Chapter 8
Modern Mathematics in Italy: A Difficult Challenge Between Rooted Tradition and Need for Innovation

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Abstract In this chapter, we first describe the Italian context when the proposals of reforms, generically indicated under the label of "new math" or "modern mathematics," were developed all around the world. The current mathematics programs dated to 1945, the main topic was geometry, taught according to a rooted tradition based on Euclid. The conference of Bologna in 1961, which followed those of Royaumont, Aarhus, and Zagreb-Dubrovnik, stimulated the Italian mathematicians to consider also for their country's reforms in the light of the proposals that emerged at the international level. With the collaboration of the Ministry of Education and under the aegis of OECD, they organized refresher courses on the new approaches suggested by modern mathematics, edited books, and supervised experiments in selected classes. The plan by the Ministry was not efficient; however, this ferment stimulated various meetings for developing new mathematics programs. These programs were never implemented and only a few notions of modern mathematics remained, but new ideas and new contacts began to circulate which slowly changed the Italian context.

Keywords CIEAEM · CIIM (Commissione Italiana per l'Insegnamento Matematico) · Emma Castelnuovo · Experimental projects · Geometry · ICMI · Italian programs of mathematics · Luigi Campedelli · Modern mathematics · New Math · Reform movement · Ugo Morin · UMI (Unione Matematica Italiana)

Traditions in Italian Mathematics Education

In 1861, Italy became a unified country and 10 years later Rome became the capital, after its annexation in 1870. These events were the final step of a process of construction of the nation, which after the Vienna Congress in 1815 had been made up of some 10 little states. One of them, the Kingdom of Sardinia (including Savoy, Piedmont, Liguria, the county of Nice, and Sardinia), was the core of this process and the king of Sardinia became the king of Italy. One of the main tasks of the new state was the construction of a system of instruction, starting from the system inherited from the Kingdom of Sardinia enacted by the Minister of Education Gabrio Casati with the Law of November 13, 1859.

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An important step was the Coppino Reform of higher education launched in 1867, which introduced the *Elements* of Euclid as a textbook (Maraschini and Menghini 1992; Vita 1986). This introduction was intended to reinforce the educational value of the Italian High School: The teaching of geometry followed the Euclidean approach and was definitely seen as "gymnastics of thought." Some paramount mathematicians, the geometer Luigi Cremona for one, were behind this choice. The Coppino Reform drastically reduced Casati's programs of 1859, to eliminate "all those theories […] whose importance lies mainly in the applications they lead to, or in their subsequent development in higher mathematics"[1] (Regio Decreto, October 10, 1867, No. 1492). In the following years, the *Elements* became non-compulsory and were gradually replaced by textbooks that still had to follow the Euclidean method. The imprint had been given: In Italy, the "traditional route" became, without doubt, deductive and synthetic teaching of geometry via axioms and theorems, as done in Euclid's *Elements*.

According to Guido Castelnuovo, loyalty to Euclid's *Elements* had overshadowed the theory of measure, just as it had happened with the Greeks, "who could not conceive of the ratio between two incommensurable measures as a number" and whose "sense of rigor rejected the use of approximations" (Castelnuovo 1919, p. 5). This position prevented teaching from penetrating "the heart of modern thought" (p. 5). For instance, Italy refused the curricular reform proposed by Felix Klein to the European countries at the beginning of the 1900s, which revolved around the concept of "functional thinking" (Schubring 1987).

Indeed, the strongly "classic" setup that inspired the school programs of the newly unified Italy—and that remained a dominant feature in Italian schools—is evident when we refer to the question of the introduction of real numbers and the concept of function. Even the main structural reform of the Italian school system, known as the Gentile Reform, launched in 1923, which also introduced the Scientific Lycée, was characterized by a marked humanistic orientation and did not solve the question. From 1925 until the programs of the Allied commission of 1945,[2] attempts to better the Gentile Reform by allowing an intuitive introduction of real numbers in the early years of High School were disregarded. Real numbers and analytic geometry continued to enter the curriculum very late, only in the third year, because as clarified above by Castelnuovo (1919), the construction of real numbers is geometric and they can only be introduced after the complete treatment of geometry (Marchi and Menghini 2013).

This is the framework that explains some of the Italian resistance to change when the reform movements of the 1960s developed all around the world. In Italy, the reference movement was "modern mathematics" inspired by Bourbakism.

In this chapter, we use the following labels to indicate the various levels of the Italian school system (see Ciarrapico and Berni 2017) for further information:

- Primary school (age range: 6–11 years; grades: 1–5).
- Secondary school:

  Middle school (age range: 11–14 years; grades: 6–8).
  High school (age range: 14–19 years).

[1] All the translations in this chapter are by the authors.

[2] After 1945 the first reform of the Lycées occurred in 2010.

The Reconstruction After World War II in Italy: National Initiatives and International Contacts in Mathematics Education

Like all nations that came out of the War, Italy had to deal with the problems of reconstruction. Although there were serious problems in the education system, chief amog them the illiteracy still present in the country, there was little attention paid to the importance of the school system, especially a good scientific education for a modern society. The first sign of an exit from stagnation was given in 1950 by the Ministry of Public Instruction, which entrusted the *Centro Didattico Nazionale per la Scuola Secondaria Superiore* [National Educational Center for High School], a body established following a law of 1942, with the task of organizing didactic specialization courses and experimental classes to inspire and encourage new teaching methods. In 1950, mathematicians such as Luigi Campedelli and Attilio Frajese, together with educators such as Aldo Agazzi and Gesualdo Nosengo, founded the *Movimento Circoli della Didattica* [Movement of Didactic Circles] and the journal *Ricerche Didattiche* [Didactical Studies] dedicated to the teaching of various disciplines. Soon after its foundation, the *Movimento* organized conferences and edited books, where the cultural background for renewing mathematics teaching was discussed (see Movimento Circoli della Didattica 1956).

The previous initiatives had a general orientation toward all disciplines taught at school. As far as mathematics education was concerned, since 1895, an association of mathematics teachers (*Mathesis*) existed and the journal *Periodico di Matematica* (from 1921 *Periodico di Matematiche*), founded in 1886, became its official organ in 1898. Another journal for mathematics teachers, *Archimede*, founded by a secondary school teacher in 1902 with the title *Il Bollettino di Matematica*, was published regularly. As a general policy, the *Periodico* mainly dealt with Italian authors and themes. *Archimede* showed some openness to developments abroad. In the 1960s, this journal published articles not only with a Bourbakist imprint but also gave voice to the unease of teachers in relation to the proposals of modern mathematics.

After World War II, the Italian community of mathematicians revived the activities of the *Unione Matematica Italiana* (UMI) [Italian Mathematical Union] and resumed relations with the international context so that in 1952 Italy hosted the first General Assembly of the reborn International Mathematical Union (IMU) (Lehto 1998). On this occasion, the old Commission on the Teaching of Mathematics founded in 1908 became a subcommission of IMU with the name "International Mathematical Instruction Commission" (IMIC). After 2 years, this name was changed into the current name "International Commission on Mathematical Instruction" (ICMI). The Italian Guido Ascoli was appointed as a Treasurer of the Executive Committee in the period 1952–1954 (*L'Enseignement Mathématique,* 1951–1954, *40,* pp. 81–82). At the meeting of Geneva on July 2, 1955, under the presidency of Heinrich Behnke, the countries belonging to IMU were invited to appoint national subcommissions for ICMI and be represented by national delegates (*L'Enseignement Mathématique*, 1955, *s. 2, 1*, p. 198).

In Italy, the *Sottocommissione Italiana per l'Insegnamento della Matematica* [Italian Subcommission for Mathematics Teaching] was established in 1954 (see BUMI 1954). The name *Commissione Italiana per l'Insegnamento Matematico* [Italian Commission for Mathematical Teaching] (from 1963 designated with the acronym CIIM) still exists, although CIIM is no longer a subcommission of ICMI: From 1975, it became a permanent subcommission of UMI. The successive presidents in the period 1954–1988 were Giovanni Sansone, Enrico Bompiani, Campedelli, Salvatore Ciampa, Vinicio Villani, Giovanni Prodi, Benedetto Scimemi, and Carmelo Mammana. They were mathematicians, but some of them showed a genuine interest in mathematics education. Usually, the president of CIIM was the ICMI representative. It was not until 1963 that school teachers were admitted to CIIM.

In the period in question, ICMI was not the only agency aimed at discussing problems related to mathematics education: A prominent role was played by the *Commission Internationale pour l'Étude* *et l'Amélioration de l'Enseignement des Mathématiques* (CIEAEM) [International Commission for the Study and Improvement of Mathematics Teaching]. Among the Italian members of CIEAEM, two teachers—Emma Castelnuovo and Angelo Pescarini—were particularly active and contributed to establishing international contacts in the school milieu.

