Chapter 20
Modernizing Mathematics Teaching: International Dialogues from Brazil

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Abstract This chapter discusses Brazil's participation in international movements on the mathematics curriculum and how it shaped local proposals for mathematics teaching. We focus on three fundamental moments in the history of mathematics education in the twentieth century. The first was the creation of the International Commission on Mathematical Instruction in the early twentieth century and Felix Klein's proposal of merging the different mathematical branches in the school curriculum. The second, known as the modern mathematics movement, resonated between the 1960s and 1980s. The third was the international mobilization launched by the World Conference on Education for All in 1990, which promoted the reorganization of the school curriculum around competencies development. By taking a look at different eras, we seek to understand how national debates and practices in mathematics teaching embraced international thinking before, during, and after the modern mathematics movement. The *international* becomes *national,* with different justifications. It appears that modernization is a recurring theme in curriculum reforms in Brazil. At different times, for changing reasons, different actors have advocated local reforms to bring mathematics teaching up to date with the most recent international trends.

Keywords Educational reforms · Euclides Roxo · Felix Klein · History of mathematics education · International Commission on Mathematical Instruction · Mathematics curriculum · Mathematics education · Modernization · Modern mathematics · Osvaldo Sangiorgi · Professional teaching knowledge · School curriculum · School Mathematics Study Group · Zoltán Dienes

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Initial Considerations

"Set and structure"[1] was the slogan repeated at the end of a demonstration class at the 4th Brazilian Congress of Mathematics Education held in 1962 in Belém do Pará, in the North of the country. Brazilian teachers called upon their colleagues to coalesce mathematical ideas circulating in international events and publications: Thus, modern mathematics in Brazil took off.

References to overseas knowledge and achievements, especially those from European nations and from America, were not new. Brazilian intellectuals have frequently and repeatedly seen foreigners as models with compelling arguments for advancing new educational proposals. Discourses on teaching, modes of institutionalizing schools and teacher training, pedagogical practice models, textbooks, school knowledge—everything, in short, has been appropriated in the local framing of discourses, educational practices, and curricula.

Appropriating international ideas is not singularly. Recent studies show that this phenomenon accompanied modern school structuring, intensifying in the nineteenth century—The *international* that became *national* (Matasci 2015). That would be no different for mathematics teaching. Perhaps school mathematics is even the discipline that has most stimulated international exchanges and debates among other countries.

This chapter highlights times of intense exchanges between Brazilian and overseas intellectuals regarding the teaching of mathematics, exchanges that took place within the scope of international movements. The first was related to the International Commission on Mathematical Instruction (ICMI) set up in the early twentieth century. The second, known as the modern mathematics movement, resonated between the 1960s and 1980s. The third, the international mobilization launched by the World Conference on Education for All, took place in the last decade of the past century.

When dealing with different eras—therefore with changing and diverse problems, cultures, and actors—we carry out a comparison to understand how national debates and practices in mathematics teaching embrace international thinking. We analyze the relationships established between certain disciplinary fields and the dialogues overseas in order to improve our understanding of how teaching in Brazilian mathematics changed throughout the twentieth century. Thus, we consider the mathematical disciplinary field and the emerging field of mathematics education, as well as the elementary school teachers who played a role in changing the curricular references for teaching mathematics. We take the theoretical construct "field" from the studies of Pierre Bourdieu, who emphasized "the role of the structures that guide scientific practices whose effectiveness is exercised on a micro-sociological scale" (Bourdieu 2001, p. 67). Thus, actors from different fields and times who played fundamental roles in transforming mathematics teaching, in terms of the production of official curricular references for its realization, are placed on the scene to dialogue with one another.

We are particularly interested in knowing the following: How did Brazil's participation in international movements on the teaching of mathematics throughout the twentieth century shape local proposals for mathematics teaching? In addressing this issue, we seek to identify how the *international* became *national* before, during, and after the modern mathematics movement. We refer to the international dialogues from Brazil as we investigate each era of the modernization of mathematics teaching.

[1] In Portuguese, "*conjunto e estrutura.*" The performance was similar to the protest marches, common at the time (Souza 2007).

Brazil and the First International Movement to Modernize Mathematics Education

In 1908, at the International Congress of Mathematicians in Rome, mathematicians were interested in discussing issues related to teaching. To this end, they created ICMI,[2] the International Commission on Mathematical Instruction. After that, a central committee was elected, led by the mathematician Felix Klein (CIEM 1908).

A relatively long time then elapsed between international discussions on mathematics and curriculum changes in Brazil. One possible explanation for this might be that there were no venues for mathematic educators, no mathematicians' associations, and no bodies that specifically dealt with educational issues at the national level.

From the end of the 1920s onward, debates on how to teach mathematics well became more intense. Tensions arose between mathematics as a disciplinary field and the teaching of mathematics itself. In reality, at that time it would constitute an anachronism to see Brazilian secondary school mathematics teachers as representatives of a "mathematical disciplinary field." Almost all mathematics teachers then were engineers. Later on, from the end of the 1930s, secondary school teachers' training became a task of the newly created universities, and teachers with a mathematics degree in Brazil started to appear (Valente 2020).

The intense debates regarding the teaching of mathematics lead us to Euclides Roxo (1890–1950), who himself was trained in engineering. Roxo held the position of director of Colégio Pedro II in Rio de Janeiro, a model institution for secondary education in Brazil, which had been modeled after French high schools (Figure 20.1). His director's post gave him status as a minister of education when there was, in fact, no such ministry. Roxo promoted changes in the secondary mathematics course during the blackout of the so-called Old Brazilian Republic,[3] a period between the Republic's Proclamation (1889) until what became known as the 1930 Revolution. A mindful supporter of inter-

Figure 20.1 Euclides Roxo, professor of mathematics and director of Colégio Pedro II, in Rio de Janeiro. (Photo from Documentation Center of GHEMAT-SP)

[2] Until the 1950s mainly indicated with the French acronym CIEM (*Commission Internationale de l'Enseignement Mathématique*) or the German IMUK (*Internationale Mathematische Unterrichtskommission*).

