Chapter 23
Modern Mathematics: An International Movement, the Experience of Morocco

Ezzaim Laabid

Abstract The implementation of modern mathematics in Morocco occurred in three main phases, from the early 1960s through the mid-1970s. In the first phase, the implementation mainly concerned the introduction of new vocabulary and symbols of set theory and algebraic structures into upper secondary education. In the second phase, which began in 1968, the latter notions were introduced in lower secondary education and were reinforced in upper secondary education by vector, affine, and analytical geometries. In the third phase, which began in 1971, there was a strengthening of modern mathematics by a restructuring of the content of the programs and by removing classical concepts of geometry from them. But, in 1975, the teaching of mathematics—which elicited a negative reaction from users of mathematics—was the subject of a national conference whose recommendations advocated changes in the mathematics programs taught in secondary schools so that they would be beneficial to the majority of students. Then, in 1978, the Minister of Education created a commission to discuss mathematics programs and made the necessary changes. Modern mathematics was abandoned gradually from 1983 to 1989. In this chapter, we describe the different stages and the peculiarities of implementation of modern mathematics in Morocco.

Keywords Affine geometry · Algebraic structure · Analytical geometry · Binary relation · Classical geometry · Concept of mapping · Curriculum · Equivalence class · First secondary cycle · France · Internal law of composition · Logical symbols · Modern mathematics · Morocco · Order relation · Second secondary cycle · Set theory · Textbook · Vector geometry

Introduction: Characterization of Modern Mathematics

After the late 1950s, and especially in the 1960s and 1970s, the teaching of mathematics underwent a major reform in several countries. This renovation revolved mainly around the introduction of "modern mathematics" in pre-university education. The reform brought a vision of mathematics that should be taught in high school. This new perception of mathematics was well explained in the official

E. Laabid (✉)
Cady Ayyad University, Marrakesh, Morocco
e-mail: ez.laabid@uca.ac.ma

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documents presenting the reform. In Morocco, for example, the *Official Programs and Instructions* published in 1973[1] presented this "new" mathematics as follows:

> It is important first to have a precise idea of what has become of mathematics. A few years ago, school curricula provided our pupils with a conception of mathematics from the ancient Greeks and the mid-nineteenth century. After about a hundred years of profound evolution, the researches and discoveries of mathematicians, and their reflections on the reality and the object of mathematics, have led to new ideas expressed in a new language. Current mathematics is no longer exclusively, nor is it essentially, what Augustus called "the science of the indirect measurement of magnitudes." (MEN 1973, p. 7)

The document also stated that the new mathematics gave more importance to the relation between objects than to the objects themselves, making of set theory and algebraic structures central topics of the curricula.

> Any "calculation," in the most general sense, has two main components: The objects on which one operates and the operative rules. Of these two constituents, only the latter is essential. Mathematics appears as the science of relations and systems of relations, the nature of the objects to which these relations apply becoming irrelevant. […] Set theory, introduced symbolism, detailed and rigorous study of structures have provided mathematicians today with a secure and flexible language; they gave them habits of rigor and conciseness often unknown to their predecessors. Finally, the study of a structure, based solely on a small number of axioms unrelated to the objects studied, is an exercise of pure reasoning. It teaches to order and enchain thoughts according to a rigorous method, admirably developing clarity of mind and rigor of judgment. (MEN 1973, p. 7)

Another element that characterized this new vision of mathematics was the abandonment of traditional geometry and its replacement by vector and affine geometries. Arguments of famous mathematicians were cited to justify this change. One of these arguments—as reported by MEN (1973)—is that of the French mathematician Jean Dieudonné:

> It seems to me that the goal is to overcome two definite psychological difficulties:
>
> 1. The student must be made aware of the necessity of an axiomatic treatment of mathematics.
>
> 2. The student must be familiarized as soon as possible with the constant handling of certain abstract notions, the most difficult to assimilate being undoubtedly that of mapping (or "transformation") and even more perhaps that of calculation on mappings.
>
> As it can be said without exaggeration that either difficulties are truly cornerstones of the entire modern mathematical edifice, all the other aspects of teaching in the first years should be consciously subordinated to the assimilation of these ideas. ... It is thus desirable to free the pupil as soon as possible from the straitjacket of traditional "figures" (point, line, plane being an exception of course) by mentioning them as little as possible, in favor of the idea of a geometric transformation of the whole plane and of space, which must be constantly emphasized and illustrated by numerous examples. (MEN 1973, pp. 121–122)

In addition, it was believed that this new conception of mathematics would be an effective means to train students intellectually, consequently achieving one of the fundamental aims of mathematical education.

> This conception leads to an exceptional economy of thought, by standardizing the mathematical tool. It created an instrument so general and powerful that it now applies to all activities of the mind, for example, to the social sciences. … Considering the essence of the mathematical method as the main aim of mathematical education, this novel approach helps to achieve intellectual training more efficiently than any other method. (MEN 1973, p. 7)

[1] We refer hereinafter to this document by MEN, an acronym for *Ministère de l'Éducation Nationale* [Ministry of National Education]. We use the second edition of 1973. The first edition was published in 1969; there were other editions in 1976 and 1979. There are no notable differences between these editions.

Implementation of Modern Mathematics in Morocco: Main Stages

When Morocco became an independent nation in 1956, mathematics education was based on French curricula, French books, and the majority of teachers and inspectors were French. The inspectors, one of whose tasks was curriculum development, followed the debates about modern mathematics and its introduction into secondary school (El Mossadeq 1989). It seems that some of these inspectors were so influenced by the stream of modern mathematics that they had to try it out in real contexts. At least that is what the following quote, from Mohammed Akkar, the former director of the national education ministry in the 1970s and university professor of mathematics, suggested.

> The needs of higher education and the concern of the public opinion for the growing gap between the evolution of science and that of education led to this reform in France. In Morocco, it was a decision of the head of programs and mathematics education, J. P. Nuss, who was strongly influenced by the ideas and movements circulating in France and Belgium at the time. The "modernization" of the programs was carried out in Morocco in two main stages: One in 1962 by Nuss and the other in 1968 by Peureux. (Akkar 2002, p. 180)

This quotation requires some clarification. Regarding the beginning of modern mathematics in Morocco, an excerpt coming from MEN (1973) traced the reform in the aftermath of independence without, however, giving a date:

> Many countries indulged in the renovation of mathematical education. The reform began in Morocco in the aftermath of independence. Our methods and programs follow a normal evolution adapted to the realities of our teaching. (p. 8)[2]

As for the stages of the implementation of modern mathematics, it seems that this was in fact a process that did not end until 1971. Starting from this period, the programs were formalized to the extreme. Several changes and corrections were made to the programs throughout the 1960s. El Mossadeq (1989) stated in this regard:

> Since 1962, the programs have undergone several changes, roughly every year there has been a certain dose of additions until 1971 when we arrived at extremely formalized programs. (p. 100)

The initiators of the implementation mentioned in the initial quotation were Jean Paul Nuss and Yves Peureux. Nuss had been ranked twenty-first in the Aggregation Exam[3] in 1939[4] and was a teacher at Lyautey high school in Casablanca, then an inspector with the Moroccan national education ministry[5]. Peureux was an inspector with the Ministry of National Education[6] who, in the late 1960s and early 1970s, led the publication of mathematics textbooks for the first and second cycle (students aged

[2] In fact, according to some oral testimonies, modern mathematics was indeed introduced into the second cycle well before 1962. The latter was said to be the year of the official introduction of modern mathematics into the curricula.

