Modeling and Simulation of Pedestrian Interaction with Moving Obstacles Using Particle Method

Parveena Shamim Abdul Salam, Sudarshan Tiwari, and Axel Klar

Abstract Modeling and simulation of pedestrian motion has been an important topic of research in recent years. In this work, we try to understand the dynamics in a shared space of pedestrians and moving obstacles. We consider a social force model coupled with an eikonal equation for pedestrian motion and appropriate kinematic equations for the obstacle motion. Firstly, we attempt to understand how the pedestrians avoid collisions with a passive obstacle. Later we analyze the interaction of pedestrians with a dynamic obstacle having a feedback interaction modeled via a repulsive potential. The hydrodynamic equations are solved using a mesh-free particle method, and the eikonal equation using the fast-marching method. The results reveal the collision avoidance strategies used which are in confirmation with existing studies. The model provides a framework to study pedestrian-vehicular traffic interactions and possibly interactions with automated vehicles in future studies.

1 Introduction

Pedestrian or crowd dynamics has been studied via varied modeling approaches, from microscopic to macroscopic scales. One of the most successful microscopic scale approaches was by modeling pedestrian motion through social or behavioural forces, see [8], which gave insights on self-organisation and collective behaviour of pedestrians like lane formation and bottlenecks. Other agent-based models have also been developed in this scale, for example, in [4]. Macroscopic modeling of the crowd using fluid dynamic equations was introduced by Henderson in [10]. Hughes, followed by others, developed this further via the idea of a potential function in the domain to incorporate more geometric information, see [6, 12]. More macroscopic

P. S. Abdul Salam (✉) · S. Tiwari · A. Klar
Technische Universität Kaiserslautern, Kaiserslautern, Germany
e-mail: parveena@mathematik.uni-kl.de; tiwari@mathematik.uni-kl.de; klar@mathematik.uni-kl.de

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models are seen in [5, 14]. Elaborate reviews of the different models along with a discussion of their advantages and limitations can be found in [1, 9].

In a social environment, humans encounter stationary or moving obstacles while maneuvering various spaces to reach their destinations. Different voluntary and involuntary strategies are used by humans to avoid collisions in such scenarios. An understanding of when and whether a collision will occur is essential, see [3]. This information forms the basis of collision avoidance models, as seen in [2]. Extensive research on pedestrian interactions with moving obstacles is still limited. In this work, we propose a model to study such interactions in shared spaces. The macroscopic model for pedestrian motion is combined with the proposed kinematic equations of obstacle motion wherein the feedback force terms are modeled via Hughes approach of potential functions obtained through an eikonal equation. For the numerical solutions, an immersed boundary approach is used along with the mesh-free particle method, as seen in [5]. In Sect. 2, we describe the models for pedestrian and obstacle motion. Sect. 3 explains the numerical method briefly. In Sect. 4, we see some results of the numerical simulation for different cases.

2 Models

2.1 Hydrodynamic Model for Pedestrian Motion

The model for pedestrian motion considered is as developed in [5], combining a social force model [8] to a Hughes-type model [12]. The hydrodynamic model equations for the evolution of density ρ and velocity u are:

$$\begin{aligned} &\partial_t \rho + \nabla_x.(\rho u) = 0\,, \\ &\partial_t u + (u.\nabla_x)u = G(x, u, \rho) + \int F(x - y, u(x) - u(y))\, \rho(y)\, dy\,. \end{aligned} \tag{1}$$

These are coupled to the eikonal equation: $f(\rho(x))\,||\nabla\phi|| = 1\,, x \in \Omega$. The force terms in (1), G and F, called the desired acceleration term and the interaction force term, respectively, are defined as:

$$G(\mathbf{x}, \mathbf{v}, \rho) = \frac{1}{T}\left(-f(\rho)\frac{\nabla\phi(\mathbf{x})}{||\nabla\phi(\mathbf{x})||} - \mathbf{v}\right), \quad F(\mathbf{x}, \mathbf{v}) = -\nabla_x U\,, \tag{2}$$

where the potential U with repulsive strength C_r and length scale l_r is given by, $U = C_r \exp\left(-\frac{|x-y|}{l_r}\right)$. We note that a more general F with dependence on both x and v can be used instead of $\nabla_x U$ in (2). The velocity-density relation used is $f(\rho(x)) = u_{\max}(1 - \rho(x)/\rho_{\max})$, where $u_{\max}$ and $\rho_{\max}$ are the maximum velocity and density. We refer to [5] for more details.