[3] In Portuguese, *República Velha.*

national proposals, Roxo was also a member of ABE (Brazilian Education Association) founded in 1924 in close dialogue with exponents and leaders conducting education at the primary level and teacher training. In addition, he taught classes at the Education Institute of Rio de Janeiro, training primary school teachers. Consequently, Roxo moved away from being a typical high school mathematics teacher, someone close to the mathematical disciplinary field, with little or no didactic-pedagogical training. Roxo, with his professional history, fought a long battle to include new, more extensive knowledge, in addition to that which was strictly mathematical, in teacher education (Valente 2004a).

Euclides Roxo endeavored to transform the secondary school mathematics curriculum in Brazil. The most crucial point in the movement headed by Felix Klein was to bring differential and integral calculus to the secondary school level. Roxo also sought to include it by initiating the study of functions as a concept that integrated arithmetic, geometry, and algebra. He aimed for a new organization for teaching and the writing of textbooks he had found in the United States of America[4] which, above all, would integrate the mathematical branches which had, until then, been taught separately. In the merging of these different branches, mathematics as a school discipline was established in Brazil, firstly in Colégio Pedro II and then within the secondary school curriculum reform in 1931.

All these innovations driven by Roxo placed him at the center of mathematical debates and tensions. On one side were the few secondary school teachers close to the educational sciences field, and, on the other, were engineers with no interest in educational discussions. From the first decades of the twentieth century, this tension triggered a public debate on mathematics and mathematics teaching,[5] a debate more focused on mathematics teaching as a professional field. Mathematics education was at the center of attention. The disputes focused on curricula, guidelines for pedagogical work, mathematics textbooks, and so on. Who would be authorized to set guidelines on what should be taught in mathematics classes and how? Brazil in the 1920s—agrarian, with a mostly illiterate population and with few isolated faculties—was just beginning to industrialize, taking advantage of the opportunities which had emerged in World War I, and aspiring to modernize. Roxo used and abused the argument of authority, evoking the mathematics teaching of more advanced countries. With Felix Klein as a major reference, Roxo proposed to follow American pedagogical practices and construct mathematics that fused arithmetic with geometry and algebra as a school subject.

In the midst of these disputes, the publishers of instructional works, who were consolidating themselves into large national companies, promoted editions of books that moved away from previous classical ones on arithmetic, geometry, and algebra, adapted from French texts. So, they placed some series of mathematics books on the publishing market. Such works did not reflect the unifying proposals of Euclides Roxo but instead assembled the different branches to be studied progressively each school year. Given the mathematics textbooks' role in teaching, we see that a conservative modernization in school mathematics took place then. The curricular revolution initiated by Klein spread in the United States through works that merged mathematical branches in teaching. Such works guided Roxo's proposals. In Brazil, however, the reform took place only partially, upholding the separation between arithmetic, geometric, and algebraic topics.

In any case, in an analysis on a broader scale, the first decades of the twentieth century led Brazil to develop a mathematics curriculum *nationally* for the first time by following an *international* example.

[4] It is worth mentioning, mainly, Ernst Breslich's writings. In fact, Roxo was guided by this author, but brought several other innovations to the organization of mathematics for secondary education, when he published the collection *Curso de Matemática Elementar* where he guides teachers to carry out the integration of arithmetic, with geometry and algebra (Valente 2004b).

[5] A very representative moment of this debate occurred through prominent publications, which occupied the front pages of the *Jornal do Commercio*, in Rio de Janeiro, between 1930 and 1931. There was great fanfare and repercussions of controversies over Euclides Roxo's proposals. Pitombeira de Carvalho described in detail the clashes waged by Roxo in the text "Euclides Roxo and the controversies about the modernization of mathematics teaching" (Carvalho 2004).

Brazil and the Modern Mathematics Movement

In the 1950s, Brazilian educators followed international debates on mathematics curricular changes at a distance. New professional positions emerged: A teacher with a degree in mathematics and a mathematics university professor interested in secondary education.

In 1953, Martha Dantas, professor of didactics of mathematics at the Federal University of Bahia, traveled to England, Belgium, and France. In Paris, she attended classes taught by Lucienne Félix, a secondary school teacher interested in "studying modern mathematics," [6] and member of the *Commission Internationale pour l'Étude et l'Amélioration de l'Enseignement des Mathématiques* (CIEAEM)/International Commission for the Study and Improvement of Mathematics Teaching. It was also in France, during a meeting with teaching inspectors at the *Centre International d'Études Pédagogiques* [International Centre for Educational Studies] in Sèvres, that the idea of organizing mathematics teaching congresses in Brazil arose (Dantas, as cited in Garnica 2008).

The first Brazilian National Congress on Mathematics Education took place between September 4 and 7, 1955, in the city of Salvador. The Congress's debate focused on the Brazilian secondary school mathematics curriculum, on which the university professors had no impact.

When the 2nd National Congress on Mathematics Education took place, news of CIEAEM's work was already circulating in Brazil. The Commission's first collective publication, entitled *L'Enseignement des Mathématiques* [The Teaching of Mathematics] (Piaget et al. 1955), brought together texts by Jean Piaget, Evert W. Beth, Jean Dieudonné, André Lichnerowicz, Gustave Choquet, and Caleb Gattegno, condensing mathematicians' concerns about aligning elementary with advanced mathematics, which they considered imperative. This didactic and epistemological question gained scientific status through Jean Piaget's studies and the assumed parallel between cognitive and mathematical structures (see Chap. 3 in this volume). This publication laid the grounds for new discussions toward proposals for a modern curriculum in mathematics teaching.

