Chapter 16
The Kolmogorov Reform of Mathematics Education in the USSR

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Abstract In the Soviet Union a reform movement in mathematics education was triggered by Andrey Kolmogorov in the 1970s, and it was followed by a counter-reform. This movement was rooted in the very different socioeconomic conditions of that time and place and followed a strategy with significant contrasts to similar programs in the United States, England, and France. This provides an interesting case study that may illuminate the way such movements arise and succeed or fail, and, at the social level, certain fundamental commonalities of constraints as well as significant differences according to local conditions. We shall show that the principal reasons for the failure of the Kolmogorov reform were political: (a) The reform ignored the reality of the socioeconomic conditions of the country; (b) The human factor was ignored, and very little attention was given to professional development and retraining of, and methodological help to, the whole army of teachers; and (c) An attempt to transfer mathematical content and methods from the highly successful advanced extension stream for mathematically strong and highly engaged children to mainstream education was an especially grievous error.

Keywords Andrey Kolmogorov · Didactic transformation · Education streams · Facultative courses · Geometry · Kolmogorov reform · New Math · Olympiad stream education · Political environment · Probability theory · Set theory · Social background · Specialist mathematics schools · Teacher training · Textbook design

Introduction

It is now widely accepted—I have never seen or heard claims to the contrary—that the reform of school mathematics education in the Soviet Union in the 1970s, initiated and led by the great mathematician Andrey Kolmogorov (1903–1988) was a fiasco. This view has been shared both by supporters of Kolmogorov and by his direct opponents.

What is still in dispute is the attribution of guilt. Who was responsible for the damage to the system of mathematics education which, as many of my colleagues in Russia feel, still has not been repaired? The reformers? Bureaucrats from the Ministry of Education? School teachers? I do not wish to take sides in this dispute, but political infighting continues, with one side of this polemic, squarely blaming

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the reformers (see, e.g., Kostenko 2013, 2014), whose writings demonstrate the biggest black spot in the study of this episode in history—specifically, the lack of access to archival documents from the highest echelons of power of that time, the Central Committee of the Communist Party, the Ministry of Education of the Soviet Union, and the Ministry of Education of the Russian Federative Republic of the Soviet Union. In the monolithic administrative structure of a totalitarian state, publicly available materials show only the tip of the iceberg of the actual decision-making processes which in the late 1960s produced the reform launched in 1970.

A rebellion against the reform in the years 1978–1980 appears to be better documented and better understood (Kolyagin and Savvina 2012). This rebellion started with the meeting of the Mathematics Division of the Academy of Sciences on December 5, 1978. The symbolic moment marking the beginning of the end of the reform was the publication of a paper by Lev Pontryagin, a famous mathematician, in *Kommunist*, the all-important political journal of the Central Committee of the Communist Party (Pontryagin 1980). The publication of a paper in *Kommunist* was the seal of approval of the author's position from the highest levels of the Party's hierarchy.

Pontryagin (1980) began his paper by quoting the definition of vector given in a reformist school textbook by Klopsky et al. (1980):

> The vector (parallel translation) defined by a pair (A, B) of distinct points is the transformation of the space which sends each point M to the point M_1 such that the ray MM_1 is co-directed with the ray AB and the distance $|MM_1|$ equals $|AB|$. (Klopsky et al. 1980, p. 42, quoted in Pontryagin 1980, p. 99)

Pontryagin continued as follows:

> This tangle of words is difficult to sort out, but, importantly—it is useless, since it cannot be applied neither in physics, or in mechanics, or in other sciences. What is this? Mockery? Or unintentional absurdity? No, the replacement in these textbooks of many relatively simple, visual formulations by cumbersome, deliberately complicated ones, it turns out, is motivated by the desire ... to improve the teaching of mathematics. If the example I gave was just an annoying exception, then the mistake would be easy to eliminate. But, in my opinion, unfortunately, the whole system of school mathematics education has come to a similar state… (p. 99)

In the political tradition of the Soviet Union, the use of this kind of language was equivalent to branding with a hot iron.

The failure of the reform was surprising on a number of counts. First, it was run by mathematicians and professionals in mathematics education of the highest class. Second, The New Math movement in the West was looked at but not imitated. The Kolmogorov reform was not, strictly speaking, the "New Math"—the nature of the reform and the sociopolitical environment were very different from that of, say, the United States of America (see, e.g., Phillips 2015). Third, the scope of the reform was relatively modest. Set-theoretic concepts were used but did not play a leading role in the exposition—but nonetheless, the set-theoretic language was used sometimes in a rather annoying way—for example, for introducing central concepts in the course of geometry, such as vector, without further use. No abstract algebra was introduced; the treatment of algebra was mostly untouched but was expanded by the inclusion of some elementary calculus. In geometry, the principal change was the introduction of vectors and the systematic use of geometric transformations; again, nothing special, at first glance—this could easily have been done without any mention of set-theoretic concepts.

There was also another aspect. As Sharygin (2002) formulated it:

> It is interesting that the Soviet system of work with mathematically gifted children, created by selfless enthusiasts and brought, oddly enough, to the level of "know-how," turned out to be almost the only market product of the Russian education system (not counting, of course, its final result—scientists). (no pagination)

The key players of the reform: Vladimir Boltyansky (1925–2019), Aleksey Markushevich (1908–1979), Naum Vilenkin (1920–1991), and Isaak Yaglom (1921–1988) happened to be major contributors to it. Moreover, Kolmogorov—one of the creators of that very "Soviet system of work with mathematically gifted children," as Sharygin highlighted, was also involved. Surprisingly, there is no umbrella term for the plethora of activities involved. I suggest using the words "Olympiad Stream" because of its historically oldest and central component: mathematical “Olympiads,” competitions for schoolchildren. Correspondingly, people who passed through the Olympiad Stream and were shaped by it are known as *olympiadniks*.

