideally, the wing would be tiltable and retractable, but there are design complications and costs to consider.

[1] Data from *The Flight Crew Operating Manual*.

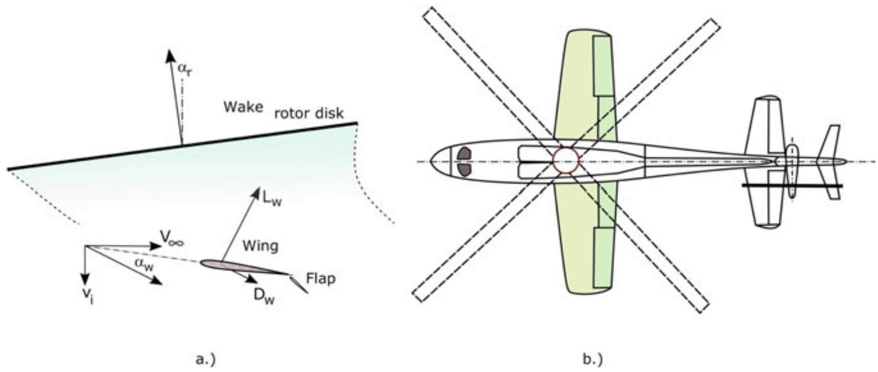


Fig. 4.18 Lift compound helicopter model

Now assume the rotor-wing combination shown in Fig. 4.18. The wing is just below the rotor. Unless the wake is highly skewed to avoid the wing (Fig. 4.9), as it happens in fast forward flight, the aerodynamic interference is too strong.

Due to the asymmetry of the rotor flow, each semi-wing may generate its own lift, which causes additional rolling/yaw moments on the aircraft. Thus, one may want to consider asymmetric wings as well, but there is no guarantee that these moments will be automatically trimmed at all flight conditions. To secure more control authority, we may need additional surfaces, such as vertical and horizontal stabilisers with elevators and rudders. The lift generated by the rotor-plus-wing configuration in steady level flight is

$$W = T\cos\alpha_r + (L_{w1}\cos\alpha_{w1} + D_{w1}\sin\alpha_{w1}) + (L_{w2}\cos\alpha_{w2} + D_{w2}\sin\alpha_{w2}) \tag{4.26}$$

where L_w and D_w are the total wing lift and drag, respectively; the two numbers indicate the wing on either side of the fuselage. If the wing is fixed, the inflow angle α_w depends on the rotor downwash, which in turns depends on the airspeed. The effective inflow on the wing is the angle between the nominal chord line and the direction of the vector $\mathbf{V} = \mathbf{v}_i + \mathbf{V}_\infty$, Fig. 4.18a.

To begin with, assume that the difference $\alpha_{w1} - \alpha_{w2}$ is negligible. In this case the rotor thrust required (in absence of fuselage and tail rotor contribution) is

$$T^* = \frac{W - L_w\cos\alpha_w + D_w\sin\alpha_w}{\cos\alpha_r} \tag{4.27}$$

with

$$L_w = qC_{L\alpha}(\alpha_w - \alpha_o)\, b_w\bar{c}_w \tag{4.28}$$

There is clearly some thrust alleviation, which could be delivered with a smaller-amplitude collective pitch or with a slowed rotor. The slowed rotor would decrease the effective tip Mach number on the advancing blade. If a detailed analysis of the rotor downwash is available, we can use strip theory on the wing and calculate the local inflow at wing section j;

$$\alpha_{wj} = \tan^{-1}\left(\frac{v_{ij}}{V_\infty}\right) \tag{4.29}$$

In any case, it should be evident at this point that the design and performance of such a rotorcraft requires the evaluation of a large parametric space with stability and performance analysis all combined.

4.15.2 Thrust Compound

Some of the difficulties highlighted at the previous points are overcome with a thrust compound vehicle. The vertical component of the thrust remains the same. Thus:

$$W = T\cos\alpha_r, \qquad T\sin\alpha_r + T_p = D \tag{4.30}$$

where T_p is the thrust generated by an aft-mounted propeller. The tilt angle of the rotor disk becomes

$$\tan\alpha_r = \frac{D - T_p}{W} \tag{4.31}$$

Unless $D > T_p$, the rotor would be tilting backward and the rotorcraft would cease to operate as we know it: *a backward tilt would essentially convert the compound helicopter to an autogyro*. This reversal condition provides the limits of the propeller thrust. In hover and low speeds, the propeller may need to be disengaged.

One drawback of this configuration, seldom advertised, is the very large noise created by the interaction of the aft rotor with the main rotor. Wake interference effects are almost inevitable, and the propeller ingests turbulence and vortices from the main rotor. Furthermore, the tonal noise components of the propeller are different from those of the main rotor and combine together to give a more complex acoustic spectrum. For example, the Sikorsky X2 has a rotor operating at 360 rpm in low speed ($V < 200$ kt), whilst the propeller rpm is 1400 (gear-ratio of 4.88); therefore, the blade passing frequency 24 Hz for the rotor 140 Hz for the six-bladed propeller (the frequency ratio is 5.83). Rotor acoustics is discussed in detail in Chap. 6.

One notable advantage of the Sikorsky X2 is that the use of counter-rotating coaxial rotors removes the need for a conventional tail rotor, and therefore there are only limited requirements on the torque balance, less tendency to yaw and roll. A coaxial rotor with rotors providing torque Q_1 and Q_2 is automatically stabilised in yaw only if $Q_1 = -Q_2$. With the lower rotor operating in the downwash of the upper rotor, a collective pitch differential is required.

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