# Some Problems in the History of Modern Mathematics

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**Personal Note**
*It is a great pleasure to take this opportunity to thank Catriona Byrne for the many ways she has helped the history of mathematics community, and for the numerous encouraging conversations I have had with her over the years, which have helped shape how I have thought about mathematics and its history. Of particular note is the work she did to bring about Springer's contribution to the Prize awarded by the European Mathematical Society. More personally, I thank her for helping to open Springer's doors to what became my four volumes on the history of mathematics in the SUMS series, which are now ably looked after by Remi Lodh.*

## 1 A Brief Historiography

I shall restrict my attention to mathematics in the West in the period from 1600 to 2000 (the modern period). The word 'mathematics' in this essay will always mean the mathematics of those four centuries. Before addressing my main theme of problems for future historians of mathematics, I would like briefly to set the context with a few historiographical remarks.

Science and technology had played a hugely significant role on both the Allied and Axis sides in the second World War and then throughout the Cold War, with implications and opportunities that needed to be understood. In the English-speaking world there was a boom in history and philosophy of science, and in logic,

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often focussed on what the key ingredients of progress might be: what was special about science, what was science anyway? Enthusiasm and funding eventually waned, but two books stand out as survivors: Tom Kuhn's [38] *The Structure of Scientific Revolutions* and Imré Lakatos's [39] *Proofs and Refutations*. Whatever their strengths or weaknesses, they both offered a point of view that many outside the field could relate to; Kuhn's paradigm shifts and Lakatos's monster barring and other ideas seemed much fresher than a trudge through the names and dates of the great scientists and mathematicians, or raking over the coals of formalism, intuitionism, and logicism.

The return to original research in the history of mathematics based on archival documents was quieter. In the U.K. there were the examples of Ivor Grattan-Guinness [23] and Tom Whiteside [57]; in the USA Carl Boyer and Morris Kline [36]; in France René Taton [55]. In East and West Germany there were a number of long-standing Chairs in the subject, and in the Soviet Union there were some good links between mathematicians and historians of mathematics (for example, Kolmogorov and Youschkevitch [37]), doubtless complicated by a desire of the communist system to produce Marxist 'history'. However, the English-speaking world also saw a growing separation between history of mathematics and history of science, as historians of science sought to orient their work more closely with historians of other topics, in whose department they worked. Scientific rationality and the experimental method became only a part of arguments about status, social acceptability, funding, and national priorities. This was not a fertile ground for historians of mathematics of any kind, for whom there are aspects of mathematics already pointed out by philosophers of the 1950s and by Leo Corry more recently in lectures, which make the writing of interesting history of mathematics difficult; chiefly, what can be said in mathematics is very tightly constrained by the standards of rigour of its time. But nor, with a few exceptions, did philosophers of mathematics sustain an interest in history, preferring set-theoretic reductionism.

Where is the history of modern mathematics today? There is no consensus about research in the field, nor should there be. Among the substantial editorial achievements that have altered our picture of modern mathematics, the most significant for the early twentieth century is the ten volumes of Hausdorff's *Werke* [31]. For the nineteenth century there have been a number of well-researched biographies, and Thomas Hawkins's exceptional book [32] *Emergence of the Theory of Lie Groups*. For the eighteenth century there is the creation of the online Euler Archive, and for the seventeenth century I would single out Henk Bos's [3] *Redefining Geometrical Exactness: Descartes' Transformation of the Early Modern Concept of Construction*, which taught us how to read Descartes with fresh eyes, and Niccolò Guicciardini's *Isaac Newton on Mathematical Certainty and Method*, which does the same for Newton. A recent, two-volume, source-based account for students by June Barrow-Green et al. [2] may serve as an introduction to much of this material and more.

Although there has been a welcome turn towards historiographical studies (Guicciardini [28], Remmert, Schneider, and Sørensen [46]) and a methodological stiffening, most openly in the groups around Catherine Goldstein and Karine

Chemla in Paris, the history of modern mathematics can too easily project a sense of worthiness that shades into seeming insignificance. That should be set against the fact that science, especially physics, celestial mechanics, and mathematics are inextricable; that on many occasions philosophy has turned to mathematics and mathematical physics to renew its pursuit of its own questions; that mathematics is a key element in the educational systems of the modern world, and that modern mathematics can be seen as a species of modernism (Gray [24]). Without detracting at all from anything that has been done, historians of mathematics might profitably reassert where their work stands at the nexus of several vital concerns in the shaping of the modern world in the last four or more centuries. Perhaps surprisingly, a way forward might begin by considering what would be involved in a social-historical approach (so often a domain bereft of serious mathematics).

