Chapter 4
The Royaumont Seminar as a Booster of Communication and Internationalization in the World of Mathematics Education

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Abstract After a succinct description of the meeting opportunities for mathematics educators up to the 1950s, this chapter describes how, in the wake of the New Math/modern mathematics reform movement, meetings have become a fundamental tool for focusing on problems and potential of reform proposals. Bodies that have played the most relevant roles are ICMI, CIEAEM, OEEC/OECD, and UNESCO. In the conferences that followed the Royaumont Seminar, particular interest was turned to the search for new axioms for geometry, with many proposals and discussions. But modern mathematics was not just this; in other places, the attention was turned to more general questions of a methodological and social nature. This congress season has fostered the creation of new traditions such as the birth of journals specialized in mathematics education, and periodic conferences on mathematics education, as exemplified by the four-year ICMEs.

Keywords Aarhus · Arab region · Athens · Belgrade · Bogotá · Bologna · Bucharest · Budapest · Cambridge · CIEAEM · Communication · Dakar · Dalat · Dubrovnik · Echternach · Egypt · Entebbe · Frascati · ICMI · International Congress of Mathematicians · Internationalization · Modern mathematics · Moscow · New Math · OECD · OEEC · Reforms · Royaumont · Saigon · UNESCO · Utrecht · Zagreb

Introduction

In the mid-twentieth century concomitant factors ranging from the increasing importance of mathematics and its applications for society to new insights on learning and teaching mathematics brought to light a need for "radical changes and improvements in the teaching of mathematics" (OEEC 1961a, p. 11) in many countries. As a result, a reform movement spread around the world, usually known as "New Math" in most English-speaking countries or "modern mathematics"—the label that we will use—in most European countries. The difference in labels is not just a question of language. Modern mathematics had its initial inspiration in the Bourbakist theories, while New Math is usually identified with the movement which sprung out of the School Mathematics Study Group (SMSG) in the

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D. De Bock (ed.), *Modern Mathematics*, History of Mathematics Education,
https://doi.org/10.1007/978-3-031-11166-2_4

USA. The two labels cover very different phenomena in the various countries involved in the movement of reform, nevertheless, these phenomena present commonalities in the period in which they developed, in some protagonists, and the intent to revising mathematics teaching and curricula. According to Kilpatrick (2012), the modern mathematics movement arose from a desire "to bring school mathematics closer to the academic mathematics of the twentieth century—to eliminate inane jargon, and make it better preparation for the mathematics being taught in the university" (p. 563).

What is striking in the reform movement of modern mathematics is the acceleration in communication and the internationalization it promoted in the milieu of school mathematics. These aspects were rather new in this area, although they had already been advocated at the beginning of the twentieth century. The journal *L'Enseignement Mathématique*, founded in 1899, identified its mission and vision in internationalization, communication, and solidarity (Furinghetti 2003). In 1908 the foundation of the International Commission on the Teaching of Mathematics, aka CIEM or IMUK[1], progenitor of the current International Commission on Mathematical Instruction (ICMI), was a further step in the path toward communication and internationalization. Despite these past events, for many years discussion on mathematics education was mainly confined to national contexts. On the contrary, the reform movement of modern mathematics, though it had different fallout in various countries, aroused the need to communicate and discuss issues of teaching and learning mathematics at the international level. The trigger of this new trend was the Seminar held in Royaumont in 1959, which stirred a series of conferences in the 1960s. The development of international bodies established with the purpose of providing institutional and financial support favored these events. In this chapter, we see how these conferences were carried out and how they contributed to rethinking mathematics education.

Meetings on Mathematics Education Before Royaumont

Nowadays attending international conferences, virtually or face-to-face, has become a common practice among scientists, but in the past, it was not so. On the occasion of the World's Columbian Exposition held in Chicago (1893) in celebration of the four-hundredth anniversary of Christopher Columbus's discovery of the "New World," alongside the usual activities of this kind of celebration, a conference of mathematicians was organized by the University of Chicago's Faculty of Mathematics with participants and reports from the United States and Europe. This was quite an exceptional event of international communication and in 1897 it was followed by the launch of the periodical International Congresses of Mathematicians (ICMs). In 1900, after the second of these congresses, the frequency became four years, with interruptions due to World Wars. For mathematicians, the ICMs provided one of the few opportunities to meet at the international level. For those interested in mathematics education, including teachers, there were even fewer opportunities. An opportunity was offered by the ICMs because, since 1900, their program included a section dedicated to the teaching of mathematics generally associated with other topics such as history, logic, or philosophy. In these sections, contributions to mathematics education were presented, usually in the form of short communications. The proceedings of the ICMs do not always contain the texts of these contributions, but rather regularly contain the titles and the names of authors. In a few ICMs there were also special contributions that were given more time, but never was a plenary talk dedicated to mathematics teaching. The only exception could be Felix Klein's plenary in Zurich in 1897, which contains the word *Unterricht* (teaching) in the title. It was mainly concerned with university teaching of advanced mathematics. Already at the beginning of the twentieth century, the anomaly of this situation was felt and CIEM launched international meetings on mathematics education that marked its initial activity. The first was in Brussels (1910), the second in Milan (1911), and the third in Paris (1914). While taking into account the dif-

[1] CIEM stands for *Commission Internationale de l'Enseignement Mathématique*, IMUK stands for *Internationale Mathematische Unterrichtskommission* (see, e.g., Howson 1984).

ferences in context, we agree with Schubring (2008) who considers these congresses, in particular, that of Paris, to be the ancestors of the present ICMEs (International Congresses on Mathematical Education).

The two World Wars and the CIEM's lethargy in the 1920s prevented the continuation of initiatives such as the meetings of the 1910s. After World War II, the context was changed and the concern about mathematics education assumed new connotations. Since the early 1950s projects regarding mathematics education were developed in the United States of America and the launch of Sputnik in 1957 accentuated the interest in both science and mathematics education. Various initiatives were undertaken but their scope remained mainly national. Even the important conference called by the Education Committee of the National Academy of Sciences at Woods Hole in September 1959, which generated Jerome Bruner's (1960) *The Process of Education,* was attended only by scientists from the United States of America. In Europe, things went differently thanks to the *Commission Internationale pour l'Étude et l'Amélioration de l'Enseignement des Mathématiques* (CIEAEM) [International Commission for the Study and Improvement of Mathematics Teaching], which created a tradition of international meetings on problems of mathematics education.

The early years of this Commission are of extreme interest for the history of mathematics education, with particular reference to modern mathematics. The group was founded by Caleb Gattegno, who in 1950 invited some 30 scholars in Debden (UK) to set up an international commission for the study and the improvement of the teaching of mathematics from nursery to university (see Vanhamme 1991). He had some 13 positive answers and, after two further meetings in the next year, in 1952, CIEAEM was officially established in La Rochette par Melun (see Félix 1986). Gattegno, a mathematician and psychologist responsible for the training of prospective teachers at London University, was able to gather scholars from mathematics, philosophy, psychology, pedagogy, and teachers (Powell 2007). Figure 4.1 shows four important protagonists in the meeting of La Rochette par Melun. As it is claimed in the *Préface* of the book by Piaget et al. (1955), the Commission aimed to take all initiatives that, through study and action, could lead to a better understanding of the problems raised by the teaching of mathematics at all levels. In his foreword to the history of CIEAEM written by Lucienne Félix (1986), Gattegno stressed the importance of studying in-depth the problems related to mathematics teaching as a necessary premise for its improvement.

Bernet and Jaquet (1998) have collected information for building an ex-post list of the founding members of CIEAEM, which includes scholars from European countries (Belgium, France, Germany, Italy, The Netherlands, Switzerland, UK) and one from Chicago. In the list of the active members of CIEAEM drawn up by Gattegno in 1957, we find, besides European members, Howard F. Fehr from

Figure 4.1 Choquet, partially hidden, Piaget, Gattegno, and Willy Servais in La Rochette par Melun in 1952. (Photo by Lucien Delmotte, collection Guy Noël)

the USA and Celestino Galli from Uruguay (Bernet and Jaquet 1998). CIEAEM thus became a truly international group. In 1960 Gattegno resigned from the Commission after ten years as its secretary (Gattegno 1988). This was the beginning of CIEAEM's gradual evolution towards new patterns, though it kept the practice of giving its annual meetings titles that should indicate the main topic. Until 1970, the number of participants in the meetings was around 40, but after that, this number increased, and as the years passed the Commission adopted better defined rules for its organization. CIEAEM still exists today and, despite having lost the innovative spirit of the early years, it has retained the initial characteristics of inclusiveness and the involvement of teachers.

Some of the titles of the meetings held in the 1950s[2] offer a kind of Manifesto of CIEAEM. In 1952, in La Rochette par Melun (France), the title "Mathematical and mental structures" referred to the lines of research developed thanks to the presence of the mathematicians Gustave Choquet, Jean Dieudonné, André Lichnerowicz, and the psychologist Jean Piaget, who are among the authors of the first book published by CIEAEM (Piaget et al. 1955). In 1953 in Calw (Germany), the title was "Connections between pupils' thinking and the teaching of mathematics," and in 1955 in Bellano (Italy), it was "The pupil coped with mathematics—a releasing pedagogy"). In each case, the themes treated were related to psychology. Possibilities for updating mathematics programs were considered in 1954 in Oosterbeek (The Netherlands), where the title was "The modern mathematics at School") and in 1955 in Ramsau (Austria), where the title was "The teaching of probability and statistics at the university and at school". In the meeting at Madrid (Spain) in 1957, the title was "Teaching materials" and the focus was on classroom practice and the use of concrete materials. That theme was developed in the second book published by CIEAEM (Gattegno et al. 1958).