I myself was a stereotypical olympiadnik, and, observing the reform unfolding (without affecting me) and collapsing, was puzzled how people whose names were known to me and my friends, whose books we read and respected, managed to botch the reform so spectacularly. In this chapter, I am trying to analyze this intriguing episode in the history of mathematics education from that particular point of view, that of a mathematician schooled in Russia during the reform period, which, I hope, deserves some attention—simply because it gives a perspective different from the ones used before.

A short contribution like this one allows me to mention only a few key actors in the events and only a few textbooks. I skip entirely the “counter-reform” movement which started in 1980, which also was very important and interesting, with many prominent mathematicians and experienced educationalists involved. Also, I do not make any comparisons with “New Math” reforms in the West—the sociopolitical environment of the Kolmogorov reform was so different, that comparisons would simply make no sense.

A few words on existing literature are due. Karp and Vogeli (2010) provided a good general survey of Russian mathematics education, and Neretin (2019) offered perhaps the best available analysis of Kolmogorov’s reform. It is 80 pages long, full of detail, and is written from a measured, rational, and non-political position. It also contains a massive bibliography and useful biographic notes. Of other sources, I would recommend Verner (2012), written by an active participant in the “counter-reform.” Kolyagin and Savvina (2012) provide a number of important documents and a well-written introduction of a summary of events. Kolyagin was an active reformer and, in particular, participated in writing books for teachers (an area of the reform which was much neglected by the reformers)—see Gusev et al. (1976), and Kolyagin (1977). Perhaps a close contact with teachers, which was inevitable in writing these books, led to Kolyagin’s conversion: He became one of the early critics of the reform. In particular, he was invited to speak at the Meeting of the Mathematics Division of the Academy of Sciences in December 1978 which can now be seen as the start of the “rebellion” of the Academy of Sciences and the Ministry of Education against the reform (Kolyagin and Savvina 2012). From my perspective, Kostenko (2013, 2014), although excessively politicized, provides a useful source of bibliographic references. Abramov (2016) offered unreserved praise to Kolmogorov.

Prehistory: The Start of the Olympiad Movement in the 1930s

To understand what happened in the reform we have to take a look at the early origins of the Olympiad Stream, more specifically, the Olympiad movement, in the 1930s. A good source for the early history of the movement is Boltyansky and Yaglom (1965).

The first Mathematical Olympiads were organized in 1934 in Leningrad by Boris Delaunay, a well-known mathematician (Chistyakov 1935; Fomin 2020), in 1935 in Moscow by Pavel Alexandrov and other Moscow mathematicians (Bonchkovsky 1936), and in 1936 in Kazan by the well-known algebraist Nikolai Chebotarev (Chebotarev 1937). The problems used in the Kazan Olympiad were quite challenging (Grigoriev 1937), which suggested an academically selective approach.

It is interesting that Delaunay’s motives for organizing the Olympiad were political. He arranged for young people who were successful in the competition to be admitted to the Mathematics Department of the Leningrad University without formal entrance examinations. This “opened the gate” to some aspiring young mathematicians of “wrong” social backgrounds, that is, children of people deprived of citizens’ rights—army officers and civil servants of the previous tsarist regime, clergy, nobility, capitalists, etc. (Zalgaller 2021). In the 1920s and early 1930s, it was forbidden to admit children of “*lishentsy*” [“deprived”] to universities—the Olympiad was a loophole.

Mathematicians invested remarkable energy and effort in this project. Why? Because Olympiads and other outreach activities were giving them some influence on who would come into mathematics and provided an opportunity to protect university mathematics from political appointees—the latter was quite prominent, for example, in biology—Trofim Lysenko being the most notorious case (Strunnikov and Shamin 1989).

Pavel Alexandrov was the chairman of the organizing committee for the first Moscow Mathematics Olympiad of 1935. Next year, he wrote in the introduction to a little booklet with problems and solutions of this Olympiad (Bonchkovsky 1936):

> The Olympiad is the first entry of future mathematicians into the mathematical arena. It should help us to select these future mathematicians from among our youth, it should help us to provide opportunities for their further mathematical development and education. (p. 4)

Here we see an unashamed elitism in a supposedly egalitarian country—but this is not so surprising. What is really astonishing is the phrase "It should help us to *select* these future mathematicians from among our youth." (p. 4, italics added).

As explained in Borovik et al. (2021), the s*election* and promotion of the "*cadre*" were the ultimate monopoly of the ruling Communist Party. The speech by Andrei Bubnov, People's Commissary of Education in 1929–1937 at the 17th Party Congress (VKP(b) 1934) is quite illuminating. Besides what would now be described as "widening participation," ensuring the steady progress of working-class children through the school system, he also emphasized a different task which could be formulated as educating the new generation of loyal to the regime, high-level specialists for the military, industry, science, medicine. Some examples are given by Bubnov in his speech:

> Look at Kamai—Professor at Kazan University, Tatar, former docker, now the author of a number of research works in the area of organic compounds of phosphorous and arsenic. (p. 114)

We should not forget that "organic compounds of phosphorous and arsenic" were an obvious euphemism for "precursors of chemical weapons." Bubnov's previous post was that of Head of the Main Political Directorate of the Red Army.

The young mathematicians Pavel Alexandrov, Boris Delaunay, Alexander Gelfond, Alexander Khinchin, Andrey Kolmogorov, Lazar Lyusternik, and Lev Schnirelmann, offered, at the right moment, their services to the Party, thus ensuring some degree of their own control over the supply of fresh blood to the top tier of the mathematical profession. In mathematics, "Red Professors," recruited from the working-class party activists (or Young Communist League activists), could not compete with people who had a deeper education because they started their development as mathematicians within the Olympiad Stream.