From Royaumont to Bologna

From November 23 to December 4, 1959, OEEC (Organisation for European Economic Co-operation, later on, OECD, Organisation for Economic Co-operation and Development)[3] organized a Seminar on the reform of mathematics teaching at the Cercle Culturel de Royaumont in Asnières-sur-Oise (France). Delegates came from 15 OEEC countries (Austria, Belgium, Denmark, France, Germany, Greece, Ireland, Italy, Luxembourg, The Netherlands, Norway, Sweden, Switzerland, Turkey, UK), Yugoslavia, Canada, and the United States. As is well known, this event was a turning point in the history of modern mathematics (see De Bock and Vanpaemel 2015; Schubring 2014).

The basis for discussion should have been the answers to a questionnaire sent in December 1959 to the countries participating in the program asking about the current conditions of mathematics education in their countries (see OEEC 1961a, p. 7),[4] but data summarizing responses to the questions were not available at the beginning of the Seminar. Answers given to the questions were nevertheless presented and analyzed in the second part of the official report (OEEC 1961a, pp. 127–210), titled "Survey of Practices and Trends in School Mathematics." The questions concerned: Number of hours of compulsory mathematics teaching in the different schools; educational qualification and teacher training; authorities competent to establish programs and select textbooks; any ongoing reforms; contents of the programs (more precisely, various topics were listed and it was asked, for each of them, in which school year it was carried out). This survey is an interesting source of information about mathematics education in the countries participating in the program before the start of modern mathematics in Europe.

We report the following information concerning Italy, based on answers given by Campedelli to the questions of the questionnaire:

- Italy was the only country, besides Ireland, in which no pedagogical studies were included in the programs for initial teacher training.
- Italy was the only country where there seemed to be an oversupply of mathematics teachers.[5]
- The part of the questionnaire relating to the training of elementary teachers referred more precisely to "arithmetic teachers"; a few countries, including Italy, specified that the training of primary teachers concerned not only arithmetic but also geometry

[3] The "Organisation for European Economic Co-operation" (OEEC), born in 1947 as "Committee for European Economic Co-operation," included 18 European countries. When the USA and Canada joined as members in the end of 1960, it was renamed "Organisation for Economic Co-operation and Development (OECD)," "*Organisation de coopération et de développement économiques*" in French. New statutes came into force on September 30, 1961. English and French are the official languages.

[4] The questionnaire (see Appendix B in OEEC (1961a), pp. 221–237) was sent to the OEEC member countries, and to Canada, the United States, and Yugoslavia. Only Spain did not answer, so the survey lists 20 countries. The results of the survey, divided by countries, are also presented in (OEEC 1961b), with the exception of Canada.

[5] This is in contradiction with various associations' agendas around the years 1962–1964 (see, e.g., R. G. 1964a).

The Italians at Royaumont

The various countries adopted different selection processes for the delegates who were to represent them. In Italy, the CIIM stated that it would participate in the Seminar with two delegates (BUMI 1960), who were Campedelli and E. Castelnuovo, both key figures in the Italian milieu of mathematics education of those years. By a twist of fate, they both were linked to G. Castelnuovo, a distinguished mathematician, founder of the Italian school of algebraic geometry. He was also engaged in the development of mathematical instruction in Italy and abroad in the early decades of the 1900s. In 1908, he chaired the International Congress of Mathematicians in Rome, when the Commission from which ICMI sprung was founded. Later, he was a member of its Central Committee between 1913 and 1920 and vice-president between 1928 and 1932.

Campedelli (1903–1978) started his academic career as an assistant professor of G. Castelnuovo in Rome. He researched in the field of algebraic geometry under the guidance of Federigo Enriques. At the end of the 1940s, when the impetus of the Italian school of algebraic geometry died out, Campedelli's interests turned almost exclusively toward mathematics education and elementary mathematics, with particula reference to geometry, logic, and foundations of mathematics. He wrote textbooks for school and university, and many articles (see Furinghetti 2019). As mentioned before, he founded the *Movimento Circoli della Didattica* and animated the first national initiatives on mathematics teaching in the 1950s. In the list drawn up by Caleb Gattegno in 1957, he was among the "*membres actifs*" [active members] of CIEAEM (see Bernet and Jaquet 1998, p. 25), which were the members who constituted the national section and promoted the work of the Commission in their countries. Campedelli contributed to the second book published by CIEAEM (Gattegno et al. 1958) with a chapter oriented to classroom practices that included extensive use of concrete materials (Figure 8.1). In his papers, Campedelli did not take an explicit position about modern mathematics, which is surprising given that he had been one of the Italian delegates at the Royaumont Seminar. In particular, it is disappointing that such a prolific author did not report adequately in Italian journals for mathematics teachers about this event.

As to E. Castelnuovo (1913–2014), daughter of Guido, the reader can find information in different papers (e.g., Furinghetti and Menghini 2014). Here we focus on her many international contacts: According to Théo Bernet and François Jaquet (1998), she was among the founding members of CIEAEM, and became the Italian representative at ICMI, starting with the 1975–1978 term (*ICMI Bulletin*, 1975, No. 5). During that term, she was also a member-at-large of ICMI. Through her text-

Figure 8.1 Emma Castelnuovo (right) and Lina Mancini Proia in Pallanza, Italy, 1973

books, she contributed to the dissemination in Italy of some of the proposals born in the milieu of CIEAEM and the Belgian schools. For her, a fundamental reference and source of inspiration were Paul Libois and his work at the *École Decroly* in Brussels, which she often visited (Castelnuovo 1965).

The attitude of E. Castelnuovo toward modern mathematics can be summed up by her exclamation at Royaumont, which she recalled in a private conversation during the forty-third meeting of CIEAEM in Locarno (1991): "But Professor [Jean Dieudonné], without the triangle, the table on which your speech sheets rest would collapse! [it was a plank placed on trestles]" (Équipe de Bordeaux 2009, p. 2). She recognized the unifying power of modern mathematics and introduced elements of set theory and transformations into her teaching. However, she did not give up the intuitive approach and her method of bringing students closer to the abstract, starting from their experiences in concrete situations. After an intensive teacher training course on mathematics teaching held in Arlon (Belgium) on July 2–4, 1960, organized by the Belgian Ministry of Public Education, she wrote that on the one hand, she felt all the charm of new studies which, thanks to their generality and simplicity, may unify the most disparate theories; on the other hand she did not consider it correct to introduce modern theories in their purest expression without making young people aware that they are the result of the work of centuries (see Castelnuovo 1960).

Bologna Conference

The themes discussed in Royaumont were revisited and extended in follow-up meetings, in particular, in Aarhus (May 30–June 2, 1960), and in Zagreb-Dubrovnik (August 21–September 19, 1960) (see Notiziario. 1960; see also Chap. 4 in this volume).

In those years, there were no Italians in the ICMI Executive Committee, but Tullio Viola participated in the ICMI meeting in Paris on December 7–8, 1959 (*L'Enseignement Mathématique, s. 2, 5,* 1959, p. 140), and Sansone and Mario Villa participated in that of Belgrade on September 21, 1960 (*L'Enseignement Mathématique*, 1959, *s. 2, 5*, p. 284). Viola represented Italy at the Aarhus meeting and contributed to the discussion (Behnke et al. 1960). Villa participated in the Zagreb-Dubrovnik meeting (BUMI 1960; OEEC 1961c), and Pescarini participated in the Budapest meeting of 1962 (Hungarian National Commission for UNESCO 1963).

A good opportunity to come in direct contact with modern mathematics was given to Italians at the international conference held in Bologna (October 4–8, 1961) entitled "A discussion of the Aarhus and Dubrovnik reports on the teaching of geometry at the secondary level" (see Chap. 4 in this volume). ICMI and the Italian CIIM sponsored the meeting, and university professors, school inspectors, educators, and delegates from UNESCO and OECD were in attendance (BUMI 1962b). The organizing committee consisted of Pietro Buzano, Marshall Stone (ICMI president), Villa (chairman), Viola, and Gilbert Walusinski (ICMI secretary). The Italian delegation consisted of six ministerial inspectors, one representative of the *Centri Didattici Nazionali* (Ruggero Roghi), and university professors (Platone 1961; Stone and Walusinski 1963). Most contributions were published in *L'Enseignement Mathématique* (1963*, s. 2, 9*).

The importance of the Bologna conference is twofold: At the international level, it offered a moment of synthesis of the various proposals and clarified the positions on modern mathematics of some famous mathematicians (Stone, Emil Artin, Henri Cartan, Hans Freudenthal); at the national level, it promoted the opening to modern mathematics in Italy.

In his talk, the Italian Ugo Morin (1901–1968) proposed to introduce tools, such as real numbers, abstract algebra, set theory, and vectors, which could be used in teaching geometry. He noted that in Italy it would be difficult to implement the axiomatic structure proposed in Zagreb-Dubrovnik because

Figure 8.2 Ugo Morin (left) and Bruno de Finetti at the UMI quadrennial Congress in Trieste, Italy, 1967. (Courtesy of *Centro Ricerche Didattiche Ugo Morin*)

of the different starting age of high school (14 years) in Italy compared to that of other countries (15 years) (Morin 1963).