Royaumont

Bodies Promoting Internationalization in Mathematics Education

Acceleration in the phenomenon of the internationality of conferences was supported by two bodies—OEEC/OECD and UNESCO—which provided institutional and financial aid. OEEC became the acronym for the "Committee for European Economic Co-operation," which was founded in 1947 and renamed the "Organisation for European Economic Cooperation" (OEEC) on April 16, 1948. It aimed to manage the substantial funds of the USA-financed Marshall Plan for implementing a European Recovery Program (ERP) addressed to the countries of Western Europe in the areas of industry, agriculture, infrastructures, energy, and technology. With this plan, the United States of America responded to the challenges that might be associated with education in socialist countries. The 18 members of OEEC were: Austria, Belgium, Denmark, France, Greece, Iceland, Ireland, Italy, Luxembourg, The Netherlands, Norway, Portugal, Sweden, Switzerland, Turkey, United Kingdom, and West Germany (originally represented by Bizonia and the French occupation zone) and the town of Trieste for a period before it returned to the Italian sovereignty. When the USA and Canada joined as members in the end of 1960, it was renamed "Organisation for Economic Cooperation and Development" (OECD). New statutes came into force on September 30, 1961. In the wake of the shock caused by the launch of Sputnik, OEEC was strongly committed to including education among its areas of intervention. In June 1958 OEEC opened an office in Paris—the Office for Scientific and Technical Personnel (OSTP)—the aim being to make science and mathematics education more effective and to promote a reform of the contents and the methods of mathematics instruction for 12- to 19-year-old students.

UNESCO (the United Nations Educational, Scientific and Cultural Organization) is a specialized agency of the United Nations (UN) that sponsors international scientific projects with the purpose of

[2] The complete list can be found in https://www.cieaem.org/index.php/en/meetings-en/previous-meetings

promoting world peace and security through international cooperation in education, the sciences, and culture. UNESCO's constitution came into force on November 4, 1946. In the 1950s, UNESCO supported initiatives in the field of mathematics education, including conferences organized by the International Mathematical Union (IMU), like for example, a conference that took place at the Tata Institute of Fundamental Research in Bombay from February 22 to 28, 1956, whose purpose was "to discuss, with special reference to South Asia, the problems of mathematical education at all levels, and to formulate plans for its sound development" (Report of the Executive Committee ..., 1956, p. 9). Members of ICMI and CIEAEM delivered talks at the Bombay conference.

During the second half of the 1960s, and particularly during the period 1969–1974, the role of UNESCO in mathematics education increased, with special attention being given to developing countries. A series of UNESCO volumes entitled *New Trends in Mathematics Teaching* (Christiansen 1978; Hodgson 2009; Jacobsen 1996) was launched. Collaboration—both scientific and organizational—with ICMI, initiated by Marshall Stone, was continued by the next ICMI President, André Lichnerowicz. UNESCO financed the new journal *Educational Studies in Mathematics* and the creation of ICME (the International Congress on Mathematical Education), launched by Hans Freudenthal, and promoted advisory meetings with ICMI for discussing ICMI programs.

The Seminar of Royaumont: Not Only "Euclid Must Go!"

According to Vanpaemel and De Bock (2019), in the 1950s, interactions between American and European actors in the field of school mathematics were limited. In 1952, the presence of Saunders Mac Lane and Stone in the Executive Committee of ICMI was the first concrete bridge between the two communities. Later on, some participants from the USA presented contributions on mathematics education to the ICMs of Amsterdam (1954) and Edinburgh (1958) (Gerretsen and de Groot 1954–1957; Todd 1960). At Edinburgh, Fehr presented a report on ICMI's first topic "Mathematical Instruction up to the Age of Fifteen Years." Moreover, in the book *Abstracts of Short Communications and Scientific Programme*, issued to members during the Congress, five speakers were nominated by the United States Committee on Mathematical Instruction to deliver talks on experiments carried out in their country. Indeed, for the two sides of the Atlantic, the first real occasion for confrontation was offered by the Seminar of Royaumont entitled "New Thinking in School Mathematics," held from November 23 to December 4, 1959, at the *Cercle Culturel de Royaumont*, Asnières-sur-Oise, France. It was organized by the OSTP and Marshall Stone, President of ICMI from 1959 to 1962, was the chair. The Royaumont Seminar can be seen as the beginning of a common reform movement to modernize school mathematics across the world, as evidenced by the OEEC recommendations to its member countries and Stone's stress on the need for a "deep and urgent" reform in his introductory address (Stone 1961, p. 29). The vulgate has emphasized the folklore of Dieudonné's claim "Euclid must go!" but, indeed, the event initiated long-distance effects which stretched well beyond geometry education.

In short, the theme of the Seminar was not only the need for new thinking in both mathematics and mathematics education—including changes in curricula and teacher training—but also the development of appropriate follow-up action (OEEC 1961a). The basis for discussion should have been answers to a questionnaire sent in December 1959 to the countries participating in the program asking about current conditions of mathematics education in their countries (see OEEC 1961a, p. 7).[3] The

[3] The questionnaire (see Appendix B in OEEC (1961a), pp. 221–237) was sent to the OEEC member countries, and to Canada, the United States, and Yugoslavia. Only Spain did not answer, so the survey lists 20 countries. The results of the survey, divided by countries, with the exception of Canada. were also presented to OEEC (1961b).

data were not available at the beginning of the Seminar, however, but answers to the questionnaire were nevertheless presented (partly summarized) and analyzed in the second part of the official report (OEEC 1961a, pp. 127–210), which was titled "Survey of Practices and Trends in School Mathematics." The questions concerned:

- The number of hours of compulsory mathematics teaching in the different schools.
- Educational qualifications of those providing teacher training.
- Which authorities were competent to establish programs and select textbooks.
- Any ongoing reforms.
- The contents of the programs (more precisely, various topics were listed and it was asked, for each of them, in which school year was it carried out).

Answers to the survey provide an interesting source of data with respect to mathematics education in the countries participating in the program before the start of modern mathematics in Europe.

Three delegates from every invited country—an outstanding mathematician; a mathematics educator or person in charge of mathematics in the Ministry of Education; and an outstanding secondary school teacher of mathematics—were invited to participate in the Seminar (OEEC 1961a). From the list of the participants (OEEC 1961a), we know that there were 33 delegates from 15 OEEC countries (Austria, Belgium, Denmark, France, Germany, Greece, Ireland, Italy, Luxembourg, The Netherlands, Norway, Sweden, Switzerland, Turkey, UK), one from Yugoslavia, three more from Canada and the United States, and three officers of OEEC. Only Norway sent three delegates. There were 13 guest speakers, of whom four were from France, three from the USA, two from the UK, and one from each of Belgium, Denmark, Germany, and The Netherlands.

CIEAEM was represented by the guest speakers Choquet, Dieudonné, Félix, and Willy Servais and the Italian delegates Luigi Campedelli and Emma Castelnuovo. ICMI was represented by the guest speakers Begle (a future Executive Committee member) and Edwin A. Maxwell (past Executive Committee member and future ICMI secretary) and by the delegates Otto Frostman (Executive Committee member), Kay W. Piene (past Executive Committee member), and the chair of the Seminar Stone (ICMI President). The 33 delegates included university professors and school teachers equally divided, and education-related officials. There were only two women, both of them secondary teachers—Castelnuovo and Félix. Among the 13 guest speakers, university professors were predominant.

The studies by De Bock and Vanpaemel (2015) and Vanpaemel and De Bock (2019), which contain information taken from Servais' archives, and by Schubring (2014a, b) based on OEEC/OECD's archives, shed new light on the development of the Seminar, but its actual program remains difficult to reconstruct due to the many discrepancies found by Schubring (2014a) between the official report (OEEC 1961a) and the documents kept in the OEEC/OECD's archives.

As written in the preface, the official report of the OEEC (1961a) meeting was "drawn up" by Fehr with the assistance of Lucas N. H. Bunt. It had the same title as the Seminar—*New Thinking in School Mathematics*. The French version was titled *Mathématiques Nouvelles* [New Mathematics]. The report included the full texts of the interventions of Dieudonné and Stone, excerpts and summaries of various contributions, and reports on the discussions, comments, and conclusions, but in an order that was not necessarily that of the presentations at the Seminar. The conference was divided into three sections, dealing with (a) New Thinking in Mathematics, (b) New Thinking in Mathematical Education, and (c) Implementation of Reform. The report did not correspond to those three sections and is instead divided into Part I (Report) and Part II (Survey) each with five sub-chapters, with Roman numbering.

According to Fehr (1961), the purpose of the Seminar was not "to formalize all mathematics instruction through an axiomatic and logical treatment," but "to focus attention on the need to introduce modern concepts, clear definitions and such methods of teaching as will lead eventually to formal mathematical structure" (p. 209). The talks of Dieudonné and Stone represented the two poles of the way to tackle the reforms in teaching. The first focused on the subject matter, bearing in mind the developments of Bourbakist research (Dieudonné 1961). Dieudonné freed geometry from Euclid's burdens and proposed a gradual program from the age of 14 upwards: After experimental work in geometry and algebra, pupils were to begin quite early to use the language of sets and to speak of axioms. Further topics would include matrices and determinants of orders 2 and 3, elementary calculus (functions of one variable), construction of the graph of a function and of a curve given in the parametric form (using derivatives), elementary properties of complex numbers, and polar coordinates. To carry out this program, the concepts of vector space, linear algebra, and quadratic forms were presented as being fundamental.

In his opening address, Stone considered whether some new topics or subjects should be introduced and whether some traditional parts of mathematics which had lost their importance should be eliminated. He did not mention the objects of the changes. Nevertheless, different from Dieudonné's talk, the heart of Stone's address was its appeal for the development of specific didactic capacities, the creation of university research institutes, and professorships. Furthermore, he stressed the need to look at programs for elementary schools, and to develop efficiently the latent mathematical talents and interests of average children in order not to remove them permanently from the study of mathematics. This would bring on improvements in secondary schools.