Mathematicians dared to ask for autonomy in the selection and development of their own. It could be seen in other documents of the epoch, for example, in Resolutions of the Second All-Union Congress of Mathematicians which took place in 1934: A special resolution was about Olympiads (VSM 1935, p. 56), and it called for "identification of gifted youths," stating that "Universities might use Olympiads for recruitment of students to mathematical, mechanical and physics departments" and requested funding from the Narkompros (the Ministry of Education) for running Olympiads and related activities.

Apparently, these requests were met by the authorities. Starting from 1935, mathematical Olympiads and associated activities, first of all, mathematical circles, flourished in Leningrad and Moscow, with *crème de la crème* of research mathematicians actively involved; what is important, new didactic approaches to exposition of non-trivial mathematics were invented and tested. A good and detailed narrative of these remarkable developments can be found in Boltyansky and Yaglom (1965).

There were two dramatic episodes in the 1930s that also helped mathematicians to gain certain autonomy in running their academic affairs and controlling the intake into the professional mathematical community. One of them was the political, by its nature, struggle around the quality of mathematical textbooks for schools: It is analyzed in Borovik et al. (2021); and some details in this section are borrowed from that paper. Another one was Luzin's Affair (Demidov and Levshin 1999; Neretin 2021a). Pavel Alexandrov and other young mathematicians launched a political attack on Nikolai Luzin, a prominent mathematician and the teacher of many of them. I will argue in another paper in preparation, that one of the reasons for the surprisingly vicious attack was Luzin's tendency to write laudatory reference letters, including letters for political appointees—potential "Red Professors," which undermined his younger colleagues' fight for the control of mathematics.

Some may say that establishing this special relationship with the totalitarian regime was entering into a pact with the Devil. I do not want to be judgmental—what mattered was that the leading mathematicians established themselves as guardians of the quality of mathematics education and also of *mathematical culture*; this term is interesting, and, I think, its specific use in Russia is not widely known abroad. It could be traced back to at least 1941, when Otto Schmidt, a mathematician, a member of the Academy of Sciences, and a high-ranking government official, formulated the role of mathematicians as specialists who maintain the strategically important *mathematical culture* of the country:

> Not only us, professionals of science, but the whole country was happy to learn about the solution of a problem set 150 years ago. This problem was solved by academician Vinogradov who proved that every odd number could be written as a sum of three prime numbers.[1] Is this needed at the practical level? No. Maybe it is not needed at all? On the contrary, it is much needed, because the culture, the mathematical culture depends on the level of these works in pure mathematics and theoretical physics. You all know that this is not needed for each of us, but we all are interested in the highest possible level of mathematical culture in our country, because it is important to be able to solve any mathematical problem and for that it is important to be able to solve problems such as Goldbach's problem. The level of mathematical culture in our country is exceptionally high. One may confidently say that in that respect our country is on the first and leading place in the world. (Schmidt 1941, quoted in Dubovitskaya 2009, p. 150)

Here, "mathematical culture" is understood as a form of the intellectual capital of the nation, the total of mathematical knowledge, understanding, skills—and, crucially, problem-solving ability, covering all mathematics, including the highest levels of mathematical research. Trickier is the meaning of "mathematical culture" when these words are applied to individual people, as in Yaglom's *Foreword* to Choquet (1970), the Russian translation of Choquet (1964): "The book by Choquet assumes that the reader has a certain mathematical culture" (p. 8), which means a general awareness of mathematics beyond the standard school or university courses, and some general content-independent mathematical traits. This concept is of course age-dependent and has a different meaning when applied to a secondary school rather than to a university student or to a research mathematician—but awareness of abstraction being used in mathematics and preparedness at least to try to handle abstract concepts is assumed at a relatively early age.

Esakov (1994) gives a tiny, but exceptionally important piece of evidence of Stalin's attitude to mathematics. The text of the speech by Lysenko at the infamous session of the All-Union Academy of Agricultural Sciences in 1948 (where Soviet genetics was totally destroyed, Vaskhnil 1948) was submitted to Stalin for approval. Stalin underlined Lysenko's statement "Every science is rooted in class [by its nature]" and wrote in the margin: "Hah-hah-hah… And mathematics? And Darwinism?" So, by 1948 (and perhaps even earlier), Stalin did not believe in the class nature of mathematics. This was a victory for mathematicians and had a profound effect on the fate of mathematics in the Soviet Union. Not every area of science was so lucky.

[1] This claim was exaggerated: Vinogradov' result was weaker than the one formulated by Schmidt, although still very impressive (Vavilov 2022).

Mathematicians in the Reform

One of the first published proposals for reform in mathematics education was put forward by Boltyansky et al. (1959); it did not introduce set-theoretic concepts, but development of geometry on the basis of geometric transformations featured in it prominently.

The new reformist program of mathematics teaching in 4th to 10th grades (ages 10–17) was approved by the Ministry of Education in 1968 and published in *Matematika v Shlole* [Mathematics in School], the mass-circulation journal for mathematics teachers (Program 1968). Implementation was to start from 1970 (not simultaneously in all grades).

A draft of the program was published earlier (Kolmogorov et al. 1967); in which it was indicated in the text that Kolmogorov and Markushevich were working on arithmetic, algebra, and elements of analysis, and Yaglom on geometry. The draft program was developed with the participation of Vladimir Boltyansky, Yuri Makarychev, Galina Maslova, Konstantin Neshkov, Alexey Semushin, Antonin Fetisov, and Aleksandr Shershevsky.

I have already highlighted the names of Vladimir Boltyansky, Andrey Kolmogorov, Aleksey Markushevich, Naum Vilenkin, Isaak Yaglom as the key players in the reform. They all were prominent and internationally renowned research mathematicians, perhaps with the exception of Isaak Yaglom, a highly competent mathematician who devoted more time to university teaching and to his (truly remarkable!) work on the popularization of mathematics than to writing research papers. He was definitely a mathematician, not a mathematics educationist; for mathematicians, he was “one of us.” Kolmogorov was the undisputed leader of the group.