2.2 *Model for Obstacle Motion*

The obstacle motion is governed by kinematic equations for position and velocity. The passive obstacles follow a fixed trajectory defined by the equation: $\frac{dx^O}{dt} = (v_x^O, v_y^O)$ with $v_x^O = -\alpha$ and $v_y^O = A\,cos(\omega t)$ where α is a positive constant for left moving obstacle and A is the amplitude and ω the frequency of oscillatory motion of the obstacle.

The equations for the dynamic obstacle, which interacts with the pedestrians and changes its trajectory or speed, following the convention in pedestrian model, are:

$$\frac{dx_i^O}{dt} = v_i^O\,, \quad \frac{dv_i^O}{dt} = \sum_{j \in N_p} F_O(x_i^O - x_j, v_i^O - v_j) + G_O(x_i^O, v_i^O, \rho_i)\,, \tag{3}$$

which are coupled to the obstacle's eikonal equation, $f_O(\rho(x))\,||\nabla\phi^O|| = 1\,, x \in \Omega$. Here, x_i^O and v_i^O are the position and velocity of the mid-point of the leading edge of the ith obstacle and ρ_i is evaluated at x_i^O by interpolating the density of pedestrians. x_j and v_j are position and velocity of jth neighbour in the list N_p of pedestrians in a circle of radius R centered at x_i^O. Also, we define F_O, G_O and f_O similar to F, G and f as above.

3 Numerical Method

The model equations for pedestrians and obstacle(s) are solved using a mesh-free particle method using least square approximations, see [17]. For this, the hydrodynamic equations in (1) are rewritten in a Lagrangian form as:

$$\begin{aligned} &\frac{dx_i}{dt} = u_i\,, \quad \frac{d\rho_i}{dt} = -\rho_i\,\nabla_x.u_i\,, \\ &\frac{du_i}{dt} = G(x_i, u_i, \delta \star \rho)\; + \sum_j F(x_i - x_j, u_i - u_j)\,\rho_j\,dV_j\,, \end{aligned} \tag{4}$$

where dV_j is the local area around a neighbouring particle. The kinematic equations of the obstacle(s) in (3) and eikonal equations are coupled to (4) to solve the system completely. An explicit Euler time discretization scheme is used for solving the systems (3) and (4).

The Lagrangian equations are solved on a mesh-free cloud of particles. Furthermore, to solve the eikonal equation, we use an independent structured or unstructured grid on the domain of interest. Information is exchanged between the mesh-free grid and the eikonal grid via interpolation techniques. The eikonal equation is solved by a fast marching method [13, 15]. The boundary conditions of the eikonal equation contain information about the environment, like the position of walls or obstacles. A moving obstacle is treated like an immersed boundary in the eikonal grid, with activation-deactivation of grid points according to the position of the obstacle.

4 Results

Using the numerical method described, we solved the above model equations to analyze the collision-avoidance behaviour of pedestrians and moving passive or dynamic obstacles. We consider a two-dimensional domain of length 100 units and width 50 units for our numerical simulations. The pedestrians are located at the left end of the domain. The right and left boundaries act as exits for the pedestrians and obstacle(s), respectively. Initial pedestrian density is taken as $\rho = 1$ ped/m^2. A fixed time step of 0.002 is used for the explicit time integration scheme.