Dieudonné's proposal drew expressions of approval and disagreement among participants, but the shared conclusion was not to remove Euclid entirely from the secondary school curriculum. Dieudonné's proposal focused on the subject matter which overshadowed other important aspects. Texts associated with these latter aspects were discovered by Schubring among the documents in preserved files labeled "Background Paper for Section IV"[4] (Schubring 2014b). Its title is "Psychological and Educational Researches into the Teaching of Arithmetic and Mathematics." The authors were William Douglas Wall, an educator director of the National Foundation for Educational Research in England and Wales (NFER) and chairman of the international project for the "Evaluation and Educational Attainment," and John B. Biggs, mathematics educator, at that time Assistant Research Officer at NFER. In OEEC (1961a) the contribution is summarized in nine paragraphs from 344–352 (pp. 101–103) under the title "Extracts from Dr. Wall's remarks." Biggs is not on the list of the participants in the Seminar and his name does not appear in the proceedings. From the extracts published we know that the paper analyzed the state of empirical research on the teaching and learning of arithmetic and mathematics: According to the authors, current research was short-term, scattered, piecemeal, and mainly focused on one or two aspects of the problem at a time without considering the complex of interrelated variables involved. The paper called for large-scale, systematic, scientific research in actual classrooms, which should involve mathematicians, educators, teachers, and psychologists, and perhaps also logicians, philosophers, physicists, engineers, statisticians, and economists. Three types of research were called for: Preliminary fundamental studies, operational research to evaluate the results of the reforms, and action research which would provide feedback on the school systems. Wall and Biggs were able to identify most of the research topics which would later be developed in the field of mathematics education. The prevailing role of university professors had shifted the attention to the contents so that in the chapter "Summary and conclusions of the Seminar" out of 507 paragraphs only two (namely the numbers 417 and 418) were dedicated to "Research in mathematics education" (OEEC 1961a, p. 120) without there being any specific mention of the contribution made by Wall or Biggs. In the following, we will see that also in subsequent conferences of those years the approach to mathematics education only focused on contents.

[4]As explained above, the official sections of the Seminar are three. The Section IV found by Schubring (2014b) may have been suppressed or may correspond to sub-chapter IV of the report, which indeed contains the "Extracts from Dr. Wall's remarks."

In the Aftermath of Royaumont

Royaumont had a significant effect on mathematics education from an international perspective, testified, among other things, by the impressive subsequent flourishing of international meetings. These meetings had an institutional character, being organized by national institutions often with the support of OECD or UNESCO. Participants were chosen by the organizers from among experts in the field and officials of the bodies supporting the meetings. There is one aspect, however, that represented a step back: In general, few or no teachers were invited to these meetings. Concerning teachers, the efforts of the reformers were mainly addressed to retraining courses, usually held at a national level: Only the Belgian training courses known as the Days of Arlon, organized annually (between 1959 and 1968) by the Belgian Society of Mathematics Teachers and the Ministry of Public Education, although primarily intended for the retraining of Belgian teachers, were over the years regularly attended by foreign guests.

Of course, the themes treated in Royaumont also influenced activities already carried out in the mathematics education milieu such as at the International Congresses of Mathematicians and CIEAEM meetings.

Aarhus, Denmark

In January 1960, the President of ICMI Stone appointed a committee consisting of Heinrich Behnke (president), Svend Bundgaard (chairman), John G. Kemeny, Piene, Ole Rindung, Servais, Stefan Straszewicz, and Erik Kristensen (secretary) with the responsibility of organizing a seminar (to be held in Aarhus, Denmark) to advance the study of the following three topics which would be discussed at the 1962 International Congress of Mathematicians in Stockholm:

(1) Which subjects in modern mathematics and which applications of modern mathematics can find a place in programs of secondary school instruction.
(2) Education of teachers for the various levels of mathematical instruction.
(3) Connections between arithmetic and algebra in the mathematical instruction of children up to the age of 15.

Three sessions were devoted to the discussion of the three points mentioned above, but, as Bundgaard wrote in the preface, a decision was taken to concentrate most of the lectures on the discussion of “Modern teaching of geometry in secondary schools with particular emphasis on ways of treatment opened up by developments lately, in particular by the algebraization of mathematics” (Behnke et al. 1960, p. ii). It was decided to publish only the lectures on geometry (by Behnke, Choquet, Dieudonné, Werner Fenchel, Hans Freudenthal, György Hajós, Günter Pickert) and the related discussions (by Bunt, Servais, János Surányi, Poul Thomsen, Tullio Viola). Kristensen was in charge of the report. Some of the authors (Bunt, Choquet, Dieudonné, and Servais) had attended the Royaumont Seminar. The meeting was held from May 30 until June 2, 1960. In the preface to the report, it is indicated that there were 30 participants from 10 countries.

Of the seven talks, five proposed new axioms for geometry: Dieudonné introduced, as in Royaumont, the plane (or space) as a two- (or three-) dimensional real vector space completed by the dot product. He showed how the concept of angle could be defined by expressing its cosine using the dot product (but also argued that the angle is not a useful notion in mathematics). The long presentation by Choquet was a summary of his book (Choquet 1964). He presented axioms for the affine and metric plane, which—like those proposed by Dieudonné—were suitable for 17-year-old pupils and also “strong” axioms for Euclidean geometry based on the primitive concepts of distance and line reflection, which were suitable for younger pupils (age 15–16). Choquet defined an angle as a rotation that is the product of two line reflections.

Pickert presented axioms based on the properties of vectors—seen as primitive concepts—and their linear combinations. Metric properties were introduced by axioms on congruence and perpendicularity.

Hajós proposed that a large number of non-independent axioms concerning properties of Euclidean geometry should be introduced which would have to be reduced as the student became aware of links between them. Fenchel presented a list of axioms defining relations such as collinearity, coplanarity, congruence, and orthogonality. His exposition made wide use of logical symbols.

Discussion of the five different proposals could not produce fruitful results, but there seemed to be some agreement on the fact that Euclidean geometry was too difficult, even in the form of transformation geometry. It was Behnke who distanced himself not only (implicitly) from Otto Botsch—who in Royaumont had spoken in favor of the German schools that experienced the geometry of transformations—but also explicitly from Felix Klein, who had always been against abstraction and in favor of intuition. Of course, Behnke recognized Klein's work and merits, but also placed him in his historical period. Behnke was totally in favor of the modern approach and said he thought that Klein would have appreciated some of its aspects (but not all of them).

There were attempts, in particular by Freudenthal, Servais, and Viola, to introduce a wider educational and pedagogical point of view. Freudenthal distinguished between a rigorous axiomatic approach and a deductive approach with local deductions in which Euclidean "facts" could be placed; he claimed that we must ask ourselves the question of why we want to teach axioms. And Servais stated that mathematization is as important as logical deduction.

The distance between Freudenthal and Dieudonné was evident. When Freudenthal deemed it necessary to take into account the processes of the pupils and their psychological and pedagogical aspects, Dieudonné exclaimed "la psychologie, je m'en fiche" [Psychology, I don't care] (Behnke et al. 1960, p. 104), and when Freudenthal claimed that with school mathematics it was necessary to see "the whole together," Dieudonné answered that "we are here to discuss mathematics and nothing else" (p. 127).

Zagreb-Dubrovnik, Yugoslavia

In 1960 OEEC organized a working session in Yugoslavia (August 21 until September 2 in Zagreb, from September 4 to 17 in Dubrovnik) in which a group of experts was charged with the study of a modern program for teaching mathematics in secondary schools. Two days later most members of this group attended the ICMI symposium on the "Coordination of the Teaching of Mathematics and Physics" held in Belgrade (September, 19–24). The experts—Emil Artin, Botsch, Choquet, Bozidar Derasimovic, Fehr, Cyril Hope, Kristensen, Đuro Kurepa, Paul Libois, Laurent Pauli, Lennart Råde, Bruno Schoeneberg, Servais, Stone Pierre Théron, and Mario Villa—were from eight OEEC countries, Yugoslavia, and the United States of America. Nine of the participants had attended the Royaumont Seminar.

The resulting proposals were collected in one volume OEEC (1961c), the preface of which explained that the programs for the first cycle were more flexible and adaptable, and those for the second cycle were aimed at students who were oriented toward scientific and technical studies. The text referred directly to school teaching. It was divided into five parts: Algebra (1st cycle); Geometry (11–15 years); Algebra (2nd cycle); Geometry (15–18 years); and Probability and Statistics. Each part contained objectives, prerequisites, programs, exercises, and comments.

The program of the 1st cycle, Algebra (11–15 years), began with elementary notions on sets, relations between sets, and cardinal numbers, and continued with more classical themes such as numerical sets and their operations, the number line, linear functions and graphs, and simple equations. Then came structures such as rings, groups, and fields. The comments of this section contained almost 50 pages on sets, with exercises and applications connected to the other topics covered. A part with arithmetic exercises followed.

The 1st cycle Geometry program was characterized by the use of algebra and by intuitive and descriptive definitions, with the use of concrete materials that could allow pupils to move on to abstraction. Geometric transformations were introduced as was a connection to algebra when treating graphs of simple equations. Simple logical deductions were proposed. In this section the programs were mixed up with the exercises: All the topics were presented through examples.

The Algebra of the 2nd cycle was an abstraction of what was seen in the first cycle, including set theory, structures, group theory, the abstract notion of vector space, and much more. In the introduction, it was indicated that this was a "maximum" program. Exercises were not given, but a division by school years was proposed together with insights into structures, group theory, applications between structures, and linear algebra.

Names of "official authors" for the individual parts of the programs were not given, but there is an "oral tradition" which attributes the geometry programs for the level 11–15 years to Libois, and to Choquet those for the level 15–18 years—actually, the second part of these programs presented axioms of the affine and the metric plane, which were very similar to those proposed by Choquet in Aarhus. The author of this second Geometry section asserted that the proposed programs represented a synthesis of the positions of the various countries. The study of geometry included vectors (and later their dot products), coordinates (with particular reference to the correspondence between the line and the set of real numbers), and properties of the Euclidean plane. The latter did not correspond to the study of geometry according to Euclid's axioms; indeed, in the opinion of the author, the classical Euclidean properties had been studied for the most part in middle school experimentally, and they did not need a systematic study at this level. The pupils would be expected to understand that in addition to Euclidean geometry, there are other geometries (the reference was to affine geometry in particular). The study of these new geometries required reference to recent developments in mathematics (such as sets, groups, vector spaces, …).