The 2nd Congress was held in Porto Alegre, in the South of the country, from June 29 to July 4, 1957. Professor Osvaldo Sangiorgi[7] cited CIEAEM's publication in a thesis titled "Classical mathematics or modern mathematics, in the elaboration of secondary education curriculum?" Following the introduction, Sangiorgi pondered:

> This is the question that has dominated, today, students of mathematics in secondary education: It seems to us that, the general purpose of education being a driving function of each era, we cannot really say the last word regarding the investigation of the best principles that should guide mathematics teaching. In fact, the affirmation of the distinguished members of the *Commission Internationale pour l'Étude et l'Amélioration de l'Enseignement des Mathématiques*, in the book *L'enseignement des mathématiques* published in Switzerland in December 1956 [sic.], is no different. This Commission assembles professors who, in various fields—psychological, methodological, and practical—seek to contribute to the improvement of mathematics teaching. (Sangiorgi 1959, p. 398)

At the 3rd Brazilian Congress on Mathematics Education held in Rio de Janeiro from July 20 to 25, 1959, the discussions regarding the curriculum's modernization made little to no progress. As in previous congresses, debates highlighted pedagogical experiences, suggestions for programs only slightly different from those already established since 1951, and methodological trials around the

[6] According to her travel and study report: Dantas, M.M.S. *O ensino da matemática na Bélgica, Inglaterra e França*. Archives of Universidade da Bahia—Faculdade de Filosofia, v. III, 1954. We thank Inês A. A. Freire for scanning it.

[7] Osvaldo Sangiorgi (1921–2017) graduated in Mathematics in 1941 from the University of São Paulo. In the 1950s, he developed a textbook series for middle school mathematics teaching which were highly successful. In 1960, Sangiorgi carried out an internship at the University of Kansas. Upon his return to Brazil, following the example of what he had seen in the United States, he led the creation of the *Grupo de Estudos do Ensino de Matemática* (GEEM) [Mathematics Teaching Study Group]. He became, therefore, an icon of the modern mathematics movement in Brazil. His textbooks on modern mathematics circulated throughout the country, making him a true bestseller (Valente 2008).

traditional contents. In explaining this, we must consider that a significant portion of the teacher leadership, those who conducted the different sessions of the first two meetings, were mathematics textbook authors. Apparently, these congress members also participated in the discussions to preserve the organization of the graded mathematics topics according to their teaching manuals and their pedagogical experiences. In this case, again, they represented editorial interests. However, it is worth mentioning a recommendation approved by the 3rd Congress:

> To request Professors to carry out experiments in the Secondary Course on the introduction of Modern Mathematics notions and bring its results to the IV Brazilian Congress on Mathematics Education. (Congresso Brasileiro do Ensino da Matemática 1959, p. 214)

Quite different was the mood at the 4th Brazilian Congress on Mathematics Education. The event was held from July 22 to 28, 1962, in Belém do Pará, in the North of the country. Following the seminar of Royaumont in 1959,[8] which was a milestone in setting up a shared political orientation for restructuring the mathematics school curriculum, the Organisation for European Economic Co-operation (OEEC; later joined by nations outside Europe to form the Organisation for Economic Co-operation and Development, OECD), had promoted a meeting of experts in Dubrovnik, Yugoslavia, 1960. The resulting text, entitled *Synopses for modern secondary school mathematics* (OEEC 1961), was by then already circulating in Brazil. In São Paulo, secondary school teachers and university professors established the *Grupo de Estudos em Ensino de Matemática* (GEEM) [Mathematics Teaching Study Group], under the leadership of Osvaldo Sangiorgi.

In repeating the slogan "Set and structure" before the 4th Congress audience, GEEM demonstrated a zealous adherence to the international movement for the modernization of mathematics education and called on the teachers present there to join in as well. GEEM members considered multiple references in this attachment. For the São Paulo University mathematicians, it was crucial that Jean Dieudonné and Gustave Choquet had endorsed and engaged in discussions about the reform and, notably, in the Royaumont Seminar. Osvaldo Sangiorgi had witnessed the projects being developed by North American groups during an internship at the Summer School in Kansas[9] in 1960; European authors' textbooks and pedagogic manuals circulating in the country presented new school mathematics; Piaget's writings validated the emphasis on mathematical structures, which was also consolidated by the Dubrovnik program.

GEEM's exchanges with educators and mathematicians from other countries were also plural: Caleb Gattegno had given lectures in São Paulo, in May 1961, divulging his proposal for the use of Cuisenaire materials; the mathematician George Springer gave a course on Mathematical Logic for secondary school teachers, in São Paulo, in 1961; in August 1962, GEEM received Lucienne Félix for a cycle of lectures for teachers, in São Paulo. A report on the ongoing reforms in Latin America, addressed in 1962 by Sangiorgi to UNESCO, indicated, on the other hand, that GEEM tried to present itself as the pioneer and leader of the all-embracing modernizing enterprise in the continent (Sangiorgi 1962).

The new National Education Law, approved in 1961, eliminated the national program of studies from being mandatory and favored instead revisions to the mathematics school curriculum. In this affable environment, GEEM proposed a new mathematics program for the first and second cycles of secondary education, which added elements of both the Dubrovnik program and the North American School Mathematics Study Group (SMSG) materials to those traditionally taught. Above all, GEEM members aimed to modernize school mathematics language:

[8] The Seminar was held in late 1959, at the *Cercle Culturel de Royaumont*, in Asnières-sur-Oise, France. The activities lasted 2 weeks and were attended by about 50 delegates from 18 countries (Guimarães 2007; Schubring 2014).