In the 1960s, Morin well understood what could be the important elements for renewing mathematics teaching and teacher training. For example, he advocated the introduction of psycho-pedagogical courses into the university curriculum for mathematics students who chose to become secondary school teachers (BUMI 1966). Morin had been Vice-President of UMI (Figure 8.2) and a member of CIIM. His actions throughout his professional life demonstrated a strong political and social commitment (Tomasi 2018). He wrote textbooks and drafted programs for secondary schools that incorporated elements of modern mathematics. Morin's intervention at the Bologna conference revealed his openness to new methods and contents, elements which were also present in his mathematical research.

L'Enseignement Mathématique included the texts of the interventions of other Italians, who did not take an explicit position on modern mathematics. In particular, the conference opened with a report by Villa (1962), who on the one hand stressed the need to modernize mathematics teaching in Italian secondary schools, which, he said, had been "standing still" for almost half a century; on the other hand, he observed that any modernization should be carried out with appropriate caution. He argued that it was necessary first to institute refresher courses for teachers and then pilot classes in which some of the topics and methods that were to be introduced into mathematical teaching could go through a necessary, prudential experimental phase. Furthermore, he added that it was clear that the tests, the experiments, and the definitive programs required an adaptation to the traditions and needs of the various countries in which the modernization of mathematics programs would be undertaken. In Villa's talk, caution and prudence were emphasized.

Other talks by Italian representatives were those by Viola about didactics without Euclid and Euclidean pedagogy, and by Lucio Lombardo-Radice about geometry and culture in humanistic schools (*Licei Classici*). Campedelli and Roghi reported respectively on the refresher courses held in Italy and on the first experiments carried out in schools. The Italian contributions seemed to stay away from the nub of the problem, specifically modern mathematics. This suggests that a criticism expressed by Cartan (1963) toward those who wanted to innovate without changing anything referred to Italians. In particular, the aloofness of Campedelli in the discussion might be interpreted as a sign of his lack of interest in (or even dislike for) modern mathematics.

First Cautious Steps Toward Modern Mathematics in Italy

In the days immediately following the conference (October 9–10, 1961), a meeting of experts appointed by OECD took place to discuss which Italian classes should test the new proposals (BUMI 1962b). The meeting was attended by several Italian university professors. In a follow-up meeting on November 12, 1961, CIIM authorized Buzano, Campedelli, Morin, Villa, and Viola to plan an experiment in schools (BUMI 1962a). The Ministry of Education set up a commission (BUMI 1962b) to organize refresher courses and pilot classes in mathematics with a modern program; costs were to be borne half by the Ministry of Education and half by OECD. These refresher courses covered topics that originated from the Bologna conference such as set theory, geometric transformations, essential elements of abstract algebra and algebraic structures, the logical structure of geometry, and applied mathematics. Two volumes were published on the contents of the courses (Villa 1965, 1966). In the school year 1962–1963, the Ministry set up some 40 "pilot classes," which concerned the last classes of the humanistic High School, the second-last classes of the scientific High School, and the High School for prospective teachers (see Notiziario. 1963). In these pilot classes, part of the mathematics program was traditional, while the other part covered topics chosen by the teachers from those developed in the refresher courses.[6] At the above-mentioned meeting of CIIM in November 1961, prudence was recommended, and a need to listen to teachers' opinions was stressed.

The pilot classes were then extended to the *istituti tecnici* (technical institutes), i.e., high schools intended to train different types of professional workers, and to a few middle schools. In these experimental classes, essentially two texts were used: *Per un Insegnamento Moderno della Matematica* [For a Modern Teaching of Mathematics], published in 1963 (Bologna, Italy: Patron) by the Ministry of Education and OECD, and *La Matematica Moderna nell'Insegnamento Secondario* [Modern mathematics in Secondary teaching] published in 1965 (Rome, Italy: Eredi V. Veschi), written by the ministerial inspector Armando Chiellini.

There was no lack of criticism of the new trends, such as that by Eugenio Togliatti expressed in a conference held in Rome in May 1962 on the theme "Scientific and humanistic teachings in the educational function of secondary school." In his report we read:

> I do not see how the average of the students, even in the last high school class, can reach such a level of knowledge and logical education that they can appreciate the high value of the scientific synthesis that is contained in an abstract "structure." […] I would not like this to end up in an arid formalism, and for the students, too difficult to appreciate. And again, how could current mathematics teachers, trained in another climate, change the direction of their teaching so radically? (Togliatti 1963, p. 44)

In 1961, a university curriculum for prospective teachers was established (Presidential Decree 26 July 1960, No. 1692). An algebra course replaced the chemistry course present in the previous curriculum of mathematics (Lombardo-Radice 1963). A new generation of mathematics teachers was emerging, for whom in the future, the problem of updating on new topics might be less pressing.

A reform that came to fruition in those years was that of the middle school, approved in December 1962 (Law No. 1859, December 31, 1962). It led to the unification of the middle cycle (11–14 years), which before was divided into various types. Mathematics and "Elements of natural sciences" were to

[6] The atmosphere of enthusiasm that animated some initiatives of that period is well expressed in the autobiography of Lina Mancini Proia (2003). It should be remembered that the teachers involved in the various experiments of those years acted mainly on a voluntarily basis.

be taught by the same teacher (something which generated a lively discussion that continues today (see Ciarrapico and Berni 2017; Furinghetti 2015). The atmosphere of modernization of mathematics teaching in those years had only a timid influence on related programs. The contents were in line with the past, if not for the invitation to give examples of "correspondences and functions" and to consider issues "whose discussion involves operational and structural analogies." The word "set" was never used. The methodological suggestions in this reform appear more modern. We read that "it is useful to refer to inductive procedures that start from observations, easy experiments, and empirical evidence" and that "constant care must be taken to harmonize arithmetic with geometry to give a unitary vision of mathematics" (Law No. 1859, December 31, 1962). Frequent use of graphs to help visualization was also recommended.

Proposals of New Programs for High School

Besides the reform of the middle school mentioned above, which strictly speaking cannot be considered a modern reform, other proposals for new mathematics programs for high schools followed one another, also in view of a reform that should have taken place in October 1966, but was never implemented.

In particular, we refer to the programs developed between 1963 and 1967 at the Italian conferences held in Gardone, Camaiore, and Frascati. Leading characters in this phase were again Morin, vice-president of UMI; Campedelli, president of CIIM; Viola, president of *Mathesis*; and Villa, director of the *Centro Didattico Nazionale*.

The programs proposed at these conferences primarily focused on the first 2 years of high school, which, should be the same for all types of schools. In the following, we will present only the topics of geometry, which have been the most discussed.

The first proposal was developed at the Gardone conference on May 27–29, 1963, as follows (Anonymous 1965):

First Year of High School

- The plane as a set of points and straight lines as its subsets. Properties of connection and parallelism. Direction of a straight line.
- Order on a straight line and partition of the plane. Segments. Convex figures. Angles. Polygons.
- The group of the congruences. Segments and angles as magnitudes.
- Perpendicular lines. Reflections.
- Euclidean properties of triangles and polygons.

Second Year of High School

- Circumference and circle.
- Area (extension) of a polygon seen as magnitude. Theorems of Pythagoras and Euclid.[7]
- Geometric constructions.
- Overview of logic schemes and methods for proving theorems and solving problems.

According to Morin (1965), who participated in the drafting process, the commission that had developed the Gardone programs was still too tied to inaccuracies characteristic of the status of mathematics 50 years ago. We do not know what exactly Morin was referring to; we notice that the change

[7] The theorems usually taught in Italy as theorems I and II of Euclid correspond to the two parts of proposition 8 of Book 6 of Euclid *Elements*. They are necessary to prove the Pythagoras theorem. All these theorems disappeared from the successive Camaiore Programs, but the Pythagoras theorem was re-introduced, along with the Thales theorem (i. e., the intercept theorem) in the Frascati Programs.

seems to be more in language than in substance. The reference to the *magnitudes* should also be emphasized. The theory of magnitudes refers to the introduction of real numbers according to Euclid's theory of proportions, that is, to a geometric introduction. In the Gardone programs, the term was used to avoid talking about *measures*, which correspond to an isomorphism between real numbers (introduced independently) and segments. The term magnitude was deleted in subsequent drafts and substituted with *lengths*.

The second proposal was developed at a conference in Lido di Camaiore on October 3–5, 1964 (Morin 1965).

First Year of High School (we only mention some of the listed points)

4. Affine properties of the plane: Incidence and parallelism, direction; translations, vectors, and point reflection.
6. Topological properties: Order on a straight line and partition of a plane. Segments, convex figures: Angles and polygons.

Second Year of High School

9. Metric properties of the plane: Perpendicular lines, reflection about a line, group of congruences. Length of a segment and width of an angle (mentioning their measures). Applications to polygons.
12. Elements of analytic geometry: Cartesian coordinates on the line and on the plane. Equations of a translation. Representation of a straight line …

There was also a first point on "elements of set theory," a second point on "inner operations on sets of numbers representing groups, rings, and fields," and an eighth point (in the second year) on "an intuitive approach to real numbers."