As for the final part of the text, the authors suggested ways for introducing probability intuitively, through games and statistics involving data collection. The main part was then dedicated to the contents for the second cycle, which coincided with those usually treated today. The comments to this part consisted of introductory paths to probability and descriptive statistics.

It has to be noted that, since the program had been discussed by experts from 10 countries, it was expected that it should satisfy most of them and effectively represent a synthesis of the various positions. Indeed, it was presented as a common program. Nevertheless, discussion continued in subsequent conferences.

Bologna, Italy

A conference entitled "A Discussion of the Aarhus and Dubrovnik Reports on the Teaching of Geometry at the Secondary Level" was held in Bologna from October 4 to 8, 1961 (BUMI 1962). It was sponsored by ICMI and the *Commissione Italiana per l'Insegnamento Matematico* (CIIM) [Italian commission for mathematical teaching]. The organizing committee consisted of the Italians Pietro Buzano, Villa, and Viola (chairman), the President of ICMI Stone, and ICMI Secretary Gilbert Walusinski. Fulvia Furinghetti and Livia Giacardi (to appear) have reported that initially Aleksandr Aleksandrov, a member of the ICMI Executive Committee, had proposed to ICMI President Stone that he organize a European meeting in the USSR in 1961 and Stone discussed this with Sergei Sobolev, but nothing came of it.

The participants in the Bologna Conference (mathematicians, educators, ministerial officials, and observers of UNESCO and OECD) were from Belgium, France, Germany, Italy, Romania, Sweden, Switzerland, the USA, The Netherlands, and India (UNESCO observer). In his opening address, Stone pointed out that geometry had been the main object of discussion at Royaumont. Although there was a consensus on the need for action on geometry, the meetings of Aarhus and Zagreb-Dubrovnik had not reached an agreement on how to realize changes and this was the reason why the meeting of Bologna was organized. Stone (1963) presented a contribution on the choice of axioms for geometry in school. His conclusions went beyond the mere examination of mathematical arguments and underlined the need to carry out and examine the results of psychological studies—both analytical and experimental—and, from a practical point of view, to prepare students from primary school to face new concepts.

In addition to those published in the journal *L'Enseignement Mathématique* (by Artin, Campedelli, Cartan, Freudenthal, Libois, Lucio Lombardo-Radice, Ugo Morin, Ruggero Roghi, Servais, Stone, Viola) there were the following contributions:

– Nyman Bertil, On a geometry text, based on Choquet's axiomatics, in preparation in Sweden;
– Pauli Laurent, Pedagogical experiences in Switzerland on the teaching of geometry in secondary schools.

The various presentations sparked lively discussions. Three contributions stood out and became a landmark of this conference. Firstly Artin (1963), who was among the experts in Zagreb-Dubrovnik, illustrated the work of mediation between three extreme positions attempted at that conference: To preserve the classical Euclidean method, to introduce Hilbert's axiomatics, and to define Cartesian space as a vector space of dimension two. The final choice was the third way, introduced by axioms. Freudenthal (1963) focused on pedagogical aspects and advocated frank didactic research in mathematics education. According to him, the geometry programs proposed in Aarhus and Dubrovnik were based on what he called an "antididactic inversion" (p. 32) since they started from the finished mathematical product, which is the result of the activity of others, rather than promoting the student's investigation and exploration for constructing the meaning of the concepts. In line with Freudenthal's ideas were the interventions of Libois and the Italian Viola.

On the opposite side, Henri Cartan (1963) was in favor of the new contents proposed in Aarhus and Dubrovnik and criticized those who, despite showing interest in reforms, in reality, did not want to make any changes. He advocated taking care not only of students who would stop their school careers after secondary school but also of those who would proceed to university. Moreover, he stressed the need to train teachers on the new mathematics content. He was convinced that in planning a radical reform, it was necessary to focus first on the mathematical content and then on educational objectives. On this point, he was not in agreement with Freudenthal (1963) who claimed that didactical problems had to be considered first.

At the end of the Bologna conference, the participants were invited to make proposals for books inspired by the new trends which had emerged from the various discussions. In Chapter 8 of this volume, Furinghetti and Marta Menghini analyze the influence of this conference on the introduction of modern mathematics in Italy.

Axiomatics had already been extensively treated by Choquet in his lecture at the Lausanne Seminar on "The teaching of analysis and relative manuals" sponsored by ICMI and the Swiss Mathematical Society and held from June 26 to 29, 1961 (see *L'Enseignement Mathématique, 1960, s. 2, 6,* 93–178).

Stockholm, Sweden. Modern Mathematics at the International Congress of Mathematicians in 1962

During the ICM held in Stockholm from August 15 to 22, 1962, reports on the three themes discussed in Aarhus were presented, based on materials submitted by different countries. Two reports referred also to modern mathematics. Kemeny (1964) presented a report "Which Subjects in Modern Mathematics and which Applications of Modern Mathematics can Find a Place in Programs of Secondary School Instruction?" based on submissions from 21 nations. He concluded that four areas of modern mathematics—elementary set theory, an introduction to logic, some topics from modern algebra, and an introduction to probability and statistics—were usually introduced, while there was no agreement on how far the axiomatic system should be applied to mathematics, and in particular to geometry. He further observed that the most frequent motivation for teaching modern mathematics was to prepare students for university and that the greatest difficulties were the shortage of qualified teachers and the lack of suitable textbooks.

In his talk, based on 11 national reports, Straszewicz (1964) identified a general tendency to bring the school education of mathematics—even in the lower grades—closer to contemporary science and its new applications by gradually introducing a modern mathematical language. It was proposed, for example, to introduce fairly early the simplest notions of the algebra of sets and propositional logic, in order to highlight the structural properties of the different number systems.

In an interview with Bernard Hodgson, Geoffrey Howson stated that in Stockholm (as well as in the previous ICM in 1958) he had been disappointed by the short amount of time dedicated to mathematics education and the lack of involvement of people such as Begle, Georges Papy, Servais, Félix … "who were doing things" (Hodgson 2008, part 1) and the too much attention paid to the words of university mathematicians not directly involved with schools. He complained about the lack of discussion between the two successive ICMs of 1958 and 1962, although he recognized that this criticism was partly answered by discussion at the Arlon meetings and CIEAEM.

Athens, Greece

The wave of renewal that swept through many nations, mainly in Europe and North America, is witnessed by the congress held in Athens from November 17 to 23, 1963 (OECD 1964). This event was the first attempt to present the achievements and repercussions of modern mathematics in OECD countries. Forty mathematicians, mathematics teachers, and education officers participated. They were from the 20 OECD countries—Austria, Belgium, Canada, Denmark, France, Germany, Greece, Iceland, Ireland, Italy, Luxembourg, Norway, Portugal, Spain, Sweden, Switzerland, The Netherlands, Turkey, UK, and the USA. Fehr wrote the preface to the proceedings, and is referred to as "Rapporteur General." The second cover says that during the conference "a forum of leaders in School Mathematics reform examined and discussed the new programs which O.E.C.D. has encouraged or actively sponsored for experimentation and adoption in Member Countries." In the Appendices, information on some national projects was given.

In the proceedings five major aspects of mathematics education were considered:

(1) The subject matter content of a modern program in mathematics for the scientific line in secondary school
(2) Methods of organizing and teaching a newly formed program
(3) The use and purpose of pilot-experimental classes in modern mathematics
(4) The role of applications in the teaching of newer branches of mathematics

(5) The education and professionalization of teachers of secondary school mathematics

As reported in the proceedings, the most significant outcome of the conference was a clear picture of what was meant by the "modernization of mathematics teaching" (p. 291). A genuine distinction appeared between (a) the updating of a traditional program, where the same sequence is maintained but modern concepts and treatments were injected into the curriculum; and (b) a new program completely modern, based on a total re-construction of the curriculum.

The resolutions and recommendations took into consideration the main issues concerning mathematics teaching: The content of the courses (sets, relations, and functions, vector spaces, calculus, probability, statistics, the use of computers, mathematical logic, updating of notations and definitions), the need for research and information, experiments in classrooms, and their evaluation, teacher education, links with other disciplines and applications.

In the United States of America: The Conference in Cambridge (MA)

Despite the contact established between the United States and Europe, Royaumont's impact was different in the two communities. We cite as an example of these fact the conference sponsored by the National Science Foundation held in Cambridge (Massachusetts) from June 18 to July 12, 1963, to discuss the future of mathematics curricula. The steering committee consisted of Begle, Bruner, Andrew Gleason, Mark Kac, William Ted Martin, Edwin Moise, Mina Rees, Patrick Suppes, Stephen White, and Samuel S. Wilks. Some 25 mathematicians and users of mathematics from universities and industries attended the conference. Among them, there were some participants in the Woods Hole Conference of 1959—including Bruner—and members of the SMSG (School Mathematics Study Group). According to the report (Goals for School Mathematics, 1964; De Finetti 1965), the main purpose "was to reconsider the structure of mathematics education and to sketch a rough outline of a possible new framework for the primary and secondary school" (p. 196). Conference recommendations dealt more with methods of presentation than with specific mathematical content, as in the case of the "spiral" approach, in which every new topic would be introduced under low pressure and would then be reconsidered repeatedly each time with more sophistication, the importance of multiple motivation for the topics introduced, and the development and implementation of discovery procedures. Among the various themes discussed, we note the reference to the importance ascribed by Freudenthal to the early development of the child's spatial intuition, and themes important in modern mathematics such as sets (without too much emphasis on theory and formal logic), elementary ideas of probability and statistical judgment. However, there were no explicit references to Royaumont nor to the follow-up meetings. Henry O. Pollack reported on this conference at the Athens meeting of 1963 (OECD 1964, pp. 90–94).