Andrey Kolmogorov

It could be conjectured that the political battles of the 1930s shaped Kolmogorov’s political stance. Perhaps he felt personally responsible for the selection, nurturing, educating, and developing professional mathematicians for the State and for the Nation. He repeatedly called for greater emphasis to be given to the training of professional research mathematicians:

> The Soviet Union nowadays needs large number of independent researchers of theoretical questions of mathematics. (Kolmogorov 1959, p. 6)
>
> Our country needs a large number of well–trained and talented mathematicians. It is very important that the professional mathematicians are chosen from those representatives of our youth who can work most productively in this area. One way of attracting gifted youth to mathematics is Mathematical Oympiads. Participation in school math circles and Olympiads may help everyone to evaluate their own ability, seriousness, and strength of their passion for mathematics. (*From the Editor*, in Vasiliev and Egorov 1963, p. 1)

These two texts were reprinted in Kolmogorov (1988).

Even more interesting is a letter from Kolmogorov to the psychologist Vadim Krutetskii (Kolmogorov 2001) with comments on the Russian original, published in 1968, of Krutetskii’s famous book *The Psychology of Mathematical Abilities in Schoolchildren* (Krutetskii 1976). Kolmogorov gave Krutetskii advice on the use of statistics and also suggested how useful it could be to develop psychometric tests which allowed an early detection of mathematical abilities in children and also predicted the ceiling of their further development as mathematicians. It was obvious that Kolmogorov was interested only in one end product of mathematics education: professional research mathematicians. In the letter, Kolmogorov also remarked (I think on the basis of his work with students in the Olympiad Stream, in particular, teaching in the specialist mathematics boarding school which he founded):

> For now, as a practitioner, I am inclined to think that the nature of mathematical development achieved in accordance with the most modern recipes of early studies in set theory and algebra, up to the age of 10–13 years, and

> with fairly good success, could be replaced by a general education of quick wit and mental activity. But a delay in mastering strict logic and special mathematical skills at the age of 14–15 is becoming difficult to compensate.

But in his textbooks for the reform, he pushed the concept of equivalence relation and equivalence classes on 7th grade students (that is, aged 13) in *mainstream* education—we shall return to that later in this chapter.

Kolmogorov invested a colossal amount of his time and effort to his reform. The bibliography in Shiryaev (2003) lists unbelievably 58 editions of school textbooks which he co-authored for the reform (and quite often, editions of the same title differed significantly from the previous ones) and 55 papers on various matters of the reform in *Mathematics in School*, a mass circulation journal for school teachers. This was really his reform. He owned it.

The Olympiad Stream

It is not the aim of this chapter to give a detailed history of what we termed the Olympiad Stream, I will give only a summary description of the state it reached in 1970, the first year of the implementation of the reform, based in part on personal experience with occasional references to contemporary and modern sources.

Kolmogorov and his comrades-in-arms created a new subculture that valued advanced-level elementary mathematics with a focus on qualitative analytic thinking. They also created a community of people who shared these cultural values. "Mathematical culture" mentioned before was the culture of this community. The social aspects of this phenomenon are captured well by the title of the paper by Gerovitch (2020): *"Mathematical Paradise": A Parallel Social Infrastructure of Postwar Soviet Mathematics*.

By the 1970s, the Olympiad Stream had reached a considerable degree of development. First of all, it was a loose informal network of academic mathematicians and school teachers of mathematics involved in organizing mathematical competitions, running mathematical circles, summer schools, Sunday schools, distance learning by correspondence schools, etc.; of undergraduate students who helped to run all these activities; and, of course, of schoolchildren who were enthusiastically taking part in them. Next, it involved competitions and Olympiads at every level: School, district/town, city/region, republican, national, as well as open national level Olympiad by correspondence; and, in many cities, "mathematical battles" between schools occurred. Boltyansky and Leman (1965) give some idea of the remarkable mathematical quality of the top layer of these competitions. There was also a plethora of mathematical circles of various kinds and levels—at many schools, but also at universities and at cultural centers for children (the so-called Houses of Young Pioneers). Most circles were run by unpaid volunteers. In some cities, there were mathematical Saturday schools, Sunday schools, winter schools during the winter vacations in January, summer schools—the latter were usually run somewhere in the countryside (Kolmogorov et al. 1971). Neretin (2021b) calls this plethora of activities the Konstantinov System, in memory of the great mathematical educator Nikolay Konstantinov (1932–2021), who was the principal contributor to its development.

This was a community with its own folklore, one that was not always properly documented. Oral mathematical problems for various kinds of oral examinations, selection interviews, for use in "mathematical battles" featured in it prominently.

Moving to a more regular part of the systems, run by secondary schools, we have to mention noncompulsory "facultative" courses on additional chapters of mathematics at schools—they were supported by special textbooks, for example (Skopets 1971), or a chapter in a textbook was reserved for facultative studies. An interesting example of the latter from the times of Kolmogorov's reform was a chapter *Logical Structure of Geometry* (which included axiomatics for planar geometry) in the geometry textbook for grade 8 (Kolmogorov et al. 1976).

Every significant town had specialist mathematics classes in some of the schools (a famous example is School 57 in Moscow (Sergeev 2008). Many cities had specialist mathematics schools.

A significant and indispensable role was played by mathematics and physics schools in the development of correspondence education: In a huge country, correspondence schools reached everywhere. The three most well known were run by the Moscow University—the first one was founded by the great mathematician Israel Gelfand in the early 1960s, and it set benchmarks for other schools (Rozov et al. 1973). Another was run by the Moscow Physical-Technical Institute (Novoselov 2010; Yumashev 2012), and the last but not the least—the correspondence school of the Novosibirsk University (Khukhro 2013). At one point, when I was a boy in Siberia, I was enrolled in all three—and this meant, first of all, that information about them somehow reached me. No one in my home school knew about that—actually many substreams of the Olympiad Stream were completely separate from the official school system.