[3] This exam is the highest level competitive exam for teachers in France.

[4] See http://rhe.ish-lyon.cnrs.fr/?q=agregsecondaire_laureats & nom= & annee_op =%3 D & annee%5Bvalue%5D= & annee%5 Bmin%5D= & annee%5Bmax%5D= & periode=4 & concours=13 & items_per_page=100 & page=12 (retrieved February 15, 2021).

[5] According to some sources, Nuss also gave courses at the Faculty of Science of the University of Rabat. More precisely, this course concerned: Mathematics course of Propaedeutics General Mathematics and Physics MGP (M. Akkar, personal communication). He was the one who developed the first Moroccan mathematics programs for the second cycle of secondary education (El Mossadeq 2007). Nuss was credited with the first Moroccan textbooks containing elements of modern mathematics (Moatassime 1978). It seems, however, that these books, which were not marketed, were intended primarily for teachers (M. Akkar, personal communication). When he returned to France, he wrote a textbook for the third year of secondary education in the Queysanne collection (Nuss 1968). Note that we have not yet managed to find copies of Moroccan textbooks by Nuss.

[6] Initially, Yves Peureux was a technical college teacher seconded to Morocco in 1950 (*Bulletin Officiel de la République Française* [Official Bulletin of the French Republic], September 1951, p. 9575). Probably when he returned to France, Yves Peureux was appointed inspector of mathematics for technical education in 1979, at the Academy of Dijon (Bottin administratif et documentaire [Administrative and documentary directory], 1979, p. 436).

12–15 and 15–18, respectively). In fact, his books, especially those for lower secondary education, were widely used during the first half of the 1970s (Aiouch et al. 1969; Fabre et al. 1969; Mercier and Mercier 1970; Mercier and Peureux 1970) (see Figure 23.1).

The initiative to introduce modern mathematics into Moroccan curricula was, in all likelihood, implicitly encouraged by a favorable context. Initially, curriculum reform affected only the second cycle of secondary education. Usually at that time, only the best students entered this cycle. The

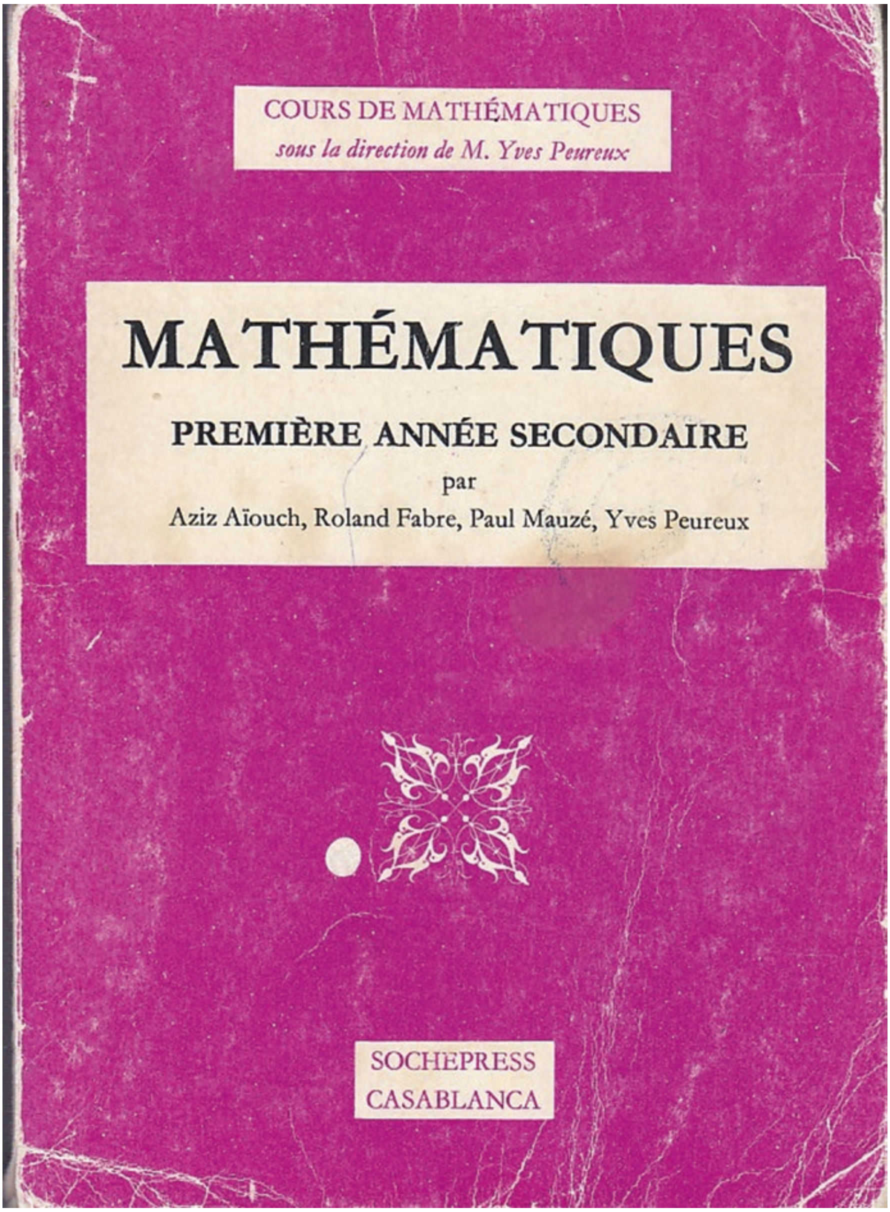

Figure 23.1 Cover of Aiouch et al. (1969), a textbook for the first year of secondary education (author's collection)

journey of the other students ended at the end of the first cycle[7]. Therefore, the number of students in the second cycle was not very large, but nearly all of them were performing well in mathematics. El Mossadeq (2007) states:

> Morocco is among the countries that adopted modern mathematics very early. Students have achieved good results from these programs. The first Moroccan baccalaureate exam was organized in 1963 (1050 candidates, including 100 in the mathematical section. For this section, there was one class in each of the cities: Marrakech, Fez, Rabat, Casablanca). (p. 27)

It follows from the above that we can distinguish three phases in the implementation of modern mathematics in Moroccan secondary education[8]: The introduction officially began in 1962 in the second cycle, was extended to the first cycle in 1968, and reached its peak in 1971[9].