Frascati, Italy

From October 8 to 10, 1964, 27 participants from six European countries and Argentina met in Frascati for discussing the topic "Mathematics at the coming to university. Real situation and desirable situation." A few school teachers were present. The conference was organized by ICMI with the collaboration of CIIM and the *Centro Europeo dell'Educazione* (CEDE) [European Center of Education] in the villa Falconieri, the headquarters of CEDE. Lectures were delivered by Jean Bass, Behnke, Bruno de Finetti, René Deheuvels, Julien Desforge, Bo Kiellberg, Arnold Kirsch, Jacqueline Lelong, Carlo Felice Manara, Papy, Pickert, André Revuz, Hans-Georg Steiner, Walusinski. There were no proceedings. The short report (R. G. 1964) contained the titles of the lectures and the summaries of the talks by Behnke, Campedelli, and Papy. There were no final resolutions, but the participants expressed a wish to fill the gap between secondary and university mathematics education. Behnke pointed out that the means for reaching this goal was to remedy the shortage of teachers and to prepare them adequately. He also recommended avoiding excessive abstraction and axioms. Modern mathematics was extensively treated by Papy who reported on Belgian schools and de Finetti when discussing his proposal for the renewal of Italian programs.

From Milano Marittima, Italy, to Echternach, Luxembourg

After 1960, CIEAEM continued with its yearly thematic meetings, many of which show a link to modern mathematics. As Félix (1986) phrased:

> It is the moment to become aware of possible post-Hilbert axioms for geometry, after the contributions of Choquet (1955–1965), Artin, H. Levi, Papy, Delessert, Revuz, Krygowska, Servais and many others who had understood the importance of the work of Dieudonné but also its limits at the educational level. This is the moment for an important reflection on the axiomatic approach, on the construction of the affine plane, on the alternative between a geometric or algorithmic solution. (p. 64)

In 1965, at the CIEAEM meeting entitled “The place of the geometry in a modern mathematical teaching” in Milano Marittima (Italy), Revuz (1965) proposed an approach to geometry that attempted to reconcile the positions expressed by Choquet and Dieudonné at the congress of Aarhus. Dieudonné had explicitly accused Choquet of being linked to naive realistic and synthetic methods (Dieudonné 1964). The solution by Revuz was based on proposals by Papy, which were quite similar to those of Choquet (1964) at the first stage, but which allowed a second stage in which the achievements of the first stage served as a base for new axioms for the further development of geometry, in agreement with Dieudonné’s positions (Vanpaemel & De Bock 2019).

The proposal by Revuz was also discussed at the colloquium of Echternach in the same year (from May 30 to June 4), the link being established by Lichnerovicz, who was a member of CIEAEM and at that time President of ICMI. The colloquium was organized by ICMI in collaboration with the Minister for Education and Cultural Affairs for the Grand Duchy of Luxembourg. The 92 participants were from eight European countries.

The subject of the seminar was “Repercussions of Mathematical Research on Teaching.” The resulting book (Conference Echternach 1965) contains the texts of the lectures by Behnke, Camille Bréard, Bunt, Choquet, Jean de Siebenthal, Paul Debbaut, André Delessert, Dieudonné, Arthur Engel, Kirsch, Papy, Pickert, Charles Pisot, Revuz, Servais, and Steiner. Many lectures were about the axiomatic method, not only as regards geometry. Dieudonné spoke about the role of linear algebra in modern mathematics, Servais about axiomatization and elementary geometry (12–15 years), and Papy about the Euclidean vector plane in teaching (15 years) (Conference Echternach 1965). Revuz’s motion, known as “*Convention de Ravenne et traité de Echternach*” [Ravenna Convention and Echternach Treaty], was signed by Choquet and Dieudonné (Félix 1986) (Figure 4.2). At the end of the colloquium, in a meeting of ICMI with the representative of UNESCO, the publication of the already mentioned series of volumes entitled *New Trends in Mathematics Teaching* was planned, and

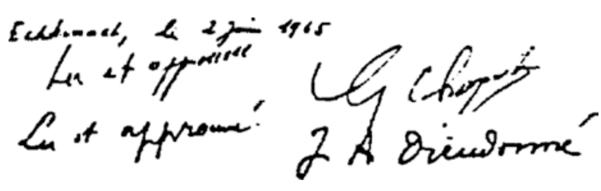

Figure 4.2 Signatures in the Echternach treaty

Anna Zofia Krygowska was entrusted with the task of overseeing the publication (see *Internationale Mathematische Nachrichten, 83*, 1966, p. 3). In 1967 the first volume (UNESCO 1967) was issued.

Modern Mathematics Goes Beyond the Iron Curtain

We may say that the Echternach conference closed the first cycle of conferences on new thinking in school mathematics—a cycle that started with the suggestions of Royaumont and continued with the proposals of Aarhus, Zagreb-Dubrovnik, Bologna, and Athens. ICMI and OEEC/OECD were the main supporting bodies.

The colloquium of Echternach was one of the last attempts to find a solution to the problem of teaching geometry, a problem which—after the intervention of Dieudonné in Royaumont—was perhaps given an excessive role. We will see that more general problems arose in other conferences.

Budapest, Hungary

From August 27 to September 8, 1962, at the invitation of the Hungarian government, UNESCO organized an international symposium in Budapest aimed at dealing with the problems involved in reforming mathematics teaching in schools in the light of the results of the meetings already held in various countries (Hungarian National Commission for UNESCO 1963). The number of participants outside Hungary was limited to 18, chosen among mathematicians, teachers, psychologists, and educators; there were six Hungarians. For the first time in this kind of meeting, there were people from Australia, Japan, and the USSR. The main themes treated were: The mathematics curriculum; teaching methods and their psychological background; training and re-training of mathematics teachers. We note the explicit mention of psychology; indeed, the British scholar, Richard R. Skemp, who participated in the conference, was the major pioneer in the psychology of mathematics education. ICMI was present through its President Stone and its officer Yasuo Akizuki.

Summaries of each day's discussions were prepared by rapporteurs and circulated on the following day (Hungarian National Commission for UNESCO 1963, p. 6). There were also "background papers" prepared before the symposium (p. 5). A chief rapporteur—Servais—prepared the "Conclusions and recommendations" (pp. 11–34). These recommendations included some nods to modern mathematics, which, nevertheless, due to the general approach to the problems adopted, was not a central issue of the symposium. In the "Introduction" (pp. 7–10)— which gave a survey of the activities—Stone claimed that "the results obtained in this Symposium are consistent with the conclusions reached by a number of recent conferences, both national and international, and represent a substantial advance beyond them" (p. 8). His conclusion was a call to establish a network of contacts and information.

Moscow, USSR. Modern Mathematics at the International Congress of Mathematicians in 1966

The four-year International Congress of Mathematicians was held in Moscow from August 16 to 26, 1966. There were communications concerning mathematics education, but the short notes published in the booklet of abstracts and the proceedings make it difficult to evaluate the topics which were considered. In Volume 3 of the one-hour and half-hour addresses, there is the text of Papy's lecture entitled "La géométrie dans l'enseignement moderne de la mathématique" [Geometry in the

Modern Teaching of Mathematics] (pp. 82–89), which reported on the Belgian experiments at the secondary level (12- to 17-year-old students). This contribution was also published in *L'Enseignement Mathématique* (1966, *s. 2, 12*, 225–233). In the same volume, Jean Leray commented on Papy's experiments in his article "L'initiation aux mathématiques" [The initiation to mathematics] (pp. 235–241). Two general reports of ICMI, presented at the Congress of Moscow, were not included in the proceedings, but were published in *L'Enseignement Mathématique*:

Pisot, Ch. 1966. Rapport sur l'enseignement des mathématiques pour les physiciens [Report on Mathematics Education for Physicists]. *L'Enseignement Mathématique, s. 2, 12*, 201–216.

Krygowska, A. Z. 1966. Développement de l'activité mathématique des élèves et rôle des problèmes dans ce développement [Development of Pupils' Mathematical Activity and the Role of Problems in this Development]. *L'Enseignement Mathématique s. 2, 12*, 293–322.

Bucharest, Romania

From September 23 to October 2, 1968, UNESCO conducted an international colloquium about "Modernization of Mathematics Teaching in European Countries" in Bucharest (Teodorescu 1968), with the cooperation of the Society of Mathematical Sciences of Romania and the Romanian National Commission for UNESCO. It was attended by 39 "experts" from 22 European countries and some 300 Romanian secondary school teachers. Among the experts were ICMI President Freudenthal and other members of the Executive Committee, Delessert, and Revuz.

Problems of mathematics teaching were raised and discussed from the point of view of the scientific-technical evolution in contemporary society. The term "modern mathematics" was used in the introduction by Nicolae Teodorescu (1968), but in a completely different sense, linked to electronic computers. "Representatives" of modern mathematics of the early 1960s, such as Revuz or Servais, were present, but their contributions related to more general questions on the teaching of mathematics. Even Papy's talk was anything but uncompromising; he proposed a progressive approach to an increasingly abstract axiomatic system. In the discussion which followed, Krygowska questioned the legitimacy of some of Papy's statements. For instance, Papy affirmed that in traditional Euclidean geometry, proofs of quite obvious facts were presented. Krygowska stated that this was not changed by the fact that we use modern axioms. In Papy's books, students also proved "intuitive theorems." The problem was a different one—it was a problem of methodology: Pupils must understand that they have to establish a logical relation between certain groups of theorems. They like to prove evident things if they know what they are proving. In turn, Sobolev criticized Papy's alleged simplification of mathematics. It was mathematicians who needed to learn to simplify their way of presenting arguments.

The final recommendations concerned: The impact of mathematics on contemporary schools; the teaching of modern mathematics; teacher education; international cooperation, including the contribution of ICMI.