Quite remarkable was the policy of publishing translations of books on mathematics, with a careful choice being made of the best works available in the world literature. For example, easily available were Russian translations of such books as Choquet (1964), Courant and Robbins (1941), Coxeter (1961), Dieudonné (1964), Faure et al. (1964), Hartshorne (1967), Niven (1961), Pólya (1962), Rademacher and Toeplitz (1957). In Soviet academic publishing, there was a specific role: a *Translation editor* supervised the translation of a particular book and usually wrote a foreword. For Courant and Robbins and for Faure et al., the translation editor was Kolmogorov, for the six other books that I listed—Yaglom. Forewords by Kolmogorov and Yaglom put these books in the context of the new role of mathematics in science and in society—and hinted at the need to develop mathematical culture and mathematics education. I doubt that these books were popular among the majority of school teachers—they were published for people within the Olympiad Stream.

The Olympiad Stream itself produced a steady flow of excellent little books for children, often of high mathematical and didactic quality.[2] I wish to mention here four booklets, which were produced as assignments for the correspondence school at Moscow University: Gelfand et al. (1968a, b), Kirillov (1970), Vasiliev and Gutenmakher (1970). They explained mathematics to children in the simplest possible way—but without losing the essence of mathematics. For the mathematically experienced adult reader, they were masterclasses of *didactic transformation*—I will say a few more words about that in the next section.

Specialist mathematics schools require a special mention. More information on this particular phenomenon can be found in a paper by Gerovitch (2019) with the precisely chosen title *"We Teach Them to Be Free."* In addition to existing high-quality day schools in big cities (first of all, in Moscow and Leningrad), four specialist boarding schools were set by a special decree of the Council of Ministers of the USSR, to be run by Moscow (Kolmogorov et al. 1981), Leningrad, Novosibirsk (Borovik 2012), and Kiev Universities. The boarding school in Moscow became known as the Kolmogorov School, and the one in Novosibirsk—as the Lavrentiev School, named after Mikhail Alekseevich Lavrentiev, an outstanding scholar who started his mathematical carrier as a student of Luzin (like Kolmogorov) but then turned to fluid dynamics and industrial mathematics (often with military applications). In the late 1950s and 1960s, Lavrentiev founded the Siberian Branch of the Academy of Sciences, built an academic campus in the forest near Novosibirsk, and founded there the Novosibirsk University and the Physics and Mathematics Boarding School (PhMSh), my *alma mater*. I talk about my school in such detail because this sheds light on the main secret of the Olympiad

[2] The full list of these books can be found on the site https://math.ru/lib/ run by the Moscow Centre for Continuous Mathematical Education, and on https://mccme.ru/, https://mccme.ru/index-e1.html which continues the proud traditions of the Olympiad Stream.

Stream—it flourished because of the warm support from the Soviet military-industrial complex. Specialist mathematics schools continue to flourish in modern Russia (Konstantinov and Semenov 2021):

> [S]chools with deeper study of mathematics (math schools) became the most important and very productive phenomenon in Russia's education of the last decades. (p. 414)

And last but not least—the *Kvant* magazine. "Kvant" means "Quantum" in Russian; it was a mass-circulation monthly magazine on physics and mathematics for schoolchildren in grades 7 to 10 (but also devoted special pages to younger children). Kolmogorov was *Kvant*'s co-founder (in 1970) and the chief editor of the mathematics component of the magazine.

This was a cultural stream that was recognized as such by most people who were actively involved in supporting it. The following are quotes from Voitishek (1973). They were chosen from lecture notes of Vaclav Voitishek (1933–2003), of the preparatory department of the Novosibirsk University:

> The author assumes that the reader has access ... to wonderful books about mathematical creativity [and refers to Russian translations of Pólya 1962 and Rademacher and Toeplitz 1957]. (p. 3)

> For the author, the models of exemplary exposition of mathematical truths are the books by Markushevich et al. (1967) on algebra, by Pogorelov (1972) on geometry, and by Boltyansky et al. (1972) on mathematics. (p. 6)

Didactic Transformation

The theoretical concept of didactic transformation from mathematics education theory could be useful for explaining the principal reason for the failure of the reform. A compact formulation of what makes mathematics education so special can be found in a paper by Hyman Bass (2005):

Upon his retirement in 1990 as president of the International Commission on Mathematical Instruction, Jean-Pierre Kahane described the connection between mathematics and mathematics education in the following terms:

- In no other living science is the part of presentation, of the transformation of disciplinary knowledge to knowledge as it is to be taught (*transformation didactique*) so important at a research level.
- In no other discipline, however, is the distance between the taught and the new so large.
- In no other science has teaching and learning such social importance.
- In no other science is there such an old tradition of scientists' commitment to educational questions. (p. 417)

The concept of didactic transformation is fairly old and can be traced back to Auguste Comte (1852):

> A discourse, then, which is in the full sense didactic, ought to differ essentially from one simply logical, in which the thinker freely follows his own course, paying no attention to the natural conditions of all communication. … On the other hand, this transformation for the purposes of teaching is only practicable where the doctrines are sufficiently worked out for us to be able to distinctly compare the different methods of expanding them as a whole and to easily foresee the objections which they will naturally elicit. (Preface)

This concept is virtually unknown in English or Russian mathematics education literature (but apparently well known in France). It should be noticed that simple conversion of content in a "psychologically acceptable form" sometimes is not enough—a more serious mathematical work may be needed, and I will show you an example in the next section, where we shall try to apply this concept to the assessment of reformist textbooks.