[9] Sangiorgi reported that he received a grant funded by the Pan American Union in collaboration with the National Science Foundation (Lima 2006).

> What we essentially desire with Modern Mathematics Programs (and this would be the recommended expression) is to study the same Mathematics subjects, known as essential in the young middle school student's education, using however a modern language that is more attractive to the new generations. (GEEM 1965, p. 89)

Therefore, adhering to the international modern mathematics movement presented itself as a secondary school teachers' initiative in alliance with mathematicians. The emergence of GEEM was followed in 1962 by the creation of the *Núcleo de Estudo e Difusão do Ensino da Matemática* (NEDEM) [Center for the Study and Dissemination of Mathematics Education], in Paraná.

These groups' actions were facilitated by government policies, which did not immediately align with modern mathematics but rather invested in the continuing educational training of secondary school teachers. The priority given to the training of teachers in mathematics and science corresponded to a shift in the constitution of a secondary education more oriented to technical-scientific training than to the traditional humanities, in alignment with the guidelines of the United States Agency for International Development (USAID). The First Inter-American Conference on Mathematics Education, held in Bogotá, Colombia, from December 4 to 9, 1961, led by North American mathematicians, contributed to the legitimization of the cooperation actions financed by USAID. Valente (2014) records that efforts encouraged by UNESCO, such as improvement activities held in Argentina in 1954 and Colombia in 1958, were ignored or devalued in the preparation and report of the Conference. The organizers did not even consult the newly created Latin American Mathematics Center based in Buenos Aires. The event produced a diagnosis of a shortage of mathematics teacher education in most Latin American countries, justifying the need for emergency action in this field (Fehr 1962).

The strategy of preparing a reform by investing in the continuing education of teachers was presented in 1962 by the president of the Brazilian Institute of Education, Science, and Culture (IBECC), an organization linked to the Ministry of Foreign Affairs and UNESCO:

> We, fortunately, passed this phase of fictional reforms. Like what is done efficiently in other countries, we have to mobilize our scientists and their [sic.] teachers. It will be necessary to change the secondary teachers' mentality and retrain them in the new curricula. Then there will be a reform. (Raw 1965, pp. xvii–xviii)

With support from government agencies and the National Science Foundation, GEEM and NEDEM developed an intense teacher-training program inspired by modern mathematics ideas, that incorporated different sources (Lima 2006; Pinto and Fischer 2011).

Government policies also signaled the shift from secondary education organized around traditional humanities to one more oriented to technical-scientific training. Between 1963 and 1965, national and local governments created six Science Teaching Centers in different regions of the country. In the city of Salvador, the Science Teaching Center of Bahia (CECIBA) housed an important group that developed modernizing activities, including teacher training and textbooks inspired by Georges and Frédérique Papy's work (Freire 2009; Macena 2013).

Modernization also took place through the establishment of experimental schools and experimental classes in existing educational institutions. Inspired by the ideas of progressive education and French *classes nouvelles*, the experiments there sought to promote students' critical thinking skills by studying the environment and the articulation between disciplines, breaking with the compartmentalization of traditional secondary education. The so-called *Colégios de Aplicação*, Educational Centers and Vocational Schools in large cities such as São Paulo, Rio de Janeiro, Salvador, Porto Alegre, and Niterói, were unique spaces for experimenting with new curricula involving different facets and readings of modern mathematics. In the late 1960s, however, which were times of worsening authoritarianism and censorship established by the 1964 military takeover, some of these experimentations were interrupted, as was the case of the Brooklyn Vocational School and the *Colégio de Aplicação* in Salvador (Búrigo 1989; Rios 2012; Soares 2001).

However, disseminating modern mathematics was not done mainly through experiments of limited scope or teacher-training courses, although these were more comprehensive. In most schools, within

a system expanding along with the country's urbanization, modern mathematics arrived through the media and the textbook series produced by authors linked to various groups (Nakashima 2007).

Taking advantage of the national legislation's easing of the mandatory curriculum, in 1965 GEEM obtained governmental consent to disseminate new curriculum guidelines in São Paulo, Brazil's main publishing center. This semi-official validation of modern mathematics facilitated the publication and adoption of new textbook series (Valente 2008). Some of these collections were collectively written. For instance, the *Grupo de Ensino de Matemática Atualizada* (GRUEMA) produced the collection entitled "Modern mathematics course for primary education" (Villela 2009), NEDEM published the collection "Modern mathematics teaching" (Pinto and Ferreira 2006), and teachers of CECIBA made up the series "Updated mathematics teaching" (Freire 2009). COLTED,[10] a government textbook distribution program implemented in 1967, helped to expand the new curriculum's outreach. Osvaldo Sangiorgi was then the president of GEEM; his instructional text "Mathematics—modern course" was included in this governmental program. Translated and adapted versions of SMSG's pedagogic series were also published since 1964 with IBECC funding (Oliveira Filho 2009).

In an atmosphere of jubilation, the 5th Brazilian Congress on Mathematics Education held from January 10 to 15, 1966, in São José dos Campos, was entirely dedicated to modern mathematics. Organized by GEEM, the event featured lectures by Marshall Stone, Georges Papy, Hector Merklen from Uruguay, Helmuth Völker from Argentina, and presentations and demonstration classes by Brazilian mathematicians and educators.