Modern Mathematics in Other Hemispheres

Latin America

In Latin America, contact with the New Math occurred initially through the textbooks of the School Mathematics Study Group (SMSG) (Barrantes and Ruiz 1998). A decisive impulse came from the First Inter-American Conference on Mathematics Education, held in Bogotá, Colombia, from

December 4 to 9, 1961. Its organization was the result of the cooperation between ICMI and CIAEM (*Comité Interamericano de Educación Matemática*) [Inter-American Committee on Mathematics Education: IACME], which was founded in 1961 by a group of mathematicians and math educators from the three Americas. The conference was sponsored by the Organization of American States (OAS), USA's National Science Foundation (NSF), the Rockefeller Foundation, the Ford Foundation, the Association of Colombian Universities, and UNESCO. Mathematicians and mathematics teachers attended as representatives or guests from American countries—23 according to Hugo Barrantes and Angel Ruiz (1998)—and there were a few European special guests from Denmark, France, and Switzerland. The keynotes addresses were delivered by Enrique Cansado, Laurent Schwartz, Guillermo Torres, and the participants in Royaumont Begle, Bundgaard, Choquet, Fehr, and Stone, who reported on what was going on in Europe and the USA and on the means to introduce innovations into Latin America: There was a need for suitable textbooks, curriculum changes, teacher training, and more. In response to the opening speech by the Education Minister, Stone, President of ICMI and the Royaumont Seminar, recalled the role of ICMI in mathematics education and encouraged collaboration with ICMI to promote regional cooperation. In fact, at the end of the conference, the first Executive Committee of CIAEM or IACME was established with Stone as *pro tempore* President (Stone remained in charge until 1972). After this event, the conferences of CIAEM were regularly held in different locations throughout Latin America.

The main ideas put forward during the conference were: Teaching geometry in a new way with a focus on modern mathematics; pursuing the unity of mathematics through the study of fundamental structures relying on modern algebra; teacher education and in-service teacher retraining. The final resolutions concerned these ideas. Some talks, in particular those of Choquet and Fehr, aroused dissension both on the mathematical contents proposed and on their inadequacy to the context in which the proposals would be inserted. Among the many interventions, we note that Torres, of Mexico, claimed that it was not advisable to abandon entire topics from classical mathematics and then fall into formal definitions and concepts that would communicate absolutely nothing to students. He suggested that mathematics should be taught in accordance with its historical development, again in opposition to Choquet who considered this method outdated. Illustrative of the context in Latin America is the statement by Omar Catunda who claimed that the formula suitable to Brazil should be "al menos la Geometría de Euclides" ["at least Euclid's geometry"] (Barrantes and Ruiz 1998, p. 13).

Africa and Asia

Science education had become an important area of cooperation with newly independent and developing countries, many of which established their agencies for curriculum development. The momentum generated by the reform movements of the 1960s resulted in projects aimed at incorporating modern approaches, methods, and materials in the area of Africa and Asia.

In 1961 the African Mathematics Program (AMP) was launched at a meeting in Dedham (Massachusetts) in which scholars in mathematics education, teachers, educators from Africa, the USA, and the UK met for planning educational innovation in African schools (Weaver 1965). Three seminars were then organized in Entebbe (Uganda) by the Entebbe Mathematical Centre, directed by Martin, to produce materials. Being linked to New Math the topics—introduced at all school levels—included sets and their use in teaching numbers, the related basic operations and their properties, intuitive geometry and measurement, statistics, probability, and motion geometry (Williams 1976). The third seminar in Entebbe, entitled "Mathematics" (Uganda, July 15–August 15), continued the work that had been started in the two preceding seminars. It was attended by 60 participants, including school teachers, from 12 countries (Ethiopia, Ghana, Kenya, Liberia, Malaya, Nigeria, Sierra Leone, Tanzania, Uganda, United Kingdom, United States, and Zambia).

The Association of South-East Asian Institutes of Higher Education organized in Saigon and Dalat (Vietnam, November 9 to 13, 1964), a "Seminar on mathematics teaching in South-East Asia." The 20 participants represented the universities of Malaya, Singapore, Thailand, and Vietnam (UNESCO 1967). The Seminar concerned the state of the art of mathematics teaching in the participating countries and the new trends in the training of teachers, engineers, economists, and researchers in mathematics. On these questions, final recommendations were made.

The *Commission Inter-Unions de l'Enseignement des Sciences* (CIES) [Inter-Union Commission for the Teaching of Science, IUCST]—of the Scientific Council of Scientific Unions (ICSU) organized in Dakar (January 14 to 22, 1965) the "Congress on science teaching and its role in economic progress." It was attended by 84 members (experts in different scientific disciplines) from nine African countries, four American countries, five Asian countries, and nine European countries. ICMI and UNESCO were represented at this conference. A parallel congress on mathematics teaching in conjunction with the teaching of other sciences was organized by ICMI in collaboration with the National Senegalese Commission of the Teaching of Mathematics (January 13 to 16, 1965).

A similar attempt to import ideas and materials from other countries was carried out by UNESCO when in 1966 it

> was requested to assist the Arab Region [including Egypt] in improving mathematics education. Seminars, syllabus determination, textbook writing sessions, and training sessions for teachers were held, which resulted in a new mathematics course for secondary schools which was implemented in most schools in the Arab States. […] The Arab League Education, Cultural and Science Organization (ALECSO) extended the reform to intermediate level students, aged 13–15 and revised the UNESCO project books. (Jacobsen 1996, p. 1246)

As Malaty (1999) noted, 8 of the 22 authors of the textbooks for senior secondary school were from "outside the Arab world" (p. 238)—mostly the United States and the United Kingdom—and the books were written in English and then translated into Arabic. A lack of prior knowledge of the local context of some of the authors of the materials hindered the development of the project.

Issues 2 and 3 of the journal *Educational Studies in Mathematics* (1978, Vol. 9) entitled "Change in Mathematics Education Since the Late 1950s—Ideas and Realization," edited by Hans Freudenthal, provided an overview of initiatives carried out in the period of the reform movements in various countries.

Toward New Horizons

Utrecht 1964, Netherlands

ICMI organized an international colloquium on "Modern curricula in secondary mathematical education" at Utrecht from December 19 to 22[5], 1964, under the presidency of Freudenthal. The organizing committee also included Behnke, Choquet, and Moise (*Internationale Mathematische Nachrichten, 80*, 1965, pp. 3, 8). There were 41 participants from 10 European countries, the United States of America, and Canada. Lectures were delivered by Max Beberman, Castelnuovo, Delessert, Jürgen Dzewas, Félix, Freudenthal, Theodorus Jacobus Korthagen, Krygowska, Servais, Steiner, Owen Storer, Straszewicz, Bryan Thwaites, Roelof Troelstra, Jacobus Van Lint, Leonardus Reinhard Joseph Westerman, and Alexander Wittenberg. There were no proceedings. UNESCO (1967) reported that the themes treated were grouped as follows: The general principles of the reform in the teaching of mathematics; reports of the work done in certain centers concerning the programs developed and

[5] In Steiner (1965) and UNESCO (1967) the final day is December 23. In the folder dedicated to Freudenthal (inventory number 1831) of the *Noord-Hollands Archief* the date is December 22.

other activities; report on experiments already completed in modern teaching and projects concerning the setting up of certain fragments of mathematics in line with the new programs. Among other things, it emerged that the reforms of the teaching of mathematics were at different stages in the countries represented, and also that they were conceived and organized in different ways concerning such questions as, for example, the introduction of axioms and the role of geometry. Only the contributions by Castelnuovo (1966) and Wittenberg (1965) were published. The latter's talk was remarkable for its insight into the teaching and learning of mathematics and for the proposals launched: The necessity of creating university chairs for mathematical education; the urgency of founding an international journal specifically dedicated to mathematics teaching.

Utrecht 1967, Netherlands

At the beginning of his ICMI presidency, Freudenthal organized the colloquium "How to teach mathematics so as to be useful," held in Utrecht from August 21 to 25, 1967. It was sponsored by the International Mathematical Union and the government of The Netherlands. The colloquium was attended by "34 active and 34 passive members" (*Internationale Mathematische Nachrichten, 91*, 1969, p. 3[6]). Lectures were given by Behnke, Anne Brailly, Delessert, Engel, Trevor Fletcher, Freudenthal, Maurice Glaymann, Brian Griffiths, John M. Hammersley, Matts Håstad, Murray Klamkin, Krygowska, Robert Cranston Lyness, Pisot, Pollak, Revuz, André Roumanet, Servais, Steiner, Jean Tavernier, Robert Walker, and Gail Young. The texts of the lectures were published in the new journal *Educational Studies in Mathematics* (1968, Vol. 1). The conference was centered on the "usefulness" of mathematics and its applications. In his intervention Hammersley (1968) openly declares his perplexities about modern mathematics. In the panel organized during the conference, besides applications of mathematics, modern mathematics was widely discussed and the panelists often referred to the concepts of structure, set theory, and logic (Behnke et al. 1968).

This colloquium was followed by a meeting of the Executive Committee of ICMI, in which some important issues which shaped the future of ICMI and affected the community of mathematics educators were discussed (Delessert 1967). Freudenthal, President of ICMI, stressed the lack of effectiveness of the contributions in the sections dedicated to mathematics education at the ICMs. He proposed to organize an ICMI congress in 1969, with invited lectures and communications. The founding of a new journal more suitable to deal with the problems of mathematics education than *L'Enseignement Mathématique,* the current organ of ICMI, was also solicited.

The two meetings of Utrecht were held under the influence of Freudenthal. More than a follow-up of Royaumont they could be seen as forerunners of the new trend in meetings and communication inaugurated by Freudenthal's presidency.

Conclusions

Dramatis Personae

We have seen that the main actors in the story of modern mathematics have been the international bodies CIEAEM, ICMI, OEEC/OECD, UNESCO, and, with a marked national impact, the Study Groups in the United States of America. But this story is, first of all, made by scholars committed to mathematics education.