An interpretation of these programs was provided by Morin (1965). The aim was to introduce, in addition to the axioms of incidence, an axiom of existence of a collineation (point reflection) which fixes a point (No. 4), and an axiom on the properties of perpendicular lines, which allowed for the introduction of reflections about a line and therefore congruences (No. 9). So we observe that axioms asserting the existence of isometries appear (as also is in Gustave Choquet's metric axioms), but geometric transformations are not yet explicit. They will appear later in the Frascati programs.

The aim of all the programs was to "adhere to the new scientific and didactic needs, but also to update notations, expressions, definitions, to the current scientific level" (Morin 1965, p. 57). It was precisely the new expressions and definitions that made some proposals hermetic. Another moot point in these programs was that a large part, relating to set theory, order structures, and algebraic structures, was supposed to precede the teaching of geometry.

At both the conferences of Gardone and Camaiore, a need was felt to renew contents and methods to promote scientific, cultural, and educational progress. Moreover, an urgency was stressed to organize refresher courses for all mathematics teachers and to make the necessary bibliographic tools available to them.

With the Camaiore programs, the debate increased, with suggestions of prudence, but not of rejection being made. There was a general attitude in favor of the reform; what was most appreciated was the possibility of new "transversal" connections between the classical topics (the concepts of structure, set, vector, function, …) and, as E. Castelnuovo (1964) suggested with reference to geometry, a treatment by type of property rather than by type of figures. In addition to this, the concept of structure, which characterized the work of Bourbaki, was recognized as unifying different mathematical branches: "This is a far-reaching fact: Undoubtedly today there would not be such a widespread awareness of the unity of mathematics if Bourbakism had not existed" (Prodi 1995, p. 416). The Camaiore programs also reflected the idea that developments in mathematics (in particular abstract algebra and topology) could not fail to influence school teaching.

The journal *Archimede* gave space to the curriculum debate. Indeed, the director of *Archimede*, Roberto Giannarelli,[8] became personally involved in the activity of reform. Starting from 1964, comments and reports on the various proposals made at the conferences and working sessions followed one another in this journal. The journal *Periodico di Matematiche* also published some contributions on this subject from 1965 onward.

The references that appear in the various papers related to the proposals always included the works of Choquet, Dieudonné, Georges Papy, etc. The Erlangen program was rarely mentioned. Francesco Tricomi (1965) complained, among other things, about the disappearance from the Camaiore programs, of the Pythagoras theorem, and theorems relating to the circumference.

A much less cautious position was taken in the account of Bruno de Finetti (1965) at another international conference, held in October 1964 at the *Centro Europeo per l'Educazione* (CEDE) [European Center for Education] in Villa Falconieri (Frascati). The conference focused on an issue that was very important at the time, namely the transition from high school to university. Of particular importance was the teaching of geometry, which, as pointed out by Jacqueline Lelong (1965) formed the basis of secondary mathematics education but did not allow any extension to higher studies.

According to de Finetti, "the main role for a radical simplification and revision of the mathematical tools and a unified vision of almost all topics, undoubtedly belongs to the notion of the linear (affine) system (and space). … Euclid's geometry, so unnatural and heavy because of the lack of distinction between affine and metric properties, can therefore be left aside" (De Finetti 1965, pp. 123–124). He greatly appreciated the programs for the middle school proposed by Papy (1966), already being implemented in Belgium, and for high school by André Revuz (1965), which constituted a rather definitive model of the ideas to be realized.

The Ministry's intent to modernize school mathematics aroused mixed reactions from teachers. Those who had followed the experiments in the pilot classes were enthusiastic, but many others, the majority, were perplexed and assumed an attitude of refusal. Among these were the older teachers and the teachers of the middle school who seemed to be unable to deal with the proposed change because most middle-school teachers held degrees in the sciences and lacked specific mathematical preparation (see Temussi 1965). The problem of teachers' lack of preparation had been mentioned by Behnke as an international problem at the aforementioned Frascati conference (see R. G. 1964a, b). The pilot classes, in which ministerial inspectors and teachers did their utmost, were not very successful for a variety of reasons including the fact that the project was experimented only in the two last school years (Ciarrapico and Berni 2017). Vincenzo Vita (1986) and Giro (1969) claimed that the results of the experiments in the pilot classes were never collected and publicized, nevertheless, their effects could be considered limited.

In lycées and high schools for prospective primary teachers—arguably the schools which most needed a change, no official reform was initiated, and—apart from the experiments—the mathematics programs remained those of 1945.

The (Blunt) Top of Modern Mathematics in Italy

During 1966 and 1967, a large commission gathered at Villa Falconieri in Frascati under the aegis of UMI and CIIM, and a long cycle of activities dedicated to programs was concluded with what became known as the Frascati programs (Giannarelli 1967; R. G. 1966).

Participants in the two conferences of 1966 (February, 23–26) and 1967 (February, 16–18) were university professors—including Campedelli, de Finetti, Prodi, and Villani—secondary school teachers invited by UMI (BUMI 1966), the president of UMI and the members of its Scientific Commission,

[8] Sometimes his contributions in the journal are signed R. G. or Giro.

members of the *Consiglio Superiore della Pubblica Istruzione* [Higher Council of Public Education], officers of the Ministry of Education, and representatives of the National Educational Center for High School. At the end of the conferences, two proposals for new programs were formulated. The first, that of the 1966 conference, was published in the journal *Archimede*, and represents a unified proposal for the first two classes of high school (R. G. 1966); the second, formulated at the end of the work in 1967, is a minimum proposal for the final 3 years of classical or scientific lycées and appeared in the same journal (Giannarelli 1967). In these programs, the aims were to educate students in mathematical thinking, avoiding teaching conceived as training. It was recommended to proceed gradually from the intuitive to the rational aspects. UMI approved these programs (BUMI 1968), which were the subject of articles and conferences (we only mention the conference promoted by CIIM and UMI in Frascati from February 6 to 8, 1969, see Notiziario 1969). Although there has been no practical implementation, Frascati's programs helped to raise awareness among the most attentive teachers of the need for renewal of mathematics teaching. De Finetti (1967), member of CIIM since 1964, who was one of the drafters of the programs, recalled the work that accompanied the discussion over the years and observed that:

> the most positively innovative aspect, if I am allowed to express myself in a paradoxical form, lies not so much in the introduction of some useful and interesting things, as in banning all the heavy and useless things with which children are tortured due to inveterate customs. (p. 80)

The program for the first 2 years presented two themes—algebra and geometry—both with aspects of modernity. It included topics such as the partition of a set and equivalence relations, structures such as ring, group, field, and—in case—lattice and metric space, presented through examples from different domains. As for geometry, in continuity with the middle school, geometric transformations (translations, rotations, and reflections) were treated with applications to segments, angles, triangles, and polygons. The program suggested an approach to geometry through several methods chosen by the teacher, including the analytical one. In the 3-year program for the final years, alongside traditional topics, new themes such as the geometric vector plane and the abstract vector space appeared. In the final year, probability and statistics appeared, timidly. It was de Finetti, a researcher in probability theory, who was opposed to introducing probability, believing that this topic could not be taught well at that level (De Finetti 1967). This was probably why the programs included only "Elements of probability and simple applications of statistics." There is also a chapter entitled "afterthoughts and complements," in which possible topics traditionally unrelated to mathematics in secondary school were listed—such as "non-Euclidean geometries," "the projective extension of affine space," "the introduction to mathematical logic and Boolean algebra," "elements of topology," "elements of game theory," "equations of the third and fourth degree."

The section on geometry for the 5 years was structured as follows:

First year

- The plane as a set of points and straight lines as its subsets. Incidence, parallelism, direction.
- Order on a straight line and partition of a plane. Segments, convex figures: Angles and polygons.

Second year

- Congruence (or isometries). Comparison of segments. Perpendicularity. Translations, rotations, and reflections. Applications to segments, angles, triangles, and polygons. Circumference and circle. Regular polygons. Intercept theorem and Pythagoras' theorem.

Third year

- The geometric vector plane: Linear combinations, coordinates, translations. Systems of linear equations in two unknowns. Cartesian equation of a straight line, systems of two lines.

- Complex numbers. Scalar product.
- Group of congruences and similarities in the plane.

Fourth year

- Cartesian equation of the circumference, ellipse, hyperbola, and parabola.
- Space as a set of points, lines, and planes as its subsets. Incidence and parallelism, half-spaces.
- Geometric vector space. Scalar product in three dimensions. Perpendicularity. Distances. Angles between lines and planes.
- Area of the plane figures: polygons, circle. Length of the circumference.

Fifth year

- Simple solids and their main properties.
- Volumes of simple solids. Areas of rotation surfaces.
- Abstract vector space. Its models and applications.

To some extent, Gardone's programs were adopted, but with an emphasis on geometric transformations. We still find, as in Camaiore's programs, an intuitive introduction of real numbers and Cartesian coordinates in the second year. In the 3-year program among the aforementioned optional topics, there are "elementary transformations and their groups," and also the "projective extension of the affine or Euclidean space," but since the affine or Euclidean spaces were not explicitly named in the programs, it may be assumed that they fell within the study of the geometric vector plane in the third year, and in its extension to space with the scalar product in the fourth year.