In December of that same year (1966), Brazil had a five-educator delegation participating in the Second Inter-American Conference on Mathematical Education, held in Lima, Peru. On the progress of mathematics teaching, there were four lectures at the Conference: one on the activities of the OAS and the others on the reforms underway in Spain, Chile, and Brazil (Barrantes and Ruiz 1998). Osvaldo Sangiorgi, in an effusive speech about the Brazilian case, mentioned the growth of mathematical research, teacher training courses, and the actions of study groups responsible for the "Mathematics Programs presently operating in Brazilian Secondary Schools" (Sangiorgi 1966, p. 107). Martha de Souza Dantas delivered the lecture "Teacher training in Brazil," mentioning the actions of the GEEM, the Science Teaching Centers, and the State Education Departments. That was all very different from the "somber picture" presented by Omar Catunda (1962, p. 58) at the First Conference in Bogotá, Sangiorgi and Dantas, and pointed toward a dynamic country, engaged in modernization (Fehr 1966; Pinto 2007). Both discourses suggested a broad convergence between the actions of different groups and official bodies. There was no reform, but it was as if there were.

The Modern Mathematics Movement and the Expansion of the School System in Brazil

The emergence of the modern mathematics movement in Brazil coincided with times of transition from a very stratified schooling model to one which was more fluid, expanded, and accessible to young people from the lower classes. That modern mathematics movement temporally coincided with a democratization of the Brazilian educational system.

In 1960, only two out of every ten children who entered school completed primary school, and one of ten continued to study beyond that stage (Cunha 1980; Gouveia and Havighurst 1969). A few prestigious schools in state capitals and other large cities offered public secondary education. An entrance examination regulated access to these schools. In the second half of the 1960s, with a view to expand-

[10] The *Comissão do Livro Técnico e do Livro Didático* (COLTED) [Technical Books and Textbooks Committee] was established in 1966. COLTED's book distribution program was financed through an agreement signed in 1967 between the Ministry of Education and Culture, the National Union of Book Publishers, and USAID (Krafzik 2006).

ing secondary education, the preparation for an educational reform that would take place in 1971 began. França (2007) explained that collaboration agreements between the Brazilian government and the United States Agency for International Development (USAID) foresaw the expansion of primary education as part of a modernization and industrialization policy in the country and aimed to form a labor force that had completed some education and training. As at the beginning of the century, the state of São Paulo was a pioneer in developing new programs for the early years of schooling, with one of its objectives being to increase the passing rates and continuity in schools. Teachers participating in GEEM were invited to develop the mathematics program; modern mathematics was considered appropriate for the time, as it advocated new teaching methods and promised to improve learning.

Other strategies also preceded and accompanied educational reform by training teachers for the first cycle of secondary education. One method adopted was the authorization of short-term degrees, which enabled teachers to teach only lower grades. The initial mathematics teacher-training programs, which in 1960 did not exceed two dozen, quickly multiplied with the establishment of small private colleges or faculties created by local government or private initiatives (Martins-Salandim 2012; Sucupira 1964). In regions with few universities and qualified teachers, governments implemented emergency programs that prepared former primary teachers—including teachers with 4 or 5 years of schooling—to teach in the new elementary schools' final grades (Costa 2013). Modern mathematics was present in all these accelerated training programs, either because it was included in the current textbooks' mathematics, which teachers had to resort to because it was considered the new mathematics of the time, or even because it was endorsed by teachers participating in the educational debate.

A second strategy to prepare for educational reform was the installation of the *Programa de Expansão e Melhoria do Ensino Médio* (PREMEM) [Middle and High School Expansion and Improvement Program]. This program, financed by USAID, the World Bank, and the Inter-American Development Bank, was created in 1968. PREMEM—later renamed PREMEN—aimed at the establishment of middle schools called "polyvalent"[11] (Arapiraca 1982). Unlike traditional secondary schools, which prepared only a small fraction of the middle class for higher education, these so-called polyvalent schools were installed in neighborhoods on the outskirts of large cities to prioritize science education, with an experimental bias, that was associated with preparation for work. They were model experiences for new secondary schools and aimed at the working class. The program trained a new contingent of teachers in intensive courses, even with remuneration, to teach in these schools. In the southeast and southern regions, this program's curriculum was coordinated by Arago Backx, who had been an intern with Georges Papy at the *Centre Belge de Pédagogie de la Mathématique* [Belgian Center for Mathematics Pedagogy] (Marins 2019). The series *Mathématique moderne*, which guided the Belgium experiment, was then a major reference for mathematics education in PREMEM teachers' training. The collection "Mathematics for Ginásio," written by Lydia Condé Lamparelli and other authors and largely inspired by the SMSG series, was adopted in polyvalent schools (Búrigo 2010). Thus, PREMEM contributed to the dissemination of different aspects of the modern mathematics movement.

In 1971, the new National Education Law combined primary education and the first cycle of secondary education into a single universal entry level of schooling lasting 8 years. New schools were created in smaller cities and on the outskirts of large cities to accommodate a large number of students. The primary course was now converted into elementary years of an extended schooling stage and was no longer committed to preparing students for the now-defunct entrance exam. The old disciplines of the secondary school first cycle, *ginásio*, gave way to new areas of knowledge, denominated by communication and expression, social studies, and sciences. New shortened teacher-training programs were also set up.

[11] In Portuguese, "polivalentes."

It became necessary to produce new curricular guidelines for the initial and final years of expanded elementary education. These were times of authoritarianism and few debates; however, the teacher groups loyal to modern mathematics remained active. In 1970, a new Mathematics Teaching Study Group was founded in Porto Alegre, in the South of the country; the following year, GEEM celebrated its 10th year of existence with a seminar coordinated by Zoltán Dienes (Figure 20.2). Teachers who were then acknowledged as experts in the field of teaching were asked to produce guidelines to steer teachers in implementing the new 8-year elementary course. So far, researchers have found no evidence of public controversy about the maintenance or reform of the old mathematics teaching program at that time. In the various widespread curricular proposals, modern mathematics was clearly present, as illustrated by the introduction of new ideas, language, and reorganization of the contents.

The new curricular documents were no longer called "programs"; rather they were presented as guidelines and orientations. In the curricular guides issued in the early 1970s for the states of São Paulo, the most industrialized and populous state, and Rio Grande do Sul in the far South, the presence of modern mathematics is clearly visible.