In this section, we present an overview of the modern mathematics introduced in each of the phases and attempt to explain the approach adopted for this implementation. We use the 3rd- and 4th-year programs and the 5th-year science programs to highlight the differences between each of the phases and thus to identify the progress in the implementation of the programs[10]. Comments related to other levels will provide the necessary additions.

Description of the Implementation Process

We assume below that the presence of modern mathematics in a program can be characterized by the presence of one or more of the following themes: Vocabulary and logical symbols; vocabulary relating to sets; binary relations; concept of mapping; internal composition laws and algebraic structures; analytical, vector, and/or affine geometry.

Stage 1: The Reform of 1962 The descriptions of the programs developed at the time shows that a chapter in the three years of the second cycle introduced the basic notions of set theory, vocabulary and logical symbols, binary relations, internal laws of composition, and their properties. The program of the 5th year did not explicitly mention the following structures: Group, ring, and field. However, they were introduced in the mathematics programs of the 6th and 7th years (mathematics section). In fact, these programs did not consider these topics as subject of study. They only gave the name of the structures after presenting their properties when studying classical sets (N,Z,Q,R, etc.). In this sense, they spoke of the ring of relative integers, the field of rational numbers, the field of real numbers, the field of complex numbers, the ring of polynomials, etc. The structure of the vector space was not

[7] At the time, there were two types of secondary education in Morocco. A short one that ended at the end of first cycle of secondary education and led to the certificate of secondary education, and a long one that led to the baccalaureate.

[8] Primary education was influenced by the wave of modern mathematics in an indirect way during the 1970s. This influence was manifested by the introduction of set language: Teaching numbers from a collection of objects and emphasizing educational intent on understanding concepts and corresponding properties instead of focusing on numeracy skills (Lakramti 2001, p. 3).

[9] The 1962 reform was carried out all at once, i.e., the programs of the three levels were changed in the same year. But for the other reforms, content was changed gradually, from year to year (Aboutir 1994, p. 12).

[10] During the period covered by this study (1962–1989), secondary education (first and second cycles) was generally made up of seven levels (four in the first cycle and three in the second cycle). Note, however, that the names given to these levels were not the same throughout this period. Between 1963 and 1972, these were for the 1st cycle (observation class, 1st year, 2nd year, and 3rd year) and for the second cycle (4th year, 5th year, and 6th year). Between 1973 and 1989, the names of the levels were: 1st year, 2nd year, 3rd year, and 4th year (for the first cycle) and 5th year, 6th year, and 7th year (for the second cycle). In this chapter we refer to these levels by the latter names. Moreover, students who entered to the second cycle were directed to one of the sections: Science, letters, or techniques. Each of these sections was divided, in the last two years of the *lycée*, into subsections. For example, for the science section, the subsections were as follows: Mathematics, mathematics and technics, experimental sciences, and economics.

referred to in the sections concerning vector algebra in the 6th and 7th years, although some of its properties were presented there (Aboutir 1994).

However, several themes studied in the programs preceding this period still had their place in the 1962 programs. For example, in addition to the elements listed above, there were several themes of classical geometry in the 5th-year program, such as fundamental axioms, segments on a line, vectors, and equal figures in space. Likewise, polyhedra, surfaces, solids, and descriptive geometry were still subjects of study in the programs of mathematics sections (6th and 7th year). The properties of the operations defined on the studied objects used the language of sets and the properties of internal composition laws. For example, it was stated that the addition defined on the segments of a line was commutative and associative and that the multiplication of a vector by a number is distributive with respect to the addition of vectors.

The approach used for the introduction of modern mathematics in Moroccan curricula was, in fact, in line with the approach of certain French textbooks of the time (see, for example, Cossart and Théron 1964; Girard and Lentin 1964; Lebossé and Hémery 1964). The latter had taken advantage, in all likelihood, of a few passages that appeared from the instructions accompanying the French programs of 1960. These passages seemed to tolerate the use of certain notions of modern mathematics, in particular, to synthesize. One of these textbooks, used in Morocco at the time, reported the following extract from the Instructions of July 19, 1960:

> The wording of the program does not explicitly mention certain simple notions about sets, nor the vocabulary currently accepted to designate them: Union, intersection, complementary sets, inclusion, belonging, etc. There is no question of banning their use; they are in fact encountered very frequently. In most theories, it is advisable to identify them little by little, to make them known, then to define them, starting from numerous examples in which they occur naturally. Their interest will thus appear through the applications that can be made of them, by the simplification or clarification that they are likely to bring in a research or in a presentation. Other notions, such as those relating to the structures of sets: groups, rings, fields can also be introduced. … They can facilitate the presentation of certain syntheses and allow useful comparisons for the future. (Cossart and Théron 1964, p. 6)

We can read in the preface of this textbook:

> This textbook has four parts, the last three of which are to some extent independent of each other[11]. The first part is intended to acquire the fundamental knowledge[12]. The notions that the program proposes to bring out are essentially the notion of set (equality, inclusion), that of relation, more particularly the relation of equivalence on the one hand, of function or mapping on the other, and those of laws of composition or group. (Cossart and Théron 1964, p. 3)

In more details, this textbook has introduced in Chapter 1 the following elements: Sets; the elements of reasoning; study of sets in plane geometry (sets of points, other sets, construction problems); relations between elements of two sets; relations between elements of the same set (generalities, equivalence relation, order relation); and laws of composition (generalities, properties of certain internal laws, various examples of laws of composition) (Cossart and Théron 1964). Therefore, it is most likely that the initiators of this reform took advantage of the relatively favorable Moroccan context (selective secondary education, small number of students, good results) to formalize what was in the making in France.

Stage 2: The Reform of 1968 In this reform, the introduction of modern mathematics affected both the first cycle and the second cycle of secondary education. For the first cycle, the descriptions of the programs and their implementation in the textbooks show that there were two levels of implementation of modern mathematics. The introduction of general notions related to set theory, binary relations, mappings, internal laws of composition, and their properties represent the explicit level. In the first two years of secondary school, these themes were introduced gradually. The first-year program

[11] The other three parts are devoted respectively to Geometry of space; equations and inequalities; and functions, numerical functions, and point transformations.

[12] This knowledge is spread over three chapters. “General notions” are dealt with in Chapter 1 (see below); “real numbers” and “vectors” are presented in Chapters 2 and 3, respectively.

introduces some elements of the vocabulary of sets (belonging, equal elements, equal sets, empty set, disjoint sets, inclusion, intersection, union), while the second-year program takes the same notions and enriches them with the notions of mapping and bijection. Table 23.1 presents an overview of the concepts mentioned and highlights their progression in the 3rd and 4th years.