Analyzing these programs we see that, despite the stimuli given by the European reforms, the Italian proposals followed a different path, in particular different from the French one. The Italian mathematicians involved in the 1960s reform did not want to deviate too much from their tradition: There was a clear common desire to treat geometry in a synthetic way, adapted to the new trend. Anyway, they accepted to anticipate the Cartesian plane and the intuitive treatment of real numbers to the second year, and also accepted to give up the traditional axiomatic approach in favor of a different axiomatic approach.

A controversial question still concerned when to introduce the "group of congruences," and the role to be given to it. The problem, as clearly expressed by Morin already in the preface to the 1958 edition of his textbook (Morin and Busulini 1963), was how to establish the equality between figures, which is the basis of elementary geometry.

In this respect, it was useful to "recover" the Erlangen program. Despite the contribution of Otto Botsch, the Erlangen program had been neglected at Royaumont (OEEC 1961a), but the study of transformation groups was present in various Italian debates on the reform of geometry programs (Menghini 2007). In the preface of the first edition of Morin and Busulini (1963), we read:

> If by suggestion we speak of movement and displacement, the geometric interest only concerns the consequent equality relation between figures, whose properties need to be postulated. This approach to the theory of equality is perfectly rational, indeed it characterizes elementary geometry following the famous Program of Klein, according to which in a geometry we study the properties of figures that are invariant for a specific group of transformations. (no pagination)

Thus, for instance, while Choquet postulated an isomorphism between the length of segments and the real numbers, Morin used the axioms of Euclidean geometry, postulating the existence of congruences that preserve segments and angles.

We can conclude that the Frascati programs represented a point of balance between innovative choices and conservative positions. After the Frascati conferences of 1966 and 1967, more and more attention was given to learning problems. Research in the field of mathematics teaching intensified and numerous experimental initiatives were undertaken in which teachers participated enthusiastically, supported by university professors. International developments in mathematics education contributed to this evolution by creating further opportunities for confronting ideas and experiments not only at the CIEAEM meetings but also in the newly established International Congress on Mathematical Education (ICME) and the meetings of the International Group for the Psychology of Mathematics Education (PME). At ICME-1 in Lyon, among the 655 active participants from 42 countries, there were 28 Italians (*ICMI Bulletin*, 1975, 5), most of them school teachers.

But, as already mentioned, none of the reform proposals for high schools, went through (Ciarrapico and Berni 2017).

Initiatives in Grades 1–8

The Primary School

Without ever appearing in official programs, modern mathematics found its place in primary schools and was even more diffused than in secondary schools. Unfortunately, lacking adequate teacher training, it was often reduced to the more superficial and folkloristic aspects of set theory and caused tensions between innovators and conservatives (Vita 1986).

One of the first encounters with set theory happened even before the Royaumont Seminar. Indeed, in 1956, thanks to the initiative of Campedelli and E. Castelnuovo, the Italian translation of the volume *Initiation au calcul* [Introduction to calculation] (Piaget et al. 1950) was published. This volume included an important chapter by Jean Piaget on the genesis of the concepts of number and measure in the child. According to Piaget, the concept of number is based on two fundamental stages: First, equipotent sets are identified—through one-to-one correspondence—later they are ordered according to the number of elements (i.e., according to the inclusion of the smallest into the largest). This chapter gave rise to the first spontaneous and isolated experiments in school (Pellerey 1989).

But the greatest dissemination occurred in the late 1960s, with the publication of several works on the teaching of mathematics in Primary School, including—between 1967 and 1969—the translation of the Nuffield Project (see Chap. 7 in this volume). This translation, published by Zanichelli (Bologna), was distributed by the *Associazione per l'Incremento dell'Educazione Scientifica* (AIES) [Association for the Increase of Scientific Education] chaired by Alba Rossi dell'Acqua and financed by Shell Italia.

As Pellerey (1989) argued, Piaget never stated that the construction of the "logic of classes" (p. 30), or set theory, should precede the concept of number and arithmetic operations. However, for the introduction of the number concept, and in general of arithmetic, the set approach was almost universally accepted as a result of a "compromise" between Piaget and the Bourbakists. In school practice, it became usual to start by introducing logical concepts and operations on sets (including intersection, union, complement, …) and only later to introduce the concept of natural number "almost as if nothing had been built by the child" (Pellerey 1989, p. 30). This is, for example, the approach of Nicole Picard (1967), which served as a reference for the programs of the Lichnerovicz Commission (see Chap. 5 in this volume). Even the Nuffield Project, in which elementary school teachers collaborated and which cannot be seen as an emanation of academic choices, did not deviate from this approach, deriving, for example, the sum from the union of disjoint sets, and the product from the Cartesian product of two sets.

According to Pellerey (1989), in those years Italian educators felt international pressure to renew the teaching contents, including those for primary schools. For this reason, the *Centro Didattico Nazionale per la Scuola Elementare* [National Educational Center for the Primary School] launched refresher courses, with contents that were inspired by the French proposals of the Lichnerovicz

Figure 8.3 Michele Pellerey (right) with Tamás Varga in Bordeaux, France, 1973. (Courtesy of Raimondo Bolletta)

Commission for Primary School, by the British proposals of the Nuffield project, and by the books and materials proposed by Zoltán Dienes (in particular, his logic blocks and his multi-base arithmetic blocks) (see Pellerey 1989).

A volume by Vittorio Duse (1969) that attempted integration between the various approaches, had a notable diffusion among primary school teachers. It was a text for teacher training, presenting a "higher point of view," and included: Introduction to set theory, primitive concepts and tautologies, Venn diagrams, Dienes logic blocks, Cartesian product of two sets, and algebraic structures.

In Italy, the influence of mathematicians led to exaggerated formalism and "rigor," and Piagetian influence led to accentuating the use of manipulative materials, such as logic blocks. The first chapter of Duse's book on set theory became the first chapter of all textbooks (not only for primary school) and this chapter—almost always detached from the rest of the book—persisted for a long time, often well beyond the end of the modern mathematics era.

Under the influence of the new proposals, some structured projects about mathematics teaching in primary schools flourished. Worth mentioning is the project *Ricerche per l'Innovazione del Curricolo Matematico nelle Elementari* (RICME) [Studies for the renewal of mathematical curriculum in Primary School], led by Michele Pellerey and born from a collaboration with the Hungarian project at the *Országos Pedagógiai Intézet* (OPI) [National Pedagogical Institute], started by Tamás Varga in 1963 (Figure 8.3). The project began in some Roman schools and then spread to other Italian schools. Logic and set theory were still present here, but not predominant. The concept of number had an eclectic character, based on a constructive approach connected to the idea of recursive functions, rather than to sets (Pellerey 1989).

A project coordinated by the University of Pavia (Ferrari et al. 1982) also envisaged a varied approach, though obviously it still included activities linked to logic and set theory for the introduction of connectives, or activities on relations between sets.

Unfortunately, these interesting projects did not have wide dissemination, but they made it possible to arrive, in 1985, at the formulation of proper programs for primary schools (Presidential Decree 104 of February 12, 1985), in which "logic and theory of sets are no longer the foundation of mathematics, but a means to analyze the mathematical discourse and guide its development" (Pellerey 1989, p. 176).

The Middle School

The reform of the middle school launched in 1963 had left some problems in mathematics teaching. To face these problems, in 1978, the Ministry of Education set up a commission of 60 members. E. Castelnuovo, Prodi, Francesco Speranza, and Villani were among the members for mathematics.

The resulting programs were promulgated in 1979 (Ministerial Decree 268 of February 9, 1979). Those concerning mathematics were interesting from the point of view of contents and pedagogical suggestions. Modern mathematics was no longer in fashion (in Italy and elsewhere), but some traces remained, as the following passages illustrate:

> The language of sets can be used as an instrument of clarification, of unitary vision, and valid help for the formation of concepts. However, a separate theoretical discussion will be avoided, as it would be inappropriate at this level ….
>
> During the three years, whenever the opportunity arises, similarities and differences between different situations will be recognized, as an approach to the ideas of relationship and structure.

These programs incorporated some novelties shared with foreign programs: Elements of logic, probability, and statistics. The use of pocket calculators was recommended. In geometry, some types of transformations were introduced (isometries and similarities, in particular, homotheties). Through the experience based on the observation of shadows, the existence of other types of transformations was suggested. The 1979 programs were at the forefront of the international context but had little application in practice due to teacher resistance to change.

Experimental Projects for Mathematics

A New Student Population

In the 1960s, there were changes in the school system, influenced by societal changes and student movements. The transition to mass schooling created a new school population.

The final exam at the end of high school (*Esame di Maturità*), which was very demanding as it required control over all subjects in the program, was reformed (Law No. 119 of April 5, 1969). The main change was that the oral examination was reduced to an interview on two subjects assigned in the previous months, one of which was chosen by the candidate. Since in Italy the diploma given by the *Esame di Maturità* was a necessary condition to enter university, this examination reform facilitated access to university. Furthermore, a new law (Law 910 of December 11, 1969) allowed access to all university faculties to students with a high school diploma, not just the lycée diploma as had previously been the case. This has changed the university population both in their basic preparation and in number.