4.1 *Case 1: Passive Obstacle*

Passive moving obstacles do not have a feedback interaction with the pedestrians and follow pre-defined trajectory. We considered two different scenarios, pure translation and translation combined with oscillation, and compared with the case of a stationary obstacle. The left and middle subfigures in Fig. 1 show the case where a pedestrian group interacts with a passive obstacle in translation. We observed that when pedestrians interact with a passive obstacle(s), they adjust their path to avoid collision with the obstacle. The path adjustment is made well in advance than the time instance of a head-on collision, using the information available via the eikonal solution. The presence of a moving obstacle slows down the pedestrians, in terms of the time taken to navigate the domain, when compared to their behaviour in the presence of a static obstacle. This implies that the pedestrians exit the domain faster

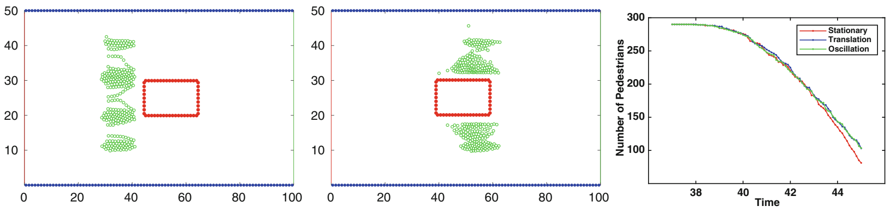

Fig. 1 Pedestrian interaction with a passive moving obstacle shown as red rectangle at time $t = 10\,s$ (left) and $t = 20\,s$ (middle). (Right) Number of pedestrians-time graph for the three different cases - stationary obstacle, passive obstacle in translation, passive obstacle in translation and oscillation

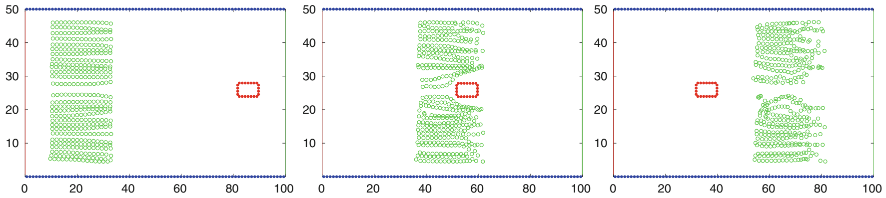

Fig. 2 Pedestrian interaction with a dynamic moving obstacle (red rectangle) at time $t = 5$ s (left), $t = 20$ s (middle) and $t = 30$ s (right). Note that the green markers denote the Lagrangian mesh-free grid points and not the physical pedestrians

in the presence of a stationary obstacle and hence the total density of pedestrians in the domain decreases faster with time as seen in the density-time plot in Fig. 1. This is expected as they have to adjust their path and speed continuously to move forward.

4.2 Case 2: Dynamic Obstacle

In the case of a dynamic moving obstacle, both the obstacle and pedestrians actively try to avoid collisions with each other since there is a feedback interaction via the force terms (cf. (1) and (3)). Figure 2 shows a scenario wherein a group of pedestrians interact with a dynamic obstacle. We observe that, though the pedestrians and obstacle(s) undergo path and speed changes, the collision avoidance mechanism is primarily via change of path by pedestrians and change of speed by obstacle(s). Owing to the two-way interactions here, in comparison to one way interaction in the case of passive obstacle, the changes in trajectory of pedestrians is more smoother, continuous and less abrupt. This leads to lesser tendency of having high density of pedestrian crowd near the corners of the leading edge of the obstacle.

5 Conclusion

We have successfully coupled a hydrodynamic model for pedestrian motion with simple kinematic equations for moving obstacles via eikonal equations. Our model satisfactorily replicates the collision-avoidance patterns observed in experimental scenarios like in [11]. But being a macroscopic model, only moderate to high-density scenarios can be studied and it is not possible to analyze microscopic behavioural patterns. We can further study the path and speed changes observed and make quantitative comparisons with other data, for example in [7, 16]. Also, exhaustive studies by changing the size or shape of the obstacle and of the domain can be conducted. We note here that the numerical method used is particularly efficient to employ in complex environments and changes in geometries. For more accurate results, parameters need to be estimated from experimental or real data. Moreover, an extension of the given model to pedestrian-vehicular traffic interactions will be presented in a more elaborate future publication.

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