The use of vocabulary related to sets in formulating the definitions of the arithmetic and geometric concepts studied was an aspect of the implicit level of implementation of modern mathematics. Here are some examples for arithmetic. The least common multiple of a, b is the smallest element of the intersection of the sets of multiples of $a$and b. The greatest common divisor of a, b is the greatest element of the intersection of the sets of the divisors of a and b. A number is a prime number if the set of its divisors is one pair (Aiouch et al. 1969, see Figure 23.1). In geometry, there is a tendency to consider basic geometric concepts (straight line, segment, half-line) as sets of points[13] and to represent the geometric figures studied using their vertices[14]. Another manifestation of this implicit aspect was the incentive to use the reciprocal image of a mapping to express the set of solutions of a first-degree equation in the fourth year.

> Let A be a part of R (which can be R) and f the mapping of A into R, such that $f(x) = ax + b$ (a, b given real numbers). Solving the equation $f(x) = d$ (d real given) is to determine the set of elements of A which admit d for image $f^{-1}(d)$ (MEN 1973, p. 131)

In fact, the principle that framed the introduction of modern mathematics in the first-cycle programs can be formulated as follows: “The concepts will not be defined a priori, but their introduction will come following the study of the examples.” The program referred to this principle by saying “to identify the notion when studying examples.” This principle was further explained in the following quotation, taken from the instructions for the first year:

Table 23.1
Progression of the introduction of the elements of modern mathematics between the 3rd and the 4th year, reform of 1968

Theme	Third year	Fourth year
Vocabulary related to sets	Belonging, inclusion, intersection, union, equality of two sets, complement to a part of a set, Cartesian product	
Vocabulary and logical symbols	Implication	Implication, equivalence, negation, conjunction, disjunction
Binary relation	Binary relation from one set to another (vocabulary, diagrams); relation in a set, properties, equivalence relation, examples of relations	
		Equivalence class, partition, order relation
Mapping	Mapping from one set to another (definition, notations), mapping in the same set, examples, bijection (definition)	Mappings, functions, numerical functions, injection, bijection, surjection, composition of mappings, reciprocal bijection
Internal composition laws and structures	Properties introduced during the study of operations on sets Z, Q (and R for the 4th year)	
“Analytical” geometry	Image of Z, Q (and R for 4th year) on a line; bipoints, notion of axis, algebraic measure of a bipoint, abscissa of a point, distance between two rational abscissa points (and real abscissa for the 4th year)	

[13] For example, the second-year textbook gave the following definitions for a half-line and for a segment:

(1) Let D be a straight line and A a point of D. A determines two half-lines d_1 and d_2 of common origin A such that $d_1 \subset D$ and $d_2 \subset D$; $d_1 \cap d_2 = \{A\}$ and $d_1 \cup d_2 = D$.

(2) We call a line segment, with ends A, B, noted $[A, B]$, the non-empty intersection of the two half-lines d_1 (of origin A) and d_2 of origin B); $d_1 \cap d_2 = [A, B]$ (Aiouch et al. 1969, pp. 227, 229).

[14] One can read in the instructions for the third year in a passage entitled “commentary on geometry”: A parallelogram whose vertices are A, B, C, D is the quadruplet (A, B, C, D) (MEN 1973, p. 60).

> The teacher will not feel obliged to "define" the meaning of words such as "objects"; "sets"; "belonging" which are primary notions. It is through their use that the mathematical content of each of these words will gradually emerge. The concepts of inclusion, intersection, union will be introduced, at the beginning of the year, quickly on sets defined in extension (i.e., finite sets), and using belonging tables. The systematic use of these concepts throughout the year, both in arithmetic and in geometry, will show their importance. The sets encountered are no longer necessarily finite. (MEN 1979, p. 33)

This principle was also particularly evident in the use of the vocabulary of internal laws of composition (commutativity, associativity, neutral element, etc.) when studying the addition and multiplication operations on sets of numbers.

For the second cycle, the programs of 1968 reinforced the choices of 1962. Since the first cycle introduced the basic notions, the programs of the second cycle emphasized the vocabulary and the symbols of mathematical logic. Also, the algebraic structures—mentioned in the previous programs when appropriate situations arose—were now studied in an abstract way and enriched by the concept of vector space and its applications in geometry (vector, affine, and analytical geometries). These concepts occupied a large part of the curricula at all levels of second-cycle secondary education. In general, in each year some of these concepts were introduced into the academic programs. The general structures (group, ring, field, vector space, and affine space) were introduced in 5th-year science and deepened in 7th-year mathematical science. The 6th-year mathematical science was devoted to the study of vector and analytic geometries. For the other branches (experimental sciences, economics), the mathematics programs were made in the same spirit of modern mathematics, but lightened of abstract concepts and enriched with exercises favoring the technical rather than the theoretical aspect.

A reading in the curriculum for the 5th year of science shows that modern mathematics occupied an important place. Besides the sections devoted to general notions (vocabulary of sets, logical symbols, notions of mappings and binary relations) and to algebraic structures (group, ring, field, vector space), other sections were dedicated to the study of what can be called fundamental examples of structures. These are the sets of numbers Z, Q, and R endowed with the usual operations, the set of parts of a set endowed with the operations Δ and $\cap$, the set of polynomials, and the set of functions with their usual laws (addition and multiplication) which, as appropriate, were presented as examples of groups, rings, or fields.

As for geometry, we note that the notions of classical geometry disappeared from the program of the 5th-year science. They were replaced by vector geometry, affine geometry, and analytical geometry. For vector geometry, the topics studied were the parametric representation of a vector line determined by a point and a directing vector and the vector plane determined by a point and two linearly independent vectors. For analytical geometry, the topics were the affine coordinate system, coordinates of a point, coordinates of the barycenter of two, three, and four points. For affine geometry, the topics were the parametric representations of a straight line and the affine plane deduced from vector representations. We notice that there was a tendency to express geometric properties analytically. Table 23.2 presents the main lines of the program for the 5th science year and shows the place occupied by modern mathematics, according to the 1968 reform.

The new vision of mathematics advocated in modern mathematics and explained in the Introduction of this chapter was present in the programs of the second cycle of 1968. In 1962, the algebraic structures were "deduced" and named following the study of examples. In 1968, these structures constituted the object of study. They were first studied for themselves and then illustrated using examples. Thus, there was a reversal of the 1962 approach. Also, when studying classical notions, the emphasis was placed on the fact that these notions were elements of an algebraic structure. The study of their intrinsic properties as mathematical objects took a back seat. For example, when studying polynomials, what came to the fore was the fact that the set of polynomials and associated operations formed a commutative, unitary, integral ring. The other aspects linked to the classical study of polynomials (divisibility, roots, factorization, etc.) seemed to be now of secondary interest.