In the state of Rio Grande do Sul, the new curricula were coordinated by the Basic Education Branch of the state Education and Culture Department. The resulting documents were released in 1972. For the first 3 years of elementary school, the curriculum was structured as a sequence of objectives, with no distinction between areas or subjects. Modern mathematics remarkably permeated the documents: For example, notions of "set" and "relations" were present in different contexts, such as in the study of animals, plants, or geographic space. The second stage of elementary school—fourth to eighth grade—was organized by areas. Zilá Maria Guedes Paim, who graduated in mathematics, coordinated the science section writing. The general strategy for mathematics teaching presented in the text mentioned order structures, topological and algebraic structures, metric spaces, and vector spaces, which reveals a significant influence of *Grupo de Estudos em Ensino de Matemática de Porto Alegre* (GEEMPA) [Mathematics Teaching Study Group in Porto Alegre] and Dienes's works (Rio Grande do Sul 1972).

In São Paulo, the curricular guides for elementary schools were published in 1973. Almerindo Marques Bastos, who had translated SMSG books, Anna Franchi, and Lydia Condé Lamparelli, all three promoters of the modernizing movement, were in charge of writing the guide's mathematics

Figure 20.2 Visit to Escola Vila Rica, in Santos, on October 7, 1971, by Zoltán Dienes, the first, from left to right. At his side, Osvaldo Sangiorgi, a key proponent of modern mathematics in Brazil. (Photo from Documentation Center of GHEMAT-SP)

section. Their text proposed continuity and unity in mathematics throughout the 8 elementary years. For example, the concept of "function" was to be implicitly present in children's activities from the 1st year and be addressed explicitly from the 5th year. The authors signaled their detachment from some modern mathematics trends:

> The use of Set Theory language in treating all themes contributes, as a unifying factor, to achieve this objective. It is only necessary to alert the teacher in the sense of not transforming this auxiliary language into the main aim of teaching the discipline. Therefore, we must use extreme care not to overdo it. (São Paulo 1973, p. 172)

The context in which curriculum documents incorporate modern mathematics was, then, very different from that of the early 1960s, when the supporters of the movement promised that from then on, everyone would understand and appreciate mathematics. The time period of implementing these new curricula was the same period as that of the extension of elementary school, which was now open to all students and demanded more locations, more schools, and more teachers. Without the corresponding increase in funds and with the abbreviated teacher-training process, this expansion led to a precariously generalized public education (Cunha 1980).

A Change in Mind

It was in this context in the mid-1970s that modern mathematics was interpreted as a failure, echoing Morris Kline's (1973) emblematic book. In the same eager way as Brazilian educators had followed the excitement of the international movement in its early days, an impression that the movement had failed in its promises and purposes spread throughout Brazil from the mid-1970s. The coinciding of time and debate contributed to the creation of a mindset that Brazilians had "plunged headfirst" into modern mathematics. In addition to the criticisms circulating internationally, many Brazilian educators felt that modern mathematics, proposed in rich countries with varying educational contexts, had, in a sense, been "smuggled" into the country. For some authors, modern mathematics in Latin America was the result of American intervention. In a period of accelerated school expansion, some felt modern mathematics could have been responsible for the drop in the quality of education. In the late 1970s, modern mathematics was often described as a frustrating attempt at change that was eventually abandoned.

Questions about this past emerged in the 1980s during a time of democratization when the mathematics education movement arose in Brazil. In 1988, at the 2nd National Mathematical Education Seminar, a round table brought together proponents of modern mathematics. At that time, the first studies on this theme were also developed (Búrigo 1989; D'Ambrosio 1987, 1991; Soares 2001). These studies gathered some evidence that Brazilian educators had neither naively nor passively submitted to external influences or copied proposals and curricular materials.

In the 2000s, an international cooperation project, coordinated in Brazil by the *Grupo de Pesquisas em História da Educação Matemática* (GHEMAT) [Research Group on the History of Mathematical Education], mobilized Brazilian and Portuguese researchers around a more ambitious investigation. They pursued a strategy of assembling a comprehensive collection of sources, producing studies on different facets of the movement, regions, and scales, and comparing them bilaterally. The resulting seminar, held in 2010, generated a synthesis of all research developed until then on the subject and addressed the emergence of modern mathematics, its proponents and their actions, as well as the proposals for new content and methodologies, and its institutionalization (Oliveira et al. 2011).

If the 1960s modern mathematics movement was characterized by a variety of relatively autonomous initiatives which eventually competed, in the 1970s it guided the production of new curricula for elementary and high school using varied and combined reconstructions of North American, European, and local productions. This broad curricular reform left lasting marks on the prescribed and practiced

curricula in Brazil. The variety of initiatives, experiments, and effects produced then are the focus of recent and ongoing research from different groups and regions in the country (Alves 2013; Arruda 2011; Borges 2011; Correia 2015; França 2012; Freire 2017; Morais 2014; Oliveira Filho 2013; Rech 2016).[12]

Brazil, the World Conference on Education for All, and Mathematics Education

Amid the international sources that gave rise to curricular and mathematics education changes and eventually led Brazil to develop the National Curriculum Parameters (PCN), the country's participation in the World Conference on Education for All is pivotal. The event took place in Jomtien, Thailand, in March 1990 and brought together the nine countries with the world's highest illiteracy rates (Bangladesh, Brazil, China, Egypt, India, Indonesia, Mexico, Nigeria, and Pakistan). Its leading promoters were UNESCO, Unicef, the United Nations Development Programme (UNDP), the United Nations Population Fund (UNFPA), and the World Bank. At the meeting, government representatives pledged to develop educational policies to face the problem of illiteracy (Shiroma et al. 2011).