Another change in student population at the end of the 1960s was generated by the marked attention for the technical institutes, that is to say, the types of schools for the age range 14–19 which were aimed at educating in applied disciplines (mechanics, electronics, chemistry, etc.). The renovation began with the programs of the *Istituto Tecnico Commerciale* (ITC) [Commercial Technical Institute] followed by those of the *Istituto Tecnico Informatico* (ITI) [Informatic Technical Institute]. In addition to some elements of computer science, the programs of these schools offered topics in statistics and operations research. These programs made just a few references to modern algebra, such as the concept of set and related operations, Venn diagrams, and laws of internal and external composition, but there was little integration with the rest of the program. On the other hand, there was no reference to synthetic geometry. In the mathematics programs of other types of technical institutes, there was nothing on modern algebra, as if the Bourbakist wave had never existed (Ciarrapico and Berni 2017).

While the programs of the technical institutes did not require parliamentary approval and were therefore defined by a commission in charge, for the lycées the obligation to pass through parliament complicated reforms, so that many reform attempts were stalled due to political disagreements. This immobility led to the *Decreti Delegati* [delegated decrees] of 1974 (Presidential Decree 419 of May 31, 1974), which allowed the first experiments in schools. In fact, they authorized individual schools to make changes to their timetables and programs. The experiments could involve different school subjects and be carried out in entire sections of a school, therefore they needed to be more fully articulated than those involving the pilot classes for mathematics of the past. The opportunity offered by the delegated decrees was well exploited by some mathematics teachers, who—fed up with legislative immobility—started important innovations in teaching. But in most schools, nothing happened. Many of the teachers were quite unaware of the innovative issues that were emerging (Ciarrapico and Berni 2017).

As far as mathematics is concerned, a strong impetus to start experiments certainly came from the Frascati programs. The opportunity provided by the delegated decrees favored the birth—around 1975—of the *Nuclei di Ricerca in Didattica della Matematica* [Units of research in didactics of mathematics] at numerous universities, in which university professors and teachers from primary and secondary schools collaborated. They promoted educational research projects, which became effective through classroom experiments. So, the Ministry began to approve proposals for experimental projects put forward by high schools and technical and professional institutes, to be implemented in one or two sections of these schools.

Subsequently, in 1976, the *Consiglio Nazionale delle Ricerche* (CNR) [National Research Council] signed three funding contracts to connect the world of university research with the experiments of curricular innovations in schools. A first contract, entitled "Research aimed at teaching mathematics in High School," was signed with UMI; it was coordinated by Villani, then president of CIIM. Research units headed by the Universities of Genoa, Naples, Parma, Pavia, Pisa, Rome, Turin, Trieste, and others participated in its realization. The experimental projects activated by this contract mainly followed the contents of the Frascati programs, with some changes, such as the anticipation of the teaching of probability and statistics, and the addition of contents related to computer science. The *Nuclei* reported on the results of their experiments at the annual UMI-CIIM conferences established in the 1970s. They became catalysts of theoretical and field research. The figure of the teacher–researcher began to emerge. Gradually, mathematics education began to take on the status of an academic discipline.

A second contract, negotiated with *Mathesis* for primary schools, put into action the already mentioned RICME Project coordinated by Pellerey. The third contract, coordinated by Paolo Boero, concerned the middle school and was signed with the University of Genoa ("Development and testing of models for motivated teaching in middle schools"). Another project was promoted on a national scale by the CEDE (European Center for Education) in 1974, at the initiative of Mario Fierli. It experimented with the introduction of computer science in the first 2 years of some secondary schools.

A few years later, the experimental programs of the various technical institutes were channeled into projects with a single denomination, practically uniform throughout the national territory.

Later, the Minister of Education Franca Falcucci started experimenting on a national scale. Of particular relevance for mathematics were the programs of the *Piano Nazionale Informatica* (PNI) [Informatics national plan] launched in 1985 and the *Progetto Brocca* (a project so called from the name of the parliamentary secretary Beniamino Brocca who developed it) launched in 1988. These programs, originating from the various experimental projects already carried out, were the result of a wide and articulate debate, which involved disciplinary associations (UMI, CIIM, *Mathesis*, …), as well as individual professors both at universities and at all school levels, engaged in experiments and educational research. The commissions that elaborated these programs were made up of experts in mathematics and education (teachers, researchers, inspectors) working inside and outside the Ministry of Education.

We emphasize once again that all these experiments provided the only pathway to innovation, given the extent of immobility at the legislative level.

From Projects to Textbooks

In the period following the Royaumont conference, Italian mathematics education experienced important cultural and institutional openings, with the translation of foreign books addressing various forms of modern mathematics, meaning that there was now much greater access to the relevant international literature. Thus, for example, Choquet's book *L'Enseignement de la Géométrie* (1964. Paris, France: Herman) was translated into Italian in 1967 (*L'insegnamento della Geometria*. Milan, Italy: Feltrinelli) with the preface of Pescarini. This book included a chapter presenting axioms for affine geometry, which could be extended to metric geometry with the introduction of the dot product, and an appendix offering a different set of axioms for metric (Euclidean) geometry. These latter axioms were well accepted by the Italian community of mathematics educators: They stated a 1–1 correspondence between the set of real numbers and the points on a line, and the existence of line reflection for every line, thereby allowing theorems to be proved using the length of a segment and isometries. Also Dieudonné's book *Algèbre Linéaire et Géométrie, Élémentaire* (1964. Paris, France: Herman) was translated into Italian in 1970 as *Algebra Elementare e Geometria Elementare* (Milan, Italy: Feltrinelli) with the preface of Pescarini.

After the translation in 1968 of Nuffield Project texts, in 1972, UMI published a translation of the first five volumes with accompanying guides of the School Mathematics Project. In 1979, two books were published in the *Quaderni dell'UMI* series: the first, edited by Candido Sitia, was a collection of significant interventions at the ICME conferences in Lyon (1969), Exeter (1972), and Karlsruhe (1976) and the second (*Cenni di Didattica della Matematica*) the Italian translation of Anna Zofia Krygowska's treatise on didactics of mathematics*, Zarys dydactyki matematyki* (Vol. 1, second edition 1975). Papy too was made known in Italy through the Italian translation of his book *Mathématique Moderne 6* (1972, Florence, Italy: Le Monnier; originally published by Labor-Didier in Brussels in 1967). Campedelli also took into account some experiences developed abroad and published by *La Nuova Italia*, a publishing house directed by him and E. Castelnuovo two books: A book by Dienes on *Mathematics in the Primary School* was published in 1977, and a translation of Trevor Fletcher's *Mathematics for Today's School* appeared in 1977–1978. In the series *Strumenti per una Nuova Scuola* [Tools for a New School] of the publisher Feltrinelli, alongside the Italian translation of the second of György Pólya's works (*Mathematical Discovery*), texts by Dieudonné, Choquet, Papy, and Dienes appeared.

Seminal Italian texts which Morin and Franca Busulini had published in 1958—thus before the Royaumont Seminar—were republished, for example, *Elementi di geometria* (Morin and Busulini 1963). This book, particularly the first part of it, was surely the most "Bourbakist" among the Italian textbooks that would appear later. It began with sets, correspondences, equivalent relations, and algebraic structures (but the authors explained that this part was not strictly necessary). Congruences were introduced through transformations, but the axioms were based on those of Hilbert, and affine geometry was treated after metric geometry.

Campedelli (1970) published the book *La Geometria dei Parallelogrammi* [The Geometry of Parallelograms], which treated affine geometry using classical synthetic methods. He recognized the need to separate affine properties from metric properties, introducing—as an axiom—the affine case of Desargues' theorem, to treat homotheties without having to resort to real numbers. The final part of the book included the classification of geometries according to Klein.

From the second half of the 1970s, textbooks were published that resulted from the projects mentioned above. The experimental work carried out within the Prodi project led to the creation between 1975 and 1982 of the volumes *Matematica come Scoperta* [Mathematics as a Discovery]. Like the other texts mentioned below, these texts had some dissemination in teacher training and played an important role in introducing innovative contents and methods, but their spread in schools was small (Ciarrapico and Berni 2017). All these books were very different from the texts traditionally adopted by teachers, and—above all—included few exercises, which is the part that teachers liked to use the most. Other textbooks published in that period were: in 1977 *Il Metodo Matematico* [The Mathematical Method] by Lombardo Radice and Mancini Proia; in 1979 *Il Linguaggio della Matematica* [Language of Mathematics] by Speranza and Rossi Dell'Acqua; in 1981 *Problemi e Modelli della Matematica*, [Problems and models of mathematics] by Walter Maraschini and Mauro Palma; and in 1982 *Matematica: Idee e metodi* [Mathematics: Ideas and methods] by Villani and Bruno Spotorno. These books featured different approaches but were all equally committed to the cultural and methodological updating of mathematical teaching, with a common focus on teaching by problems. They included the introduction of geometric transformations and the basic concepts of probability and statistics. They also introduced sets, in different contexts ranging from probability to the solutions of equations, and to relations and functions. Similarly, groups were defined in all texts, based on the composition of geometric transformations or the properties of numerical sets. In geometry, the distinction between affine and metric properties was always underlined. To prove geometric properties, the texts by Prodi, Lombardo Radice and Mancini Proia, and Maraschini and Palma used Choquet's metric-based axioms, with some variations. In the third volume of the text by Lombardo Radice and Mancini Proia, Choquet's affine axioms were also presented.