Stage 3: The Reform of 1971 The programs of the early 1970s consolidated and reinforced the orientations taken by the programs of the 1960s. Reading them shows that they were aimed at highlighting the modern aspect of the curriculum by placing more emphasis on topics related to modern

Table 23.2
Elements of the program of the 5th-year science, reform of 1968

Theme	Notions studied
Vocabulary and logical symbols	Logical conjunction, disjunction, implication, equivalence, reciprocal, quantifiers, negation (on examples), reasoning by recurrence, reduction to the absurd
Vocabulary related to sets	Sets, elements, belonging, equality, subsets, inclusion, union, intersection, empty set, disjoint sets, complementary sets, Cartesian product of two sets
Binary relation	Equivalence relation, equivalence class, quotient set, relation of order, congruence on *Z*, *Z*/*nZ* set of integers modulo *n*
Concept of mapping	Mapping of a set in (or on) a set, equality of two mappings, injective mapping, surjective mapping, one-to-one mapping, reciprocal mapping, composition of two mappings, concept of function
Internal composition laws and structures	Group, ring, field, homomorphism, isomorphism, relation compatible with a law, examples ($\mathcal{P}$ (E), Δ, ∩), *N*, *Z*, *Q*, *R*, provided, respectively, with the usual laws; the set of functions, the set of polynomials with real coefficients, concept of vector space
Geometry	Affine space attached to a real vector space: Points, bipoints, equipollent bipoints. Affine coordinate system (straight line, plane, usual space): Coordinates of a point, components of a vector associated with a bipoint. Vector parametric representation of a line determined by a point and a directing vector, of the plane determined by a point and two linearly independent vectors. Parametric representations deduced from vector representations of the line and the plane. Barycenter of two, three, and four points. Coordinates of the barycenter.

mathematics. In the first cycle, the programs of the 3rd and 4th year explicitly introduced the laws of internal composition. In addition to the use of certain symbols and a logical vocabulary, the notions of binary relation and mapping were developed. The explicit study of internal composition laws began from the 3rd year when definitions, notations, tables, and all properties were introduced (commutativity, associativity, neutral element, symmetrical elements) by studying many examples. In the 4th year, these notions were supplemented by the notions of regular element and the distributivity of one law in relation to another. A synthesis summarizing the properties of the usual laws defined on the sets of numbers (*N*, *Z*, *Q*, *R*) was recommended, as well as the use of the term "group."

For the second cycle, the 1971 programs do not seem to have made any significant changes in terms of mathematical content compared to those of 1968. The change was essentially to make the presentation more rigorous and to adopt as much as possible the "general to particular" approach in presenting concepts. For example, in the 5th-year science program, notions of logic were treated in an abstract manner and were enriched by the study of various types of reasoning (deduction, successive deductions, contraposition, recurrence, disjunction of cases, reduction to absurdity). Notions related to sets were presented as illustrations of general notions of logic. The operations on sets (intersection, union, and symmetric difference) were considered as operations in the ring ($\mathcal{P}$ (E), Δ, ∩). In addition, in the chapter on equations, where the learning outcome was to solve first- and second-degree equations, the program recommended first posing the equation problem in general terms. The textbook for the 5th year broke down the beginning of this chapter as follows: I. General: 1. Concept of equation—examples, a) Concept of equation, b) Equation and classification of applications; 2. Various generalizations of the concept of equation, a) Equations with several unknowns, b) Systems of equations, c) Generalization of the concept of equation[15] (MEN 1976a).

[15] This general introduction is followed by the following sections: II. First-degree equations and systems of equations; III. First-degree inequalities and systems of inequalities.

Notes and Comments

The programs included instructions[16] to guide the teachers by giving them details about the key concepts and indicating the general orientation of the program. Several passages in the instructions highlighted the benefits that secondary education may derive from the introduction of modern mathematics. The recommendations were intended to show the importance of certain concepts. These passages could be grouped into two categories. The passages in the first category related to the advantages of using the language of modern mathematics, the unifying character of general notions, and the importance of certain particular notions. Those in the second category concerned the relevance of using vector concepts for the study of geometry and the role that the latter can play in an initiation to mathematical reasoning. We present in this section some of these passages.

The first passage quoted listed the advantages of the language of set theory:

> The language of this theory is the best fit for mathematics. It allows a clear, precise, and simple expression, better and simpler than ordinary language. The teacher, having become aware of the vagueness of the traditional mathematical language, will make a very wide use of the "new" vocabulary thus placed at his disposal. Certainly, he will retain, with regret, certain words consecrated by custom. (MEN 1973, p. 118)

A second passage encouraged the use of the precise vocabulary introduced by modern mathematics in geometry instead of the approximate one of ancient mathematics, starting in the 3rd year.

> It is important to improve the mathematical language used in previous classes while reducing the number of "learned" words as much as possible. The students like to remember, to use the complicated words whose deep meaning escapes them. We will replace the too approximate vocabulary (figure, cut, slide, etc.) by a precise "modern" vocabulary (set, union, intersection, inclusion, etc.). (MEN 1973, pp. 129–130)

A third passage considered that the general notions constituted the most important point of the program of the 4th year and that in particular, the internal laws of composition should retain the full attention of the teacher.

> The 4th-year program is characterized by the development or introduction of essential general concepts: Binary relations, equivalence relations, quotient sets, order relations, mappings, internal composition laws[17]. The study of this first chapter is of paramount importance. You should set aside enough time for it and offer lots of exercises throughout the year (MEN 1973, p. 131)

As for the unifying nature of certain concepts, we mention a passage from the official instructions for the second year, which explained the sense of introducing the concept of isometry (bijection that preserves distances) from the beginning of the first secondary cycle. It explained in particular that this notion may exempt the student from remembering the names of several geometric transformations.

> The geometry chapter is of considerable importance. Students invited to do many manipulations (use of the compass, tracing paper, etc.) are naturally led to the notion of bijection and to the isometries of the plane. They do not have to remember the different names of these isometries: Translation, rotation, etc. They only need to realize experimentally that there are bijections of the plane that "keep" the distance. (MEN 1973, pp. 120–121)

[16] The instructions appeared with the 1968 programs.

[17] The document adds in parentheses: "This last point should retain all the attention of the teacher." It is probably to justify the importance of this theme that an exercise textbook (with solutions) was compiled on the internal laws of composition. This textbook, consisting of two booklets, one of which contains the statements of the exercises and the other is devoted to solutions and indications, contains more than 500 exercises spread over 8 chapters (I to VIII). The distribution is as follows: I. Internal composition laws (133 exercises); II. Associativity (78 exercises); III. Neutral element (42 exercises); VI. Symmetrical elements (66 exercises); V. Regular elements (29 exercises); VI. Commutativity (59 exercises); VII. Distributivity (19 exercises); VIII. Group concept (75 exercises) (El Mossadeq and Peureux 1971, see Figure 23.2).