## 2 The Problems

### *2.1 Write a Social-Historical Book in the History of Mathematics*

We currently lack a book in the history of mathematics rooted in its contemporary social developments, although the forthcoming six-volume *Bloomsbury Cultural History of Mathematics* edited by J.W. Dauben and D.E. Rowe should be an excellent resource for many audiences (full disclosure, I have an essay in volume 5). What does it mean that mathematics was the province of a few gifted individuals in the seventeenth century, often outside the small university world? Or that these small numbers persisted into the eighteenth century, the age of the Academies and the Republic of Letters? Or that matters then passed to the ever-expanding universities of the nineteenth and twentieth centuries, concurrent with the rise of modern capitalism, and the number of mathematicians grew by perhaps a factor of 50? Soon, reform of the school educational system became a feature of every country that aspired to advanced mathematics, but there is no recent study of the connection between the rise of the universities after 1800 and any truly interesting account of the effect this had on mathematics, although there was a flurry of interest in neohumanism in Germany (see e.g. Pyenson [45]), there have been close studies of mathematics in nineteenth century Cambridge (Craik [9] and (Warwick [56]), and more recently there have been arguments about the first century of the École Polytechnique. Rowe's [48], a look at the Göttingen tradition, grounded in archival sources, offers a fresh indication of how things could be done, as do the dense and valuable books by Reinhard Siegmund-Schultze (for example, his [51] *Mathematicians Fleeing from Nazi Germany: Individual Fates and Global Impact*). Even if there are no significant connections between advanced pure mathematics and any social need, that would be a valuable part of the story (see Harris [30]).

It is time for the history of statistics to be integrated with the rest of the history of mathematics, despite the fact that in many universities mathematics and statistics remain in different departments. There are two good books on the history of statistics (Porter [44], Stigler [52]), but right now the subject seems to be lying fallow. There are two books on measure theory and the axiomatic approach to probability of Kolmogorov and others (Hochkirchen [33], von Plato [43]). Measure theory went in two directions, one towards probability (in the work of Doob) and the other towards functional analysis and ultimately also quantum mechanics. We now live at a time when probabilistic thinking is enriching several domains of analysis (notably number theory and partial differential equations) and while it would be absurd to imagine writing a history of such a fast-moving field today this might create the opportunity for a fresh look at the history of probabilistic thinking in mathematics. A start could be made with a fresh look at the history of thermodynamics.

## 2.2 Write a Book on the History of Applied Mathematics

Most fundamentally, we need to rethink the term 'applied mathematics', not just because it has various meanings in different countries even today, but because it emerged as a term only at the start of the nineteenth century; previously there had been a division into pure and mixed mathematics. Truesdell spoke of the rational mechanics of the eighteenth century, referring to the striking absence of experimental work in science in the period and the reliance on untested mathematics (inevitable, of course, in the dominant field of celestial mechanics). If it is true that experimental physics only took off at the start of the nineteenth century, perhaps with the study of electricity, and theoretical physics only came later (see Jungnickel and McCormmach [34]), there still needs to be a historical investigation of the associated mathematics in this context.

We lack a book on the complicated relationship between modern mathematics and modern physics, although it is widely believed that this was strangely attenuated in the early years of quantum mechanics. There is much we know about each side, see (Schneider [50]) and e.g., from an enormous literature on Einstein, (Renn [47]), but less on the interaction between the fields. There has been no shortage of solid work in the history of applied mathematics, much of it concentrated on the nineteenth century, and much of it in the form of biographies. We also have the pioneering books by Olivier Darrigol on electrodynamics [10], hydrodynamics [11], and optics [12]. More recently there has been at least two books on mathematics and the first world war (Aubin and Goldstein [1], Royle [49]), and books on mathematics and the early history of flight. But, for example, we still lack a book on the major British applied mathematicians that Klein so appreciated in his [35] *Vorlesungen über die Entwicklung der Mathematik im 19. Jahrhundert (1928)*. From the physics side, we have Buchwald's [6] and [7] concerning responses to Maxwellian physics and the rise of the wave theory of light.

Any such book would have to consider such topics as Maxwell's equations, the Einstein field equations, Schrödinger's equation and other partial differential equations, which underlines the importance of the next problem.

## 2.3 A History of Differential Equations

There is a number of accounts of one or another equation, and an extensive mathematical folklore, but my attempt to say something in a book (Gray [25]) aimed at final-year students convinced me that there is a need for a more thorough history with a more sophisticated methodology and less constrained by mathematical difficulty. It seems very likely that a good treatment of the subject will change our ideas about the growth of mathematical analysis, and move it away from an over-emphasis on rigour towards what might be called rigour for good reasons. A history of partial differential equations would be particularly valuable, especially if it could engage with the fundamental changes introduced by 1920. After that, the field becomes highly diverse and complicated and may defy historians for some time (see, however, Lützen [40]). Two topics that I had to omit stand out: Maxwell's equations (but see Buchwald [6]) and perturbation theory, so important in celestial mechanics and other fields. The twentieth century also saw the introduction, following ideas of Poincaré, of an abstract theory of flows, probabilistic ideas, and the ergodic theorems of Birkhoff and von Neumann.

A few isolated equations aside, theories of ordinary and partial differential equations began with Euler, Laplace, Lagrange, and Monge in the eighteenth century, and that leads into the next problem.