[6] We refer here to the paper version of the first issue of 1969 of the *Internationale Mathematische Nachrichten*, which has not been digitalized. The number 91 of the volume was repeated for the first issue of 1970.

While certain meetings were open to teachers and other agents in schools, the voice of mathematicians prevailed. Among the participants and speakers, there were some frequent names. The Bourbaki group was always well represented, in the first place by Choquet and Dieudonné, and sometimes by other Bourbakists (Cartan in Bologna and Schwartz in Bogotá). The Belgians Servais and Papy were frequent participants. Fehr was also very active and had institutional responsibilities in drawing up important reports. ICMI was present with its Presidents Stone and Lichnerowicz, and Behnke, vice-President, former Secretary, and President.

The lists of participants in the meetings described above provide interesting data about the emergence of women on the international stage of mathematics education. Before the meetings under Freudenthal's presidency, we identified only four women delivering talks—Castelnuovo, Félix, Krygowska, and Lelong. In the list of participants in the Utrecht meeting of 1964, there are Castelnuovo, Félix, Krygowska, and the French C. Tcherkawsky (*Internationale Mathematische Nachrichten, 80*, 1965, p. 8). She is probably the secondary school teacher Colette Tcherkawsky, author of textbooks (*L'Archicube*, 2017, February, 149–152). At the Frascati meeting of 1964, there was the Italian mathematics teacher Maria Albanese, a member of CIIM. The list of participants in the colloquium of Bucharest (1968) included Castelnuovo, Krygowska, Lina Mancini Proia, Frédérique Papy-Lenger, and Marie-Antoinette Touyarot (teacher at the *École Normale* de Caen). In the available records of the conferences considered above, we have not found other mentions of women, except those participating as observers of UNESCO or secretaries. We know that there were no women officially participating in the conference of Bogotá (Barrantes and Ruiz 1998). We do not have enough information about feminine participants in the meetings linked to projects addressed to developing countries (Entebbe, Saigon, Dalat, Dakar, and Arab countries). The proceedings of the Echternach meeting in 1965 do not show a list of participants, but the photo published there (Figure 4.3) is an iconic representation of the poor rate of women to men. The scant presence of females is mainly due to the fact that the invited experts or simple participants were manly university professors, a category where women were poorly

Figure 4.3 Colloquium at Echternach in 1965. (Courtesy of Robert Kennes)

represented. We note that the CIEAEM meetings, which have not been the object of our analysis, had a slightly different trend regarding the presence of women. It was at ICME-1 in 1969 that a clear opening to the "other side of the moon" started, as shown by the informal list of participants in ICME-1 sent to us by Erich Wittmann. This opening increased with each subsequent ICME and in other conferences such as those organized by the International Study Group on the Relations between the History and Pedagogy of Mathematics (HPM) and the International Group for the Psychology of Mathematics Education (PME).

Much Ado About Nothing?

The wishes expressed in Utrecht 1967 were fulfilled: In May 1968, the first issue of the journal *Educational Studies in Mathematics* came out, and from August 24 to 30, 1969, the first ICME was held in Lyon, France (Figure 4.4). UNESCO contributed financially to both initiatives. Another story was beginning.

The roots of the ferments in the milieu of mathematics education that marked the 1960s are to be sought in socio-political and cultural factors affecting the 1950s. About socio-political factors, it is illuminating what Howson (December 2021, personal communication to Furinghetti) claims about the School Mathematics Project, which started in 1961:

> In some ways it [the School Mathematics Project] was generated by the great part mathematics had played in the war, code-breaking, the beginnings of operational research, …. There was seen to be a need to update what was taught in schools to meet new demands for mathematical knowledge. […] significantly, the money to launch this [the School Mathematics Project] came almost entirely from industry and commerce where the need for people with advanced mathematics was increasing.

As to the cultural factors, there was a new approach to the problems of mathematics education which led to new projects and new working groups, for example, in the USA, the University of Illinois Committee on School Mathematics (UICSM) first and the School Mathematics Study Group (SMSG) later, and in Europe CIEAEM.

The chapters in the second part of this book show the impact of modern mathematics in various countries. At the end of the 1960s, the decline of modern mathematics was beginning. Its definitive end was sanctioned in 1972 by René Thom's talk at ICME-2 entitled "Modern Mathematics: Does it

Figure 4.4 ICME-1, Lyon: Krygowska, Steiner, Papy-Lenger, Zoltán Dienes. (Courtesy of Gert Schubring)

exist?" (Thom 1973), and later by Morris Kline's book *Why Johnny Can't Add: The Failure of the New Math* (Kline 1973).

However, not all was lost and there have been initiatives, which still exist, which owe their birth to modern mathematics. The most remarkable of these initiative are: The French *Instituts de Recherche sur l'Enseignement des Mathématiques* in 1969 (IREMs) [Mathematics Education Research Institutes], the German *Zentrum für Didaktik der Mathematik* (ZDM) [Centre for didactics of mathematics] in Karlsruhe in 1968, and *Institut für Didaktik der Mathematik* (IDM) [Institute for the Didactics of Mathematics] in Bielefeld in 1972. The establishment of IREMs sprang from the work of the Commission Lichnerowicz, a commission born to promote the reform of modern mathematics when it was realized that any discussion on mathematics teaching had to be based on a solid program of mathematics teacher education and research. As stressed by Maurice Glaymann, the IREMs were bodies independent from the Commission (Artigue 2008). This independence allowed them to survive the Commission Lichnerovicz, which ended its work in 1973. About the ZDM and the IDM, their creation was a kind of reaction to modern mathematics stimulated by the observation that renewal in mathematics teaching was above all a matter of methods of teaching and learning, rather than a matter of mathematical content (Steiner, personal communication to Menghini, 1989).

Despite the failure, at least in its most radical form, of modern mathematics in various countries, we agree with Kilpatrick (2012) that "By the time the new math era ended […] everyone concerned with school mathematics had a much better sense of what was going on around the world" (p. 569). And indeed, it was the Royaumont Seminar that, by promoting communication in the discussion of mathematics education, generated the driving force behind most initiatives in the following decades.

References

Artigue, M. (2008). *Interview with Maurice Glaymann*. Retrieved November 1, 2020, from https://www.icmihistory.unito.it/clips.php

Artin, E. (1963). Les points de vue extrêmes sur l'enseignement de la géométrie [The extreme points of view on geometry teaching]. *L'Enseignement Mathématique, s. 2, 9,* 1–4.

Barrantes, H., & Ruiz, A. (1998). *The history of the Inter-American Committee on Mathematics Education* (Bilingual Spanish and English edition). Bogotá, Colombia: Academia Colombiana de Ciencias Exactas, Físicas y Naturales.

Behnke, H., Choquet, G., Dieudonné, J., Fenchel, W., Freudenthal, H., Hajós, G., & Pickert, G. (1960). *Lectures on modern teaching of geometry and related topics*. Aarhus, Denmark: Matematisk Institut (Aarhus Universitet), Elementaer Afdeling. Nr. 7.

Behnke, H., Hammersley, J. M., Krygowska, A. Z., Pollak, H., Revuz, A., Servais, W., Sobolev, S., & Freudenthal, H. (1968). Panel discussion. *Educational Studies in Mathematics*, *1,* 61–79.

Bernet, T., & Jaquet, F. (1998). *La CIEAEM au travers de ses 50 premières rencontres* [The CIEAEM through its first 50 meetings]. Neuchâtel, Switzerland: CIEAEM.

Bruner, J. (1960). *The Process of Education.* Cambridge, MA: Harvard University Press.

BUMI. (1962). Il convegno di Bologna promosso dalla Commissione internazionale dell'insegnamento matematico [The Bologna conference promoted by the International Commission on Mathematical Instruction]. *Bollettino della Unione Matematica Italiana, s. 3, 17,* 199–214.

Cartan, H. (1963). Réflexions sur les rapports d'Aarhus et Dubrovnik [Reflections on the Aarhus and Dubrovnik reports]. *L'Enseignement Mathématique, s. 2, 9*, 84–90.

Castelnuovo, E. (1966). Un enseignement modern des mathématiques dans le 1er cycle secondaire [Modern teaching of mathematics in lower secondary education]. *L'Enseignement Mathématique, s. 2, 12*, 195–199.

Choquet, G. (1964). *L'enseignement de la géométrie* [The teaching of geometry]. Paris, France: Hermann.

Christiansen, B. (1978). The cooperation between ICMI and UNESCO. *ICMI Bulletin, 10,* 4–10.

Conférence Echternach. (1965). *Les répercussions de la recherche mathématique sur l'enseignement. Textes originaux des conférences faites au séminaire organisé par la C.I.E.M. à Echternach (G.-D. de Luxembourg) été 1965* [The impact of mathematical research on teaching. Original texts of the conferences given at the seminar organized by the C.I.E.M. in Echternach (G.-D. of Luxembourg) summer 1965]. Echternach, Luxembourg: Institut Grand-Ducal Section des Sciences Naturelles, Physiques et Mathématiques.

De Bock, D., & Vanpaemel, G. (2015). Modern mathematics at the 1959 OEEC Seminar at Royaumont. In K. Bjarnadottir, F. Furinghetti, J. Prytz, & G. Schubring (Eds.), *"Dig where you stand" 3. Proceedings of the third International Conference on the History of Mathematics Education* (pp. 151–168). Uppsala, Sweden: Department of Education, Uppsala University.

De Finetti, B. (1965). Programmi e criteri per l'insegnamento della matematica alla luce delle diverse esigenze [Programs and criteria for teaching mathematics in the light of different needs]. Periodico di Matematiche, s. 4, 43, 119–143.

Delessert, A. (1967). Compte rendu de la séance de la C.I.E.M. tenu à Utrecht, le 26 Août 1967 [Minutes of the ICMI meeting held in Utrecht on August 26, 1967]. *L'Enseignement Mathématique, s. 2, 13,* 243–246.