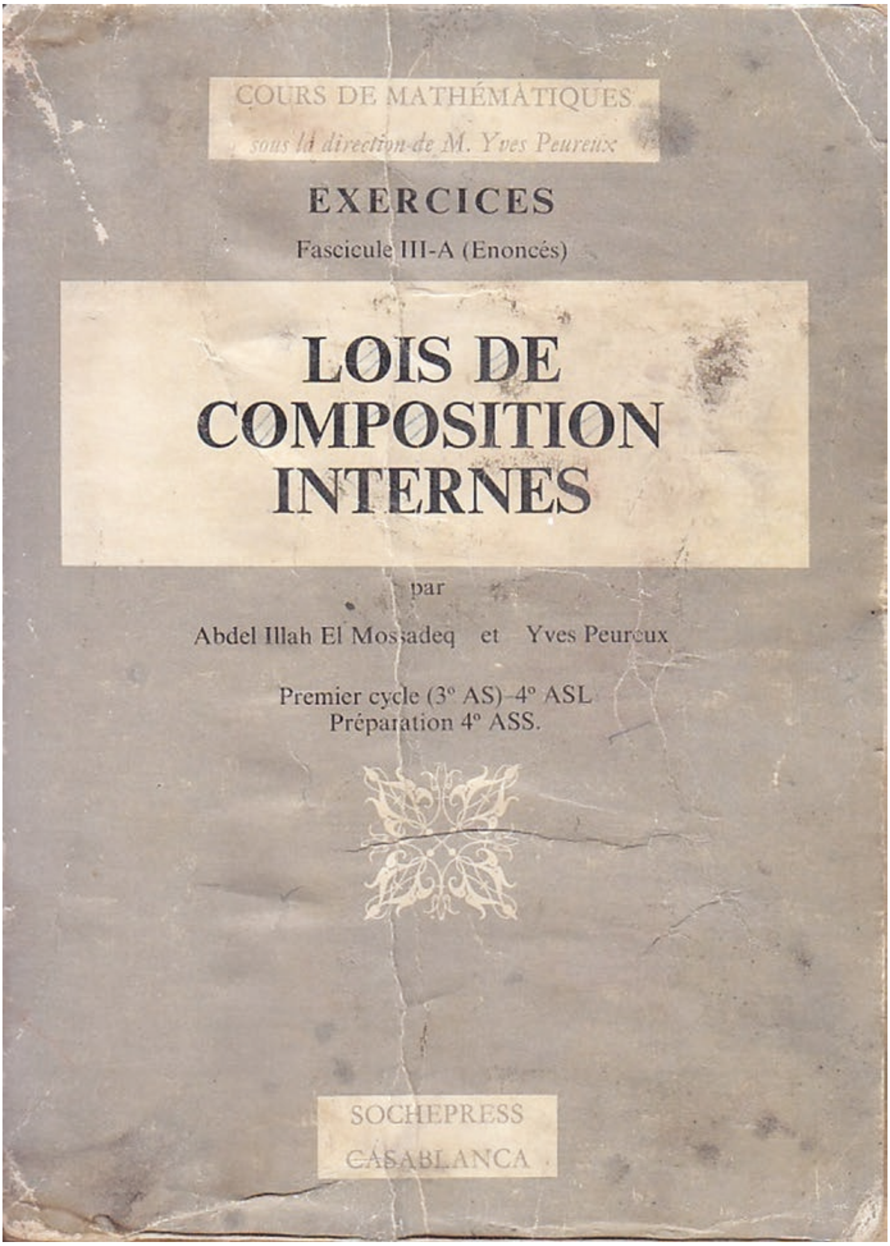

Figure 23.2 Cover of El Mossadeq & Peureux (1971), a textbook with exercises on internal laws of composition intended to prepare students for the second cycle of secondary education (author's collection)

On the other hand, the relevance of studying geometry using vector concepts was emphasized. The idea of starting with metric notions as a transitional step toward vector notions was raised in a reflection by the French mathematician Gustave Choquet about the teaching of geometry[18]:

[18] In fact, this quote is made up of two passages appearing in different pages of Choquet's book (1964). The first passage "It is well established ... in textbooks" appears on page 10 (in the introduction). The second passage "some teachers ... with metrics" appears on page 153 (in Appendix 1).

> It is well established that Euclid's "axiomatics" no longer meets our logical demands; the same can be said of many "axiomatics" found in textbooks. Some teachers think that metric notions are more intuitive and more easily understood by a young student than vector notions. …[19] and only prolonged experimentation with various methods can lead to a conclusion. However, it might be better to start the development of geometry with metrics. (MEN 1973, p. 123)

One of the aspects of the expected success of these reforms was to lead the student, from the start of secondary school, to be no longer satisfied with the observations of properties on examples or following experimental verifications, but to be aware of the need for proofs. The official instructions of the 2nd-year secondary class stated in this sense:

> The student must conceive that certain properties are admitted after observation and experimental verification, and that others are established and proved from the previous ones. Success will be achieved when the student who became aware of the difference between the simple observation of a fact … and its proof, will not be satisfied with the first and will demand the second. (MEN 1973, p. 119)

One of the roles assigned to undergraduate geometry education is the initiation into reasoning. The latter will be achieved by a gradual transition from manipulative geometry to demonstrative geometry. This transition should be framed by a balance between practical manipulations and reasoning in the teaching of geometry in the first cycle. Two quotations further explain this. The first recommended:

> The study of each of the chapters of the geometry program in this class is accompanied by practical work intended to present a concept, illustrate a definition, or suggest a property. … The use of essential instruments—ruler, compass, set square, protractor, etc.—should not be the occasion for a session of pure manual work. On each occasion, the teacher will force the student to observe, to ask questions, to reason. (MEN 1973, p. 121)

As for the second quotation, it recommended:

> What should be assimilated as soon as possible is that from a proposition admitted for any reason whatsoever, and whose provenance is not taken into account, other propositions can be derived only by reasoning. In short, the teaching in the first secondary years should be a skillfully measured mixture of well-chosen "geometric experiments" and partial reasoning on the results of these experiments; something analogous to learning physics. (MEN 1973, p. 122)

Textbooks Accompanying the Implementation of Modern Mathematics

As we mentioned above, at the time of Morocco's independence, the teaching of mathematics was based on French textbooks. This situation did not change much in practice over the next decade, even though the first Moroccan mathematics textbooks were produced in the early 1960s, when modern mathematics was implemented in the second cycle of secondary education. French textbooks continued to be used especially as some of these textbooks, such as the one by Cossart and Théron (1964) mentioned above, introduced some notions of modern mathematics into their content at the same time. In fact, the implementation of modern mathematics in Morocco was accompanied by three attempts to produce textbooks: The first in 1961, the second in 1968, and the third in 1975. About the first and second attempt, Moatassime (1978) reported:

> The first, in 1961, was conducted by J. P. Nuss. It was made for the second cycle: In fact, we started by reforming the program of the second cycle by introducing so-called "modern" mathematics. The printing was made in Morocco (in Mohammedia). Then, in 1969, the Y. Peureux collection was adopted for the first and second cycle, in the meantime the program for the first cycle had also been modified. The purpose of the books was to retrain the teachers and at the same time serve as textbooks for the students! … The authors were all French. The printing was carried out in Morocco, in Tangier. (p. 51)