## 2.4 A Book on the History of Mathematics in the Eighteenth Century

Researching the history of mathematics in the eighteenth century sits uneasily between the preferences of mathematics and history of science departments. Two intertwined themes are the advances in celestial mechanics, and the reformulations of the calculus. Laplace's intimidating *Mécanique Céleste* was widely taken to have removed all doubts about the workings of the solar system, but the history of planetary astronomy still needs to be properly included in the history of mathematics, despite Gillispie's [22] *Pierre-Simon Laplace 1749–1827* and the work of Curtis Wilson; see, e.g. his [58] and [59]. The Euler Archive is a valuable initiative in this direction.

Scholarship on calculus in the century is almost bracketed by Guicciardini's two books [26, 27] on Newton and various studies of aspects of Cauchy's rigorization of analysis. In between, Euler brought about a shift in the foundations of the subject by successfully introducing the concept of a function, in whatever limited a form (see Ferraro [16]), but attempts by Lagrange to provide rigorous foundations of the calculus failed, as Ferraro and Panza [17] have shown. Another figure on whom scholarship has just begun, is Johann Heinrich Lambert, a member of every section of the Académie Royale des Sciences in Berlin, who wrote on many subjects and about whom we have several fragmented accounts.

Recently, Andrea del Centina (see e.g. his paper of 2020 [8]), and Jean-Yves Briend and Marie Anglade (see a series of papers starting with their paper [4] in 2017), have been revising our understanding of what falls under the heading of projective geometry in the seventeenth century. But we lack recent accounts of geometry in the eighteenth century, although there is (Bruneau [5]) on MacLaurin, and De Risi's innovative work on the foundations of geometry and attitudes to Euclid's *Elements*, much of which is still to appear (but for an early yet valuable work see his [14]). Presently, it seems as if, books by Euler and Cramer notwithstanding, geometry went into something of a decline. There was, for example, surprisingly little differential geometry of surfaces in the eighteenth century.

Historians and other intellectuals have given the idea of a progressive Enlightenment a rough time in the last 20 years, but apart from Hankins' book [29] there has been very little written on the involvement of mathematicians in the Enlightenment project and, for example, the production of the great *Encyclopédie*. As one example to be integrated into the history of mathematics, there is the controversy between d'Alembert, Rameau, and Rousseau about the new theory of music. This is only one reason for a fresh examination of mathematics of the age of the Academies.

## *2.5 The History of Mathematics from a New Philosophical Perspective*

Such were the crises of mathematics around 1900 that Hilbert was driven to say that to solve their problems mathematicians had to become philosophers. The highest standards were required of the rigour, reliability, and perhaps meaning of mathematics, and three families of ideas emerged: intuitionism, formalism, and logicism. More than anyone else, Gödel and Tarski answered many of the questions raised at the start of the twentieth century. More recently, the idea that mathematics is about axiomatically defined structures in various inter-relations (structuralism) has been a rival to a feeling that, at base, mathematics is an outgrowth of set theory and logic, a view that leaves most mathematicians cold. In the last 10 years, however, in the work of Mancosu [41, 42], Tappenden, in his [53] and [54] (to appear), and others, attempts have been made to engage philosophically with questions that mathematicians do ask themselves: What makes an idea fruitful? What is the right definition of a new concept? What is meant by purity of method and why is it valuable? What characterises a 'right' proof? How does advanced mathematics emerge from the seemingly incontrovertible elementary arithmetic, and with what consequences (here, see Ferreirós [19])? We now have a variety of sophisticated tools, such as Epple's epistemic objects (Epple [15]).

All this refocuses history of mathematics on how are discoveries made, which has a weakness of assuming too easily that some discoveries just get made, as if they were ordained in advance. There is a need for histories of mathematics that deliberately play down the eventual successes in favour of the seeking after new results, and recent developments in the philosophy of mathematical practice may provide the tools for such a thing.

It is time to take seriously the questions of why mathematics matters and why it is convincing, as opposed to taking these questions for granted. From a social perspective, and for the nineteenth century, a lot might hang on the opinions of astronomers and certain kinds of physicists, but the question is worth asking philosophically. To what extent is rigour a spur to mathematical discovery? This raises the question of what various mathematicians have been trying to do, why, under what constraints, and with what success. Such a book would be more Lakatosian than Kuhnian. Ultimately, we need a methodological approach to the mathematics of the previous centuries that is not a survey of results that reads like *Mathematics Reviews* for the past, and a philosophy of mathematical practice may well offer such a thing. Aside from the books mentioned above, one could also draw inspiration from Ferreirós's [18], and two books [20, 21] by Marcus Giaquinto.

## 3 Concluding Remarks

I heard recently of a 600-page history of modern Germany that mentioned Bach precisely once. This was not to find fault, but merely to indicate how much necessarily gets left out of such a book, never mind how Bach's legacy remains more alive than a lot of what was included. I haven't been able to check, but I doubt if Euler's name was mentioned at all in the book, and that is the problem of the history of mathematics, as it is of music and art. The challenge is to find places to stand, and organising principles, that make the history of mathematics not only accessible but vital.[1]

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[1] I would like to thank Tom Archibald, David Rowe, and José Ferreirós for helpful comments on earlier drafts of this essay.

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