A First Case Study: Vectors

In the ideal world, vectors (displacement, velocity, acceleration, force, etc.) are best introduced in physics courses. This was the approach in pre-reform Soviet schools, where vectors were only briefly mentioned in mathematics classes. This was the way of teaching in my years at the specialist boarding school at Novosibirsk by our physics lecturer Evgeny Bichenkov, whose lecture notes were later turned into a cute little book (Bichenkov 1999), and his approach was borrowed from the famous Feynman's Lectures in Physics (Feynman et al. 1964). This was why my fellow-students and I were puzzled by Kolmogorov's definition of vectors as parallel translations.

Boltyansky and Yaglom (1963), a few years prior to the reform, discussed equivalent forms of vector definitions:

> Of course, no matter what definition we take, a *vector* from the elementary geometry's point of view *is a geometric object characterized by direction … and length*. But this definition is excessively general and does not trigger any geometric images. According to this general definition, a parallel translation is a vector … since a parallel translation is characterized by its direction and length. Indeed, we could accept a definition: "a vector is any parallel translation." This definition is logically perfect, and could be taken as a basis for development of the entire theory of actions over vectors and their applications. However this definition, despite its full correctness, also cannot satisfy us since thinking about vector as a geometric transformation appears to be insufficiently intuitive, distant from physical interpretations of vector magnitudes. (pp. 293–294, italics in the original)

This was an interesting warning; to show that it was justified we reproduce here a definition of parallel translation given by Lobeeva (1963) in a contemporary discussion of vectors in school education run by the magazine *Mathematics in School*:

> A parallel translation is a point transformation of the plane, in which points trace equal, parallel, and equally directed segments. (p. 64)

It is a psychologically convincing description of a parallel translation as a process developing in time: moving points leave behind their traces showing their positions at intermediate moments of time. The trouble starts as soon as we consider other geometric transformations, for example, rotations (which also can be seen as processes developing in time, so points are leaving traces behind them) and axial symmetries (where points just jump instantly across the axis—what are their traces?). The composition of two rotations through the same angle of 180 degrees, but around two different points is a parallel translation (this is easy to prove). But look at the trace of a point: It is the union of two half-circles. The composition of two axial symmetries in two parallel axes is also a parallel translation (this is even easier)—but where do traces of points come from?[3]

This is an example of a didactic transformation gone wrong—because a concept was handled ignoring the wider mathematical context. In more specific terms, an element of the group of isometries of the Euclidean plane was treated by Lobeeva on its own, ignoring the rest of the group. Of course, Boltyansky and Yaglom knew this group and understood difficulties arising from defining vectors as parallel translations.

Verner (2012) explained that in the reform, the cautious approach of Boltyansky and Yaglom was overruled by Kolmogorov:

> A. N. Kolmogorov volunteered to write a [geometry] textbook for the grades 6–8 forms. … He did not entrust writing "Geometry 6–8" to the well-known geometers V. G. Boltyansky and I. M. Yaglom who were in his commission. (pp. 19–20)

Kolmogorov preferred to stick to the definition of vector as a parallel translation. The wording in Kolmogorov et al. (1977) was very casual, almost off the cuff:

[3] An exercise for the reader: What is the composition of two axial symmetries with intersecting axes?

> In this chapter, we shall specifically deal with parallel translations, calling them by a new name: **vectors**. (p. 59, bold in the original)

Unfortunately, the definition of parallel translation starts in Kolmogorov et al. (1979) with a definition of equivalence relation and then immediately states, without proof, that an equivalence relation on a set partitions this set into equivalence classes. This is followed by this example:

> The parallelity relation between straight lines in the plane defines a partition of the set of straight lines in the plane into classes. Each of these classes consists of straight lines, parallel to each other …. These classes are *bundles of parallel lines*. Another example of these classes is *directions*… (p. 128, italics in the original)

No other examples are given, and a definition of direction appears later:

> The set of rays, each of which is co-directed with the same ray, is called *direction.* (p. 130, italics in the original)

Finally, a definition of parallel translation is given on p. 132—quite similar in wording to the one given in Klopsky et al. (1980), mentioned in the Introduction and quoted by Pontryagin in his famous attack on the reform (Pontryagin 1980).

It appears that at least some of Kolmogorov's collaborators understood the difficulty of his approach. In a book for teachers by Gusev et al. (1976), five pages (pp. 6–11) were devoted to explaining, to teachers, Kolmogorov's definitions and its versions used in various textbooks. The equivalence relation is featured prominently (p. 8). An advanced set-theoretic approach was also invoked:

> To summarize, we considered a possibility of introducing the concept of vector as a set of pairs of points defining the same parallel translation, that is, the set of all pairs (X, Y), for which $T(X) = Y$, as a vector. The set of pairs (X, Y) is sometimes called the graph of the parallel translation.
>
> In the modern treatment it is conventional to identify the graph of the mapping with the mapping itself. Everything said before has led to the identification, in the school course of mathematics, of a parallel translation and a vector as synonyms, denoting exactly the same concept. (p. 9)

In the reform, the set theory was confined to non-compulsory enhancement courses—but I was unable to find anywhere in them the delicate and abstract identification of a function (that is, a mapping from a set to a set) with its graph. A proper definition of the graph required a definition of the direct product of two sets—which was also nowhere to be found.

Nowadays in England, my university colleagues consider the theorem about an equivalence relation partitioning a set into equivalence classes as *pons asinorum* of undergraduate abstract mathematics. Alas, many graduates from English universities obtain their bachelor's degree in mathematics without grasping this concept. At that time, circa 1970, in Russia, the equivalence/partition duality was perhaps one of the boundary markers between the Olympiad Stream and the mainstream school mathematics. Kolmogorov himself, and the reformer mathematicians of his circle were experts in the education of students in the Olympiad Stream. But they crossed the boundary into mainstream education without caring about the didactic transformation of the new material which they brought with them.

In short, at the methodological level, the principal reason for the failure of the reform was the absence (or failure) of appropriate didactic transformation of the new mathematical content. This was even more surprising because inside the Olympiad Stream, the didactic transformation was used quite successfully—I have already given a few examples.