Dieudonné, J. (1961). New thinking in school mathematics. In OEEC, *New thinking in school mathematics* (pp. 31–46). Paris, France: OEEC.

Dieudonné, J. (1964). Algèbre linéaire et géométrie élémentaire [Linear algebra and elementary geometry]. Paris, France: Hermann.

Fehr, H. F. (1961). The significance of this report. In OEEC, *New thinking in school mathematics* (pp. 207–210). Paris: OEEC.

Félix, L. (1986). *Aperçu historique (1950–1984) sur la Commission Internationale pour l'Étude et l'Amélioration de l'Enseignement des Mathématiques (CIEAEM). 2ème édition revue et augmentée* [Historical overview (1950–1984) on the International Commission for the Study and Improvement of Mathematics Teaching (CIEAEM). 2nd revised and expanded edition]. Bordeaux, France: IREM de Bordeaux. Retrieved December 31, 2017, from http://math.unipa.it/~grim/cieaem_files/CIEAEM_histoire_FLucienne_1985.pdf

Freudenthal, H. (1963). Enseignement des mathématiques modernes ou Enseignement moderne des mathématiques [Teaching modern mathematics or modern teaching of mathematics]. *L'Enseignement Mathématique, s. 2, 9*, 28–44.

Furinghetti, F. (2003). Mathematical instruction in an international perspective: The contribution of the journal *L'Enseignement Mathématique*. In D. Coray, F. Furinghetti, H. Gispert, B. R. Hodgson, & G. Schubring (Eds.), *One hundred years of L'Enseignement Mathématique* (Monographie 39 de *L'Enseignement Mathématique*), pp. 19–46. Genève, Switzerland: L'Enseignenement Mathématique.

Furinghetti F., & Giacardi, L. (to appear). ICMI in 1950s and 1960s: Reconstruction, settlement, and "revisiting mathematics education". In F. Furinghetti & L. Giacardi (Eds.), *The International Commission on Mathematical Instruction 1908–2008: People, events, and challenges in mathematics education*. Cham, Switzerland: Springer.

Gattegno, C., Servais, W., Castelnuovo, E., Nicolet, J. L., Fletcher, T. J., Motard, L., Campedelli, L., Biguenet, A., Peskette, J. W., & Puig Adam, P. (1958). *Le matériel pour l'enseignement des mathématiques* [Materials for the teaching of mathematics]. Neuchâtel, Switzerland: Delachaux et Niestlé.

Gattegno, C. (1988). Reflections on forty years of work on mathematics teaching. *For the Learning of Mathematics, 8*(3), 41–42.

Gerretsen, J. C. H., & de Groot, J. (Eds.). (1954–1957). *Proceedings of the International Congress of Mathematicians*. Groningen, The Netherlands: E. P. Noordhoff N. V./ Amsterdam, The Netherlands: North-Holland.

Goals for school mathematics. (1964). *The American Mathematical Monthly, 71*(2), 196–199.

Hammersley, J. M. (1968). On the enfeeblement of mathematical skills by modern mathematics and by similar soft intellectual trash in schools and universities. *Educational Studies in Mathematics*, *1*, 17. The full text is published in 1968, *Bulletin of the Institute of Mathematics and its Applications, 4*, 66–85.

Hodgson, B. R. (2008). *Interview with Geoffrey Howson*. Retrieved November 1, 2020, from https://www.icmihistory.unito.it/clips.php

Hodgson, B. R. (2009). ICMI in the post-Freudenthal era: Moments in the history of mathematics education from an international perspective. In K. Bjarnadóttir, F. Furinghetti, & G. Schubring (Eds.), *"Dig where you stand": Proceedings of the conference on "On-going research in the History of Mathematics Education"* (pp. 79–96). Reykjavik, Iceland: The University of Iceland.

Howson, A. G. (1984). Seventy-five years of the International Commission on Mathematical Instruction. *Educational Studies in Mathematics*, *15*, 75–93.

Hungarian National Commission for UNESCO. (1963). *Report on the work of the international symposium on school mathematics teaching*. Budapest, Hungary: Akadémiai Kiadó.

Jacobsen, E. (1996). International co-operation in mathematics education. In A. J. Bishop, K. Clements, C. Keitel, J. Kilpatrick, & C. Laborde (Eds.), *International handbook of mathematics education* (pp. 1235–1256). Dordrecht, The Netherlands/Boston, MA/London, United Kingdom: Kluwer.

Kemeny, J. G. (1964). Which subjects in modern mathematics and which applications of modern mathematics can find a place in programmes of secondary school instruction?. *L'Enseignement Mathématique, s. 2, 10*, 152–176.

Kilpatrick, J. (2012). The new math as an international phenomenon *ZDM—Mathematics Education, 44,* 563–571.

Kline, M. (1973). *Why Johnny can't add: The failure of the new math*. New York, NY: St. Martin's Press Random House.

Malaty, G. (1999). The third world mathematics education is a hope for the world mathematics education development in the 21st century. In A. Rogerson (Ed.), *Proceedings of the International Conference Mathematics Education into the 21st Century: Societal challenges, Issues and Approaches* (pp. 231–240). Cairo, Egypt: The mathematics education for the future project.

OECD. (1964). *Mathematics to-day. A guide for teachers*. Paris, France: OECD.
OEEC. (1961a). *New thinking in school mathematics*. Paris, France: OEEC. French edition: *Mathématiques nouvelles*. Paris, France: OECE.
OEEC. (1961b). *School mathematics in OEEC countries—Summaries*. Paris, France: OEEC.
OEEC. (1961c). *Synopses for modern secondary school mathematics*. Paris, France: OEEC.
Piaget, J., Beth, E. W., Dieudonné, J., Lichnerowicz, A., Choquet, G., & Gattegno, C. (1955). *L'enseignement des mathématiques* [The teaching of mathematics]. Neuchâtel, Switzerland: Delachaux et Niestlé.
Powell, A. B. (2007). Caleb Gattegno (1911–1988): A famous mathematics educator from Africa? *Revista Brasileira de História da Matemática, Especial n° 1–Festschrift Ubiratan D'Ambrosio,* 199–209.
R. G. (1964). Seminario matematico internazionale Villa Falconieri – Frascati, 8–10 ottobre 1964) [International mathematical seminar (Villa Falconieri – Frascati, October 8–10, 1964)]. *Archimede, 16*, 314–330.
Report of the Executive Committee to the national adhering organizations. Covering the period April 21, 1955–May 31, 1956. (1956). *Internationale Mathematische Nachrichten, 47–48*, 1–16.
Revuz, A. (1965). Pour l'enseignement de la géométrie, la route est tracée [For the teaching of geometry, the road is drawn]. *Mathematica & Paedagogia, 28*, 74–77.
Schubring, G. (2008). The origins and the early history of ICMI. *International Journal for the History of Mathematics Education, 3*(2), 3–33.
Schubring, G. (2014a). The original conclusions of the Royaumont Seminar 1959, edited and commented by Gert Schubring. *International Journal for the History of Mathematics Education, 9*(1), 89–101.
Schubring, G. (2014b). The road not taken—The failure of experimental pedagogy at the Royaumont Seminar 1959. *Journal für Mathematik-Didaktik, 35*(1), 159–171.
Steiner, H.-G. (1965). Internationales Kolloquium in Utrecht ober modernen mathematischen Unterricht an der höheren Schule [International colloquium in Utrecht on modern mathematical education at high school]. *Mathematisch-physikalische Semesterberichte, n. s., 12*(1), 127–128.
Stone, M. H. (1961). Reform in school mathematics. In OEEC, *New thinking in school mathematics* (pp. 14–29). Paris, France: OEEC.
Stone, M. H. (1963). Le choix d'axiomes pour la géométrie à l'école [The choice of axioms for geometry at school]. *L'Enseignement Mathématique, s. 2, 9*, 45–55.
Straszewicz, S. (1964). Relations entre l'arithmétique et l'algèbre dans l'enseignement des mathématiques pour les enfants jusqu'à l'âge de quinze ans [Connections between arithmetic and algebra in the mathematical instruction of children up to the age of 15]. *L'Enseignement Mathématique, s. 2, 10*, 271–293.
Teodorescu, N. (Ed.). (1968). *Colloque International UNESCO. Modernization of mathematics teaching in European countries.* Bucharest, Romania: Editions didactiques et pédagogiques.
Thom, R. (1973). Modern mathematics: Does it exist? In A. G. Howson (Ed.), *Developments in mathematical education. Proceedings of the Second International Congress on Mathematical Education* (pp. 194–209). Cambridge, United Kingdom: University Press.
Todd, J. A. (Ed.). (1960). *Proceedings of the International Congress of Mathematicians*. Cambridge, United Kingdom: Cambridge University Press.
UNESCO. (1967). *New trends in mathematics teaching—Tendances nouvelles de L'enseignement des mathématiques. Vol. 1*. Paris, France: UNESCO.
Vanhamme, J. (1991). Un peu d'histoire/A short historical background. In A. Warbecq (Ed.), *Proceedings of the 41st CIEAEM Meeting* (*Role and conception of mathematics curricula*, Bruxelles, 1989) (pp. V–XV). Bruxelles: Le Comité organisateur.
Vanpaemel, G., & De Bock, D. (2019). New Math, an international movement? In E. Barbin, U. T. Jankvist, T. H. Kjeldsen, B. Smestad, & C. Tzanakis (Eds.), *Proceedings of the Eighth European Summer University on History and Epistemology in Mathematics Education* (pp. 801–812). Oslo, Norway: Oslo Metropolitan University.
Weaver, J. F. (1965). African mathematics program *The Arithmetic Teacher, 12,* 472–480.
Williams, G. A. A. (1976). The development of a modern mathematics curriculum in Africa. *The Arithmetic Teacher, 23,* 254–261.
Wittenberg, A. (1965). Priorities and responsibilities in the reform of mathematical education: An essay in educational meta-theory. *L'Enseignement Mathématique, s. 2, 11,* 287–308.