As a result, the participating countries shared a commitment by signing the "World Declaration on Education for All," and the need to create the Ten-Year Education Plan arose in Brazil. To subsidize such an action, the "Education for All National Week" was held May 10–14, 1993, in Brasília, the country's capital.

In a macro analysis, the precept of the neoliberal ideology of the 1990s influenced various countries in Latin America, imposing values based on the economic market as the educational process needed to meet the demands of productive restructuring and efficacy parameters. Adhering to this logic, the educational reforms vowed to meet modern society's demands by following quality standards and guidelines formulated by major international agencies (Châtel 2011; Portela 2013).

By the end of the twentieth century, Brazil was seeking to incorporate itself into the globalized economy; however, the anticipated democratization of education was slow: In 1990, only 19% of the population had completed the initial stage of 8 years of schooling. On the other hand, international organizations offered financing to the poorest countries' social sectors under the condition they would adapt their education systems to the labor market's needs according to the desired economic development. Brazil joined in. In this scenario, the World Bank presented itself as an intermediary to make the transformations required by the new world order viable (Fonseca 2009).

Brazil also committed itself to expand the population's basic education through the New Delhi Declaration, signed on December 16, 1993. Through this declaration, the countries' leaders assumed to pursue vigorously and decisively the goals defined by the World Conference on Education for All and the World Summit for Children, held in 1990. They called on international collaborators to finance their actions.

The above information summarizes the national and global context of the planned directions for educational reforms from an already vast amount of literature that analyzes the emergence of Brazilian PCNs on a macro scale.

Bringing the lenses of observation closer so that a better understanding of the new proposals for the teaching of mathematics as they were formulated in the 1990s is presented, we focus our attention on the year 1995, when Fernando Henrique Cardoso assumed the Presidency of the Republic. That year, the president entrusted further development of PCNs to the Secretary of Elementary Education.

[12] We highlight here some of the doctoral and master's theses published after 2010; other works are mentioned along the section. Preliminary findings of ongoing research have also been published in journals and in the annals of events in the area of the History of Mathematics Education.

Article 210[13] in the Brazilian Constitution of 1988 defines neither who nor which instrument would fix the mandatory school curriculum topics and enforce the Decennial Education Plan guidelines. In the midst of this vagueness, the Ministry of Education (MEC) assumed the leading role in designing the PCNs (Cury 2002).

The first structuring of the PCNs took place by creating teacher groups in each area or subject. Maria Tereza Perez Soares, experienced in training public school teachers, and Maria Amábile Mansutti, specialist in Didactics of Mathematics, participant in advisory bodies of education secretariats for several years, were first asked to develop the mathematics section of PCNs (Figure 20.3). Later Célia Maria Carolino Pires, PhD in Education, member of curriculum development teams in the state of São Paulo, was invited to join the team. They began constructing a mathematics curricular proposal for the first level of schooling, which corresponded to elementary school years 1–4. The PCNs for the so-called first and second cycles of elementary education (initial 4 years) were the first to be defined by the Ministry of Education, and a preliminary version was prepared in 1996. The preliminary version of the PCNs for the final years of elementary school was released in 1997. The final texts were published in 1997 and 1998, respectively. It was only in 1999 that the PCNs for high school were established.

Once the PCNs had been drawn up, the aim was to present a ministerial proposal for constructing a common national basis for elementary education which would be an instrument to guide the shaping of school curricula. The final version was submitted to the National Education Council (CNE) in September 1996, which instead chose to produce a document with broader guidelines, the National Curricular Guidelines. Thus, in October 1997, the then president of the Republic, Fernando Henrique Cardoso, announced the document's distribution to all teachers as a production of the Ministry of Education (Galian 2014). Under Brazilian legislation approved in 1996, schools would have the autonomy to build their pedagogical projects, so the CNE decided that the PCNs would not be mandatory. In practice, however, the PCNs were central in the evaluation of textbooks distributed throughout the country and in the construction of tests by which the teaching quality in each school, municipality,

Figure 20.3 Maria Amábile Mansutti, teacher with extensive experience in the early school years, member of the team that prepared the National Mathematical Curriculum Parameters. (Photo from Documentation Center of GHEMAT-SP)

[13] Art. 210. Minimum contents will be set for elementary education, in order to ensure common basic training and respect for national and regional cultural and artistic values. In Portuguese: "Serão fixados conteúdos mínimos para o ensino fundamental, de maneira a assegurar formação básica comum e respeito aos valores culturais e artísticos, nacionais e regionais."

and region was evaluated. So the PCNs impacted the curriculum more or less the same as if they were mandatory programs,[14] and, by these means, educational policies regulated the teaching action.

In reading the mathematics PCNs, we see problem solving as the main focus of mathematics class activities. The document refers to "An agenda for action," a publication of the National Council of Teachers of Mathematics (NCTM) in the United States of America. The PCN guidelines for mathematics teaching thus differed from those previously proposed by the modern mathematics movement. Instead of valuing mathematical content for its own sake, problem solving became the main goal. The problems were to originate in activities that students were to solve using mathematics as a tool, and thus develop competencies, a central term for the new proposals.

As a result, there was a change in the teaching proposals. Understandably, mathematical content became less important and even placed second in favor of activities that would develop competencies. This change became accompanied by reforms in teacher training. If teacher training was centered previously on knowledge in the field of mathematics, from 2000 on, professional teaching competencies came to the forefront. In this scenario, learning mathematical content was not the goal of the training, but instead an instrument for developing professional teaching knowledge. Thus, mathematical themes became relevant (or not) depending on the cognitive competencies students would develop in solving problems arising from pedagogical activities. And these would not, as a rule, be purely mathematical activities. They could also be related to situations considered familiar to students, in which mathematics was used as a tool to solve problems.