The famous geometer Aleksandr D. Aleksandrov (1980, reprinted as Aleksandrov 2008a) gave a rather harsh assessment of the new course of geometry:

> It is hard to find something more harmful for the spiritual—mental and moral—development, than train a person to pronounce words which meaning he does not really understand, and, when necessary, is guided by other concepts. (p. 307)[4]

[4] I would not quote Aleksandrov here, if his words did not fully apply to the undergraduate mathematics teaching in English universities.

It is interesting that in his speech at the meeting of the Academic Council of the Institute of Mathematics of the Siberian Branch of the Academy of Sciences USSR on December 25, 1980 (printed as Alexandrov 2008b), he harshly criticized the paper by Pontryagin (1980) as an attack on "abstract" mathematics and put the blame for the botched reform on Kolmogorov's collaborators (Zalman Skopets was the only one named—perhaps because he was not a mathematician, but a mathematics educationist).

> We talk not about a set-theoretic approach, not about some special abstractions and sophistry, but about very simple things, like crude mistakes in Russian language in *Geometry 6* or a ridiculous definition of a polytope in a textbook for grades 9–10. It is not abstraction in mathematics, but, in the final count, abstracting from responsibility, abstracting from conscientiousness are the root of mistakes and absurdities in school teaching as well as in public pronouncements about mathematics. (p. 319)

A Second Case Study: Probability Theory

Kolmogorov was one of the founders of modern probability theory. His ground-breaking work *Grundbegriffe der Wahrscheinlichkeitsrechnung* [Basic concepts of probability theory] (Kolmogoroff 1933) was a masterpiece of exposition of mathematics.

So it could be surprising that in the reform, probability theory (and some elementary combinatorics needed) were limited in scope and confined to a short chapter in (non-compulsory) facultative courses in grade 9. The theory was restricted to the finite frequentist setting and Bernoulli trials, it just barely touched conditional probability, and a lot of attention was given to the direct computation of probabilities with the help of combinatorial formulae; however, it included some simple examples of geometric probabilities (Kotii and Potapov 1971).

This modest original treatment was soon developed to include random variables and one of the simplest versions, due to Chebyshev, of the law of large numbers (Antipov et al. 1979; Firsov et al. 1977). Still the content appeared to be rather unimaginative. There was no sign of a contribution from Kolmogorov to the probability theory chapters of textbooks. Why? Perhaps, being *the* expert in probability theory, he understood that any step away from the elementary—and frequently artificial—material led to serious conceptual difficulties.

David Corfield, a mathematician and well-known philosopher of mathematics, made the following incisive comments in the context of debates around teaching probability and statistics in schools in England (D. Corfield, personal communication, October 12 and 13, 2010):

> One intriguing problem about teaching probability theory is that there are at least four distinct interpretations of probability (an objective and a subjective Bayesianism, a propensity theory, and a frequency theory), along with various pluralist positions. Unless you work in artificial situations with, say, perfect dice, these differences, which I imagine most school teachers are unaware of, will confuse one's teaching.
>
> Presumably an analysis of decisions to play lotteries could be done in a fairly uncontroversial way, though the relative utility of losing a pound and gaining so many millions is far from obvious.
>
> Then there's the question of various forms of optional insurance. Here we enter the problems of assessing likelihoods of events when data only covers certain groups. E.g., how do I calculate the probability of suffering a heart attack when all I have is data for, say, non-smoking 40-year-old males. Maybe there's no data for those with my diet, exercise, income, job satisfaction, marital happiness, etc. Ultimately there's only one me. This is the "reference class" problem.

For a more detailed discussion, see Gillies (2000).

Perhaps we have to conclude that Kolmogorov treated probability theory very differently from geometry.

Social Blindness

Insufficient attention to didactic transformation of new material, making it accessible if not to all, but at least to the majority of mainstream students may be linked to reformers' surprising social blindness in many other aspects of the reform.

For example, the need to re-educate, and retrain the whole army of teachers was somehow overlooked. I made a systematic search for books for teachers and students in pedagogical colleges which were produced in the support of the reform. Not much was found. It was not surprising that Kolyagin (2001) characterized the provision of advice to teachers, of didactic materials, etc., as essentially nonexistent. Kolyagin also pointed to the disruption of teaching the new generation of teachers in pedagogical colleges—something that could be foreseen before the launch of the reform and appropriately alleviated.

As an unexpected side effect, the neglect of teachers damaged the Olympiad Stream. During the period 1974–1978, I was a university student, but I was involved in running regional Mathematics Olympiads in Siberia and the Soviet Far East and had a chance to see the negative effects of the reform on the participants. The overwhelmed teachers could no longer give the students enough attention and time.

We provide another example of blindness to social realities. In their experimental textbook Boltyansky et al. (1979) gave the following advice to 12-year-old students (grade 6):

> You will not find answers to the problems at the end of the book. After all, we want you to learn to reason in the right way, to have confidence in the correctness of the logical arguments, in the rationality of the solution found. We want you to be able to apply your geometric and logical knowledge in life, in your future work—but life does not provide answers to questions, easy or difficult, that it poses. Therefore, get used to solving problems without "peeping" in solutions. And if sometimes your solution turns out to be not entirely correct, imprecise, you will be corrected by your comrades and the teacher. (pp. 3–4)

Alas, after 5 years of attempts to learn mathematics most students were well aware that no one would correct them if answers had not been given to their teachers in advance. Advising students to seek help from teachers was a breach of one of the unwritten traditional rules for mathematics textbooks in Russia: A good student should be able to learn mathematics directly from a textbook without help from a teacher. However, within the Olympiad Stream, it was fine not to give answers to problems in mathematics and physics; moreover, it was a normal practice. I myself was taught in that tradition; for example, Bichenkov (1999), the book which grew out of Bichenkov's lectures in Novosibirsk PhMSh which I attended—had no answers. Problems were wonderful—but I still do not know how to solve many of them.