[19] The omitted sentence in MEN (1973) is "In fact, no regular vector-based education has been given to young pupils so far" (Choquet 1964, p. 153)

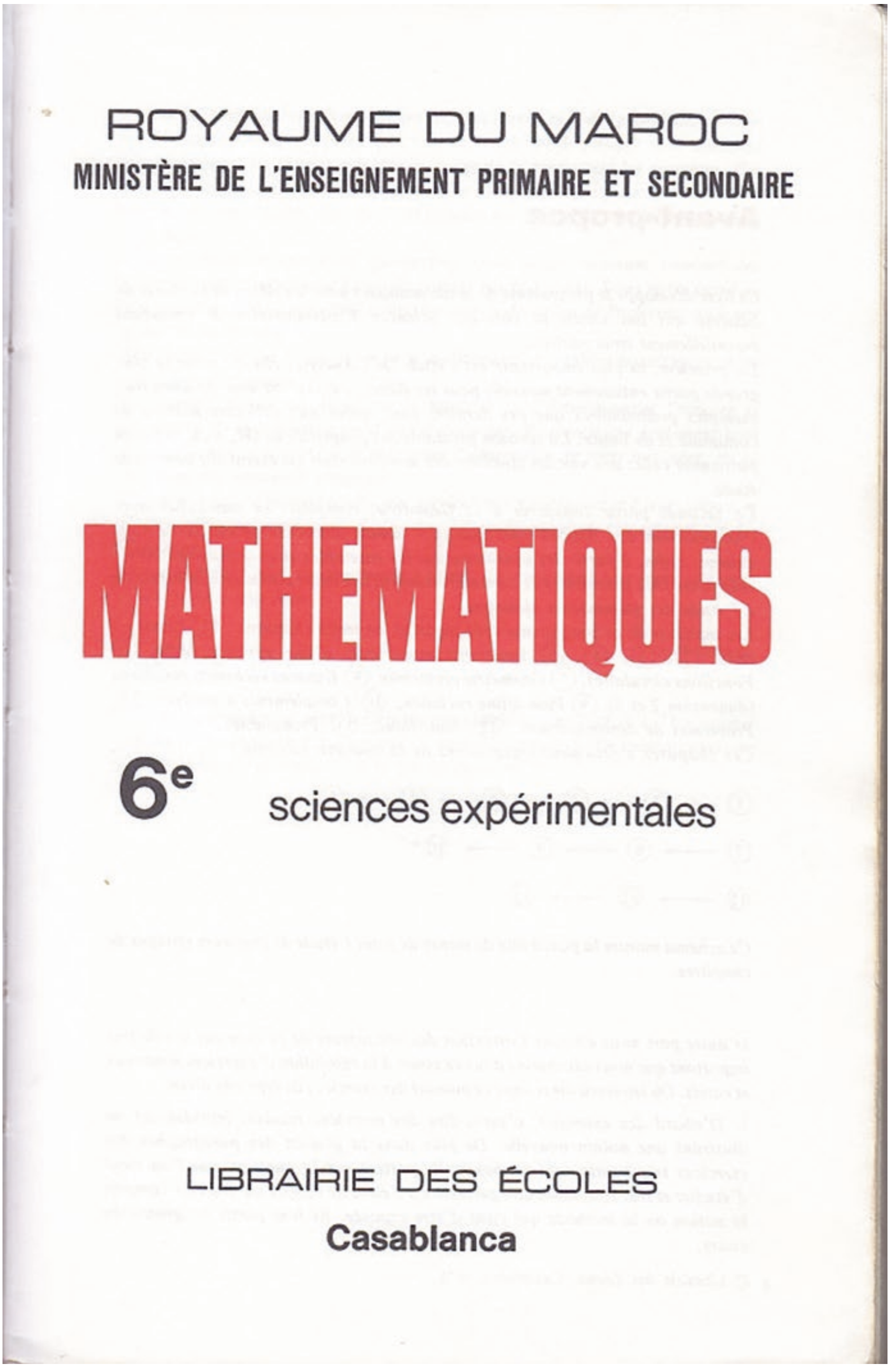

Figure 23.3 Cover of MEN (1976b), a textbook intended to 6th year of secondary education (experimental science section) (author's collection)

The textbooks produced by these two attempts have been mentioned above. The collection of Yves Peureux textbooks did not cover all options of the second cycle. Therefore, the ministry adopted the collection entitled *Kingdom of Morocco, Ministry of Primary and Secondary Education* (see Figure 23.3). These books were identical to the French collection "Queysanne and Revuz," except for a few minor readjustments, mainly relating to the institutional nomenclature[20] within the texts. This is what Moatassime (1978) said about this edition:

> A last attempt, quite recently, resulted in an authorless collection, published under the stamp of the Ministry of Primary and Secondary Education. The least that can be said about it is that it is a step backwards from the previous ones. The Ministry has, in fact, ordered the editors of the "Queysanne and Revuz collection" to adapt their textbooks to Moroccan programs. But can we speak of adaptation when, in statistics, all the examples are taken from a French context—as if there was not sufficient material for this in Morocco—when, in the 7th-grade books, there is not a single subject of the Moroccan baccalaureate, which does not date from yesterday, etc. (p. 51)

Despite the publishing of Moroccan textbooks, several collections of French textbooks and annals (books that brought together the subjects of the baccalaureate from different French academies, see, e.g., Vuibert-SMER 1978) were circulating in Morocco at the time. It should be noted in passing that several French annals included the subject of the Moroccan baccalaureate.

Criticisms of the Reform of Modern Mathematics

In Morocco, the process of abandoning modern mathematics began in the mid-1970s. In addition to international trends[21], at least two factors seem to have fueled the questioning of modern mathematics. The first was the mismatch between the ambitions targeted by these programs and the training of teachers responsible for carrying them out. This period saw a great diversity in the profiles of mathematics teachers. To remedy the shortage of qualified teachers, the Ministry had to resort to primary school teachers and even to scientific students having the level of the baccalaureate to teach mathematics in the first cycle. In addition, after France was unable to meet Morocco's demand for teachers, Morocco turned to Bulgarians, Romanians, Belgians, and Canadians to provide second-cycle mathematics education (El Mossadeq 1989). Thus, many of those responsible for teaching these programs were unaware of the changes in the mathematics curricula.

The second factor is related to the criticisms of several users of mathematics (such as physics teachers, technical fields' teachers, etc.). They criticized both the content and the pedagogical approach adopted in high school curricula. These criticisms maintained that the programs did not benefit the majority of students and, at best, met only the needs of highly scientific professions, such as engineering and mathematical research[22]. These negative evaluations were expressed in particular during a conference organized by the Ministry of National Education in Rabat in 1975[23]. This conference brought together officials from different levels of education (from primary to university) and from different disciplines (mathematics, physics, economics, etc.). Among the arguments raised at this conference was the fact that the teaching of mathematics did not achieve the objectives set. It neither targeted a sufficient number of students nor prepared students for higher education in subjects other than math-

[20] The names of the secondary levels are not the same in France and in Morocco. In France, the secondary levels are designed as follows: 6th, 5th, 4th, 3rd, 2nd, 1st, and terminale, but in Morocco these levels are named as follows: 1st, 2nd, 3rd, 4th, 5th, 6th, and 7th.