This paradigm of mathematics education has circulated in the public school network. Today, in a movement that reaffirms curricular guidance in mathematics teaching, we find that instructional materials developed very differently from previously. Indexes of these materials no longer indicate the mathematical topics or subjects to be taught. Instead, there are activities, and sequences of problems or situations, which students solve using mathematical means. The didactic material *Educação Matemática nos Anos Iniciais do Ensino Fundamental* (EMAI) [Mathematics Education in the Initial Years of Elementary Education] provides a remarkable example of such guidance for the teaching of mathematics; it is a book series distributed to all public schools in the state of São Paulo.

Final Considerations

Here, this chapter's guiding consideration should be mentioned again: How did Brazil participate in the international movements on mathematics teaching throughout the twentieth century? We try to answer the main question considering a macro scale, the international contexts, and connections, as well as aspects more directly related to the proposals coming from these movements that focus on mathematics classrooms. Such an analysis, we believe, can be contemplated to a reasonable extent by considering the relationships between different disciplinary fields and the professional field of teaching.

In the first international movement, which followed the creation of ICMI, under Felix Klein's leadership, Euclides Roxo was a national reference seeking to develop a didactic viewpoint for mathematics teaching. Following Klein's footsteps, he sought to bring elementary mathematics closer to a more superior level, that is elementary mathematics from a higher standpoint. The professional field of teaching looked for pedagogical methods for the discipline of mathematics. The role of intuition, the merging of mathematical branches, and the introduction of the concept of function reveal elements of proposals for the mathematics classroom inspired by Klein. All these elements, although incomplete, were present in Brazilian mathematical textbooks from the 1930s on. A comprehensive rubric entitled

[14] Later, in 2017, the National Common Curricular Base established a mandatory program that encompasses competencies and content lists.

"mathematics" was created to replace the existing school disciplines of arithmetic, algebra, and geometry.

The modern mathematics movement in Brazil can also be seen as an attempt to change a school culture, where mathematics was far distant from that valued by the academic field. It was no longer a question of seeking a pedagogical approach to the discipline, but rather of reconfiguring the teaching of mathematical content in favor of mathematics learning. To a reasonable extent, the mathematics field outweighed the professional field of teaching. Higher education mathematics professors were called upon to give courses and update school teachers on new mathematics. Different Brazilian groups moved to consolidate an elementary version of advanced mathematical content using algebraic structures in elementary textbooks, as had occurred in the United States. Translations and adaptations of American and European teaching materials were references for teachers' training and new textbooks.

Finally, as seen through the macro observation lens, the third international movement showed itself not to be solely focused on mathematics teaching; it was also an attempt to detach from the disciplinary fields and focus on competencies instead of content. The disciplinary fields opposed this movement. Thus, mathematics clashed with mathematics education, an emerging disciplinary field, where experts held a positive view of competencies' development. The competencies were to trigger a change in the professional teachers' knowledge. And this new knowledge would not be taken directly from the disciplinary field of reference, mathematics. New professional knowledge was to be drawn from mathematics education references. The language justifying the inclusion of this mathematical content involved the development of cognitive competencies: Numerical, algebraic, geometric, and probabilistic thinking. The PCNs would be the official expression of the result of these clashes.

How then can we say that Brazil participated in the international movements on mathematics teaching throughout the twentieth century? In the first international movement, Brazil's proposals were based on the argument of modernization, following European countries and the United States, all considered references for education. Following the lead of those countries considered to be the most advanced, new proposals were to be implemented. Brazil was not present as an active member of these international debates. Almost 30 years after the creation of ICMI, Euclides Roxo, inspired by mathematics textbooks in the United States, proposed newly organized textbooks. In addition, he mentioned debates at the beginning of the century to justify the changes in the teaching of mathematics in Brazil that would break with the tradition he then considered backward.

Yet, in modern mathematic times, these changes came about as education adjusted to the changing world, the new era of computers, and changes in the mathematical field itself, which gave voice to content relevant to this new era. Again, the United States and Europe were considered examples to be followed, as they maintained contact with mathematical communities worldwide. Brazil followed international debates wholeheartedly, with Brazilian teachers and even mathematicians attending congresses on mathematics teaching; teachers went abroad to learn about new proposals, new textbooks, and new courses of school modern mathematics. It was no longer a question of breaking with tradition, but rather of being at the forefront of the race—the Western world in dispute with the East in the context of the Cold War. And for that, the modernization of science teaching, in general, was necessary.

In this current third international movement, Brazil is an active participant, as a representative of an emerging country. Governmental personalities integrate international organizations, such as the World Bank, participating, albeit as secondary actors, in forming the country's agendas for the changes considered imperative for education. The most recent reforms in the country involve the leading role of specialists in the field of mathematical education. For these specialists, mathematics is foundation material, but their disciplinary and professional field of action does not identify with the disciplinary field of mathematics.

Modernization is a recurring theme in the curriculum reforms of mathematics teaching in Brazil: The country's intention is to be always up-to-date with the most recent international trends. However,

justifications for the resulting changes are varied. The arguments that Euclides Roxo used for modernization were aimed at including Brazil in the group of "civilized countries." Looking at the reforms that have occurred elsewhere and incorporating these, Brazil would then have adopted modern teaching of mathematics. The justifications for the changes brought about by the modern mathematics movement took into account scientific progress, from where the "physical-mathematical sciences continually received new and substantial impulses," as Osvaldo Sangiorgi wrote (1959, p. 399). Thus, the teaching of sciences and mathematics was expected to keep pace with such advances. In more recent times, changes in mathematics teaching are seen as a way to reduce inequalities, now renamed "equity." Everyone must study math, everyone can learn math, everyone can learn to solve problems... Promises are taken up, at every turn, by different actors. By their readings of the international movements, the modernizing agenda keeps being updated.

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