And the last example has some curiosity value. For the promotion of the reform, Boltyansky and Levitas (1973) tried to appeal directly to parents and wrote a book in the form of a dialogue between a mathematician and a few parents. Alas, the parents in their book are not very representative of the general population. In the book, they (and the reader) are offered "homework." Here is one of the problems:

> On return from school, your son asked you a question:
>
> We have been told that axioms cannot be proven, and gave an axiom: "There is only one straight line that passes through two points." Why couldn't this be proven? Apply the ruler and check that the second line goes along the first one. This is a proof!
>
> What would you say to your son? (p. 41)

How many real parents, even after reading the book, were able to answer this question coherently?

I have a feeling that the reformers were not aware of many aspects of the socioeconomic situation in the country even those which directly affected education. Perhaps they sincerely believed in the official dogma of the social homogeneity of the Soviet society. This was an illusion. In the next section, I will try to explain the roots of this illusion.

The Golden Age of Soviet Mathematics Education

This was the principal mistake of the reformers: They continued to live in the Golden Age of Soviet mathematics—as the period from the 1950s through the 1970s is frequently called (Gerovitch 2013, 2020), and they did not realize that it was coming to an end. I was lucky to see the last days of the Golden Age, and I have seen how it ended. And I cannot blame them for their illusion.

In the Soviet Union, the society was not almost homogeneous, as the official propaganda insisted, it was deeply stratified, and had an etacratic social structure—the social position of people was almost entirely determined by their place within the dictatorial autocratic state and the centrally planned economy (Radaev and Shkaratan 1992, 1995; Shkaratan 2012).

In the beginning of the 1970s, all aspects of life in the Soviet Union were still dominated by the tidal wave of social mobility unprecedented in the history of humanity. Social mobility ensured the stability of the society and of the totalitarian system which ruled the society. No matter how hard life was, every family could have realistic expectations that their children would have better lives if they got education—and, for that reason, people were prepared to forgive the blunders and even the crimes of their rulers. But social mobility was something that was centrally planned, as was everything in the economy. It was planned, for example, how many people will leave villages and move to cities to work in the industry.

It was a plain numerical *horizontal* expansion of the economy, not accompanied by a growth in productivity. In the case of education, especially mathematics education, the expansion could be seen with exceptional clarity. Expanding school education required more school teachers, more pedagogical colleges and universities, and more university teachers, with demand feeding back into the need for further improvement and expansion of schools. It was driven by insatiable demand from the military-industrial complex for educated (and, above all, mathematically literate) workers and engineers, and from the armed forces for soldiers and officers with good mathematical skills.

In more politically sensitive areas, the process of social mobility was skillfully manipulated. The authorities were running an elaborate system of positive discrimination and promotion of young people from politically safe strata of the society—and this involved institutionalized anti-Semitism.

Of course, the economic expansion without a matching growth in productivity could not last, and, in the 1970s, the music stopped. So far as social mobility was concerned there was no more room at the top. The Soviet Union collapsed very soon afterward.

It can be conjectured that the reformers were either blinded by the official propaganda or forced to behave as if they believed it.

The limited space of this chapter does not allow me to provide an in-depth analysis of my observations—I hope this will eventually be done by professional historians and politologists. My role was simple: I wished to attract these experts' attention to an exciting object of study.

A Lesson for Our Times?

The principal lesson of Kolmogorov's reform: The methods of the Olympiad Stream are not transferable to mainstream education on the cheap, as it was attempted by the reformers. What was needed for success was a serious investment in a proper reform, with sociological studies, with more time spent on developing and testing textbooks, books for teachers, didactic material, textbooks for pedagogical colleges, with systematic re-education and professional development of teachers, with a much larger and better-coordinated team of developers, and proper project management. In the economic situation of the Soviet Union in 1968, all that was unfeasible.

In conclusion, I offer to the reader's attention a short fragment from a blatantly self-promotional film from Yandex, the Russian IT and Internet giant (Yandex 2020)—it has English subtitles. I recommend the reader watches just the segment 3:22–4:35, where Tigran Khudaverdyan, General Director and Director of Operations at Yandex, answers a question from a reporter: "What is yandexoid?" The word "yandexoid" entered Russian IT jargon, it is the proud self-designation of Yandex employees who apparently feel themselves as *the salt of the earth* (Matthew 5:13).

> Alexei Pivovarov (the reporter): What is yandexoid?
> Tigran Khudaverdyan: yandexoid... yandexoid is ... it's such an environment... Lesha[5], have you ever participated in Olympiads, some school ones?
> Alexei Pivovarov: There was a case. It didn't end well.
> Tigran Khudaverdyan: My memories are: You are the first guy in the village, in your class, in your school, such a great fellow. You think you know the topic best. You come there, for example, to the city Olympiad ... but it doesn't matter where, from the city one to the republican one, and you understand that in general everyone there is smarter than you, that all your greatness is broken, just by the fact that the strongest have gathered there. Well, *to be an yandexoid is to be an olympiadnik every day* [italics added], you must come to work every day, you have to be able to be wrong, not knowing, and then you come up with something absolutely brilliant. They may show you very reasonably that it won't work there—or vice versa. And if you can't stand that, then you won't be able to work.

This is the new reality: The selective Olympiad Stream, with its intrinsic competitiveness, is welcome in the corporate world.

Meanwhile, nowadays in Russia, in the new technological and socioeconomic environment, the debate about the balance between the selective stream and mainstream, and the content of the mainstream in school mathematics education is very much alive (Borovik et al. 2022; Khalin et al. 2022; Konstantinov and Semenov 2021). It is important to ensure that it is informed by lessons from Kolmogorov's reform.

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[5] Lesha is a familiar form of the name Alexei.

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