The experimental programs promoted by the Ministry of Education in 1985 and 1988 (PNI and *Brocca*) took off from the aforementioned texts and from the related projects. They left the choice to teachers about how to introduce geometry "using the geometry of transformations or following a more traditional path" (PNI, Ministerial Circular No. 24 of February 6, 1991). Sets remained, but groups disappeared due to a—perhaps excessive—reaction to abstract structures. Somehow, through the aforementioned books and the experimental programs, certain aspects of modern mathematics survived in Italy more than in many other countries. In particular, the book by Maraschini and Palma, the most widely adopted, fostered the spread of new and modern ideas, even among teachers less committed to renewal. Nevertheless, it can be said that these books did not open a new era, but rather gradually concluded what was—in the words of Paolo Linati—"the era of forgotten experiments and missed opportunities" (Linati 2012, p. 63).

Conclusions

In 1965, the journal *Archimede* published a collection of notes about the introduction of modern mathematics written by a group of teachers of lycées and technical institutes. These teachers claimed to be aware of the importance of certain aspects of modern mathematics, but criticized the proposals made at the aforementioned meeting of Camaiore on the new contents of geometry. The main point of criticism was the absence of adequate teacher preparation. Furthermore, they stressed that there were very few textbooks dealing with the new approach to algebra. We add that the special books published for the pilot classes were not made available to all schools and that the results of the experiments were not publicized (Giro 1969). In Correale (1965) the criticisms were not limited to the institutions: We catch a veiled criticism of university professors who discussed modern mathematics in theory, but who failed to face the real problems of the school in practice. Despite these perplexities expressed by teachers, it must be recognized that the initiatives of the 1970s in which the mathematicians played an important role (UMI congresses on mathematics teaching, the projects launched by mathematicians, and the network of *Nuclei* with active involvement of teachers) were the trigger for going forward toward innovation. Times had changed, as had society, and perhaps the events of the 1970s would have happened anyway, but modern mathematics was an accelerator, something which provided suggestions on how to act to achieve results.

We may say that the only remains of modern mathematics in Italian schools are the language of set theory and geometric transformations. Other innovations suggested by the foreign projects, such as the introduction of probability and statistics, have also been accepted but with difficulty. However, it should also be considered the other side of the coin: The substantial refusal to adopt modern mathematics in the Italian school has made it possible to preserve a tradition of geometric teaching that has disappeared in other countries.

History has shown that the Italian school system is by its very nature conservative: From the first reform of Casati in 1859 to the years when modern mathematics appeared internationally, Italy has had only the Gentile Reform in 1923, with little variation brought in at the end of the Fascist period (Bottai Reform) and later on at the end of World War II (by the Allied military Government). As told at the beginning, there was a tradition that was difficult to eradicate. The point was, in William Sawyer's (1955) words, that "The main difficulty in many modern developments of mathematics is not to learn new ideas but to forget old ones" (p. 65).

References

Anonymous. (1965). Troppo «ermetici» i nuovi programmi [Too "hermetic" the new programs]. *Archimede, 17,* 102–103.

Behnke, H., Choquet, G., Dieudonné, J., Fenchel, W., Freudenthal, H., Hajós, G., & Pickert, G. (1960). *Lectures on modern teaching of geometry and related topics*. Aarhus, Denmark: Matematisk Institut (Aarhus Universitet), Elementaer Afdeling. Nr. 7. (E. Kristensen was in charge of the report).

Bernet, T., & Jaquet, F. (1998). *La CIEAEM au travers de ses 50 premières rencontres* [The CIEAEM through its first 50 meetings]. Neuchâtel, Switzerland: CIEAEM.

BUMI. (1954). Notizie [News]. *Bollettino della Unione Matematica Italiana s. 3, 9*, 208–232.

BUMI. (1960). Notizie [News]. *Bollettino della Unione Matematica Italiana, s. 3, 15*, 85–104; 305–355.

BUMI. (1962a). Notizie [News]. *Bollettino della Unione Matematica Italiana*, s. 3, 17, 120–157.

BUMI. (1962b). Il convegno di Bologna promosso dalla Commissione internazionale dell'insegnamento matematico [The Bologna conference promoted by the International Commission of Mathematical Teaching]. *Bollettino della Unione Matematica Italiana, s. 3, 17*, 199–214.

BUMI. (1966). Notizie [News]. *Bollettino della Unione Matematica Italiana, s. 3, 21*, 102–118.

BUMI. (1968). Notizie [News]. *Bollettino della Unione Matematica Italiana, s. 4, 1*, 154–171.

Campedelli, L. (1970). *La geometria dei parallelogrammi* [The geometry of parallelograns]. Florence, Italy: Le Monnier.

Cartan, H. (1963). Réflexions sur les rapports d'Aarhus et Dubrovnik [Reflections on the Aarhus and Dubrovnik reports]. *L'Enseignement Mathématique, s. 2, 9*, 84–90.

Castelnuovo, E. (1960). Le matematiche moderne nell'insegnamento secondario al convegno di Arlon (Belgio) [Modern mathematics in secondary education at the Arlon conference (Belgium)]. *Archimede, 12*, 265–266.

Castelnuovo, E. (1964). *Didattica della Matematica* [Mathematics Education]. Firenze, Italy: La Nuova Italia.

Castelnuovo, E. (1965). Matematica d'avanguardia [Avant-garde mathematics]. *Riforma della Scuola, 11*, 3–19. Translated as "Mathématique d'avant-garde" in the volume *La mathématique moderne* [modern mathematics] (pp. 1–14) and as *Belgique: Mathématique d'avant-garde* [Belgium: Pioneering mathematics] (booklet).

Castelnuovo, G. (1919). La riforma dell'insegnamento matematico secondario nei riguardi dell'Italia [The reform of secondary mathematics education in Italy]. *Bollettino della Mathesis, 11*(1–4), 1–5.

Ciarrapico, L., & Berni, M. (2017). *I curricoli di matematica, gli ordinamenti scolastici e le riforme dal 1940 al 2015* [The mathematics curricula, school systems and reforms from 1940 to 2015]. Bologna, Italy: Unione Matematica Italiana.

Correale, G. (1965). «Fateci vedere come si fa» ["Let us see how it's done"]. *Archimede, 17*, 104–107.

De Bock, D., & Vanpaemel, G. (2015). Modern mathematics at the 1959 OEEC Seminar at Royaumont. In K. Bjarnadottir, F. Furinghetti, J. Prytz, & G. Schubring (Eds.), *"Dig where you stand" 3. Proceedings of the third International Conference on the History of Mathematics Education* (pp. 151–168). Uppsala, Sweden: Department of Education, Uppsala University.

De Finetti, B. (1965). Programmi e criteri per l'insegnamento della matematica alla luce delle diverse esigenze [Programs and criteria for teaching mathematics in the light of different needs]. *Periodico di Matematiche, s. 4, 43*, 119–143.

De Finetti, B. (1967). Le proposte per la matematica nei nuovi licei: informazioni, commenti critici, suggerimenti [The proposals for mathematics in the new lycées: information, critical comments, suggestions]. *Periodico di Matematiche, s. 4, 45*, 75–153.

Duse, V. (1969). *Per un insegnamento moderno della matematica nella scuola elementare* [For a modern teaching of mathematics in primary school]. Brescia, Italy: La Scuola.

Équipe de Bordeaux – AMPERES (2009). Pourquoi enseigner le triangle? [Why to teach the triangle]. Retrieved January 19, 2021, from http://educmath.ens-lyon.fr

Ferrari, M., Bazzini, L., Borghi, L., De Ambrosis, A., Isella Massara, C., Mosconi Bernardini, P., Trivelli Ricci, P., Vittadini Zorzoli, M., & Germani, L. (1982). Un progetto di educazione matematico–scientifica per il primo ciclo elementare [A mathematical–scientific education project for the first primary cycle]. *L'Insegnamento della Matematica e delle Scienze Integrate, 5*(1), 56–73.

Furinghetti, F. (2015). Matematici e insegnamento della matematica a livello pre-universitario: Enrico Bompiani terzo presidente della CIIM [Mathematicians and the teaching of mathematics at pre-university level: Enrico Bompiani third president of CIIM]. *La Matematica nella Società e nella Cultura, Rivista dell'Unione Matematica Italiana, s.1, 8,* 75–109.

Furinghetti, F. (2019). Riforme internazionali e innovazioni nazionali: Luigi Campedelli quarto presidente della CIIM dal 1964 al 1972 [International reforms and national innovations: Luigi Campedelli fourth president of CIIM from 1964 to 1972]. *Matematica, Cultura e Società. Rivista dell'Unione Matematica Italiana, s. 1, 4*, 269–288.