[21] For example, the congress of African mathematicians in Rabat in 1976 made a recommendation that "we do not accept programs used in foreign countries, but we must design programs suitable for our development problems," which in a way contradicted the recommendation made in Cairo in 1969, which advocated the introduction of modern mathematics as in developed countries (El Mossadeq 1989).

[22] The programs do not meet the needs of the majority of students, but only those who will be oriented in mathematical sciences. This has led some teachers who use mathematics to remark: The programs work as if all students would become mathematicians in the future (Aboutir 1994).

[23] The main remarks and recommendations resulting from this conference are reported in MEN (1983).

ematics. In particular, it did not meet the needs of occupations in demand in the labor market. One of the participants in this conference asserted:

> All this made the teaching of mathematics in Morocco start to pose serious problems and the orientation toward the section of mathematical sciences became very difficult. There has been a lot of academic failure in the discipline among young people. Without too much exaggeration, we can say that this poorly initiated and executed reform has done considerable damage and does not prepare students, neither for mathematical research, nor for the use of mathematics in other areas of activity. (Akkar 2002, p. 181)

Other points discussed in this meeting included the following:

- The excess of formalism and the abundance of vocabulary emptied mathematics of its meaning
- Solving problems involving computations or geometric configurations created enormous problems for students
- The exercises and problems on algebraic structures were often superficial and devoid of meaning[24]
- The "disappearance" of the classical geometries of the plane and space and their replacement by affine geometry was to be regretted

All these observations and recommendations led the Ministry of National Education to create in 1978 a commission entitled "National Commission for Reflection and Reform of Mathematics Education." This Commission consisted of representatives from all sectors interested in mathematics education. Representatives of central services, teachers from higher education, those responsible for teaching mathematics in lower and upper secondary schools and in the training of executives, and those responsible for physics and technical education all participated in this commission. The points to be discussed by the commission, as defined by the Minister, were as follows:

- Coordination between mathematics programs of different teaching cycles;
- Coordination between the teaching of mathematics and that of other disciplines.

The Commission made general recommendations regarding the objectives of mathematics education, as well as specific remarks aimed at improving implemented curricula. The recommendations insisted that the objectives of mathematics at the secondary level and the minimum level required to enter the labor market must be clearly defined. With regard to the programs, the Commission recommended that the number of general concepts and logical symbols be reduced, that the teaching of geometry be enriched, that more attention be given to developing skills in numerical calculation, and the necessary time be devoted to practice and exercises, and to considering examples inspired by other disciplines. The programs recommendations by the Commission were considerably less "formal" than the previous ones. They were relieved of excessive logical rigidity and conceptual abstraction and, in particular, they reincorporated classical geometry.

Conclusion

The reform of 1962 introduced modern mathematics partially in the programs of the 2nd cycle. This introduction was limited to the insertion of a chapter containing the basics of set theory, vocabulary and logical symbols, binary relations, internal laws of composition, and their properties. The new programs did not consider algebraic structures as a subject of study. Only the name of the structures were given after presenting the properties when studying classical sets (*N*, *Z*, *Q*, *R*, etc.).

[24] Often at the level of high school, the exercises given to the students were superficial and concerned properties that the majority of students already knew in other forms, which made them unable to see the point of such exercises.

In the reform of 1968, the introduction of modern mathematics affected both the first and the second cycle of secondary education. For the first cycle, reading the descriptions of the programs and their implementation by the textbooks shows that there were two levels of implementation of modern mathematics. The introduction of general notions related to set theory, binary relations, mappings, internal laws of composition, and their properties represented the *explicit* level. The use of a vocabulary related to sets in the formulation of the definitions of the arithmetic and geometric concepts studied was an aspect of the *implicit* level of the implementation of modern mathematics. A new vision of mathematics, one in which relationships between objects were more important than the nature of the objects themselves, was embodied in the modern mathematics programs of the second cycle of 1968. In 1962, the algebraic structures had been "deduced" and named following the study of examples, but in 1968, these structures constituted the object of study. They were studied first for themselves and then illustrated using examples. Thus, there was a reversal of the 1962 approach.

The programs resulting from the reform of 1971 consolidated and reinforced the orientations taken by the programs of the 1960s. Reading them suggests that they were aimed at improving the modern aspect of the curriculum by placing more emphasis on topics related to modern mathematics. In the first cycle, the programs of the 3rd and 4th years introduced explicitly the laws of internal composition. In addition to the use of certain symbols and a logical vocabulary, there was also a development of the notions of binary relation and mapping. For the second cycle, the programs of 1971 did not seem to have undergone any significant changes compared to those of 1968 in terms of mathematical content. The change was essentially to make the presentation more rigorous so that the "general to particular" approach was adopted in presenting the concepts.

It can be noted that the process of abandoning modern mathematics was accompanied by another process, specifically the *Arabization* of the teaching of scientific disciplines (mathematics, physics, chemistry, natural sciences). As the Arabization process started from the 3rd primary year, during the 1980–1981 school year, it reached the secondary school in 1983–1984. Then, the abondonment of modern mathematics was done in secondary school over a period of seven year (from the school year 1983–1984 until the 1989–1990 school year). In fact, removing traces of modern mathematics from curricula was not something easily achieved. It was completed only after the reform that took place at the turn of the century (Laabid 2019).

We cannot close this chapter without mentioning an idea that circulated in Morocco. According to the proponents of this idea, the implementation of modern mathematics in Morocco at that time was not the result of a national need. Some Moroccan authors believed the reform to be an experiment in which France, the former colonial power, tested the introduction of modern mathematics in Morocco before including it in French curricula[25]. For instance, for Al Jabri (1986) this reform illustrated the dependence of the Moroccan educational system on its French counterpart. In this regard, he denounced:

> The almost complete subordination to the educational system in France and its developments. This dependence manifests itself in two basic aspects. The first is the imitation of the developments happening in France blindly, without taking into account our reality, our circumstances, and our specific needs. The second is the desire of French circles to conduct a particular experiment in Morocco, and study its results, in order to apply it in France if found valid. This is what happened when Morocco—before France—started teaching modern mathematics in all high-school sections. (Al Jabri 1986, pp. 80–81)

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[25] From this perspective, some researchers believe that the 1968 programs were of an experimental nature. They even add that "Inspector J. P. Nuss, who was among those responsible for the preparation of school programs and textbooks, considered Morocco as a space for experimentation with these programs before their application in France" (Mawfik 2001, p. 2).

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