Furinghetti, F., & Menghini, M. (2014). The role of concrete materials in Emma Castelnuovo's view of mathematics teaching. *Educational Studies in Mathematics, 87,* 1–6.

Gattegno, C., Servais, W., Castelnuovo, E., Nicolet, J. L., Fletcher, T. J., Motard, L., Campedelli, L., Biguenet, A., Peskett, J. W., & Puig Adam, P. (1958). *Le matériel pour l'enseignement des mathématiques* [Materials for the teaching of mathematics]. Neuchâtel, Switzerland: Delachaux & Niestlé.

Giannarelli, R. (1967). Progetto di programma minimo di matematica per il triennio liceale (Villa Falconieri, February 16–18, 1967) [Minimum math program project for the last three years of High School]. *Archimede, 19*, 60–63.

Giro (1969). Perché nessun consuntivo sulle «classi pilota»? [Why no balance sheet on the "pilot classes"?]. *Archimede, 21*, 185–186.

Hungarian National Commission for UNESCO. (1963). *Report on the work of the international symposium on school mathematics teaching*. Budapest, Hungary: Akadémiai Kiadó.

Lehto, O. (1998). *Mathematics without borders: A history of the International Mathematical Union*. New York, NY: Springer Verlag.

Lelong, J. (1965). Sur l'harmonisation des enseignements secondaire et supérieurs, et le rôle de la géométrie élémentaire [On the harmonization of secondary and higher education and the role of elementary geometry]. *Periodico di Matematiche, s. 4, 43*, 235–240.

Linati, P. (2012). *L'algoritmo delle occasioni perdute. La Matematica nella scuola della seconda metà del Novecento* [The algorithm of lost opportunities. Mathematics in the school of the second half of the twentieth century]. Trento, Italy: Erickson Live.

Lombardo-Radice, L. (1963). Il nuovo insegnamento di algebra al primo anno di matematica [The new course of algebra in the first year of mathematics]. *Archimede, 15*, 1–7.

Mancini Proia, L. (2003). Autobiografia di un'insegnante di matematica [Autobiography of a mathematics teacher]. In M. Menghini & M. R. Trabalza (Eds.), *Geometrie in cielo e in terra* [Geometries in heaven and on earth] (pp. 19–47). Foligno, Italy: Edizioni dell'Arquata.

Maraschini, W., & Menghini, M. (1992). Il metodo euclideo nell'insegnamento della geometria [The Euclidean method in the teaching of geometry]. *L'Educazione Matematica, 13*, 161–180.

Marchi, M. V., & Menghini, M. (2013). Italian debates about a modern curriculum in the first half of the 20th century. *The International Journal for the History of Mathematics Education, 8*(2), 23–47.

Menghini, M. (2007). La geometria nelle proposte di riforma tra il 1960 e il 1970 [Geometry in the reform proposals between 1960 and 1970]. *L'Educazione Matematica, 28*, 29–40.

Morin, U. (1963). Geometria elementare classica e metodi moderni [Classical elementary geometry and modern methods]. *L'Enseignement Mathématique, s. 2, 9*, 76–83.

Morin, U. (1965). Interpretazione di un progetto di matematica per il biennio liceale [Interpretation of a mathematics project for the first two years of the Lycée. *Archimede, 17*, 57–68.

Morin, U., & Busulini, F. (1963). *Elementi di geometria* [Elements of geometry] (Third edition). Padova, Italy: CEDAM.

Movimento Circoli della Didattica (Ed.) (1956). *Didattica della matematica.* [Didactics of mathematics]. Rome, Italy: Signorelli.

Notiziario. (1960). Seminario della commissione internazionale per l'insegnamento matematico (I.C.M.I.) in Danimarca [Seminar of the International Commission for Mathematical teaching (I.C.M.I.) in Denmark]. *Archimede, 12*, 268.

Notiziario. (1963). Classi pilota per l'insegnamento della matematica e della fisica [Pilot classes for the teaching of mathematics and physics]. *Archimede, 15*, 331.

Notiziario. (1969). Riunione della C.I.I.M. a Villa Falconieri (Frascati) [Meeting of C.I.I.M. at Villa Falconieri (Frascati)]. *Archimede, 21*, 111–112.

OEEC. (1961a). *New thinking in school mathematics*. Paris, France: OEEC.

OEEC. (1961b). *School mathematics in OEEC countries—Summaries*. Paris, France: OEEC.

OEEC. (1961c). *Synopses for modern secondary school mathematics*. Paris, France: OEEC.

Papy, G. (1966). La geometria nell'insegnamento moderno della matematica [Geometry in the modern teaching of mathematics]. *Archimede, 18*, 81–91.

Pellerey, M. (1989). *Oltre gli insiemi* [Beyond the sets]. Napoli, Italy: Tecnodid.

Piaget, J., Bosher, B., & Châtelet, A. (1950). *Initiation au calcul* [Introduction to calculation]. Paris, France: Baurelier. (Italian translation 1956. *Avviamento al calcolo* [Introduction to calculation]. Florence, Italy: La Nuova Italia).

Picard, N. (1967). *À la conquête du nombre: Classe de C. P. Livre du maître* [Conquering the number: Class of C. P. Teacher's book]. Paris, France: OCDL.

Platone, G. (1961). Convegno internazionale sull'insegnamento matematico [International conference on mathematical teaching]. *Archimede, 13*, 313–319.

Prodi, G. (1995/1982). Tendenze attuali nell'insegnamento della matematica [Current trends in mathematics teaching]. *L'Insegnamento della Matematica e delle Scienze Integrate*, Numero speciale in onore del Prof. Giovanni Prodi [Special issue in honor of Prof. Giovanni Prodi]*, 18*, 413–430 (First published in 1982. *Rendiconti. Accademia Nazionale delle Scienze detta dei XL. Memorie di Matematica e di Scienze fisiche e Naturali, 7*(17), 183–196.

Revuz, A. (1965). La geometria in un insegnamento moderno della matematica [Geometry in a modern teaching of mathematics]. *Periodico di Matematiche, 43*, 242–245.

R. G. (1964a). Riforme fatte e riforme in studio, con i resoconti dei due colloqui di Villa Falconieri [Reforms made and reforms being studied, with the reports of the two meetings at Villa Falconieri]. *Archimede, 16*, 9–42.

R. G. (1964b). Seminario matematico internazionale (Villa Falconieri – Frascati, 8–10 ottobre 1964) [International mathematical seminar (Villa Falconieri – Frascati, October 8–10, 1964)]. *Archimede, 16*, 314–330.

R. G. (1966). Nuovi programmi di matematica per i licei [New High School mathematics programs]. *Archimede, 18*, 124–129.

Sawyer, W. W. (1955). *Prelude to mathematics.* Harmondsworth, United Kingdom: Penguin Books.

Schubring, G. (1987). *The cross-cultural 'transmission' of concepts—The first international mathematics curricular reform around 1900, with an Appendix on the biography of F. Klein*. Occasional paper Nr. 92, IDM (Universität Bielefeld).

Schubring, G. (2014). The road not taken—The failure of experimental pedagogy at the Royaumont Seminar 1959. *Journal für Mathematik-Didaktik, 35*(1), 159–171.

Stone, M. H., & Walusinski, G. (1963). International Commission on Mathematical Instruction. Report of the period 1959–62. *L'Enseignement Mathématique, s. 2, 9*, 105–112.

Temusssi, S. (1965). «Guardare in faccia la realtà e andare cauti» [Face reality and be cautious]. *Archimede, 17*, 112–114.

Togliatti, E. G. (1963). L'insegnamento della matematica [The teaching of mathematics]. In *Atti del Convegno: Insegnamenti scientifici e insegnamenti umanistici nella funzione formativa della scuola secondaria. Roma, 8–10 maggio 1962* [Proceedings of the Conference: Scientific teachings and humanistic teachings in the formative function of secondary school] (pp. 31–46). Rome, Italy: Accademia Nazionale dei Lincei.

Tomasi, L. (2018). Un ritratto di Ugo Morin nel cinquantesimo anniversario della sua scomparsa. *L'Insegnamento della Matematica e delle Scienze Integrate, 41B*, 403–431.

Tricomi, F. G. (1965). Accordare «progressisti» e «conservatori» [Matching "progressives" and "conservatives"]. *Archimede, 17*, 101.

Villa, M. (1962). Il discorso del Prof. Villa [The speech of Prof. Villa]. *Bollettino della Unione Matematica Italiana, s. 3, 17*, 200–207.

Villa, M. (Ed.). (1965). *Matematica moderna nella scuola media* [Modern mathematics in middle schools]. Bologna, Italy: Patron.

Villa, M. (Ed.). (1966). *Matematica moderna nelle scuole secondarie superiori* [Modern mathematics in high schools]. Bologna, Italy: Patron.

Vita, V. (1986). *I programmi di matematica per le scuole secondarie dall'unità d'Italia al 1986. Rilettura storico-critica* [Mathematics programs for secondary schools from the unification of Italy to 1986. Historical-critical reinterpretation]. Bologna, Italy: Pitagora.